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Light meson spectroscopy at BESIII

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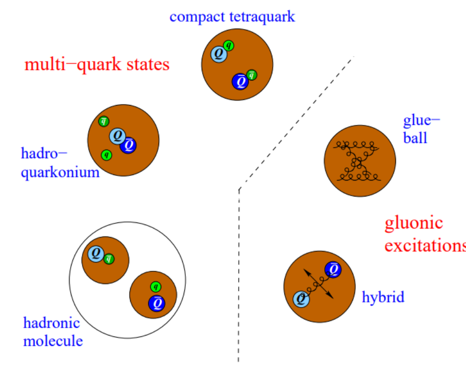
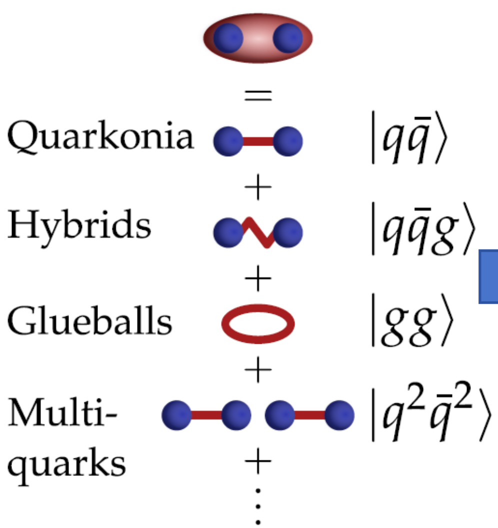
light QCD physics

- Well-known classes of hadrons: meson($q\bar{q}$), baryon(qqq)
- QCD allows for hadrons beyond quark model so-called exotics:
  - **•Multi-quark states ; Hybrids ; Glueballs**
  - •Strong evidences for multi-quark in heavy quark sector

    A new "particle zoo": https://qwg.ph.nat.tum.de/exoticshub/
  - •Evidence for gluonic excitations remains sparse
- **Light meson spectroscopy**
  - •Key tool to study/develop QCD in nonperturbative region

Phys.Rept. 873 (2020) 1





Identification is challenging

Manifestly exotic: with forbidden QN

Flavor exotic : $Z_c, T_{cc}, T_{\psi\psi}\ldots$

Spin exotic: $J^{PC} = 0^{--}, even^{+-}, odd^{-+}$

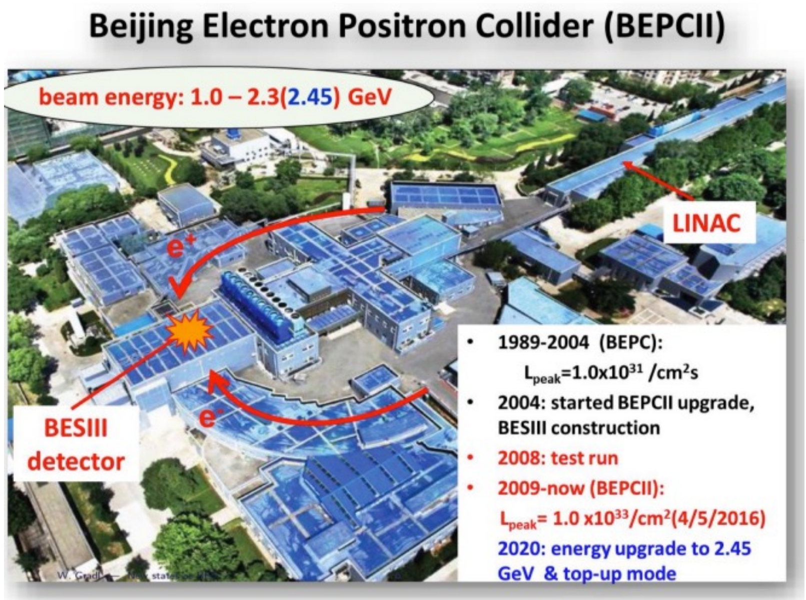
Crypto exotic : with QN as $q\bar{q}$

Supernumerary states

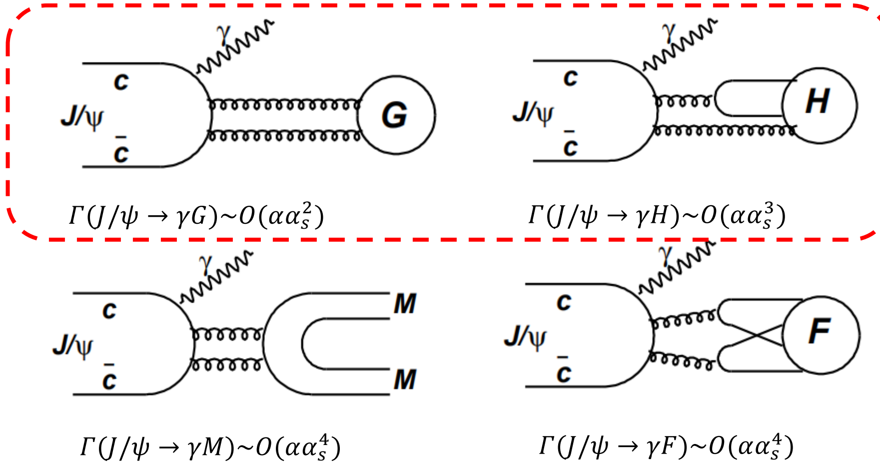
Abnormal properties

+ Kinematic effects

World's Largest τ –charm Data Sets in e^+e^- Annihilation



$$\Gamma(J/\psi \to \gamma G) > \Gamma(J/\psi \to \gamma H) > \Gamma(J/\psi \to \gamma M) \geq \Gamma(J/\psi \to \gamma F)$$



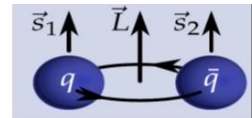
Glueballs and hybrids are expected to have a larger yield compared to mesons.

Charmonium radiative decays provide an ideal laboratory for gluonic states

- Gluon-rich process
- Well defined initial and final states
  - Kinematic constraints
  - $I(J^{PC})$ filter :
- Clean high statistics data sample : 10×10^9 J/ψ and 2.9×10^9 $\psi(2S)$ @BESIII

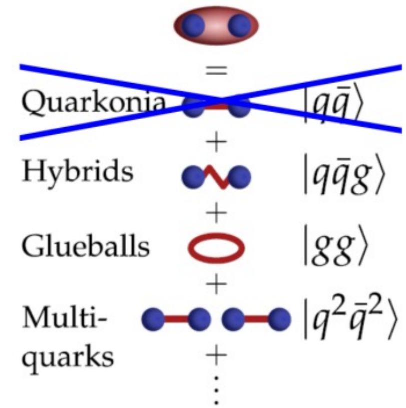
Light hadrons with exotic quantum numbers

- Unambiguous signature for exotics
  - ✓ Light Flavor-exotic hard to establish
  - ✓ Efforts concentrate on Spin-exotic
    - Forbidden for $(q\bar{q})$: 0^{--}, even^{+-}, odd^{-+}



$\vec{J} = \vec{L} + \vec{S}$ $P = (-1)^{L+1}$ $C = (-1)^{L+S}$

Allowed J^{PC}: $0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{++}, \ldots$



- Only 3 spin exotic candidate so far $\Rightarrow$ all 1^{-+} isovectors :

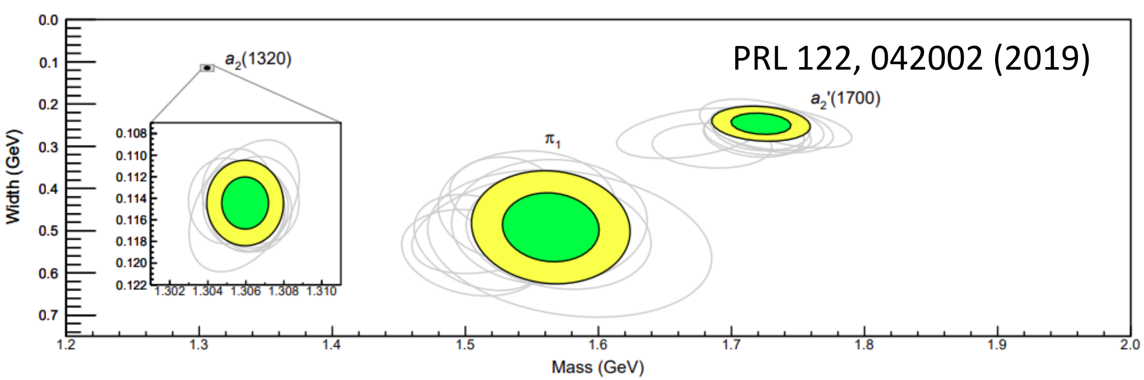
$\pi_1(1400)$: seen in $\eta\pi$

$\pi_1(1600)$: seen in $\rho\pi, \eta'\pi, b_1\pi, f_1\pi$

$\pi_1(2015)$: seen in $b_1\pi, f_1\pi$

✓ $\pi_1(1400)$, $\pi_1(1600)$ can be explained as one pole

Phys. Rev. D 105, 012005 (2022)



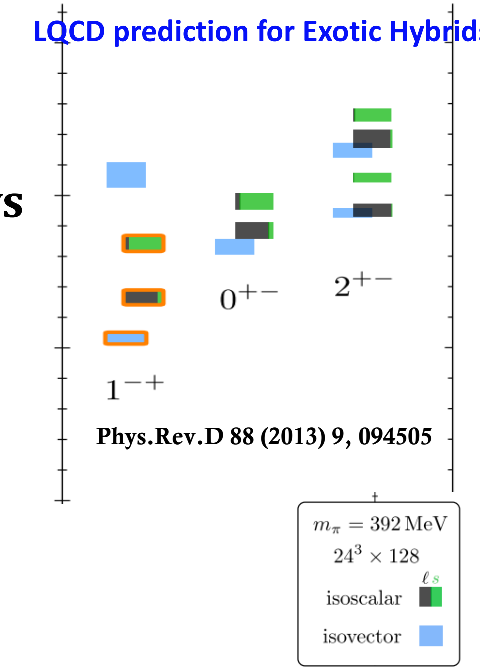
	Decay mode	Reaction	Experiment
$\pi_1(1400)$	$\eta\pi$	$\pi^-p \to \pi^-\eta p$ $\pi^-p \to \pi^0\eta n$ $\pi^-p \to \pi^-\eta p$ $\pi^-p \to \pi^0\eta n$ $\bar{p}n \to \pi^-\pi^0\eta$ $\bar{p}p \to \pi^0\pi^0\eta$	GAMS KEK E852 E852 CBAR CBAR
	$\rho\pi$	$\bar{p}p \to 2\pi^+2\pi^-$	Obelix
$\pi_1(1600)$	$\eta'\pi$	$\pi^-Be \to \eta'\pi^-\pi^0 Be$ $\pi^-p \to \pi^-\eta' p$	VES E852
	$b_1\pi$	$\pi^-Be \to \omega\pi^-\pi^0 Be$ $\bar{p}p \to \omega\pi^+\pi^-\pi^0$ $\pi^-p \to \omega\pi^-\pi^0 p$	VES CBAR E852
	$\rho\pi$	$\pi^-Pb \to \pi^+\pi^-\pi^- X$ $\pi^-p \to \pi^+\pi^-\pi^- p$	COMPASS E852
	$f_1\pi$	$\pi^-p \to p\eta\pi^+\pi^-\pi^-$ $\pi^-A \to \eta\pi^+\pi^-\pi^- A$	E852 VES
$\pi_1(2015)$	$f_1\pi$	$\pi^-p \to \omega\pi^-\pi^0 p$	E852
	$b_1\pi$	$\pi^-p \to p\eta\pi^+\pi^-\pi^-$	

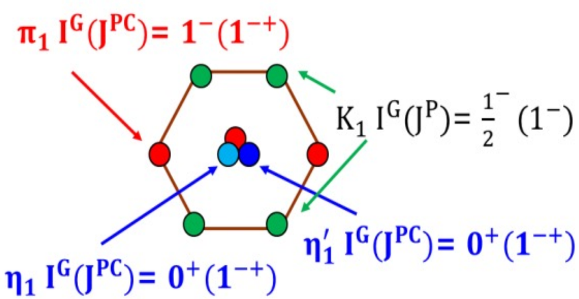
Light hadrons with exotic quantum numbers

- Lightest spin-exotic: 1^{-+} **hybrid** $\Longrightarrow$ $1.7 \sim 2.1 GeV/c^2$

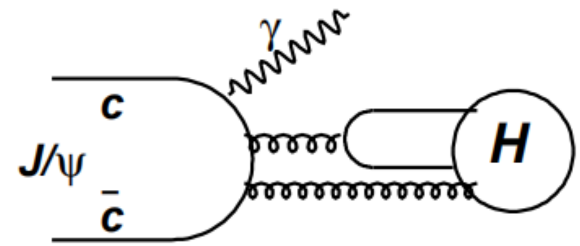
- **Isoscalar 1^{-+} is critical to establish the hybrid nonet**
  - Can be produced in the gluon-rich charmonium decays
  - Can decay to $\eta\eta'$ in P-wave

[PRD 83,014021 (2011), PRD 83,014006 (2011), EP.J.P 135, 945(2020)]





Search for $\eta_1(1^{-+})$ in $J/\psi \to \gamma\eta\eta'$



$\Gamma(J/\psi \to \gamma H) \sim O(\alpha\alpha_s^3)$

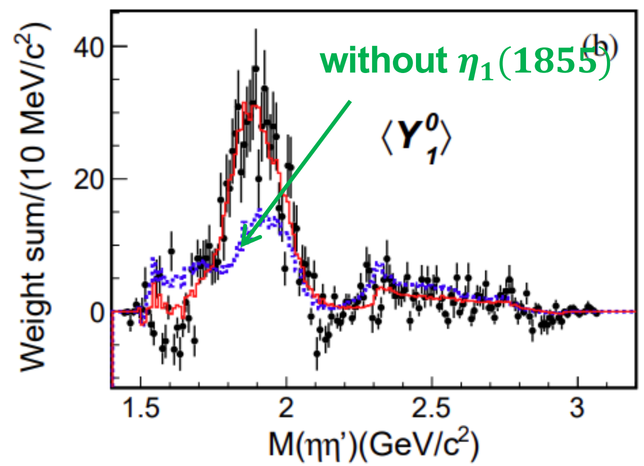
Observation of Exotic Isoscalar State $\eta_1(1855)$ in $J/\psi \to \gamma\eta\eta'$

- **An isoscalar 1^{-+} state, $\eta_1(1855)$, has been observed with statistical significance larger than 19σ**

PRL 129, 192002 (2022) ; PRL 130, 159901 (2023) (erratum)
PRD 106,072012 (2022) ; PRD 107,079901 (2023) (erratum)

$$M = (1855 \pm 9^{+6}_{-1})\ MeV/c^2;\quad \Gamma = (188 \pm 18^{+3}_{-8})\ MeV$$

$$B(J/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta') = (2.70 \pm 0.41^{+0.16}_{-0.35})\times10^{-6}$$



- **Mass is consistent with hybrid on LQCD**

- **Inspired many interpretations:**
  - Hybrid?
  - Molecule?
  - Tetraquark?

Phys.Rev.D107(2023)7,074028;
Rept.Prog.Phys.86(2023)026201;
CPC46,051001(2022);
CPL39,051201(2022);
PLB834,137478(2022);
PRD106,074003(2022);
PRD 106, 036005(2022)

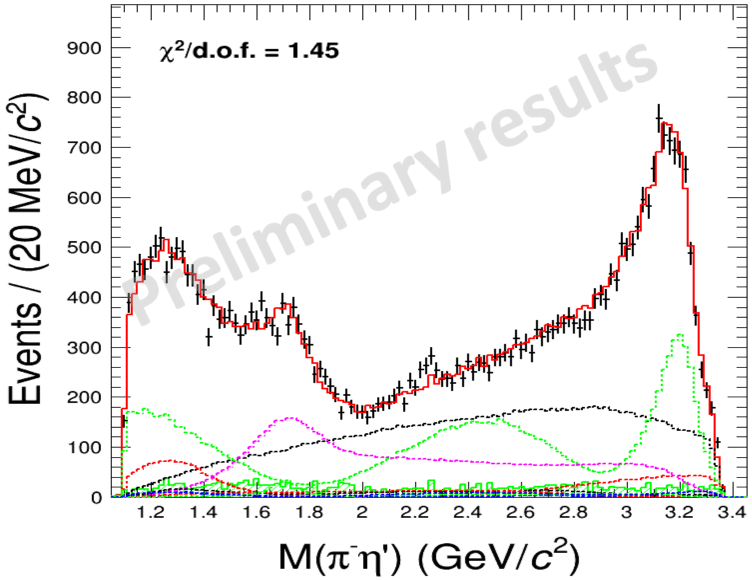
- **Further more, suppression of $f_0(1710) \to \eta\eta'$ supports it has a large overlap with glueball**

$$\frac{Br(f_0(1500) \to \eta\eta')}{Br(f_0(1500) \to \pi\pi)} = (1.66^{+0.42}_{-0.40})\times10^{-1} \qquad \frac{Br(f_0(1710) \to \eta\eta')}{Br(f_0(1710) \to \pi\pi)} < 2.87\times10^{-3} @90\% C.L$$

Opens a new direction to completing the picture of spin-exotics

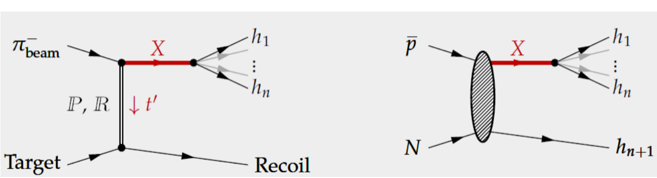
Observation of $\pi_1(1600)$ in $\chi_{c1} \to \eta^{'}\pi^{+}\pi^{-}$

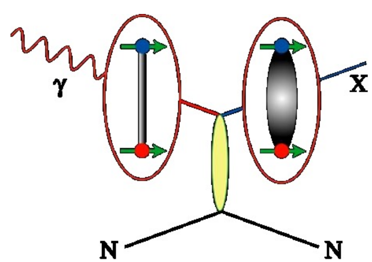
$2.7 \times 10^{9}\ \psi(3686)$ @BESIII [preliminary]

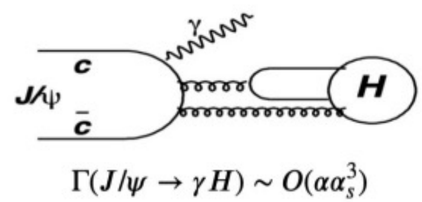


- **Amplitude analysis of $\chi_{c1} \to \eta^{'}\pi^{+}\pi^{-}$ is performed**
- **$\pi_1(1600)$ is observed with significances $> 10\sigma$**
- **The J^{pc} of $\pi_1(1600)$ is measured to be exotic 1^{-+}**
  - ✓ better than other assignments
- **With significant Breit-Wigner phase motion**
  - ✓ Evidence of $\pi_1(1600) \to \eta^{'}\pi$ at CLEO-c is confirmed [PR D84 112009 (2011)]

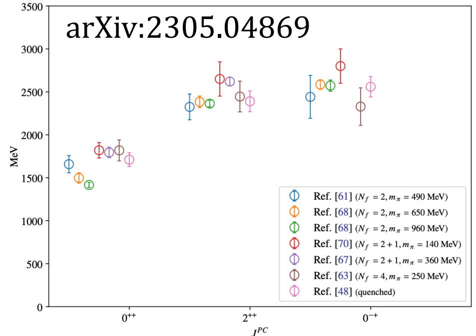
Observations of π_1 and η_1 in charmonium decays provide a new path to study 1^{-+}







Glueball



Light Yang-Mills glueballs on lattice (quenched and unquenched results)

- **Glueballs: the most direct prediction of QCD**
- **Low-lying glueballs with ordinary J^{PC} $(0^{++}, 2^{++}, 0^{-+})$**
  - ✓ **mixing with $q\bar{q}$**
    - Challenge: reveal the exotic admixture.

- **Non $q\bar{q}$ nature difficult to be established**
  - ✓ Overpopulation, but QM assignment is difficult
  - ✓ Identification is model-dependent

⇒Systematical study is needed in the identification

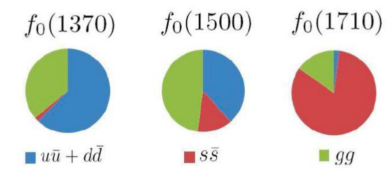
- **Lattice QCD predictions for glueball masses and BR:**
  - 0^{++} ground state: 1.5-1.7 GeV/c^2 ; $B(J/\psi \to \gamma G_{0^{++}}) = 3.8(9)\times10^{-3}$
  - 0^{-+} ground state: 2.3-2.4 GeV/c^2 ; $B(J/\psi \to \gamma G_{0^{-+}}) = 2.31(80)\times10^{-4}$
  - 2^{++} ground state: 2.3-2.6 GeV/c^2 ; $B(J/\psi \to \gamma G_{2^{++}}) = 1.1(2)\times10^{-2}$

Scalar Glueball

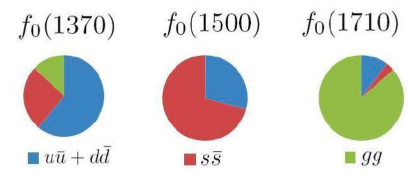
- Observed $f_0(1370), f_0(1500), f_0(1710)$
  - ✓ supernumerary scalars suggest additional degrees of freedom
  - ✓ However, mixing scenarios are controversial
- Measured $B(J/\psi \to \gamma f_0(1710))$ is x10 larger than $f_0(1500)$

  BESIII [PRD 87 092009, PRD 92 052003, PRD 98 072003]

Close and Kirk, PLB483 (2000) 345



Cheng et al, Phys. Rev. D74 (2006) 094005



- Flavor-blindness of glueball decays

  $\Gamma(G \to \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3:4:1:0:1$

  - $G \to \eta\eta'$ decay is expected to be suppressed
  - Scalar glueball expected to be suppressed $\Gamma(G \to \eta\eta')/\Gamma(G \to \pi\pi) < 0.04$

    [PR D 92, 121902; PR D 92, 114035]

- New inputs from $J/\psi \to \gamma\eta\eta'$

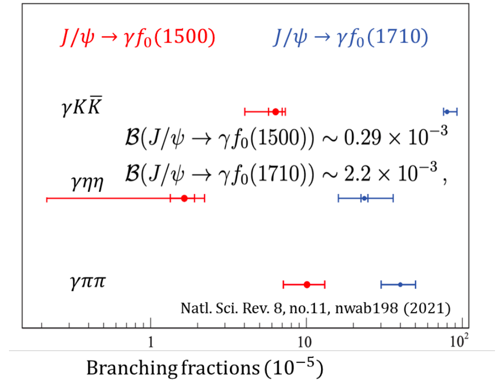
PRL 129, 192002 (2022) ; PRL 130, 159901 (2023) (erratum)
PRD 106,072012 (2022) ; PRD 107,079901 (2023) (erratum)

- Significant $f_0(1500)$

$$\frac{Br(f_0(1500) \to \eta\eta')}{Br(f_0(1500) \to \pi\pi)} = (1.66^{+0.42}_{-0.40}) \times 10^{-1}$$

- Absence of $f_0(1710)$

$$\frac{Br(f_0(1710) \to \eta\eta')}{Br(f_0(1710) \to \pi\pi)} < 2.7 \times 10^{-3} @90\% C.L$$



Supports to the hypothesis that $f_0(1710)$ overlaps with the ground state scalar glueball

Tensor Glueball

$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22) keV$$
$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1 \times 10^{-2}$$

CLQCD, Phys. Rev. Lett. 111, 091601 (2013)

- **Experimental results**

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta\eta) = (3.8^{+0.62}_{-0.65}{}^{+2.37}_{-2.07})\times 10^{-5}$$

BESIII PRD 87,092009 (2013)

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\phi\phi) = (1.91 \pm 0.14^{+0.72}_{-0.75})\times 10^{-4}$$

BESIII PRD 93, 112011 (2016)

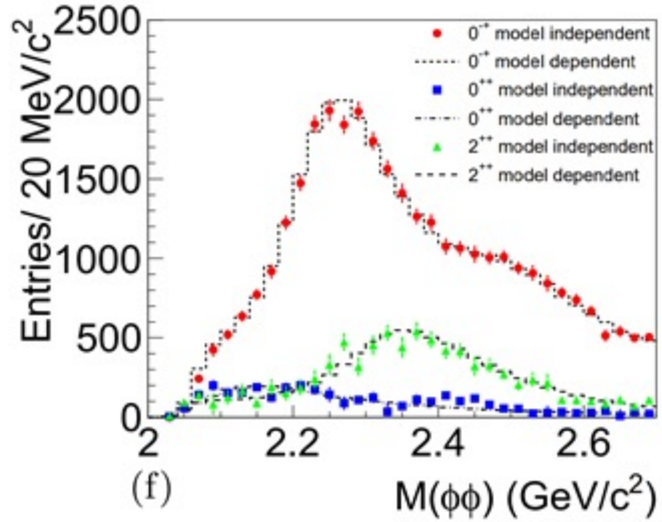
$$Br(J/\psi \to \gamma f_2(2340) \to \gamma K^0_s K^0_s) = (5.54^{+0.34}_{-0.40}{}^{+3.82}_{-1.49})\times 10^{-5}$$

BESIII PRD 98,072003 (2018)

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta'\eta') = (8.67 \pm 0.7^{+0.16}_{-1.67})\times 10^{-6}$$

BESIII PRD 105,072002 (2022)

BESIII $J/\psi \to \gamma\phi\phi$ with 1.3B J/ψ



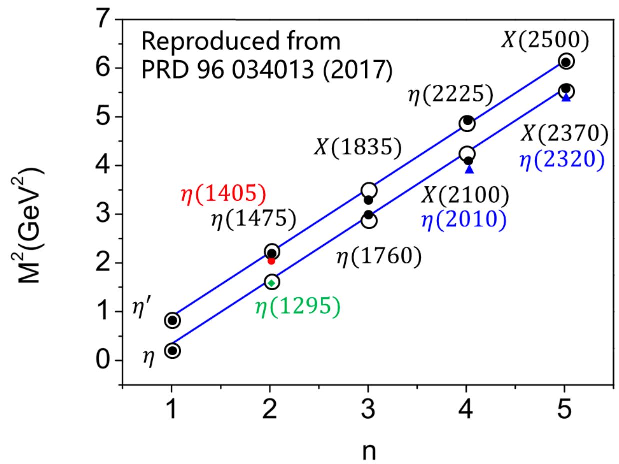
- $f_2(2010)$, $f_2(2300)$, $f_2(2340)$ in πp reactions are all observed in $J/\psi \to \gamma\phi\phi$ **with a strong production of $f_2(2340)$**
- Consistent with double-Pomeron exchange from WA102@

More complicated due to the large number of tensor states

→ More decay modes are desired

Pseudoscalar Glueball

- **Pseudoscalar meson spectrum**
  - ✓ Only η and η' (& radial excitations) from quark model
  - ✓ A promising place to search for extra states



- **LQCD predicts: 0^{-+} glueball (2.3~2.6 GeV)**
- **Little experimental information above 2 GeV**
  - ✓ A glueball-like state **X(2370)** [Zhang Peng's talk]

- **$\eta(1405)/\eta(1475)$ puzzle (found by MarkIII)**
  - ✓ Quark model predicts :only one pseudoscalar meson near 1.4 GeV
  - ✓ Theoretical interpretations :

    $\eta(1475)$ ⇨ the first radial excitation of η'

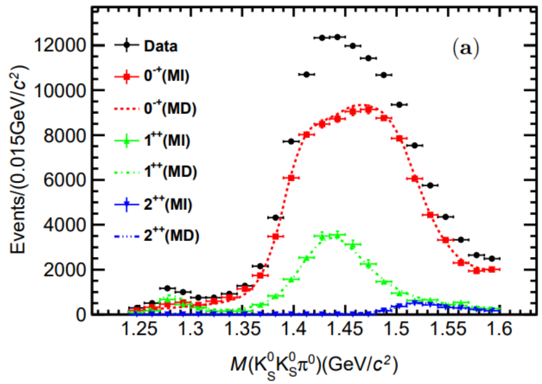
    $\eta(1405)$ ⇨ the glueball candidate &&**Mass incompatible with LQCD**

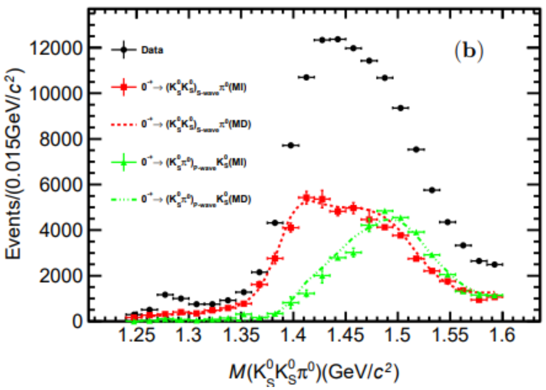
$\eta(1405)$ and $\eta(1475)$ are two separate states or just one pseudoscalar state, namely η(1440), in different decay mode?

What's the nature of the outnumbered $\eta(1405)$?

Partial Wave Analysis of $J/\psi \to \gamma K_s^0 K_s^0 \pi^0$

JHEP03(2023)121





Resonance	M(MeV/c^2)	Γ(MeV)
$\eta(1405)$	$1391.7 \pm 0.7^{+11.3}_{-0.3}$	$60.8 \pm 1.2^{+5.5}_{-12.0}$
$\eta(1475)$	$1507.6 \pm 1.6^{+15.5}_{-32.2}$	$115.8 \pm 2.4^{+14.8}_{-10.9}$
$f_1(1285)$	$1280.2 \pm 0.6^{+1.2}_{-1.5}$	$28.2 \pm 1.1^{+5.5}_{-2.9}$
$f_1(1420)$	$1433.5 \pm 1.1^{+27.9}_{-0.7}$	$95.9 \pm 2.3^{+13.6}_{-10.9}$
$f_2(1525)$	$1515.4 \pm 2.5^{+3.2}_{-7.6}$	$64.0 \pm 4.3^{+2.0}_{-6.1}$

- **Mass Independent PWA : Disentangle J^{PC} in each bin**
  - **Valuable inputs to develop models**
  - Two 0^{-+} around 1.4 GeV/c^2 in $(K_s^0 K_s^0)_{s-wave}\pi^0$ and $(K_s^0\pi^0)_{p-wave}K_s^0$ partial waves
- **Mass Dependent PWA with BW to extract resonances**
- **Consistency between MI and MD results**
- **Dominated by 0^{-+}**
  - Two BWs around 1.4 GeV is needed
- **Theorists attempt to reveal $\eta(1405)/\eta(1475)$ pole structure**

Phys.Rev.D 107，L091505 (2023)
Phys.Rev.D 109，014021 (2024)

- **further study is needed**

Partial Wave Analysis of $J/\psi \to \gamma\gamma\phi$

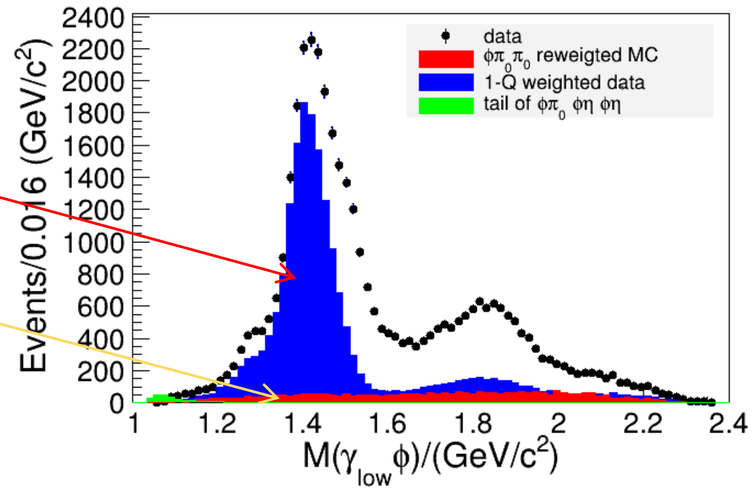
The decays $J/\psi \to \gamma X, X \to \gamma V$ $(V = \rho, \omega, \phi)$ serve as flavor filter

•unravelling quark contents of the intermediate resonances

arXiv: 2401.00918

■ main challenges

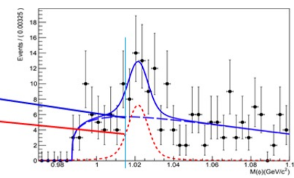
- high background level (55%)
  - non-ϕ background (46.7%)
  - ϕ background (8.2%)



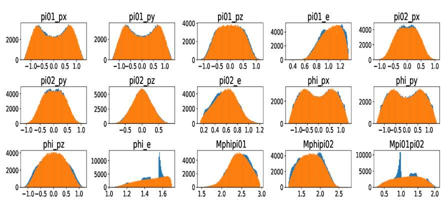
■ innovative point

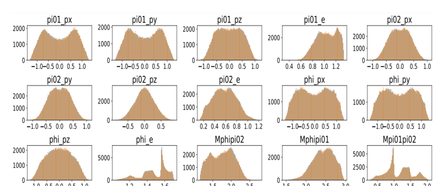
- non-ϕ background (Q factor method，CLAS)

$$Q_i = \frac{F_s(\vec{\xi}_r, \hat{\alpha}_i)}{F_s(\vec{\xi}_r, \hat{\alpha}_i) + F_b(\vec{\xi}_r, \hat{\alpha}_i)}$$



- ϕ background (ML based multi-dimensional reweighting method, BESIII)

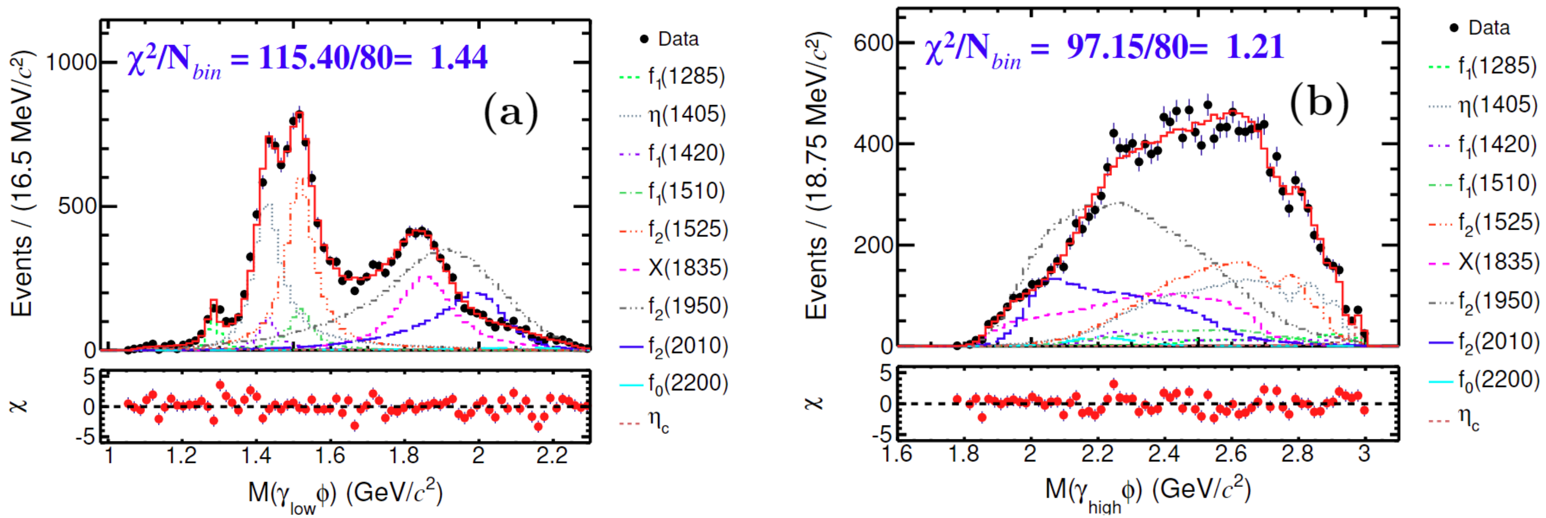




Partial Wave Analysis of $J/\psi \to \gamma\gamma\phi$

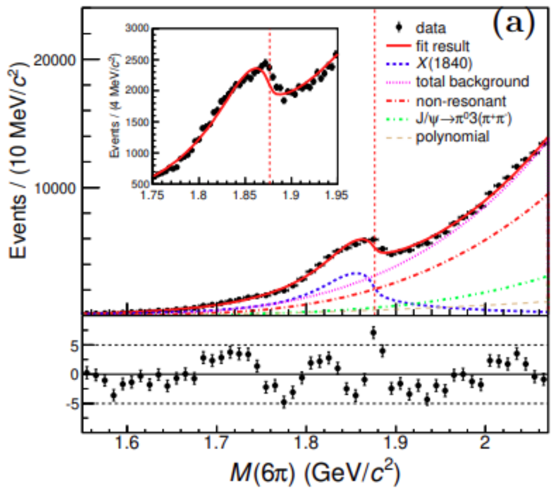
arXiv: 2401.00918

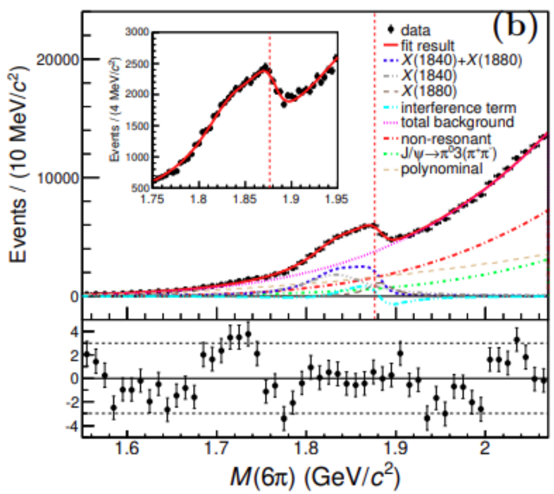
The decays $J/\psi \to \gamma X, X \to \gamma V$ $(V = \rho, \omega, \phi)$ serve as flavor filter
•unravelling quark contents of the intermediate resonances



- $\eta(1405)$ is observed, while $\eta(1475)$ can not be excluded
- $X(1835) \to \gamma\phi$ suggests its assignment of **second η' excitation**
- **$\eta_c \to \gamma\phi$ is observed for the first time, the first radiative decay mode of η_c**
- **Observation of $f_2(1950)$ and $f_0(2200) \to \gamma\phi$ unfavored their glueball interpretations.** [PRD 108, 014023 (2013); arXiv: 2404.01564]
- **No evidence of $\eta_1(1855)$ and X(2370), well consistent with the predictions for hybrid/ glueball.** [PRD 107, 114020(2023); NPA 1037, 122683]

X(1880): A New State Observed in $J/\psi \to \gamma 3(\pi^+\pi^-)$





- **Anomalous shape observed in J/ψ → γ3(π⁺π⁻) near $M(p\bar{p})$ threshold**
- **Two models:**
  - Flatte model
  - Interference between two resonances
- **Observed line shape consistent with two overlapping resonant structure: X(1840) and X(1880) (10σ)**
  - According to angular distribution, the J^{pc} for X(1840) and X(1880) tend to be 0^{-+}
  - X(1880): interpretation of a $p\bar{p}$ bound state

⇒**complex resonant structures near the $p\bar{p}$ threshold**

Parameters	Solution I	Solution II
$M_{X(1840)}$ (MeV/c^2)	$1832.5 \pm 3.1 \pm 2.5$	
$\Gamma_{X(1840)}$(MeV)	$80.7 \pm 5.2 \pm 7.7$	
$\mathcal{B}_{X(1840)}(\times 10^{-5})$	$1.19 \pm 0.30 \pm 0.15$	$2.07 \pm 0.50 \pm 0.36$
$M_{X(1880)}$ (MeV/c^2)	$1882.1 \pm 1.7 \pm 0.7$	
$\Gamma_{X(1880)}$(MeV)	$30.7 \pm 5.5 \pm 2.4$	
$\mathcal{B}_{X(1880)}(\times 10^{-5})$	$0.29 \pm 0.20 \pm 0.09$	$1.19 \pm 0.31 \pm 0.18$

much narrower than X(1835)

Summary and outlook

BESIII experiment is an excellent laboratory to study light hadron physics and search for light QCD exotic states

Exciting results from new J/ψ and ψ' data are presented

- **pesudoscalar state : $\eta(1405)$, X(2370)， X(1880)**
- **isoscalar/isovector 1^{-+}spin exotics state: $\eta_1(1855)$ /$\pi_1(1600)$**

BESIII is taking data since 2008. It will continue to run ~2030

- **BEPCII-U: 3x upgrade on luminosity; Ecms expanded to 5.6 GeV (summer 2024)**

More interesting results are expected!!!

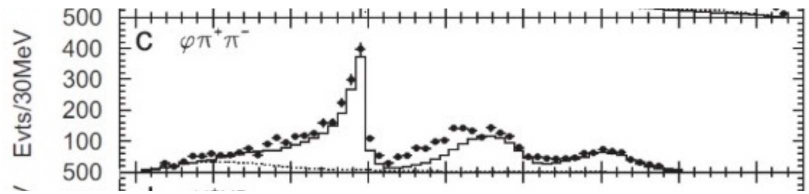
Back Up

Background treatment

The remaining background can be divided into two categories

- **ϕ background**

Dominated by J/ψ →$\phi\pi^0\pi^0$, which has rich structures. Difficult to be modeled with MC.



- **non-ϕ background**

ϕ sideband is tricky:

$e.g.$ $J/\psi \to \gamma\eta(1405)$,$\eta(1405) \to \pi_0 K^+K^-$, $\pi_0 \to \gamma\gamma$

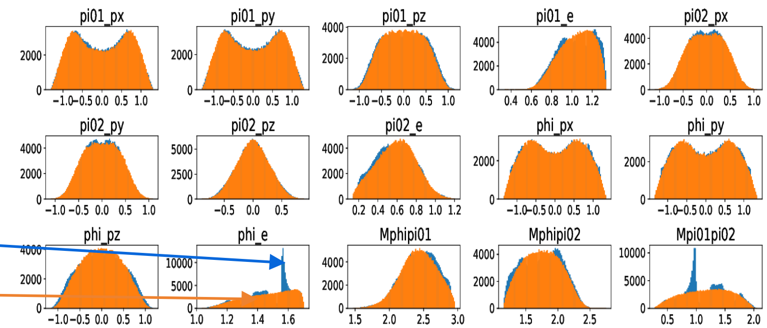
(1) If one of the photons γ_2 $\gamma_3$from the $\pi_0$decay is soft (say, γ_3), the γ_2 will be energetic and M($\gamma_2 K^+K^-$) will be at the $\eta(1405)$ mass.

(2)The K^+K^- mass distribution from $\eta(1405) \to \pi_0 K^+K^-$ peaks nears K^+K^- threshold, which is very close to ϕ mass.

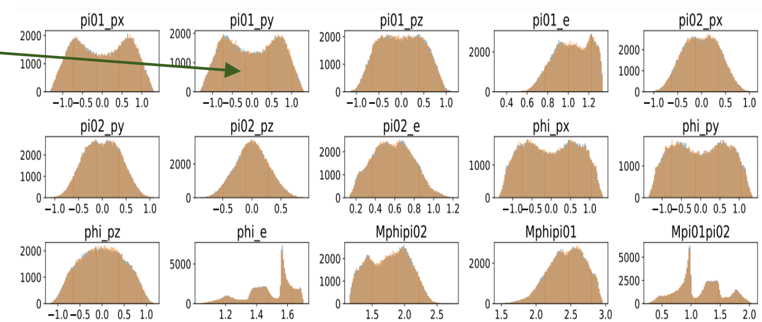
Background treatment

➢ ϕ background

Using a Machine learning based multi-dimensional reweighting method to get "data-like" MC of $J/\psi \to \phi\pi^0\pi^0$



- Select $J/\psi \to \phi\pi_0\pi_0$ events from data
- Generate $J/\psi \to \phi\pi_0\pi_0$ PHSP MC
- Perform multi-dimensional reweighting (ML)
- The distributions of weighted MC are well consistent with data



- The weighted $J/\psi \to \phi\pi_0\pi_0$ MC after $\boldsymbol{J/\psi \to \gamma\gamma\phi}$ event selection will be used for background estimation

* Beijiang. Liu, Xian Xiong, Guoyi Hou, Shiming Song, and Lin Shen. PoS ICHEP2018, 160 (2019). http://doi.org/10.5281/zenodo.1451985

Background treatment

- **Q-factor method: multi-dimensional sideband subtraction** [JINST 4 P10003 (2009)

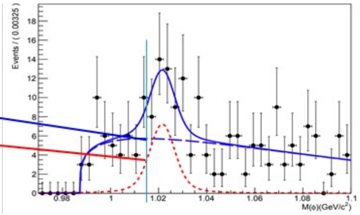
Generalize the ‘sideband’ subtraction method to higher dimensions without requiring the data to be divided into bins, successfully used in BAM-00221, Phys. Rev. D 100, 052012, by Malte Albrecht et al.

- In multi-dimensional phase space of $J/\psi \to \gamma\gamma\phi$, a so called Q-weight is given event-by-event , representing the probability of signal.
- A set of coordinates $\vec{\xi}$ must be defined($cos\theta(\gamma_{rad})$,$cos\theta(\phi)$, $cos\theta(K^{+})$, $M^2(\gamma_{high}\phi)$, $M^2(\gamma_{low}\phi)$). For event i, we find 200 of its nearest neighboring events in PHSP. The normalization Δ_k by default is set to the largest possible distance between two events in the coordinate ξ_k, and fit the reference coordinate $\vec{\xi}_r = M(K^+K^-)$.

$$d_{i,j}^2 = \sum_{k \neq r} \left[\frac{\xi_k^i - \xi_k^j}{\Delta_k} \right]^2$$

- Q-factor for event i, is determined by the fitting results and its $M(K^+K^-)$.

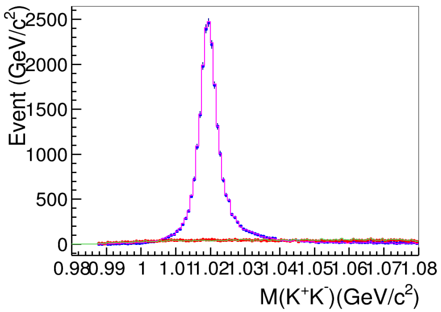
$$Q_i = \frac{F_s(\vec{\xi}_r, \hat{\alpha}_i)}{F_s(\vec{\xi}_r, \hat{\alpha}_i) + F_b(\vec{\xi}_r, \hat{\alpha}_i)}$$

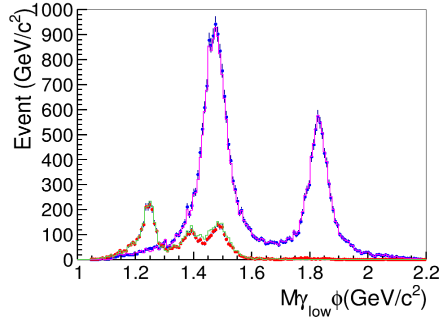


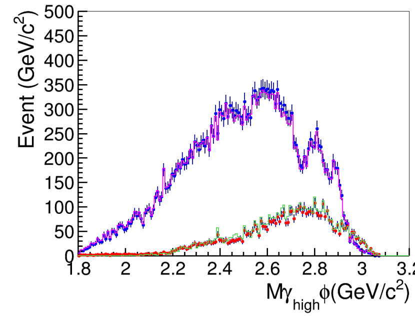
*CLAS experiment: Journal of Instrumentation, 4(10):P10003, 2009.
*CB/LEAR experiment: The European Physical Journal C, 75(3), 2015.

Background treatment

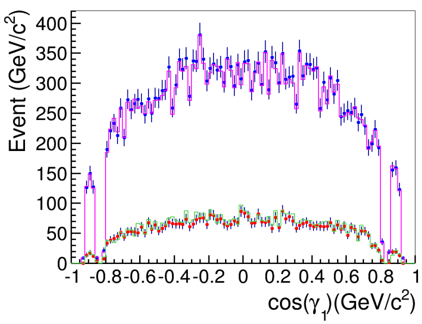
☐ Q-factor method check

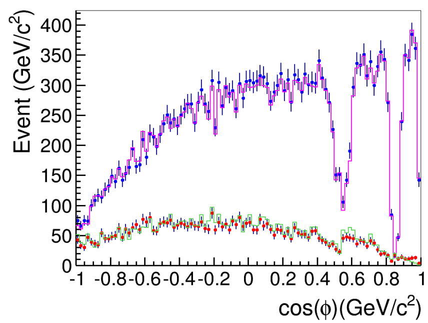






Coordinate	Δ_k
$cos\theta(\gamma_{rad})$	2
$cos\theta(\phi)$	2
$cos\theta(K^+)$	2
$\varphi(K^+)$	6.28
$\varphi(\phi)$	6.28
$M^2(\gamma_{high}\phi)$	0.15
$M^2(\gamma_{low}\phi)$	0.1





MC

Background channels

$J/\psi \to \gamma\eta(1405)$,$\eta(1405) \to K^+K^-\pi_0$

$J/\psi \to \gamma\eta(1285)$,$\eta(1285) \to K^+K^-\pi_0$

$J/\psi \to \gamma f(1510)$,$f(1510) \to K^{*+}K^-$

$J/\psi \to \gamma f(1510)$,$f(1510) \to K^{*-}K^+$

Signal channels

$J/\psi \to \gamma\eta(1405)$,$\eta(1405) \to \gamma\gamma\phi$

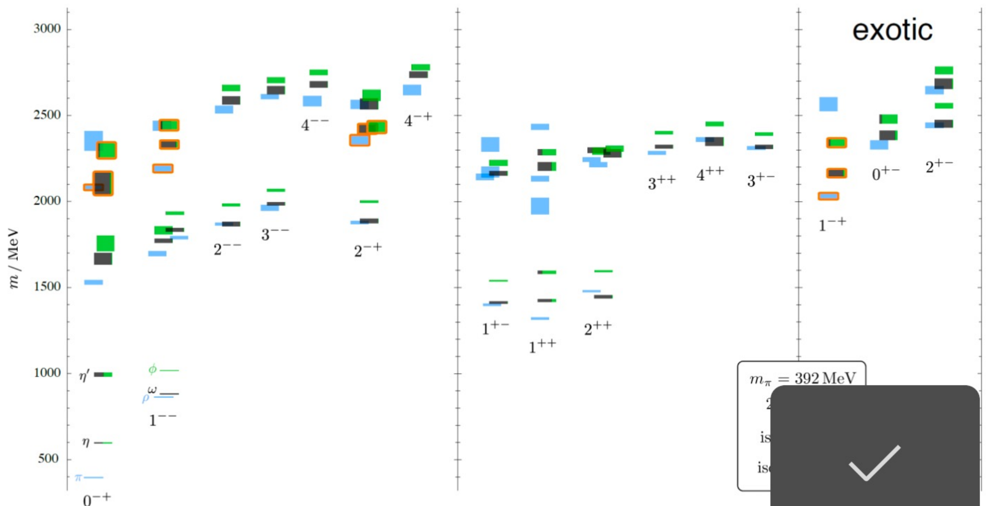
$J/\psi \to \gamma X(1835)$,$X(1835) \to \gamma\gamma\phi$

$J/\psi \to \gamma\eta(1475)$， $\eta(1405) \to \gamma\gamma\phi$

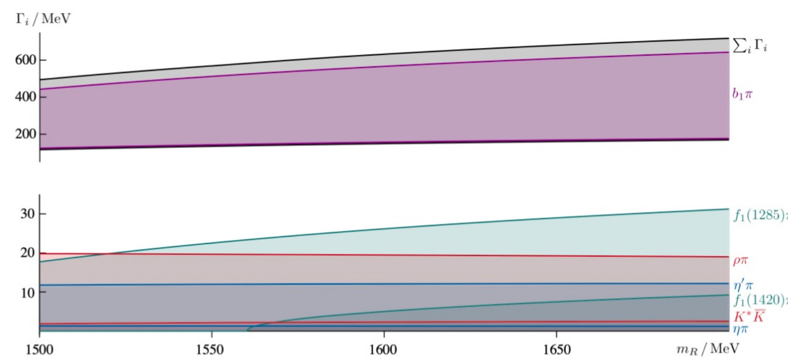
These figures show the generated distribution for signal(Magenta line) and background (green line) , the signal(blue dots) and background(red dots) estimated by Q-factor.

➢ **The Q-factor method can well present the true signal spectrum.**

Hybrid from LQCD



PRD 88 094505(2013)



PRD 103, 054502(2021)

Decay width of $\mathbf{1^{-+}}$ hybrid π_1

Lightest spin-exotic state: $\mathbf{1^{-+}}$

Observation of $\pi_1(1600)$ in $\chi_{c1} \to \eta' \pi^+ \pi^-$

- **Spin-parity of the $\pi_1(1600)$**
  - 1^{-+} assignment fit is better than that for 0^{++}, 2^{++} or 4^{++} assignments with significances well over 10σ

- **Significance of the Breit-Wigner phase motion**
  - Replace the resonant $\pi_1(1600)$ with a non-resonant $\pi\eta'$ P-wave described by the Breit-Wigner function without phase motion
    - $f = \frac{1}{\sqrt{(m^2-s)^2+m^2\Gamma^2}}$
  - The fit yields that: $\Delta M = +6.9\ \mathrm{MeV}/c^2$, $\Delta\Gamma = -96.4\ \mathrm{MeV}$
  - We observed significant phase motion with a statistical significance greater than 10σ