Crossing-symmetric dispersive analyses for meson-meson scatterings from lattice QCD data

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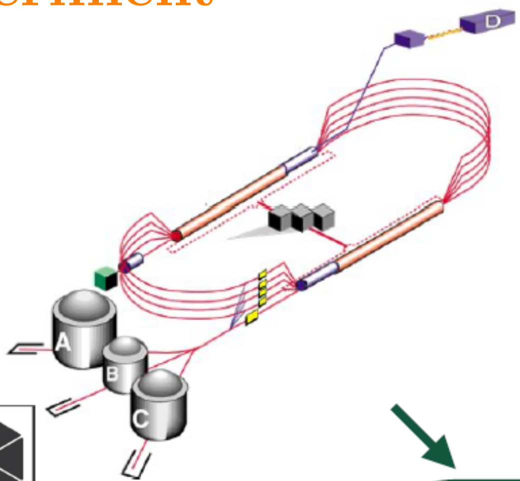
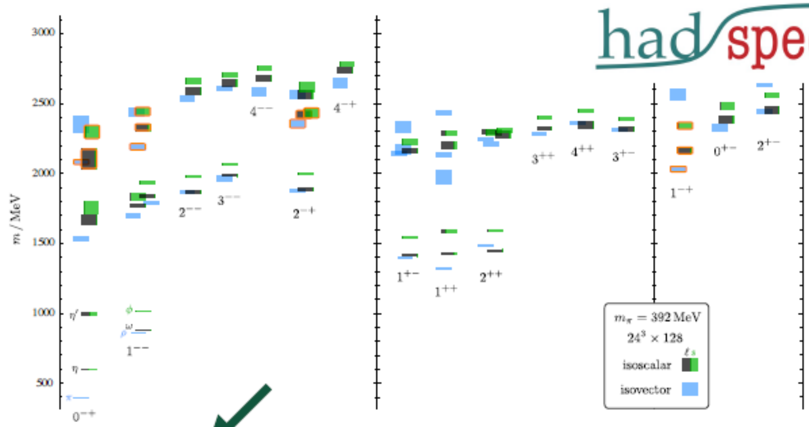
Why lattice QCD?

Experiment



Lattice QCD



Amplitude analyses

- Unitarity
- Analiticity
- Crossing

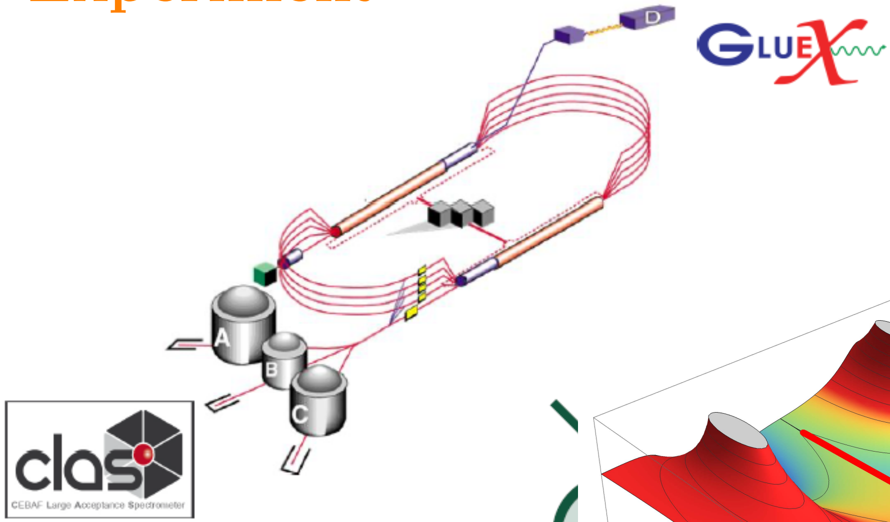
↔ **QCD**

↓

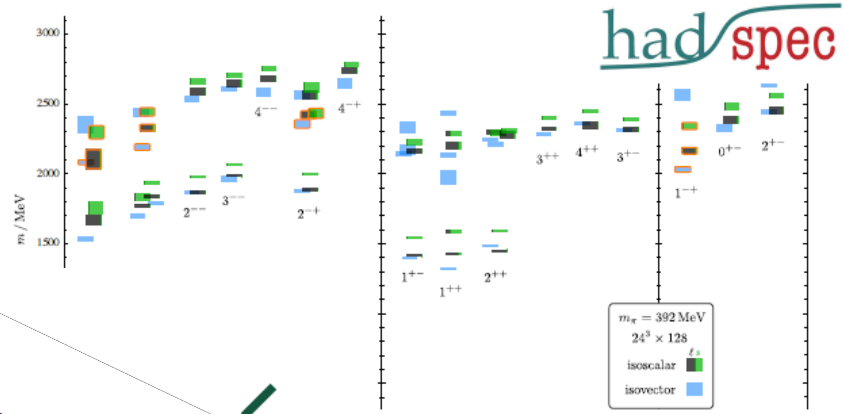
Observables

Why lattice QCD?

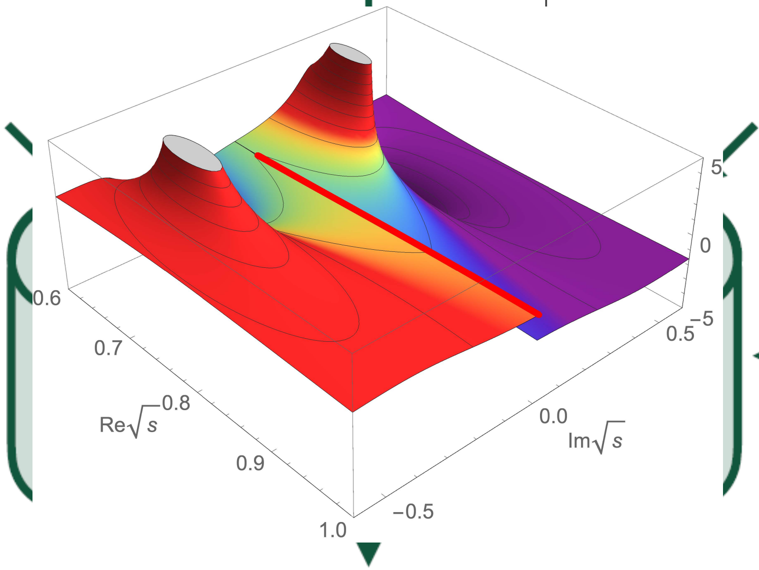
Experiment



Lattice QCD



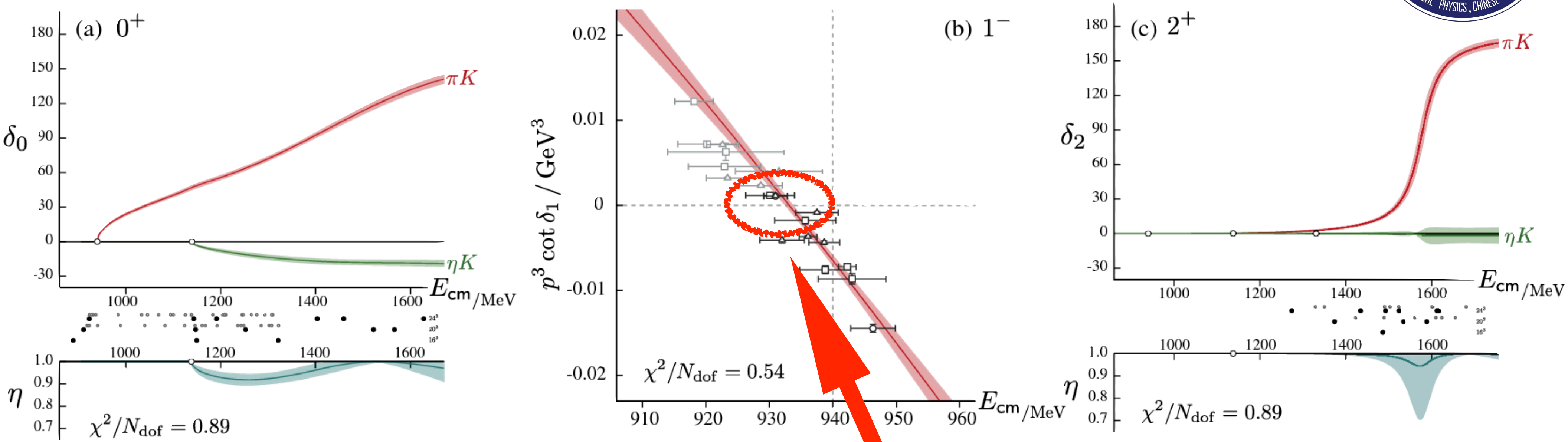
Resonances manifest as the poles of the amplitudes



↔ QCD

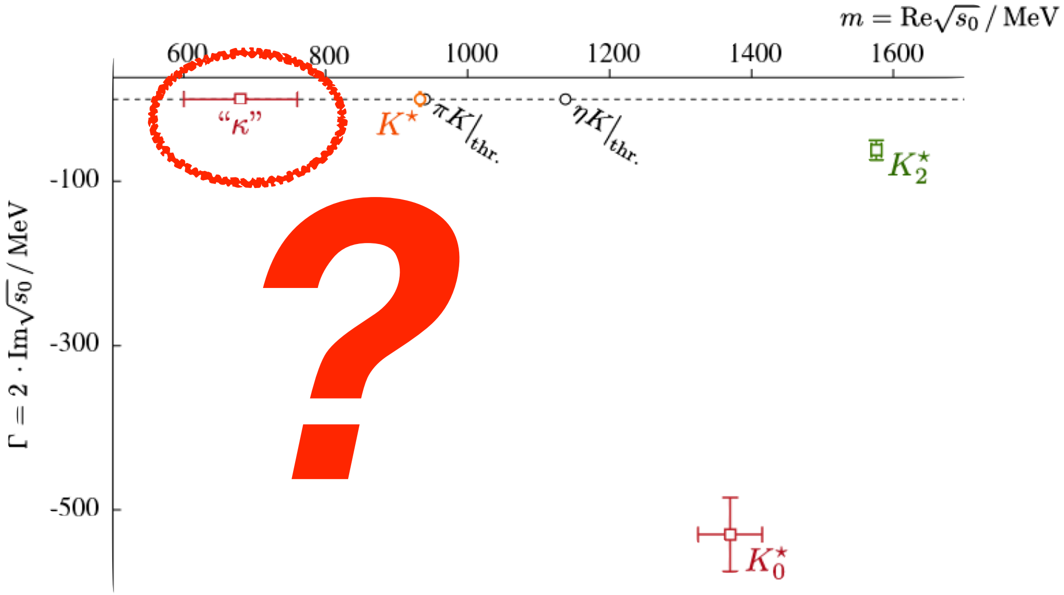
Observables

πK scattering at $m_\pi = 391$ MeV



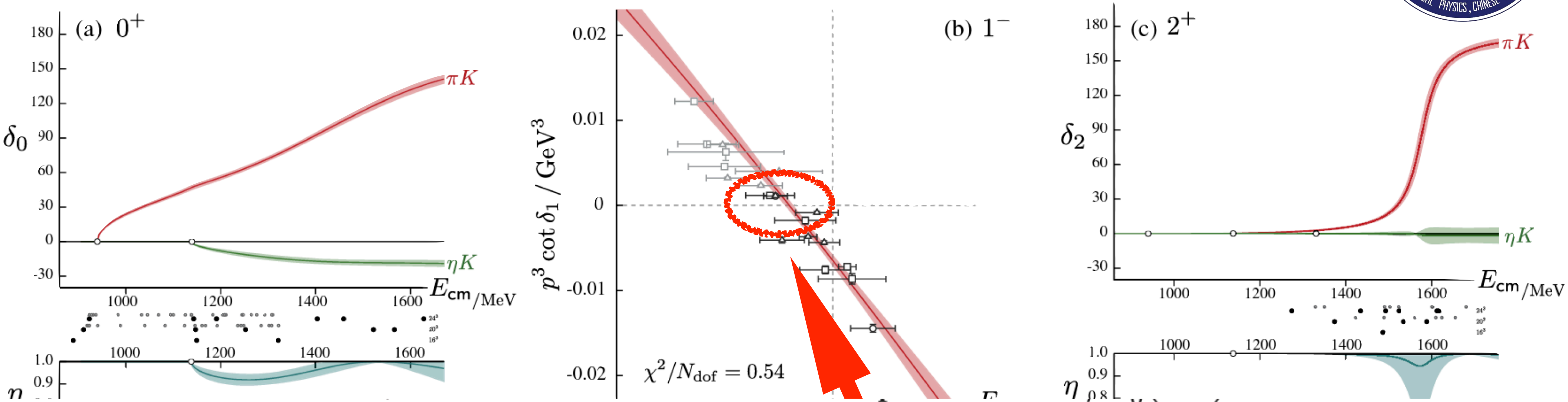
Shallow bound state pole

HSC, PRL (2014); PRD (2015)

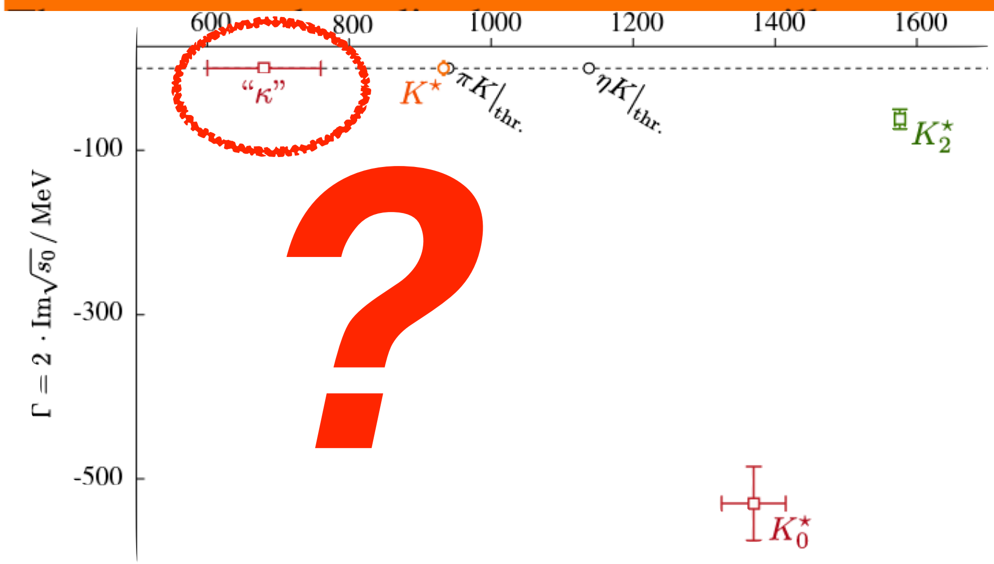


- $m_\pi \sim 390$ MeV, K-matrix fits:
  - $\kappa / K_0^*(700)$: **one virtual state???**
  - $K^*(892)$: **a shallow bound state**

πK scattering at $m_\pi = 391$ MeV



considered quark masses. As has been found in analyses of experimental scattering data, simple analytic continuations into the complex energy plane of precisely determined lattice QCD amplitudes on the real energy axis are not sufficient to model-independently determine the existence and properties of this state.



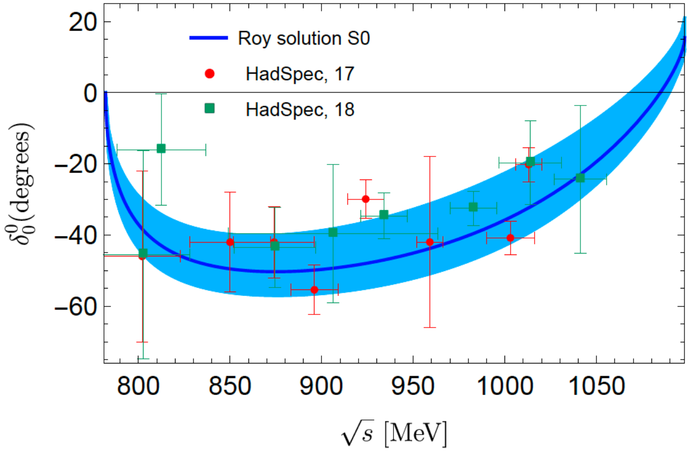
HSC et.al., PRL (2019)

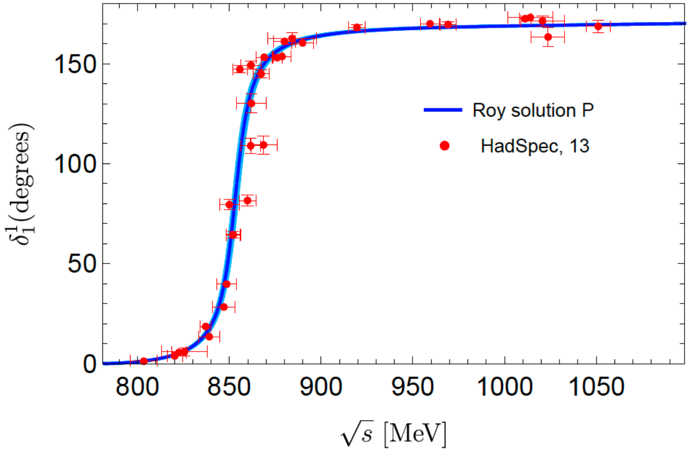
- $m_\pi \sim 390$ MeV, K-matrix fits:
  - $\kappa/K_0^*(700)$: **one virtual state???**
  - $K^*(892)$: **a shallow bound state**

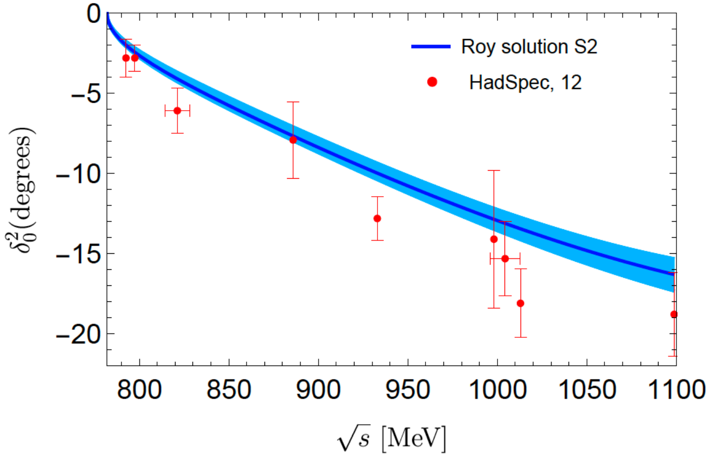
Ex.: $\pi\pi$ scattering at $m_\pi = 391$ MeV

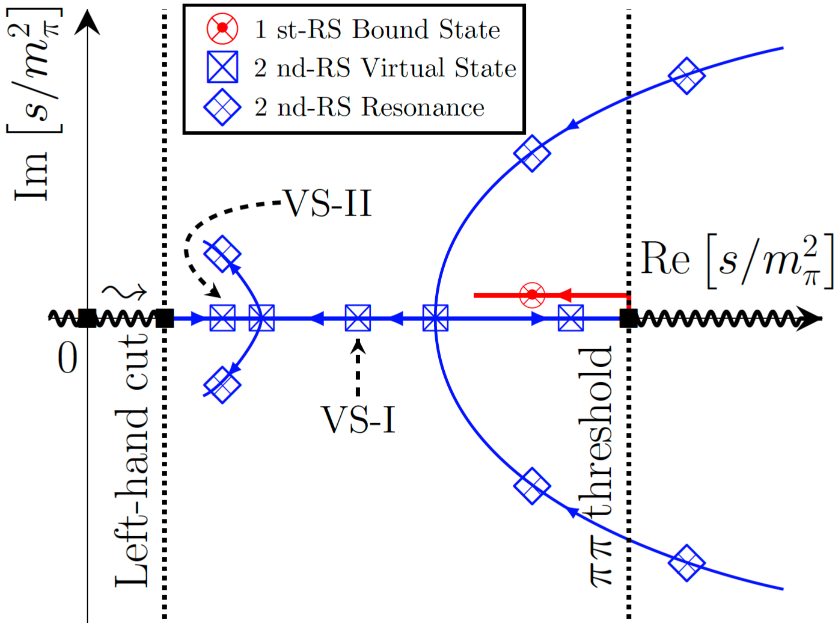
$\sigma/f_0(500)$

$m_\pi = 391$ MeV $\Rightarrow$ bound state σ pole!









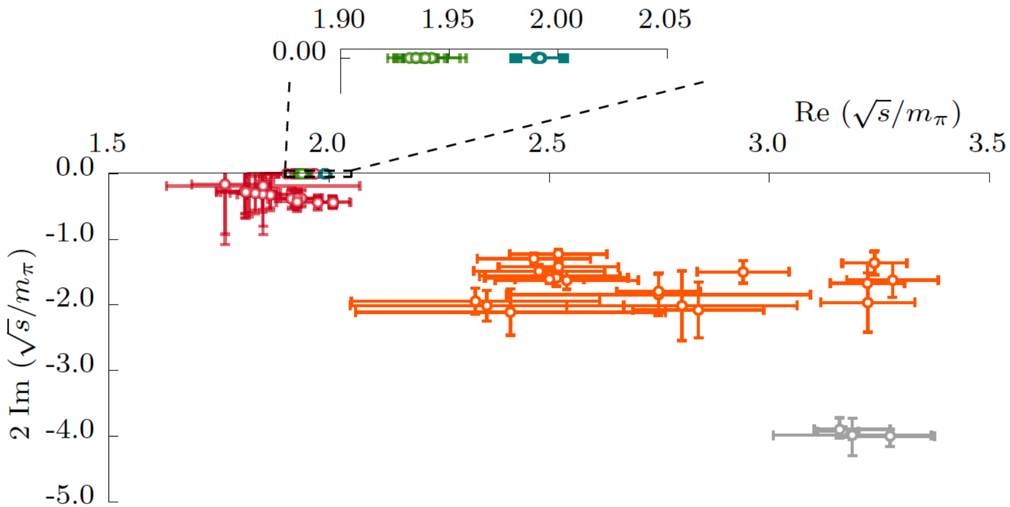
☑ Bound state and virtual state pole trajectories of σ as a function of m_π

XHC et.al., PRD (2023); Y.-L. Lyu et.al., PRD (2024)

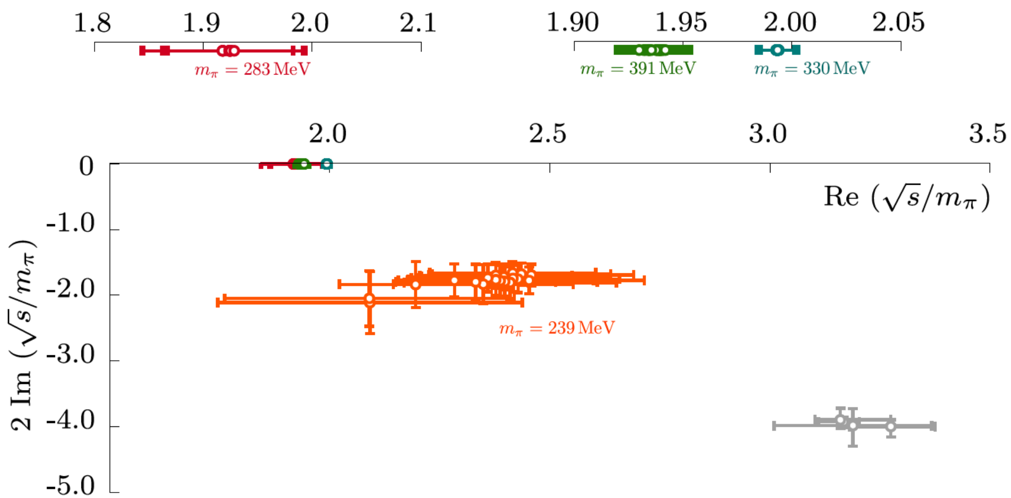
K-matrix analyses v.s. dispersive analyses

$$\sigma/f_0(500)$$

New σ pole positions via preliminary Roy equation analyses **XHC** et.al., PRD (2023); HSC, PRD (2024)



K-MATRIX



ROY EQUATION

What is Roy or Roy-Steiner type equation?

Roy-Steiner type equations = Analyticity (Causality) + Crossing symmetry + Unitarity

Crossing-symmetric dispersive analyses

S. Roy (1941–)

F. Steiner (194?–)

- Renaissance caused by the development of χPT

$\pi\pi$
- G. Colangelo, et al., NPB (2001); B. Ananthanarayan, et. al., Phys. Rept. (2001); **I. Caprini, et al., PRL (2006)**; B. Moussallam, EPJC (2011); Garcia-Martin, et al.,PRD (2011); PRL (2011); I. Caprini, et al., EPJC (2011); J. Pelaez, Phys.Rept. (2016); XHC et.al., PRD (2023); HSC, PRD (2024)…

πK
- P. Buettiker, et al., EPJC (2004); **S. Descotes-Genon, et al., EPJC (2006)**; J. Pelaez and A. Rodas, EPJC(2018); PRL (2020); Phys.Rept. (2022); J. Pelaez et.al., PRL (2023)…

πN
- C. Ditsche, et al., JHEP (2012); M. Hoferichter et.al., JHEP (2012); M. Hoferichter, et al., PRL 115, 092301(2015); PRL 115, 192301 (2015); Phys. Rept. (2016); PLB (2016); EPJA (2016); J. Ruiz de Elvira et.al., JPG (2018); M. Hoferichter, et al., PRL (2018); **XHC, et.al., JHEP (2022)**; **M. Hoferichter, et al., PLB (2024)**…

- $\gamma\pi \to \pi\pi$: T. Hannah, NPB (2001); M. Hoferichter et.al., PRD (2012); $\gamma\gamma \to \pi\pi$: M. Hoferichter et.al., EPJC (2011); $\gamma^*\gamma^* \to \pi\pi$: M. Hoferichter and P. Stoffer, JHEP (2019)…

Roy-Steiner type equations

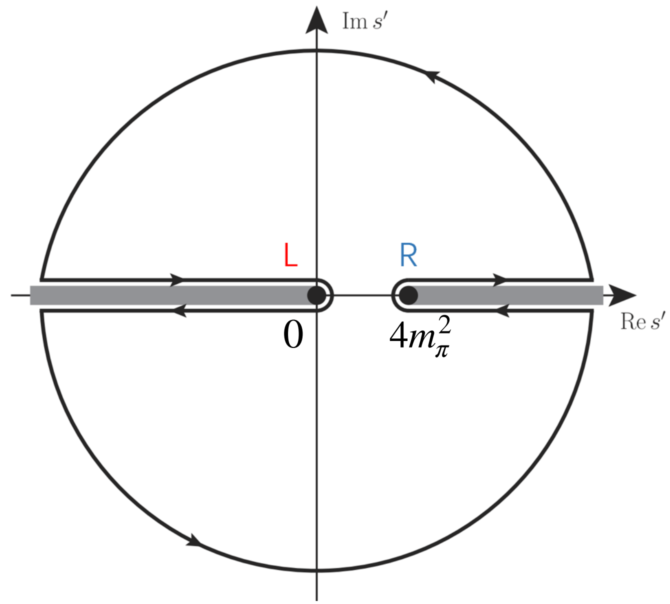
$$\mathrm{Re}\, t_J^I(s) = k_J^I(s) + \sum_{I'} \sum_{J'} \mathscr{P}\int_{4m_\pi^2}^{\infty} \mathrm{d}s' \quad \underbrace{K_{JJ'}^{II'}(s',s)}_{\frac{1}{\pi}\frac{\delta_{JJ'}\delta_{II'}}{s'-s} + \bar{K}_{JJ'}^{II'}(s,s')} \quad \mathrm{Im}\, t_{J'}^{I'}(s')$$

ROY EQUATION

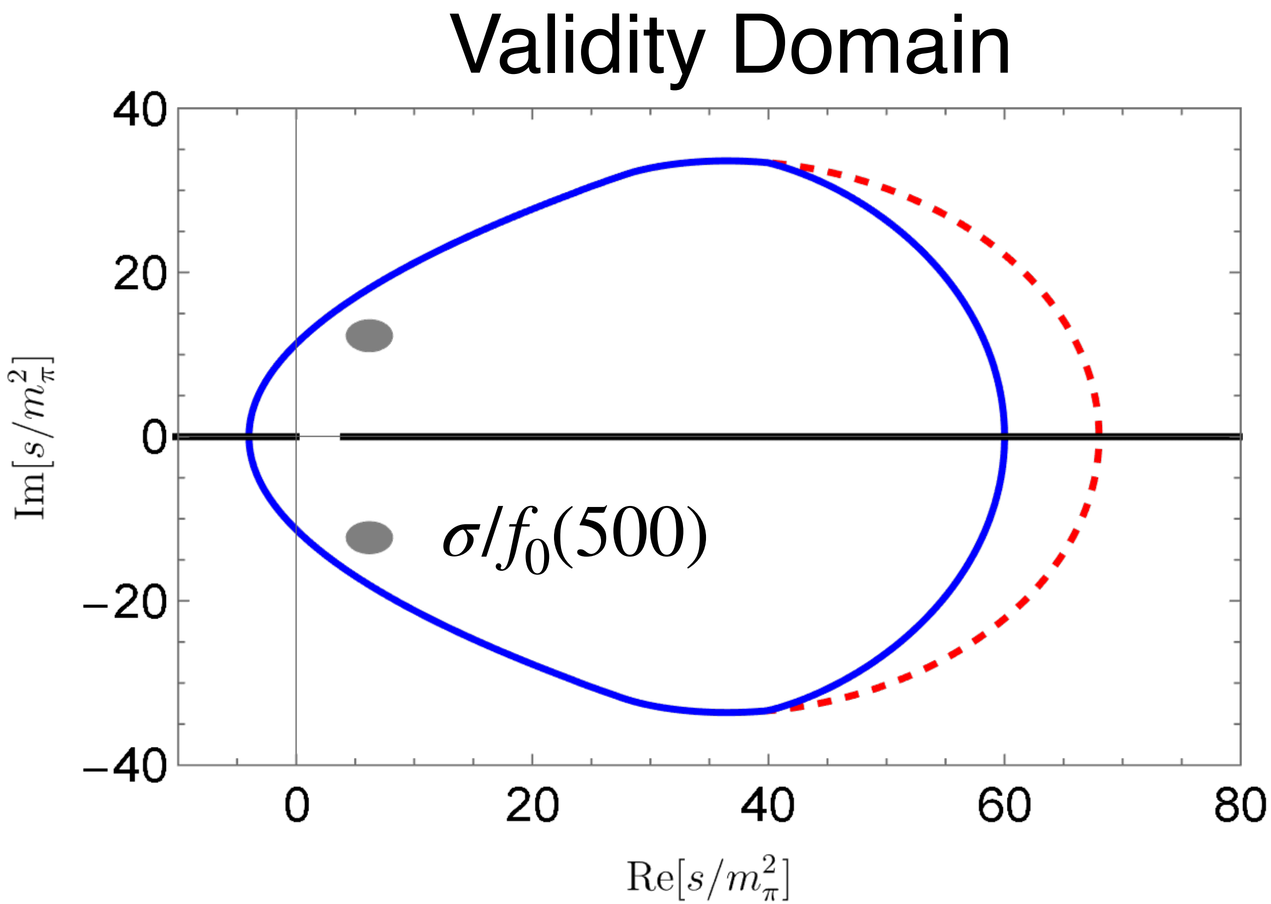
$$K_{00}^{00}(s,s') = \frac{1}{\pi\,(s'-s)} + \frac{2\ln\left(\frac{s+s'-4M_\pi^2}{s'}\right)}{3\pi\left(s-4M_\pi^2\right)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'\left(s'-4M_\pi^2\right)}$$

$$k_0^0(s) = a_0^0 + \frac{s-4m_\pi^2}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)$$

Left-hand cuts



Roy equation for $\pi\pi$ scattering



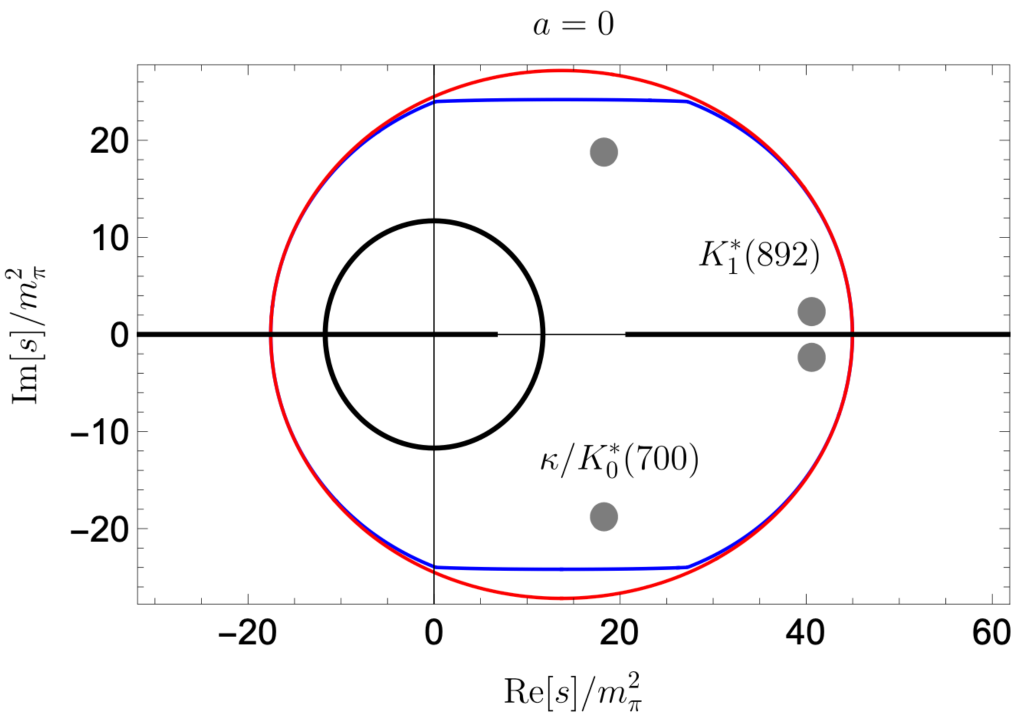
$$m_\sigma = 441^{+16}_{-8}\mathrm{MeV}$$

$$\Gamma_\sigma = 544^{+18}_{-25}\mathrm{MeV}$$

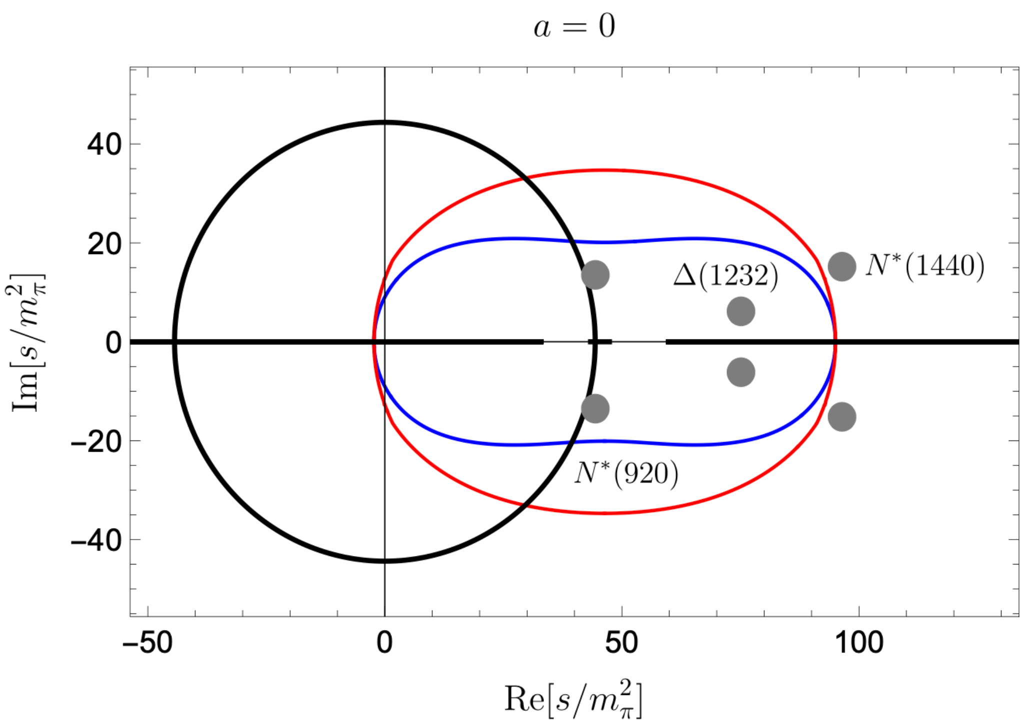
I. Caprini, et al., PRL (2006)

Roy-Steiner type equations

πK



πN



$$m_\kappa = 658 \pm 13\text{MeV}$$
$$\Gamma_\kappa = 557 \pm 24\text{MeV}$$

Descotes-Genon and Moussallam, EPJC (2006)

$$m_{N*} = 918 \pm 3\text{MeV}$$
$$\Gamma_{N*} = 326 \pm 18\text{MeV}$$

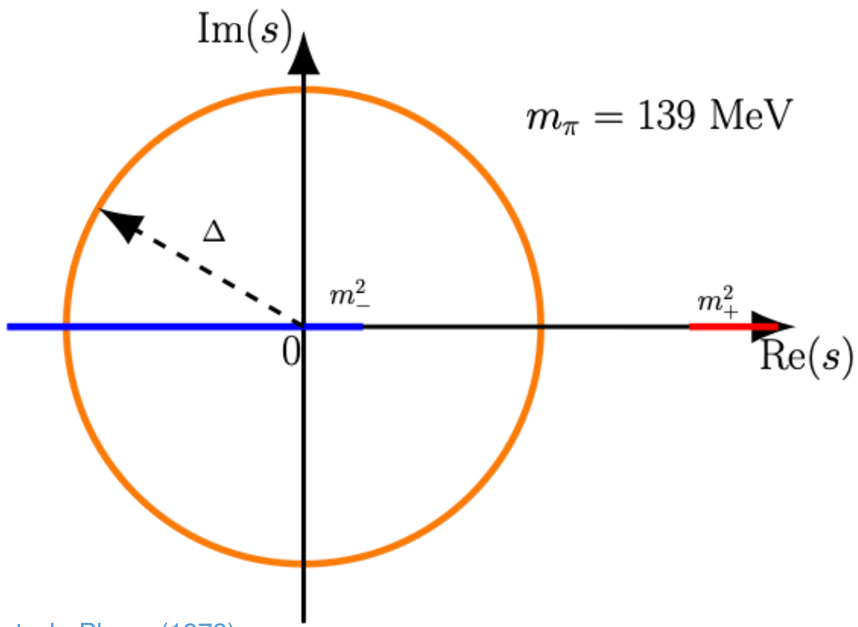
XHC, Q.-Z. Li and H.-Q. Zheng, JHEP (2022)

$$m_{N*} = 913.9 \pm 1.6\text{MeV}$$
$$\Gamma_{N*} = 337.7 \pm 6.2\text{MeV}$$

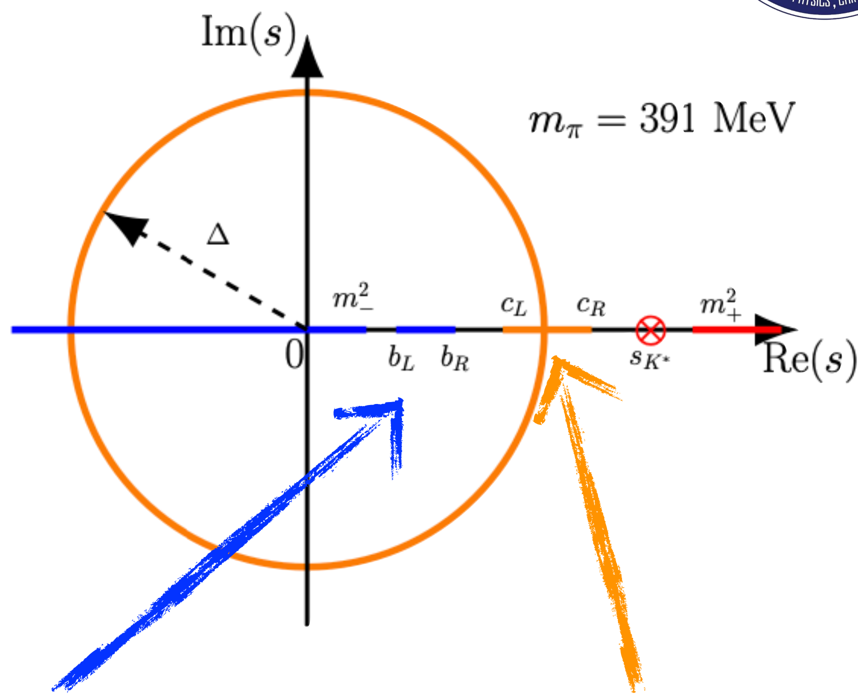
Hoferichter, et al., PLB (2024)

πK scattering at $m_\pi = 391$ MeV

The cut structure of the πK partial-wave amplitudes

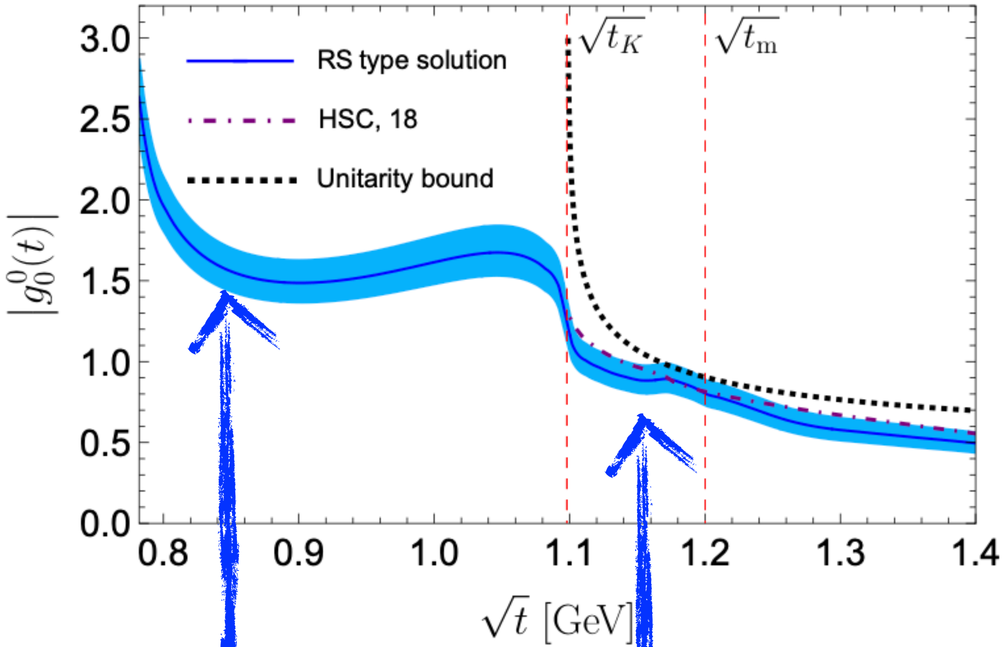


Lang, Fortsch. Phys. (1978)



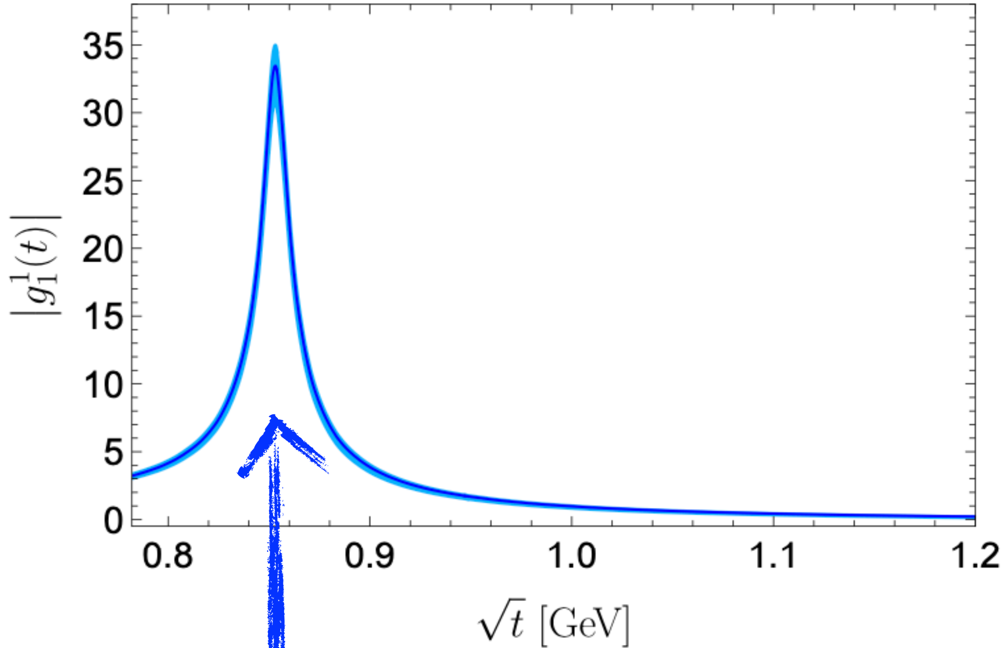
t-channel $\pi\pi \to K\bar{K}$ partial wave amplitudes

- t-channel solution from Roy-Steiner equations



σ bound state pole

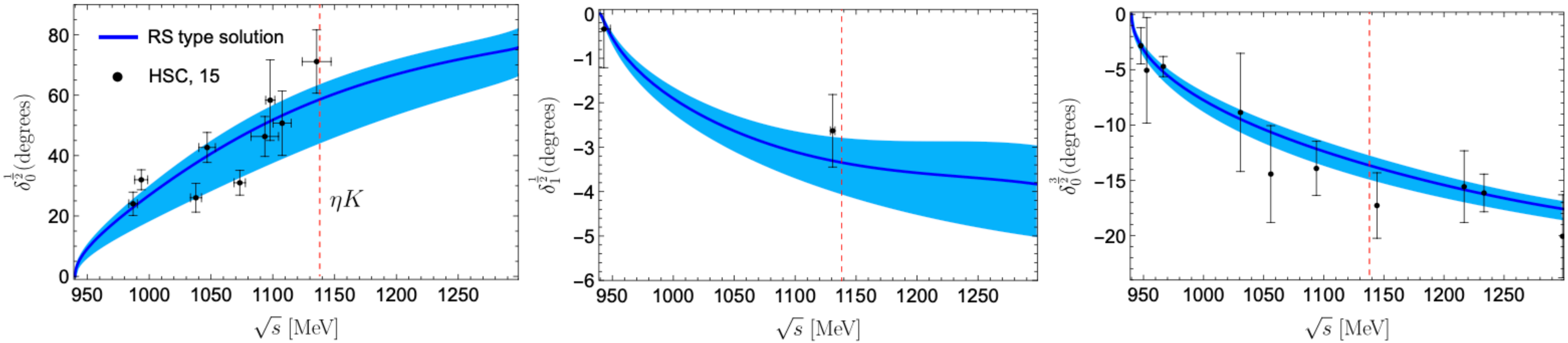
$f_0(980)$



$\rho(770)$ dominance

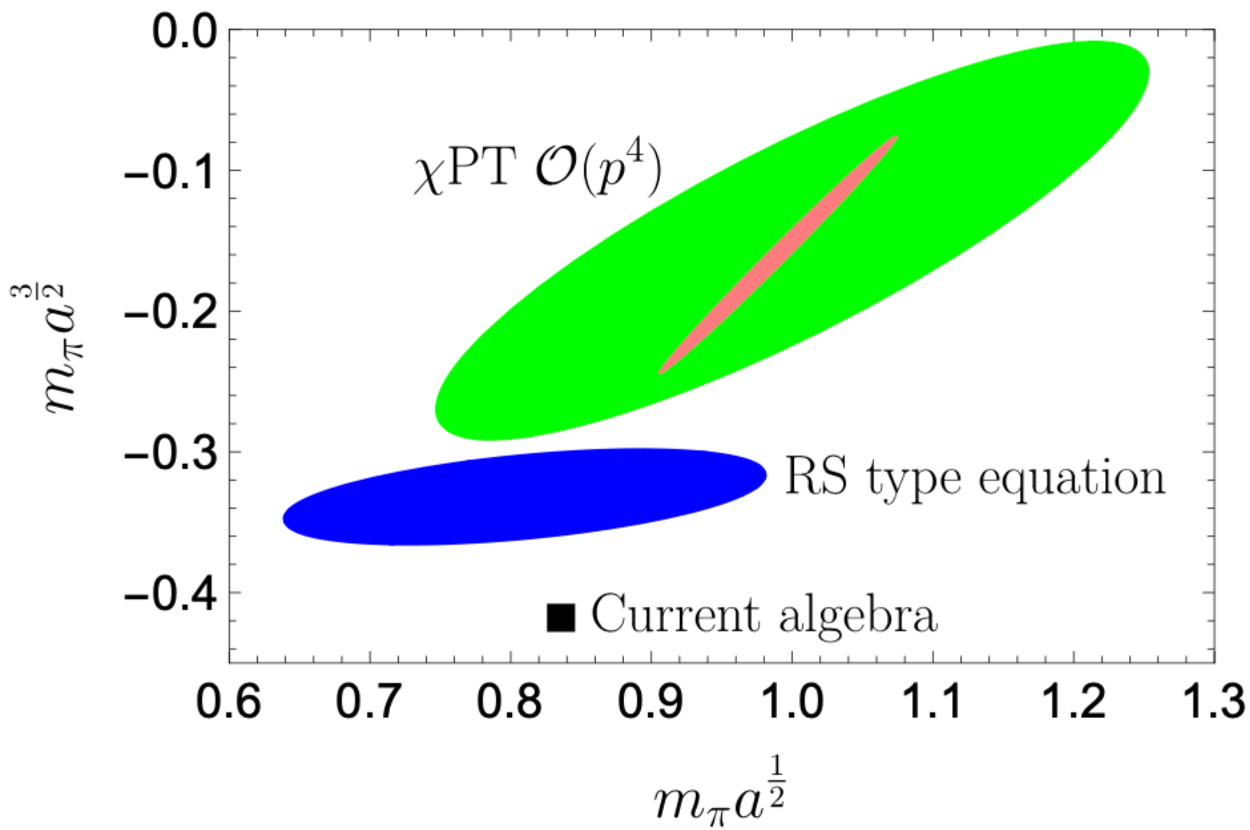
s-channel $\pi\pi \to K\bar{K}$ partial wave amplitudes

☑ s-channel solution from Roy-Steiner equations

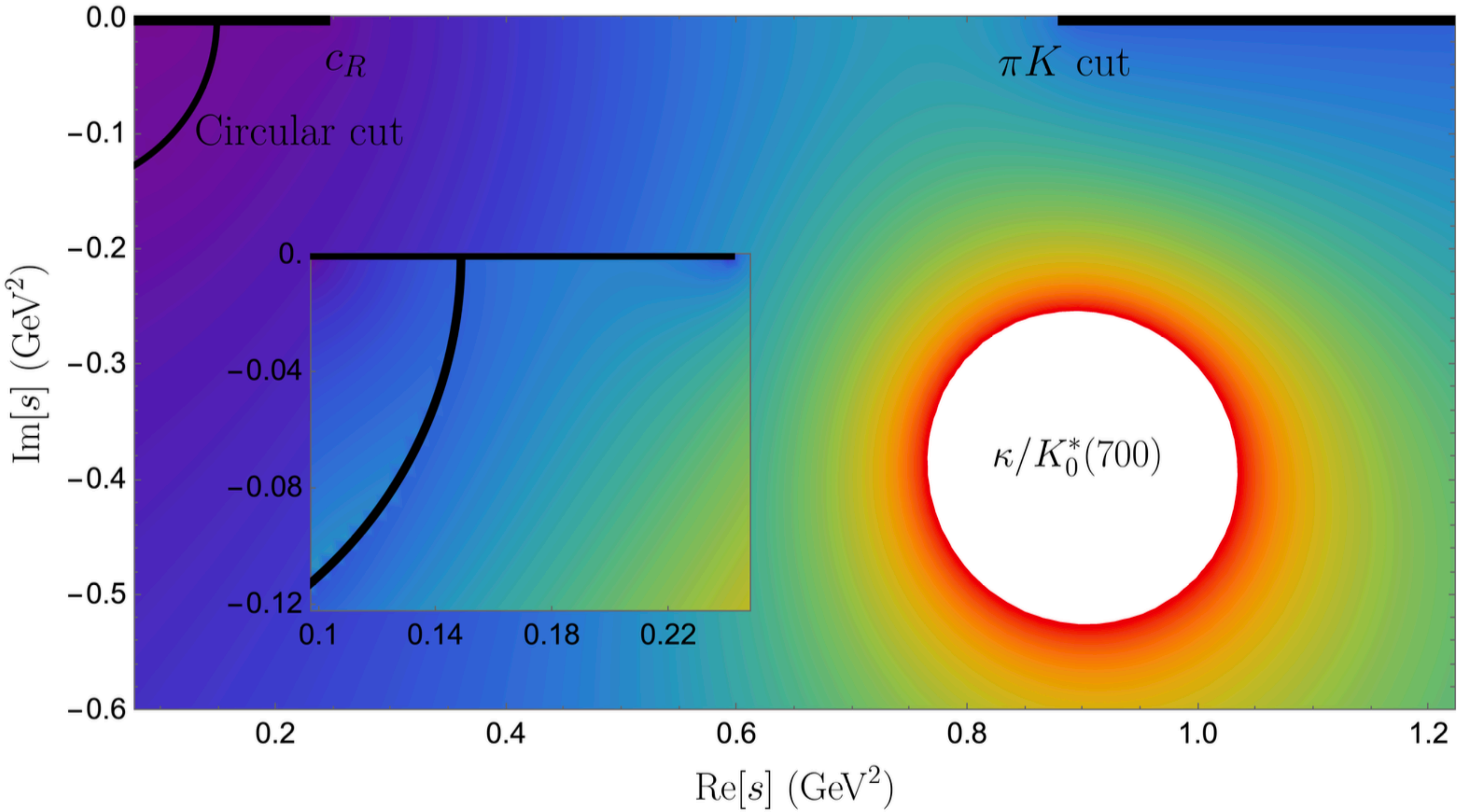


- $I = \dfrac{1}{2}$ S-wave: no sharp features that signal the presence of a nearby pole
- $I = \dfrac{1}{2}$ P-wave: shallow vector bound state $K^*(892)$
- $I = \dfrac{3}{2}$ S-wave: repulsive channel

S-wave scattering lengths



Dispersive determination of $\kappa/K_0^*(700)$ from LQCD data



A broad resonance instead of a deeply bound virtual state pole

Summary and outlook

- [x] The unity of dispersive techniques and lattice QCD data is powerful to investigate low energy hadron physics

- [x] Widely-used unitarization methods such as K-matrix, etc., are not good in light meson & baryon studies

- [x] Dispersive approaches, Muskhelishvili-Omnès formalism, Roy-Steiner type equations, etc. are necessary

- [ ] πD scattering at physical & unphysical m_π: $D_0^*(2300)$, two pole?

- [ ] KN & $\bar{K}N$ scatterings: $\Lambda(1405)$, two pole?

- [ ] Dispersive determination of three-body resonances?

Thank you for your attention!