A recent study on hadronic contributions to muon g-2

Lingyun Dai
Hunan University

with B.H. Qin, Jia-Yu Zhou, S.J.Wang, W. Qin, J. Portoles, *et.al.*

Based on: arxiv: 2403.14294, JHEP07 (2023) 037,
RPP84 (2021) 076201, JHEP03 (2021) 092,
PRD99 (2019) 114015, PRD97 (2018) 036012,
PRD95 (2017) 056007, PRD94 (2016) 116061,
PRD90 (2014) 036004, PLB736 (2014) 11,
PRD88 (2013) 056001, et. al.

第十四届全国粒子物理学术会议
Aug 2024, Qingdao

湖南大學
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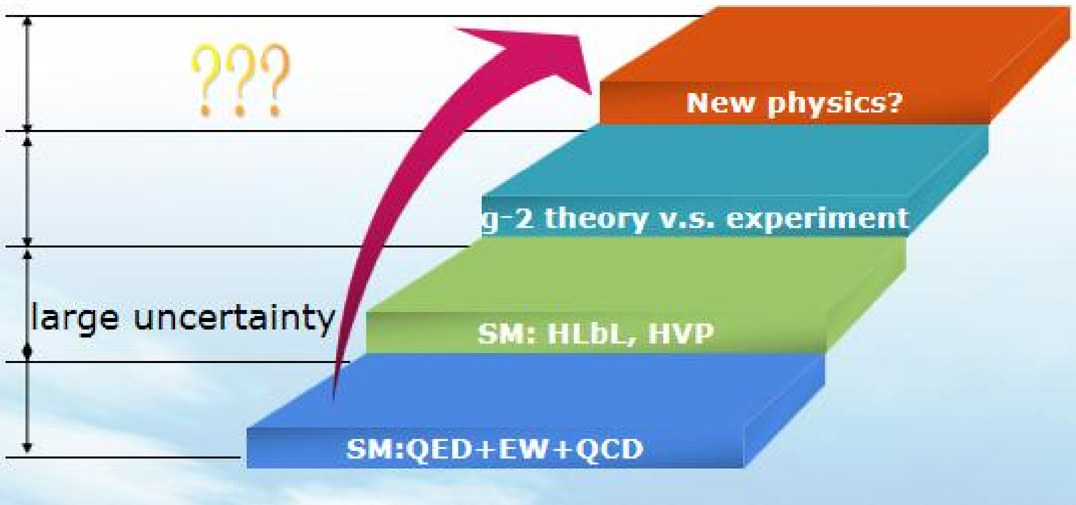
Outlines

1. Introduction
2. HVP
3. HLBL
4. Summary

1、Introduction



$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm QCD}$$

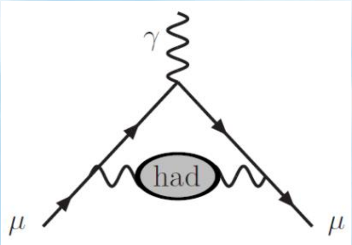
- HVP, HLbL?

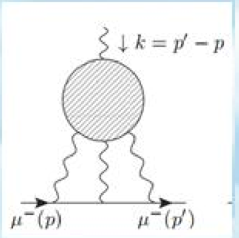
	values $(\times 10^{-11})$
QED	116584718.931(104)
EW	153.6(1.0)
HVP	6845(40)
HLBL	92(18)
SM	116591810(43)
exp.(BNL)	116592089(63)
exp.(FNAL)	116592040(54)
exp.(avg.)	116592061(41)
$a_\mu^{\rm SM}$-$a_\mu^{\rm exp}$	251(59)

Phys.Rev.Lett.126, 141801 (2021)
Phys.Rev.D 73, 072003 (2006).

J-PARC, FNAL, CSNS, HIAF?

Phys.Rept.887(2020)1





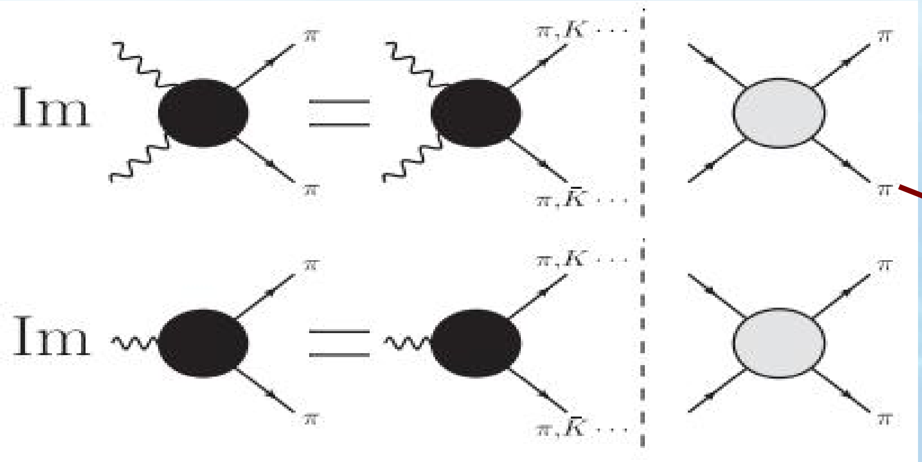
Methods from SM

- Hadronic Part: Methods from SM
  - LQCD
  - Data-driven solutions from experiment
  - Amplitude analysis: model independent

- Only one physical amplitude!
- It should satisfy the fundamental QFT principles
- It should be compatible with the exp results

Amplitude analysis: FSI

- Most resonances decays into light pseudoscalars
- FSI needs to be taken into account to perform an amplitude analysis
- Methods: KM, N/D, AMP, Roy equation, PKU, Pade, LSE, BSE, ChEFT, *et.al.*



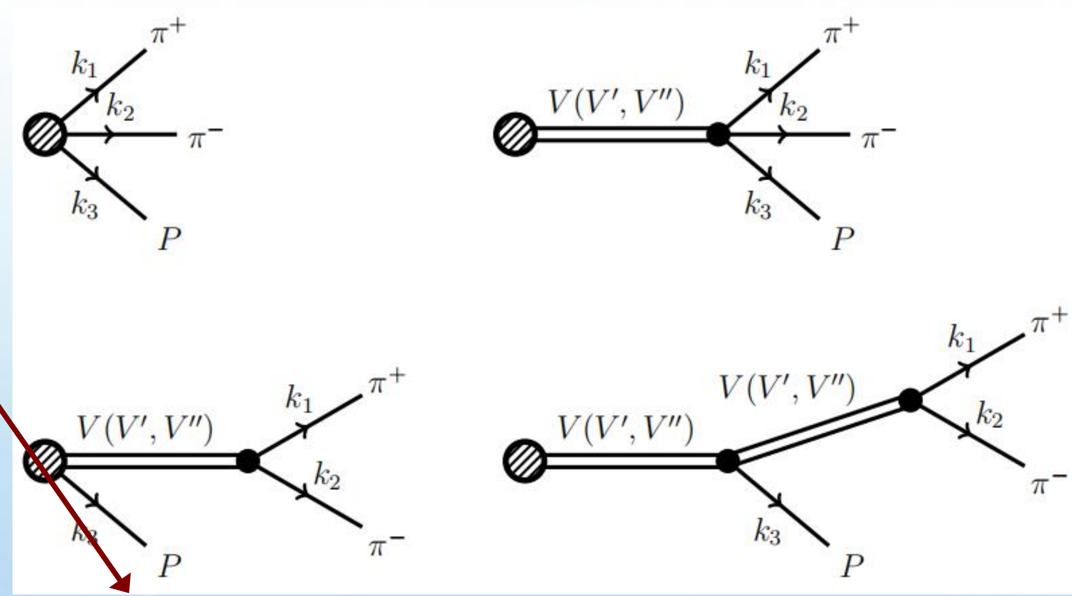
Yao, Dai#, Zheng, Zhou, RPP84(2021)076201

2、HVP

- RChT in the resonance region, excited states?
  - resonances included as new degrees of freedom
  - QCD high energy constraints to reduce LECs
  - 1/Nc expansion

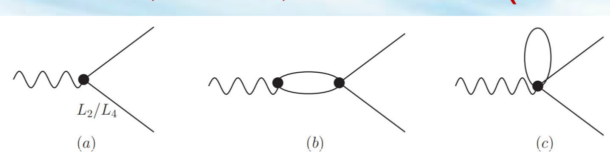
- V', V'' has the same topologies as the ground states

- ππ-KK FSI part by matching with Omens functions and ChPT



$$\frac{1}{M_V^2 - x} \rightarrow \frac{1}{M_V^2 - x} + \frac{\beta'_\pi}{M_{V'}^2 - x} + \frac{\beta''_\pi}{M_{V''}^2 - x}$$

Guerrero, et.al., PLB 412 (1997) 382



Wang, Fang, Dai, JHEP07 (2023) 037

Dai, et.al., PRD88 (2013) 056001

Building amplitudes

- We give a combined analysis on several channels:

$$\pi^+\pi^-, K^+K^-, \pi^+\pi^-\pi^0, \pi^+\pi^-\eta \quad \pi^0\gamma \text{ and } \eta\gamma.$$

  - ρ-ω mixing, origined from Gasser&Leutwyler's

$$F_V^\pi = \Big(1 + \frac{F_V G_V}{F^2} Q^2 \big(BW(M_\rho, \Gamma_\rho, Q^2) + \beta'_{\pi\pi} BW(M_{\rho'}, \Gamma_{\rho'}, Q^2) + \beta''_{\pi\pi} BW(M_{\rho''}, \Gamma_{\rho''}, Q^2)\big) \left(\frac{1}{\sqrt{3}} \sin\theta_V \sin\delta^\rho + \cos\delta\right)\cos\delta - \frac{F_V G_V}{F^2} Q^2 \big(BW(M_\omega, \Gamma_\omega, Q^2) + \beta'_{\pi\pi} BW(M_{\omega'}, \Gamma_{\omega'}, Q^2) + \beta'''_{\pi\pi} BW(M_{\omega''}, \Gamma_{\omega''}, Q^2)\big) \left(\frac{1}{\sqrt{3}} \sin\theta_V \cos\delta - \sin\delta^\omega\right)\sin\delta^\omega\Big) \exp\left[\frac{-s}{96\pi^2 F^2}\left(\mathrm{Re}\left[A[m_\pi, M_\rho, Q^2] + \frac{1}{2} A[m_K, M_\rho, Q^2]\right]\right)\right]$$

- Not much freedom for Fit

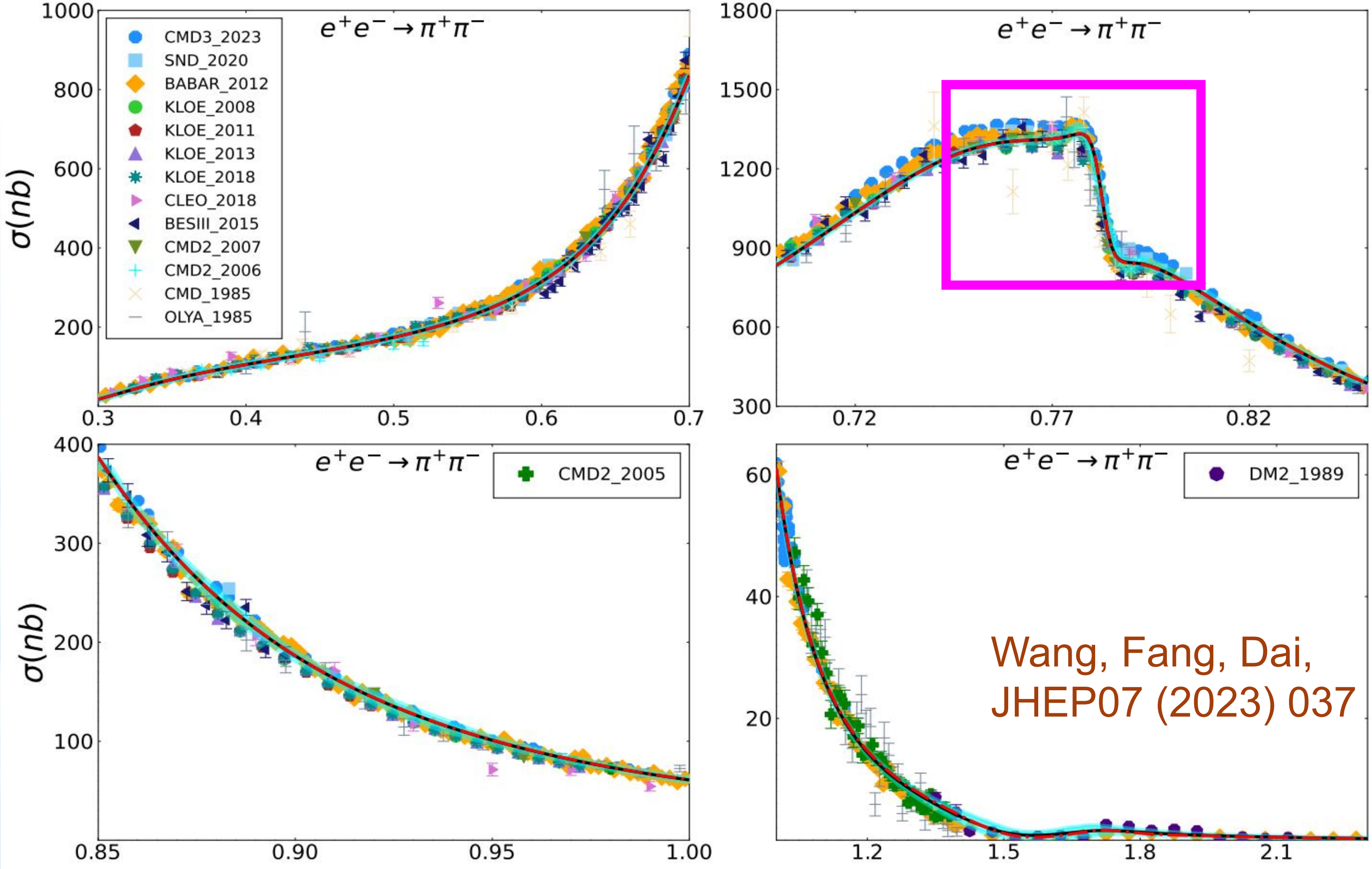
It is 1, from QCD as well as disersion relation constraints

Gasser&Leutwyler, Phys.Rept.87 (1982) 77

Guerrero&Pich, PLB 412 (1997) 382

ππ

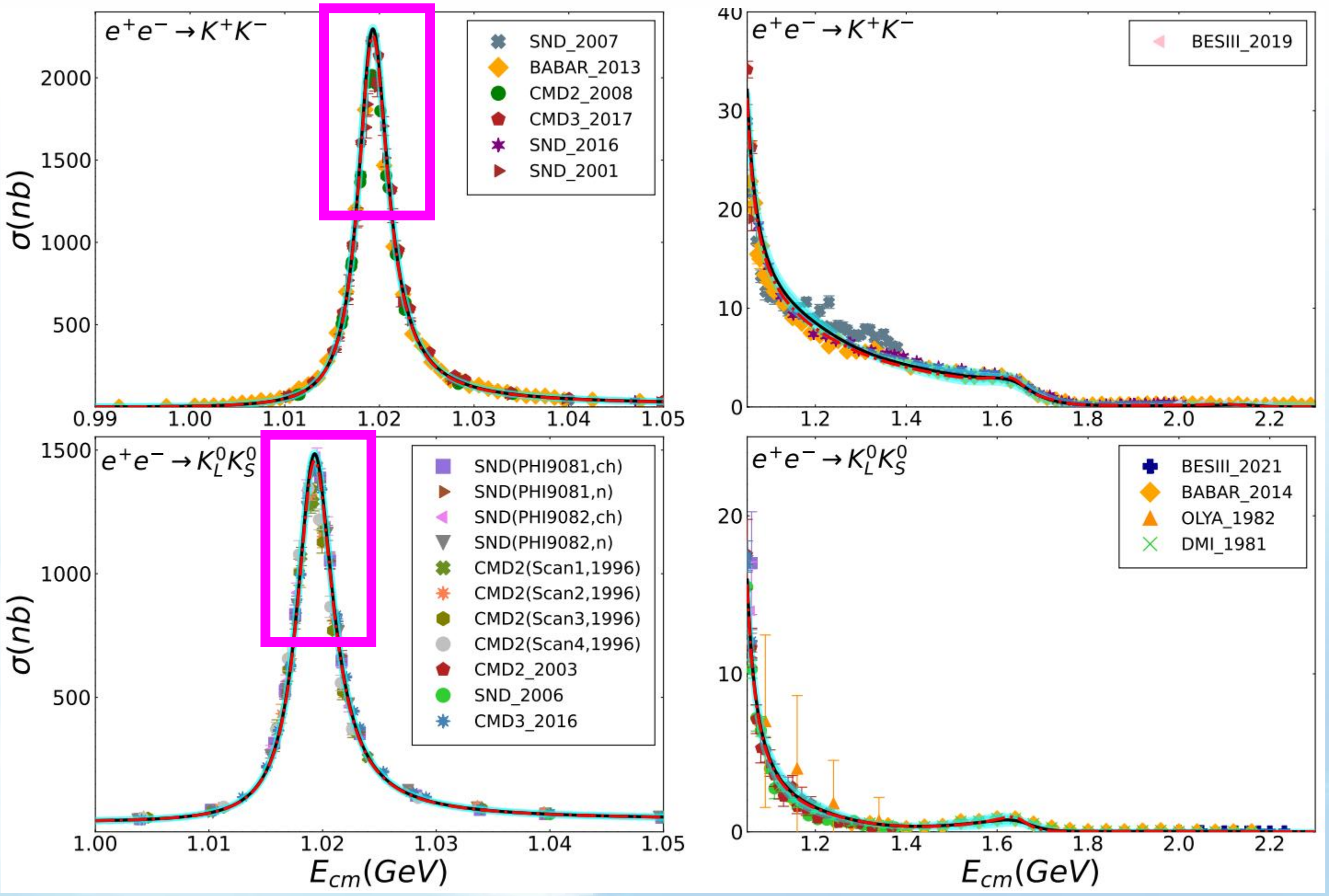
- ππ: Now closer to KLOE and BESIII's
- Latest exp: CMD-3, large discrepancy



Wang, Fang, Dai, JHEP07 (2023) 037

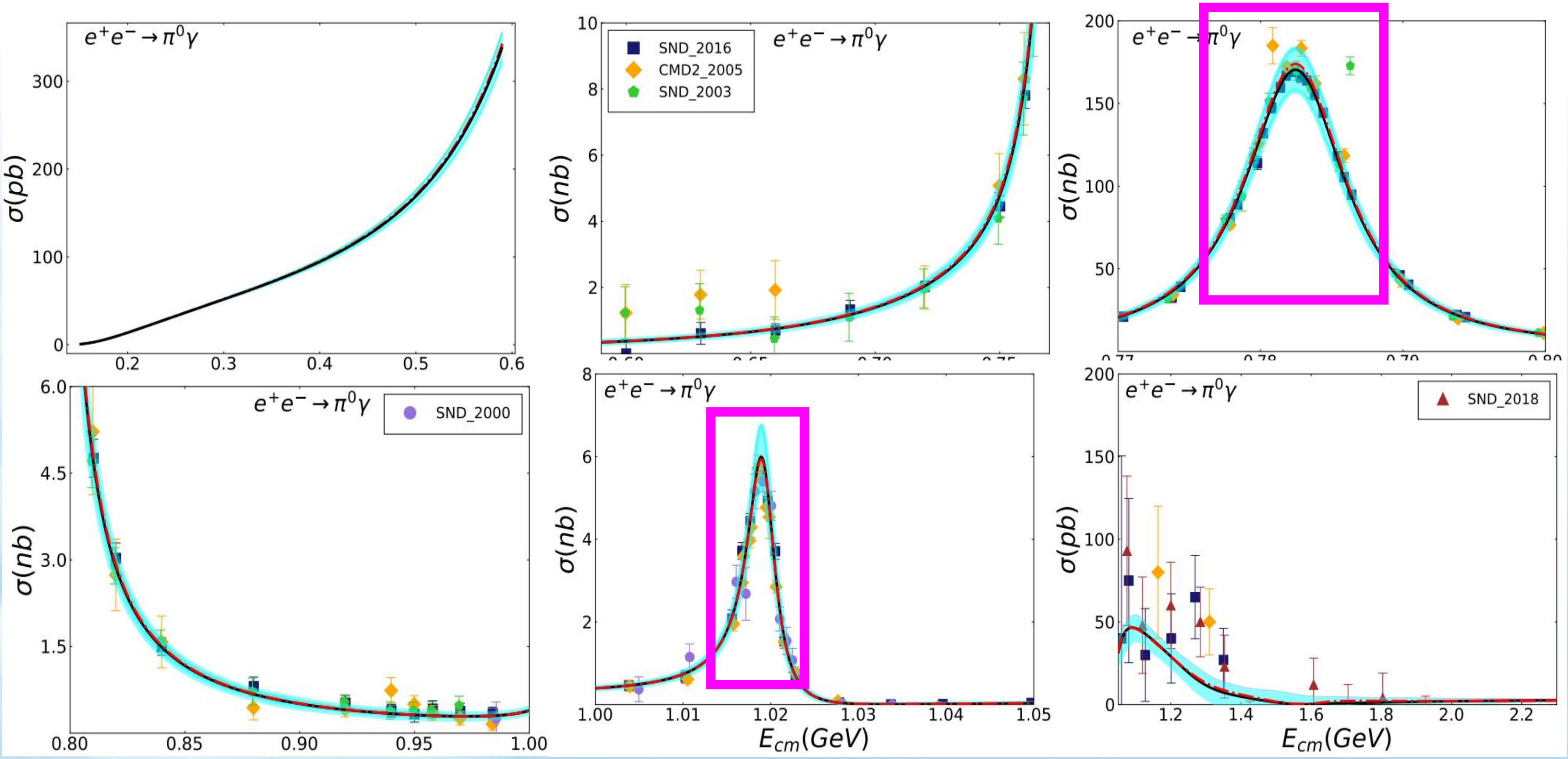
KK

- KK: data in the ϕ 'peak' have large discrepancy
- $K_L K_S$: further direct constraints on $\pi\pi$, KK channels



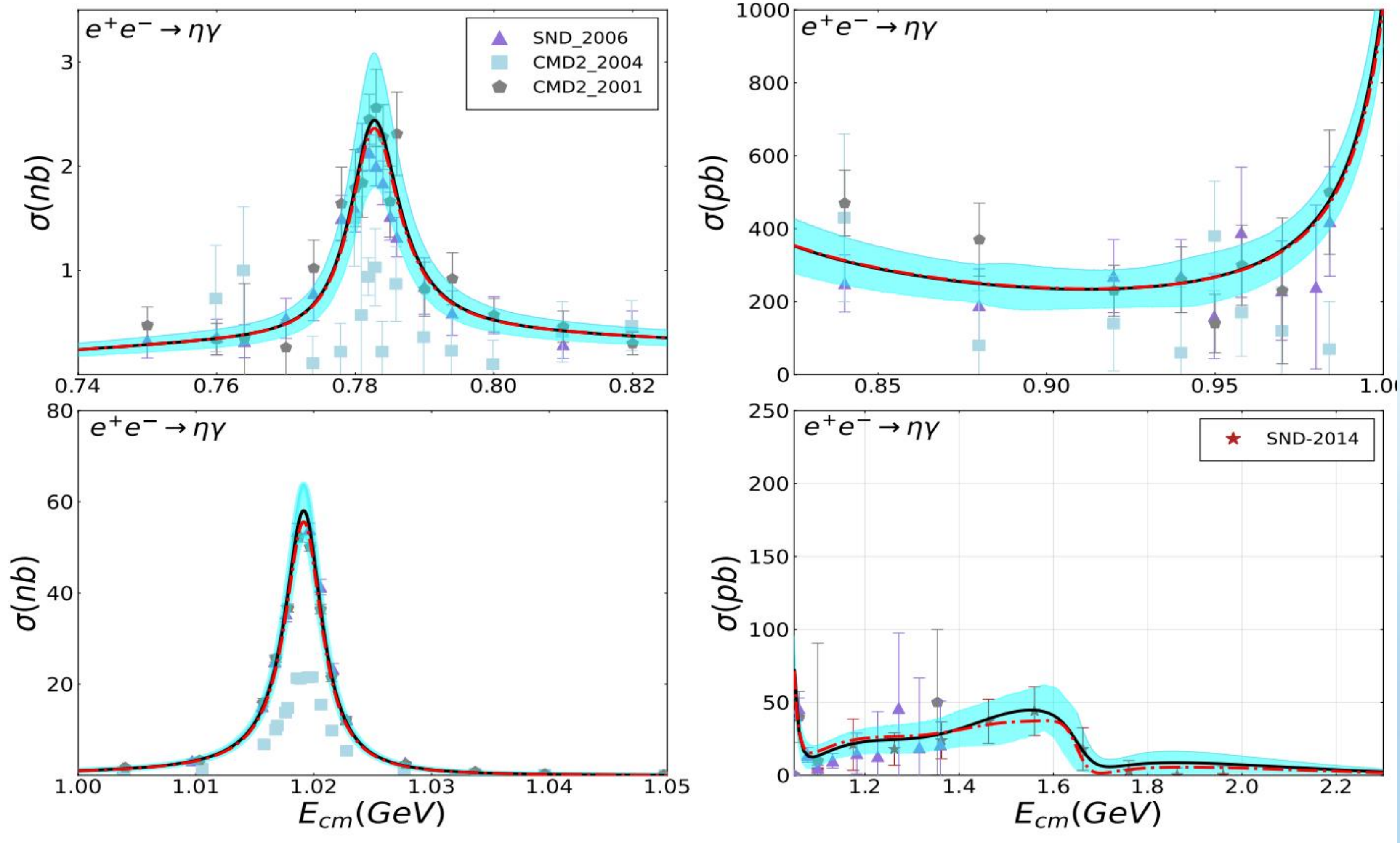
πγ

- πγ: helps to constrain ππ, KK channels: ρ, ω, ϕ



ηγ

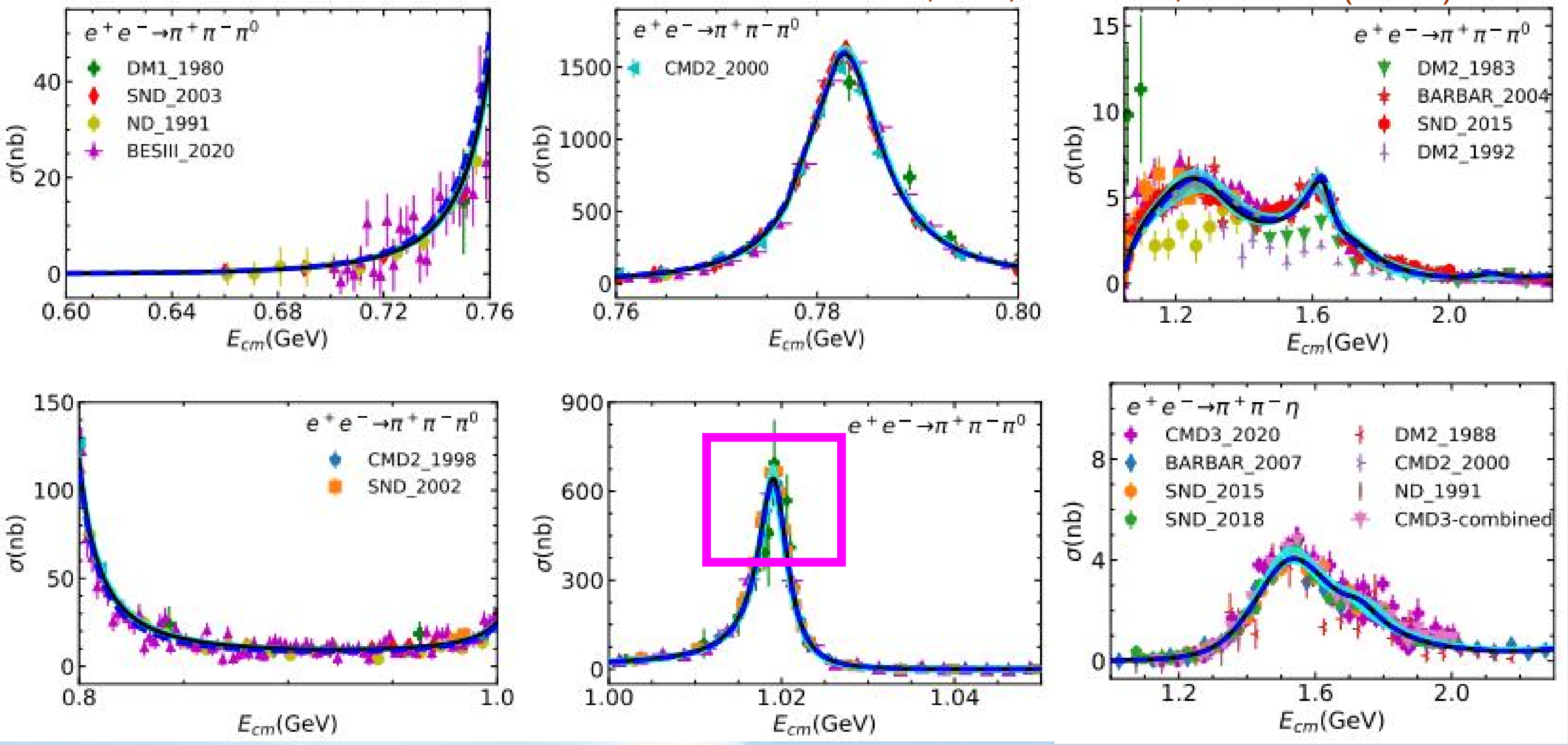
- $\eta\gamma$: helps to constrain KK, and parameters of ρ, ω, ϕ



ππ π, ππη

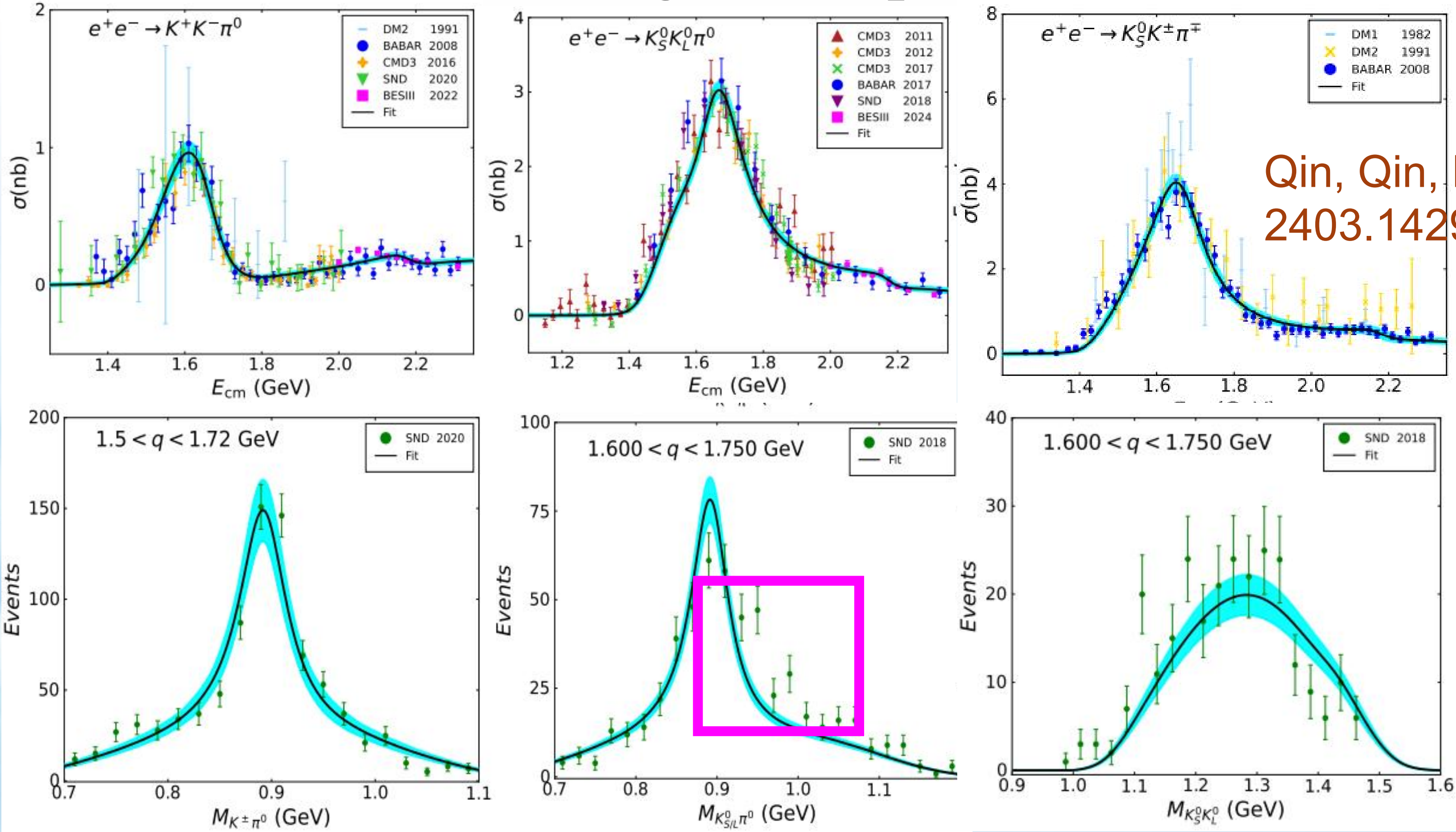
- πππ: needs more precise data in the ω ϕ region
- ππη: check our model

Qin, Dai, Portoles, JHEP03(2021)092



KKπ

- KKπ: angular distributions are helpful to constrain amplitudes
- Three body rescattering can improve it

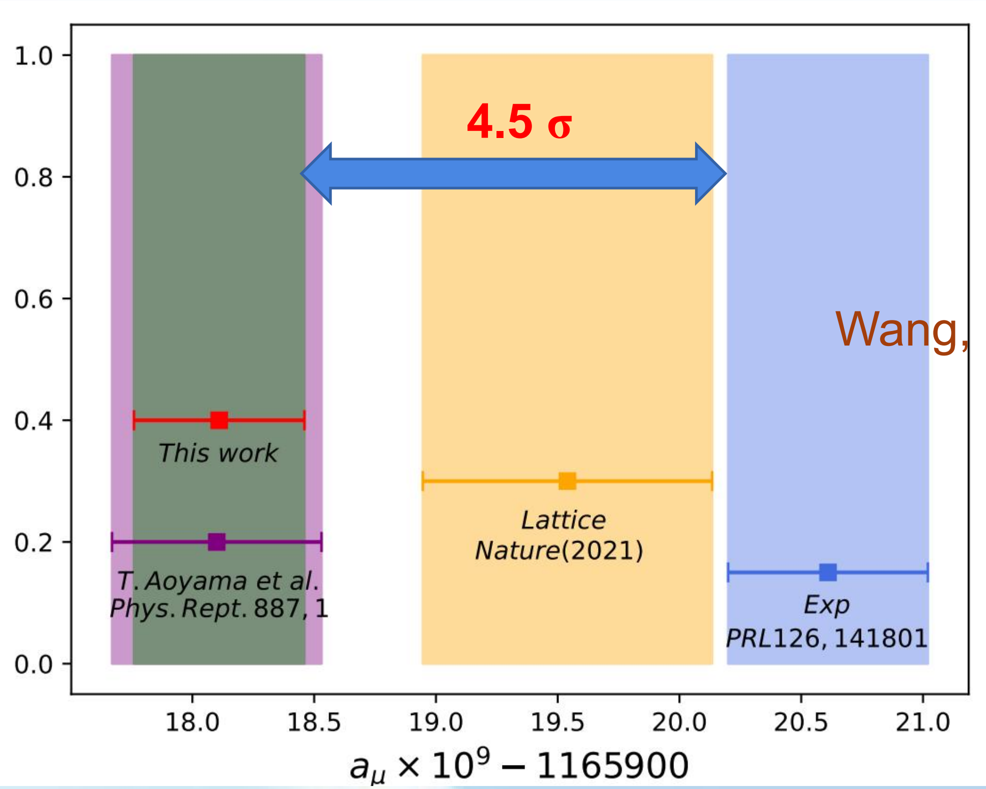


Qin, Qin, Dai, arxiv: 2403.14294 [hep-ph]

Discrepancy

- Ours: $a_\mu = 11659181.1 \pm 3.5 \times 10^{-11}$
- It differs 4.5σ from latest experiment's
  - 3.9σ If HLBL part repleaced with latest LQCD's

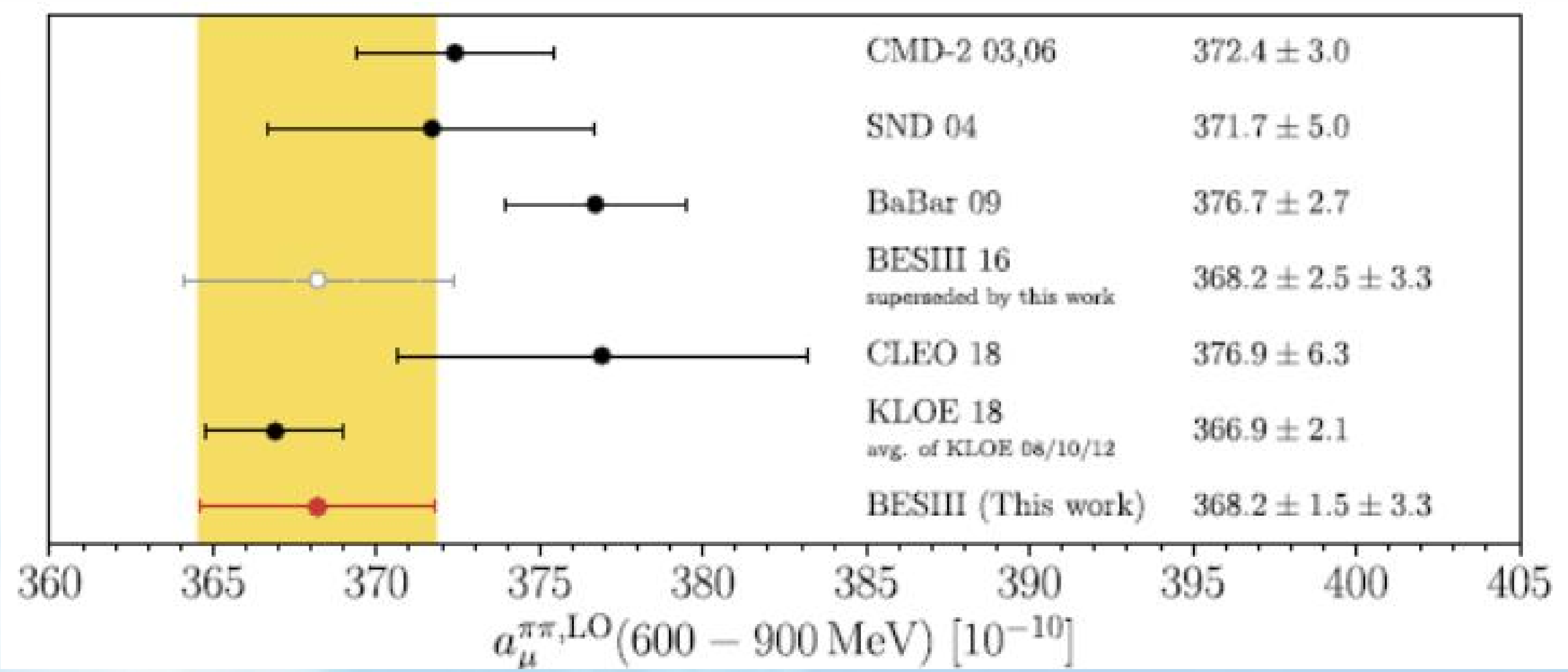
T. Blum, et.al., arxiv:2304.04423 [hep-lat]



Wang, Fang, Dai

Future experiment?

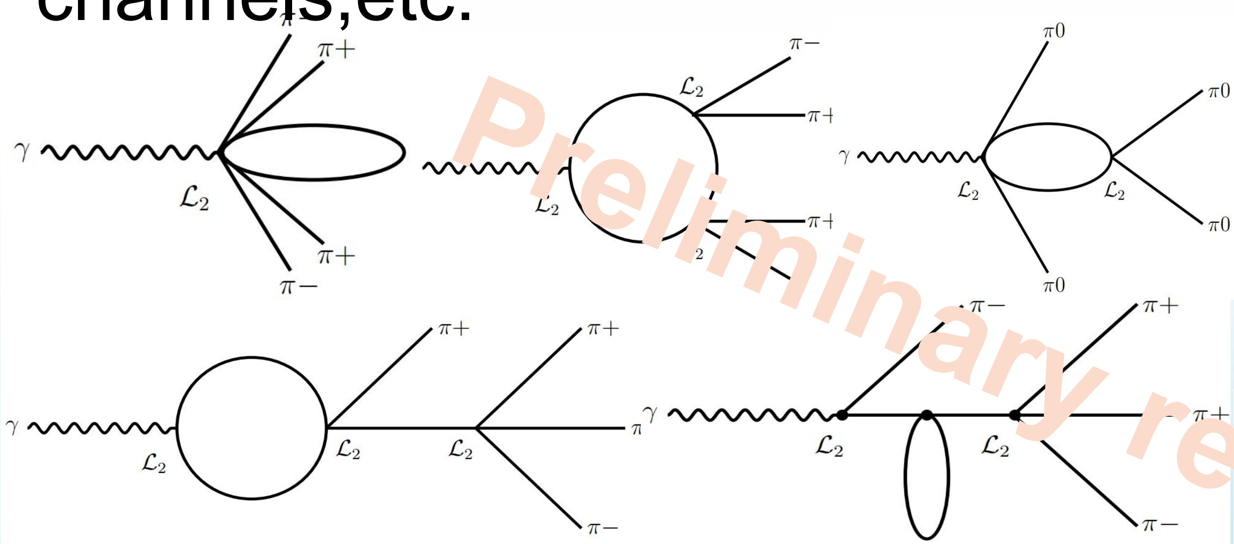
- Future experiments?

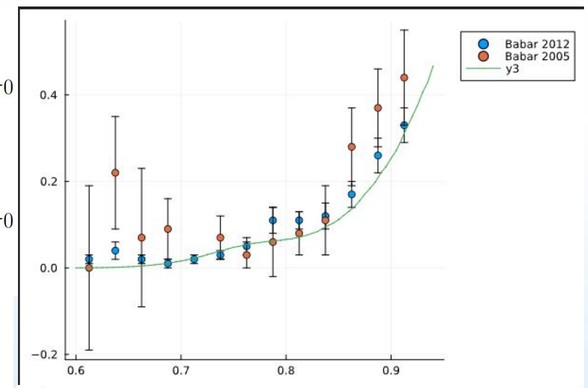


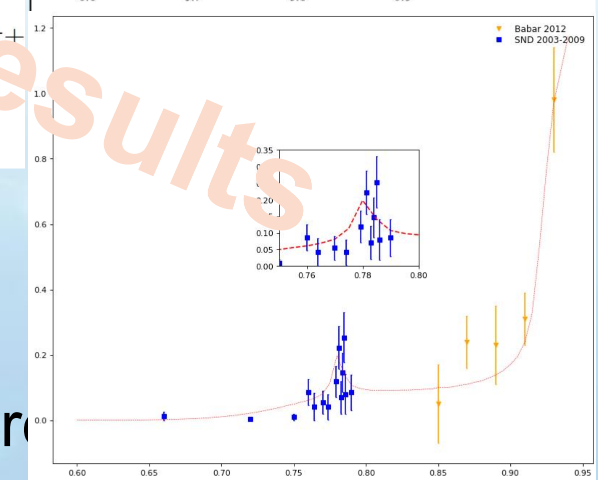
Guangshun Huang, talk at HNU

Four body final states?

Four body final states are important: ππππ, ππKK channels,etc.





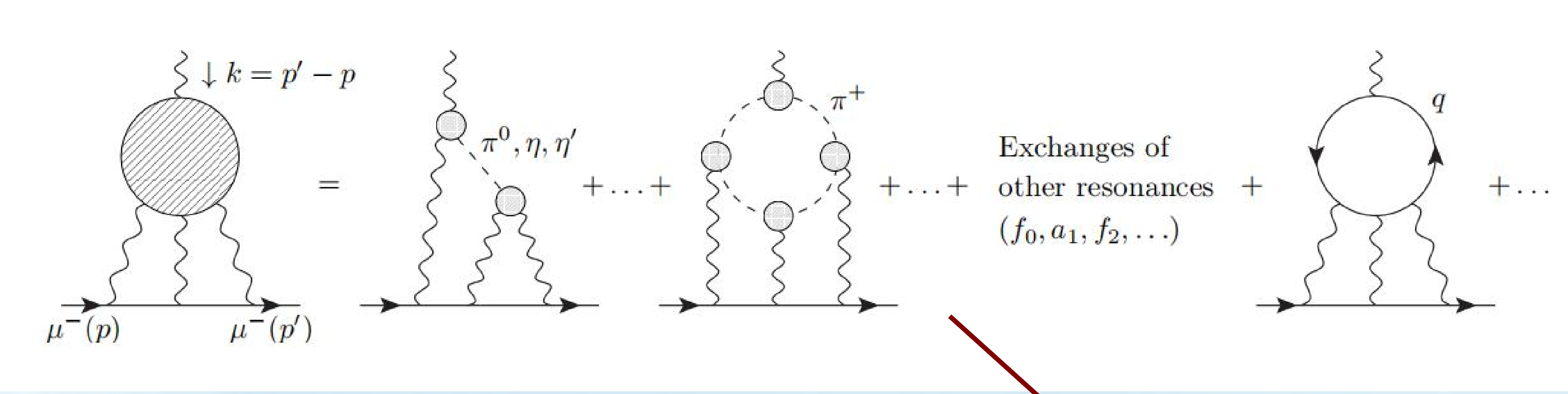


Preliminary results

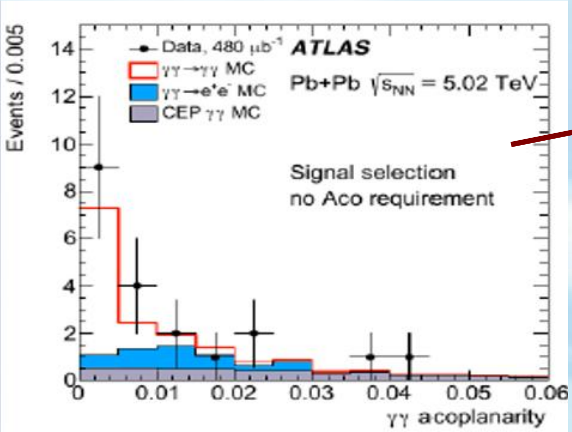
- ChPT's << data, in resonance energy r
- FSI?
- Resonances?

3、HLBL

- $\gamma\gamma^*\to\gamma^*\gamma^*$ has the clean background, a typical example for amplitude analysis



Phys.Rept.887(2020)1



Natrure Phys. 13 (2017) 852,

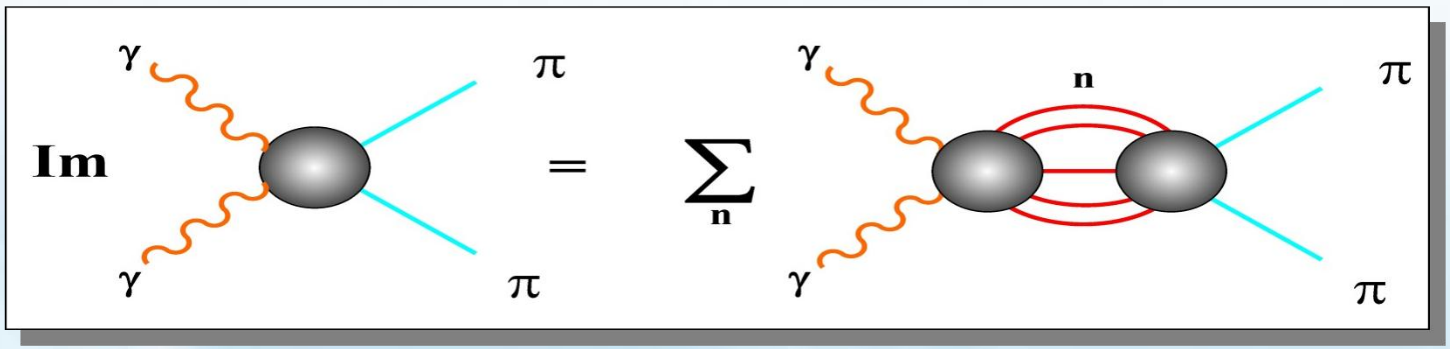
$\gamma\gamma\to\gamma\gamma$, only 13 events

Small yeild, but the result has already been used to set new limits on the Born infeld extension of the SM

Phys. Rev. Lett. 118 (2017) 261802

Building amplitudes

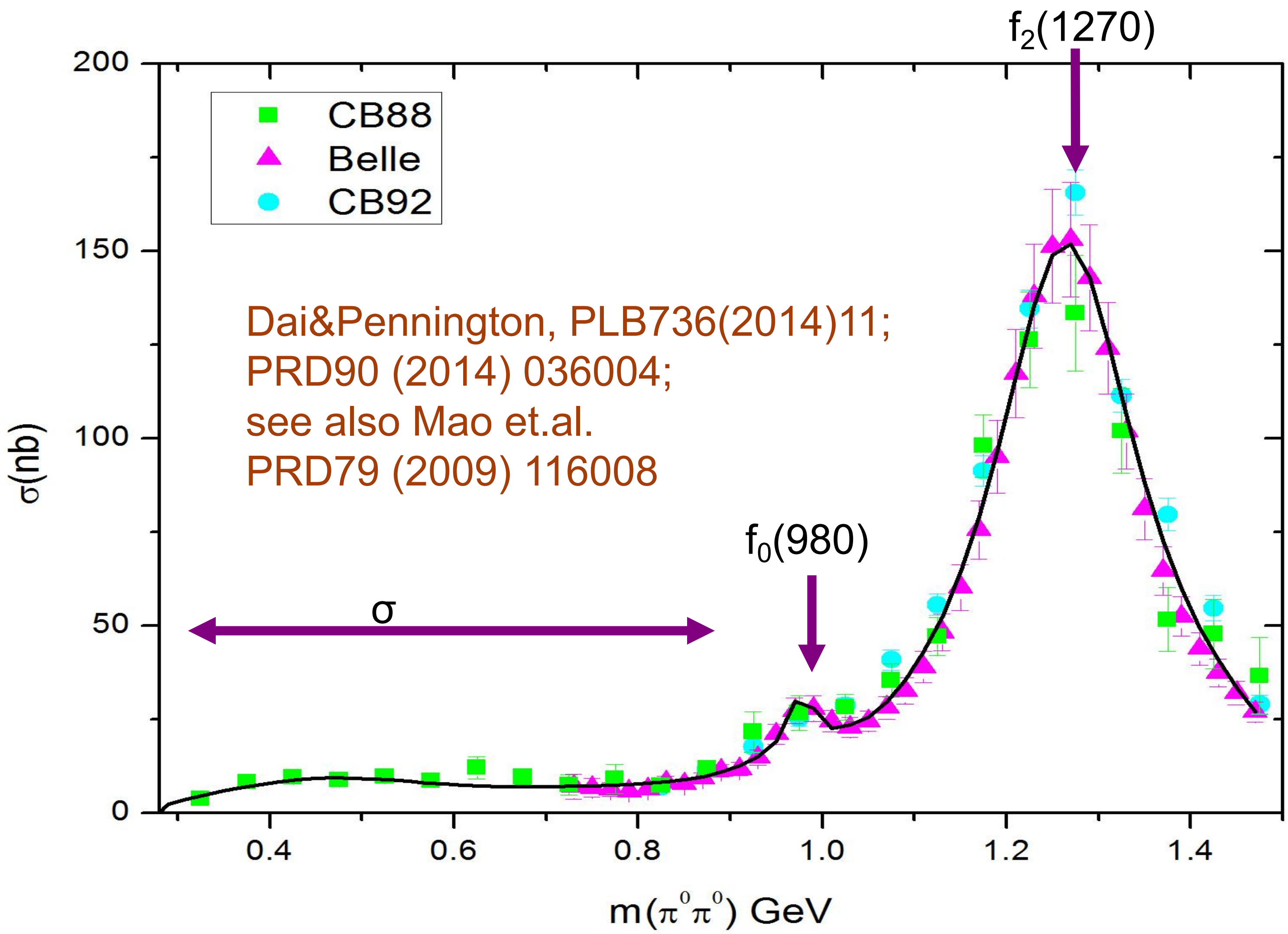
- Final State Interaction Theorem
- Dispersion relations
- ChPT constraints



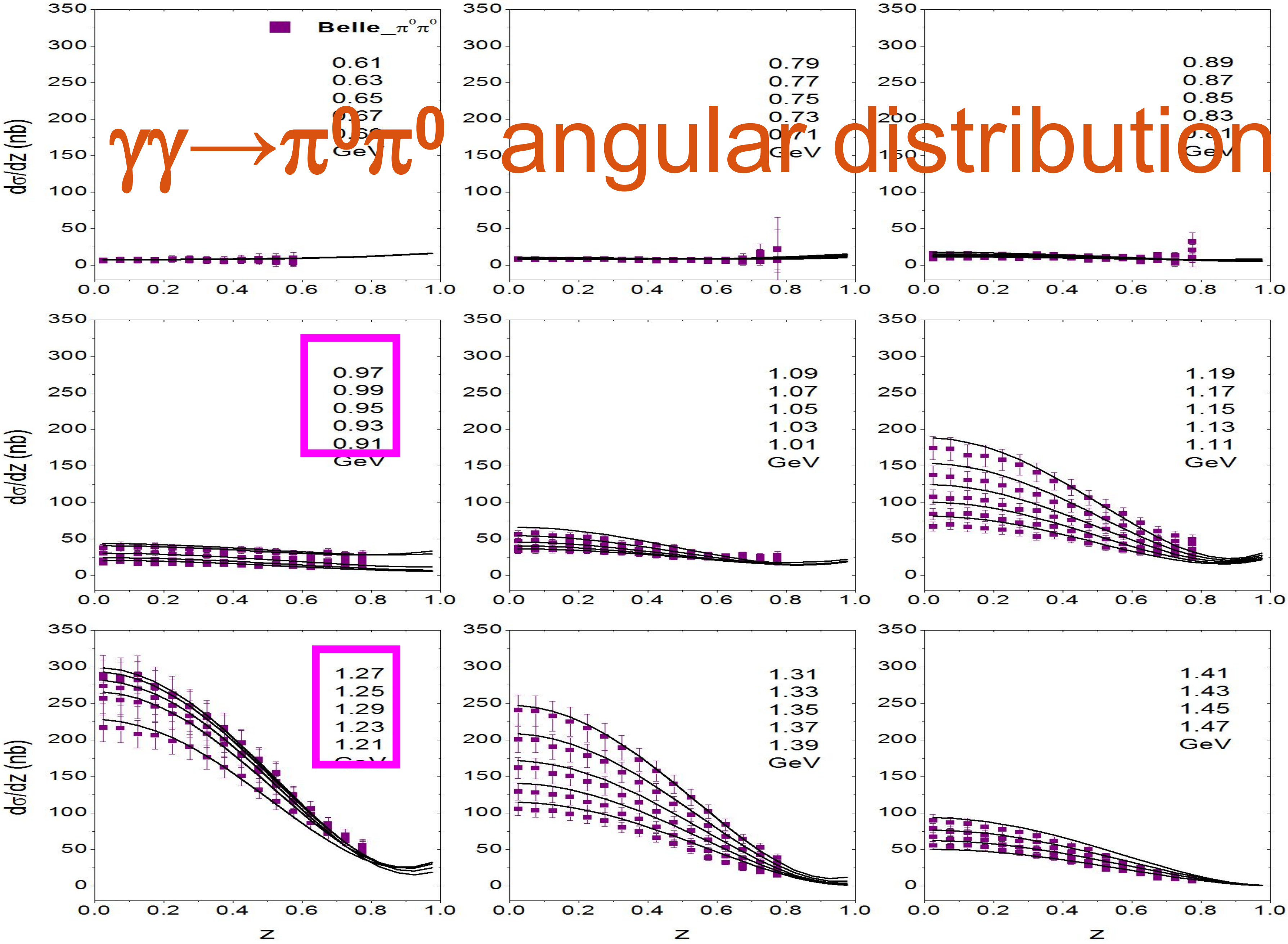
$$\mathcal{F}^I_{00}(s) = \mathcal{B}^I_{00}(s) + b^I s\, \Omega^I_{00}(s) + \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{00}(s')\right] \Omega^I_{00}(s')^{-1}}{s'^2(s'-s)} - \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_R ds' \frac{\mathcal{B}^I_{00}(s')\, \mathrm{Im}\left[\Omega^I_{00}(s')^{-1}\right]}{s'^2(s'-s)}$$

Solved by ChPT

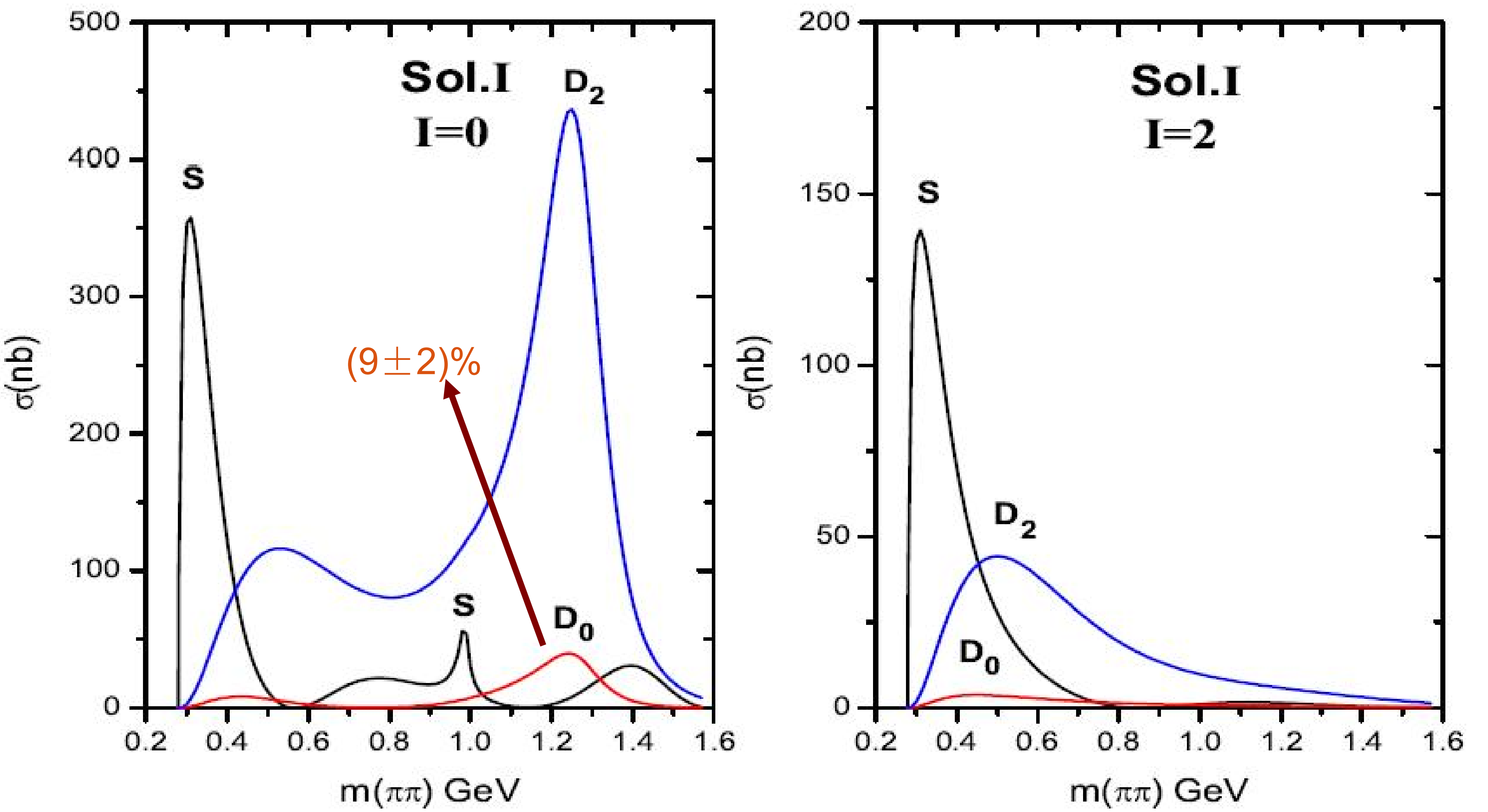
$\gamma\gamma\rightarrow\pi^0\pi^0$ integrated cross section



The angular distribution is helpful to seperate each partial wave.

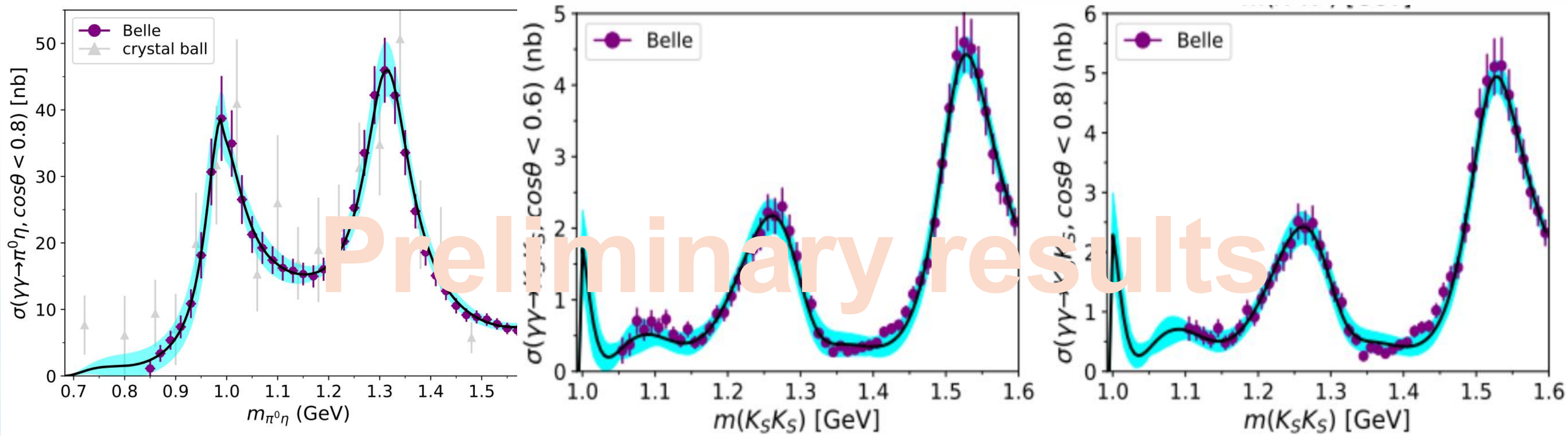


γγ→ππ individual partial waves



Other $\gamma\gamma$ collisions

- πη-KK-πη' coupled channel scatterings



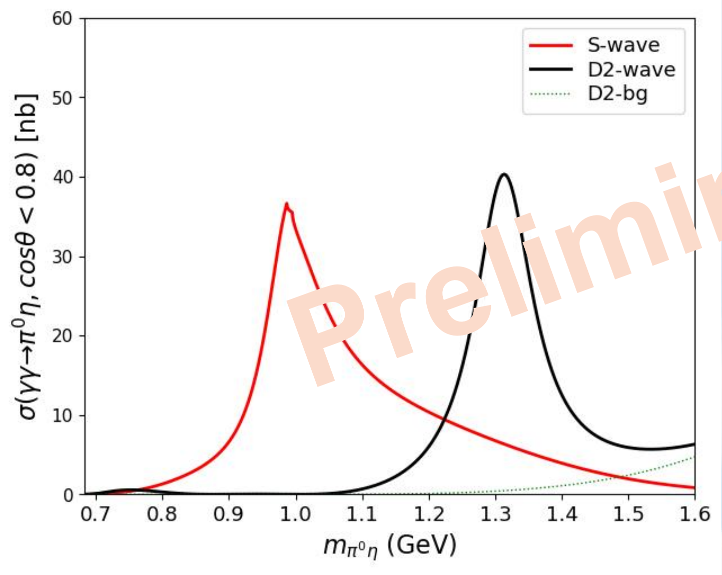
Kuang, Dai *et.al.*, in preparation

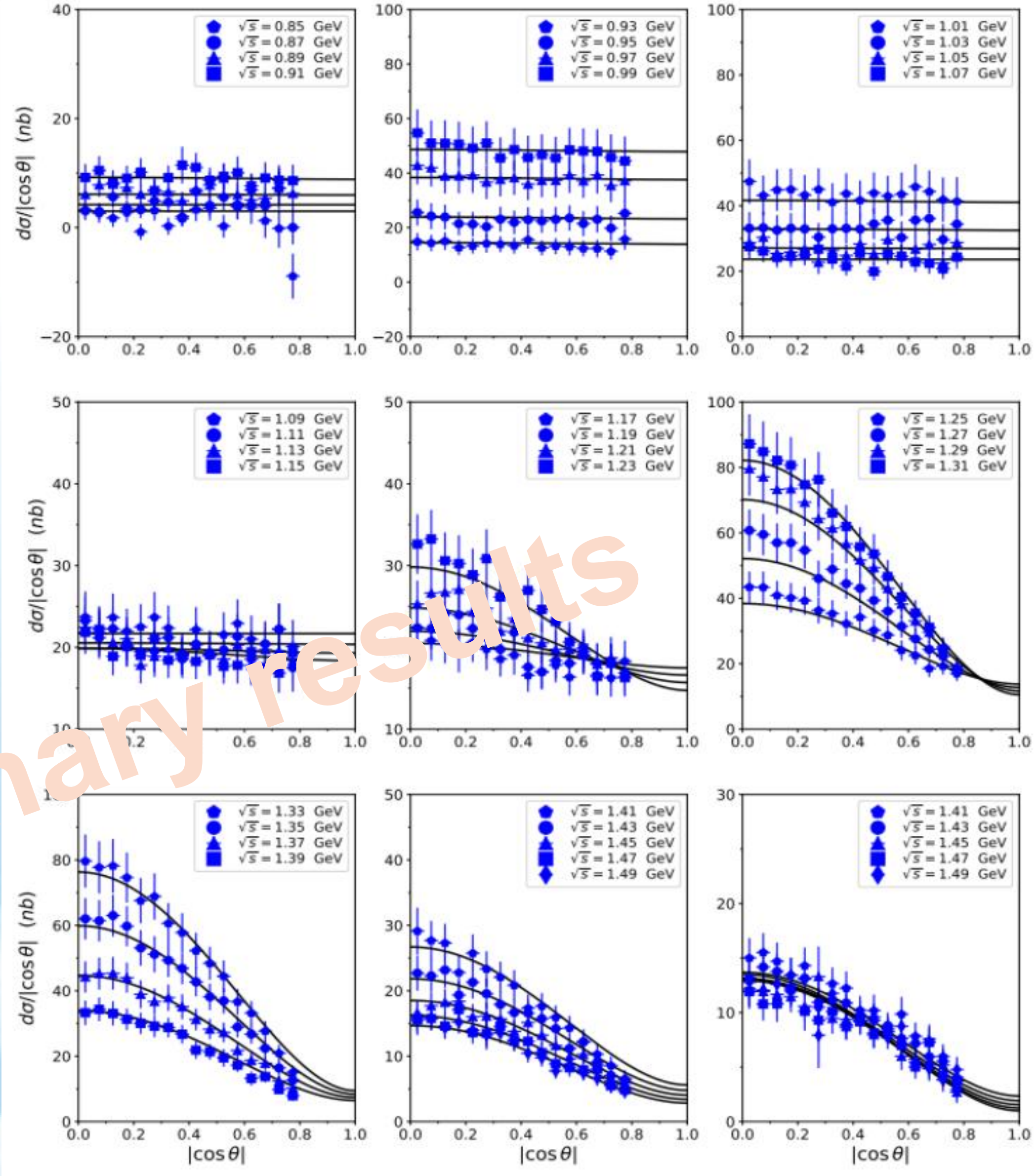
- DR+ChEFT constraints
- AMP: FSI

Experiment	Process	Data-points	$\chi^2_{average}$
Belle/Crystal ball	$\gamma\gamma\to\pi^0\eta$	680	
CB(AGS)/A2 MAMI-B	$\eta\to\pi^0\gamma\gamma$	21	
TPC/Argus/Belle	$\gamma\gamma\to K^+K^-$	18	
TASSO/CELLO	$\gamma\gamma\to\bar{K}^0K^0$	5	
Belle	$\gamma\gamma\to\bar{K}^0_SK^0_S$	315	
BESIII	$\eta'\to\pi^0\gamma\gamma$	13	

angular distribution

- $a_0(980)$?
- HLBL constraints for I=1





Preliminary results

Constraints to light-by-light sumrule

- The contribution to PV sumrule is certainly not zero
- We can refine the Isovector one: It won't change much

evaluation of $\Delta^I(4m_\pi^2, \infty, Z=1)$	$I=0$	$I=1$	$I=2$
$\gamma\gamma \to \pi^0$ [6] (nb)	-	-190.9±4.0	-
$\gamma\gamma \to \eta, \eta'$ [6] (nb)	-497.7±19.3	-	-
$\gamma\gamma \to a_2(1320)$ [6] (nb)	-	*135.0±12±25* †	-
$\gamma\gamma \to \pi\pi$ (nb)	308.0±41.5	-	-44.2±6.1
$\gamma\gamma \to \overline{K}K$ (nb)	23.7±7.5	18.1±4.9	-
SUM (nb)	-166.0±46.4	-37.8±28.4	-44.2±6.1

Dai&Pennington, PRD95 (2017) 056007;

4、Summary

FSI

Amplitude analsysis connects QFT principles and Exp. FSI needs to be considered when performing amplitude analysis.

HVP

Ours has a significant discrepancy with the latest FNAL's. Processes of multi-body channels needs to be studied.

HLBL

We have strong constraints to HLBL amplitudes. 4π's can not be ignored. $\pi\pi\pi\pi$, $\pi\pi KK$?

Next?

Further study of multi-body FSI of light hadrons is neccessary; Discrepancy between LQCD v.s. data driven+ChEFT+FSI?

Thank You For your patience !