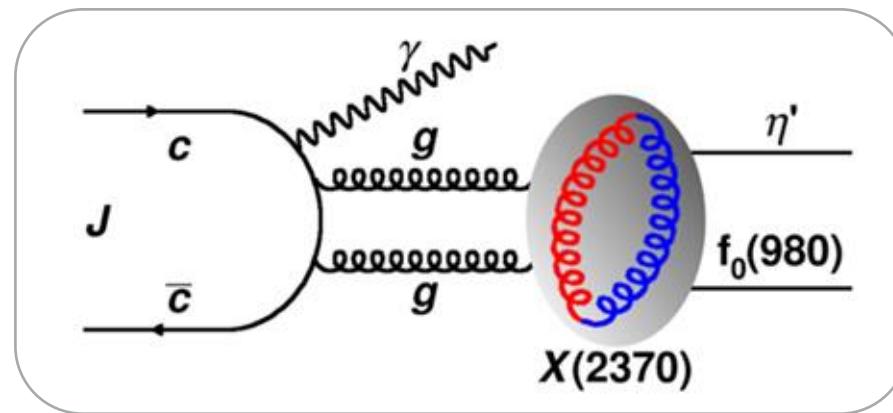




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BESIII

# Discovery of a Glueball-like particle $X(2370)$

第十四届全国粒子物理学学术会议

张鹏 (中国科学院高能物理研究所)

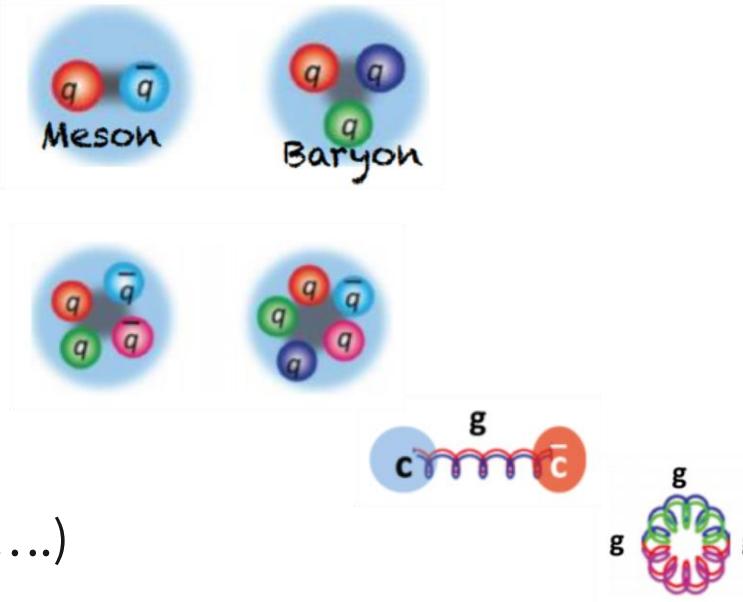
BESIII Collaboration, PRL 132, 181901 (2024)

2024.08.15 山东大学 青岛

# Forms of hadrons

- The basic theory for strong interactions is quantum chromodynamics (QCD). Hadrons are the bound states of QCD
- Hadrons consist of 2 or 3 quarks in conventional quark model

- Mesons (quark-antiquark states)
- Baryons (3-quark states)



- New forms of hadrons allowed by QCD:

- **Multi-quark:** quark number  $\geq 4$
- **Hybrid state:** the mixture of quark and gluon
- **Glueball:** composed of gluons (gg, ggg, gggg ...)

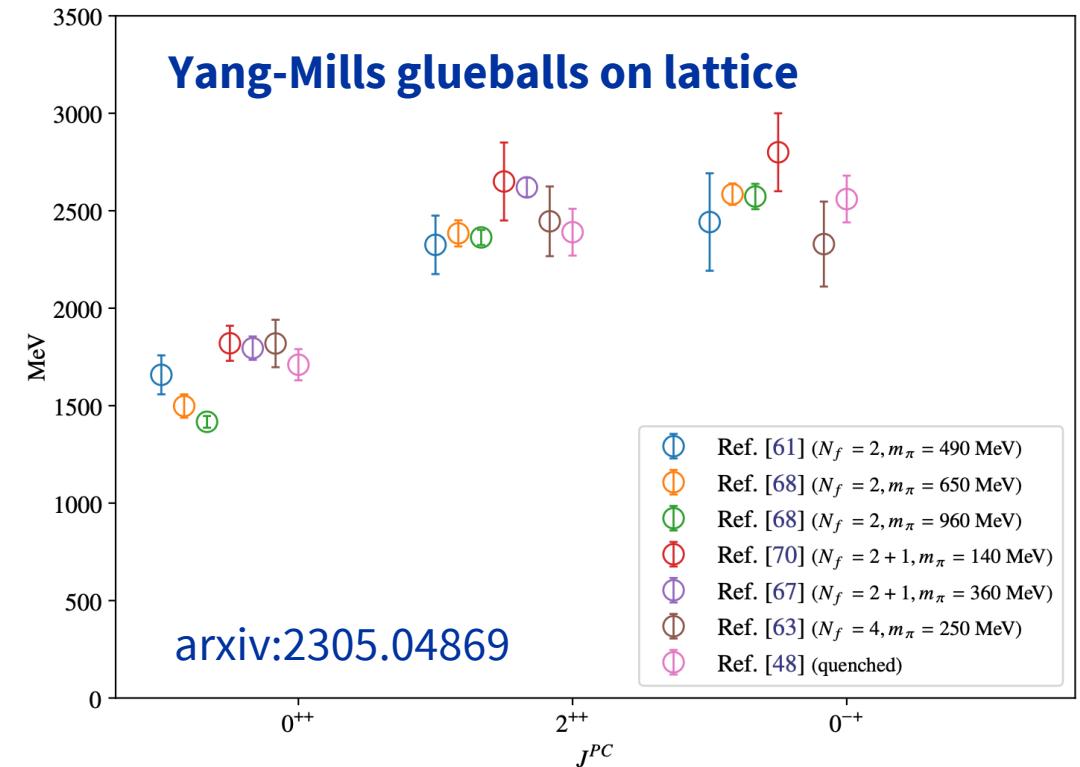
# Glueball

- The self interaction of gluons —— gauge bosons of strong interactions, is unique property of QCD. Glueball formed by self-interacted gluons.

## → Direct test of QCD

- Lattice QCD predictions on glueball masses:
  - $0^{++}$  ground state:  $1.3 \sim 2$  GeV/c $^2$
  - $2^{++}$  ground state:  $2.2 \sim 2.8$  GeV/c $^2$
  - $0^{-+}$  ground state:  $2.3 \sim 2.8$  GeV/c $^2$

Light-mass glueball:  $0^{++}, 2^{++}, 0^{-+}$



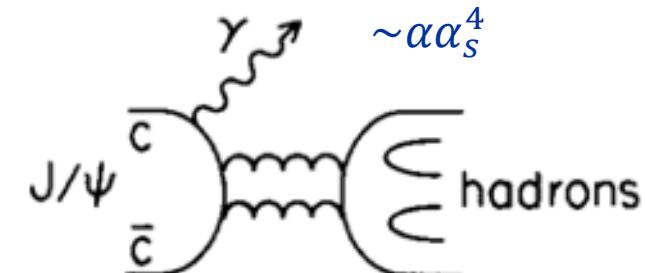
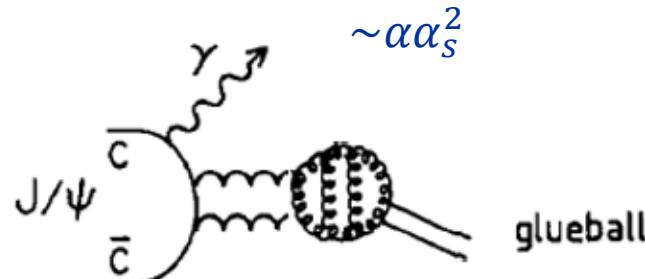
# Glueball Search

- Many experiments searched for glueballs over the past 4 decades.
- Many historical glueball candidates, but also some **difficulties/controversy**.
  - Scalar Glueball candidate ( $0^{++}$ ):  $f_0(1710)$
  - Tensor Glueball candidate ( $2^{++}$ ):  $f_2(2340)$
  - Pseudoscalar Glueball candidate ( $0^{-+}$ ):  $\eta(1405)$



# Glueball Search in J/ $\psi$ radiative decays

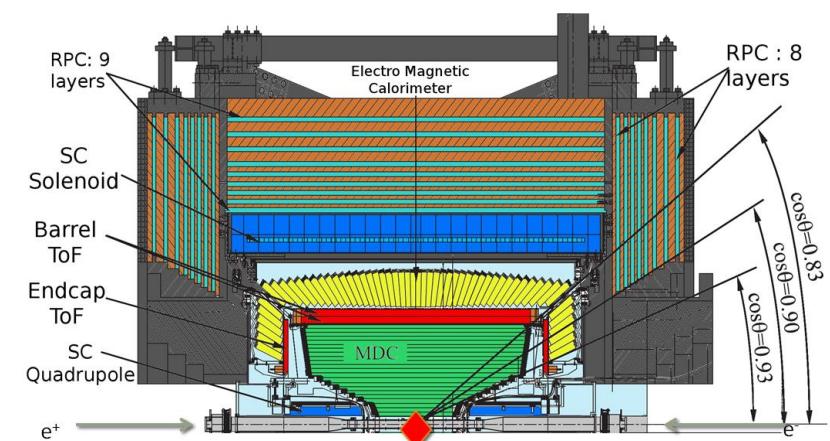
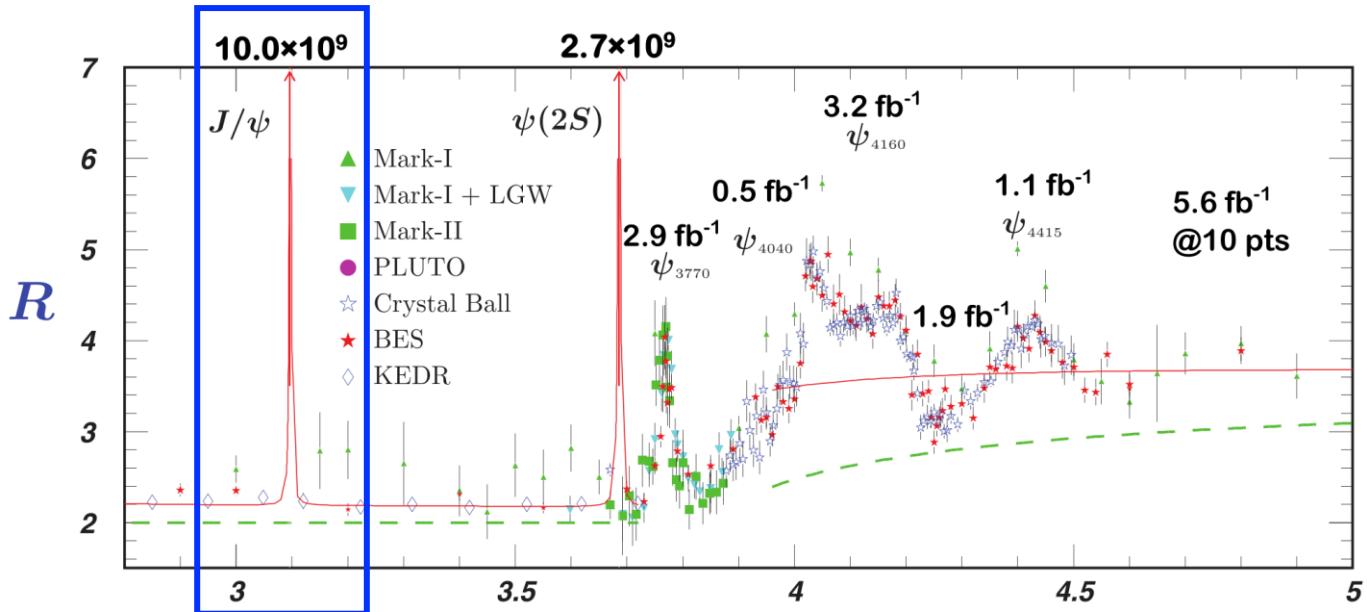
- **Gluon rich environment**
- **Isospin filter:** final states dominated by I=0 processes  $C(X) = +$  for  $J/\psi \rightarrow \gamma X$
- **Spin-parity filter:** C parity must be +, so  $J^{PC} = 0^{-+}, 0^{++}, 1^{++}, 2^{-+}, 2^{++} \dots$
- **Clean environment** in electron-positron collision
  - very different from proton-antiproton collision
- **Glueballs are expected to be richly produced** in J/ $\psi$  radiative decays
  - Glueball production rate in J/ $\psi$  radiative decays could be higher than normal hadrons



→ J/ $\psi$  radiative decays are believed to be an ideal place to search for glueballs.

# BESIII data collections

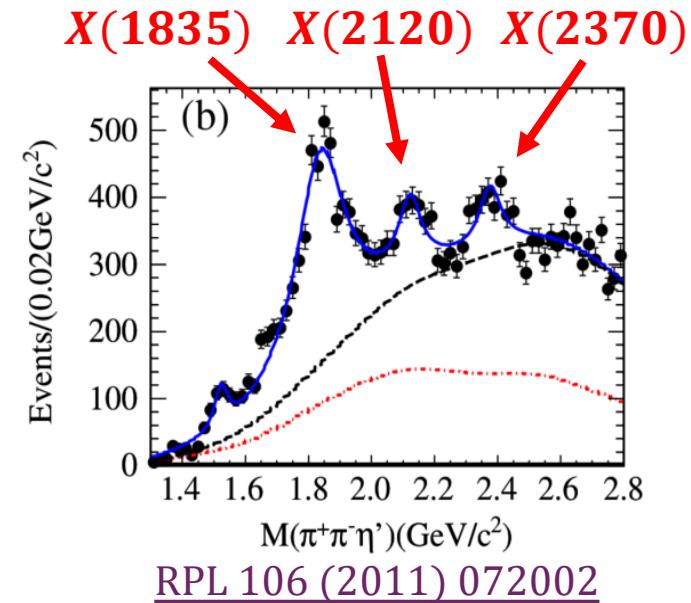
World largest J/ $\psi$  data sample : ~10 billion.  
Provide a good opportunity to search for glueball



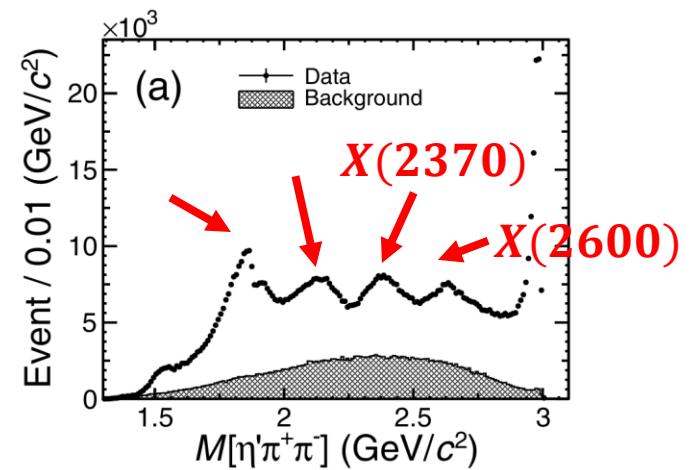
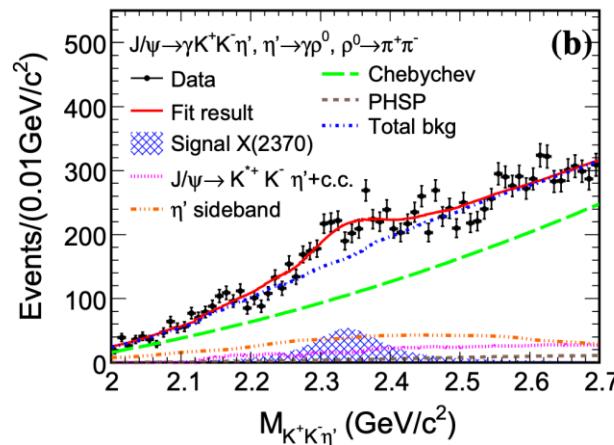
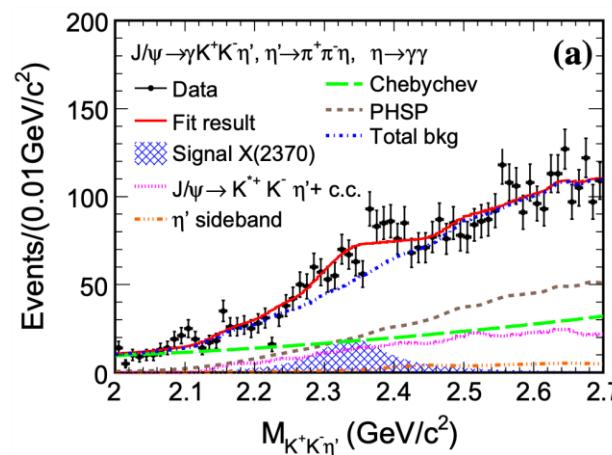
# $X(2370)$

- First observation by BESIII of the  $X(2370)$  in  $J/\psi \rightarrow \gamma\pi^+\pi^-\eta'$

	$M(\text{MeV}/c^2)$	$\Gamma(\text{MeV})$	Sig.
X(1835)	$1836.5 \pm 3.0^{+5.6}_{-2.1}$	$190.1 \pm 9.0^{+38}_{-36}$	$>20\sigma$
X(2120)	$2122.4 \pm 6.7^{+4.7}_{-2.7}$	$83 \pm 16^{+31}_{-11}$	$7.2\sigma$
X(2370)	$2376.3 \pm 8.7^{+3.2}_{-4.3}$	$83 \pm 17^{+44}_{-6}$	$6.4\sigma$



- Confirmed by BESIII in  $J/\psi \rightarrow \gamma K\bar{K}\eta'$  (new mode),  $\gamma\pi^+\pi^-\eta'$



# **$X(2370)$ --- good candidate of $0^{-+}$ glueball**

- Its mass is consistent with LQCD prediction on the lightest  $0^{-+}$  glueball
- Observed in the best place to search for the  $0^{-+}$  glueball:
  - in  $J/\psi$  radiative decays
  - Flavor symmetric decay modes of  $\pi^+\pi^-\eta'$  and  $K\bar{K}\eta'$  —— favorite decay modes of  $0^{-+}$  glueball
- Determination of its spin-parity is crucial

# Spin-Parity determination of the $X(2370)$ in $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta'$

Journal: PRL 132, 181901 (2024)

## Advantages of this channel:

- ~10B clean  $J/\psi$  events
- Almost no background:
  - Possible dominant background processes of  $J/\psi \rightarrow \pi^0 K_S^0 K_S^0 \eta'$  and  $J/\psi \rightarrow K_S^0 K_S^0 \eta'$  are forbidden by exchange symmetry and C-parity conservation.
- High efficiency and precise resolution of charged particles and photons:
  - good reconstruction for  $K_S^0$  and  $\eta$
  - good reconstruction for  $\eta'$  for two dominant decay modes of  $\eta' \rightarrow \gamma \pi^+ \pi^-$  and  $\eta' \rightarrow \pi^+ \pi^- \eta$

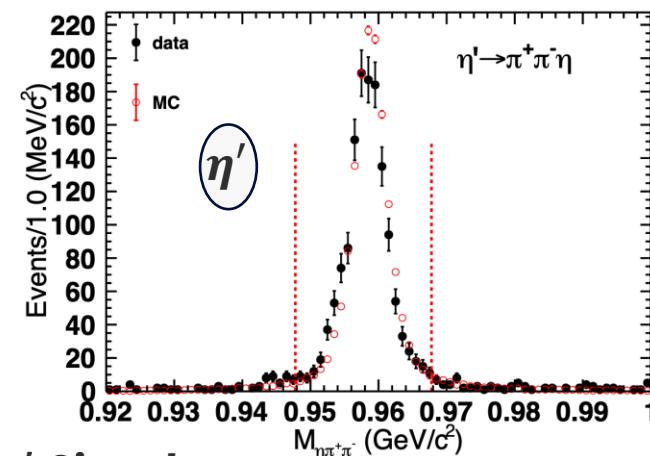
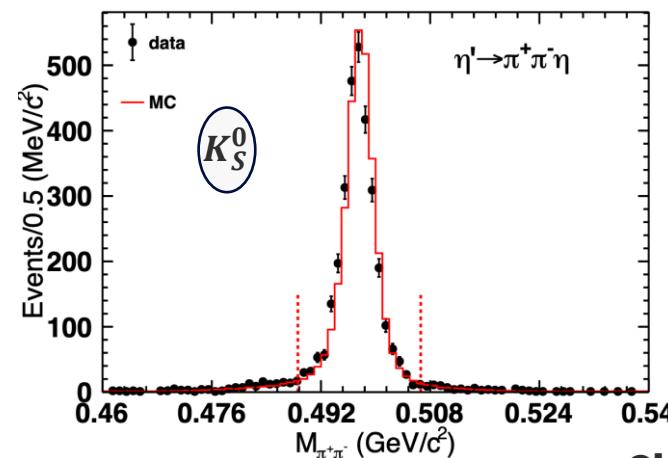
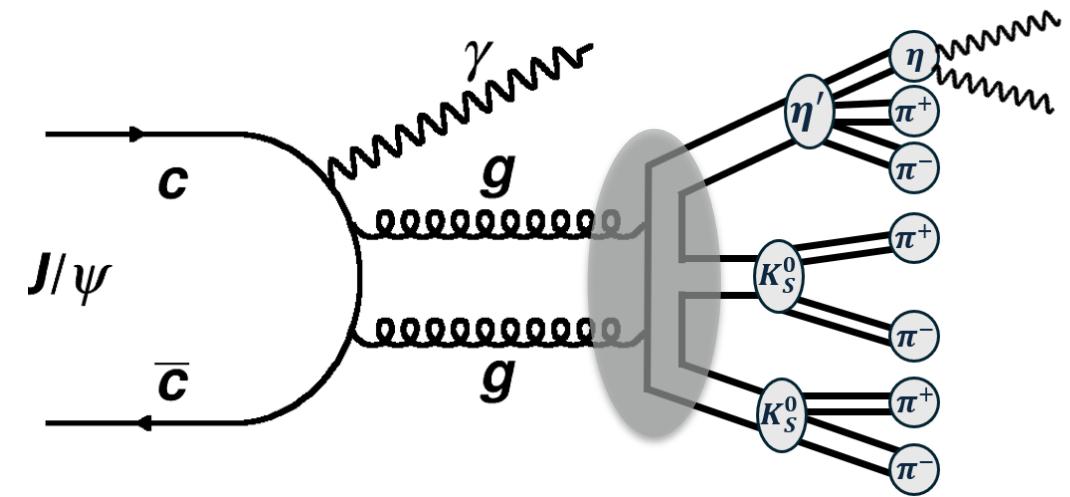
# Selection for $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta', \eta' \rightarrow \pi^+ \pi^- \eta$

## ➤ Signal selection:

- At least 3 charged pairs + 3 photons
- Constraint kinematic fit with energy-momentum conservation
- $K_S^0$  reconstruction:  $|M_{\pi^+ \pi^-} - M_{K_S^0}| < 9 \text{ MeV}/c^2$
- $\eta'$  reconstruction:  $|M_{\pi^+ \pi^- \eta} - M_{\eta'}| < 10 \text{ MeV}/c^2$

## ➤ Background veto:

- $\pi^0$  veto:  $|M_{\gamma\gamma} - M_{\pi^0}| > 20 \text{ MeV}/c^2$



Clean  $K_S^0$  and  $\eta'$  Signal



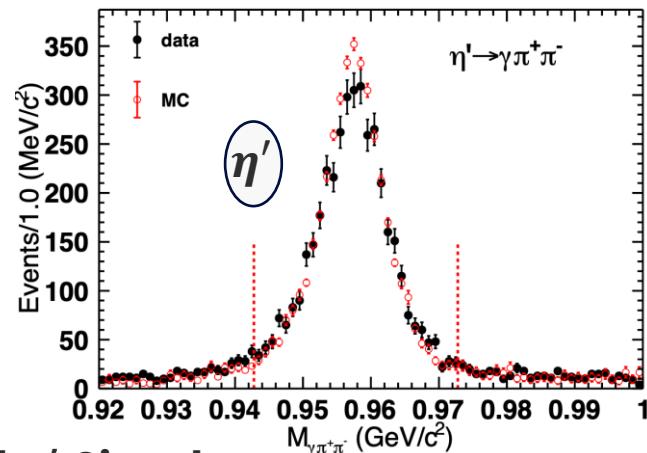
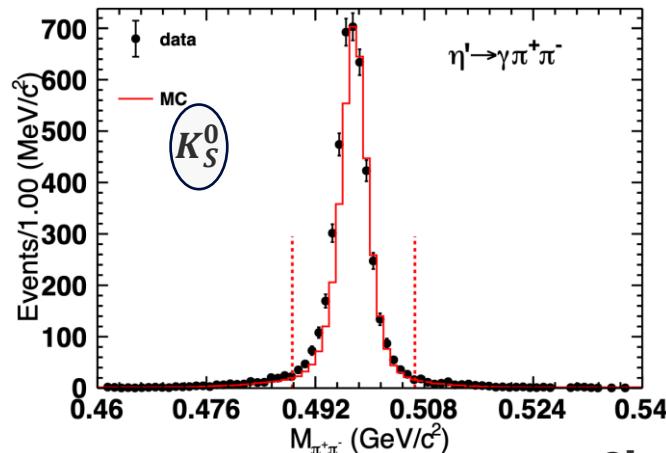
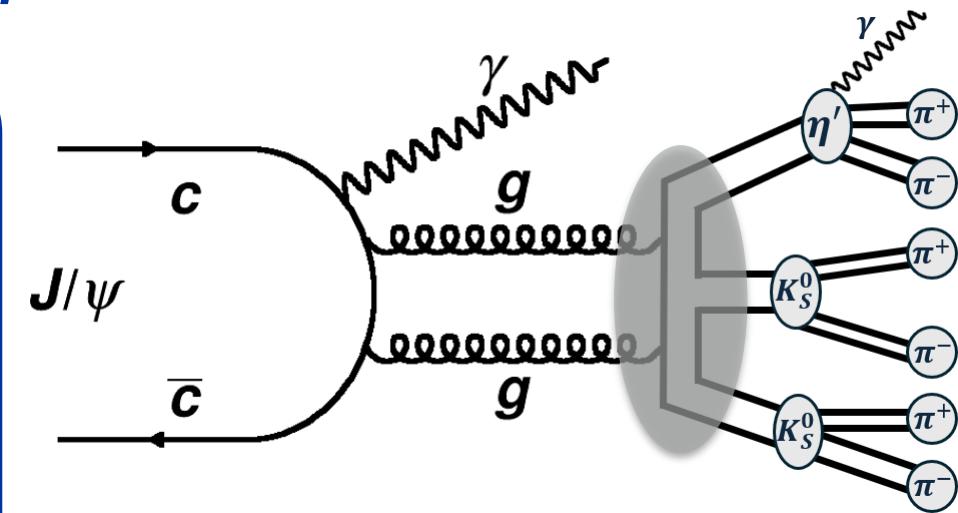
# Selection for $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta'$ , $\eta' \rightarrow \gamma \pi^+ \pi^-$

## ➤ Signal selection:

- At least 3 charged pairs + 2 photons
- Constraint kinematic fit with energy-momentum conservation
- $K_S^0$  reconstruction:  $|M_{\pi^+\pi^-} - M_{K_S^0}| < 9 \text{ MeV}/c^2$
- $\eta'$  reconstruction:  $|M_{\gamma\pi^+\pi^-} - M_{\eta'}| < 15 \text{ MeV}/c^2$

## ➤ Background veto:

- $\pi^0/\eta$  veto:  $|M_{\gamma\gamma} - M_{\pi^0}| > 20 \text{ MeV}/c^2$ ,  $|M_{\gamma\gamma} - M_\eta| > 30 \text{ MeV}/c^2$



Clean  $K_S^0$  and  $\eta'$  Signal

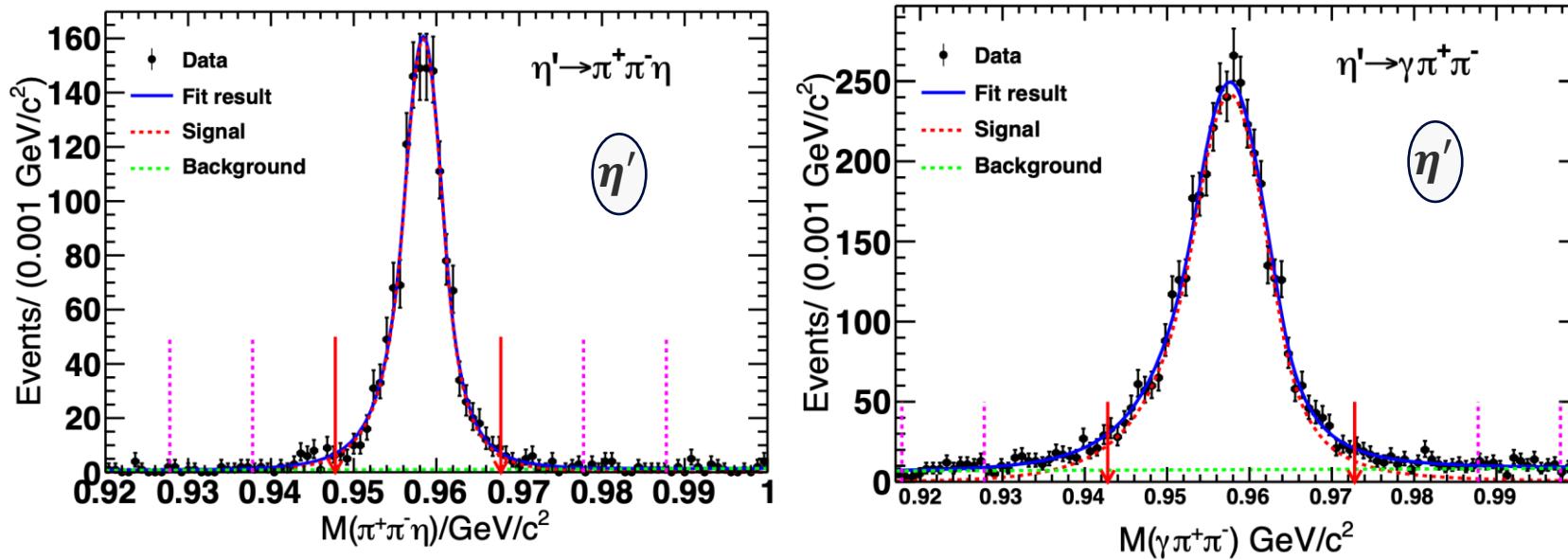


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# Background estimation

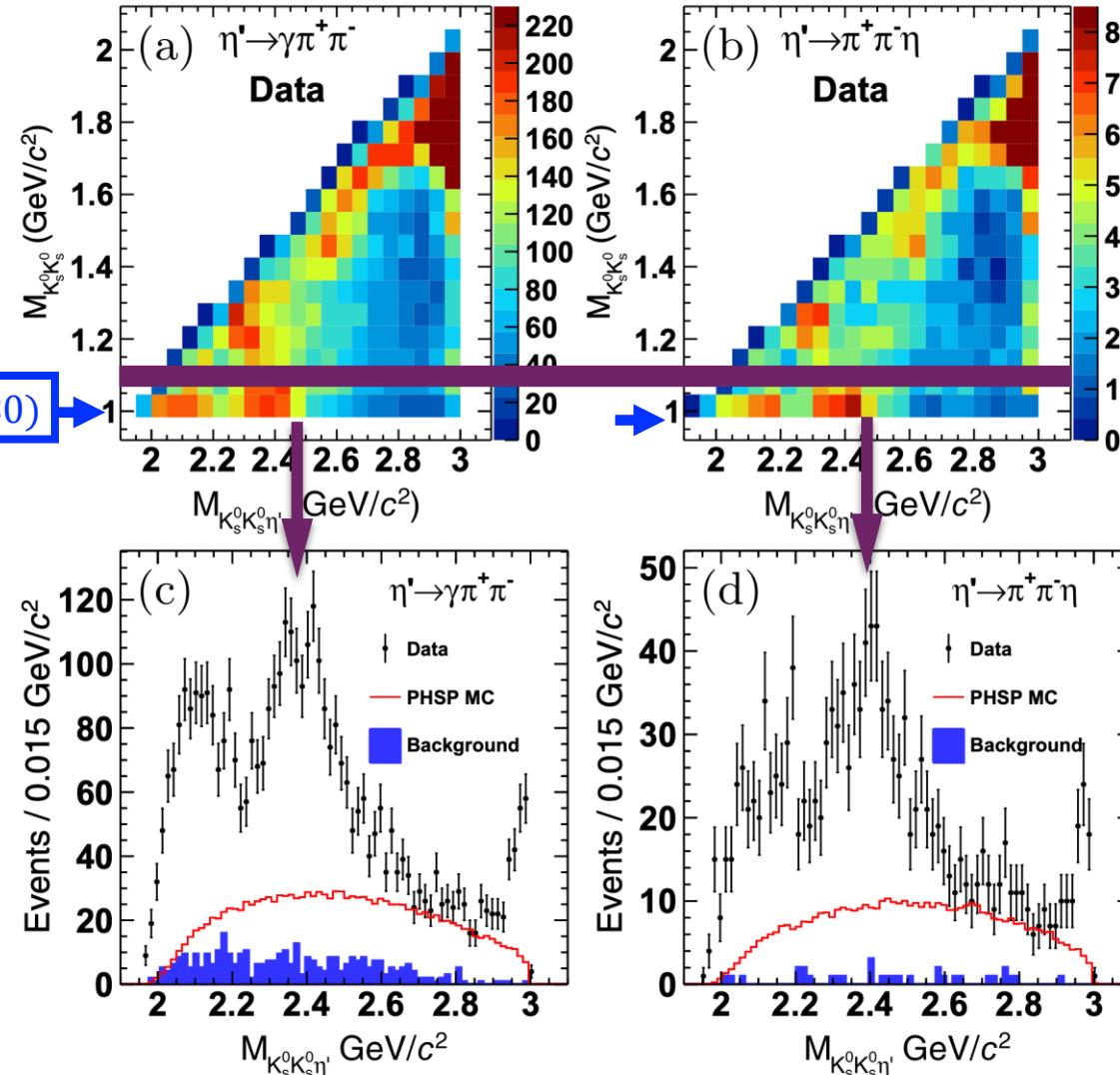
- Negligible mis-combination for  $K_S^0$  reconstruction ( $<0.1\%$ )
- No background from  $J/\psi \rightarrow \pi^0 K_S^0 K_S^0 \eta'$ : further validation directly from data
- Little background from non- $\eta'$  processes: estimated directly from  $\eta'$  mass sideband region:
  - No peaking background
  - Non- $\eta'$  background fraction: 1.8% for  $\eta' \rightarrow \pi^+ \pi^- \eta$ ; 6.8% for  $\eta' \rightarrow \gamma \pi^+ \pi^-$



Spin-Parity determination benefits from low background level



# Mass spectra after final selection

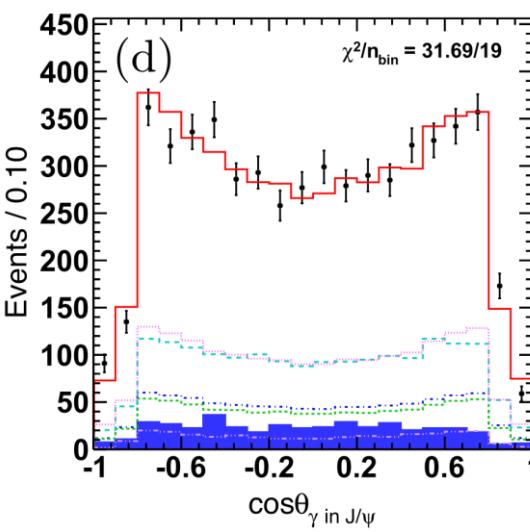
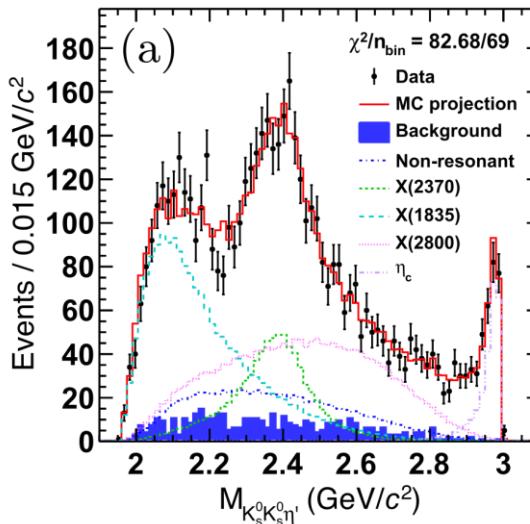


PRL 132 (2024) 181901

- Similar structures in  $\eta' \rightarrow \pi^+\pi^-\eta$  and  $\eta' \rightarrow \gamma\pi^+\pi^-$  modes
  - Evident  $f_0(980)$  in  $K_S^0 K_S^0$  mass threshold
  - A clear connection between the  $f_0(980)$  and  $X(2370)$
- $f_0(980)$  in selection with  $M_{K_S^0 K_S^0} < 1.1 \text{ GeV}/c^2$ 
  - Clear signal of the  $X(2370)$  and  $\eta_c$
  - Reduce PWA complexities from additional intermediate processes



# PWA Fit



PRL 132 (2024) 181901

- Best fit can well describe the data including resonances ( $>5\sigma$ ):  $X(1835), X(2370), X(2800), \eta_c$
- Spin-parity of the  $X(2370)$  is determined to be  $0^{-+}$  with significance larger than  $9.8\sigma$  w.r.t. other  $J^{PC}$  assumptions
- $X(2800)$ : a broad structure for the effective contributions from possible high mass resonances

state	$J^{PC}$	Decay mode	Mass (MeV/ $c^2$ )	Width (MeV)	Significance
$X(2370)$	$0^{-+}$	$f_0(980)\eta'$	$2395^{+11}_{-11}$	$188^{+18}_{-17}$	$14.9\sigma$
$X(1835)$	$0^{-+}$	$f_0(980)\eta'$	1844	192	$22.0\sigma$
$X(2800)$	$0^{-+}$	$f_0(980)\eta'$	$2799^{+52}_{-48}$	$660^{+180}_{-116}$	$16.4\sigma$
$\eta_c$	$0^{-+}$	$f_0(980)\eta'$	2983.9	32.0	$> 20.0\sigma$
PHSP	$0^{-+}$	$\eta'(K_S^0 K_S^0)_{S-wave}$	—	—	$9.0\sigma$
		$\eta'(K_S^0 K_S^0)_{D-wave}$	—	—	$16.3\sigma$



# PWA Validations

- Intermediate process: significance  $< 3\sigma$  and impact is ignored
  - $J^{PC}$  and decay modes for each components:  $f_0(1500)\eta'$ ,  $f_2(1270)\eta'$ ,  $K^*(1410)K_S^0$ ,  $K_0^*(1430)K_S^0$ ,  $K_2^*(1430)K_S^0$ ,  $K^*(1680)K_S^0$ ,  $(K_S^0 K_S^0)_S \eta'$ ,  $(K_S^0 K_S^0)_D \eta'$ ,  $(K_S^0 \eta')_P K_S^0$ ,  $(K_S^0 \eta')_D K_S^0$
- Additional resonance checks: significance  $< 5\sigma$ 
  - No evidence of the  $X(2120)$  in the  $K_S^0 K_S^0$  mass threshold region for  $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta'$  only
  - The significance of  $X(2600) \rightarrow f_0(980)\eta'$  is  $4.2\sigma$
  - Impact from the  $X(2120)$  and  $X(2600)$  is considered as systematic uncertainty
- The  $X(2800)$  with a mass of 2799 MeV and width of 660 MeV:
  - Used to described **effective contributions from high mass region**
  - **Strongly rely on the description of  $\eta_c$  lineshape:** different variations are included into the systematic uncertainty
  - **Statistical uncertainties of the  $X(2800)$  mass and width** are included in the systematic uncertainties on the  $X(2370)$  measurements

# Results

## X(2370) measurements:

[PRL 132 \(2024\) 181901](#)

- $J^{PC} = 0^{-+}$  with significance  $> 9.8\sigma$
- $M = 2395^{+11}_{-11}(stat.)^{+26}_{-94}(syst.) \text{ MeV}/c^2$
- $\Gamma = 188^{+18}_{-17}(stat.)^{+124}_{-33}(syst.) \text{ MeV}$
- $B[J/\psi \rightarrow \gamma X(2370)] \cdot B[X(2370) \rightarrow f_0(980)\eta'] \cdot B[f_0(980) \rightarrow K_s^0 K_s^0] = 1.31^{+0.22}_{-0.22}(stat.)^{+2.85}_{-0.84}(syst.) \times 10^{-5}$

## LQCD prediction on lightest pseudoscalar glueball:

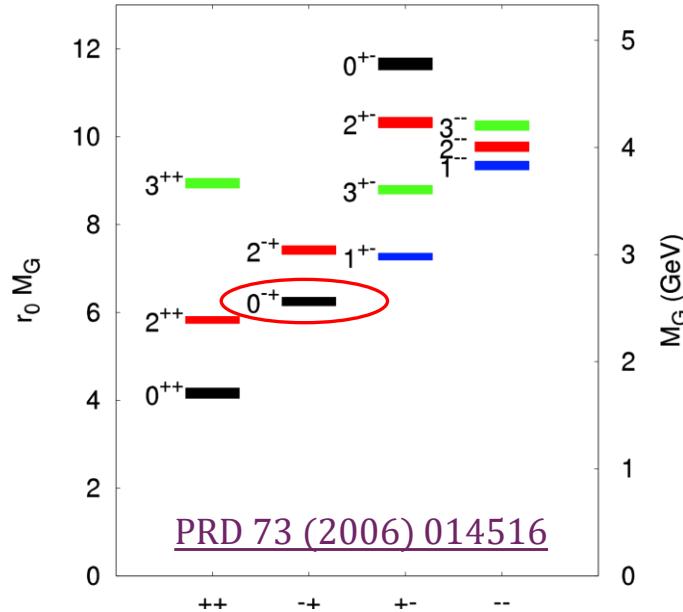
- $J^{PC} = 0^{-+}$
  - $M = 2395 \pm 14 \text{ MeV}/c^2$
  - $B[J/\psi \rightarrow \gamma G_{0-+}] = (2.31 \pm 0.80) \times 10^{-4}$
- [PRD 100 \(2019\) 054511](#)

- The measurements **agree with the predictions on lightest pseudoscalar glueball**
  - The spin-parity of the  $X(2370)$  is determined to be  $0^{-+}$  for the first time
  - Mass is consistent with LQCD predictions
  - The estimation on  $B[J/\psi \rightarrow \gamma X(2370)]$  and prediction on  $B[J/\psi \rightarrow \gamma G_{0-+}]$  are at the same level



# Discovery of a Glueball-like Particle: $X(2370)$

[1]  $f_0(980) \rightarrow K_S^0 K_S^0$   
[2]  $a_0^0(980) \rightarrow \pi^0 \eta$

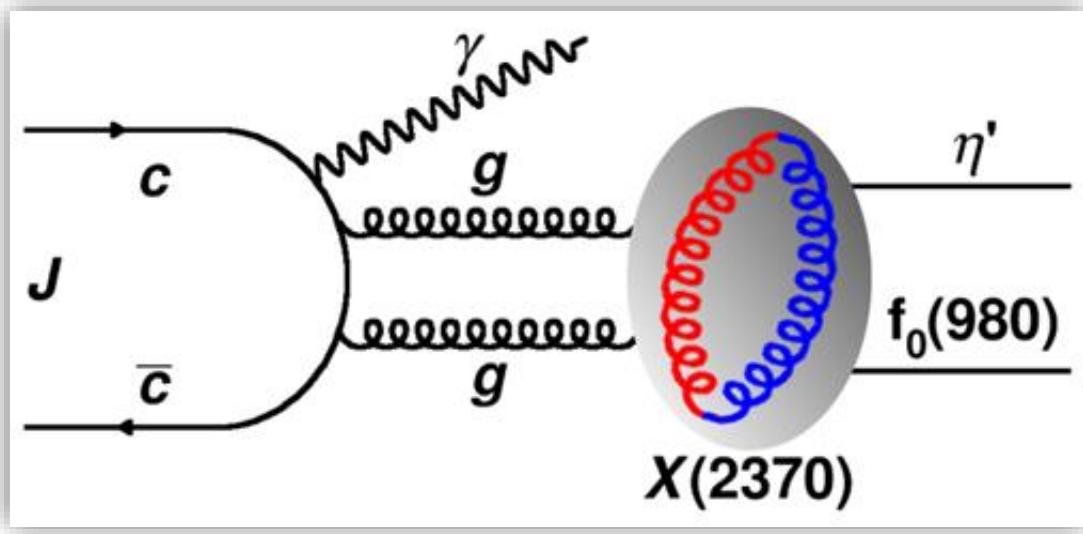


Decay mode	Significance of $X(2370)$	
$\pi^+ \pi^- \eta'$	$6.4\sigma$	<u>PRL 106 (2011) 072002</u>
$K_S^0 K_S^0 \eta$	seen	<u>PRL 115 (2015) 091803</u>
$K \bar{K} \eta'$	$8.3\sigma$	<u>Eur. Phys. J. C 80, 746 (2020)</u>
$f_0(980) \eta'$ [1]	$11.7\sigma$	<u>PRL 132 (2024) 181901</u>
$K_S^0 K_S^0 \pi^0$	$\gg 5\sigma$	
$\pi^0 \pi^0 \eta$	$\gg 5\sigma$	<u>ICHEP2024</u>
$a_0^0(980) \pi^0$ [2]	$\gg 5\sigma$	

- Only one  $0^{-+}$  resonance observed with mass, spin-parity, production rate and decay property consistent to  $0^{-+}$  glueball expectation
- In the mass range of 2.3 - 2.8GeV: consistent with LQCD prediction
  - Production rate in the  $J/\psi$  radiative decays: consistent with LQCD prediction
  - Decay property like  $\eta_c$ : two favorite decay modes decay modes of  $\pi^+ \pi^- \eta'$  and  $K \bar{K} \eta'$

# Summary

- Glueballs are important predictions from LQCD:
  - Unique particles formed by gluons (force carriers) due to self-interactions of gluons
- The  $X(2370)$  is the first particle that matches the theoretical expectations for a glueball
  - $J^{PC} = 0^{-+}$
  - Measurements and predictions on mass are consistent within uncertainties
  - observed in  $J/\psi$  radiative decay and its production rate meets expectation
  - flavor symmetric decay modes (favorite decay modes of  $0^{-+}$  glueball)
- Glueball-like particle,  $X(2370)$  is discovered by BESIII



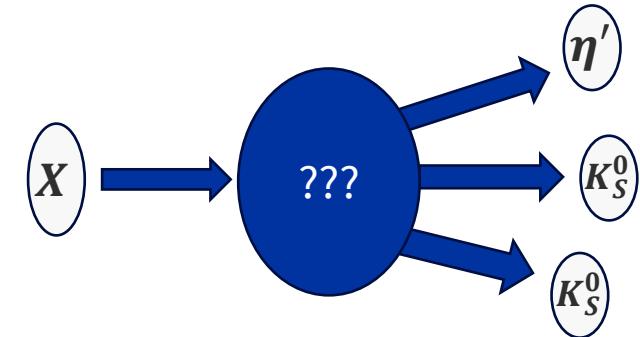
Thanks!



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# Partial wave analysis (PWA)

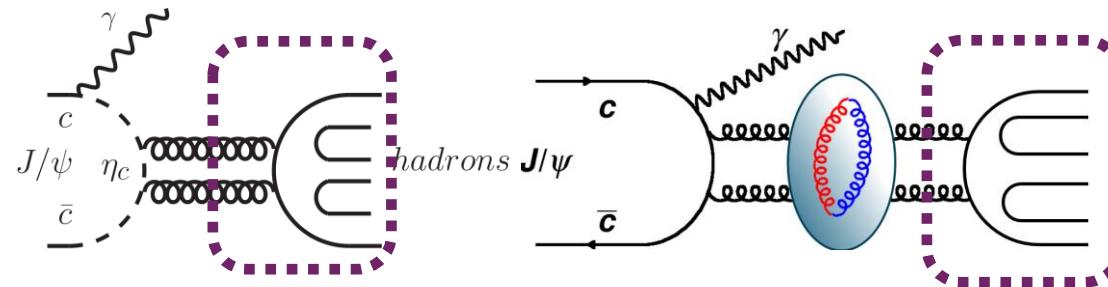
- PWA is a key tool to hadron spectroscopy
  - Input: four-momenta of the final-state particles
  - Measure the resonances' **spin-parity**, resonance parameters, production and decay properties, ...
- PWA of  $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta'$  is performed using **covariant tensor formalism** [1]. The signal amplitudes are parametrized as quasi-sequential two-body decays:
  - $J/\psi \rightarrow \gamma X, X \rightarrow Y \eta', Y \rightarrow K_S^0 K_S^0$  or  $X \rightarrow Z K_S^0, Z \rightarrow K_S^0 \eta'$ 
    - ✓  $K_S^0$  and  $\eta'$  constructed with daughter particles as they are unstable states
  - Differential cross section is observable:  $\omega(\xi, \alpha) = \frac{d\sigma}{d\Phi} = \left| \sum_j A_j \right|^2$
- An unbinned maximum likelihood fit is performed on the data.



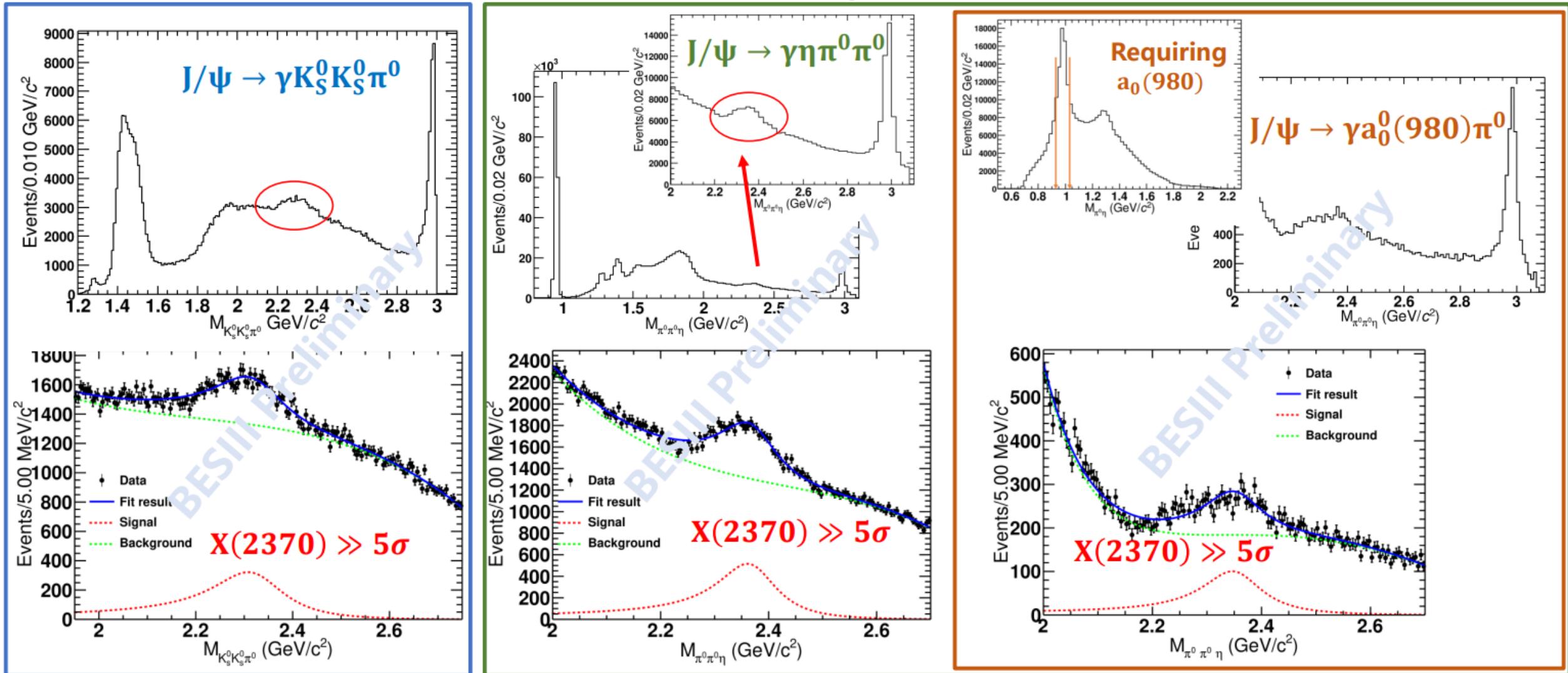
[1] Zou & Bugg, Eur. Phys. J. A 16, 537–547 (2003)

# Glueball Decays

- Via gluons —— flavor symmetric decays
- No rigorous predictions on glueballs' decay patterns and their branching fractions
- The decay patterns of glueballs may resemble those of the Charmonium family, as they also decay through gluons.
  - The  $0^{-+}$  glueball could have similar decays of  $\eta_c$
  - One of the largest decay modes of  $\eta_c$  is  $\pi\pi\eta'$ , so  $J/\psi \rightarrow \gamma\pi\pi\eta'$  could be a good place to search for the  $0^{-+}$  glueball



# Observation of new decay modes of X(2370)



- $X(2370) \rightarrow K_S^0 K_S^0 \pi^0, * \eta \pi^0 \pi^0, a_0^0(980) \pi^0$  firstly observed, all accompanied with  $\eta_c$



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# Glueball Decays

- The decay modes of glueballs may resemble those of the Charmonium family decays, as both can only decay via gluons.

$\pi^+ \pi^- K^+ K^-$ .<sup>[12]</sup> For a glueball, say, a  $J^{PC} = 2^{++}$  glueball, which is made of two gluons, its decay proceeds via the two-gluon hadronization, which is similar to the second step of the  $\chi_{c2}$  decay. The difference between the  $2^{++}$  glueball and  $\chi_{c2}$  in their decays is that the two gluons are hadronized at different energy scales, and consequently in the two cases the branching ratio for a given final state can be different. At the higher energy scale like the  $\chi_{c2}$

From Kuang-Ta Chao 1995 Commu. Theor. Phys. 24.373

ple equally to all flavors. Since there has been no glueball confirmed by experiments, the best way looking into the flavor symmetry should be to study the decay processes which proceed through a two gluon intermediate state [10]. Fortunately, a lot of experiments have already studied such processes as the decays of charmonium family. One example is, the two

it is worth noticing that there are not any other particles showing such properties [12] as  $\xi$  except for the particles with pure OZI suppressed decay modes such as  $J/\psi$ ,  $\chi_{c0}$ ,  $\chi_{c2}$ , etc. The flavor-symmetric couplings

The knowledge [12] about the hadronic decays of  $J/\psi$ ,  $\eta_c$ ,  $\chi_{c0}$  and  $\chi_{c2}$  which proceed through pure gluon intermediate state suggests that the glueballs

From Tao Huang, Kuang-Ta Chao et al. PLB 380 (1996) 189-192

# Interpretation

	X(2370)	$\eta_c$	Interpertation on the X(2370)
$f_0(980)\eta'$	✓	✓	Disfavors $q\bar{q}$ meson with pure $u\bar{u}/d\bar{d}$ component
$f_0(980)\eta$	Suppressed	Suppressed	Disfavors $q\bar{q}$ meson with pure $s\bar{s}$ component
$f_0(1500)\eta$	✓	✓	Disfavors $q\bar{q}$ meson with pure $s\bar{s}$ component

## ➤ The X(2370) decay properties observed: disfavor the interpretation of $q\bar{q}$ meson

- Observed decay modes ( $\eta_c$  dominant decays) and suppressed decay modes are consistent between the  $X(2370)$  and  $\eta_c$
- A good agreement with the glueball interpretation

## ➤ The X(2370) production properties observed:

- richly produced in  $J/\psi$  radiative decays as the glueball expectation
- In the mass region larger than 2 GeV, the only particle  $X(2370)$  for the  $0^{-+}$  glueball candidate in  $\eta_c$  radiative decays and two golden decay modes  $\pi\pi\eta'$  and  $K\bar{K}\eta'$

## ➤ Mass, spin-parity: consistent with $0^{-+}$ glueball prediction

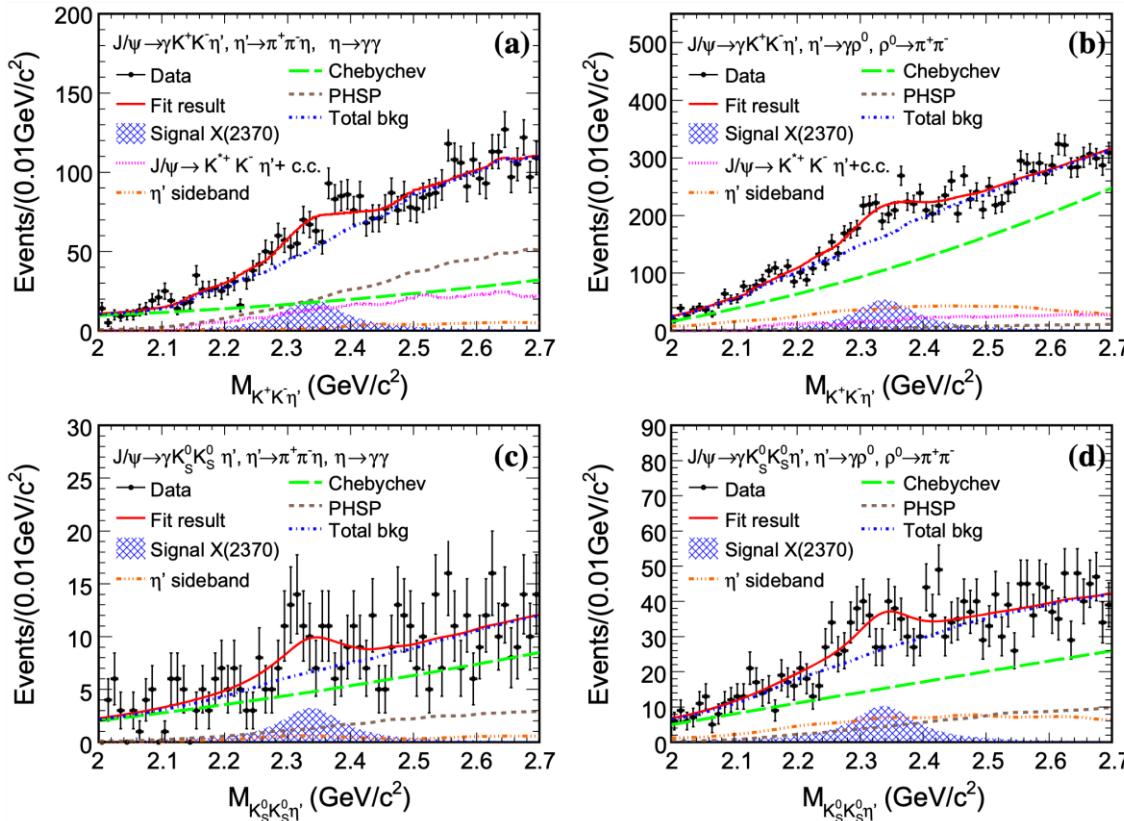
# Golden Decay Modes in $0^+$ Glueball Searches

- Typically, PPP (3 pseudoscalar mesons, such as  $\pi\pi\eta$ ,  $\pi\pi\eta'$ ,  $KK\pi$ ) modes are believed as golden decay modes in  $0^+$  glueball searches.
  - S wave decays for  $0^+$  mesons, no suppression factor, dominant decay modes
  - PPP modes are strongly suppressed in  $0^{++}$ ,  $2^{++}$  mesons decays — spin-parity filter
- PP (2 pseudoscalar mesons) modes are mostly forbidden for  $0^+$  mesons
- VV modes (2 vector mesons, such as  $\omega\omega$ ,  $\phi\phi$ ,  $\rho\rho$ ,  $K^*K^*$ )
  - P wave decays for  $0^+$  mesons — suppressed decays, especially near mass threshold
  - All  $J^{PC}$  mesons allowed, not a spin-parity filter
- Baryon modes
  - All  $J^{PC}$  mesons allowed, not a spin-parity filter

# Confirmation of the $X(2370)$ in $J/\psi \rightarrow \gamma K\bar{K}\eta'$

➤  $J/\psi \rightarrow \gamma K\bar{K}\eta'$  channel was analyzed with  $\sim 1.31B$   $J/\psi$  events

Eur. Phys. J. C 80, 746 (2020)



➤ Combined following channels

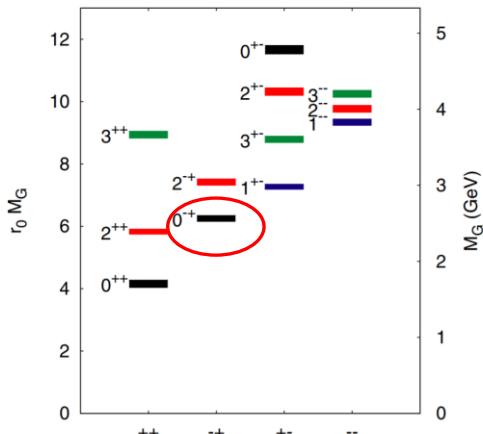
- $J/\psi \rightarrow \gamma K^+ K^- \eta'$  and  $J/\psi \rightarrow \gamma K_S^0 \bar{K}_S^0 \eta'$
- $\eta' \rightarrow \gamma \pi^+ \pi^-$  and  $\eta' \rightarrow \pi^+ \pi^- \eta$

➤ Confirmation of the  $X(2370)$  with  $8.3\sigma$

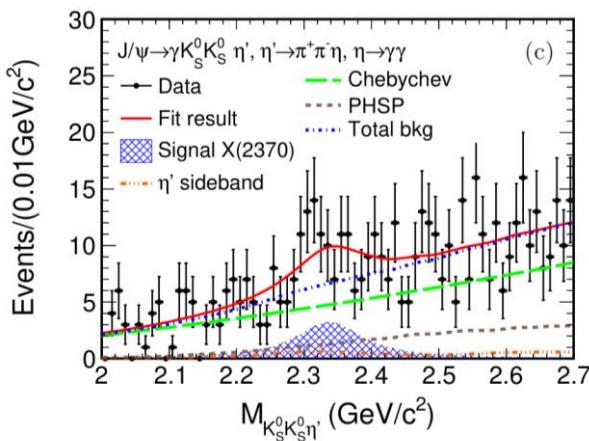
- $M = 2341.6 \pm 6.5(\text{stat.}) \pm 5.7(\text{syst.}) \text{ MeV}/c^2$
- $\Gamma = 117 \pm 10(\text{stat.}) \pm 8(\text{syst.}) \text{ MeV}$

Observation:  $X(2370)$  new decay mode of  $K\bar{K}\eta'$

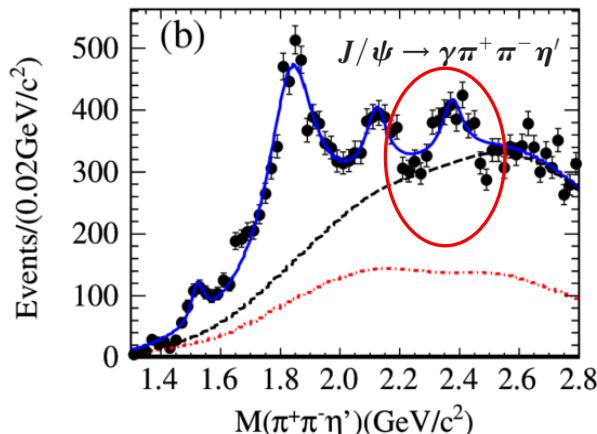
# Overview



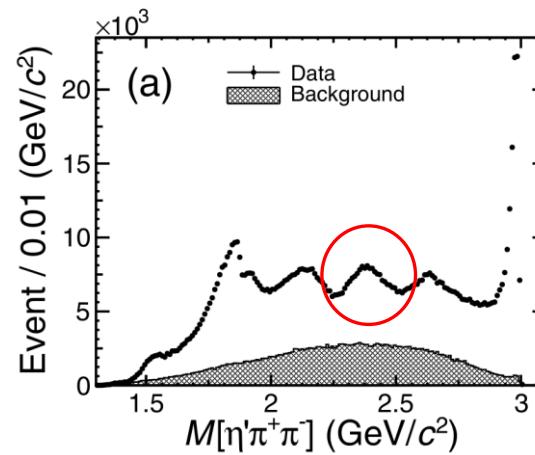
PRD 73 (2006) 014516



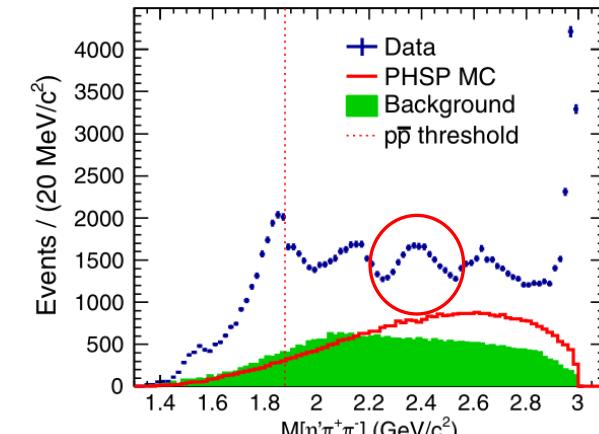
Eur. Phys. J. C 80, 746 (2020)



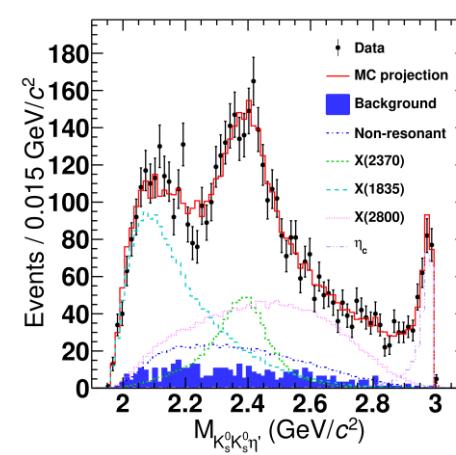
RPL 106 (2011) 072002



PRL 129 (2022) 042001



PRL 117 (2016) 042002



PRL 132 (2024) 181901

- The  $X(2370)$  was first observed in  $J/\psi \rightarrow \gamma \pi^+ \pi^- \eta'$  channel and confirmed in  $J/\psi \rightarrow \gamma K \bar{K} \eta'$  channel. The  $J^{PC}$  of  $X(2370)$  is determined to be  $0^{-+}$  in  $J/\psi \rightarrow \gamma K_s^0 K_s^0 \eta'$  channel.
- The mass, spin-parity, production and decay properties of the  $X(2370)$  are consistent with the features of the lightest pseudoscalar glueball.

# $X(2370)$ seen in $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta$

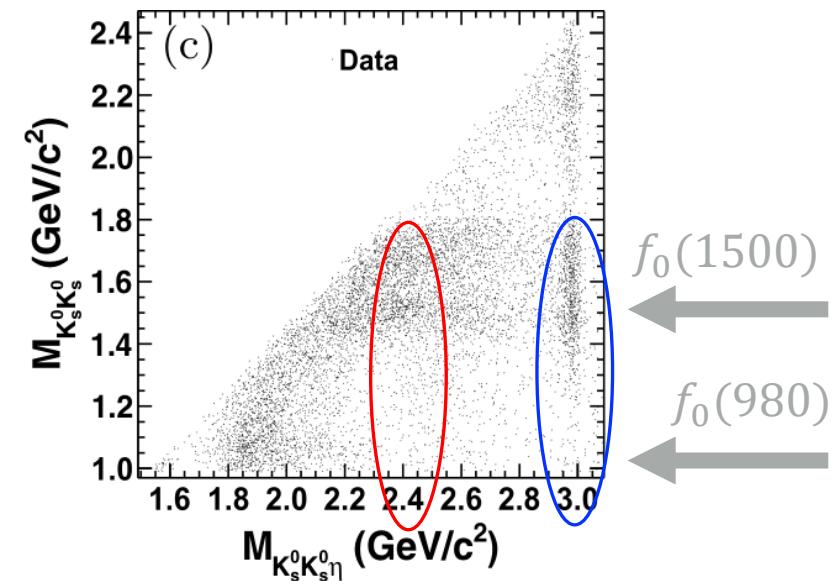
Journal: Phys. Rev. Lett. 115 (2015) 091803

- In the 2D scatter plot of  $M(K_S^0 K_S^0)$  vs.  $M(K_S^0 K_S^0 \eta)$ , **qualitatively**, we can clearly observe same decay patterns between the  $X(2370)$  and  $\eta_c$  if phase space allows

Observation and Spin-Parity Determination of the  $X(1835)$  in  $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta$

## With 1.31B $J/\psi$ events

- In the upper  $M(K_S^0 K_S^0)$  mass band of 1.5-1.7 GeV range, clear signals of both  $X(2370)$  and  $\eta_c$ .
- In the lower  $M(K_S^0 K_S^0)$  mass band of  $f_0(980)$ , no  $X(2370)$ , nor  $\eta_c$ .

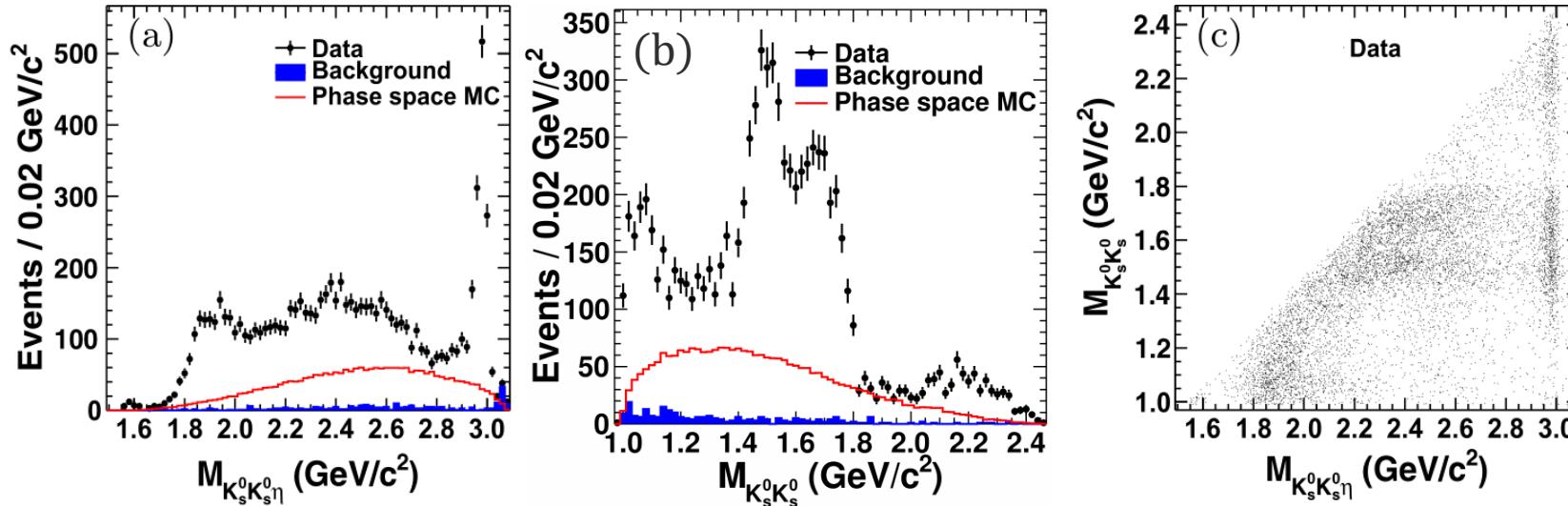


- The similarity between the  $X(2370)$  and  $\eta_c$  decay modes supports the glueball interpretation of the  $X(2370)$

# $X(2370)$ seen in $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta$

## ➤ Mass spectra

With 1.31B  $J/\psi$  events



PRL 115 (2015) 091803

Observation and Spin-Parity Determination of the  $X(1835)$  in  $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta$

# $X(2370)$ observed in $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta'$

PWA projections

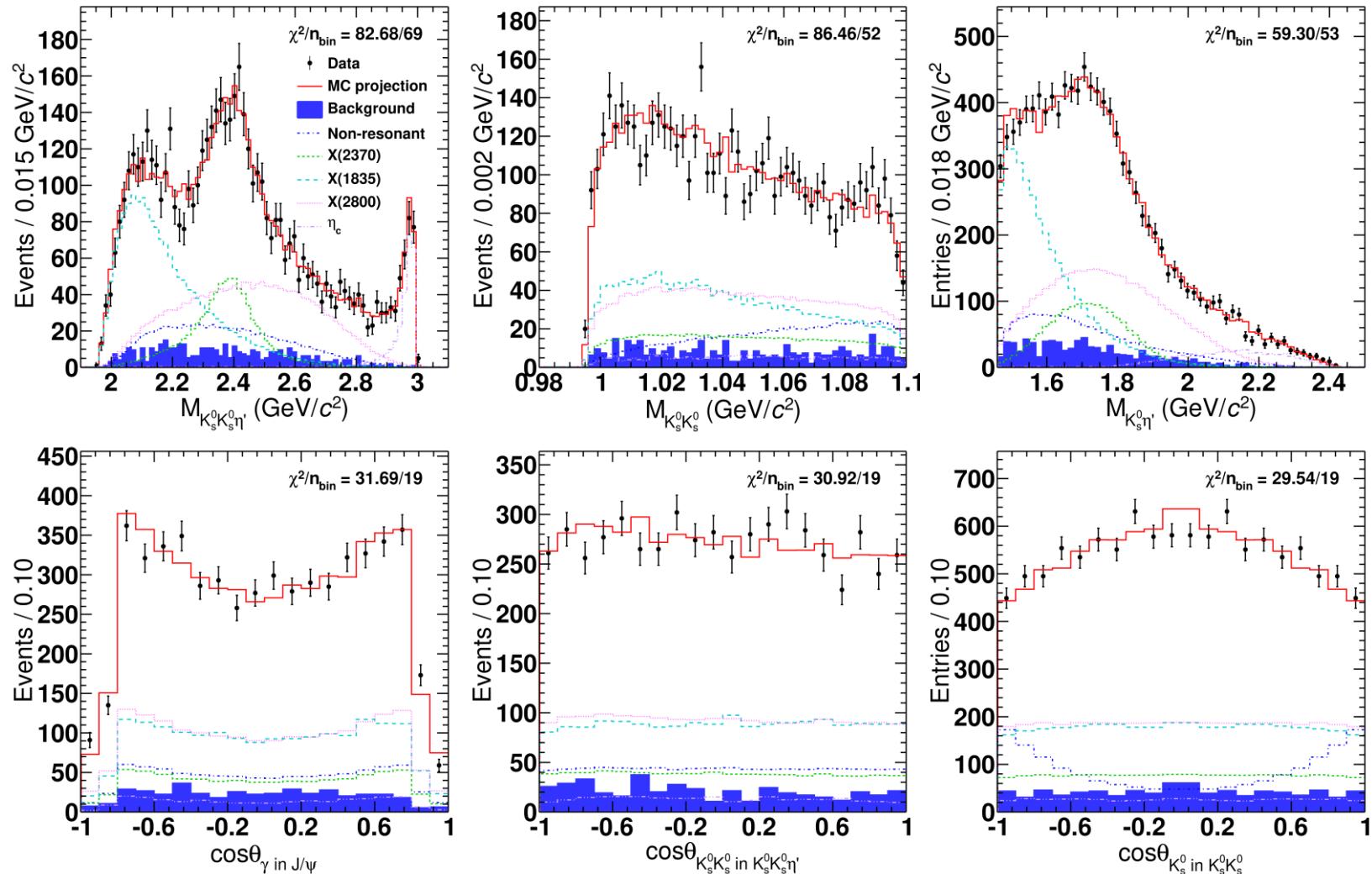


Table 8: Possible waves for the decay of  $Y \rightarrow K_s^0 K_s^0$  and  $Z \rightarrow K_s^0 \eta'$ .

# Possible intermediate states

Table 6: Possible waves for the decay of  $J/\psi \rightarrow \gamma X$ .  $L$  is the quantum number of the orbital angular momentum.

Decays	$J/\psi \rightarrow \gamma 0^{-+}$	$J/\psi \rightarrow \gamma 1^{++}$	$J/\psi \rightarrow \gamma 2^{-+}$	$J/\psi \rightarrow \gamma 2^{++}$
$L$	1	0, 2	1, 3	0, 2

Table 7: Possible waves for the decay of  $X \rightarrow Y\eta'$  or  $X \rightarrow ZK_s^0$  with different  $J^{PC}$  of  $X$ . Label “--” means that this decay is forbidden.

$J^{PC}$	Decays	$X \rightarrow f_0\eta'$	$X \rightarrow f_2\eta'$	$X \rightarrow K^*K_s^0$	$X \rightarrow K_0^*K_s^0$	$X \rightarrow K_2^*K_s^0$
$0^{-+}$		0	2	1	0	2
$1^{++}$		1	1, 3	0, 2	1	1, 3
$2^{-+}$		2	0	--	2	0
$2^{++}$		--	1, 3	--	--	1, 3