



$B^+ \rightarrow K^+ \nu \bar{\nu}$  excess@Belle II,

**(Dark) SMEFT and NP flavour structure**

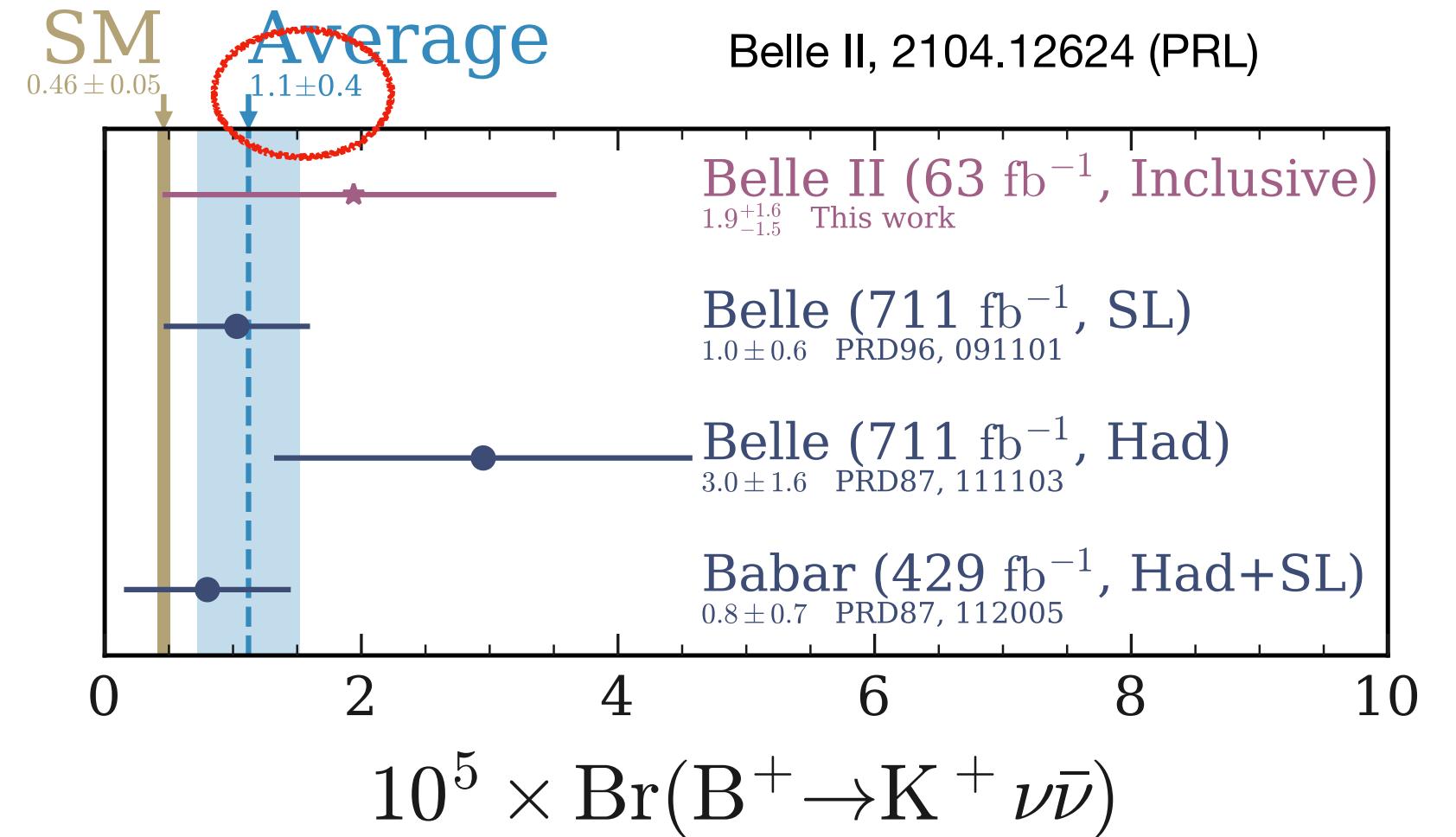
Xing-Bo Yuan (袁兴博)

Central China Normal University (华中师范大学)

arXiv: 2402.19208, Biao-Feng Hou(侯镖锋), Xin-Qiang Li(李新强), Meng Shen(沈萌), Ya-Dong Yang(杨亚东), XBY

# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

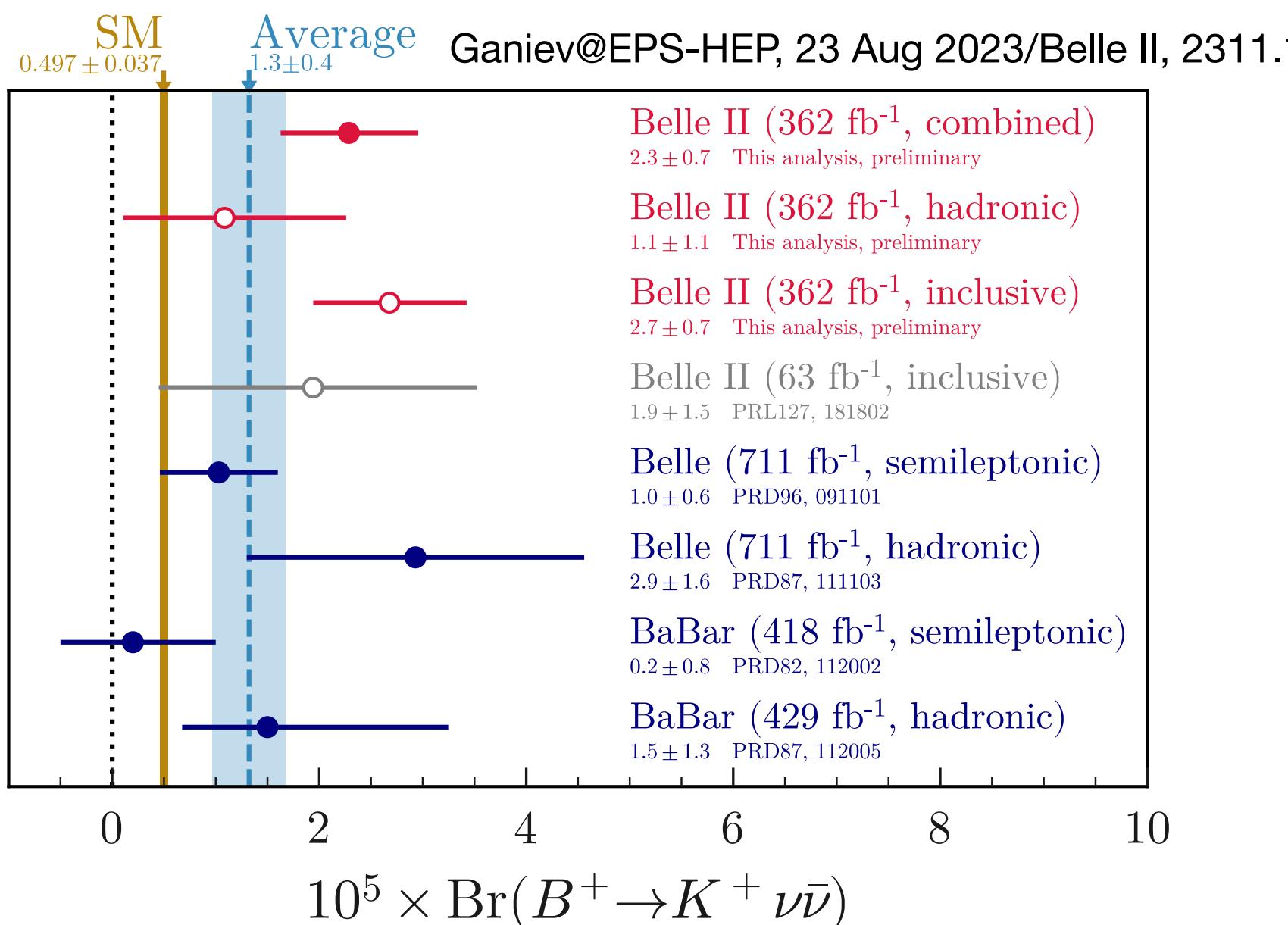
► 2021 Apr



30+ theory papers !

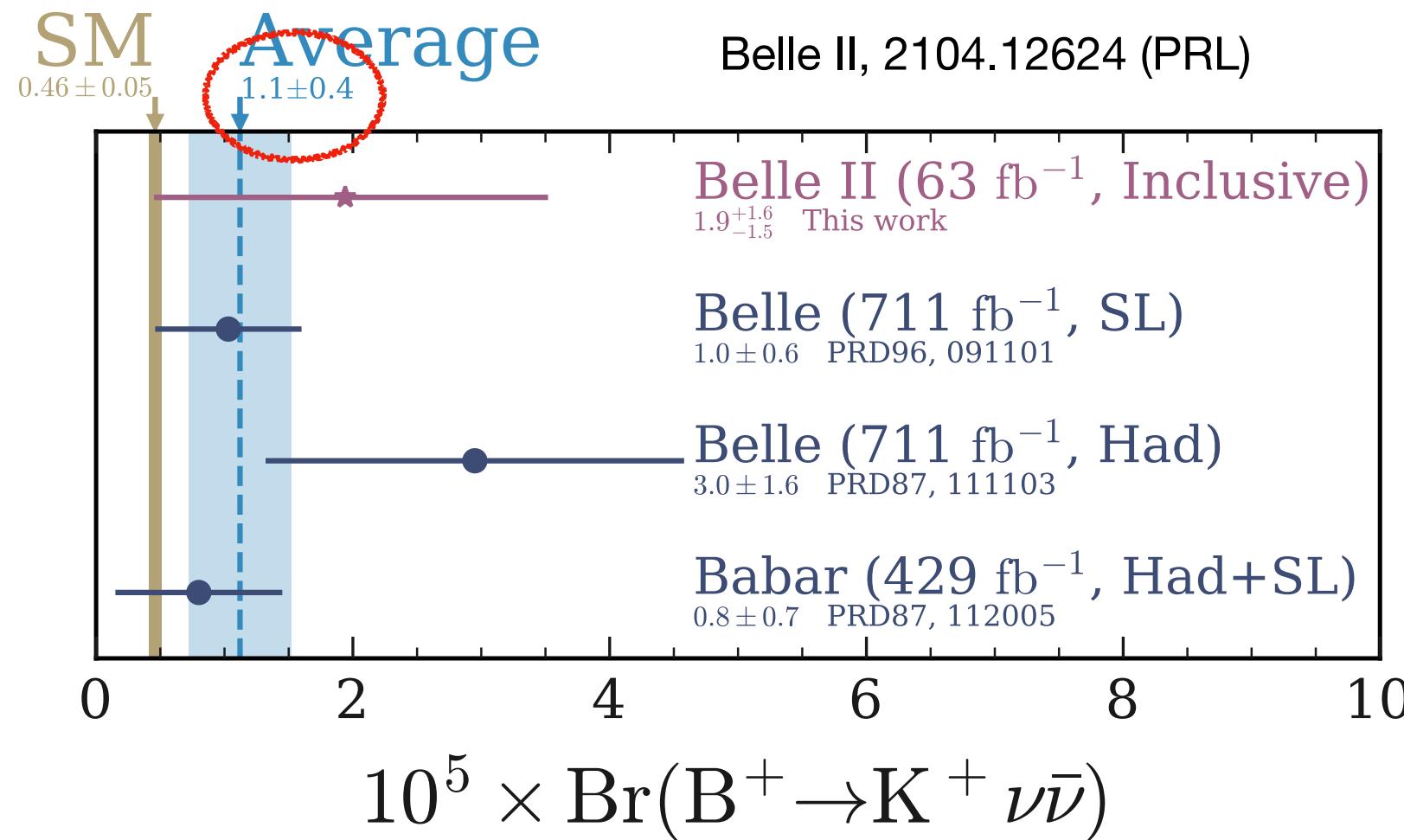
- Impact of  $B \rightarrow K\nu\nu$  measurements on beyond the Standard Model theories #69  
Thomas E. Browder (Hawaii U.), Nilendra G. Deshpande (Oregon U.), Rusa Mandal (Siegen U.), Rahul Sinha (IMSc, Chennai and Bhubaneswar, Inst. Phys.) (Jul 2, 2021)  
Published in: *Phys.Rev.D* 104 (2021) 05, 053007 • e-Print: 2107.01080 [hep-ph]
- A tale of invisibility: constraints on new physics in  $b \rightarrow s\nu\bar{\nu}$  #65  
Tobias Felkl (New South Wales U.), Sze Lok Li (New South Wales U.), Michael A. Schmidt (New South Wales U.) (Nov 8, 2021)  
Published in: *JHEP* 12 (2021) 118 • e-Print: 2111.04327 [hep-ph]
- Explaining the  $B^+ \rightarrow K^+\nu\bar{\nu}$  excess via a massless dark photon #16  
E. Gabrielli, L. Marzola, K. Müürsepp, M. Raidal (Feb 8, 2024)  
e-Print: 2402.05901 [hep-ph]
- Phenomenological study of a gauged  $L_\mu - L_\tau$  model with a scalar leptoquark #42  
Chuan-Hung Chen (Taiwan, Natl. Cheng Kung U. and NCTS, Taipei), Cheng-Wei Chiang (Taiwan, Natl. Taiwan U. and NCTS, Taipei), Chun-Wei Su (Taiwan, Natl. Taiwan U.) (May 16, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 5, 5 • e-Print: 2305.09256 [hep-ph]
- Decoding the  $B \rightarrow K\nu\nu$  excess at Belle II: kinematics, operators, and masses #27  
Kåre Fridell, Mitrajyoti Ghosh, Takemichi Okui, Kohsaku Tobioka (Dec 19, 2023)  
e-Print: 2312.12507 [hep-ph]
- Higgs portal interpretation of the Belle II  $B^+ \rightarrow K^+\nu\nu$  measurement #29  
David McKeen (TRIUMF), John N. Ng (TRIUMF), Douglas Tuckler (TRIUMF and Simon Fraser U.) (Dec 1, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 7, 075006 • e-Print: 2312.00982 [hep-ph]
- Light new physics in  $B \rightarrow K^{(*)}\nu\nu$ ? #30  
Wolfgang Altmannshofer (UC, Santa Cruz, Inst. Part. Phys.), Andreas Crivellin (Zurich U.), Huw Haigh (Vienna, OAW), Gianluca Inguglia (Vienna, OAW), Jorge Martin Camalich (IAC, La Laguna) (Nov 24, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 7, 075008 • e-Print: 2311.14629 [hep-ph]
- $B \rightarrow K\nu\nu$ , MiniBooNE and muon g - 2 anomalies from a dark sector #31  
Alakabha Datta (Mississippi U. and SLAC and UC, Santa Cruz), Danny Marfatia (Hawaii U.), Lopamudra Mukherjee (Nankai U.) (Oct 23, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 3, L031701 • e-Print: 2310.15136 [hep-ph]
- Implications of an enhanced  $B \rightarrow K\nu\nu$  branching ratio #39  
Rigo Bause (Tech. U., Dortmund (main)), Hector Gisbert (INFN, Padua and Padua U.), Gudrun Hiller (Tech. U., Dortmund (main) and Sussex U.) (Aug 31, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 1, 015006 • e-Print: 2309.00075 [hep-ph]
- $B$  meson anomalies and large  $B^+ \rightarrow K^+\nu\bar{\nu}$  in non-universal  $U(1)'$  models #40  
Peter Athron (Nanjing Normal U.), R. Martinez (Colombia, U. Natl.), Cristian Sierra (Nanjing Normal U.) (Aug 25, 2023)  
Published in: *JHEP* 02 (2024) 121 • e-Print: 2308.13426 [hep-ph]
- $B \rightarrow K^*M_X$  vs  $B \rightarrow KM_X$  as a probe of a scalar-mediator dark matter scenario #33  
Alexander Berezhnoy (SINP, Moscow), Dmitri Melikhov (SINP, Moscow and Dubna, JINR and Vienna U.) (Sep 29, 2023)  
Published in: *EPL* 145 (2024) 1, 14001 • e-Print: 2309.17191 [hep-ph]
- Flavor anomalies in leptoquark model with gauged  $U(1)_{L_\mu - L_\tau}$  #34  
Chuan-Hung Chen (Taiwan, Natl. Cheng Kung U. and Unlisted, TW), Cheng-Wei Chiang (Taiwan, Natl. Taiwan U. and Unlisted, TW) (Sep 22, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 7, 075004 • e-Print: 2309.12904 [hep-ph]
- Revisiting models that enhance  $B^+ \rightarrow K^+\nu\nu$  in light of the new Belle II measurement #35  
Belle-II Collaboration • Xiao-Gang He (Tsung-Dao Lee Inst., Shanghai and Taiwan, Natl. Taiwan U.) et al. (Sep 22, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 7, 075019 • e-Print: 2309.12741 [hep-ph]
- A new look at  $b \rightarrow s$  observables in 331 models #18  
Francesco Loparco (Jan 22, 2024)  
e-Print: 2401.11999 [hep-ph]
- Correlating  $B \rightarrow K^{(*)}\nu\bar{\nu}$  and flavor anomalies in SMEFT #19  
Feng-Zhi Chen, Qiaoyi Wen, Fanrong Xu (Jan 21, 2024)  
e-Print: 2401.11552 [hep-ph]
- Recent  $B^+ \rightarrow K^+\nu\bar{\nu}$  Excess and Muon g - 2 Illuminating Light Dark Sector with Higgs Portal #20  
Shu-Yu Ho, Jongkuk Kim, Pyungwon Ko (Jan 18, 2024)  
e-Print: 2401.10112 [hep-ph]
- SMEFT predictions for semileptonic processes #4  
Siddhartha Karmakar, Amol Dighe, Rick S. Gupta (Apr 15, 2024)  
e-Print: 2404.10061 [hep-ph]
- Implications of  $B \rightarrow K\nu\bar{\nu}$  under Rank-One Flavor Violation hypothesis #5  
David Marzocca, Marco Nardeccia, Alfredo Stanzione, Claudio Toni (Apr 9, 2024)  
e-Print: 2404.06533 [hep-ph]
- The quark flavor-violating ALPs in light of B mesons and hadron colliders #20  
Tong Li (Nankai U.), Zhiou Qian (Hangzhou Normal U.), Michael A. Schmidt (Sydney U. and New South Wales U.), Man Yuan (Nankai U.) (Feb 21, 2024)  
Published in: *JHEP* 05 (2024) 232 • e-Print: 2402.14232 [hep-ph]
- Scalar dark matter explanation of the excess in the Belle II  $B^+ \rightarrow K^+ + \text{invisible}$  measurement #9  
Xiao-Gang He, Xiao-Dong Ma, Michael A. Schmidt, German Valencia, Raymond R. Volkas (Mar 19, 2024)  
e-Print: 2403.12485 [hep-ph]
- Status and prospects of rare decays at Belle-II #10  
Elisa Manoni (Mar 12, 2024)  
Published in: *PoS WIFAI2023* (2024) 024 • Contribution to: *WIFAI 2023*, 024
- Rare  $B$  and  $K$  decays in a scotogenic model #11  
Chuan-Hung Chen, Cheng-Wei Chiang (Mar 5, 2024)  
e-Print: 2403.02897 [hep-ph]

► 2023 Aug



# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

► 2021 Apr

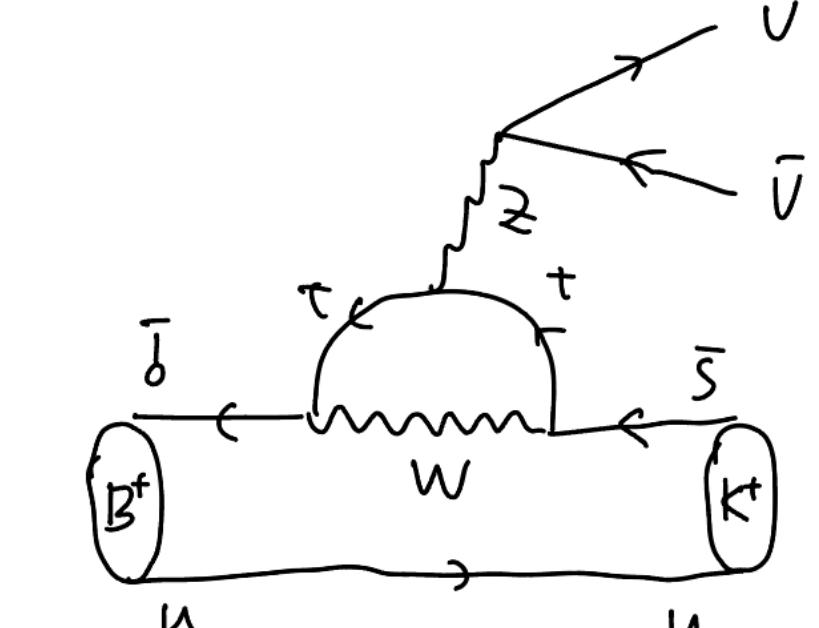


► Exp vs SM [10<sup>-6</sup>]

$$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{SM}} = 4.16 \pm 0.57$$

$$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{exp}} = 23 \pm 7$$

$$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{exp}} \gtrsim 10 \text{ (2}\sigma \text{ lower bound)}$$



2.7 $\sigma$  difference  
NP/SM  $\gtrsim 2$

► Theoretical prediction

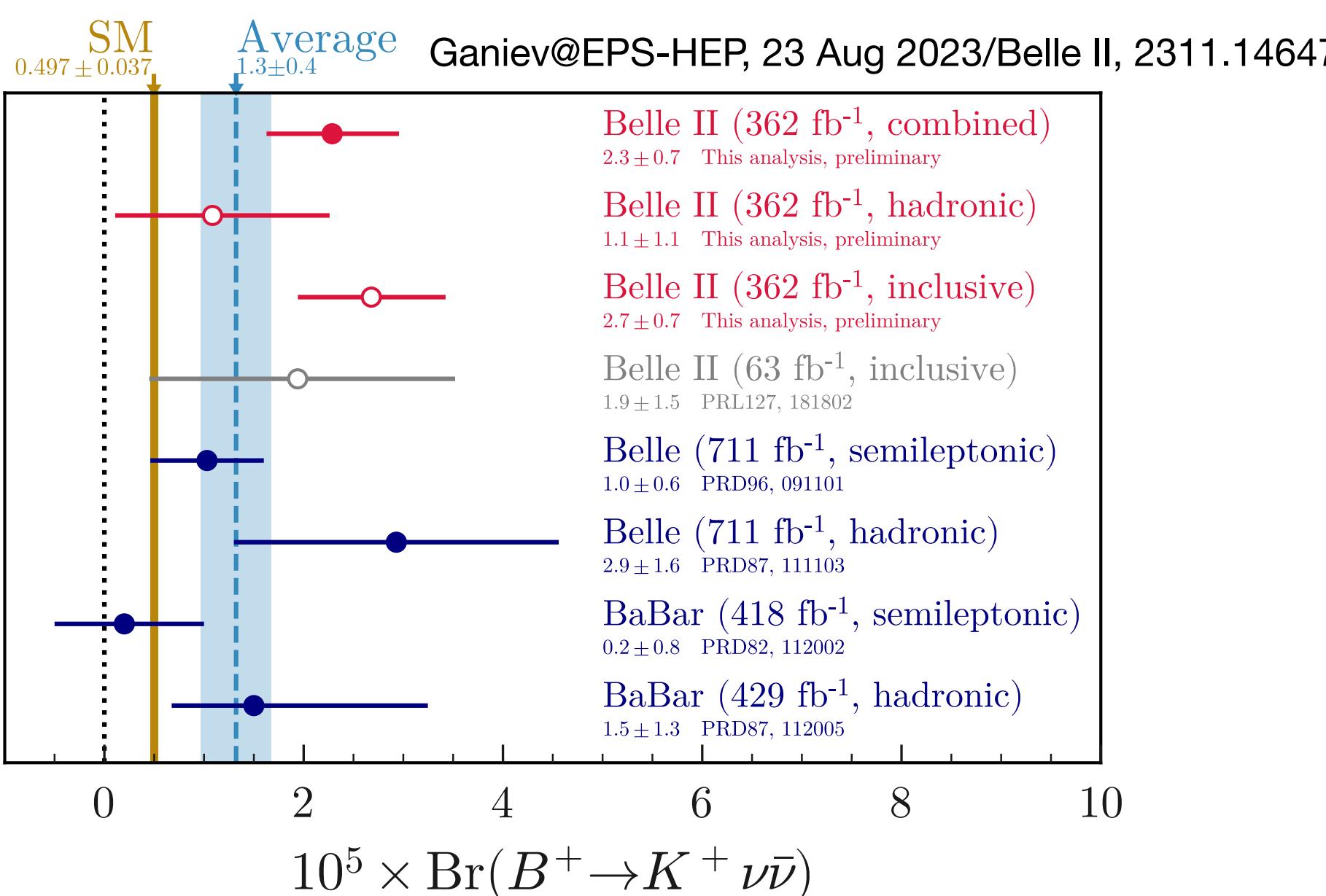
## Factorization

$$\mathcal{A} \propto C_L \cdot \langle K | \bar{s} \gamma^\mu b | \bar{B} \rangle \cdot \bar{\nu} \gamma_\mu \nu$$

Wilson coef    quark current    neutrino current

theoretically, simple and clean  
one of the cleanest channels in  
flavour physics

► 2023 Aug



$$\mathcal{O}_L = (\bar{s} \gamma_\mu P_L b)(\bar{\nu} \gamma^\mu P_L \nu) \text{ in the SM}$$

$$\mathcal{O}_R = (\bar{s} \gamma_\mu P_R b)(\bar{\nu} \gamma^\mu P_L \nu) \text{ possible in BSM}$$

operator structure highly  
constrained by LH neutrino

$$\mathcal{O}_L = (\bar{s} P_L b)(\bar{\nu} P_L \nu) \times$$

$$\mathcal{O}_R = (\bar{s} P_R b)(\bar{\nu} P_R \nu) \times$$

$$\mathcal{O}_T = (\bar{s} \sigma_{\mu\nu} b)(\bar{\nu} \sigma^{\mu\nu} \nu) \times$$

$$\mathcal{O}_{T5} = (\bar{s} \sigma_{\mu\nu} \gamma_5 b)(\bar{\nu} \sigma^{\mu\nu} \nu) \times$$

# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

$b \rightarrow s$

Observable	SM	Exp	Unit
$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})$	$4.16 \pm 0.57$	$23 \pm 5^{+5}_{-4}$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^0\nu\bar{\nu})$	$3.85 \pm 0.52$	$< 26$	$10^{-6}$
$\mathcal{B}(B^+ \rightarrow K^{*+}\nu\bar{\nu})$	$9.70 \pm 0.94$	$< 61$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})$	$9.00 \pm 0.87$	$< 18$	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \phi\nu\bar{\nu})$	$9.93 \pm 0.72$	$< 5400$	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \nu\bar{\nu})$	$\approx 0$	$< 5.9$	$10^{-4}$

$b \rightarrow d$

$\mathcal{B}(B^+ \rightarrow \pi^+\nu\bar{\nu})$	$1.40 \pm 0.18$	$< 140$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \pi^0\nu\bar{\nu})$	$6.52 \pm 0.85$	$< 900$	$10^{-8}$
$\mathcal{B}(B^+ \rightarrow \rho^+\nu\bar{\nu})$	$4.06 \pm 0.79$	$< 300$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \rho^0\nu\bar{\nu})$	$1.89 \pm 0.36$	$< 400$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \nu\bar{\nu})$	$\approx 0$	$< 1.4$	$10^{-4}$

$s \rightarrow d$

$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$	$8.42 \pm 0.61$	$10.6^{+4.0}_{-3.4} \pm 0.9$	$10^{-11}$
$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu})$	$3.41 \pm 0.45$	$< 300$	$10^{-11}$

Why such a large NP effect has not shown up  
in other  $b \rightarrow s$  decays ?  
in  $b \rightarrow d, s \rightarrow d$  decays ?

## ► Exp vs SM [10<sup>-6</sup>]

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**2.7 $\sigma$  difference**  
**NP/SM  $\gtrsim 2$**

## ► Theoretical prediction

### Factorization

$$\mathcal{A} \propto C_L \cdot \langle K | \bar{s}\gamma^\mu b | \bar{B} \rangle \cdot \bar{\nu}\gamma_\mu \nu$$

Wilson coef    quark current    neutrino current

**theoretically, simple and clean**  
**one of the cleanest channels in**  
**flavour physics**

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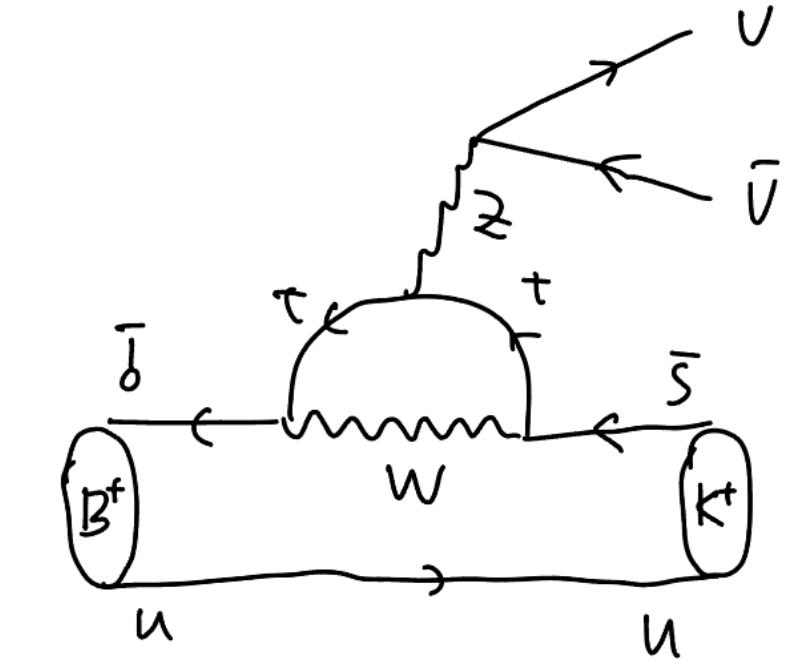
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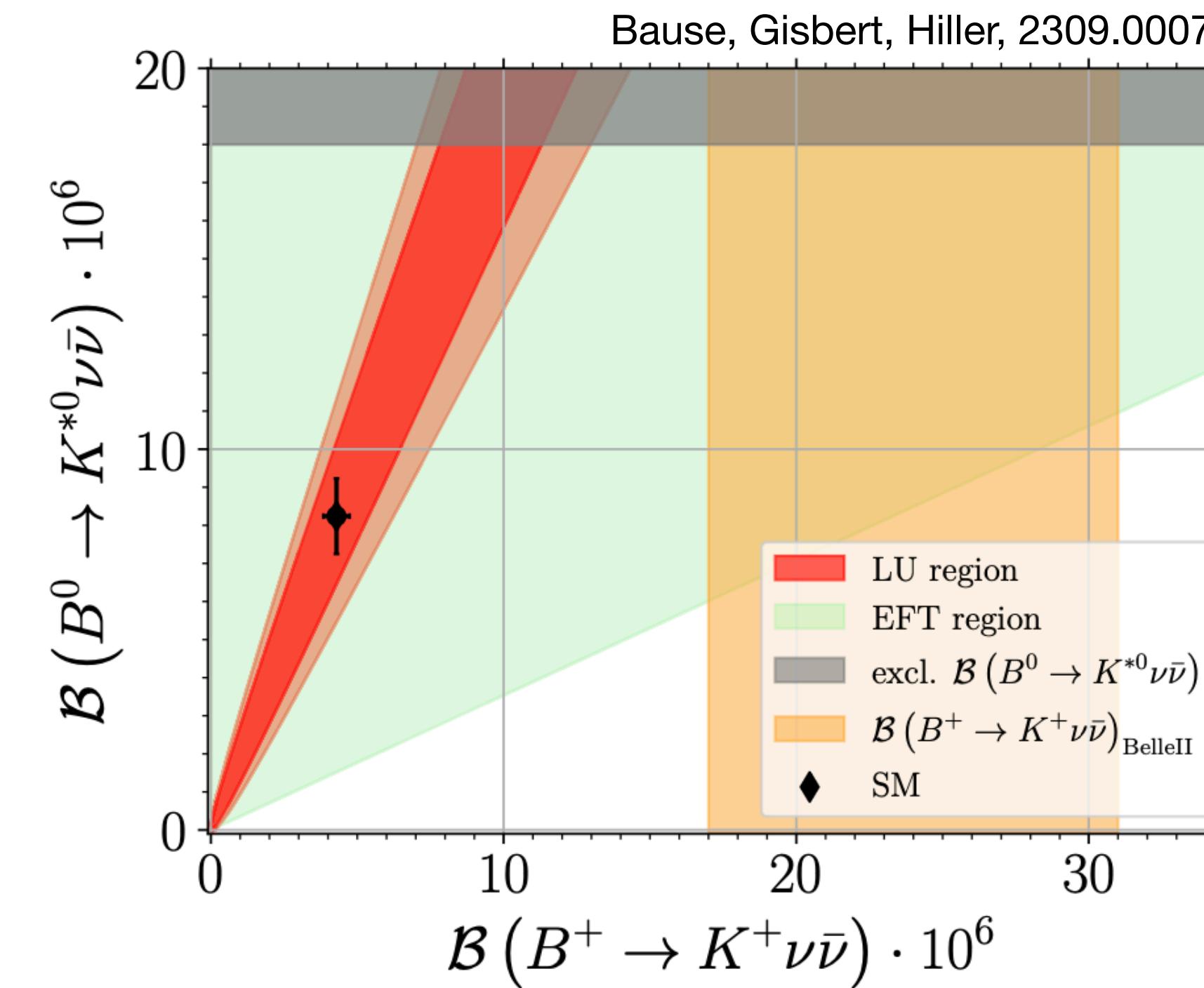
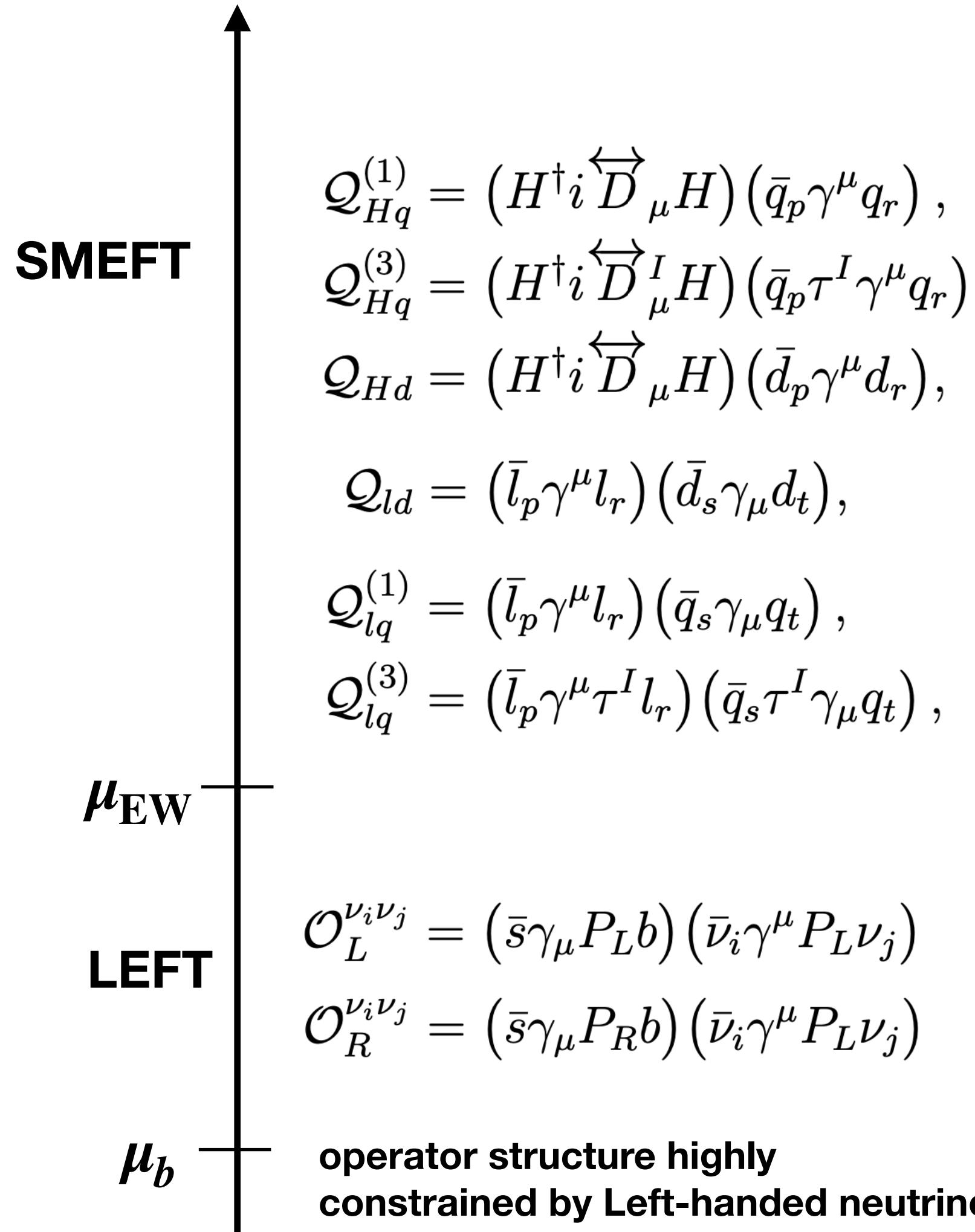
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# $b \rightarrow s\nu\bar{\nu}$ : SMEFT



$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = A_+^{BK} x^+,$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu}) = A_+^{BK^*} x^+ + A_-^{BK^*} x^-,$$

$$x^\pm = \sum_{\nu, \nu'} |C_L^{\nu \nu'} \pm C_R^{\nu \nu'}|^2,$$

Bause, Gisbert, Hiller, 2309.00075  
 Allwicher, Becirevic, Piazza, Rosauro-Alcaraz, Sumensari, 2309.02246  
 Chen, Wen, Xu, 2401.11552

# $b \rightarrow s\nu\bar{\nu}$ : SMEFT

	<b>SMEFT</b>
$\mu_b$	$\mathcal{Q}_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q}_p \gamma^\mu q_r),$
$\mu_{EW}$	$\mathcal{Q}_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{q}_p \tau^I \gamma^\mu q_r),$
	$\mathcal{Q}_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d}_p \gamma^\mu d_r),$
	$\mathcal{Q}_{ld} = (\bar{l}_p \gamma^\mu l_r) (\bar{d}_s \gamma_\mu d_t),$
	$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma^\mu l_r) (\bar{q}_s \gamma_\mu q_t),$
	$\mathcal{Q}_{lq}^{(3)} = (\bar{l}_p \gamma^\mu \tau^I l_r) (\bar{q}_s \tau^I \gamma_\mu q_t),$
$\mathcal{O}_L^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_L b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$	
$\mathcal{O}_R^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_R b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$	operator structure highly constrained by Left-handed neutrino

$b \rightarrow s$

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in other  $b \rightarrow s$  decays ?  
in  $b \rightarrow d, s \rightarrow d$  decays ? **NP flavour structure**

# Minimal Flavour Violation

- Flavour symmetry without Yukawa

$$G_{\text{QF}} = SU(3)_q \otimes SU(3)_u \otimes SU(3)_d$$

- Flavour symmetry breaking only from SM Yukawa

$$-\mathcal{L}_Y = \bar{q} Y_d H d + \bar{q} Y_u \tilde{H} u + \text{h.c.}$$

- Flavour symmetry recovering: Yukawa coupling  $\implies$  spurion field

$$Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \quad Y_d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$$

D'Ambrosio, Giudice, Isidori, Strumia, 2009

- EFT with MFV: operators, constructed from SM and Yukawa spurion fields, are invariant under CP and  $G_{\text{QF}}$

$$\mathcal{C}^{\text{MFV}} = \begin{cases} f(A, B) & \text{for } \bar{q}\gamma^\mu C q, \\ f(A, B)Y_d & \text{for } \bar{q}Cd, \bar{q}\sigma^{\mu\nu}Cd, \\ \epsilon_0 \mathbb{1} + Y_d^\dagger g(A, B)Y_d & \text{for } \bar{d}\gamma^\mu Cd, \end{cases} \quad \begin{aligned} f(A, B) &= \epsilon_0 \mathbb{1} + \epsilon_1 A + \epsilon_2 B + \epsilon_3 A^2 + \epsilon_4 B^2 + \epsilon_5 AB + \dots \\ A &= Y_u Y_u^\dagger \\ B &= Y_d Y_d^\dagger \end{aligned}$$

# Minimal Flavour Violation

- ▶ Spurion function

$$f(A, B) = \epsilon_0 \mathbb{1} + \epsilon_1 A + \epsilon_2 B + \epsilon_3 A^2 + \epsilon_4 B^2 + \epsilon_5 AB + \dots \dots$$

- ▶ Cayley-Hamilton identity for  $3 \times 3$  invertible matrix  $X$

$$X^3 = \text{Det}X \cdot \mathbb{1} + \frac{1}{2}[\text{Tr}X^2 - (\text{Tr}X)^2] \cdot X + \text{Tr}X \cdot X^2$$

- ▶ Spurion function after resummation

$$\begin{aligned} f(A, B) = & \epsilon_0 \mathbb{1} + \epsilon_1 A + \epsilon_3 A^2 + \epsilon_5 AB + \epsilon_7 ABA + \epsilon_{10} AB^2 + \epsilon_{12} A^2B^2 + \epsilon_{14} B^2AB + \epsilon_{15} AB^2A^2 \\ & + \epsilon_2 B + \epsilon_4 B^2 + \epsilon_6 BA + \epsilon_9 BAB + \epsilon_8 BA^2 + \epsilon_{13} B^2A^2 + \epsilon_{11} ABA^2 + \epsilon_{16} B^2A^2B. \end{aligned}$$

Colangelo, Nikolidakis, Smith, 2009  
Mercolli, Smith, 2009

- ▶ assumption #1: neglect tiny imaginary parts of  $\epsilon_i$
- ▶ assumption #2: neglect spurion B (suppressed by  $\mathcal{O}(\lambda_d^2)$ )

$$f(A, B) \approx \epsilon_0 \mathbb{1} + \epsilon_1 A + \epsilon_2 A^2$$

# Minimal Flavour Violation

- MFV coupling      **FCNC controlled by CKM**

$$C^{\text{MFV}} = \begin{cases} \epsilon_0 1 + \epsilon_1 \Delta_q & \text{for } \bar{d}_L \gamma^\mu C d_L \\ \epsilon_0 \hat{\lambda}_d + \epsilon_1 \Delta_q \hat{\lambda}_d & \text{for } \bar{d}_L C d_R, \bar{d}_L \sigma^{\mu\nu} C d_R \\ \epsilon_0 1 & \text{for } \bar{d}_R \gamma^\mu C d_R \end{cases} \quad \Delta_q = V^\dagger \hat{\lambda}_u^2 V$$

**No Right-handed down-type FCNC !**

- Numerics

$$\Delta_q = \begin{pmatrix} 0.8 & -3.3 - 1.5i & 79.3 + 35.4i \\ -3.3 + 1.5i & 16.6 & -397.5 + 8.1i \\ 79.3 - 35.4i & -397.5 - 8.1i & 9839.0 \end{pmatrix} \times 10^{-4}$$

$$\Delta_q \hat{\lambda}_d = \begin{pmatrix} 0.0021 & -0.18 - 0.08i & 191.3 + 85.4i \\ -0.009 + 0.004i & 0.88 & -958.7 + 19.6i \\ 0.21 - 0.10i & -21.1 - 0.4i & 23728.1 \end{pmatrix} \times 10^{-6}$$

# $b \rightarrow s\nu\bar{\nu}$ : SMEFT with MFV

## ► Prediction

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})_{\text{SM}}}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu\bar{\nu})_{\text{SM}}} = 0.46 \pm 0.07$$

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})}{\mathcal{B}(B^+ \rightarrow \pi^+ \nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})_{\text{SM}}}{\mathcal{B}(B^+ \rightarrow \pi^+ \nu\bar{\nu})_{\text{SM}}} = 29.7 \pm 5.6$$

## ► prediction

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu\bar{\nu})_{\text{SM}} = (9.00 \pm 0.87) \times 10^{-6}$$

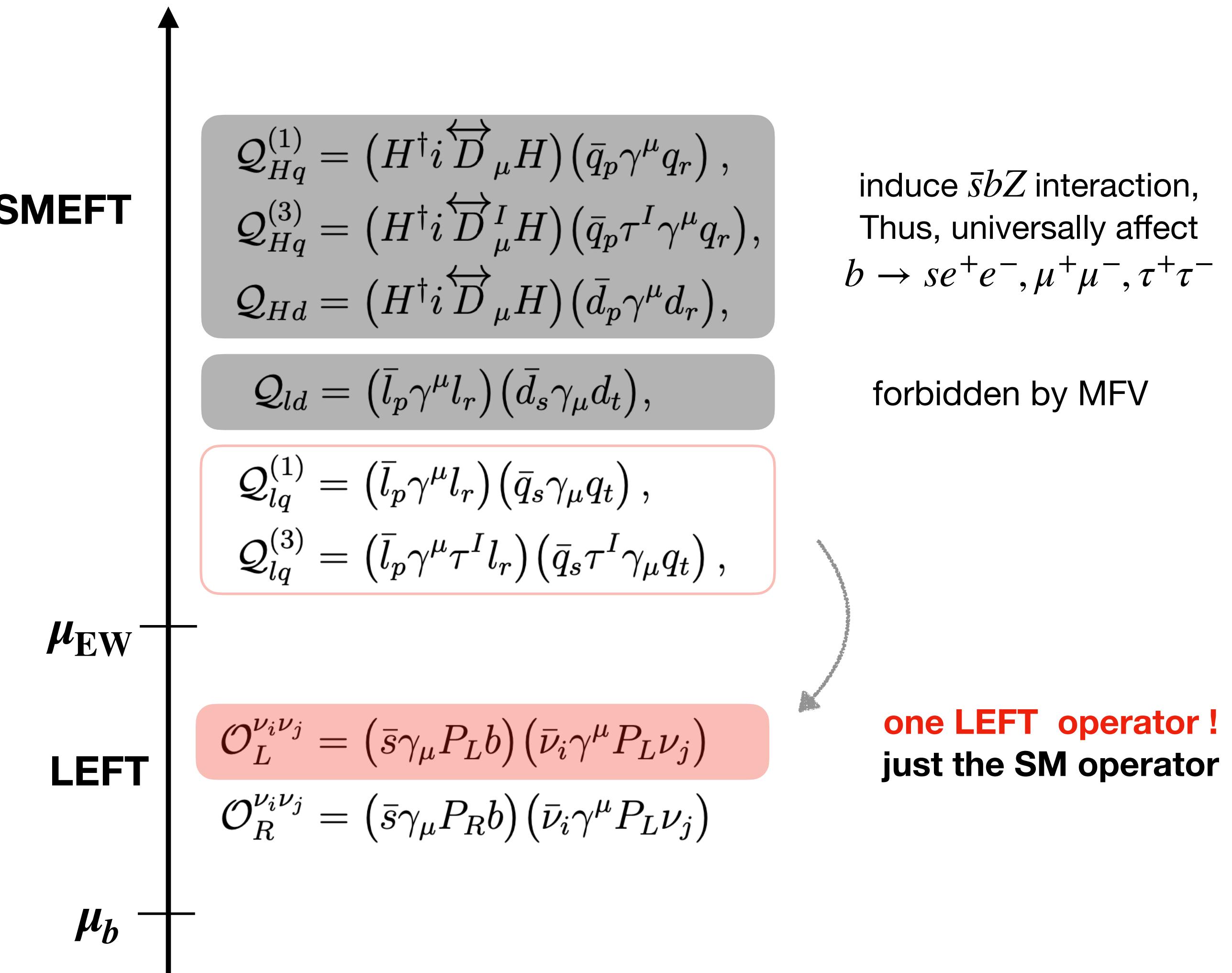
$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu\bar{\nu})_{\text{MFV}} = (50^{+17}_{-16}) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu\bar{\nu})_{\text{exp}} < 18 \times 10^{-6}$$

$$\mathcal{B}(B^+ \rightarrow \pi^+ \nu\bar{\nu})_{\text{SM}} = (1.40 \pm 0.18) \times 10^{-7}$$

$$\mathcal{B}(B^+ \rightarrow \pi^+ \nu\bar{\nu})_{\text{MFV}} = (7.8^{+2.8}_{-2.6}) \times 10^{-7}$$

$$\mathcal{B}(B^+ \rightarrow \pi^+ \nu\bar{\nu})_{\text{exp}} < 140 \times 10^{-7}$$



# $b \rightarrow s\nu\bar{\nu}$ : SMEFT with MFV

## ► Prediction

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$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{SM}} = (9.00 \pm 0.87) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{MFV}} = (50^{+17}_{-16}) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{exp}} < 18 \times 10^{-6}$$

Inconsistent



Belle II excess (if confirmed in the future) implies:

- impossible to explain in SMEFT with MFV
- NP flavour structure is highly non-trivial
- NP structure in quark sector is beyond MFV**
- flavour violation is beyond Yukawa coupling**

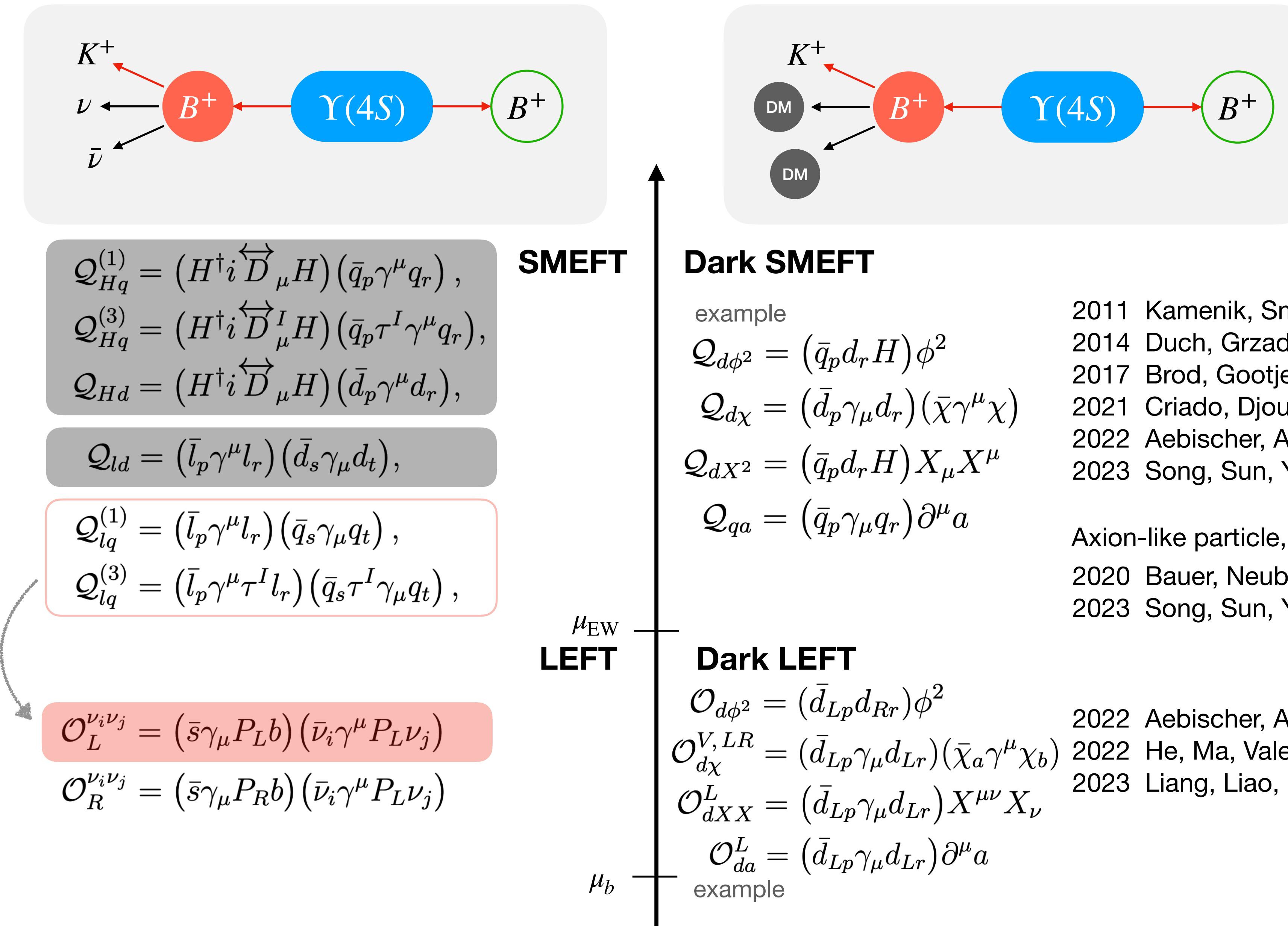
This conclusion only assumes the quark MFV.  
No lepton flavour structure is assumed.

$$\mathcal{B}(B^+ \rightarrow \pi^+\nu\bar{\nu})_{\text{SM}} = (1.40 \pm 0.18) \times 10^{-7}$$

$$\mathcal{B}(B^+ \rightarrow \pi^+\nu\bar{\nu})_{\text{MFV}} = (7.8^{+2.8}_{-2.6}) \times 10^{-7}$$

$$\mathcal{B}(B^+ \rightarrow \pi^+\nu\bar{\nu})_{\text{exp}} < 140 \times 10^{-7}$$

# $b \rightarrow s\nu\bar{\nu}$ : exp picture



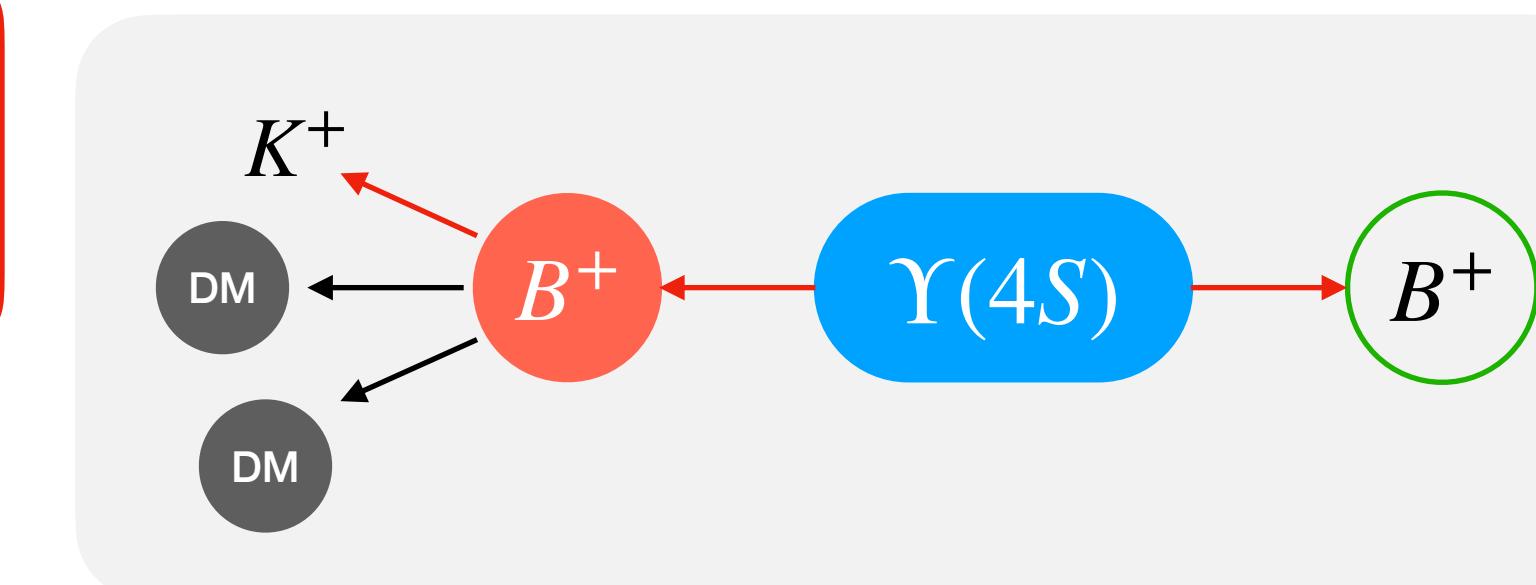
# $b \rightarrow s\nu\bar{\nu}$ : DSMEFT

Can DSMEFT operators explain the Belle II excess,  
while satisfy other  $b \rightarrow s$  bounds ?

Observable	SM	Exp	Unit
$\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})$	$4.16 \pm 0.57$	$23 \pm 5^{+5}_{-4}$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^0 \nu\bar{\nu})$	$3.85 \pm 0.52$	$< 26$	$10^{-6}$
$\mathcal{B}(B^+ \rightarrow K^{*+} \nu\bar{\nu})$	$9.70 \pm 0.94$	$< 61$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^{*0} \nu\bar{\nu})$	$9.00 \pm 0.87$	$< 18$	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \phi \nu\bar{\nu})$	$9.93 \pm 0.72$	$< 5400$	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \nu\bar{\nu})$	$\approx 0$	$< 5.9$	$10^{-4}$
$\mathcal{B}(B^+ \rightarrow \pi^+ \nu\bar{\nu})$	$1.40 \pm 0.18$	$< 140$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \pi^0 \nu\bar{\nu})$	$6.52 \pm 0.85$	$< 900$	$10^{-8}$
$\mathcal{B}(B^+ \rightarrow \rho^+ \nu\bar{\nu})$	$4.06 \pm 0.79$	$< 300$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \rho^0 \nu\bar{\nu})$	$1.89 \pm 0.36$	$< 400$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \nu\bar{\nu})$	$\approx 0$	$< 1.4$	$10^{-4}$
$\mathcal{B}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$	$8.42 \pm 0.61$	$10.6^{+4.0}_{-3.4} \pm 0.9$	$10^{-11}$
$\mathcal{B}(K_L \rightarrow \pi^0 \nu\bar{\nu})$	$3.41 \pm 0.45$	$< 300$	$10^{-11}$

$\mu_{EW}$

$\mu_b$



## Dark SMEFT

$$\mathcal{Q}_{d\phi} = (\bar{q}_p d_r H)\phi + \text{h.c.}, \quad \mathcal{Q}_{d\phi^2} = (\bar{q}_p d_r H)\phi^2 + \text{h.c.},$$

$$\mathcal{Q}_{\phi q} = (\bar{q}_p \gamma_\mu q_r)(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2), \quad \mathcal{Q}_{\phi d} = (\bar{d}_p \gamma_\mu d_r)(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2),$$

$$\mathcal{Q}_{q\chi} = (\bar{q}_p \gamma_\mu q_r)(\bar{\chi} \gamma^\mu \chi), \quad \mathcal{Q}_{d\chi} = (\bar{d}_p \gamma_\mu d_r)(\bar{\chi} \gamma^\mu \chi),$$

$$\mathcal{Q}_{dHX} = (\bar{q}_p \sigma_{\mu\nu} d_r) H X^{\mu\nu} \quad \mathcal{Q}_{dX^2} = (\bar{q}_p d_r H) X_\mu X^\mu$$

$$\mathcal{Q}_{qa} = (\bar{q}_p \gamma_\mu q_r) \partial^\mu a \quad \mathcal{Q}_{da} = (\bar{d}_p \gamma_\mu d_r) \partial^\mu a$$

scalar: 4

fermion: 2

vector: 1+13

ALP: 2



## Dark LEFT

$$\mathcal{O}_{d\phi} = (\bar{d}_{Lp} d_{Rr})\phi + \text{h.c.}, \quad \mathcal{O}_{\phi d}^L = (\bar{d}_{Lp} \gamma_\mu d_{Lr})(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2),$$

$$\mathcal{O}_{d\chi}^{V,LR} = (\bar{d}_{Lp} \gamma_\mu d_{Lr})(\bar{\chi}_a \gamma^\mu \chi_b), \mathcal{O}_{d\chi}^{V,RR} = (\bar{d}_{Rp} \gamma_\mu d_{Rr})(\bar{\chi}_a \gamma^\mu \chi_b),$$

$$\mathcal{O}_{dX}^T = (\bar{d}_{Lp} \sigma_{\mu\nu} d_{Rr}) X_a^{\mu\nu} \quad \mathcal{O}_{dXX}^L = (\bar{d}_{Lp} \gamma_\mu d_{Lr}) X^{\mu\nu} X_\nu$$

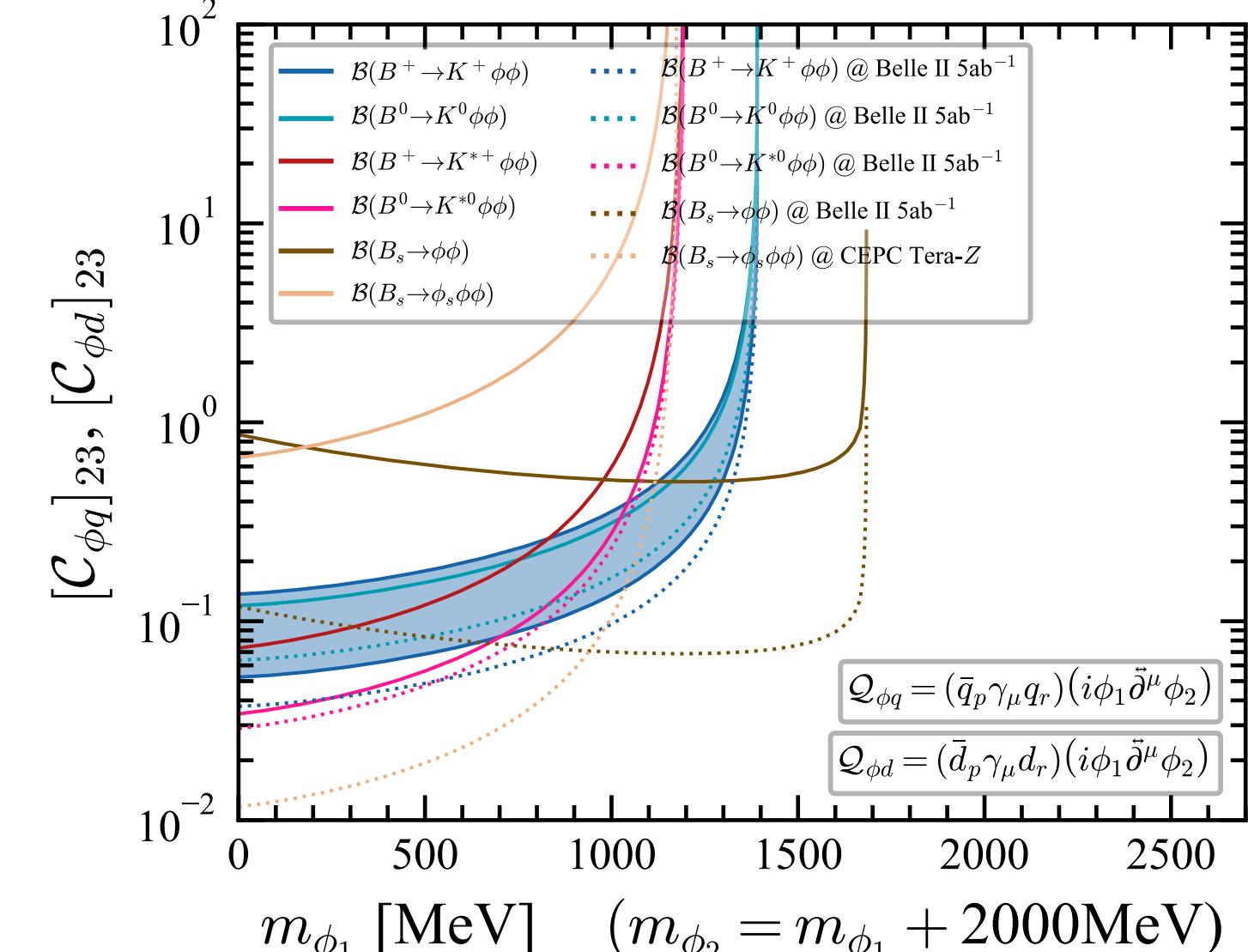
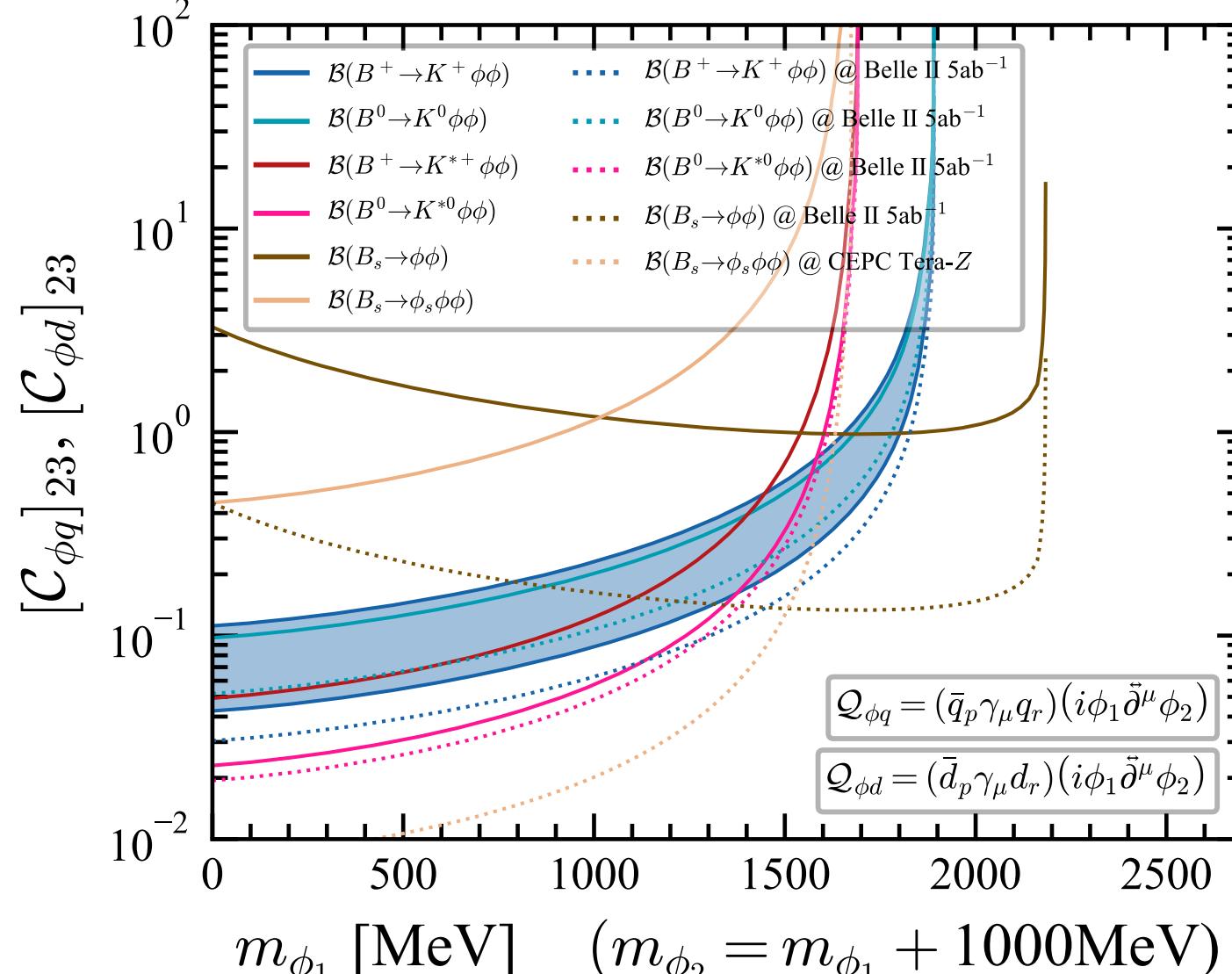
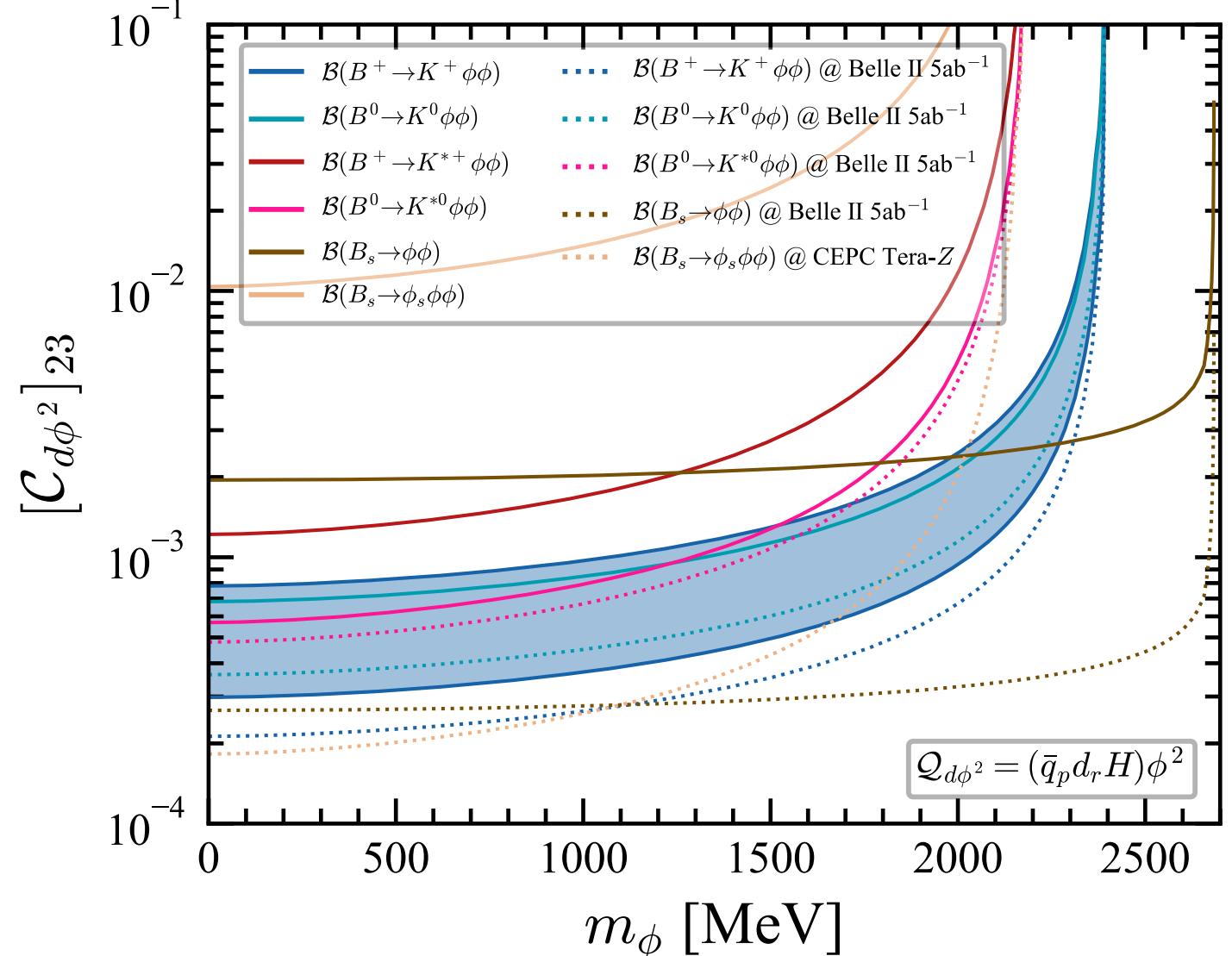
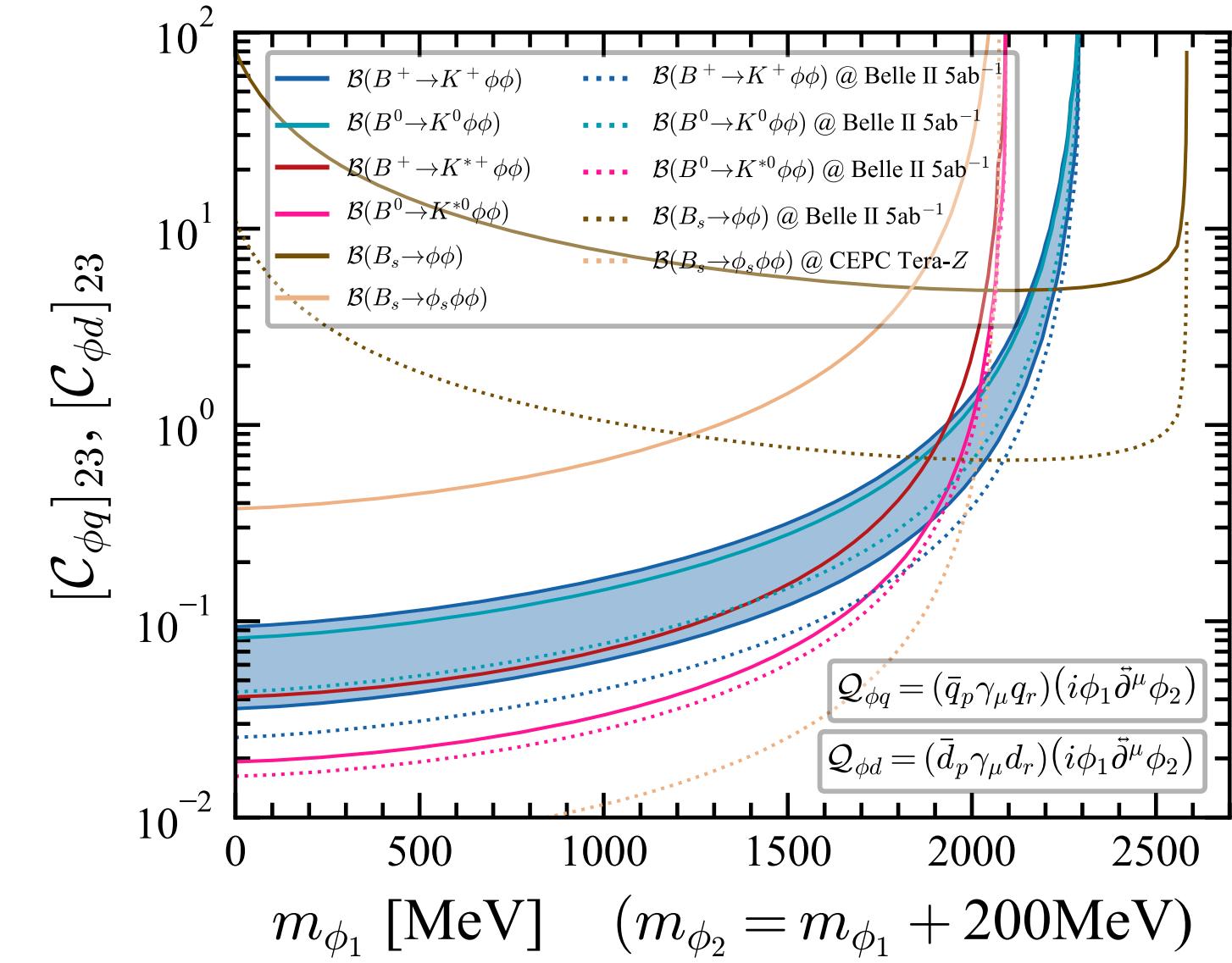
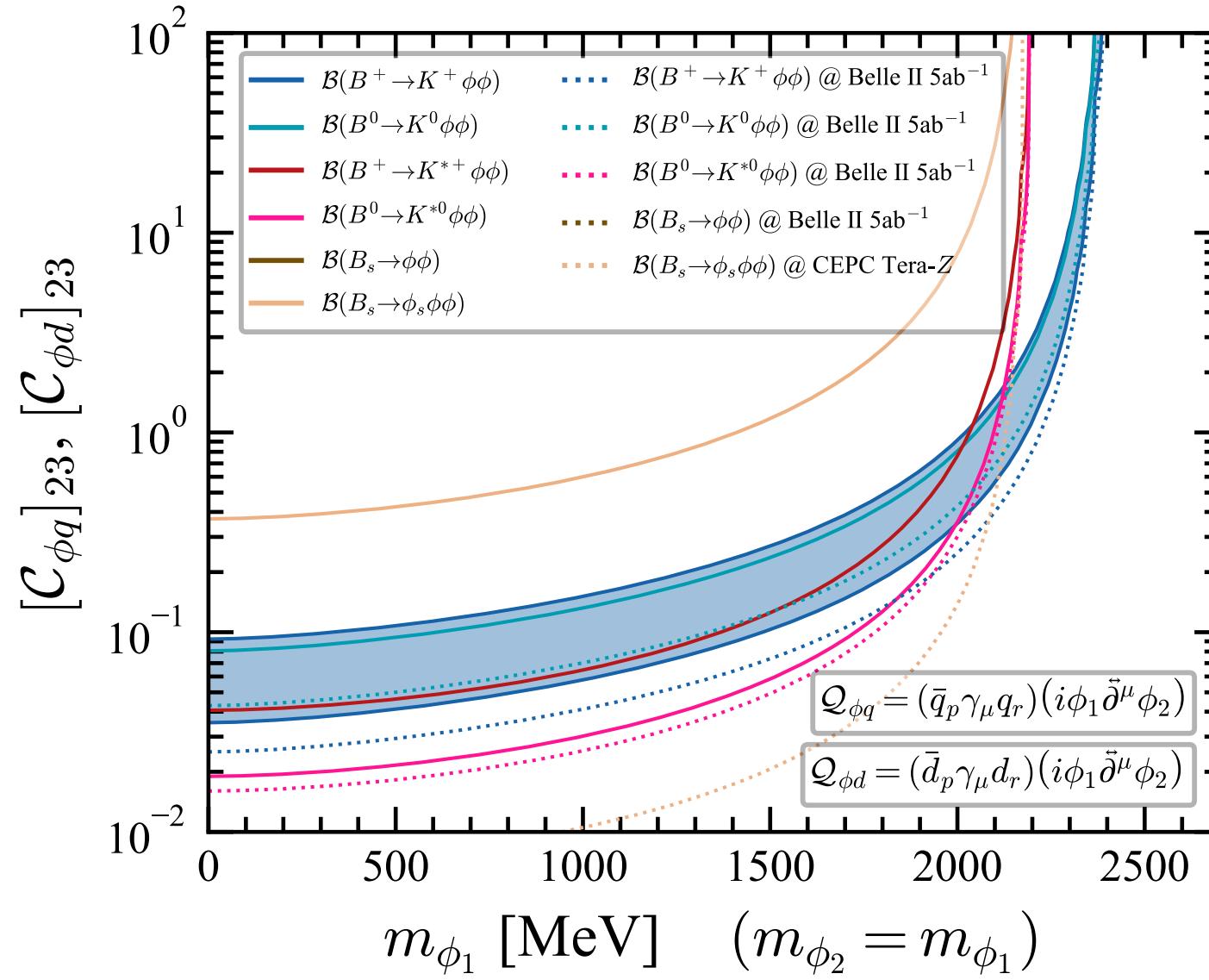
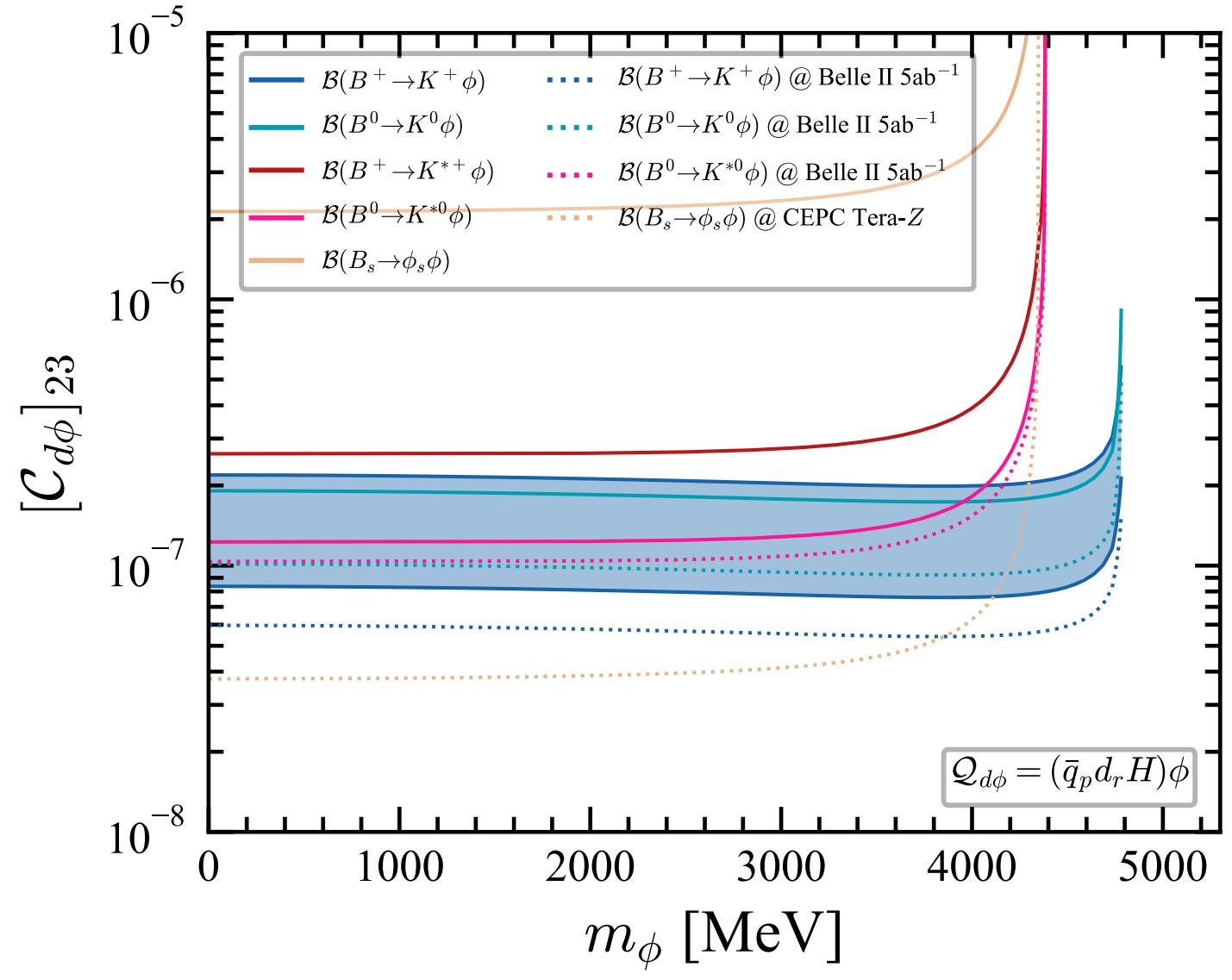
scalar: 4

fermion: 5

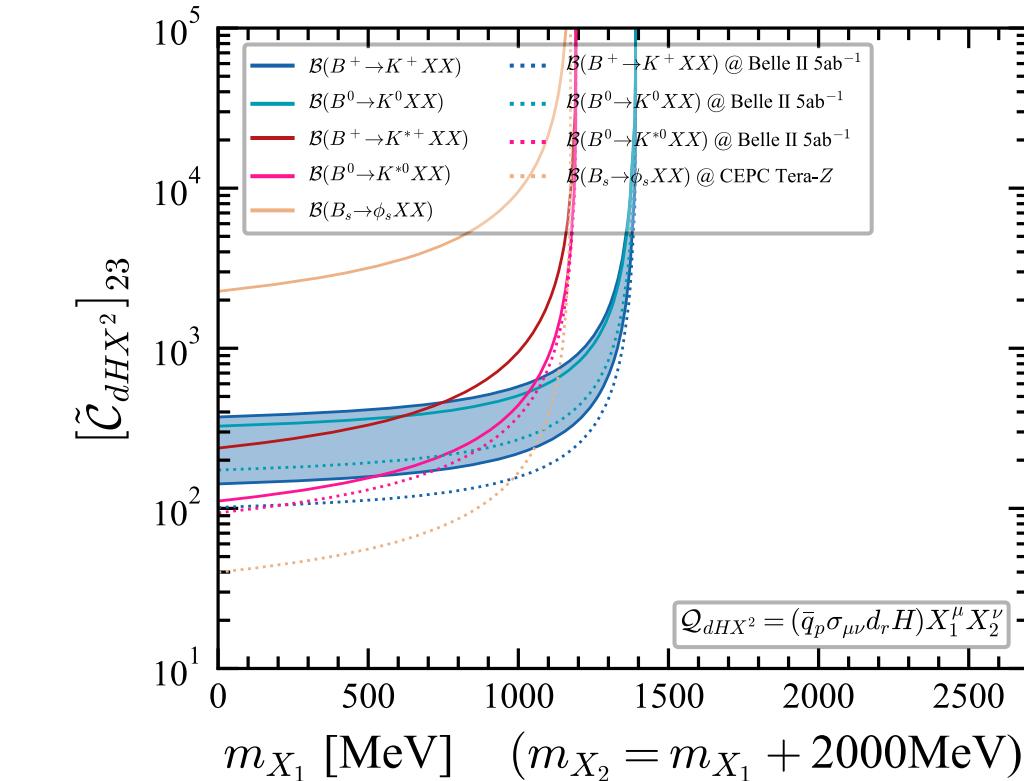
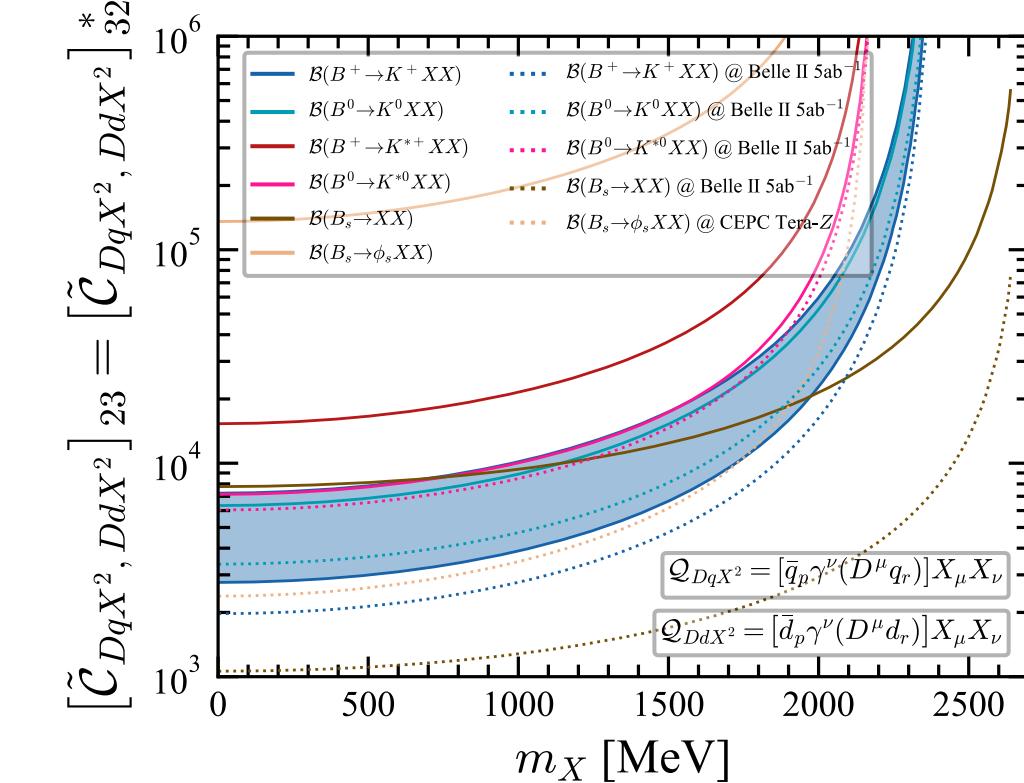
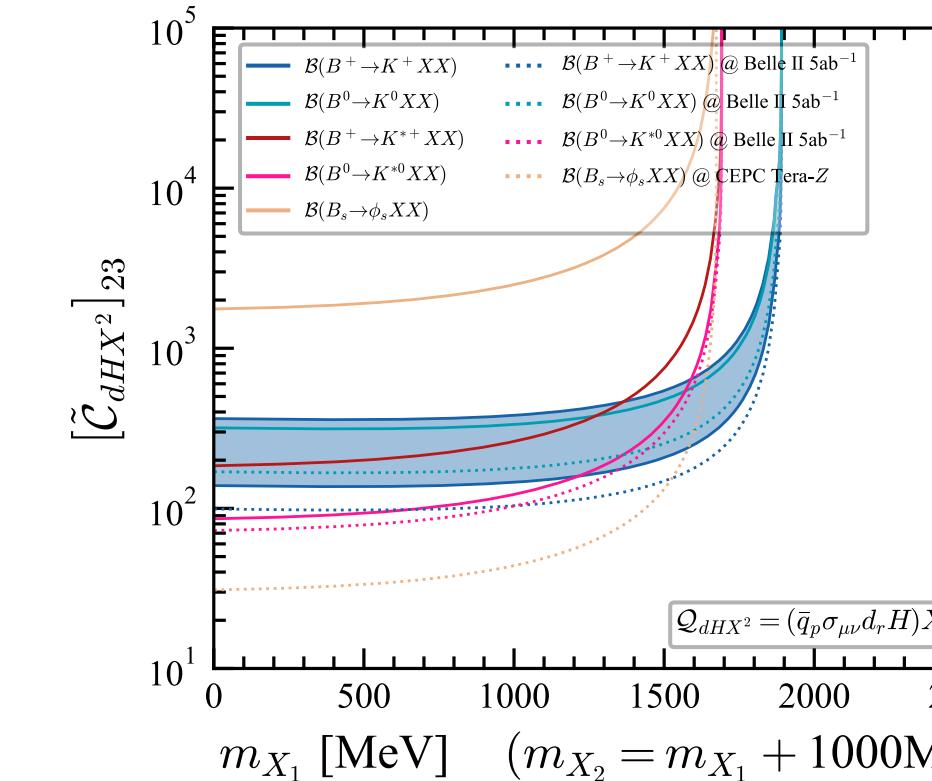
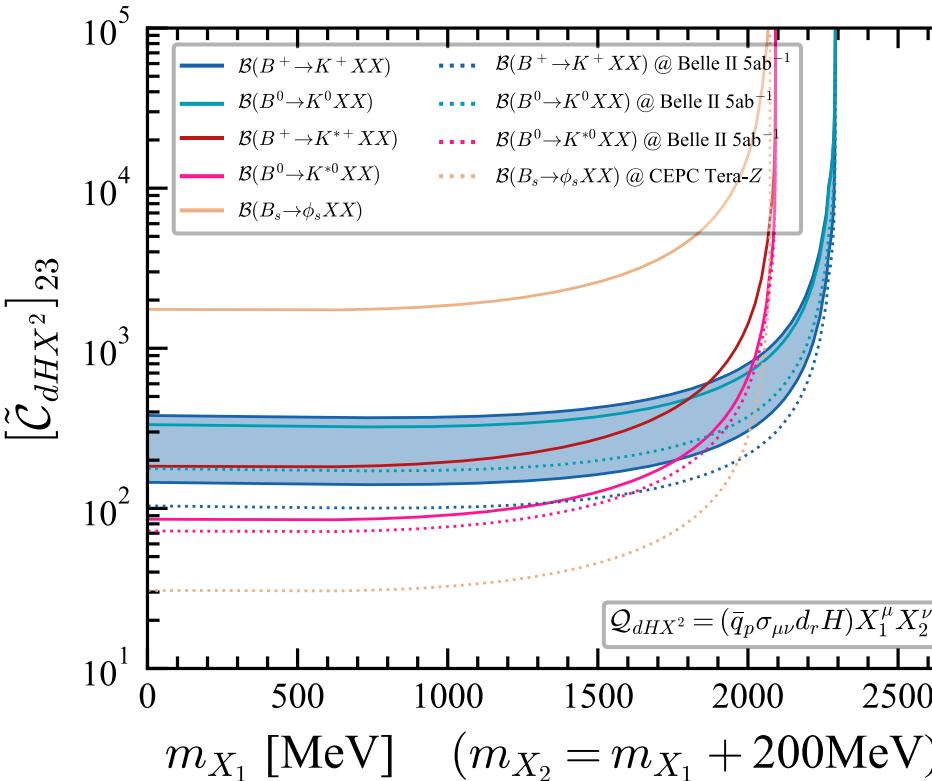
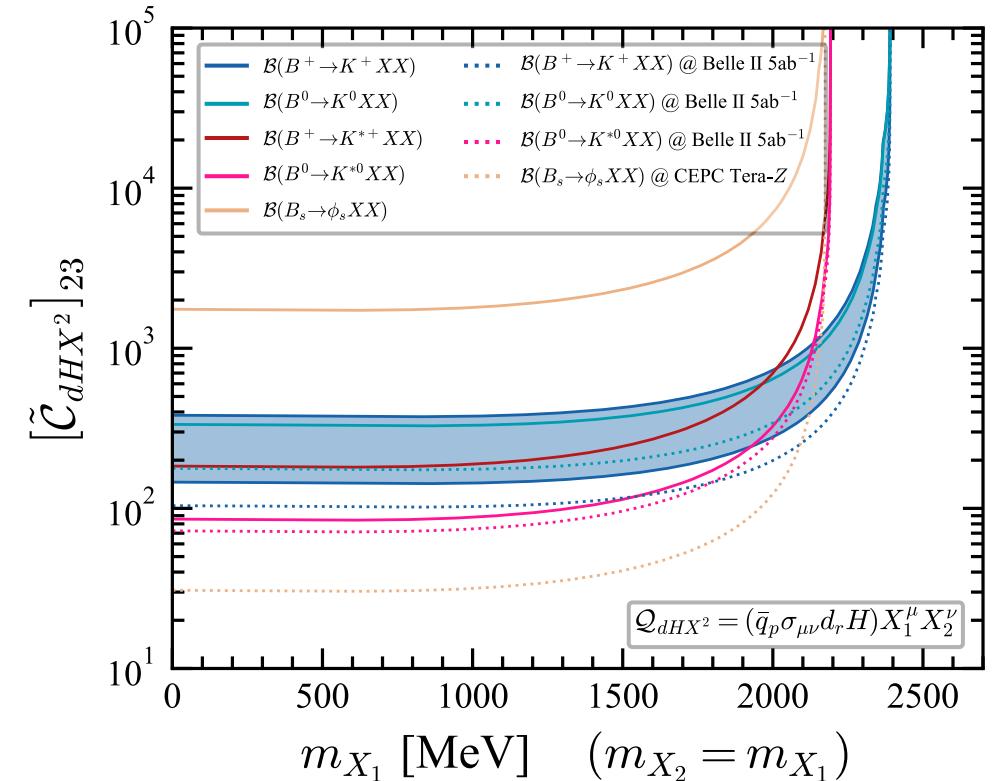
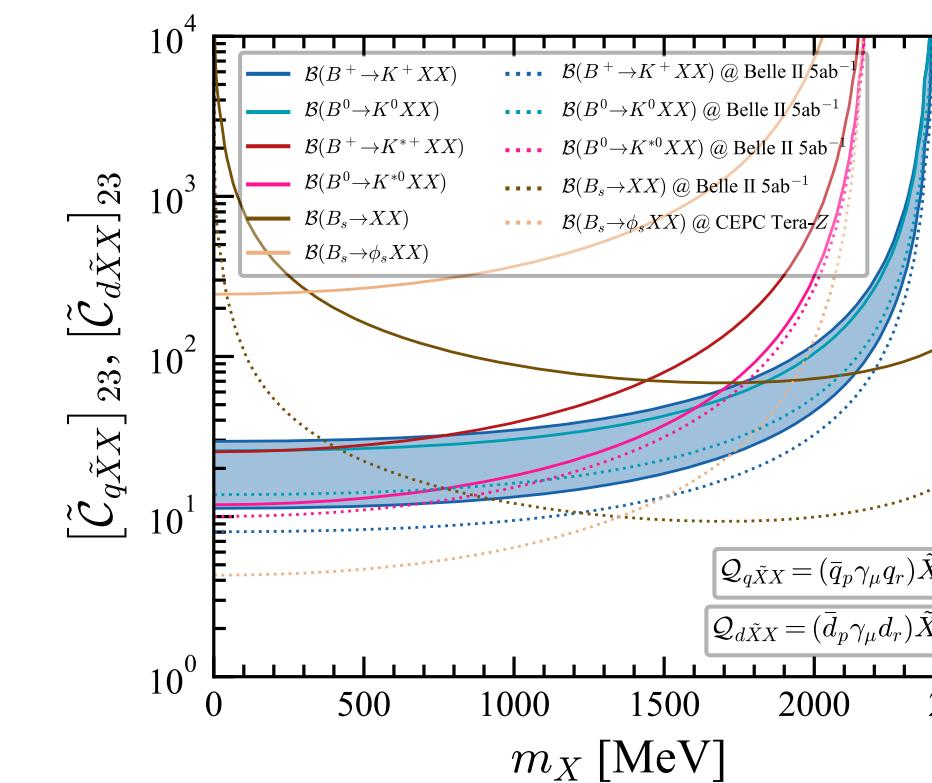
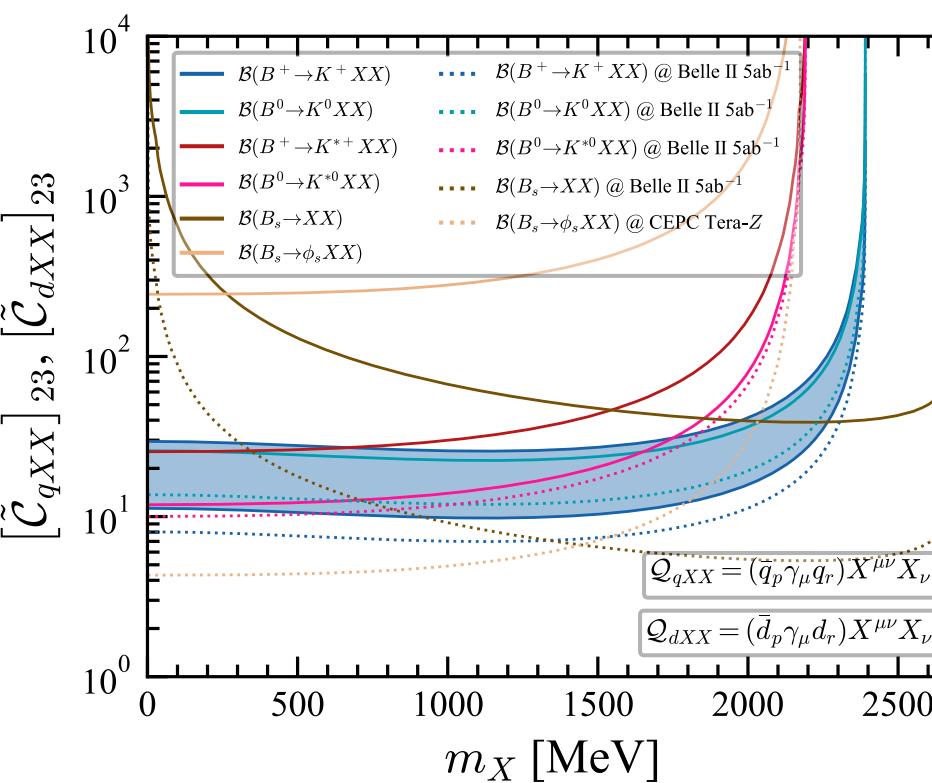
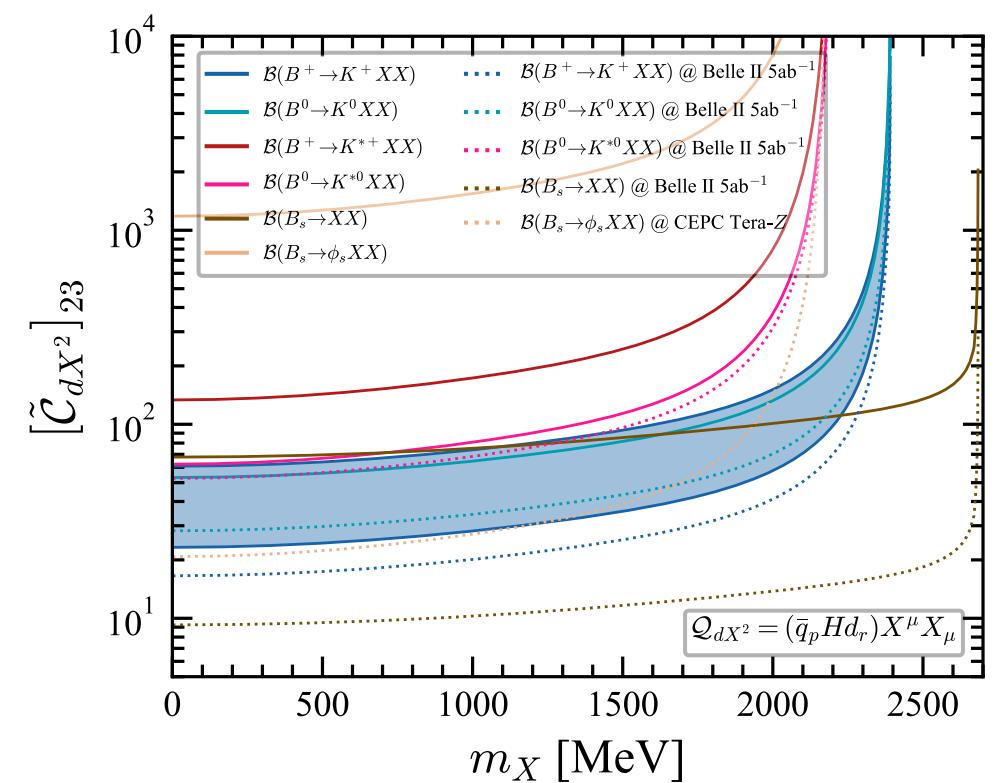
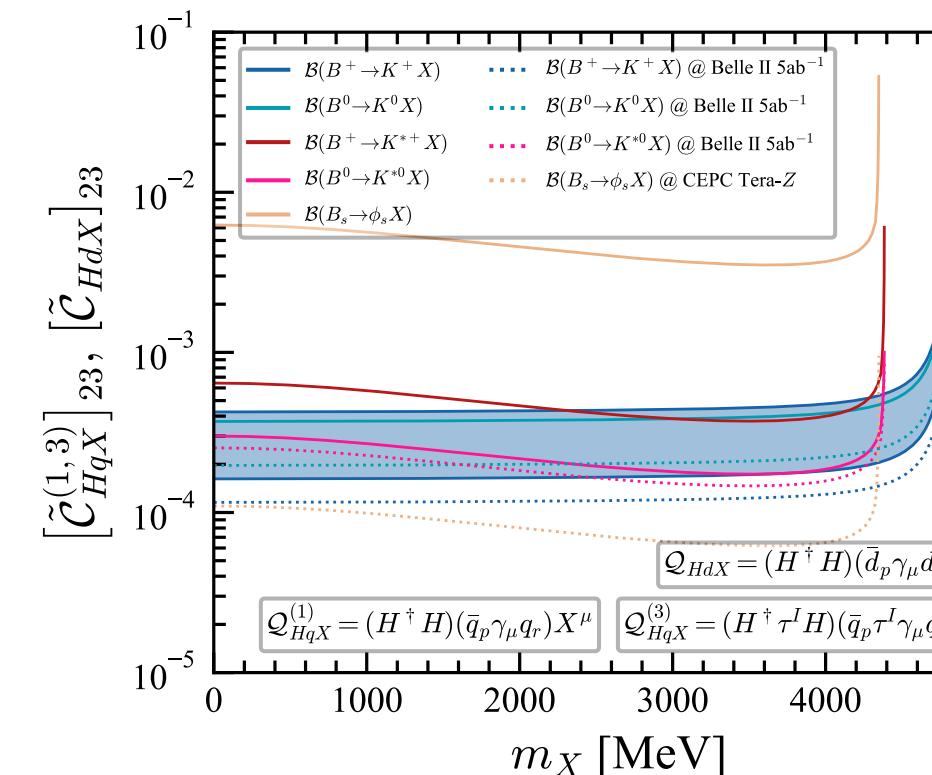
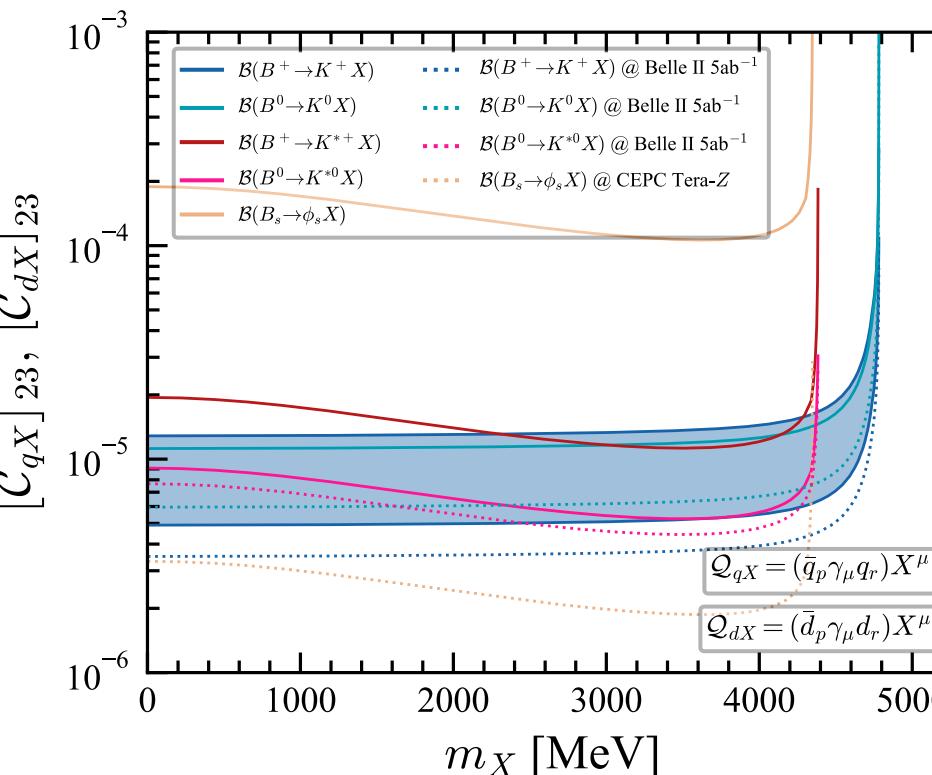
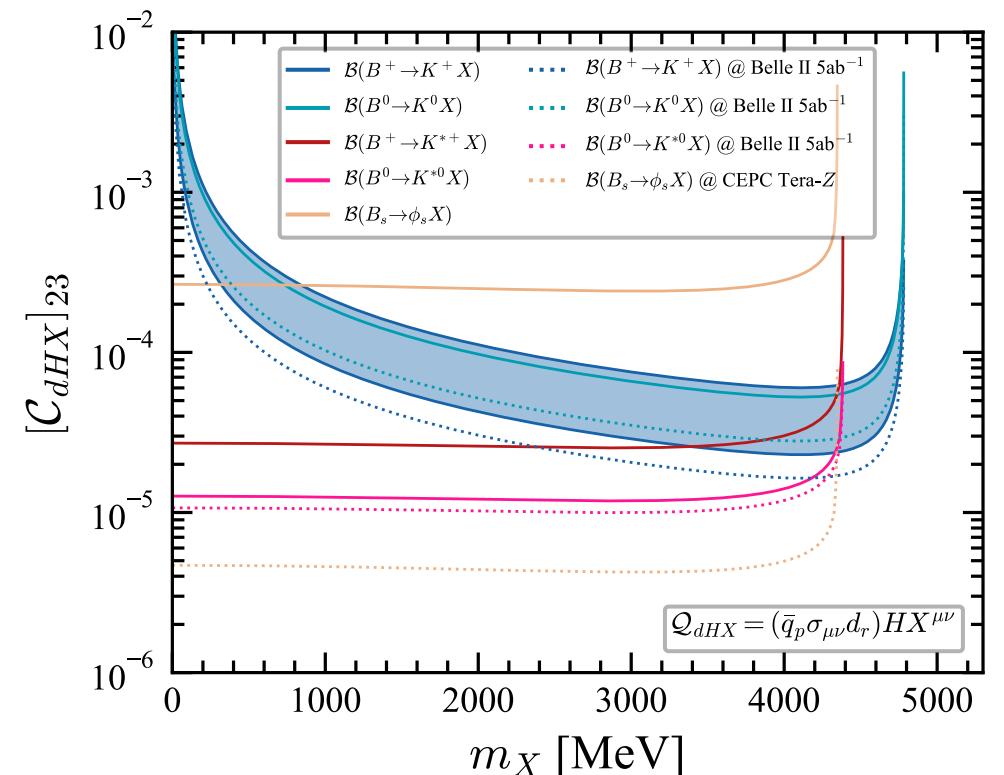
vector: 1+10

ALP: 2

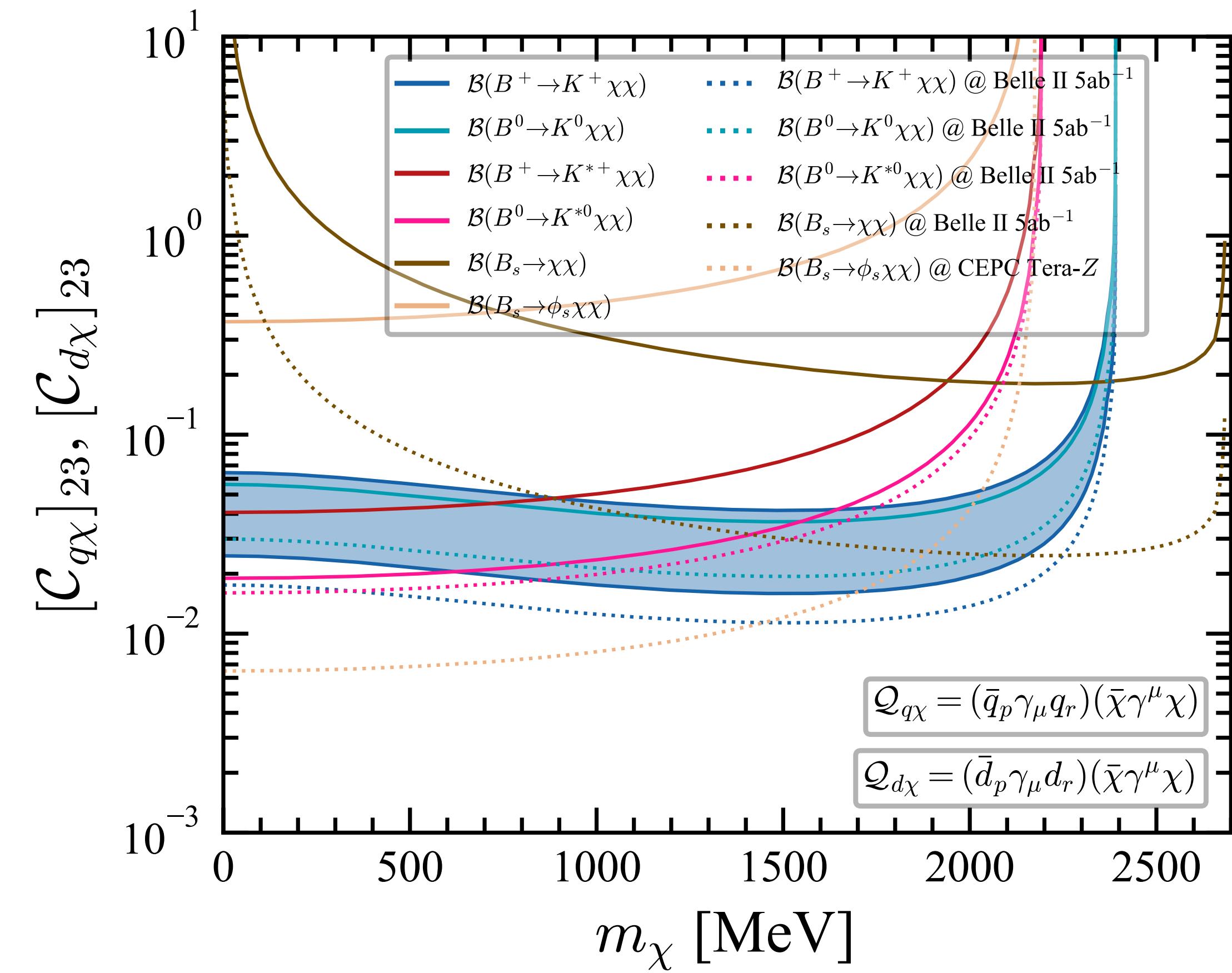
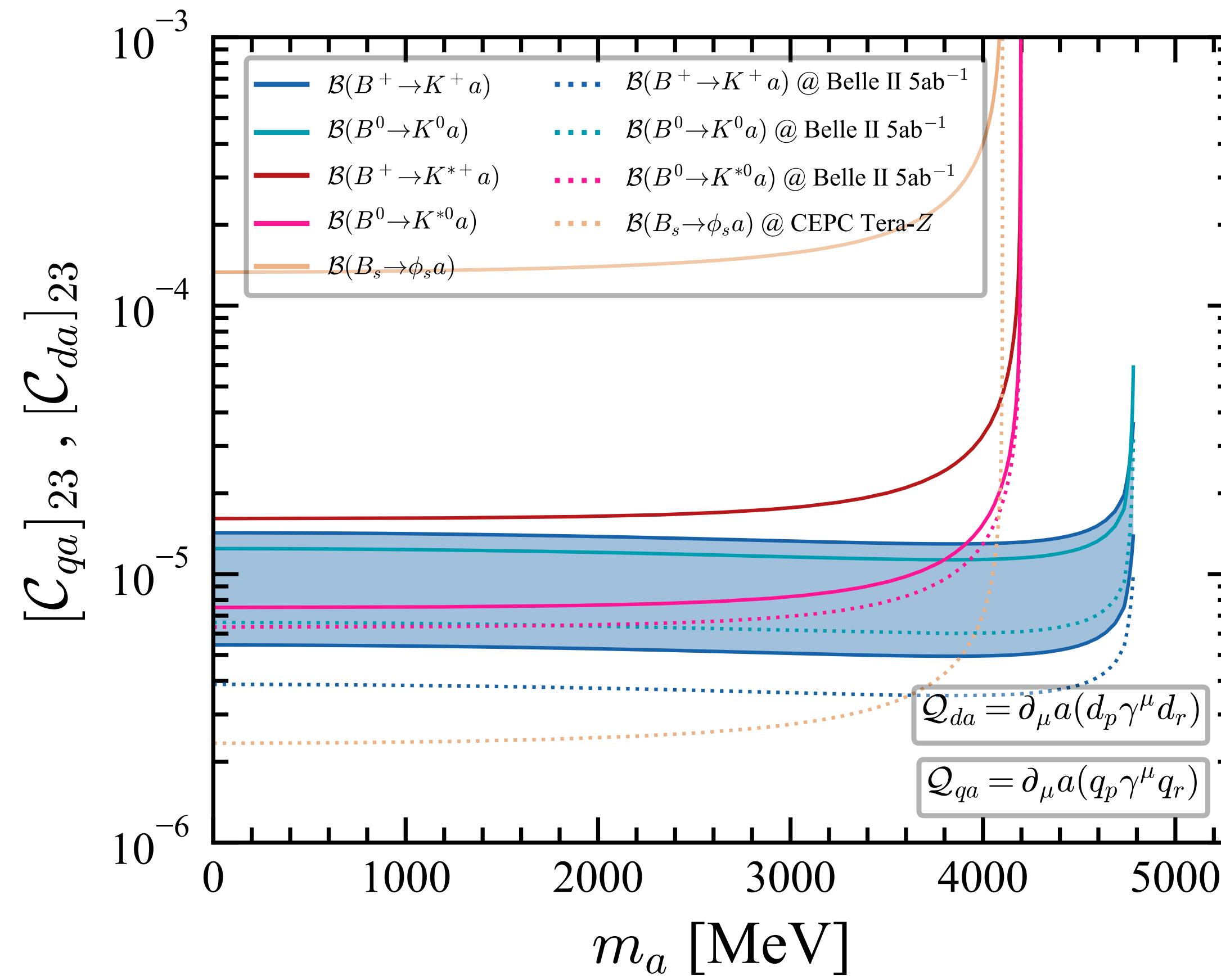
# Dark SMEFT: Scalar



# Dark SMEFT: Vector



# Dark SMEFT: Fermion, ALP



All the operators survive from the constraints of the various FCNC decays.

In the future, all the parameter space to explain the Belle II anomaly can be covered by combining the Belle II (e.g.,  $B^0 \rightarrow K^0 + \text{inv}$ ) and CEPC (e.g.,  $B_s \rightarrow \phi + \text{inv}$  and  $B_s \rightarrow \text{inv}$ ) measurements.

# Dark SMEFT with MFV

- MFV coupling  $b \rightarrow s, b \rightarrow d, s \rightarrow d$  are connected with each other.

$$\mathcal{C}_i^{\text{MFV}} = \begin{cases} \epsilon_0^i \hat{\lambda}_d + \epsilon_1^i \Delta_q \hat{\lambda}_d & \text{for } \mathcal{Q}_i = \mathcal{Q}_{d\phi}, \mathcal{Q}_{d\phi^2}, \mathcal{Q}_{dHX}, \mathcal{Q}_{dHX^2}, \mathcal{Q}_{dX^2}, \\ \epsilon_0^i \mathbb{1} + \epsilon_1^i \Delta_q & \text{for } \mathcal{Q}_i = \mathcal{Q}_{\phi q}, \mathcal{Q}_{q\chi}, \mathcal{Q}_{qXX}, \mathcal{Q}_{q\tilde{X}X}, \mathcal{Q}_{DqX^2}, \mathcal{Q}_{qX}, \mathcal{Q}_{HqX}^{(1,3)}, \mathcal{Q}_{qa}, \\ \epsilon_0^i \mathbb{1} & \text{for } \mathcal{Q}_i = \mathcal{Q}_{\phi d}, \mathcal{Q}_{d\chi}, \mathcal{Q}_{dXX}, \mathcal{Q}_{d\tilde{X}X}, \mathcal{Q}_{DdX^2}, \mathcal{Q}_{dX}, \mathcal{Q}_{HdX}, \mathcal{Q}_{da}, \end{cases}$$

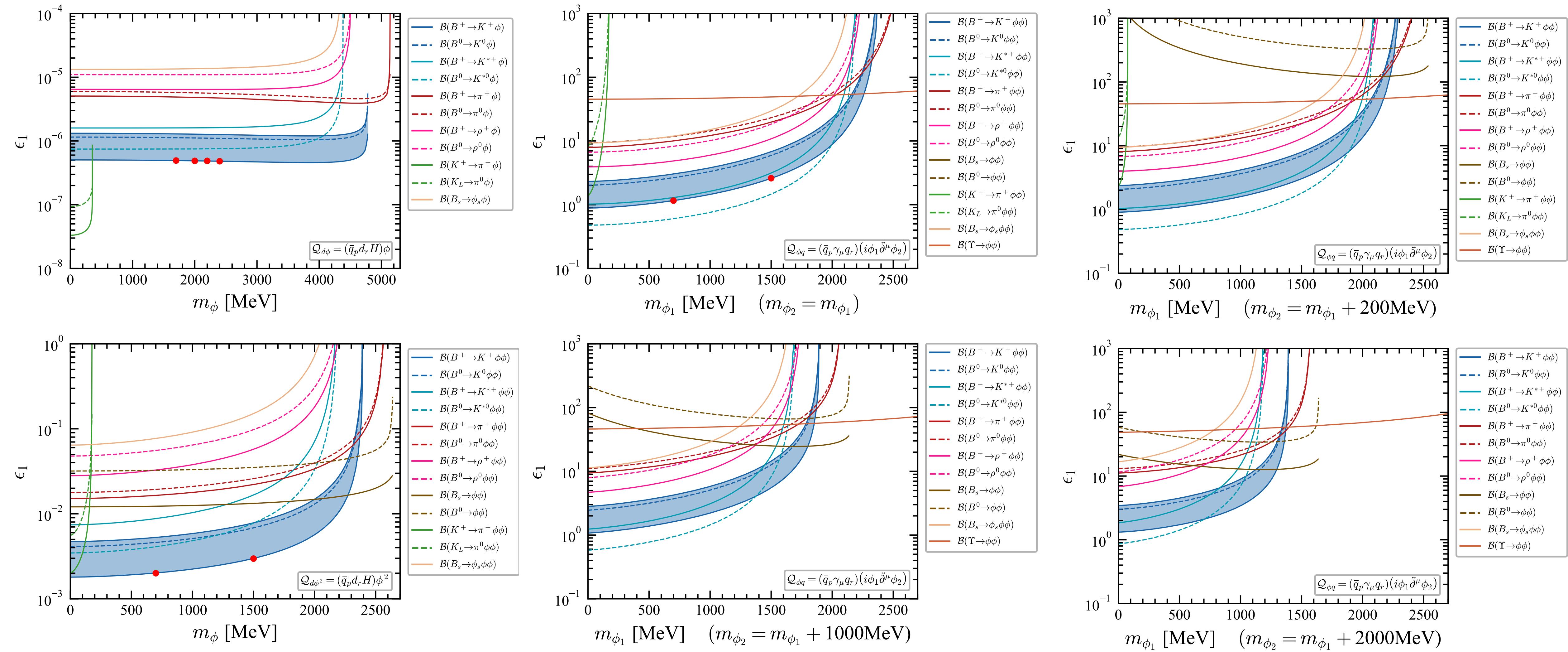
**8 operators are eliminated**

- Numerics

$$\Delta_q = \begin{pmatrix} 0.8 & -3.3 - 1.5i & 79.3 + 35.4i \\ -3.3 + 1.5i & 16.6 & -397.5 + 8.1i \\ 79.3 - 35.4i & -397.5 - 8.1i & 9839.0 \end{pmatrix} \times 10^{-4}$$

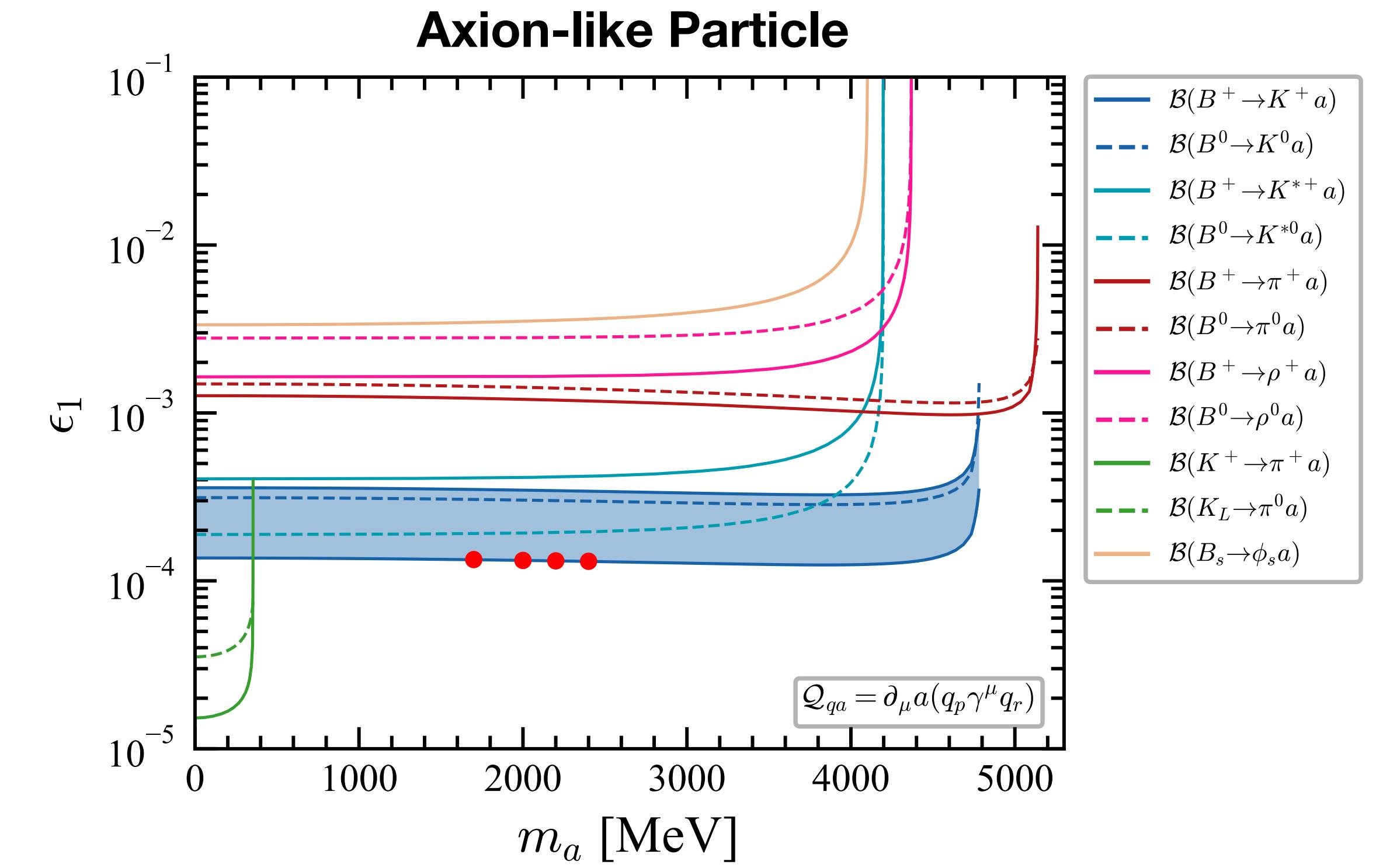
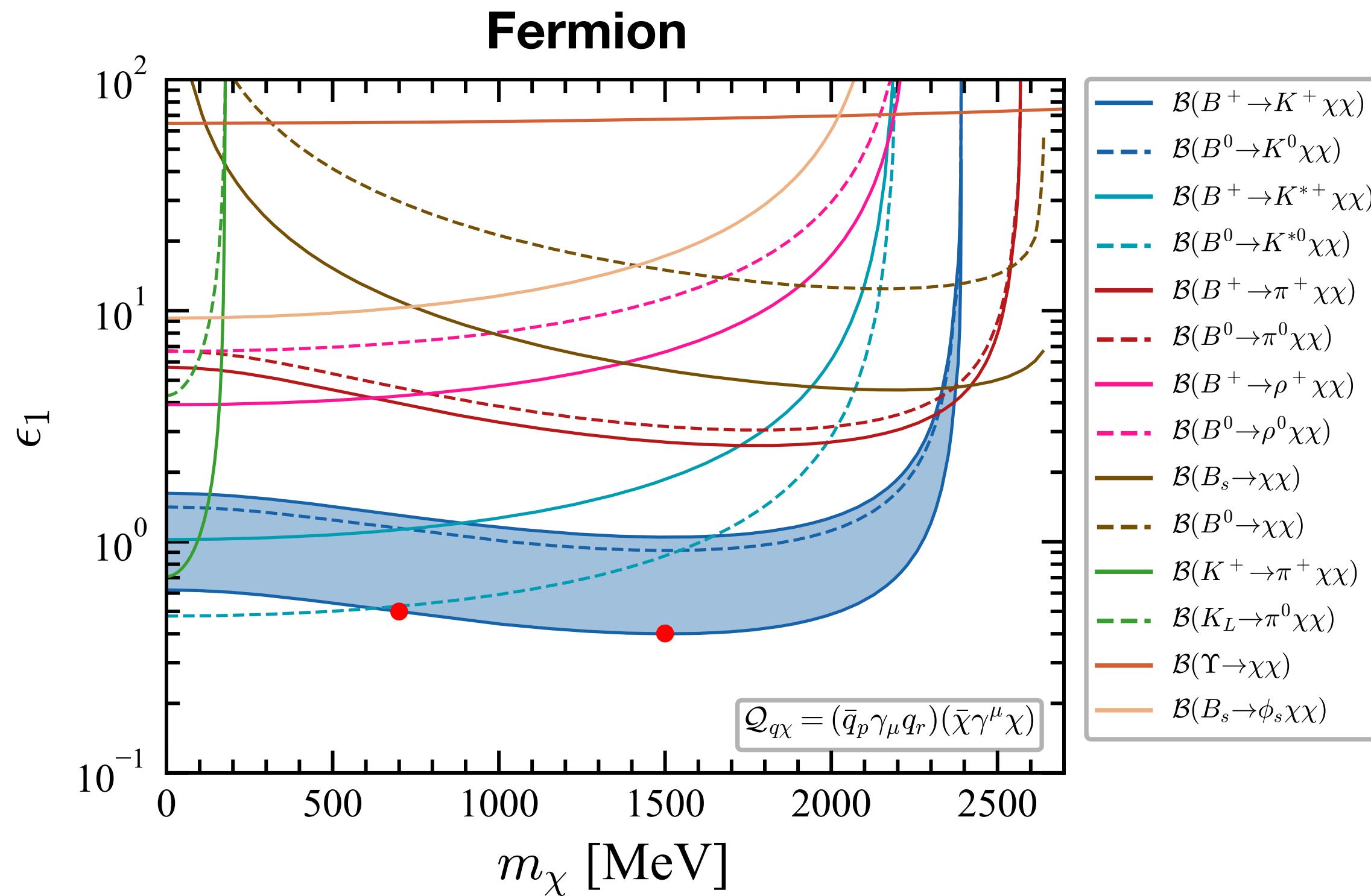
$$\Delta_q \hat{\lambda}_d = \begin{pmatrix} 0.0021 & -0.18 - 0.08i & 191.3 + 85.4i \\ -0.009 + 0.004i & 0.88 & -958.7 + 19.6i \\ 0.21 - 0.10i & -21.1 - 0.4i & 23728.1 \end{pmatrix} \times 10^{-6}$$

# Dark SMEFT with MFV: Scalar



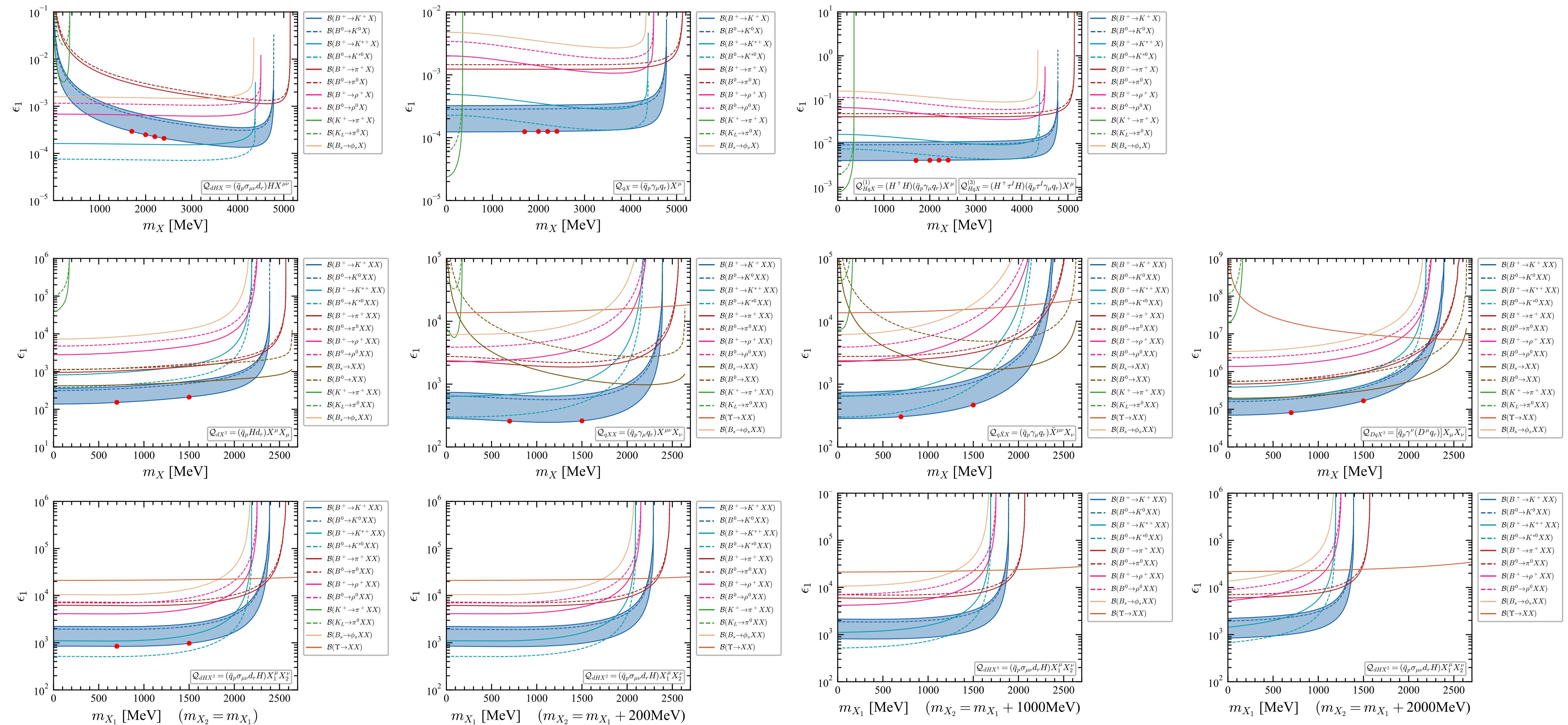
all the operators survive  
some ones highly constrained

# Dark SMEFT with MFV: Fermion, ALP



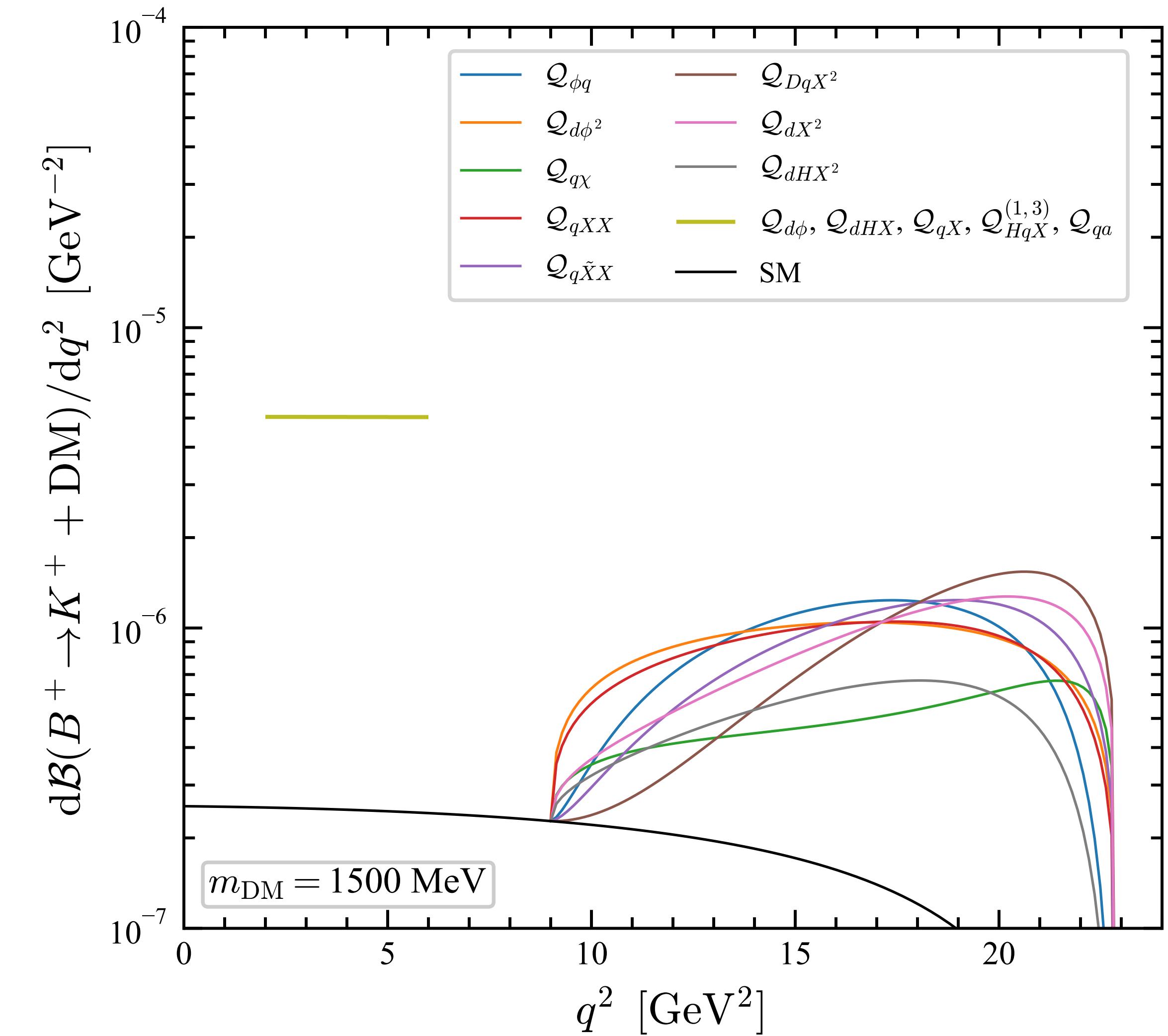
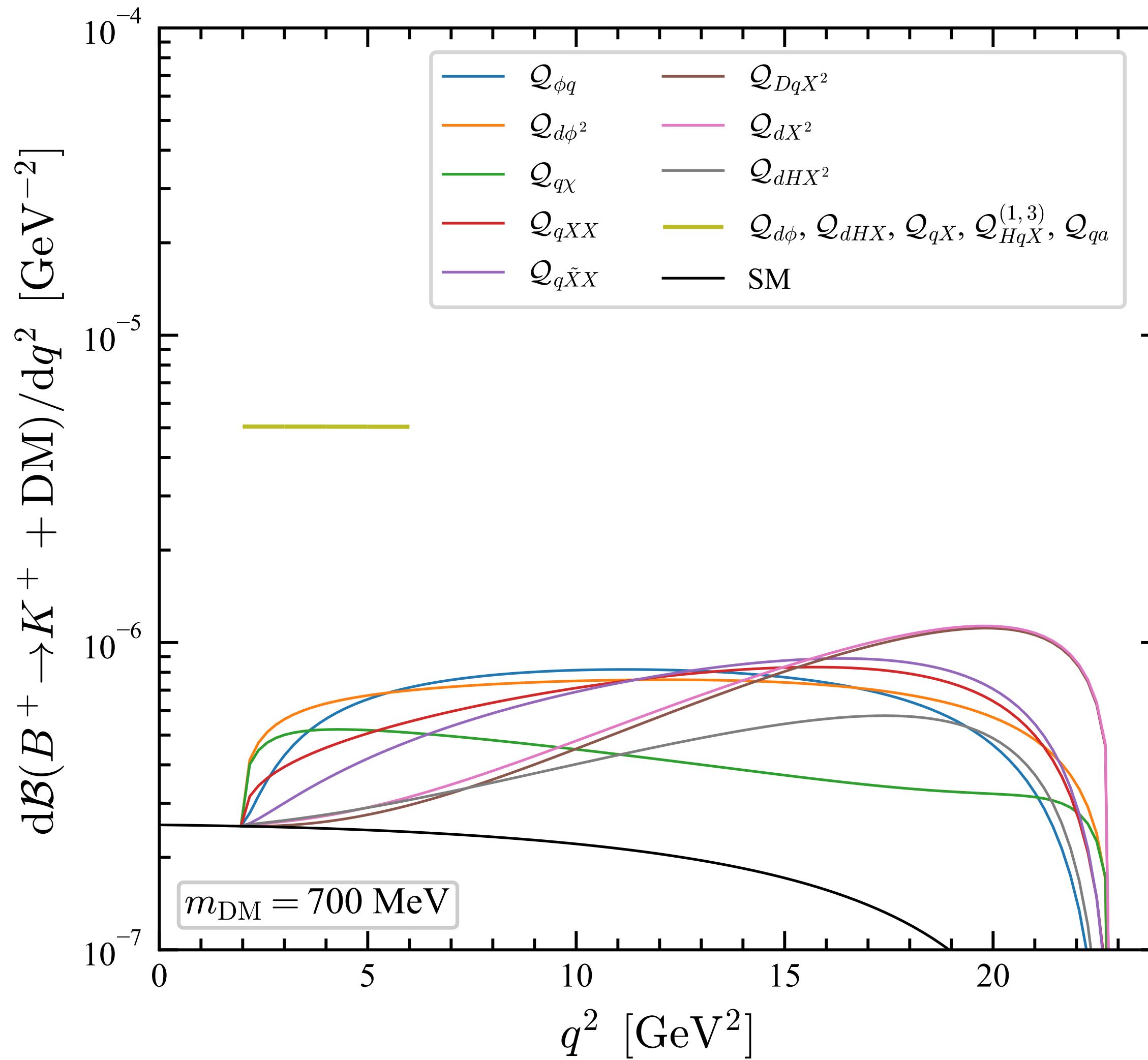
all the operators survive

# Dark SMEFT with MFV: Vector



**all the operators survive, some ones highly constrained**

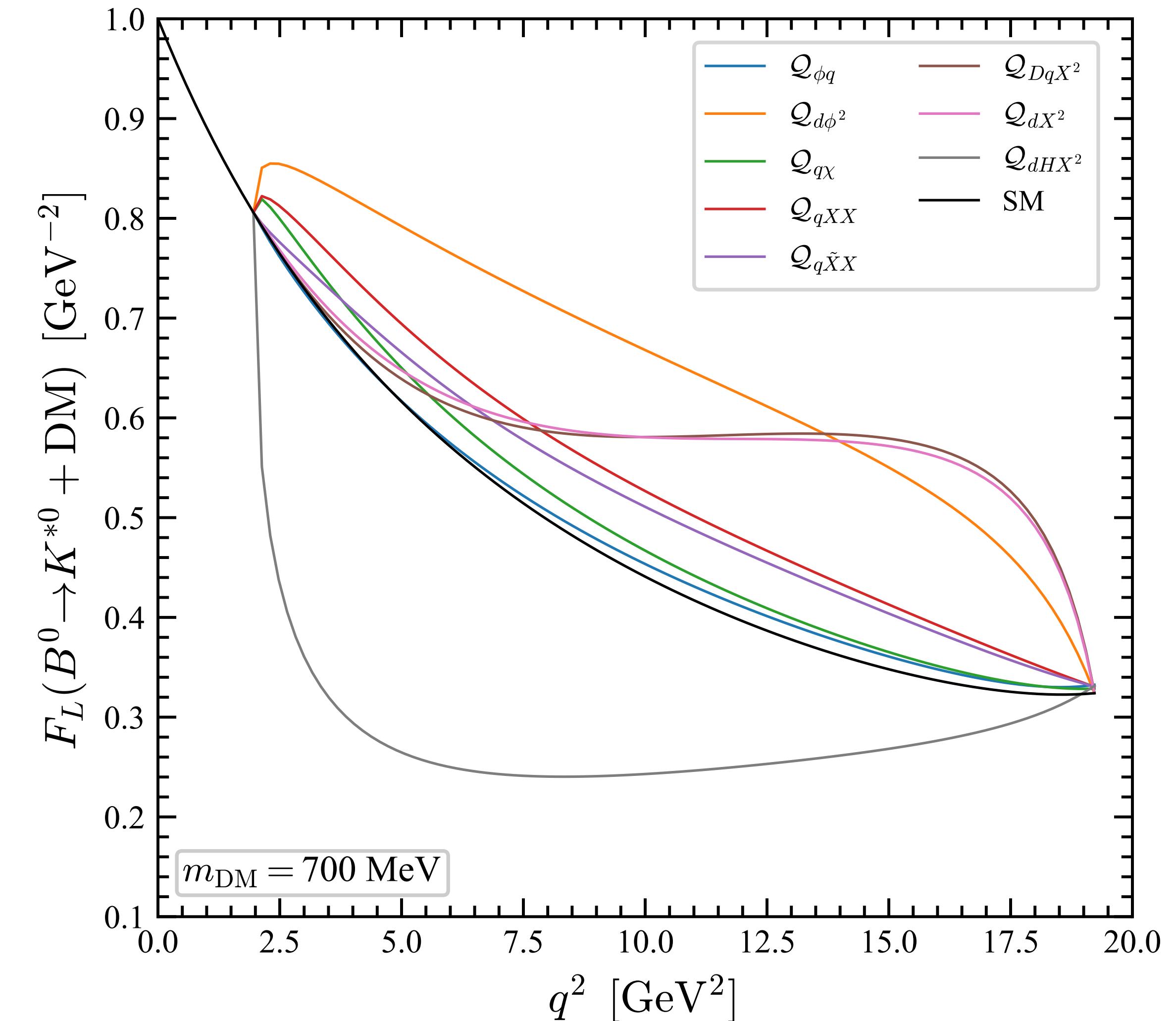
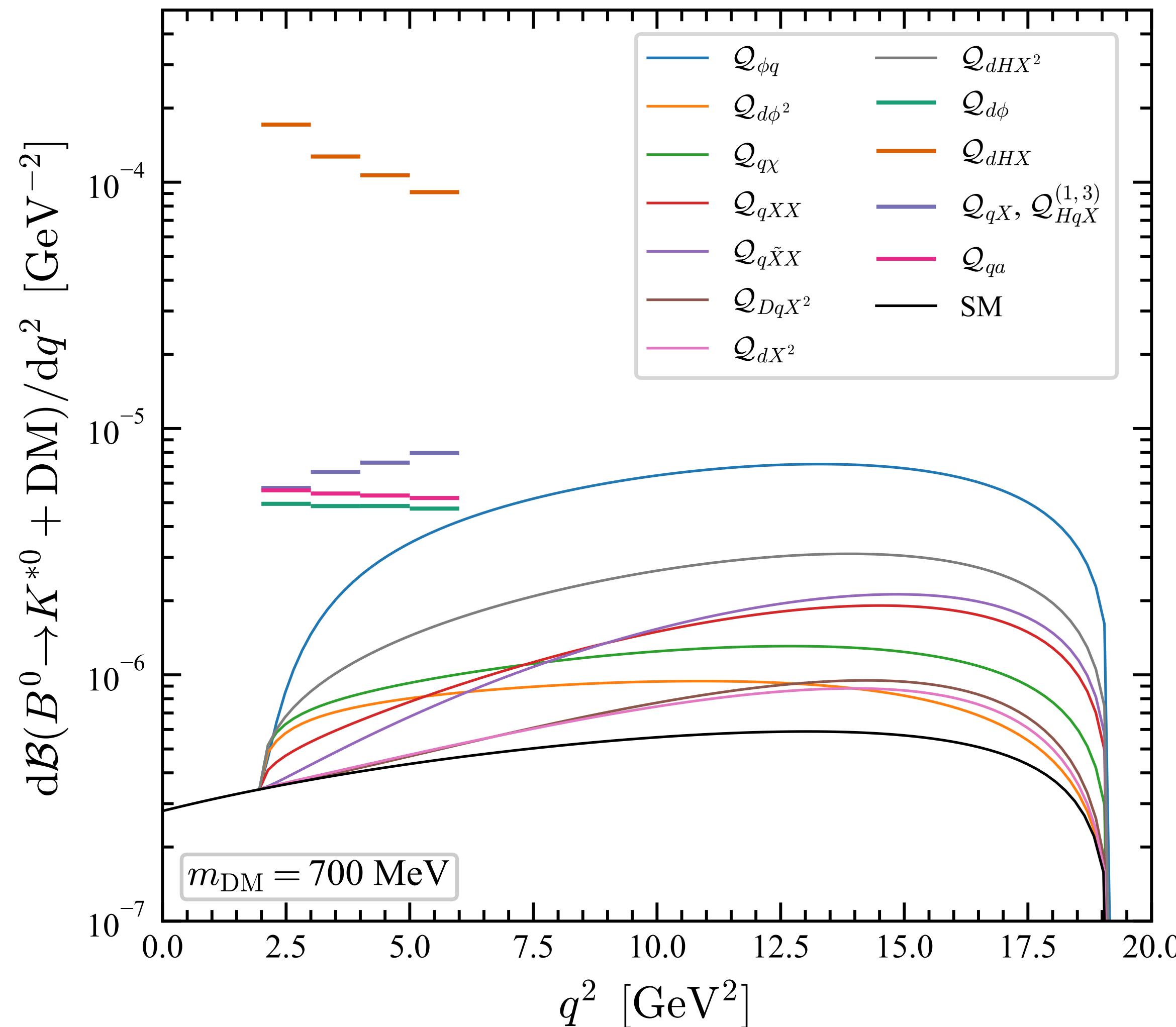
# Dark SMEFT: $dB/dq^2$



Difficult to distinguish the DSMEFT operators by considering only the  $B^+ \rightarrow K^+ \nu \bar{\nu}$  decay. However,

# Dark SMEFT: $dB/dq^2, F_L$

$m_{\text{DM}} = 700 \text{ MeV}$

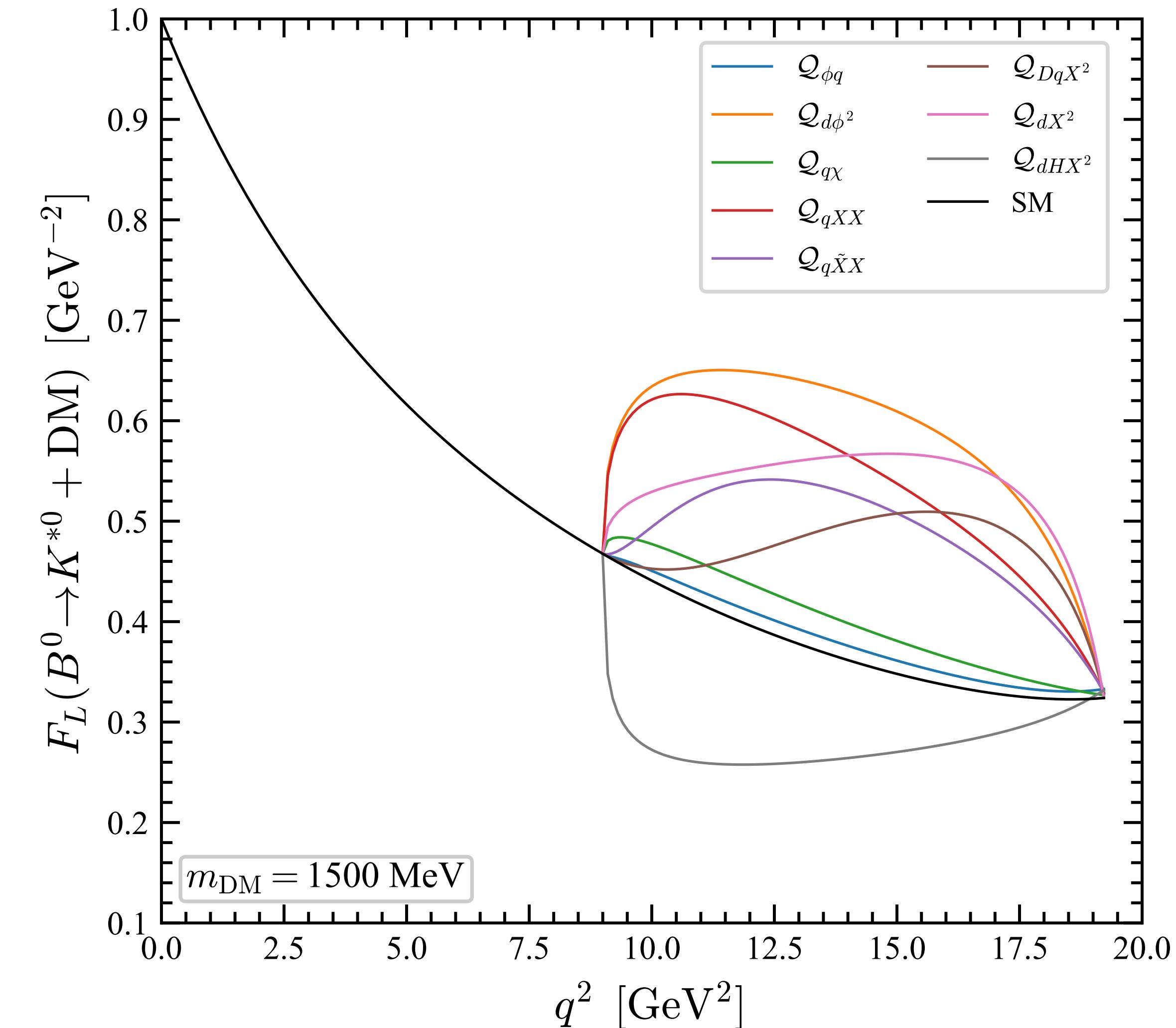
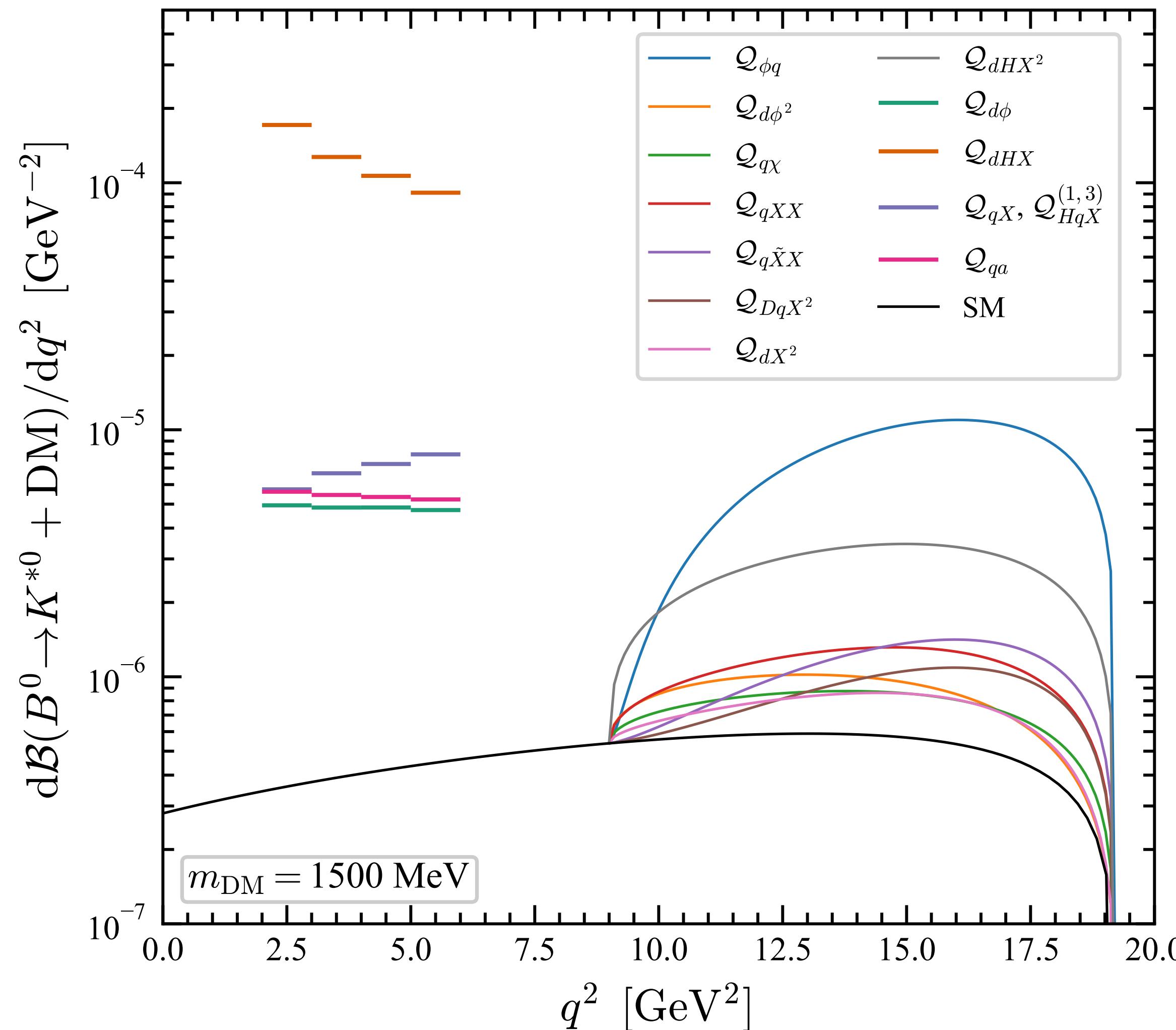


All the operators are distinguishable from each other by combining these observables, except

$\mathcal{Q}_{qXX}$  and  $\mathcal{Q}_{qX\tilde{X}}$   
 $\mathcal{Q}_{dX^2}$  and  $\mathcal{Q}_{DqX}$

# Dark SMEFT: $dB/dq^2, F_L$

$m_{\text{DM}} = 1500 \text{ MeV}$



All the operators are distinguishable from each other by combining these observables, except

$\mathcal{Q}_{qXX}$  and  $\mathcal{Q}_{q\tilde{X}X}$   
 $\mathcal{Q}_{dX^2}$  and  $\mathcal{Q}_{DqX}$

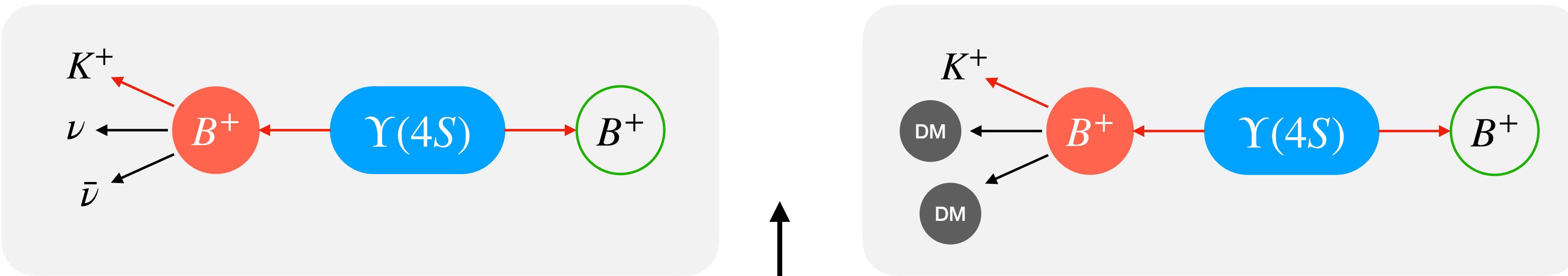
# Conclusion

**HadronToNP**: a package to calculate decay of hadron to new particles

$B \rightarrow K + \text{DM}$ ,  $B \rightarrow \rho + \text{DM}$ ,  $\Lambda_b \rightarrow \Lambda + \text{DM}$ ,  $\Upsilon \rightarrow \text{DM}$ , ...

*to be finished*

$D \rightarrow \pi + \text{DM}$ ,  $D \rightarrow \rho + \text{DM}$ ,  $\Xi_c \rightarrow \Xi + \text{DM}$ ,  $J/\psi \rightarrow \text{DM}$ , ...



SMEFT

Dark SMEFT

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}}}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}}} = 0.46 \pm 0.07$$

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}}}{\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}}} = 29.7 \pm 5.6$$

Belle II excess (if confirmed in the future) implies:

- impossible to explain in SMEFT with MFV
- NP flavour structure is highly non-trivial
- **NP structure in quark sector is beyond MFV**
- **flavour violation is beyond Yukawa coupling**

$\mu_{\text{EW}}$   
LEFT

$\mu_b$

Dark LEFT

All DSMEFT operators survive in general and MFV flavour structure  
 $d\mathcal{B}/dq^2$  and  $F_L$  are useful to distinguish them  
future work: interplay with DM direct detection and relic density

Observable	SM	Exp	Unit
$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$	$4.16 \pm 0.57$	$23 \pm 5^{+5}_{-4}$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^0 \nu \bar{\nu})$	$3.85 \pm 0.52$	< 26	$10^{-6}$
$\mathcal{B}(B^+ \rightarrow K^{*+} \nu \bar{\nu})$	$9.70 \pm 0.94$	< 61	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})$	$9.00 \pm 0.87$	< 18	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \phi \nu \bar{\nu})$	$9.93 \pm 0.72$	< 5400	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \nu \bar{\nu})$	$\approx 0$	< 5.9	$10^{-4}$
$\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})$	$1.40 \pm 0.18$	< 140	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \pi^0 \nu \bar{\nu})$	$6.52 \pm 0.85$	< 900	$10^{-8}$
$\mathcal{B}(B^+ \rightarrow \rho^+ \nu \bar{\nu})$	$4.06 \pm 0.79$	< 300	$10^{-7}$
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$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$8.42 \pm 0.61$	$10.6^{+4.0}_{-3.4} \pm 0.9$	$10^{-11}$
$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$	$3.41 \pm 0.45$	< 300	$10^{-11}$

# **Backup**

# $b \rightarrow s\nu\bar{\nu}$ and $b \rightarrow s\ell\ell$

B.F.Hou, X.Q.Li, M.Shen, Y.D.Yang, **XBY**, 2402.19208

SMEFT notation:  $l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ ,  $q = \begin{pmatrix} u \\ d \end{pmatrix}_L$ ,  $d = d_R$

## ► Prediction

$$\frac{\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})}{\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{SM}}}{\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{SM}}} = 0.46 \pm 0.07$$

## ► prediction

$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{SM}} = (9.00 \pm 0.87) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{SMEFT}} = (50^{+17}_{-16}) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{exp}} < 18 \times 10^{-6}$$

## ► Only $\mathcal{O}_{lq}^{(3)}$ is relevant with $R_{D^{(*)}}$

## ► $\mathcal{O}_{ld}$ can explain the $B^+ \rightarrow K^+\nu\bar{\nu}$ data

## ► $\mathcal{O}_{ld}$ also induce $O'_{9,ij}$ and $O'_{10,ij}$

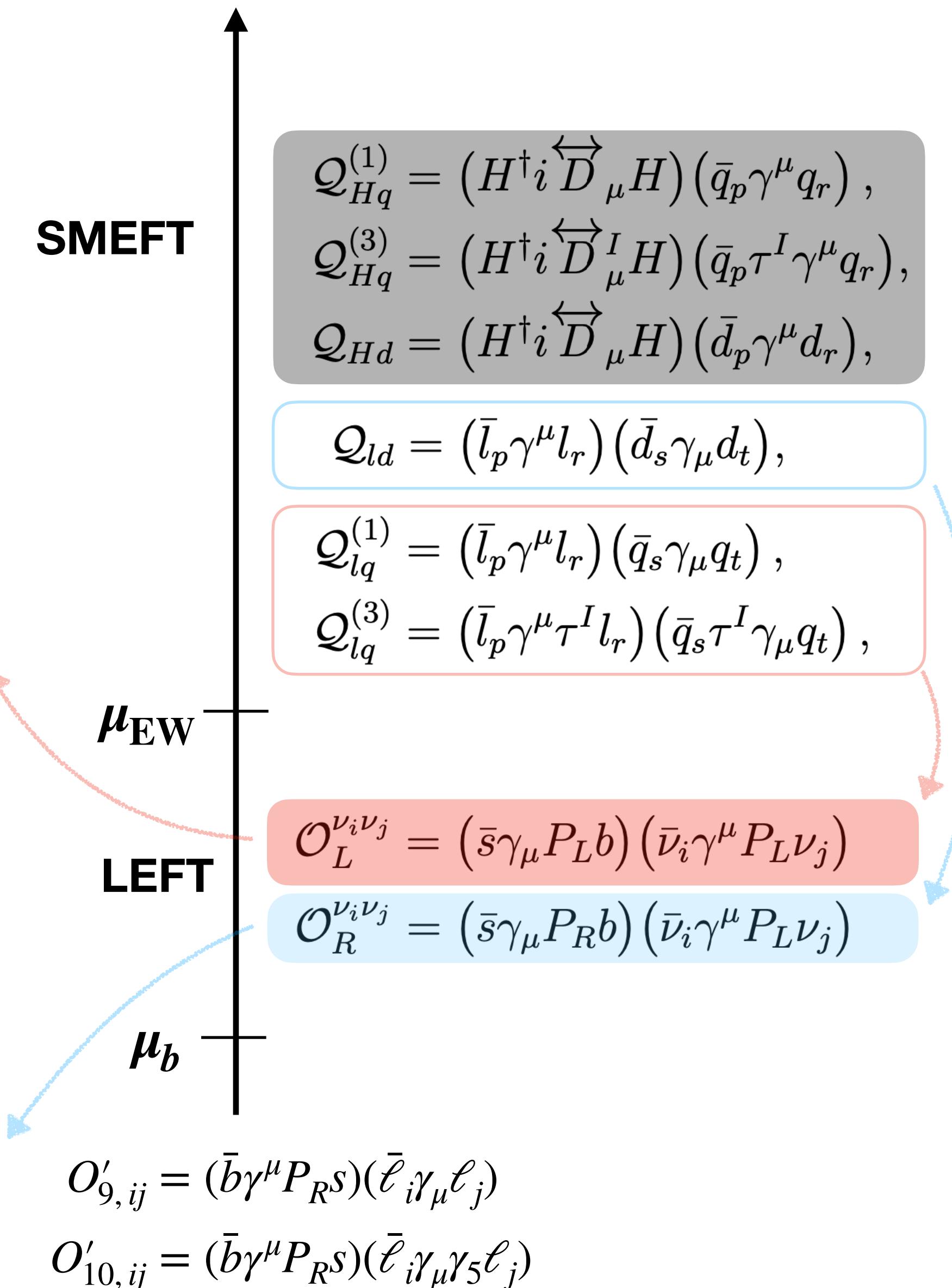
## ► They can't improve the $b \rightarrow s\ell\ell$ fit

►  $O'_{9e}$  and  $O'_{10\mu}$  worsen the fit. **weird** (LFV,  $\tau\tau \gg ee, \mu\mu$ )

►  $O'_{9,ij}$  and  $O'_{10,ij}$  with  $i = j = \tau$  has no effect.

►  $O'_{9,ij}$  and  $O'_{10,ij}$  with  $i \neq j$  (i.e. LFV) has no effect.

conflict



induce  $\bar{s}bZ$  interaction,  
Thus, universally affect  
 $b \rightarrow se^+e^-, \mu^+\mu^-, \tau^+\tau^-$

one LEFT operator!  
just the SM operator

# Backup

$$\begin{aligned}\mathcal{Q}_{d\phi} &= (\bar{q}_p d_r H) \phi + \text{h.c.}, & \mathcal{Q}_{d\phi^2} &= (\bar{q}_p d_r H) \phi^2 + \text{h.c.}, \\ \mathcal{Q}_{\phi q} &= (\bar{q}_p \gamma_\mu q_r) (i \phi_1 \overleftrightarrow{\partial}^\mu \phi_2), & \mathcal{Q}_{\phi d} &= (\bar{d}_p \gamma_\mu d_r) (i \phi_1 \overleftrightarrow{\partial}^\mu \phi_2),\end{aligned}\quad (4.2)$$

$$\mathcal{Q}_{q\chi} = (\bar{q}_p \gamma_\mu q_r) (\bar{\chi} \gamma^\mu \chi), \quad \mathcal{Q}_{d\chi} = (\bar{d}_p \gamma_\mu d_r) (\bar{\chi} \gamma^\mu \chi), \quad (4.3)$$

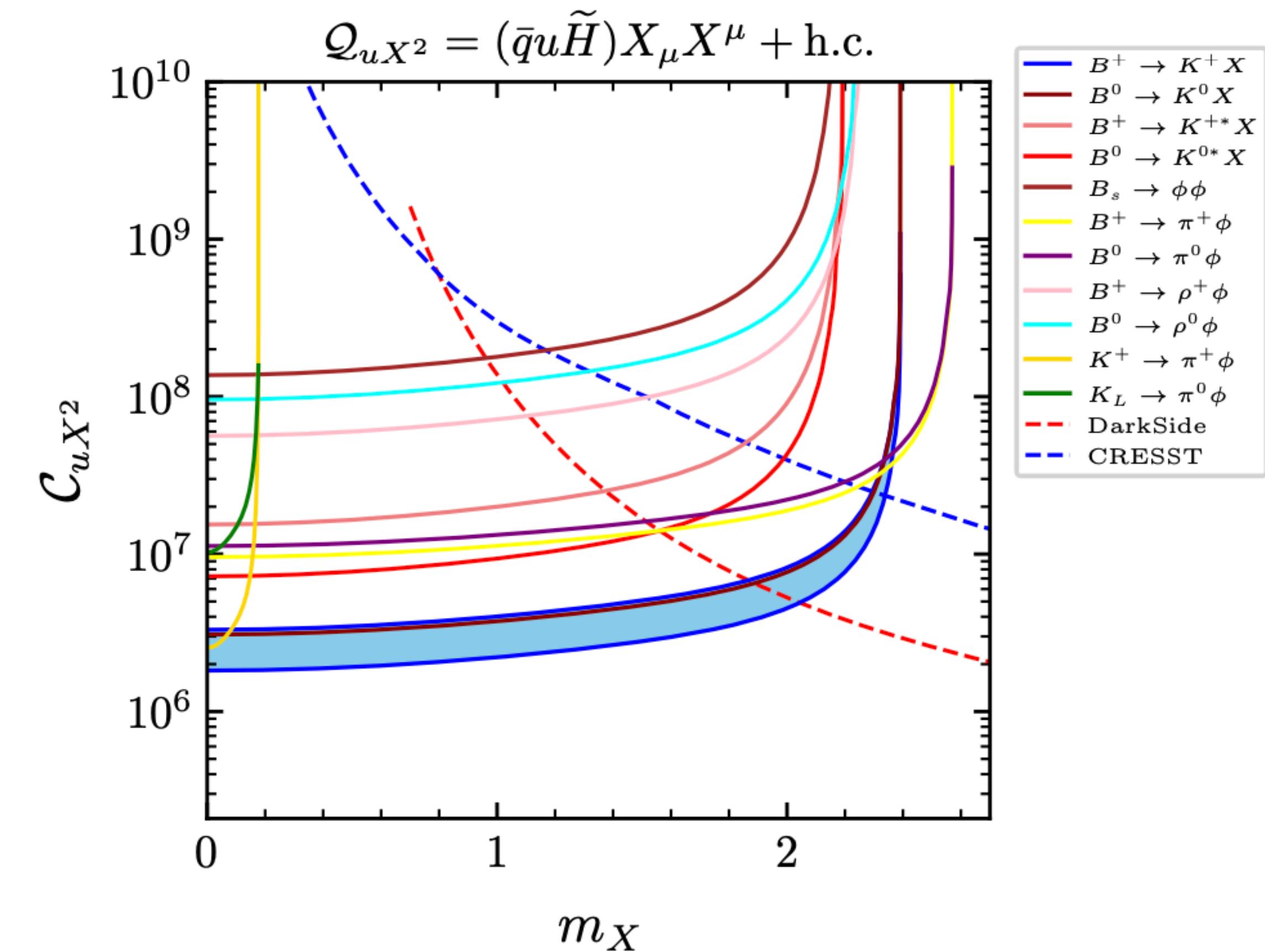
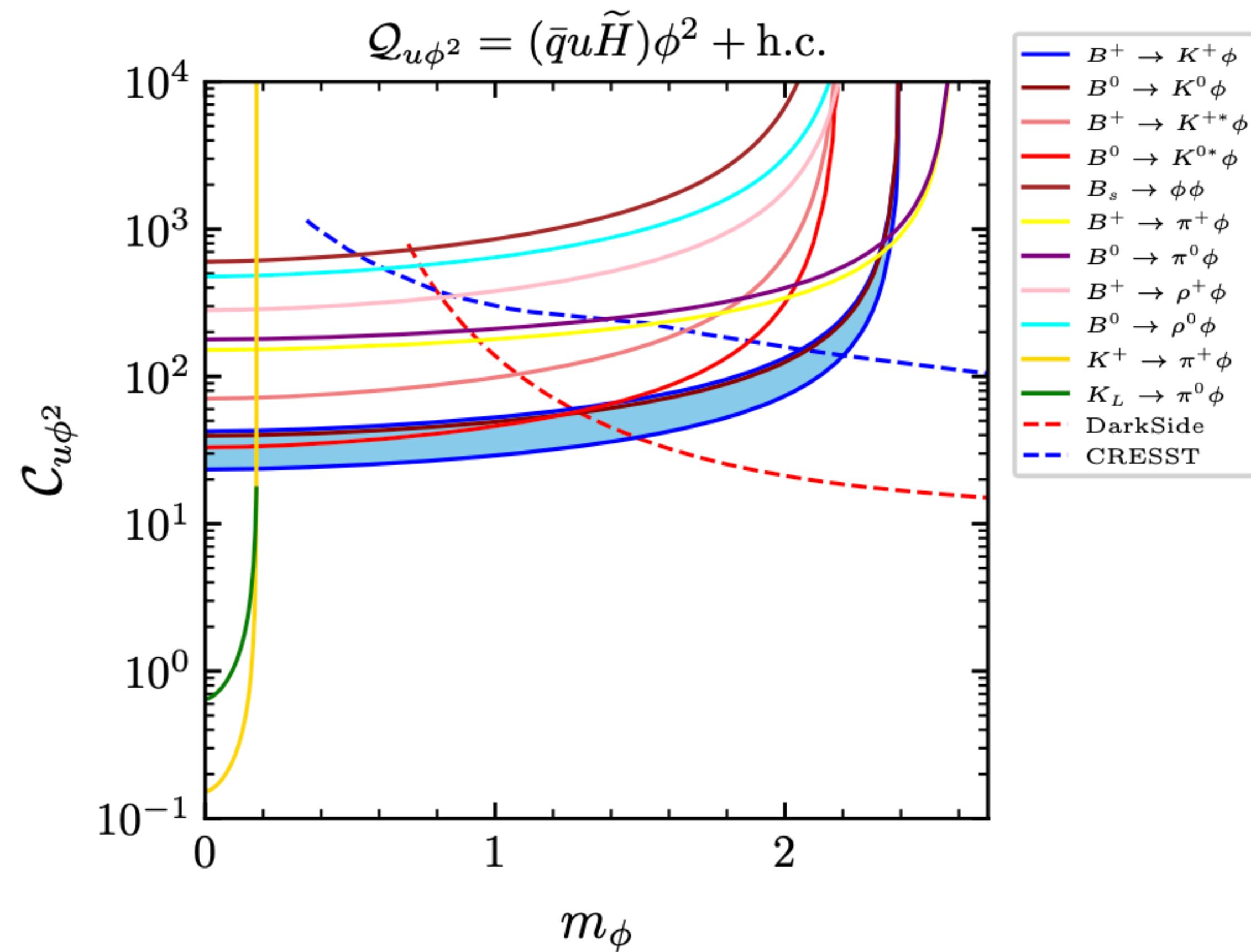
$$\mathcal{Q}_{dHX} = (\bar{q}_p \sigma_{\mu\nu} d_r) H X^{\mu\nu} + \text{h.c.}, \quad (4.4)$$

$$\begin{aligned}\mathcal{Q}_{dX} &= (\bar{d}_p \gamma_\mu d_r) X^\mu, & \mathcal{Q}_{HdX} &= (H^\dagger H) (\bar{d}_p \gamma^\mu d_r) X_\mu, \\ \mathcal{Q}_{qX} &= (\bar{q}_p \gamma_\mu q_r) X^\mu, & \mathcal{Q}_{HqX}^{(1)} &= (H^\dagger H) (\bar{q}_p \gamma^\mu q_r) X_\mu, \\ \mathcal{Q}_{dX^2} &= (\bar{q}_p d_r H) X_\mu X^\mu + \text{h.c.}, & \mathcal{Q}_{HqX}^{(3)} &= (H^\dagger \tau^I H) (\bar{q}_p \tau^I \gamma^\mu q_r) X_\mu, \\ \mathcal{Q}_{qXX} &= (\bar{q}_p \gamma_\mu q_r) X^{\mu\nu} X_\nu, & \mathcal{Q}_{dXX} &= (\bar{d}_p \gamma_\mu d_r) X^{\mu\nu} X_\nu, \\ \mathcal{Q}_{q\tilde{X}X} &= (\bar{q}_p \gamma_\mu q_r) \tilde{X}^{\mu\nu} X_\nu, & \mathcal{Q}_{d\tilde{X}X} &= (\bar{d}_p \gamma_\mu d_r) \tilde{X}^{\mu\nu} X_\nu, \\ \mathcal{Q}_{DqX^2} &= i(\bar{q}_p \gamma^\mu D^\nu q_r) X_\mu X_\nu + \text{h.c.}, & \mathcal{Q}_{DdX^2} &= i(\bar{d}_p \gamma^\mu D^\nu d_r) X_\mu X_\nu + \text{h.c.},\end{aligned}\quad (4.5)$$

$$\mathcal{C}_i = \tilde{\mathcal{C}}_i \cdot \begin{cases} (m_X/\Lambda)^2 & \text{for } \mathcal{Q}_i = \mathcal{Q}_{dX^2}, \mathcal{Q}_{DdX^2}, \mathcal{Q}_{DqX^2}, \mathcal{Q}_{dHX^2}, \\ (m_X/\Lambda) & \text{for } \mathcal{Q}_i = \text{others}. \end{cases}$$

$$\mathcal{Q}_{qa} = (\bar{q}_p \gamma_\mu q_r) \partial^\mu a, \quad \mathcal{Q}_{da} = (\bar{d}_p \gamma_\mu d_r) \partial^\mu a, \quad (4.7)$$

# Backup



very preliminary result for top-philic DM

# Backup

One can also apply the MFV hypothesis to the lepton sector. However, since the mechanism of neutrino mass generation is still unknown, there are different approaches to formulate the leptonic MFV [73–79]. Here, we consider the realization of leptonic MFV within the so-called minimal field content [73, 74], in which the neutrino masses are generated by the Weinberg operator. In this case, the Yukawa interactions in the lepton sector can be written as

$$-\Delta\mathcal{L} = \bar{e}Y_eH^\dagger l + \frac{1}{2\Lambda_{LN}}(\bar{l}^c\tau_2 H)Y_\nu(H^T\tau_2 l) + \text{h.c.}, \quad (2.18)$$

where  $l$  denotes the left-handed lepton doublet with the charge conjugated field given by  $l^c = -i\gamma_2 l^*$ , and  $e$  is the right-handed charged lepton singlet.  $\Lambda_{LN}$  denotes the breaking scale of the lepton number symmetry  $U(1)_{LN}$ .  $Y_e$  and  $Y_\nu$  stand for the  $3 \times 3$  Yukawa coupling matrices in flavour space. In the absence of these Yukawa couplings, the lepton sector respects the flavour symmetry

$$G_{LF} = SU(3)_l \otimes SU(3)_e. \quad (2.19)$$

finite polynomial of  $A_\ell$  and  $B_\ell$ . After neglecting all the terms involving  $B_\ell$ , which are suppressed by the small lepton Yukawa couplings  $Y_e$ , we obtain

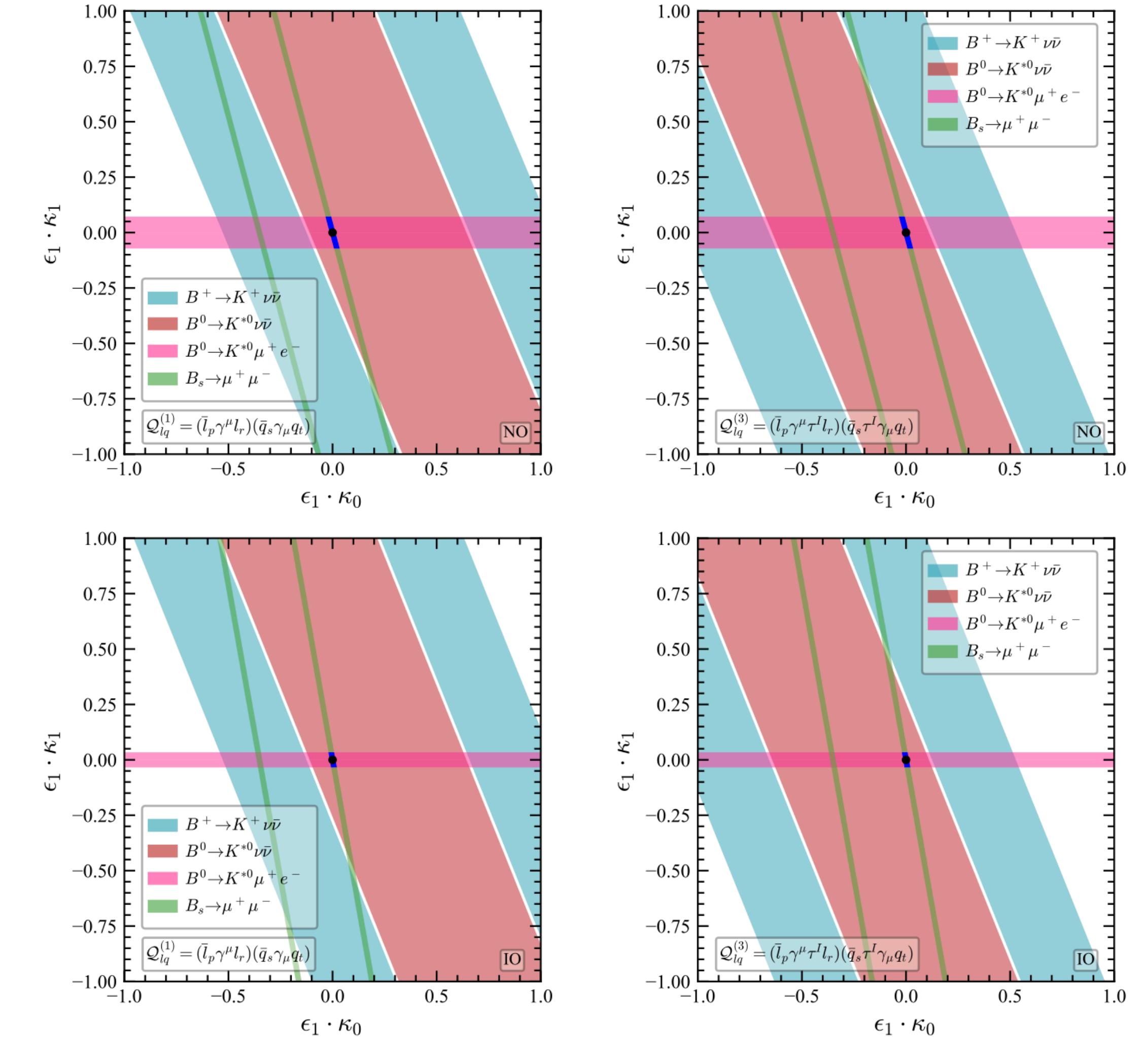
$$\mathcal{C}_{MFV} \approx \kappa_0 + \kappa_1 A_\ell + \kappa_2 A_\ell^2, \quad (2.21)$$

where the coefficients  $\kappa_{0,1,2}$  are free real parameters. In the numerical analysis, we keep only the leading lepton flavour violation term  $A_\ell$  for simplicity, i.e.,  $\kappa_2 = 0$ . Turning to the lepton mass eigenbasis, the current  $\bar{l}\gamma^\mu Cl$  gives in the MFV hypothesis the following interactions:

$$\bar{e}_L\gamma^\mu(\kappa_0\mathbb{1} + \kappa_0\Delta_\ell)e_L + \bar{\nu}_L\gamma^\mu(\kappa_0\mathbb{1} + \kappa_0\hat{\lambda}_\nu^2)\nu_L, \quad (2.22)$$

where the basic LFV coupling  $\Delta_\ell$  can be obtained from  $A_\ell$  and takes the form

$$\Delta_\ell = U\hat{\lambda}_\nu^2 U^\dagger, \quad (2.23)$$



$$\Delta_\ell^{\text{NO}} = \begin{pmatrix} -0.19 - 0.01i & -0.25 - 0.02i & 0.31 - 0.04i \\ 0.12 + 0.01i & 0.28 - 0.00i & 0.29 + 0.04i \\ -0.37 - 0.01i & 0.21 - 0.05i & -0.03 + 0.01i \end{pmatrix}, \quad \Delta_\ell^{\text{IO}} = \begin{pmatrix} 0.21 + 0.09i & -0.34 + 0.05i & 0.03 + 0.11i \\ 0.31 + 0.12i & 0.19 + 0.00i & -0.15 - 0.14i \\ 0.12 - 0.02i & 0.04 - 0.19i & 0.34 - 0.10i \end{pmatrix}$$