

# Pion Axioproduction Revisited

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# Origin of Axion

SM: other terms +  $\bar{\theta} G \tilde{G}$

CP-violated

SM extended with axion: other terms +  $\frac{1}{2} \partial_\mu a \partial^\mu a + \left( \frac{a}{f_a} + \bar{\theta} \right) G \tilde{G}$

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CP-violated

SM extended with axion: other terms +  $\frac{1}{2} \partial_\mu a \partial^\mu a + \left( \frac{a}{f_a} + \bar{\theta} \right) G \tilde{G}$

minimizing the vacuum energy leads to  $\langle a/f_a \rangle = -\bar{\theta}$   
perform the axion field shift,  $a \rightarrow a - \bar{\theta} f_a$ , and other field transformations

SM extended with axion: other terms +  $\frac{1}{2} \partial_\mu a \partial^\mu a + \frac{1}{2} m_a^2 a^2 + \dots$

CP-symmetric

tiny mass  $m_a \propto \frac{1}{f_a}$ , candidate for DM

# Axion Models

## Visible Axion (PQWW)

- $10^3 \text{ GeV} \leq f_a \leq 10^6 \text{ GeV}$
- Seems to be ruled out by experiments on astrophysical grounds. [Phys. Rev. D 18, 1829 (1978), Phys. Rev. D 22, 839 (1980)]
- Still attempts to make the experimental data compatible with the original model. However, these attempts require a lot of additional assumptions. [JHEP 07 (2018) 092]

# Axion Models

## Invisible Axion (KSVZ and DFSZ)

- $10^9 \text{GeV} \leq f_a \leq 10^{12} \text{GeV}$
- QCD Lagrangian including axion below the PQ scale:

$$\mathcal{L}_{\text{QCD},0} - (\bar{q}_L M_a q_R + \text{h. c.}) + \bar{q} \gamma^\mu \gamma_5 \frac{\partial_\mu a}{2f_a} (X_q - Q_a) q$$

$M_a = \exp\left(i \frac{a}{f_a} Q_a\right) M$ ,  $Q_a$  is the chiral rotation matrix,  $M$  is the quark mass matrix

$X_q$  is the model-dependent coupling matrix

$$X_q^{\text{KSVZ}} = 0$$

$$X_{u,c,t}^{\text{DFSZ}} = \frac{1}{3} \sin^2 \beta, X_{d,s,b}^{\text{DFSZ}} = \frac{1}{3} \cos^2 \beta$$

# Motivation

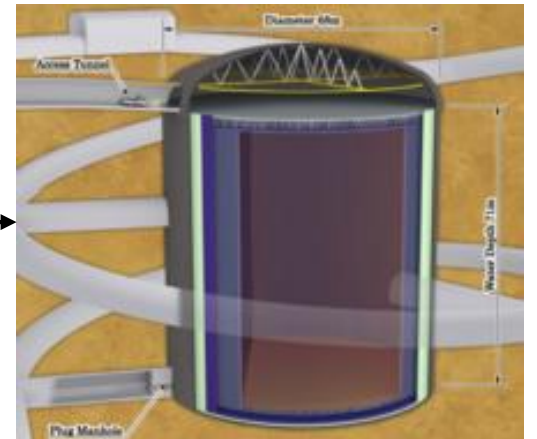
Supernova



$NN \rightarrow aNN$

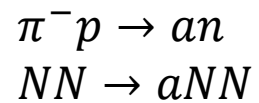
$an \rightarrow \pi^- p$

Water Cherenkov Detector



# Motivation

Supernova



[Carenza *et al.*, Phys. Rev. Lett. 126, 071102 (2021)]

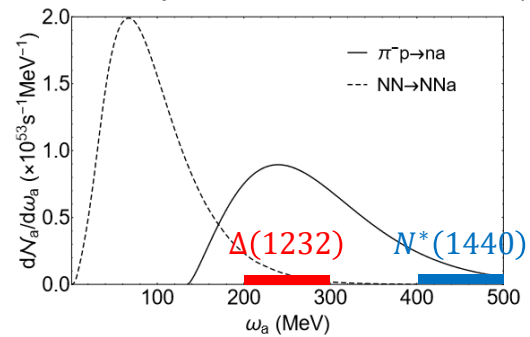
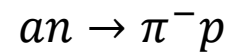
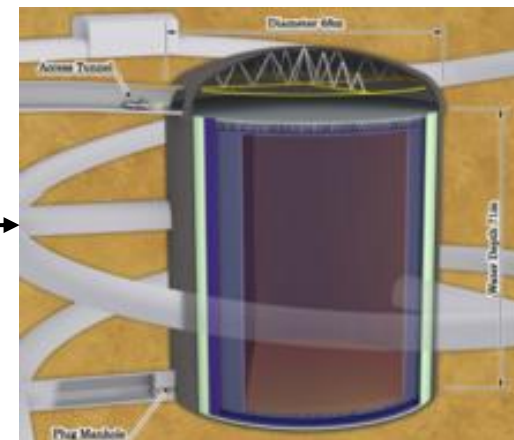


FIG. 1: The number spectra of axions for  $\pi N$  (solid curve) and  $NN$  (dashed curve) processes for our benchmark axion model at a post-bounce time  $t_{pb} = 1$  s.



Water Cherenkov Detector



# Motivation

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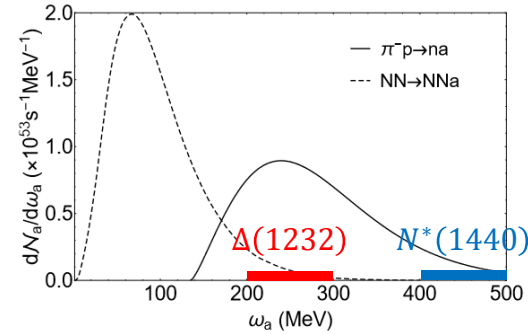
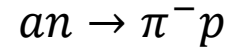
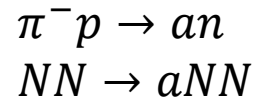


FIG. 1: The number spectra of axions for  $\pi N$  (solid curve) and  $NN$  (dashed curve) processes for our benchmark axion model at a post-bounce time  $t_{pb} = 1$  s.

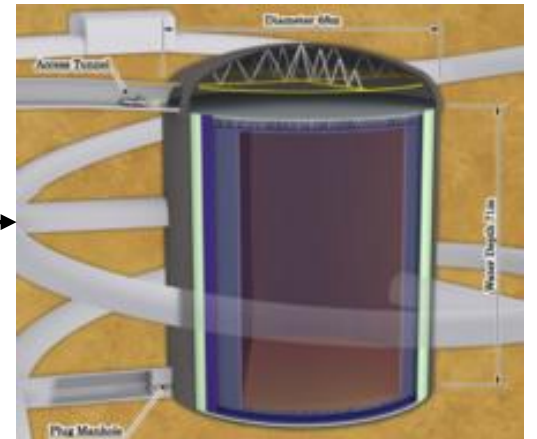


$$\Delta(1232): J^P = \frac{3}{2}^+$$

$$\sigma_{an \rightarrow \pi^- p} \approx \sigma^{S_1} + \sigma^{P_1} + \sigma^{P_3}$$

$$N^*(1440): J^P = \frac{1}{2}^+$$

Water Cherenkov Detector



$$f_a^2 \sigma_{an \rightarrow \pi^- p} \approx F_\pi^2 \sigma_{\pi N \rightarrow \pi N} \approx 1 \text{ mb} (\text{GeV}/f_a)^2 \text{ around the } \Delta(1232) \text{ region}$$



# Chiral Lagrangian Framework

## Pion Field

$$U(x) = \exp\left(i \frac{\pi_i(x) \tau_i}{F}\right)$$

## Nucleon and Roper Field

$$\Psi_{N^{(*)}}(x) = \begin{pmatrix} p^{(*)}(x) \\ n^{(*)}(x) \end{pmatrix}$$

## Delta Field

$$\Delta_\mu(x) = \begin{pmatrix} \Delta_\mu^{++}(x) \\ \Delta_\mu^+(x) \\ \Delta_\mu^0(x) \\ \Delta_\mu^-(x) \end{pmatrix}$$

## Axion Field

$$\chi = s + ip = M_a$$

$$a_\mu = c_{u-d} \frac{\partial_\mu a}{2f_a} \tau_3$$

$$a_{\mu,i}^{(s)} = c_i \frac{\partial_\mu a}{2f_a} \mathbb{I}, i = \{u + d, s, c, b, t\}$$

$$c_{u\pm d} = \frac{1}{2} \left( X_u \pm X_d - \frac{1 \pm z}{1 + z + w} \right), c_s = X_s - \frac{w}{1 + z + w}, c_{c,b,t} = X_{c,b,t}$$

$$z = \frac{m_u}{m_d}, w = \frac{m_u}{m_s}$$

[Phys. Lett. 169B, 73 (1986), J. High Energy Phys. 03 (2020) 138]

# Chiral Lagrangian Framework

## Pion-Nucleon Interaction

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - \dot{m}_N + \frac{\dot{g}_A}{2} \not{u}\gamma_5 + \frac{\dot{g}_0}{2} \not{u}_i\gamma_5 \right\} \Psi_N$$

$$\begin{aligned} \dot{g}_A &\rightarrow g_A = \Delta u - \Delta d, \\ \dot{g}_0^{u+d} &\rightarrow g_0^{u+d} = \Delta u + \Delta d, & s^\mu \Delta q &= \langle p | \bar{q} \gamma^\mu \gamma_5 q | p \rangle \\ \dot{g}_0^q &\rightarrow g_0^q = \Delta q, \quad \text{for } q = s, c, b, t \end{aligned}$$

[Eur. Phys. J. C 82, 869 (2022)]

## Delta-Pion-Nucleon Interaction

$$\mathcal{L}_{\Delta\pi N} = \frac{g}{2} \bar{\Delta}_\mu T^{a\dagger} (g^{\mu\nu} + z_0 \gamma^\mu \gamma^\nu) \langle \tau_a u_\nu \rangle \Psi_N + \text{H.c.}$$

## Roper-Pion-Nucleon Interaction

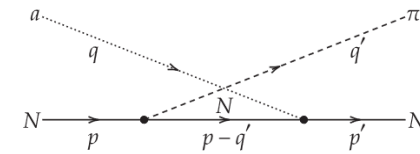
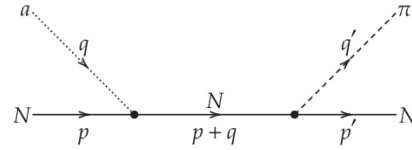
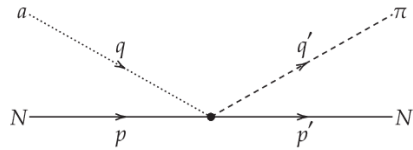
$$\mathcal{L}_{N^*\pi N} = \frac{\sqrt{R}}{2} \bar{\Psi}_{N^*} \left\{ \frac{g_A}{2} \not{u}\gamma_5 + \frac{g_0^i}{2} \not{u}_i\gamma_5 \right\} \Psi_N + \text{H.c.}$$

|                     | Fit 1              | Fit 2              | Fit 3                     | Fit 4               |
|---------------------|--------------------|--------------------|---------------------------|---------------------|
| $g_{\pi\Delta N}$   | 2.05               | 2.04               | 2.05                      | 2.05                |
| $Z$                 | -0.16              | -0.08              | -0.05                     | -0.05               |
| $\sqrt{R}$          | 0.67               | 0.79               | 0.79                      | 0.79                |
| $\tilde{T}_+$       | -0.15              | -0.26              | -0.09                     | -0.09               |
| $\bar{c}_d$ [MeV]   | 26.2 (25.,50.)     | 25.0 (25.,50.)     | $5.9 \cdot 10^5$          | 115.8               |
| $\bar{c}_m$ [MeV]   | 50.0 (25.,50.)     | 43.9 (25.,50.)     | $1.0 \cdot 10^6$          | 199.8               |
| $M_S$ [MeV]         | 1560 (840,1560)    | 1560 (840,1560)    | $3.8 \cdot 10^7$          | 63681.              |
| $g_S$               | 100.2 <sup>†</sup> | 104.3 <sup>†</sup> | $2.4 \cdot 10^{6\dagger}$ | 35159. <sup>†</sup> |
| $G_V$ [MeV]         | 67*                | 53*                | 67 *                      | 53*                 |
| $g_\rho$            | 5.00 (5,8)         | 5.00 (5,8)         | 6.08                      | 7.70                |
| $\kappa_\rho$       | 5.70 (5.7,6.5)     | 5.70 (5.7,6.5)     | 3.53                      | 3.53                |
| $a$                 | 0.16               | 0.16               | 0.16                      | 0.16                |
| $\chi^2/\text{dof}$ | 21.5/355           | 15.6/355           | 12.9/355                  | 12.0/355            |

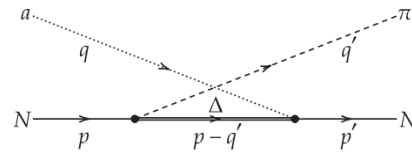
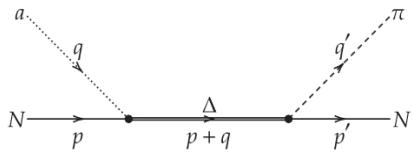
[Nucl. Phys. A673, 311 (2000)]

# Relevant Feynman Diagrams

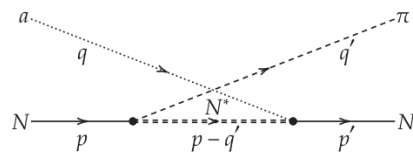
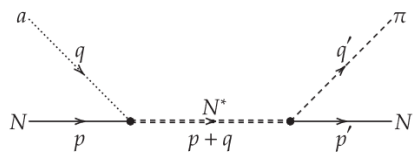
## Contact and Nucleon-Mediated Diagrams



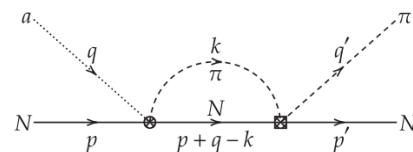
## Delta-Mediated Diagrams



## Roper-Mediated Diagrams



## Pion Rescattering Diagrams

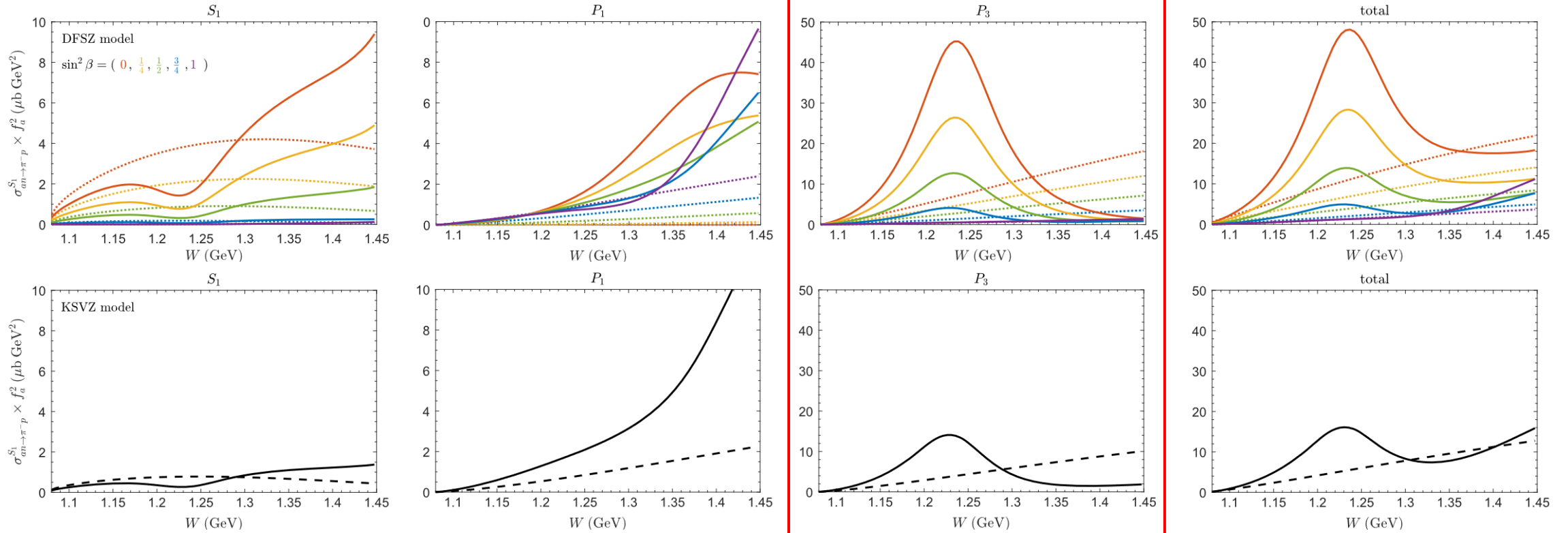


$$= T_{aN \rightarrow \pi N}(s) \times g(s) \times T_{\pi N \rightarrow \pi N}(s), \text{ with } g(s) \text{ usual two-point loop function}$$

Use Breit-Wigner propagators to avoid pole singularities.

A more refined treatment could be given, e.g. by including the resonance self-energy in the complex mass scheme [Phys. Rev. C 72, 055203 (2005)], but that is not required here.

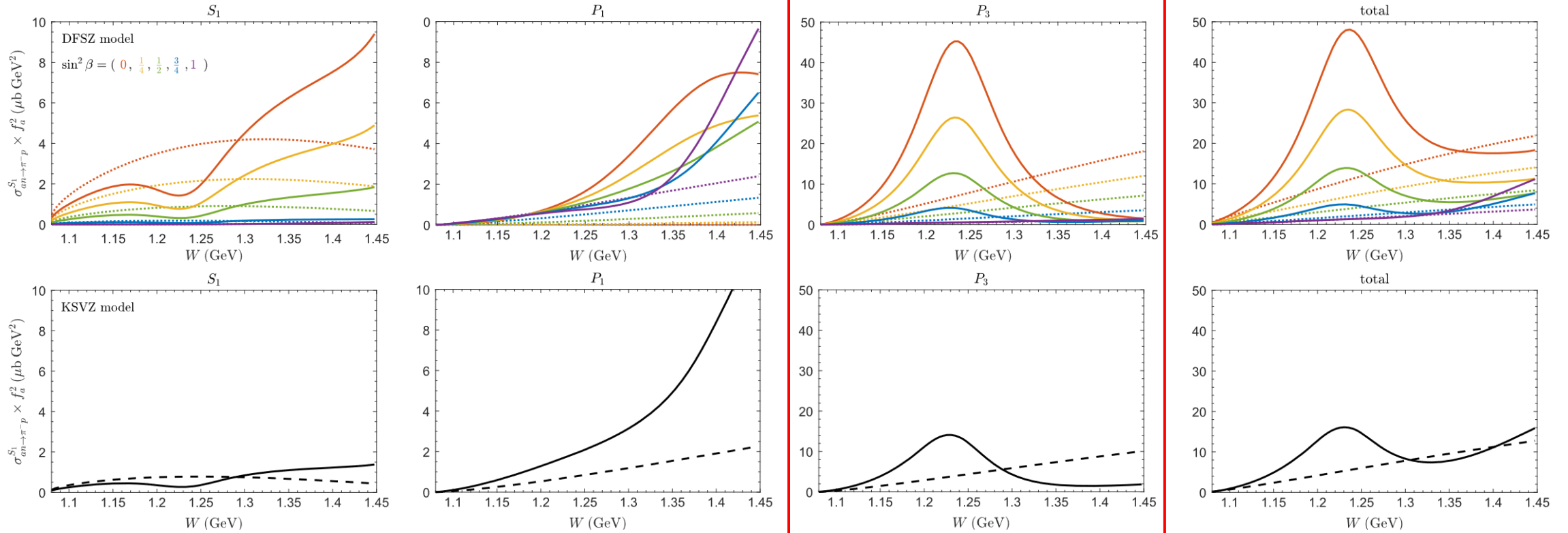
# Results



Resonance peak in  $\sigma^{P_3}$  decreases and almost vanishes as  $\sin^2 \beta \rightarrow 1$  in the DFSZ model

$$\mathcal{M}_{\Delta_S}^{P_3} \propto c_{u-d}, |c_{u-d}^{\text{DFSZ}}(\sin^2 \beta)| = (1.0116 - \sin^2 \beta)/3$$

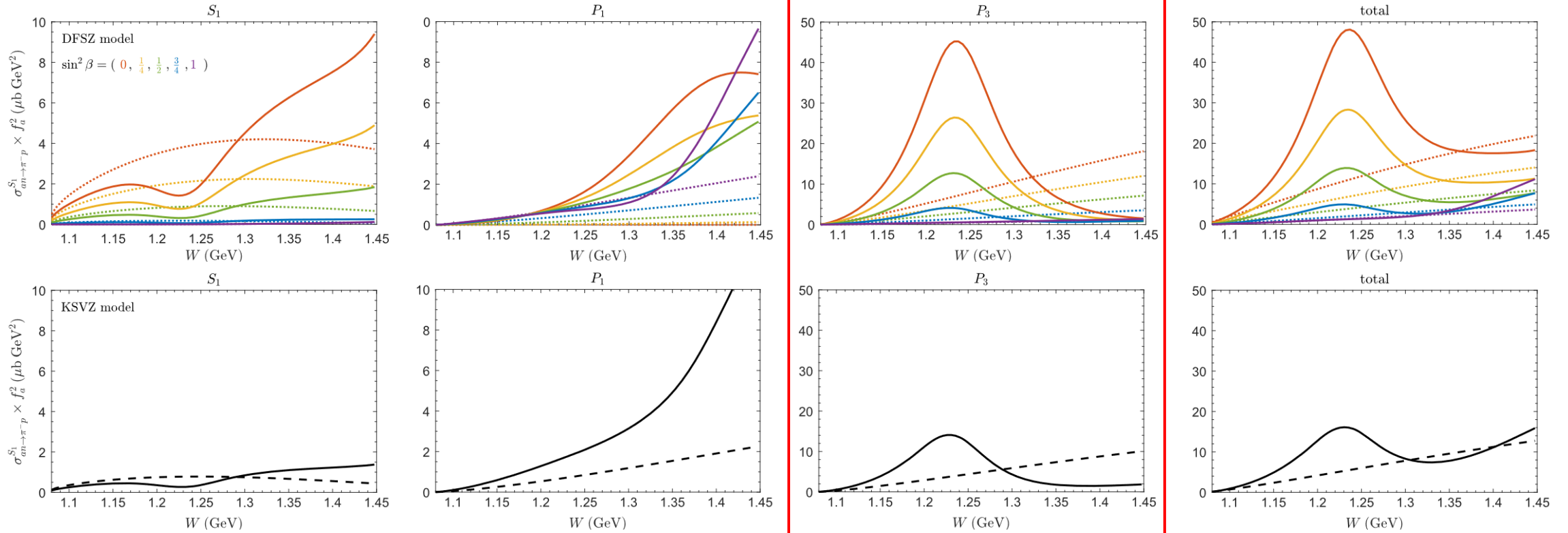
# Results



$\sigma^{P_3}$  of the KSVZ model closely aligns with that of the DFSZ model when  $\sin^2 \beta = 1/2$

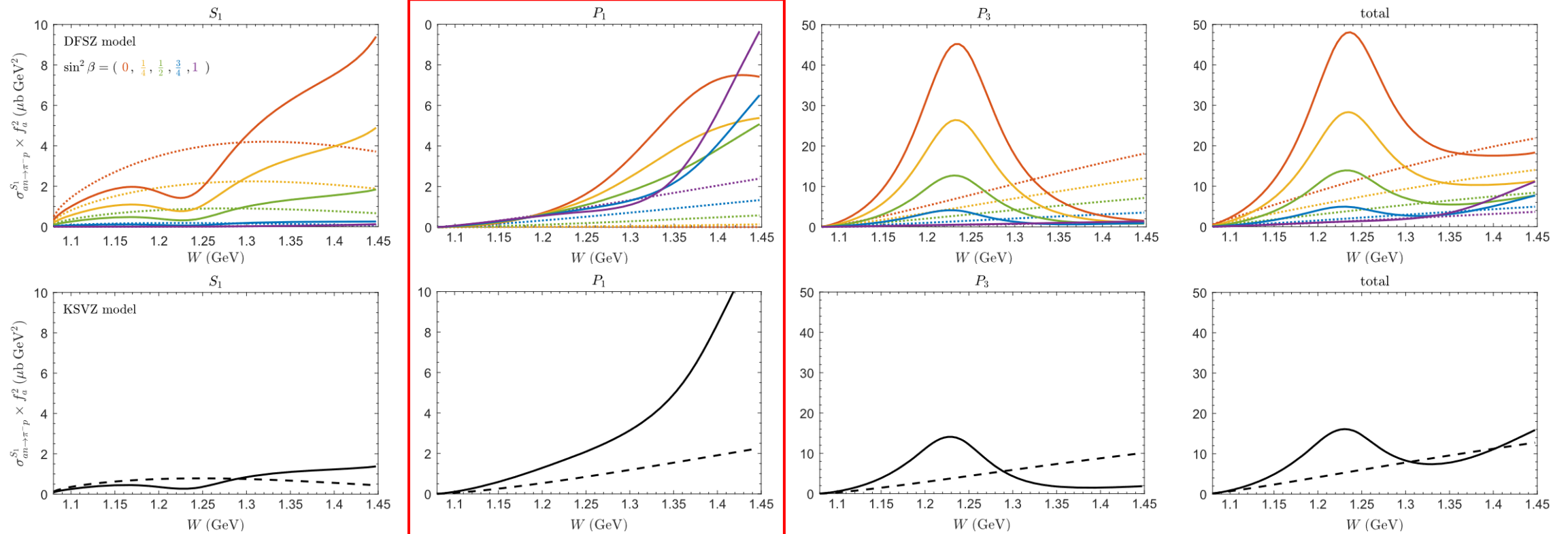
$$\mathcal{M}_{\Delta_S}^{P_3} \propto c_{u-d}, |c_{u-d}^{\text{DFSZ}}(\sin^2 \beta = 1/2)| = |c_{u-d}^{\text{KSVZ}}|$$

# Results



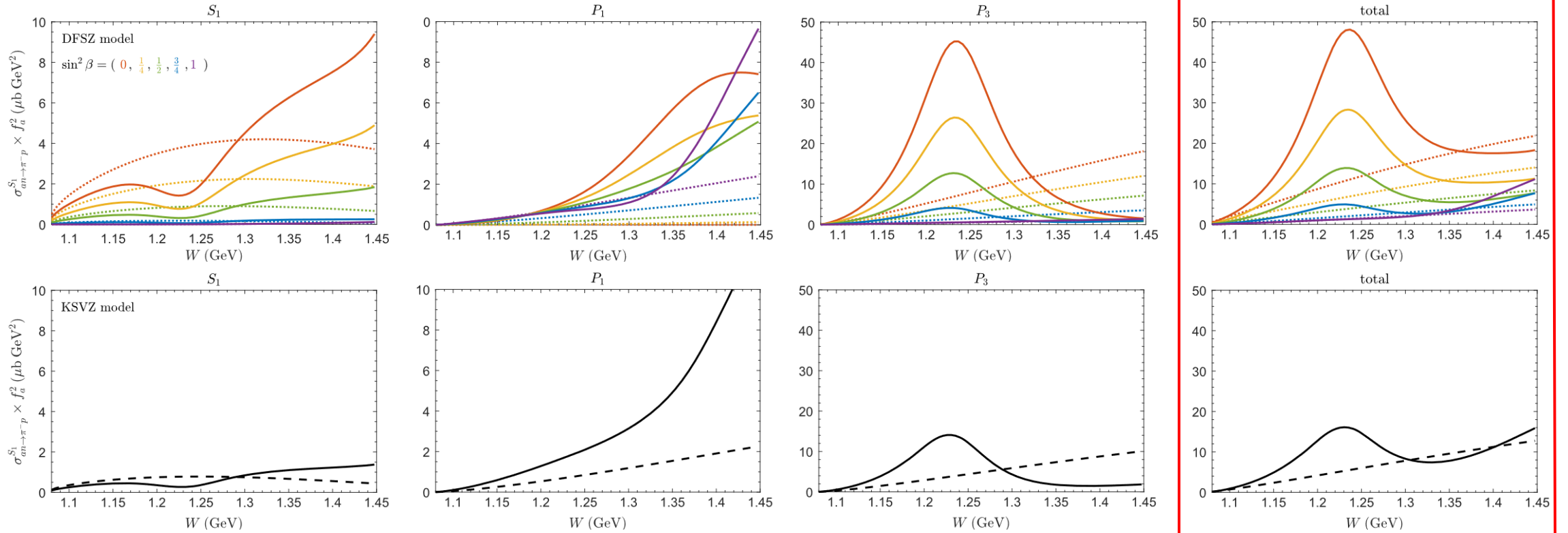
$f_a^2 \sigma_{an \rightarrow \pi^- p} \approx 50 - 1 \mu\text{b}(\text{GeV}/f_a)^2$  is about 20 – 1000 smaller than the naive estimate by Carena *et al.*

# Results



Resonance peak in  $\sigma^{P_1}$  remains relatively stable with variations in  $\sin^2 \beta$  in the DFSZ model

# Results

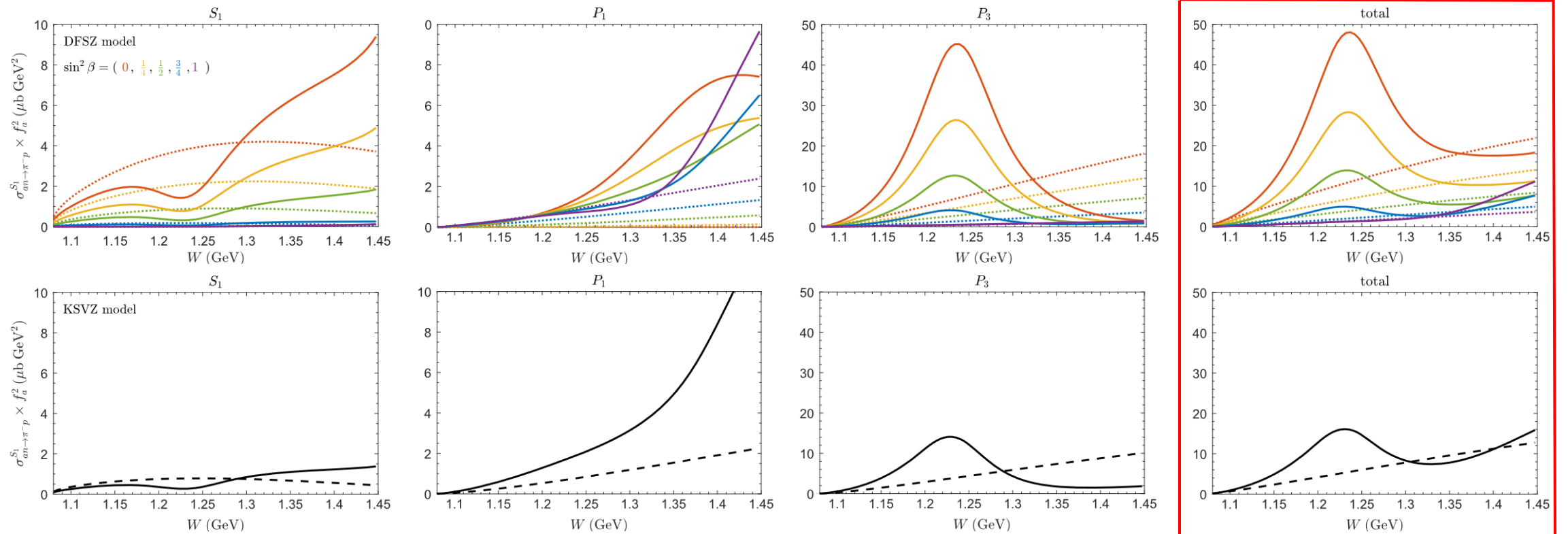


If  $\sin^2 \beta \approx 1/2$  in the DFSZ model, it would be hard to distinguish from the KSVZ model

$$f_a^2 \sigma_{an \rightarrow \pi^- p}^{\text{DFSZ}}(\sin^2 \beta = 1/2) = 7 \mu\text{b}(\text{GeV}/f_a)^2, f_a^2 \sigma_{an \rightarrow \pi^- p}^{\text{KSVZ}} = 15 \mu\text{b}(\text{GeV}/f_a)^2 \text{ at } W = m_{N^*}$$



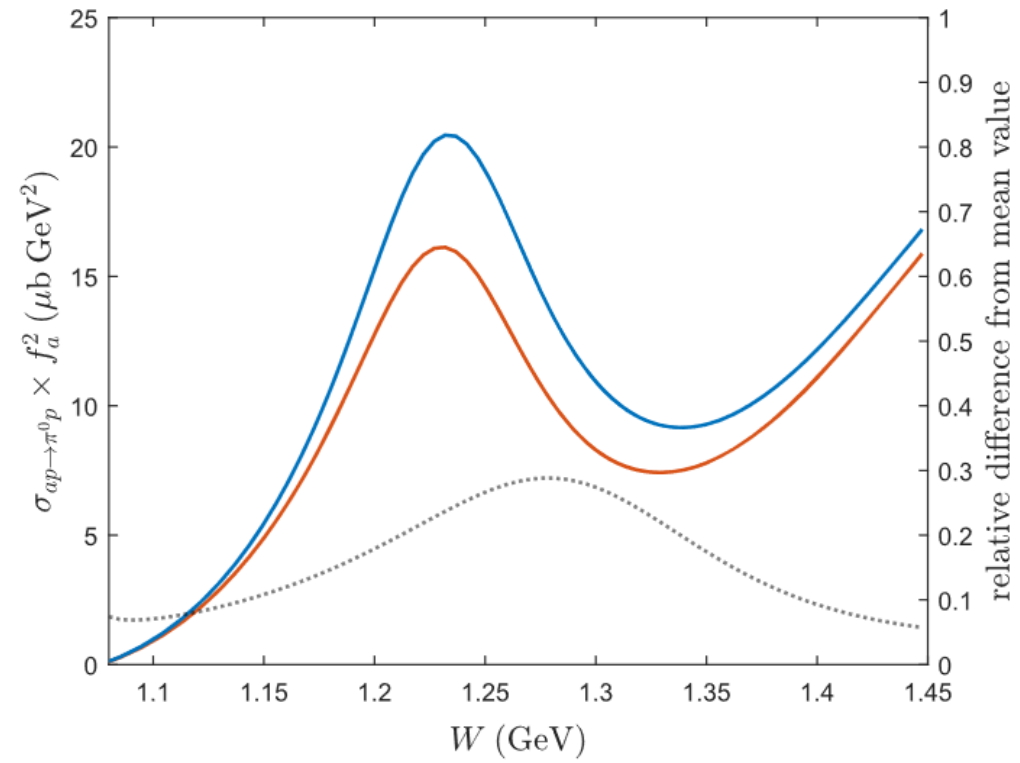
# Results



One may distinguish the DFSZ model, with  $\sin^2 \beta$  sizably deviating from  $1/2$ , from the KSVZ model.

At  $W = m_\Delta$ ,  $\mathcal{O}(10)$  pions would be generated in a megaton water Cherenkov in the KSVZ model using  $f_a = 10^9$  GeV, whereas the count would be noticeably higher ( $\sin^2 \beta \rightarrow 0$ ) or lower ( $\sin^2 \beta \rightarrow 1$ ) in the DFSZ model.

# Results



**Thanks For Your Attention**