中国物理学会高能物理分会
HIGH ENERGY PHYSICS BRANCH OF CPS

山东大学
SHANDONG UNIVERSITY

第十四届全国粒子物理学术会议

2024/08/12-18, 山东大学（青岛）

微扰QCD和精确计算研究进展

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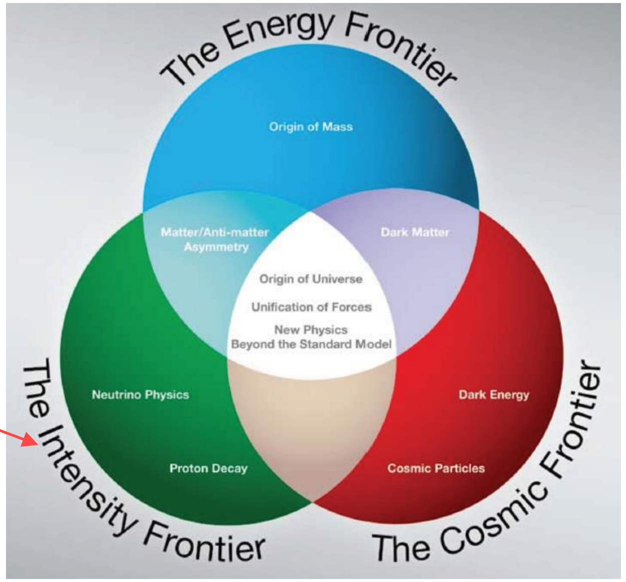
北京大学

Outline

Precision: gateway to discovery

➢ **New particles/physics have not been discovered yet at LHC**

- Currently main strategy: search anomalous deviations from theory
- Interplay between exp. and th.



To make full use of data: theoretical errors should be much smaller than experimental errors, ideally:

$$Error_{th} < \frac{1}{3} Error_{exp}$$

10 years of Higgs

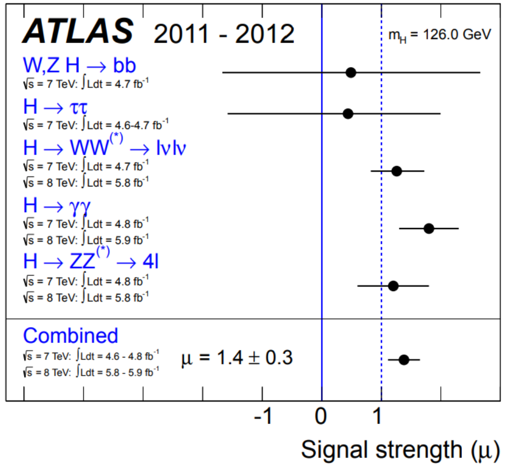
➢ At the time of discovery

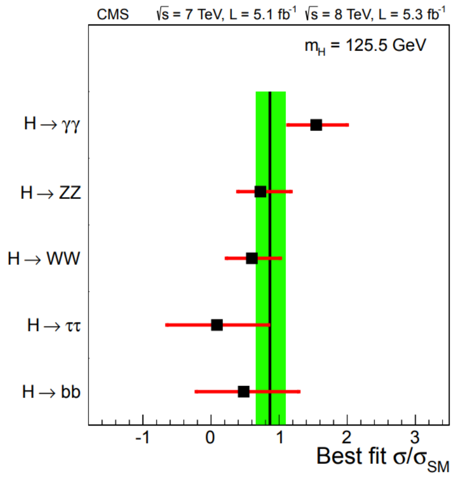
ATLAS ,PLB2012 $\mu = 1.4 \pm 0.3$

CMS, PLB2012 $\mu = 0.87 \pm 0.23$

Theoretical error: negligible

$$\mu \equiv \frac{\sigma_{\text{Exp}}}{\sigma_{\text{SM}}}$$



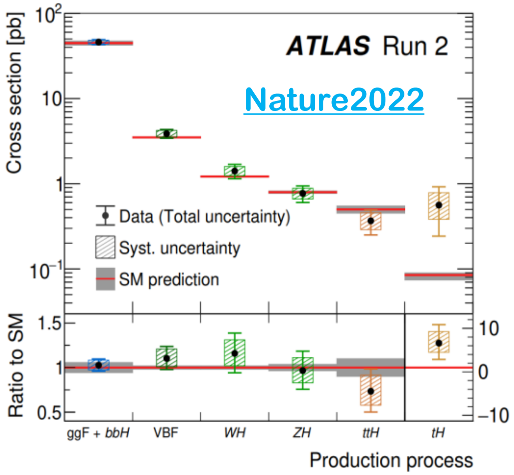


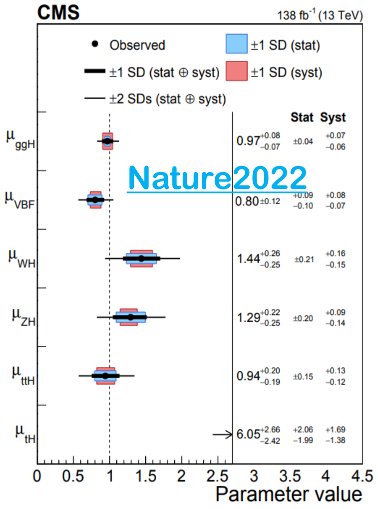
➢ 10 years later

ATLAS: $\mu = 1.05 \pm 0.06 = 1.05 \pm 0.04(\text{th}) \pm 0.03(\text{exp}) \pm 0.03(\text{stat})$

CMS: $\mu = 1.002 \pm 0.057 = 1.002 \pm 0.036(\text{th}) \pm 0.033(\text{exp}) \pm 0.029(\text{stat})$

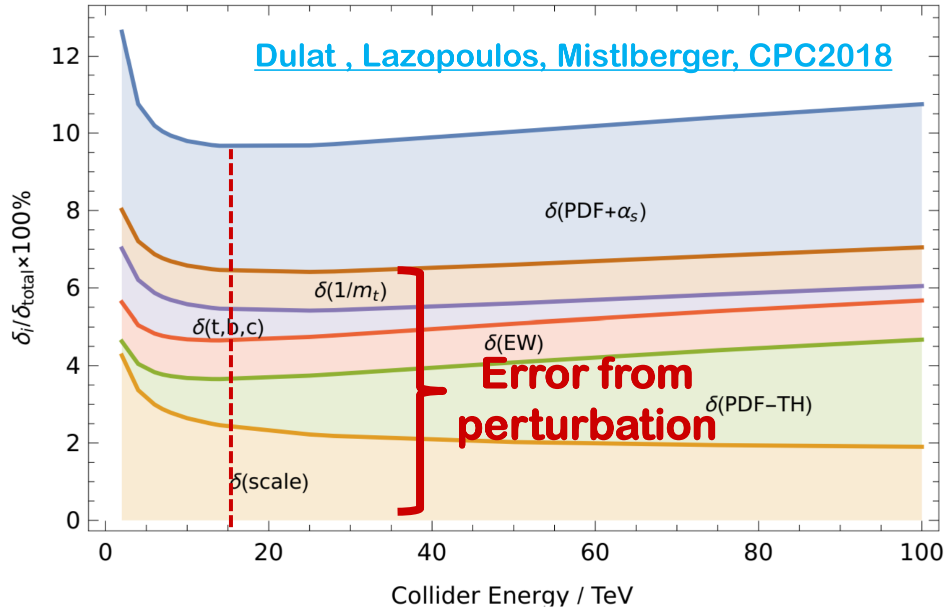
- Theoretical error dominant!
- Theorists also working hard, but experimentalists are excellent…





Uncertainty budget

$$
\begin{aligned}
\delta(\text{theory}) &= \,^{+0.13pb}_{-1.20pb} \quad \left(^{+0.28\%}_{-2.50\%}\right) \quad \delta(\text{scale}) \\
&+ \pm 0.56pb \quad (\pm 1.16\%) \quad \delta(\text{PDF-TH}) \\
&+ \pm 0.49pb \quad (\pm 1.00\%) \quad \delta(\text{EWK}) \\
&+ \pm 0.41pb \quad (\pm 0.85\%) \quad \delta(\text{t,b,c}) \\
&+ \pm 0.49pb \quad (\pm 1.00\%) \quad \delta(1/m_t) \\
&= \,^{+2.08pb}_{-3.16pb} \quad \left(^{+4.28\%}_{-6.5\%}\right), \\
\delta(\text{PDF}) &= \pm 0.89\text{pb} \quad (\pm 1.85\%), \\
\delta(\alpha_S) &= \,^{+1.25pb}_{-1.26pb} \quad \left(^{+2.59\%}_{-2.62\%}\right).
\end{aligned}
$$



$$\sigma = \sum_{i,j} dx_1 dx_2 \boxed{f_{i/p}(x_1) f_{j/p}(x_2)} \, \boxed{\hat{\sigma}_{ij}} + O(\Lambda^2/M^2)$$

PDFs
More input data, other methods

$\alpha_s = 0.118 \pm 0.001$
Lattice, other methods

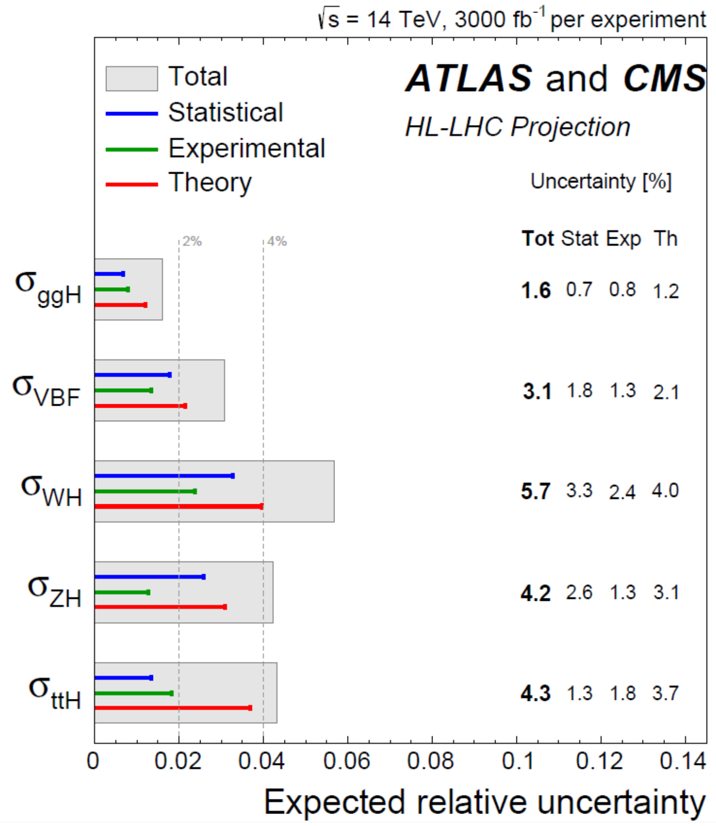
Theory
Calculate higher orders in perturbation theory

Looking ahead

➢ High-precision data expected

- Run III of LHC
- High luminosity LHC: expect $O(1\%)$ uncertainty
- Requirement: reducing theoretical uncertainties by at least a factor of 5-10 (1-2 higher orders in α_s !)
- Usually, 1000 times more computational resource for each higher order!

Can theorists keep up?

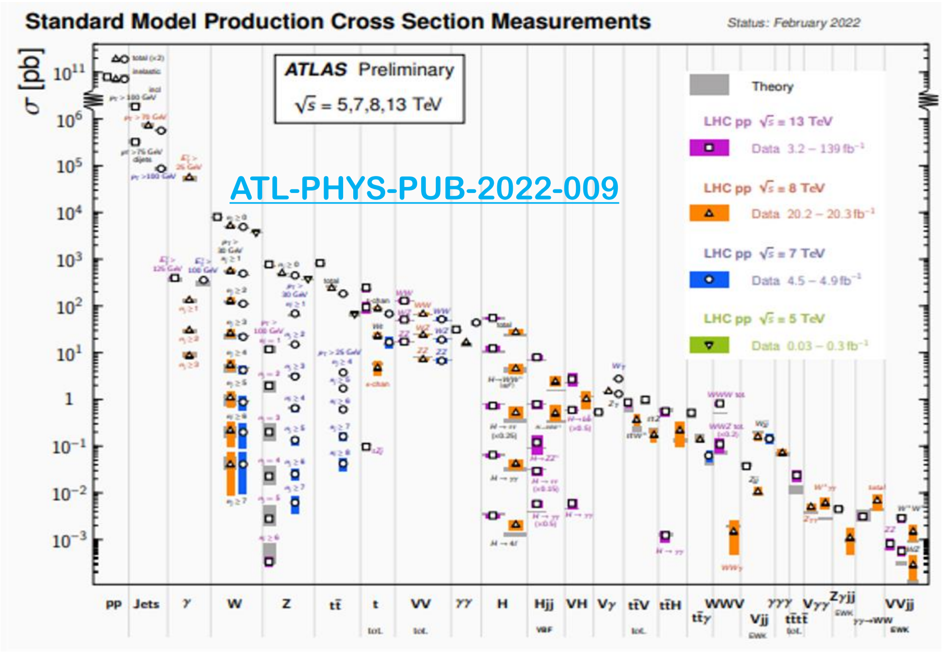


CERN2019
Assuming theoretical errors halved

Era of precision physics at the LHC

➢ High-precision data

- Many observables probed at precent level precision
- At least NNLO QCD and NLO EW corrections generally required (plus parton shower, resummation, etc.)



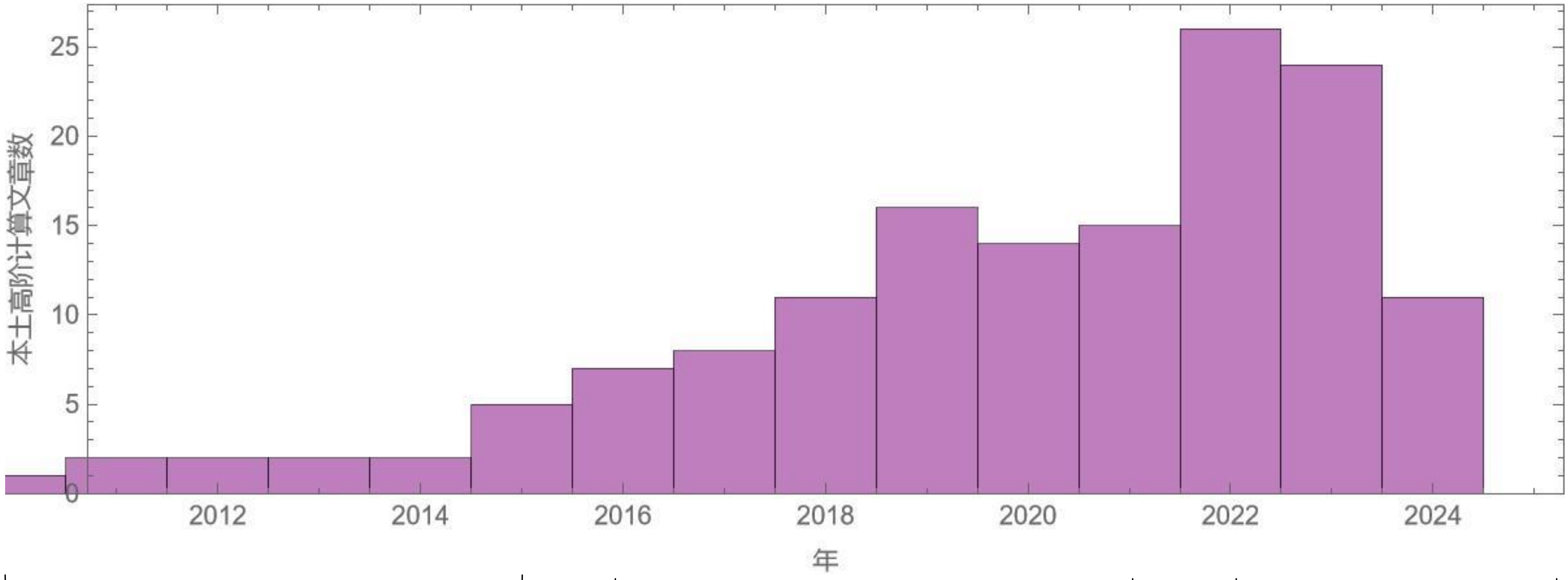
Automatic NNLO (and higher) correction is highly demanded

Note: Automatic NLO correction obtained 15 years ago: MadGraph, Helac, etc

➢ A “billion-dollar project”

- Halving total uncertainty ≈ building another LHC
- Note: LHC cost about 10 billion dollars

High-order community from the Mainland

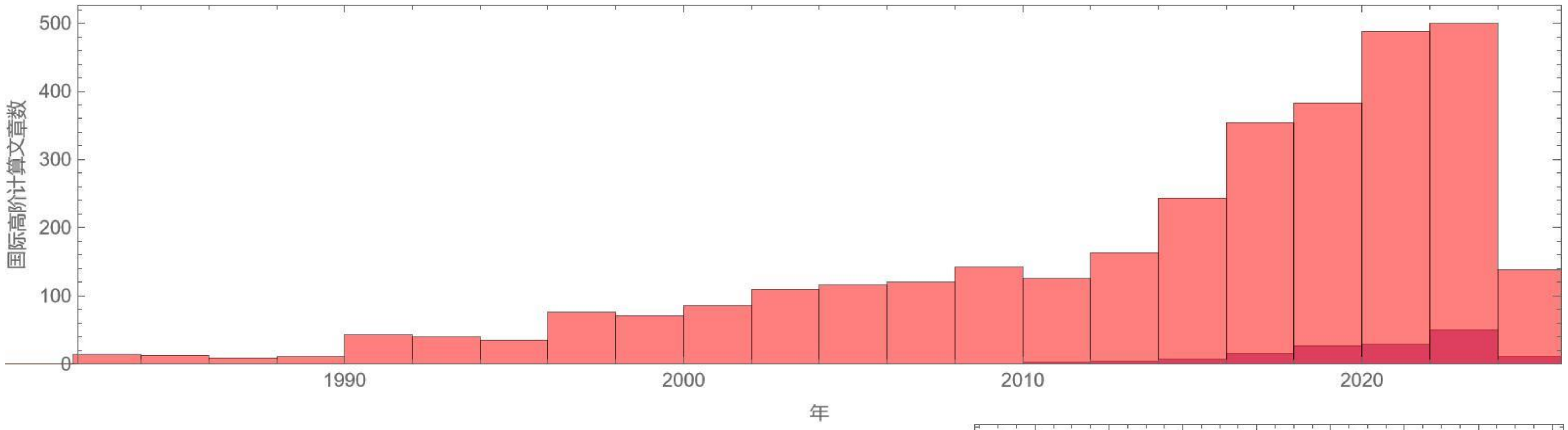


1990-2015 萌芽期：吴丹迪、郑汉青、廖益、李新强、冯波、李重生团队、乔从丰团队、贾宇团队等

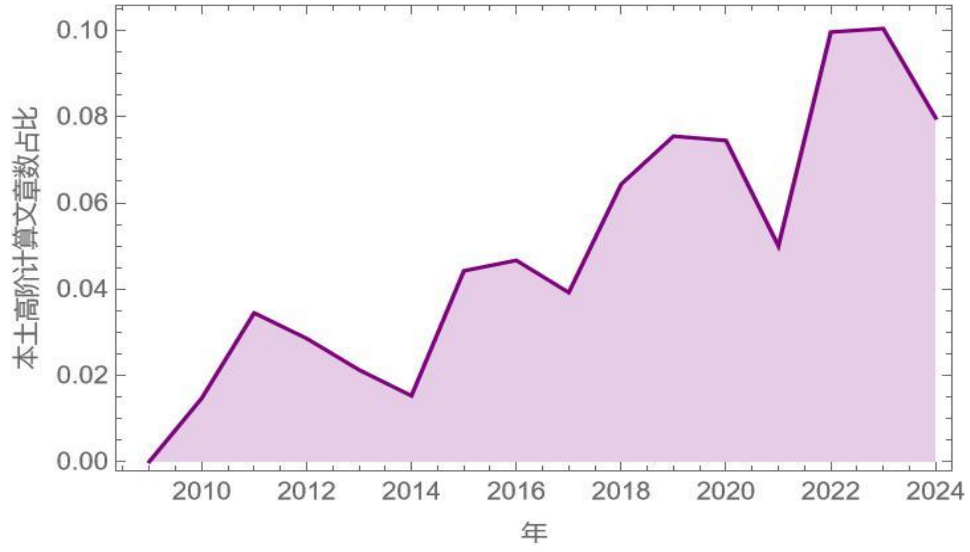
2016-2021 追赶期：大量回国年轻人及本土培养年轻人加入

2022-202x并跑期：产生多个国际影响力工作

The whole community



- 本土团队：起步晚、发展快、已达**10%**比重



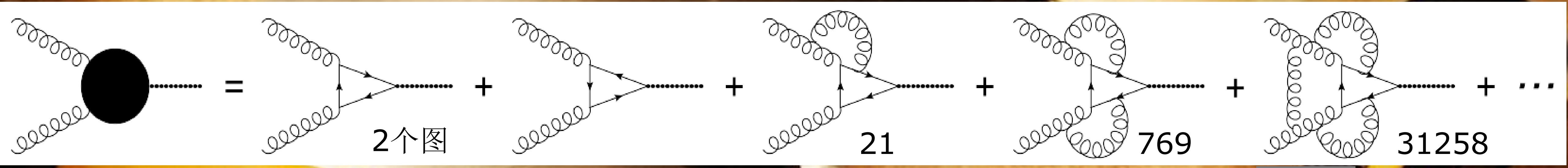
Outline

I. Overview

II. Methodology

III. Phenomenology

IV. Summary and outlook



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Packages!

工欲善其事，必先利其器。

何为器？

实验→对撞机

芯片→光刻机

Packages for perturbative calculations

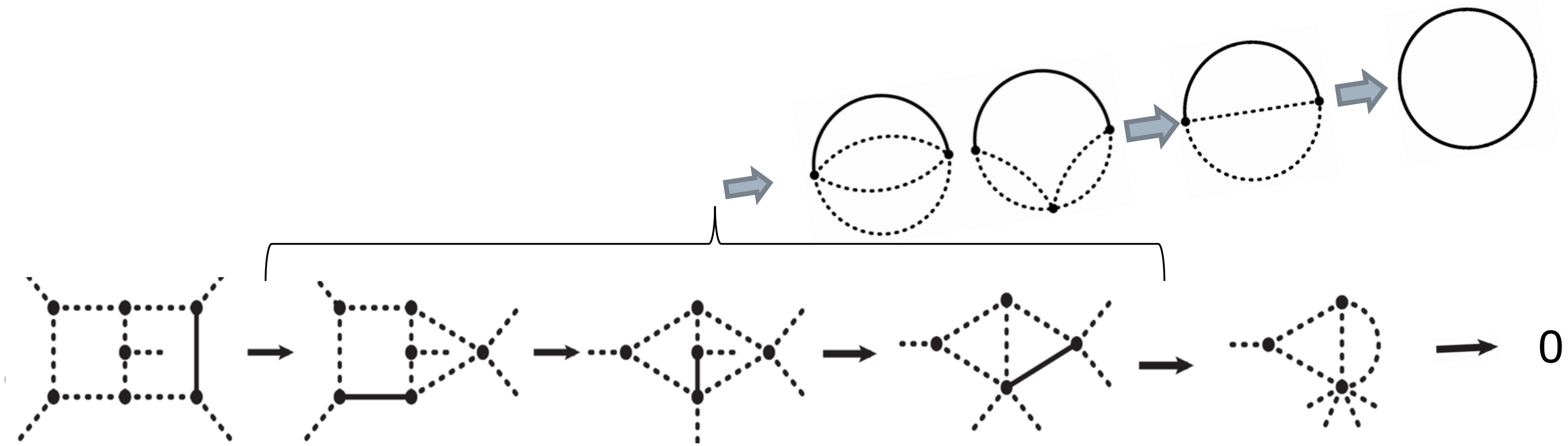
- Selected contributions from the Mainland
  - 2004 FDC: automatic NLO computation Wang, NIMA2004
  - 2012 Apart: multiloop partial fraction Feng, CPC2012
  - 2021 pfd-parallel: general partial fraction Bendle, et al. CPC2024
  - 2021 FastGPL: compute GPL functions Wang, Yang, Zhou, 2112.04122
  - 2021 HepLib: Amplitude computation Feng, et al. CPC2021
  - 2022 AMFlow: Feynman integrals computer Liu, YQM, CPC2023
  - 2023 NeatIBP: IBP generator Wu, et al. CPC2024
  - 2024 FeAmGen.jl: generate amplitude Wu, Li, CPC2024
  - 2024 Blade: Feynman integrals reducer Guan, Liu YQM, Wu, 2405.14621
  - 2024 AmpRed: Amplitude computation Chen, 2408.06426

Incredible progress!!!

Don't suppress for any future shocking pheno. works

Package AMFlow

➢ **In principle, compute any Feynman integral**



X.Liu, YQM, Wang, PLB2018

X.Liu, YQM, PRD2022

Z.F.Liu, YQM, PRL2022

- **With only input from linear algebra (linear relations between Feynman integrals)**
- **Works for any-loop order**
- **AMFlow: used by the community almost every week** Liu, YQM, CPC2023

The first solution for the 40-years-long challenging problem

Package NeatIBP

➢ Syzygy constraint for IBPs

Gluza, Kajda, Kosower, PRD2011

$$0 = C \int \mathrm{d}z_1 \cdots \mathrm{d}z_n \sum_{i=1}^{n} \frac{\partial}{\partial z_i} \left(a_i(z) P^{\gamma} \frac{1}{z_1^{\alpha_1} \cdots z_n^{\alpha_n}} \right)$$

M1: $b(z)P + \sum_{i=1}^{n} \left(a_i(z) \frac{\partial P}{\partial z_i} \right) = 0$

M2: $a_i(z) = b_i(z) z_i, \quad \text{for } i \in \{j | \alpha_j > 0\}$

➢ Module intersection

Laplace expansion M1: $\sum_j P_{ij} \frac{\partial P}{\partial P_{kj}} - \delta_{ik} P = 0$

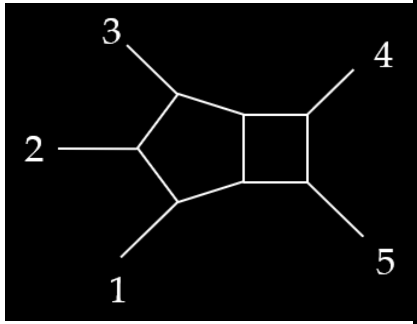
$$M = M_1 \cap M_2$$

Böhm, Georgoudis, Larsen, Schulze, Zhang, PRD2018
Bosma, Larsen, Zhang, PoS LL2018
Boehm, et al., PoS MA2019

➢ NeatIBP: the first public implementation

Wu, et al. CPC2024

- Much less equations
- Much less resources
- Very promising for pheno.



\# of IBP (FIRE6): 11207942

Max numerator degree: 5
Max denominator power: 1
\# of MI: 61
\# of IBP: 14120

Time used: 27m at

CPU cores: 10
RAM: 128GB

Package AmpRed

Define the kinematics

```
SetKinematics[{P} → {p[1], p[2]}, {FA["MH"]} → {0, 0}];
FA["MH"] = FA["EL"] = FA["GS"] = 1;
FA["MT"] = 2; FA["MW"] = 1 / 2;
FA["SW"] = 1 / 2;
```

Convert the amplitudes

```
amp = FA2AR[Get["HiggsDecay_TwoLoop_Amplitude.m"], ToFeynmanInt → True] /. {_Incoming
```

Tensor algebras

```
amp1 = amp // SpinorChainSimplify // ColourSimplify;
```

Integral reduction

```
amp2 = AlphaReduce[amp];
```

Numerical evaluation of master integrals

```
amp3 = AlphaIntEvaluateN[amp1, 8, {}, PrecisionGoal → 40, Print → Print];
```

Result

```
Chop[Collect[amp3[[1]] // epsSeries, {_Pair, eps}], 10^-20]
```

$$\left(\frac{0.655596971963698933992715692909 77\, i\, C_F}{\epsilon} + 63.108561600245894557465525711 8952\, i\, C_F\right) p_1\cdot\epsilon^*_{p_2}\, p_2\cdot\epsilon^*_{p_1} + \left(\frac{0.0956155694439145384129444592241 12\, i\, C_F}{\epsilon} - 0.181268820276402967819607796164 8\, i\, C_F\right) p_1\cdot\epsilon^*_{p_1}\, p_2\cdot\epsilon^*_{p_2} + \left(-\frac{0.327798485981849466996357846454 9\, i\, C_F}{\epsilon} - 31.554280800122947278732762855947 6\, i\, C_F\right) \epsilon^*_{p_1}\cdot\epsilon^*_{p_2}$$

陈文, **8.15**下午

Foundations of **AmpRed**

- Integral reductions through the Feynman-parameter representation
  Chen (JHEP) 2020
  Chen (EPJC) 2020
  Chen (EPJC) 2021
- Recursive calculations of parametric integrals
  Chen, Luo, Yang, Zhu (JHEP) 2023
  Chen 2406.12051

Chen, 2408.06426

- **Computing two-loop $H \to \gamma\gamma$ in a few lines**
- **An almost standalone package**

New methods

- Compute Feynman integral using generating functions
  - General one-loop reduction Feng, CTP2023; Feng, Hu, Sheng, Yang, 2403.16040; 胡畅, 8.15下午
  - Multi-loop scalar reduction Guan, Li, YQM, PRD2023
- Find more symmetry relations
  Wu, Zhang, 2406.20016
  - Momentum transformation with rational functions, rather than constant
  - Some times reduce the number of master integrals
- Systematic way to deal with γ^5
  - Refined Kreimer scheme Chen, JHEP2023
  - Avoid finite renormalization

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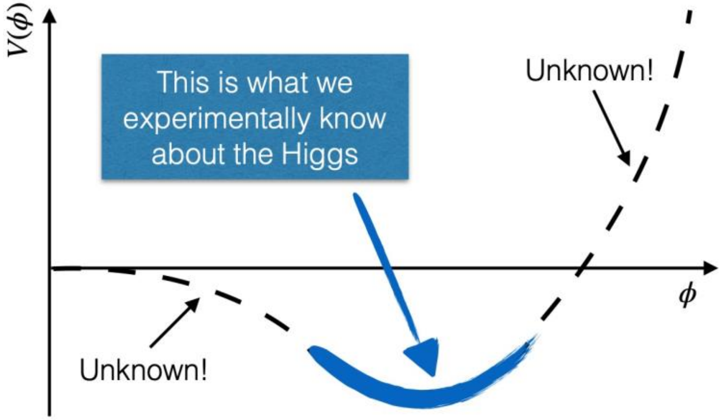
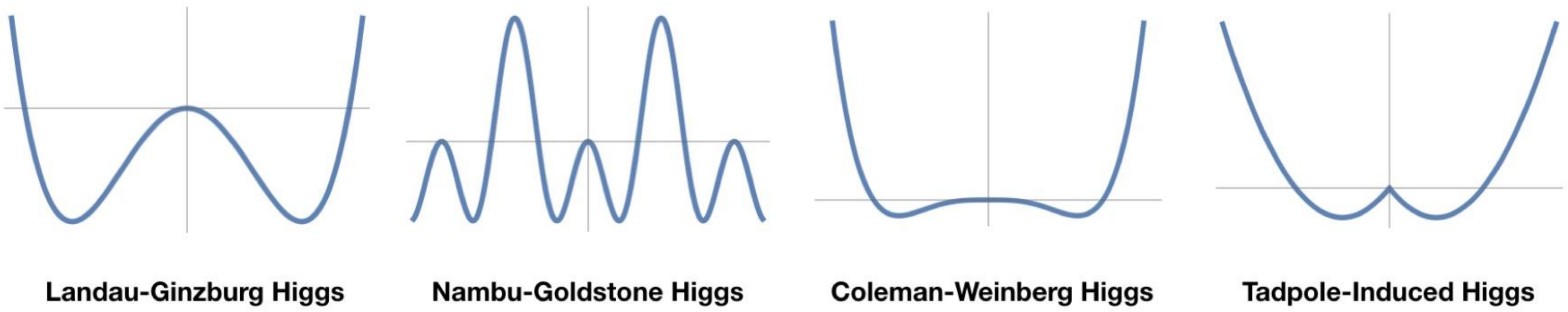
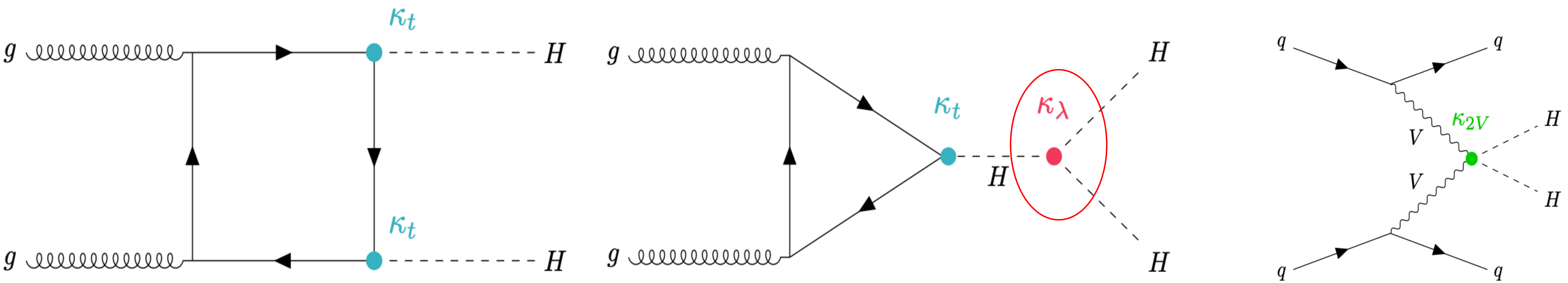
Higgs Potential Not Determined Yet



Agrawal, et al., PRD2020



- **Higgs pair production: direct probe the potential, discrimination between different scenarios**
- **6 experimental talks on this topic**



Rigorous constraint on Higgs boson self-couplings

- **Naïve functional form not correct due to higher corrections**

$$\sigma_{HH} = A + B\kappa + C\kappa^2$$

Rescaling κ results in wrong EW parameters: e, m_H, m_t, m_W, m_Z

- **Propose a modified renormalizable Lagrangian**

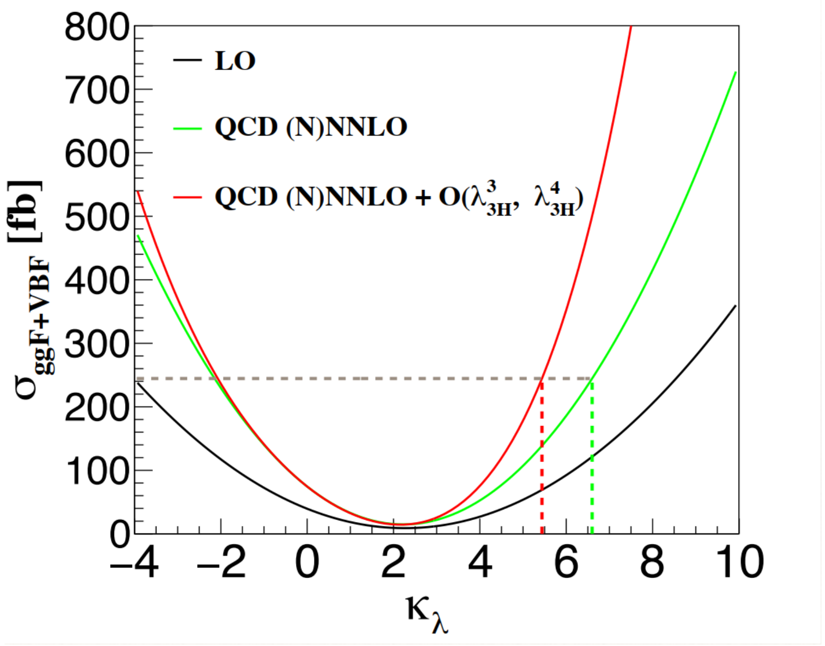
张晓, 8.15下午

$$\mathcal{L}_{\rm H} = \frac{1}{2}Z_\phi(\partial_\mu H)^2 - \left(-\frac{1}{2}Z_{\mu^2}Z_\phi Z_v^2\mu^2 v^2 + \frac{1}{4}Z_\lambda Z_\phi^2 Z_v^4 \lambda v^4\right) - (Z_\lambda Z_\phi^2 Z_v^3 \lambda v^3 - Z_{\mu^2}Z_\phi Z_v \mu^2 v)H$$
$$- \left(\frac{3}{2}Z_\lambda Z_\phi^2 Z_v^2 \lambda v^2 - \frac{1}{2}Z_{\mu^2}Z_\phi \mu^2\right)H^2 - Z_{\kappa_{3\rm H}} Z_\lambda Z_\phi^2 Z_v \lambda_{3\rm H} v H^3 - \frac{1}{4}Z_{\kappa_{4\rm H}} Z_\lambda Z_\phi^2 \lambda_{4\rm H} H^4 + \cdots ,$$

Li, Si, Wang, Zhang, Zhao, 2407.14716

More stringent constraint for ATLAS (CMS) limit:

6.6 (6.49) → 5.4 (5.37)

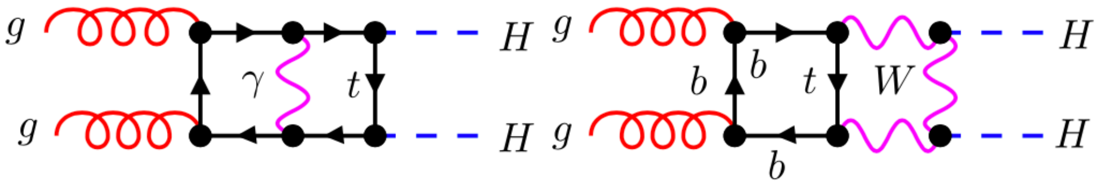


EW correction for $pp \to HH$

➢ EW: the largest theoretical uncertainty

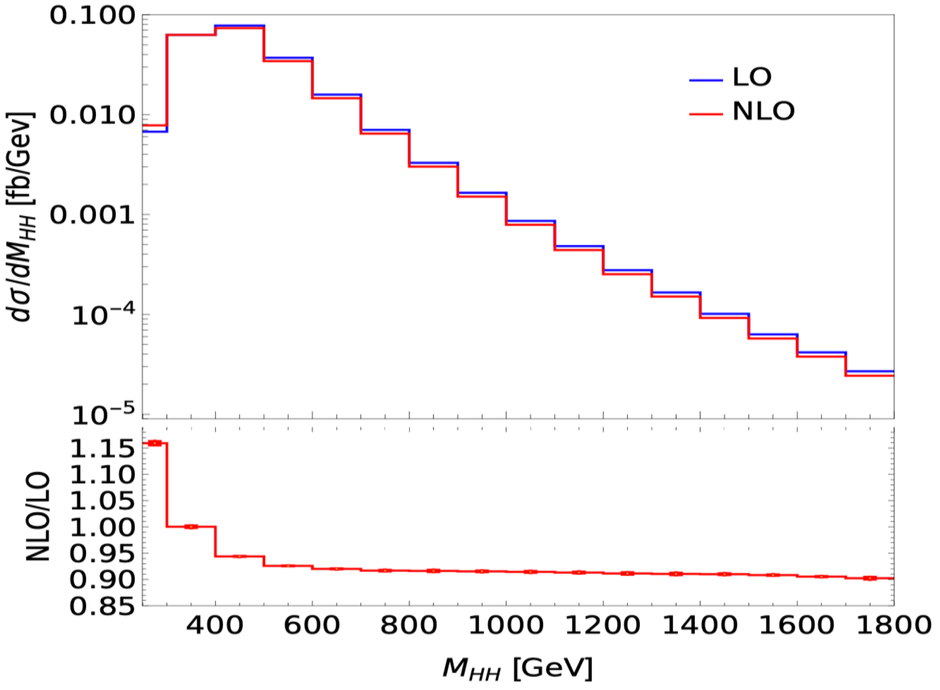
Bi, Huang, Huang, YQM, Yu, PRL2024

- QCD corrections known to NNNLO, 3% uncertainty



- Based on packages AMFlow+Blade+CalcLoop

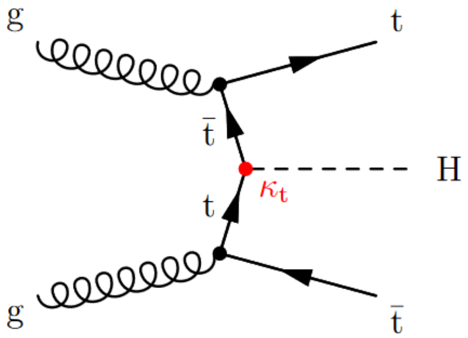
μ	$M_{HH}/2$	$\sqrt{p_T^2+m_H^2}$	m_H
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$-factor	0.958(1)	0.957(1)	0.954(1)



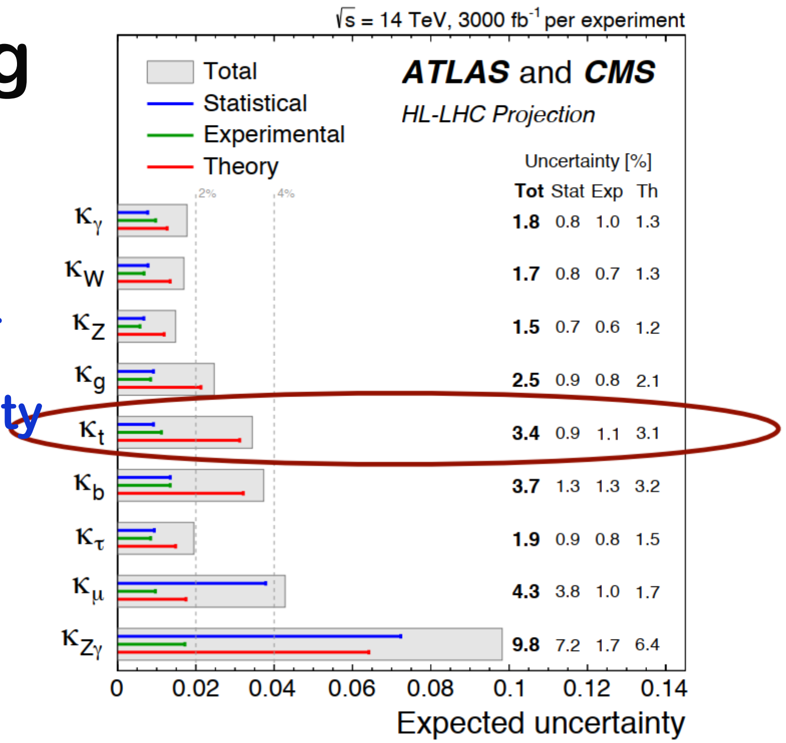
- EW corrections reduce the LO results by 4%
- More than 10% corrections for some regions
- Uncertainty of EW less than 1%

Towards NNLO calculation for $t\bar{t}H$ production

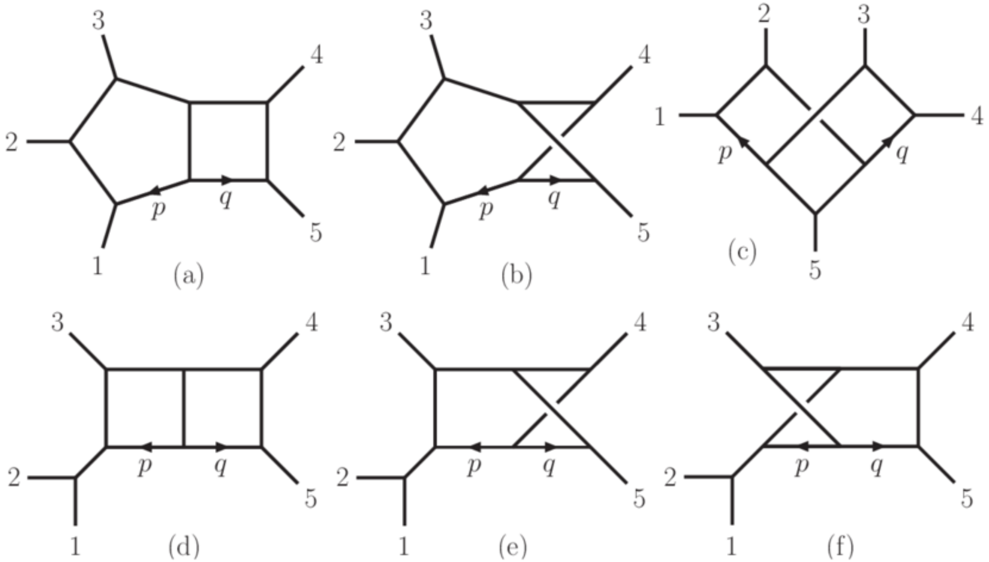
- **Direct probe of top quark Yukawa coupling**



Theoretical uncertainty is much larger than expected experimental uncertainty



Bottleneck: two-loop amplitudes



- **NNLO in soft H approximation**
  - **Two-loop estimated 100% error**
  - **Unreliable for differential cross section**

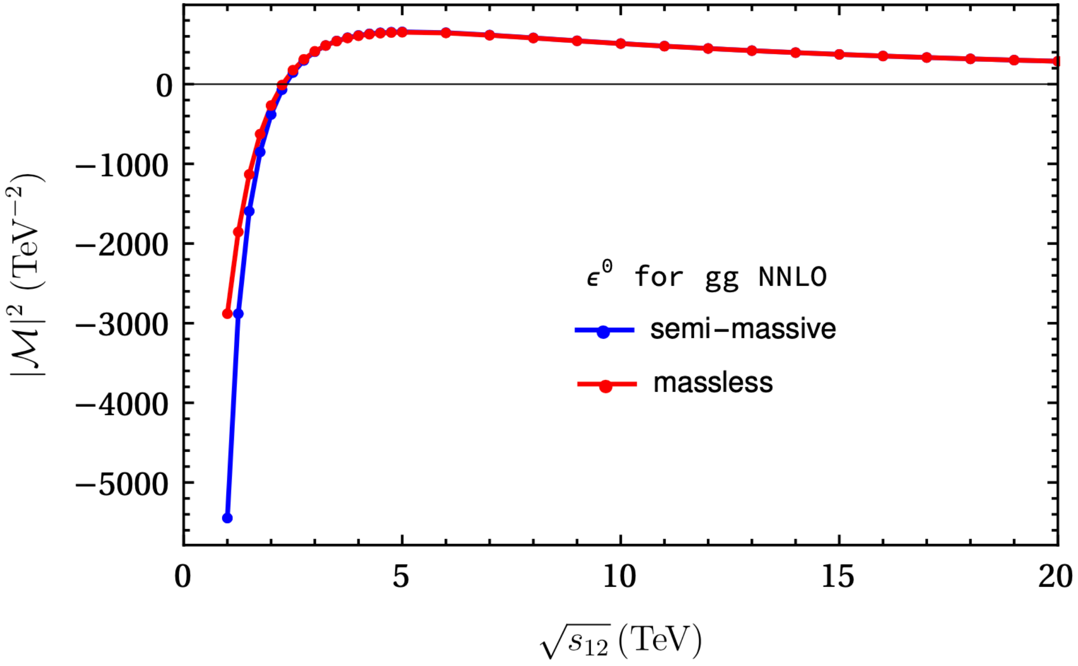
Catani et al., PRL2023

Towards NNLO calculation for $t\bar{t}H$ production (con.)

➢ Approximation in the high energy limit

杨李林, 8.15下午

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$



Wang, Xia, Yang, Ye, 2402.00431

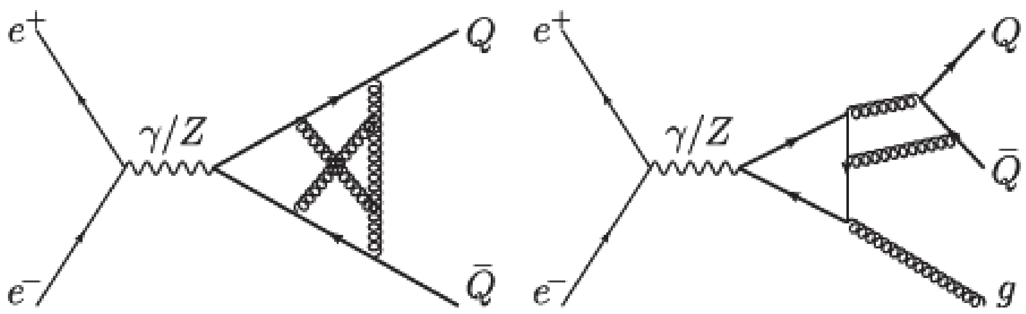
- **Massless amplitudes: much simpler, computed using standard techniques**
- **Two-loop amplitudes at high energies are ready**
- **Combine with low energy approximations (threshold / soft Higgs) in future**

Precise prediction for $e^+e^- \to Q\bar{Q}$

➢ For R-ratio

- Light particles production: known to N^4LO in QCD
- $Q\bar{Q}$ production: only N^2LO in QCD is known

➢ NNNLO QCD calculation

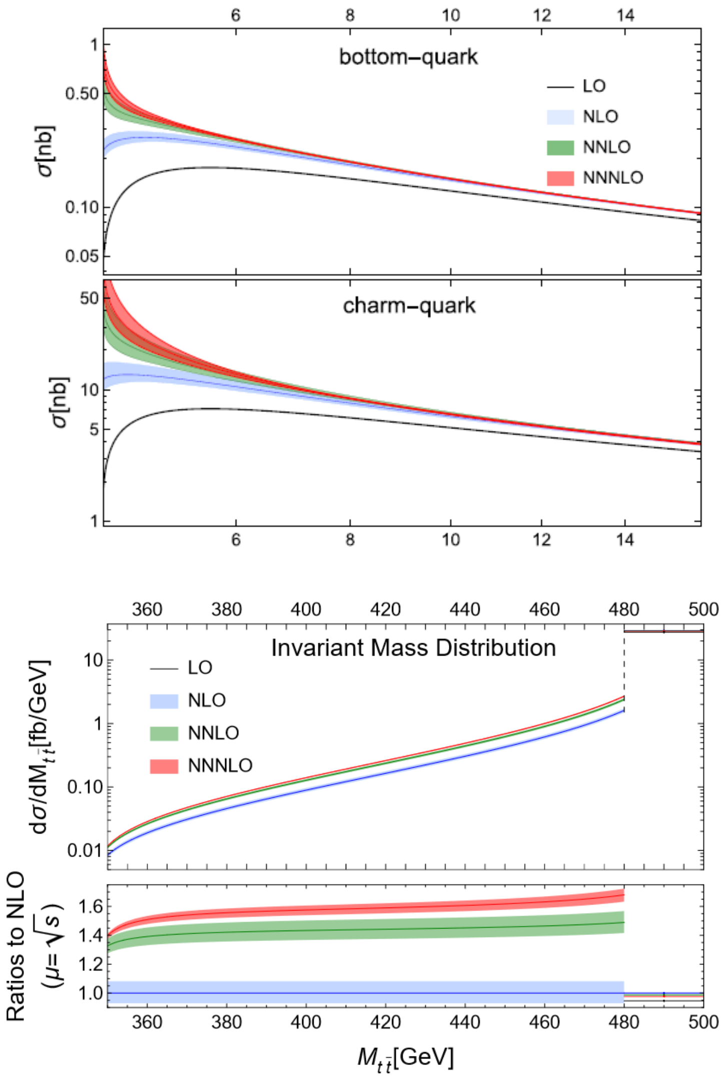


Based on packages AMFlow+Blade+CalcLoop

E.g. top: 0.1% correction for total,

10% correction for differential

Chen, Guan, He, Liu, YQM, PRL2024



Precise prediction for $e^+e^- \to Q\bar{Q}$ (con.)

➢ Factorized out Coulomb part and resum

吴兴刚, 8.15上午

$$\sigma = \sigma_0 \times \mathcal{R}_{\rm NC} \times \mathcal{R}_{\rm C}$$

exactly non-conformal term

$$\mathcal{R}_{\rm C} = 1 + C_F \frac{\pi}{2v}\alpha_s^{\rm V}(sv^2) + C_F^2 \frac{\pi^2}{12v^2}\alpha_s^{\rm V,2}(sv^2) + C_F\left(\frac{\pi^3}{3v}\beta_0^2 - C_F\frac{2\zeta_3}{v^2}\beta_0\right)\alpha_s^{\rm V,3}(sv^2) + \cdots$$

PMC

$$\mathcal{R}_{\rm C}\Big|_{\rm PMC} = 1 + C_F \frac{\pi}{2v}\alpha_s^{\rm V}(Q^2_{*,\rm C}) + C_F^2 \frac{\pi^2}{12v^2}\alpha_s^{\rm V,2}(Q^2_{*,\rm C}) + 0 \times \alpha_s^{\rm V,3}(Q^2_{*,\rm C}) + \cdots$$

resum

$$\frac{X}{1-\exp(-X)}$$

Sommerfeld-Gamow-Sakharov factor

$$X = \pi C_F \frac{\alpha_s^{\rm V}(Q^2_{*,\rm C})}{v}$$ Obtained by solving NR Schrödinger equation

$$\frac{X}{1-\exp(-X)} = 1 + \frac{X}{2} + \frac{X^2}{12} - \frac{X^4}{720} + \cdots$$ The X^3-coefficient is exactly zero!

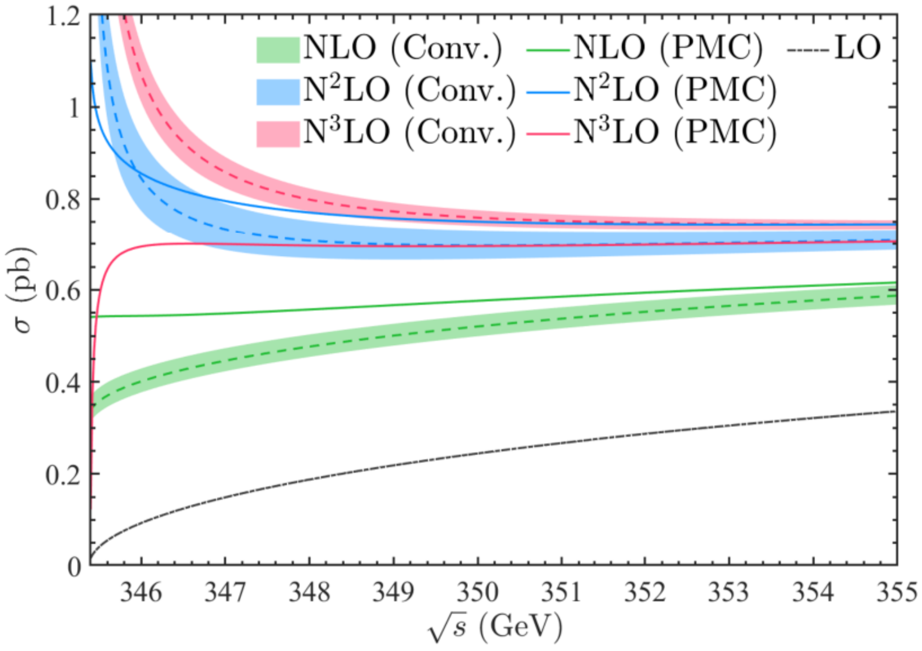


FIG. 3. (Color online) Total cross section $\sigma_{t\bar{t}}$ with different QCD corrections under conventional (dashed line) and PMCs (solid line) scale-setting approaches, respectively.

Yan, Wu, Wu, Shan, Zhou, PLB2024

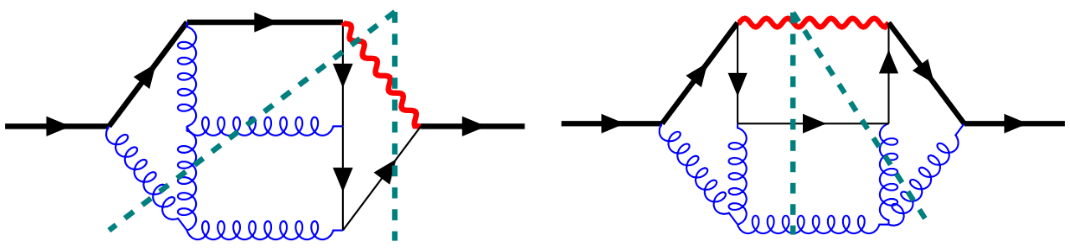
NNNLO QCD prediction for $t \to b + W$

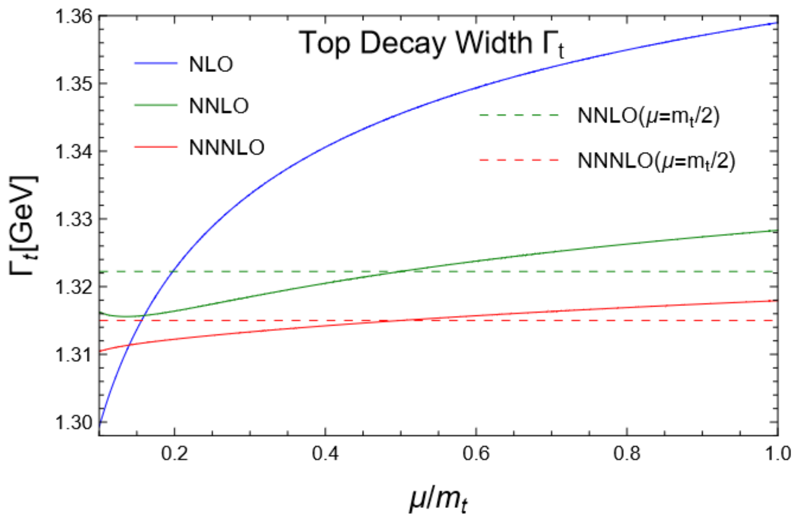
- Top decay width Γ_t: sensitive to new physics

王烨凡, 8.15上午

$\Gamma_t = 1.36 \pm 0.02\ (\text{stat.})^{+0.14}_{-0.11}\ (\text{syst.})$ GeV [CMS, 2014].

- In the future e+e− collider an uncertainty of 30 MeV will be achieved, needs NNNLO QCD





- Analytic for leading color: Chen, Li, Li, Wang, Wang, Wu, PRD2024
- Numeric for full color: Chen, Chen, Guan, Ma, 2309.01937

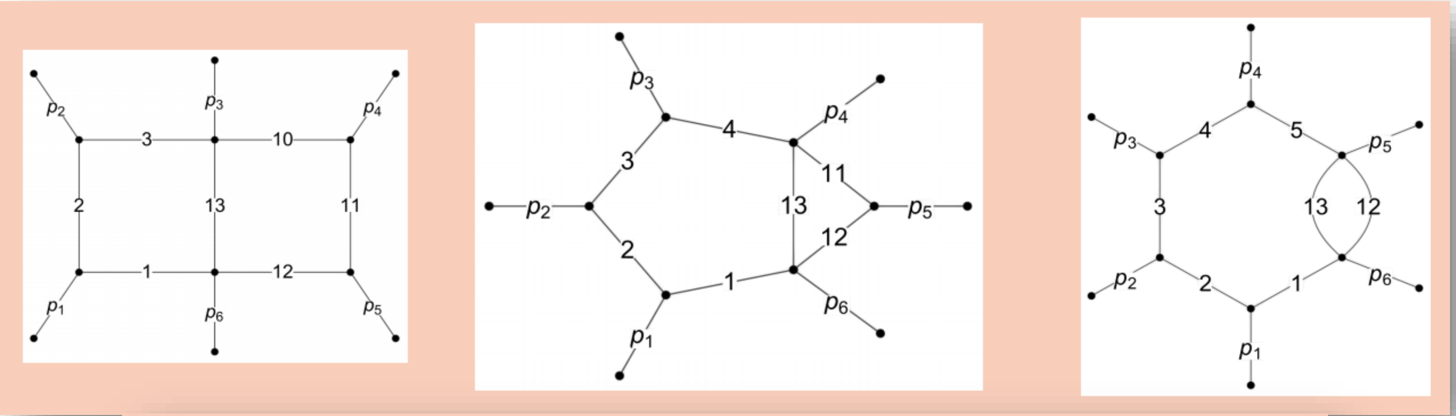
$$\Gamma_t = 1.3148^{+0.003}_{-0.005} + 0.027\,(m_t - 172.69)\,\text{GeV}$$

	$\delta_b^{(i)}$	$\delta_W^{(i)}$	$\delta_{\text{EW}}^{(i)}$	$\delta_{\text{QCD}}^{(i)}$	Γ_t [GeV]
LO	-0.273	-1.544	–	–	1.459
NLO	0.126	0.132	1.683	-8.575	$1.361^{+0.0091}_{-0.0130}$
NNLO	≈ 0.03	0.030	*	-2.070	$1.331^{+0.0055}_{-0.0051}$
N^{3}LO	*	0.009	*	-0.667	$1.321^{+0.0025}_{-0.0021}$

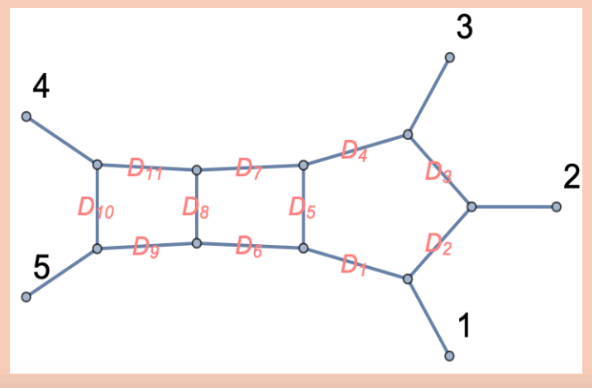
- Further improment using PMC scale settings: Jiang, Wu, Zhou, Li, Shan, PRD2024

Analytic computation of muliloop multileg integrals

张扬, 8.14下午



J. Henn, A. Matijasic, J. Miczajka, T. Peraro, Y. Xu, YZ, *JHEP08(2024)027*



Liu, Matijasic, Miczajka, Peraro, Xu, Xu, YZ, *to appear*

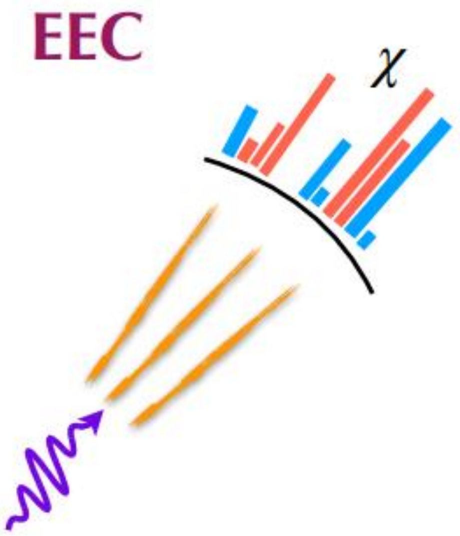
Canonical DEs: $\frac{\partial}{\partial x_i} I(x,\epsilon) = \epsilon A_i(x) I(x,\epsilon)$

Alphabet searching: $A_i = \frac{\partial}{\partial x_i}\tilde{A}, \quad \tilde{A} = \sum_k \tilde{a}_k \log(W_k)$

- **Analytic computation of 2loop 6point and 3loop 5point Feynman integrals for the first time**

- **The analytic computation of many more multi-loop multi-leg multi-scale Feynman integrals may be possible**

Energy Correlators



$$\Sigma_{\mathrm{EEC}} = \frac{1}{\sigma}\int d\sigma \sum_{ij} \frac{E_i E_j}{Q^2}\delta(\chi - \theta_{ij})$$

Sterman, 1975

Bashman, et al. 1978

Conformal collider physics:
Energy and charge correlations

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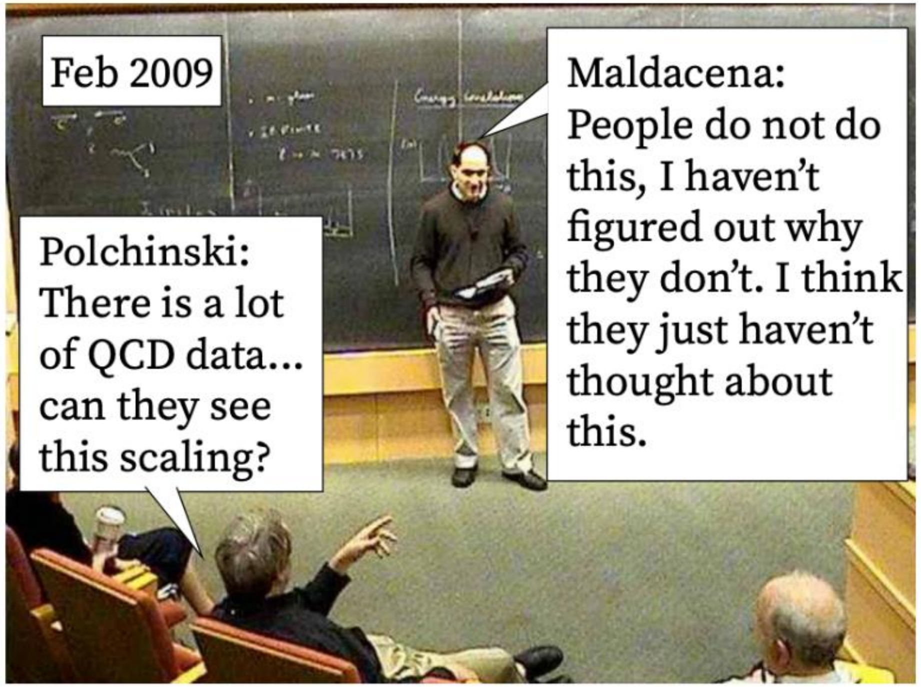
[a] Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA
[b] School of Natural Sciences, Institute for Advanced Study
Princeton, NJ 08540, USA

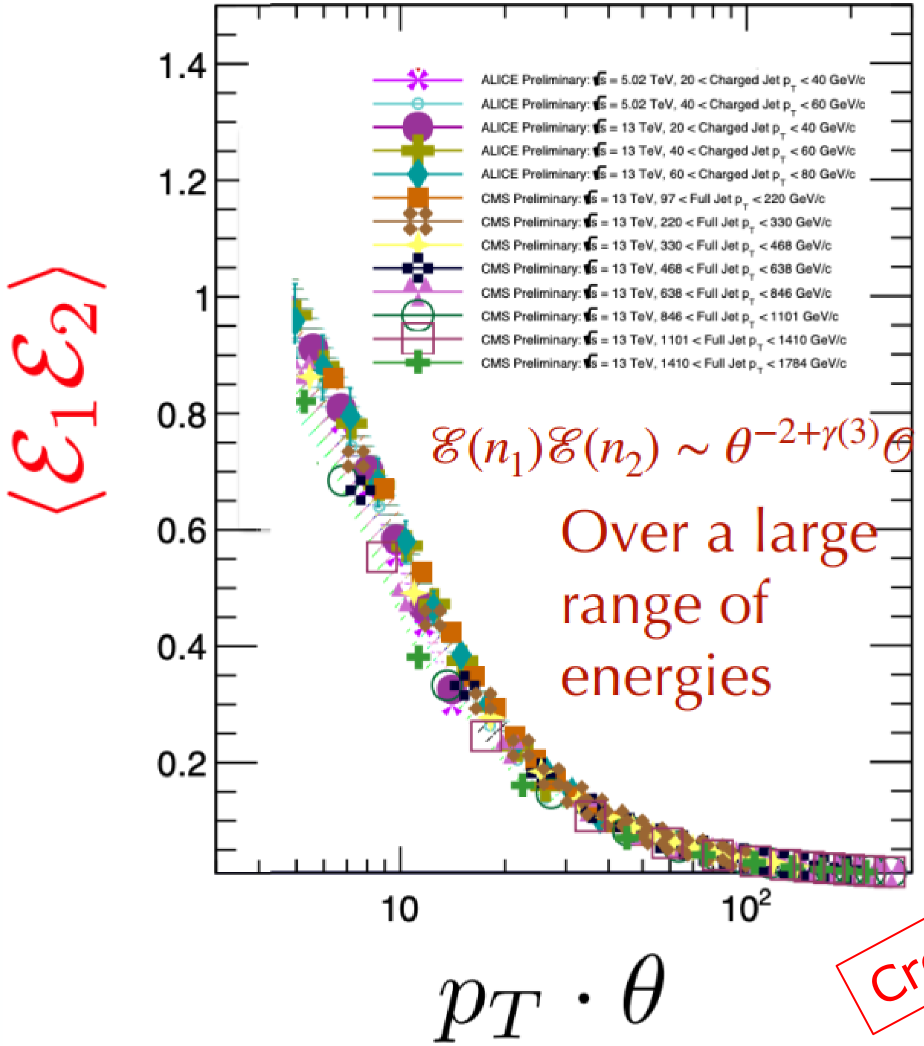


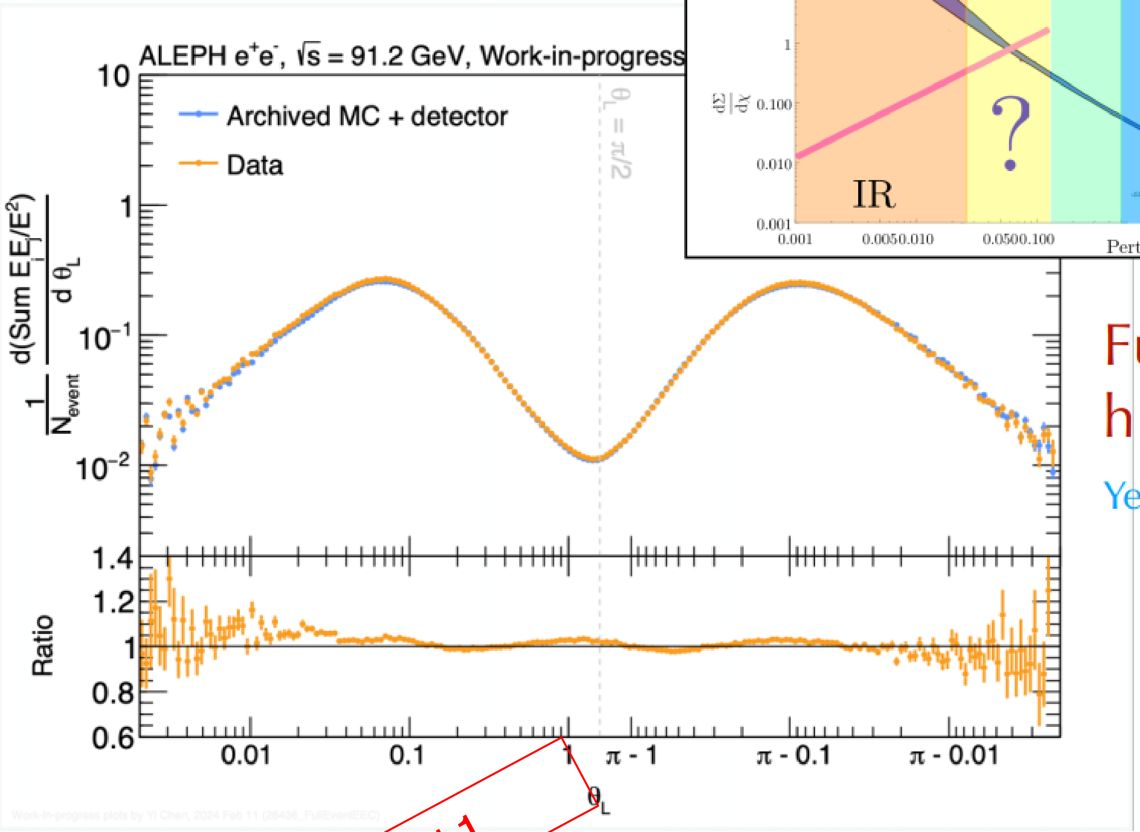
$$\mathcal{E}(n_1)\mathcal{E}(n_2) \sim \theta^{-2+\gamma(3)}\mathcal{O}$$

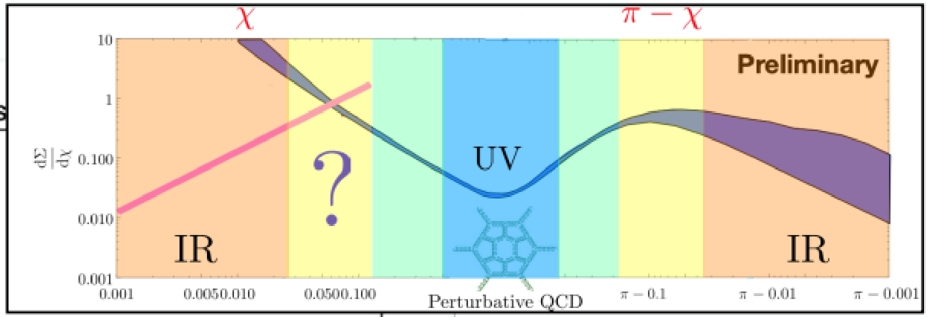
Scaling rule by conformal theory Hofman, Maldecena

Energy Correlators

As of 2024







Full Spectrum with high precision

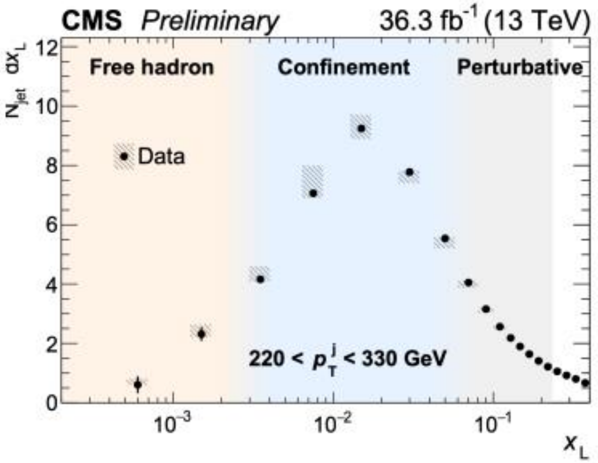
Yen-Jie Lee, MITP talk 2024

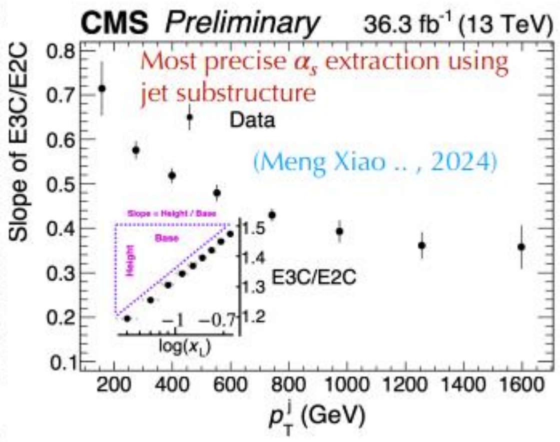
+Hao Chen, Yibei Li, Hua Xing Zhu; Jun Gao; Hai Tao Li; Dingyu Shao; Wang Wei; Meng Xiao; Kai Yan …

Rethinking jet using Energy Correlator

Dixon, Luo, Shtabovenko, Yang, Zhu, PRL 2018 **Editors' Suggestion**

Gao, Hai Tao Li, Moult, Zhu, PRL 2019 + ….

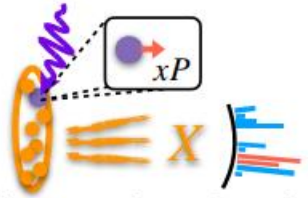


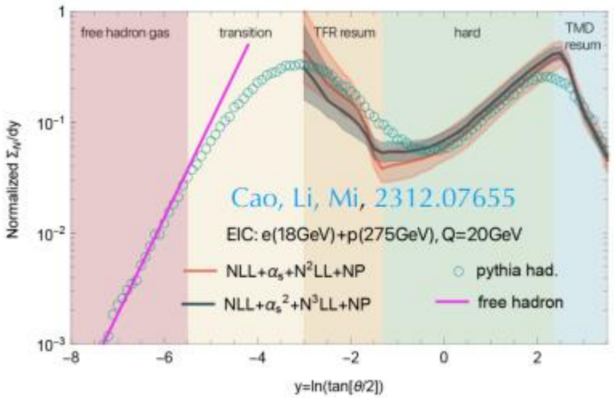


Nucleon Energy Correlator for hadron structures

Liu, Zhu, PRL 2023

Liu, Liu, Pan, Yuan, Zhu, PRL 2023 Physics + ….





A Different Angle on the Color Glass Condensate Physics

June 12, 2023 • Physics 16, s89

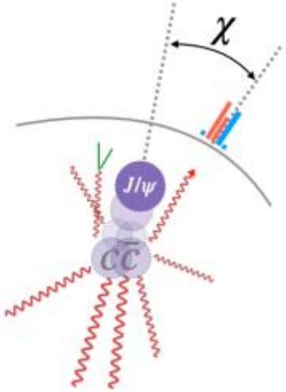
Predictions indicate that a new type of measurement at the future electron-ion collider could spot an elusive high-density regime of gluons called the color glass condensate.

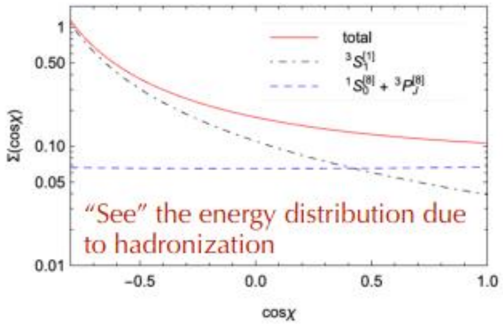
A new probe of QGP

Yang, He, Moult, Wang, 2024 PRL + …

Quarkonium Energy Correlator

Chen, Liu, YM, 2405.10056





+ Hao Chen, Yi Bei Li; Jun Gao; Hai Tao Li; Jian-Ping Ma; Dingyu Shao; Wang Wei; Kai Yan …

总结及展望

- 为了达到精确检验目的，高阶微扰计算亟需大力发展
- 过去两年多，中国微扰论团队在理论方法、程序开发、唯象应用方面，均取得巨大的进展
- **10**年时间，微扰论团队从萌芽发展至与国际并跑阶段
- 未来期待：从量变到质变，逐渐取得国际引领地位

谢谢！