Two-body Hadronic B-meson Decays in QCD Factorization Approach

李新强

华中师范大学

In collaboration with G. Bell, M. Beneke, T. Huber, and S. Krankl

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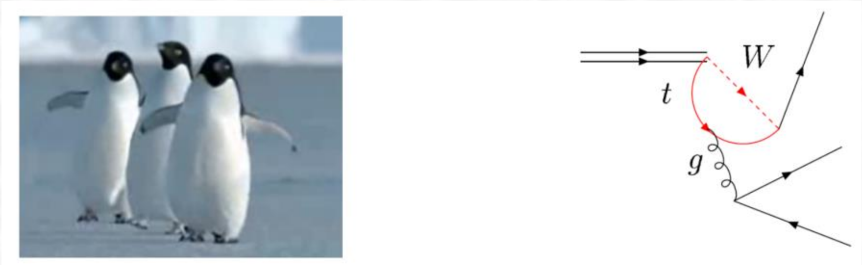
Outline

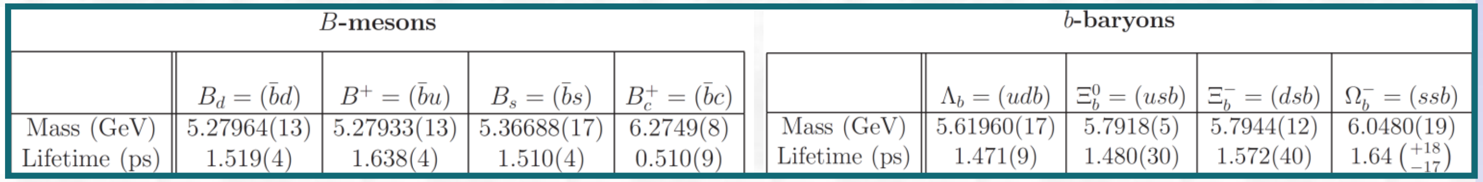
- Introduction & motivation
- Theoretical framework & QCDF approach for hadronic B decays
- NNLO perturbative QCD corrections to hadronic matrix elements
- Possible higher-order power corrections motivated by data
- Summary



Introduction & Motivation

B physics and B decays

- ☐ **B physics: productions & decays of various b hadrons**



	B-mesons			
	$B_d = (\bar{b}d)$	$B^+ = (\bar{b}u)$	$B_s = (\bar{b}s)$	$B_c^+ = (\bar{b}c)$
Mass (GeV)	5.27964(13)	5.27933(13)	5.36688(17)	6.2749(8)
Lifetime (ps)	1.519(4)	1.638(4)	1.510(4)	0.510(9)

	b-baryons			
	$\Lambda_b = (udb)$	$\Xi_b^0 = (usb)$	$\Xi_b^- = (dsb)$	$\Omega_b^- = (ssb)$
Mass (GeV)	5.61960(17)	5.7918(5)	5.7944(12)	6.0480(19)
Lifetime (ps)	1.471(9)	1.480(30)	1.572(40)	$1.64\,(^{+18}_{-17})$

- ☐ **b-hadron weak decays:** at the quark level, all governed by flavor-changing charged-currents mediated by W-boson

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} J^\mu_{\rm CC}\, W^\dagger_\mu + \text{h.c.}$$

$$J^\mu_{\rm CC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\,\gamma^\mu \begin{pmatrix} e_{\rm L} \\ \mu_{\rm L} \\ \tau_{\rm L} \end{pmatrix}$$

$$+ (\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L})\,\gamma^\mu\, V_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix}$$

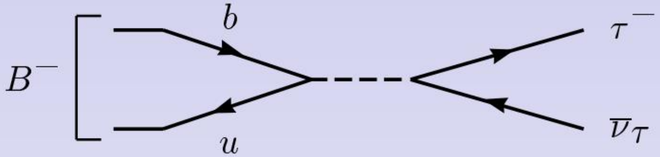
g: $SU(2)_L$ gauge coupling

V_{CKM}: CKM matrix for quark mixing

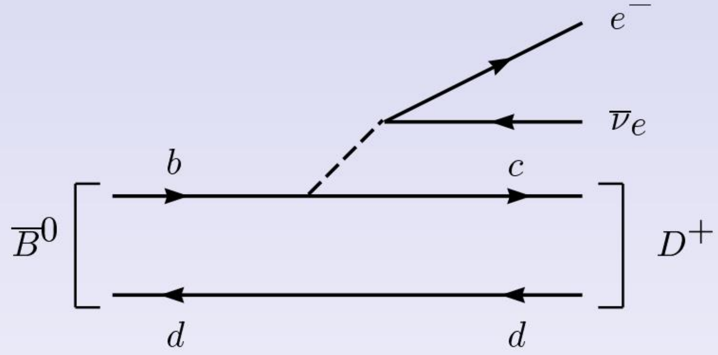
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- ☐ **Classification of b-hadron weak decays:** three classes

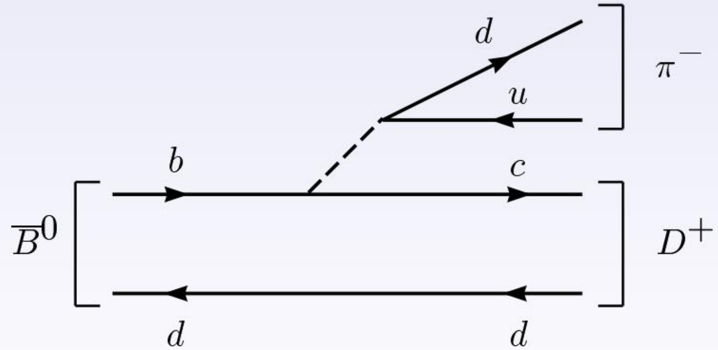
  purely leptonic, semi-leptonic, non-leptonic (hadronic)



purely leptonic



semi-leptonic



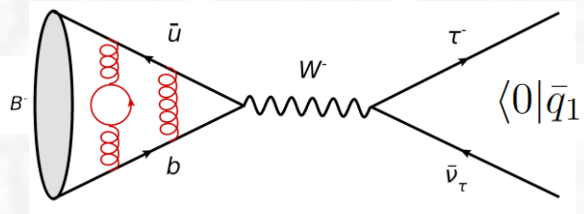
non-leptonic (hadronic)

Interplay between weak & strong forces

☐ **QCD effect always matters:** in real world, quarks confined inside hadrons and no free quarks;

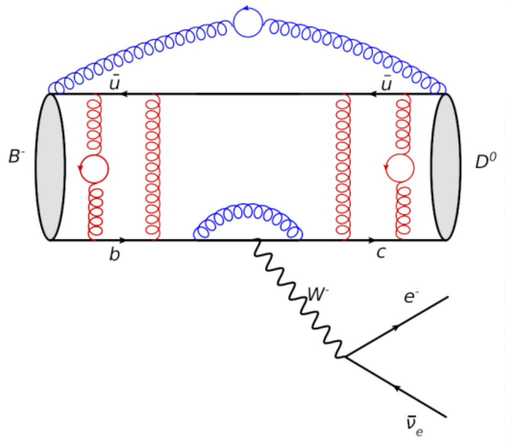
the simplicity of weak interactions overshadowed by the complexity of strong interactions

☐ **Purely leptonic decays:** decay constant



$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = ip_\mu f_P$$

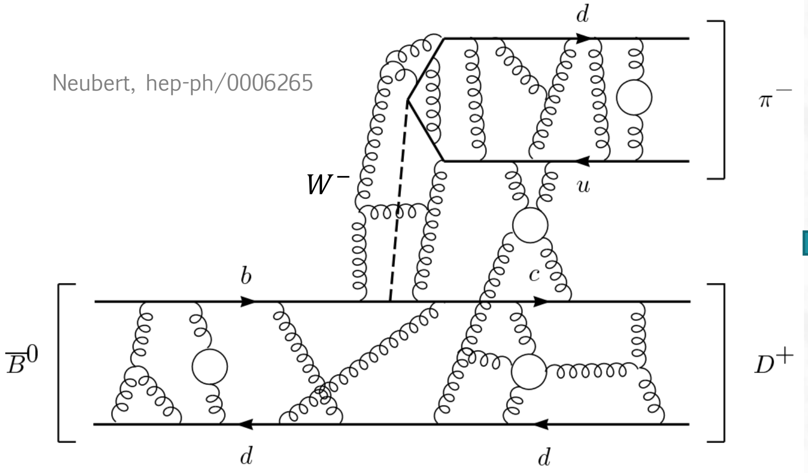
☐ **Semi-leptonic decays:** transition form factors



$$\langle D|\,\bar{c}\gamma^\mu b\,|\bar{B}\rangle \equiv f_+(q^2)(p_B+p_D)^\mu$$
$$+\,[f_0(q^2)-f_+(q^2)]\,\frac{m_B^2-m_D^2}{q^2}\,q^\mu$$

LQCD or LCSR etc.

☐ **Hadronic decays:** hadronic matrix elements


Neubert, hep-ph/0006265

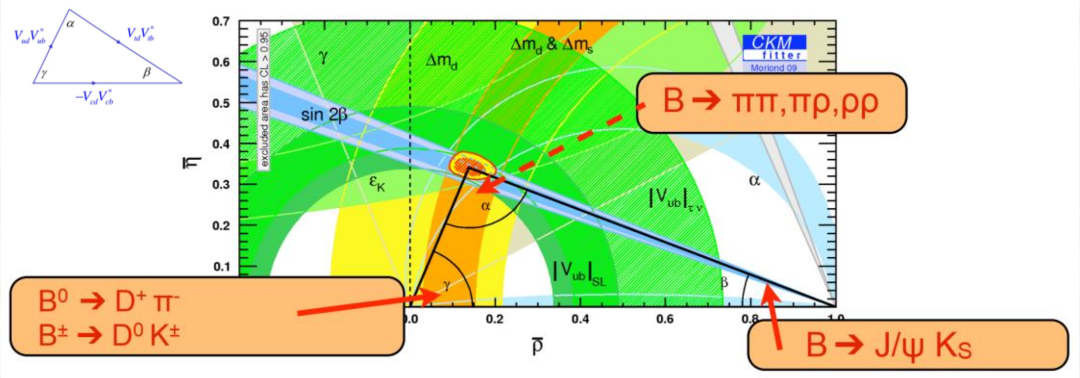
multi-scale problem with highly hierarchical scales!

EW interaction scale	≫	ext. mom'a in B rest frame	≫	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	≫	$m_b \sim 5$ GeV	≫	$\Lambda_{\rm QCD} \sim 1$ GeV

the most complicated case, but very important!

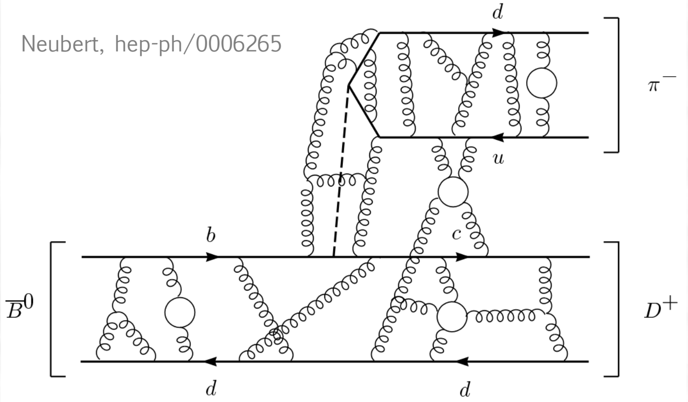
Why hadronic B decays

- ☐ **direct access to the CKM parameters, especially to the three angles of UT**



- ☐ **further insight into the strong-interaction effects involved in hadronic weak decays**

factorization? strong phase origin?...



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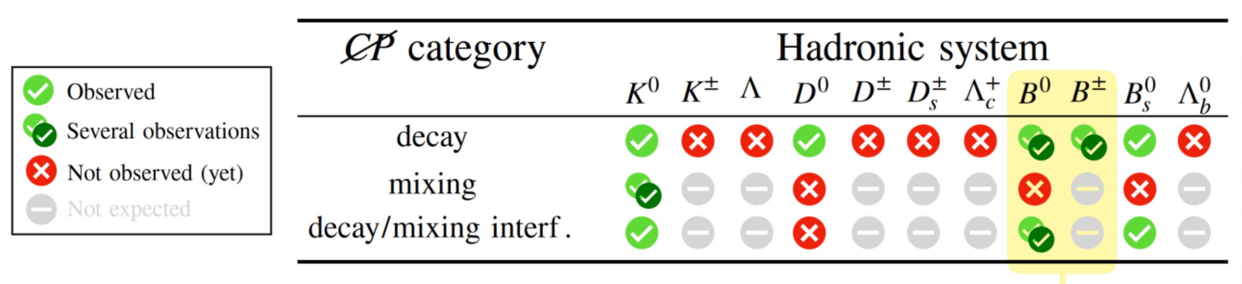
- ☐ **deep insight into the hadron structures: especially exotic hadronic states**

- ☐ **deepen our understanding of the origin & mechanism of CPV**



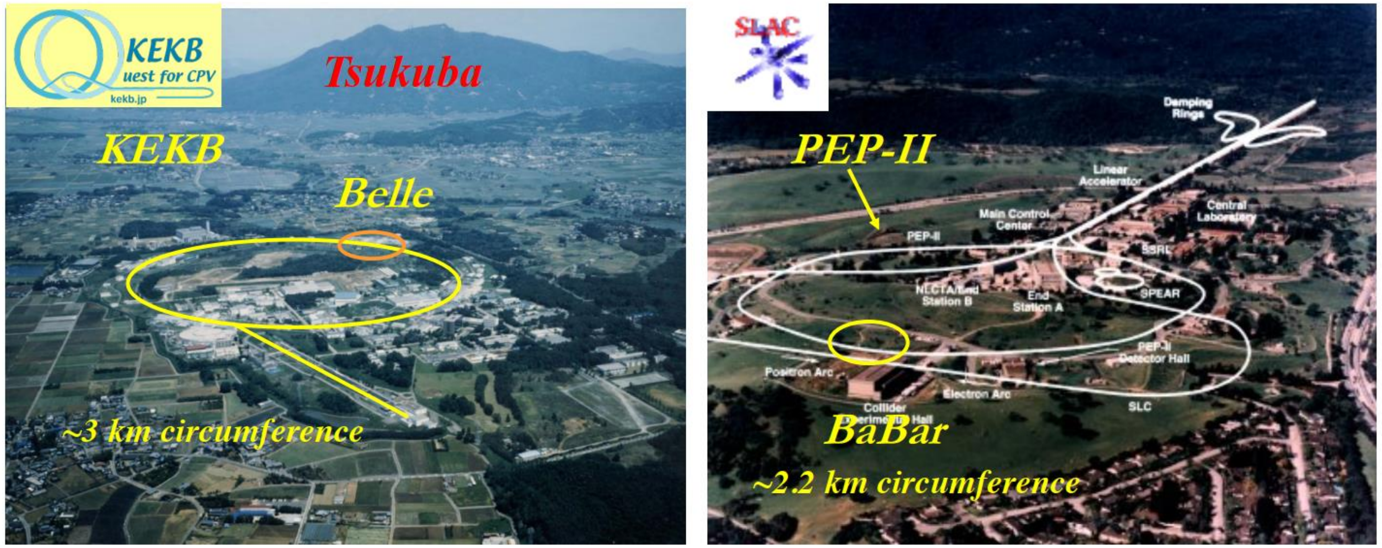
✓ Observed
✓✓ Several observations
✗ Not observed (yet)
⊖ Not expected

$\not{C}\not{P}$ category	K^0	$K^\pm$	Λ	D^0	$D^\pm$	$D_s^\pm$	Λ_c^+	B^0	$B^\pm$	B_s^0	Λ_b^0
decay	✓	✗	✗	✓	✗	✗	✗	✓✓	✓✓	✓	✗
mixing	✓✓	⊖	⊖	✗	⊖	⊖	⊖	✗	⊖	✗	⊖
decay/mixing interf.	✓	⊖	⊖	✗	⊖	⊖	⊖	✓✓	⊖	✓	⊖

➡ ***although very complicated but necessary both theoretically and experimentally!***

Exp. status of B physics

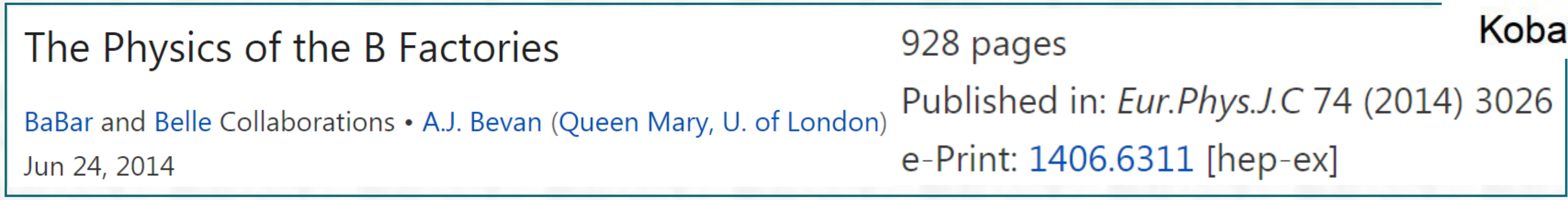
□ **B-factories (e^+e^-): Belle & BaBar**



3.5 GeV e^+ 8 GeV e^-

3.1 GeV e^+ 9 GeV e^-

BaBar & Belle confirmed the KM mechanism of CPV in the SM!


The Physics of the B Factories

BaBar and Belle Collaborations • A.J. Bevan (Queen Mary, U. of London)

Jun 24, 2014

928 pages

Published in: *Eur.Phys.J.C* 74 (2014) 3026

e-Print: 1406.6311 [hep-ex]

□ **Hadron colliders ($p\bar{p}$): CDF & D0 @ Tevatron**

https://www-d0.fnal;
https://www-cdf.fnal.gov/gov/

Observation of B_s mixing

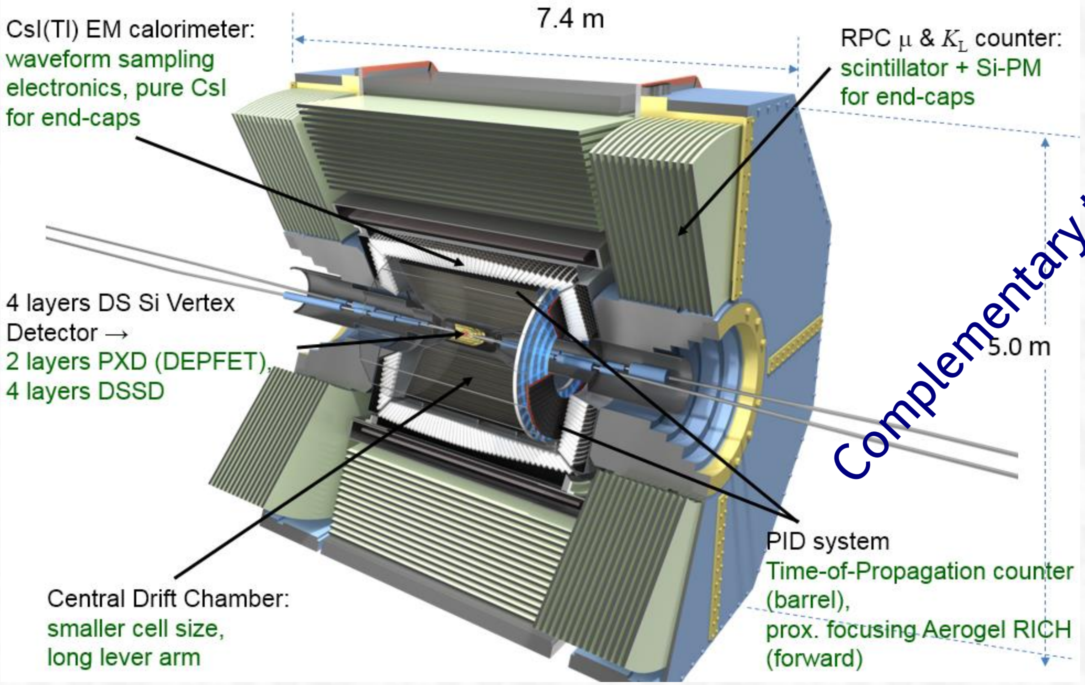
Nobel Prize 2008 for

Makoto Kobayashi

Toshihide Maskawa

Exp. status of B physics

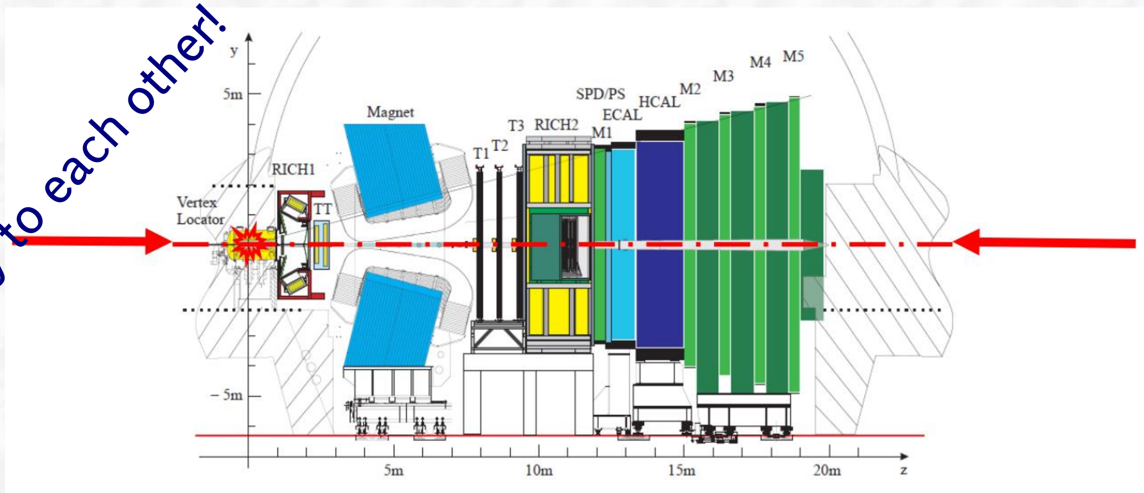
☐ **Super B-factories (e^+e^-): Belle II**



[E. Kou *et al.* [Belle II], PTEP 2019 (2019) 123C01]

☐ **Hadron colliders (pp): LHCb @LHC**

Complementary to each other!



[R. Aaij *et al.* [LHCb Collaboration], arXiv:1808.08865]

LHCb & Belle II: the two currently running experiments aimed at heavy flavor physics!

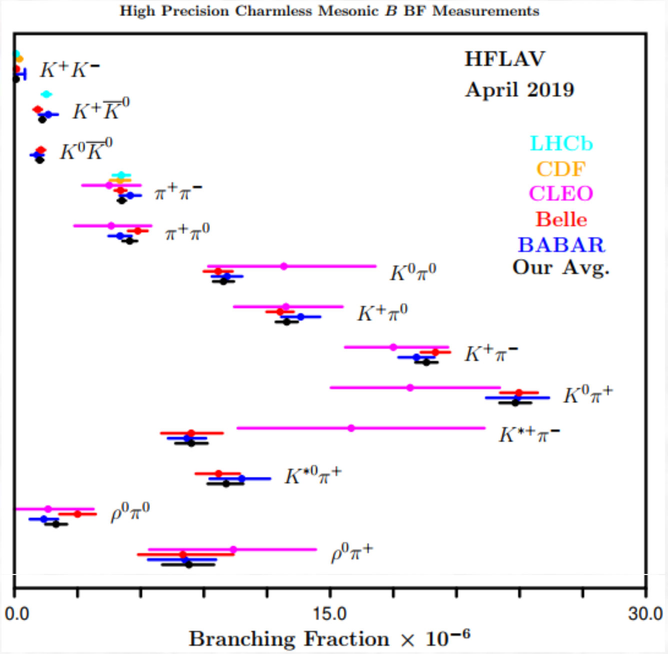
☐ **Two main goals among others:**

- **Check if there are any extra new CP-violation mechanisms beyond the KM?**
- **Check if there are new particles/interactions that are sensitive to flavor structures?**

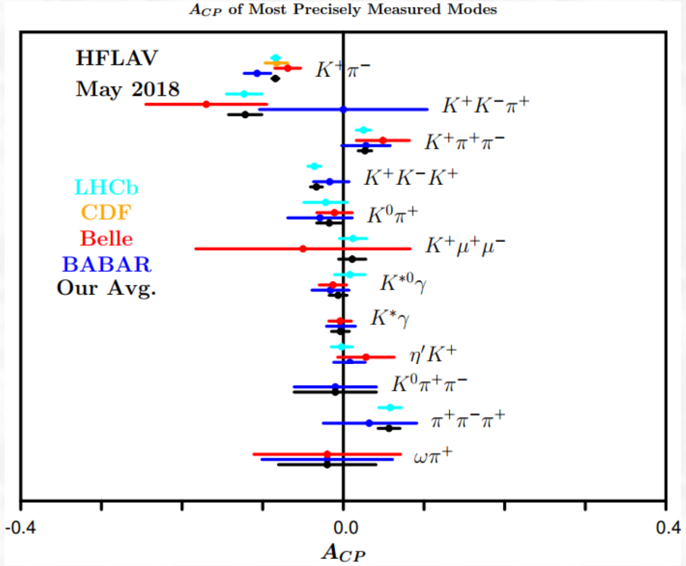
Precision era of B physics

- ☐ **More precise data from these dedicated experiments**

High Precision Charmless Mesonic B BF Measurements



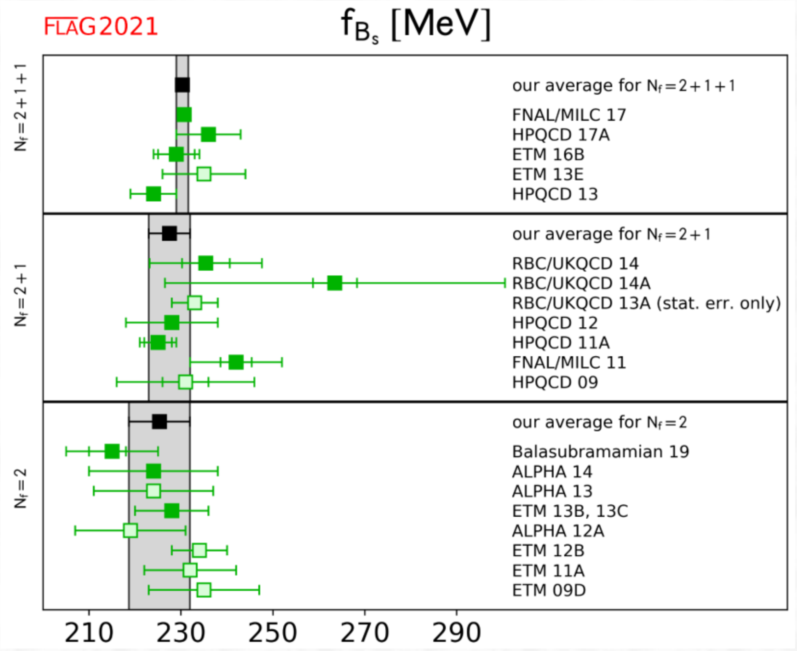
A_{CP} of Most Precisely Measured Modes

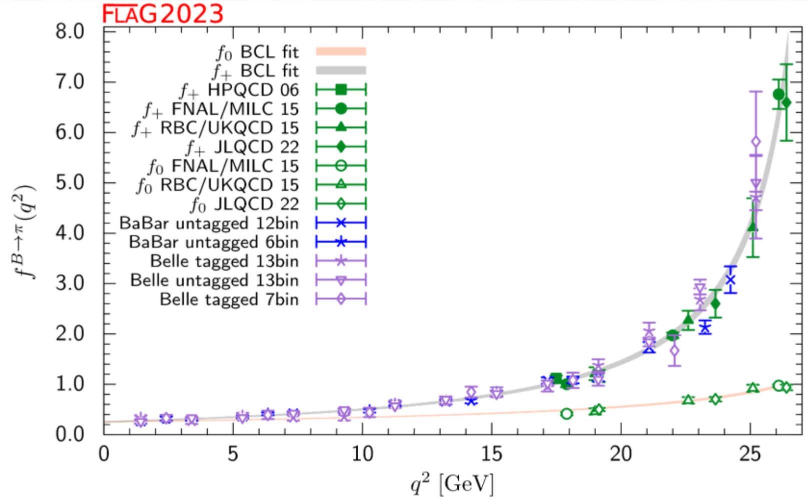


https://hflav.web.cern.ch/

- ☐ **Lattice QCD & LCSR etc. also provide more precise results for the non-pert. hadronic parameters**

we are entering an *era of precision flavor physics!*



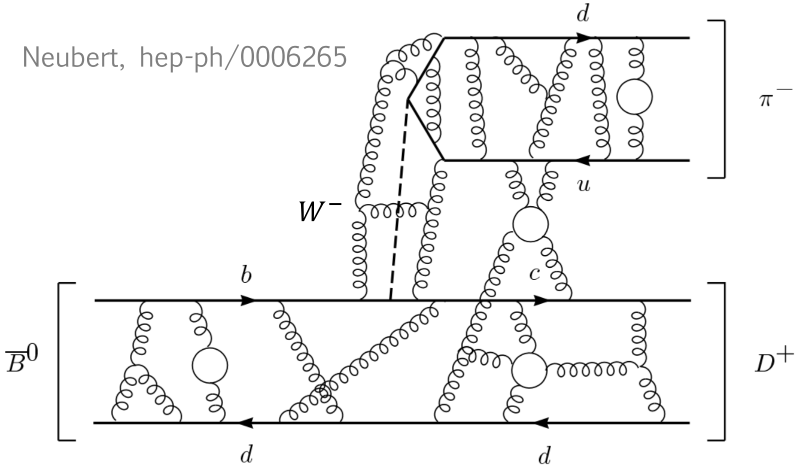


http://flag.unibe.ch/2021/

Theoretical framework & QCDF approach for hadronic B-meson decays

Effective Hamiltonian for hadronic B decays

- **For hadronic B decays: typical multi-scale problem;** → **EFT formalism more suitable!**



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multi-scale problem with highly hierarchical scales!

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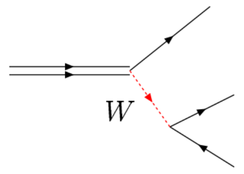
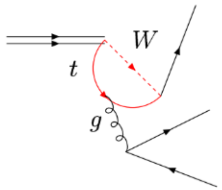
$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \left(C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + \sum_{i=\rm pen} C_i \mathcal{O}_{i,\rm pen} \right)$$

- **Starting point $\mathcal{H}_{\rm eff} = -\mathcal{L}_{\rm eff}$: obtained after integrating out heavy d.o.f. ($m_{W,Z,t} \gg m_b$)**

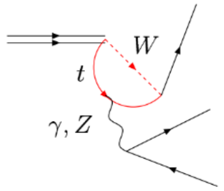
  [Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Munz '98]



tree



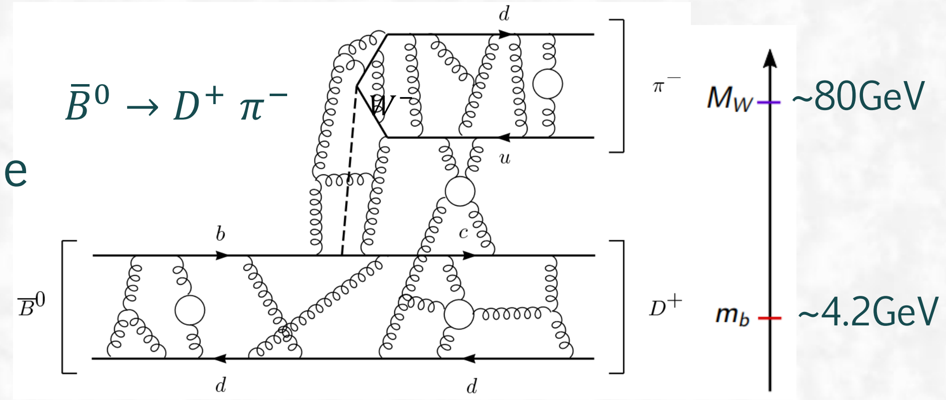
QCD penguin



EW penguin

- **Wilson coefficients C_i: all physics above m_b; perturbatively calculable & NNLL program now complete!** [Gorbahn, Haisch '04; Misiak, Steinhauser '04]

Calculation of $C_i(\mu_b)$



☐ **Problem:** well-separated multiple scales would spoil the perturbative convergence due to large logs

$$\boxed{P(M_W, m_b) = 1 + \alpha_s \left(\# \ln \frac{M_W}{m_b} + *\right) + \alpha_s^2 \left(\# \ln^2 \frac{M_W}{m_b} + *\right) + \ldots}$$

☐ **Solution:** the perturbative series needs to be re-organized, and all $(\alpha_s \ln \frac{m_W}{m_b})^n$ re-summed!

- **step1:** through matching to achieve a separation of scales, sometimes also called "factorization";

$$\left[1 + \alpha_s \left(\# \ln \frac{M_W}{\mu} + *\right) + \ldots\right] \cdot \left[1 + \alpha_s \left(\# \ln \frac{\mu}{m_b} + *\right) + \ldots\right]$$

$$P(M_W, m_b) = C(M_W, \mu) D(m_b, \mu)$$

$\boxed{\mu \text{ arbitrary}}$

at the cost of introducing a "factorization scale" μ.

- **step2:** solve RGE and evolve

$$\text{RGEs:} \left\{ \begin{array}{rcl} \mu\frac{d}{d\mu}C(M_W,\mu) & = & \gamma(\mu)\,C(M_W,\mu) \\ \mu\frac{d}{d\mu}D(M_W,\mu) & = & -\gamma(\mu)\,D(M_W,\mu) \end{array} \right\} \Rightarrow \mu\frac{d}{d\mu}(CD) = 0$$

["C and D run with μ."]

$\boxed{\mu_{\text{high}} \sim m_W}$

$$\begin{array}{rcl} C(M_W,\mu) & = & C(M_W,\mu_{\text{high}})\,U(\mu_{\text{high}},\mu) \\ D(m_b,\mu) & = & D(m_b,\mu_{\text{low}})\,U(\mu,\mu_{\text{low}}) \end{array}$$

$\boxed{\mu_{\text{low}} \sim m_b}$

☐ **Final result:**

RG-improved P.T.

$$P(M_W, m_b) = \boxed{\underbrace{C(M_W,\mu_{\text{high}})\,U(\mu_{\text{high}},\mu_{\text{low}})}_{C_{\text{RGimproved}}(M_W,\mu_{\text{low}})}}\,D(m_b,\mu_{\text{low}})$$

$U(\mu_{\text{high}}, \mu_{\text{low}})$ is generally an exponential, and hence re-sums large logs $(\alpha_s \ln\frac{\mu_{\text{high}}}{\mu_{\text{low}}})^n$!

Calculation of Wilson coefficients $C_i(\mu_b)$

☐ Three steps to get $C_i(\mu_b)$:

- Matching calculation of $C_i(m_W)$ in fixed-order perturbation theory:

$$C_i(m_W) = C_i^{(0)}(m_W) + \frac{\alpha_s}{4\pi} C_i^{(1)}(m_W) + \cdots$$

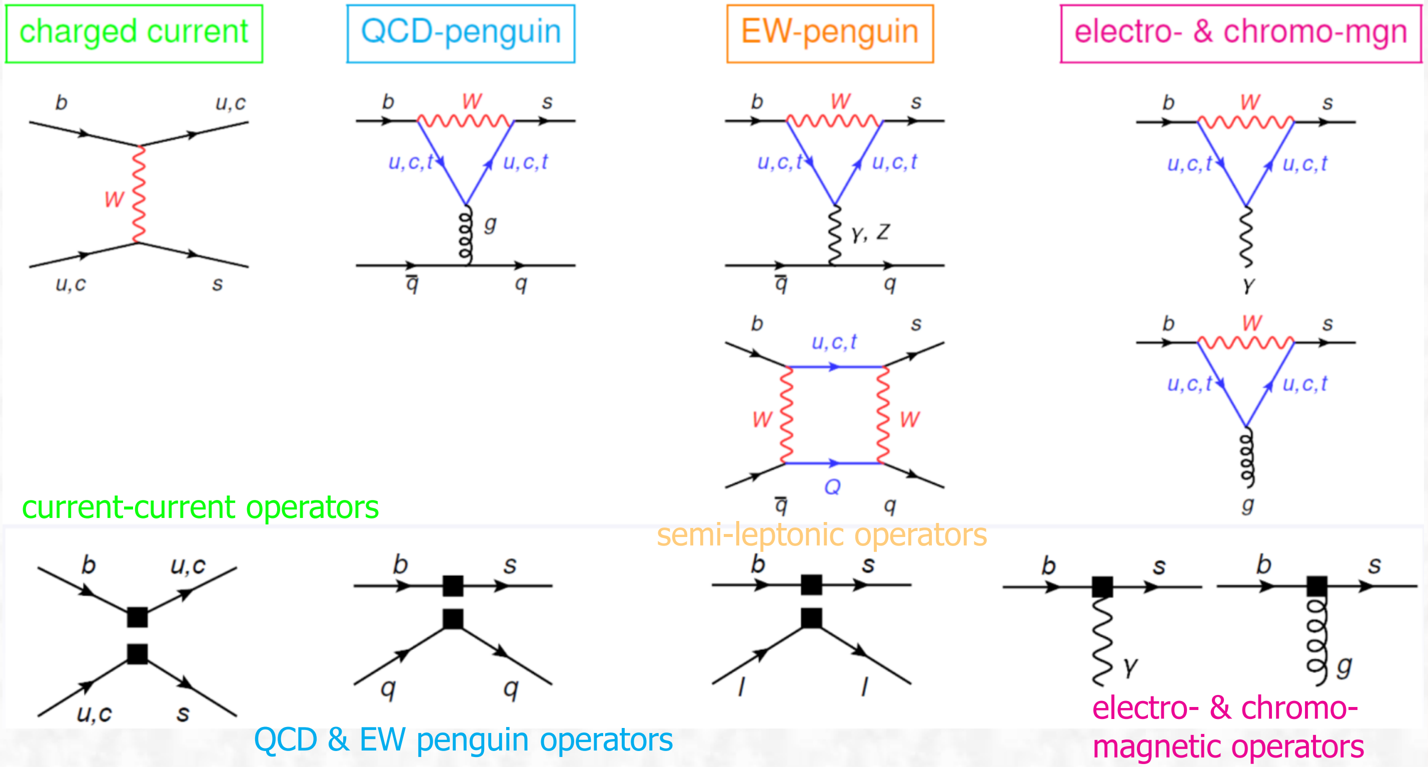
- Calculation of anomalous dimensions γ_{ij} of local operators in $\mathcal{H}_{\text{eff}}$:

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\alpha_s}{4\pi} \gamma_{ij}^{(1)} + \cdots$$

- Use renormalization group to evolve the Wilson coefficients from the high to the low scale:

$$C_i(m_W) \to C_i(m_b) = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right)^{-\gamma_{ij}^{(0)}/2\beta_0} C_j(m_W) + \cdots$$

☐ Local operators $\mathcal{O}_i$:



$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10 \quad |C_i(m_b)| \sim 4 \end{cases}$$

Hadronic matrix elements

- ☐ **For a typical two-body decay $\bar{B} \to M_1 M_2$:**

$$\boxed{\mathcal{A}(\bar{B} \to M_1 M_2) = \sum_i [\lambda_{\text{CKM}} \times C_i \times \langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle]}$$

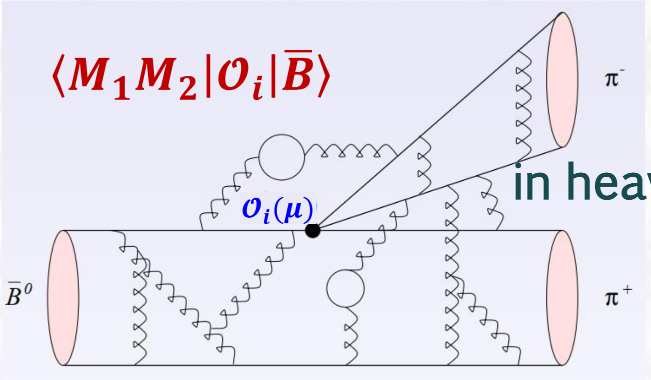
- ☐ $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$: depending on spin & parity of $M_{1,2}$; final-state re-scattering introduces strong phases, and hence non-zero direct CPV; ➡ *A quite difficult, multi-scale, strong-interaction problem!*

- ☐ **Different methods proposed for dealing with $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$:** naïve fact., generalized fact., ……

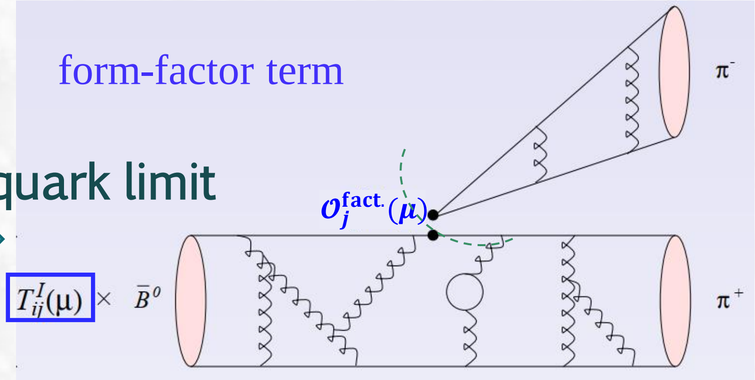
  - Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, · · ·
    [Keum, Li, Sanda, Lü, Yang '00;
    Beneke, Buchalla, Neubert, Sachrajda, '00;
    Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
    how to include higher-order perturbative and power corrections?

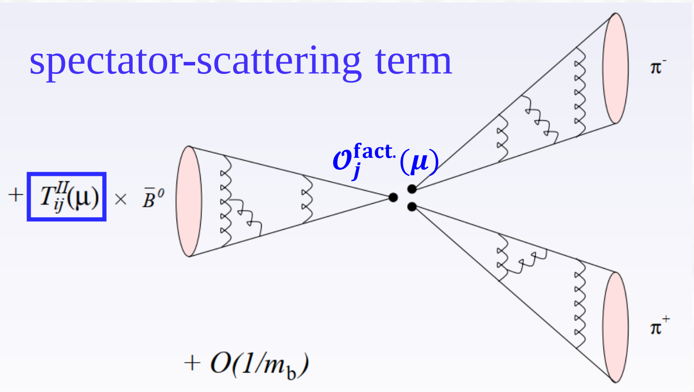
  - Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, · · ·
    [Zeppenfeld, '81;
    London, Gronau, Rosner, He, Chiang, Cheng *et al.*]
    how to systematically estimate symmetry-breaking effects?

- ☐ **QCDF/SCET:** systematic framework to all orders in α_s, limited by Λ_{QCD}/m_b corrections [BBNS '99-'03]



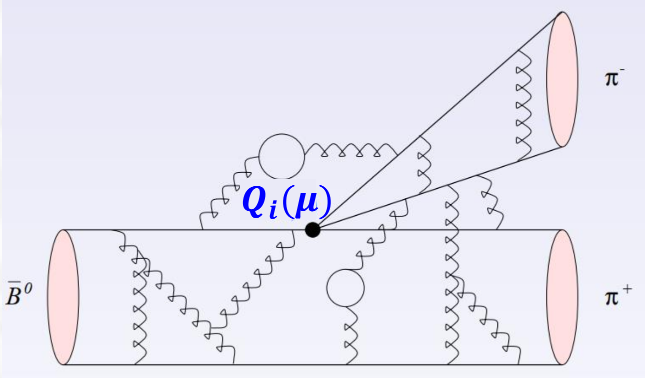
in heavy-quark limit ➡





QCDF formula for charmless B decays

☐ **QCDF formula:** [BBNS '99-'03]



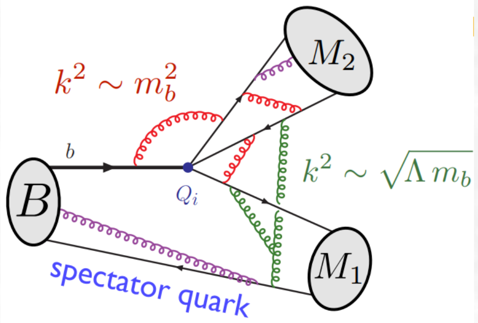
$$\langle M_1 M_2|Q_i|\bar{B}\rangle \sim F^{B\to M_1}(q^2=0)\int_0^1 \mathrm{d}x\,\mathbf{T}_i^{\mathrm{I}}(x)\,\phi_{M_2}(x) \quad \text{form-factor term}$$

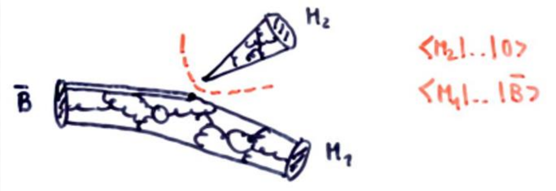
$$+\int_0^\infty \frac{\mathrm{d}\omega}{\omega}\int_0^1 \mathrm{d}x\,\mathrm{d}y\;\mathbf{T}_i^{\mathrm{II}}(x,y,\omega)\,\phi_{M_1}(y)\,\phi_{M_2}(x)\,\phi_B^+(\omega)$$

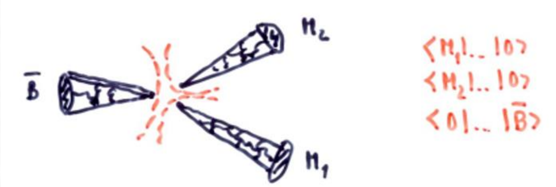
spectator-scattering term

☐ **How to proof QCDF formula:**

- based on diagrammatic factorization [BBNS '99-'03]
- method of expansion by regions [Beneke, Smirnov '97]
- heavy-quark & collinear expansion for hard processes [Lepage, Brodsky '80]





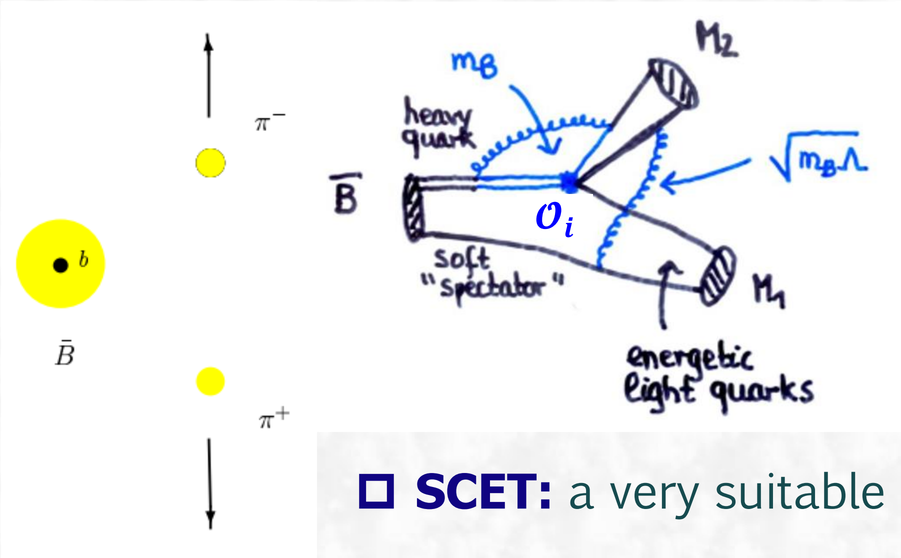


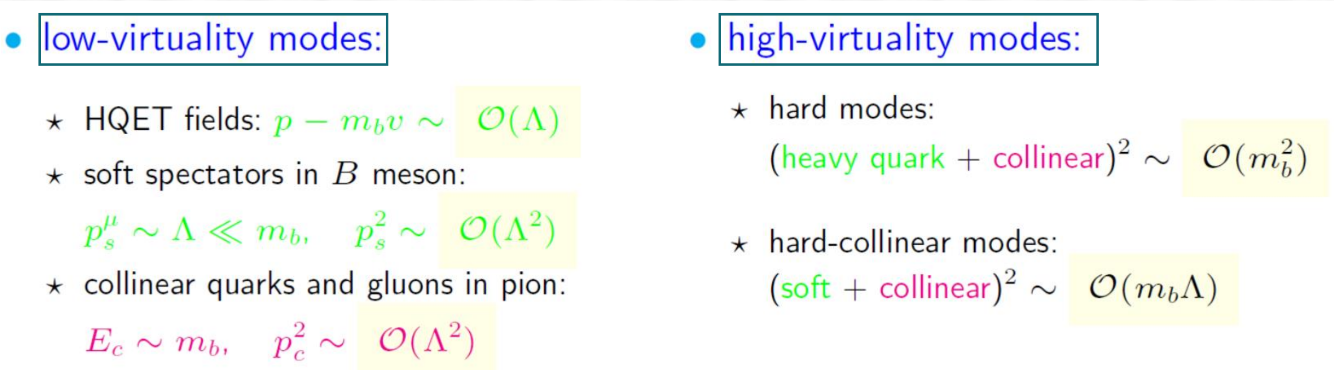
universal non-perturbative hadronic parameters

➡ $\langle M_1 M_2|Q_i|\bar{B}\rangle$ factorized into $\langle M|j_\mu|\bar{B}\rangle$ (transition form factors), $\langle M|j_\mu|0\rangle$, $\langle 0|j_\mu|\bar{B}\rangle$ (decay constants & LCDAs)

Soft-collinear factorization from SCET

☐ **For a two-body decay:** simple kinematics, but complicated dynamics with several typical modes





- low-virtuality modes:
  - ⋆ HQET fields: $p - m_b v \sim$ $\mathcal{O}(\Lambda)$
  - ⋆ soft spectators in B meson:
    $p_s^\mu \sim \Lambda \ll m_b$, $p_s^2 \sim$ $\mathcal{O}(\Lambda^2)$
  - ⋆ collinear quarks and gluons in pion:
    $E_c \sim m_b$, $p_c^2 \sim$ $\mathcal{O}(\Lambda^2)$
- high-virtuality modes:
  - ⋆ hard modes:
    (heavy quark + collinear)$^2 \sim$ $\mathcal{O}(m_b^2)$
  - ⋆ hard-collinear modes:
    (soft + collinear)$^2 \sim$ $\mathcal{O}(m_b\Lambda)$

☐ **SCET:** a very suitable framework for studying factorization and re-summation for processes involving energetic & light particles/jets [Bauer *et al.* '00; Beneke *et al.* '02]

☐ **From SCET point of view:** introduce different fields/modes for different momentum regions, and SCET diagrams must reproduce precisely QCD diagrams in collinear & soft momentum region!

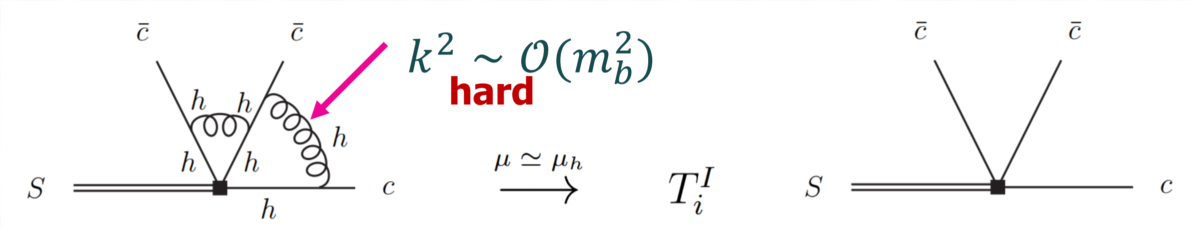
↳ achieve *soft-collinear factorization* & hence QCDF formula via QFT machinery [Beneke, 1501.07374]

Soft-collinear factorization from SCET

☐ **QCDF formula from SCET:** hard kernels $T^{I,II}$ = matching coefficients from QCD to SCET

$$\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2} \Longrightarrow \boxed{\text{QCD - SCET} = T^I \ \&\ T^{II}}$$

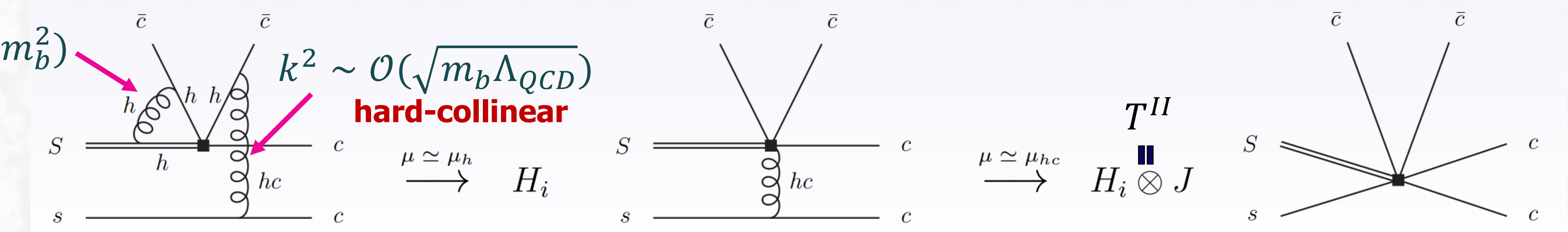
☐ **For T^I:** only hard scale involved, one-step matching from QCD → $\mathrm{SCET_I(hc, c, s)}$!



$k^2 \sim \mathcal{O}(m_b^2)$
hard

☐ **For T^{II}:** two scales involved, two-step matching from QCD → $\mathrm{SCET_I(hc, c, s)}$ → $\mathrm{SCET_{II}(c, s)}$!

$k^2 \sim \mathcal{O}(m_b^2)$
hard



$k^2 \sim \mathcal{O}(\sqrt{m_b \Lambda_{QCD}})$
hard-collinear

☐ **SCET formalism reproduces exact QCDF formula, but more apparent & efficient;** [Beneke, 1501.07374]

$$\boxed{\langle M_1 M_2 | Q_i | \bar{B} \rangle = T^I(\mu_h) * \phi_{M_2}(\mu_h) f_+^{BM_1}(0) + H_i(\mu_h) * U_\parallel(\mu_h, \mu_{hc}) * J(\mu_{hc}) * \phi_{M_2}(\mu_h) * \phi_{M_1}(\mu_{hc}) * \phi_B(\mu_{hc})}$$

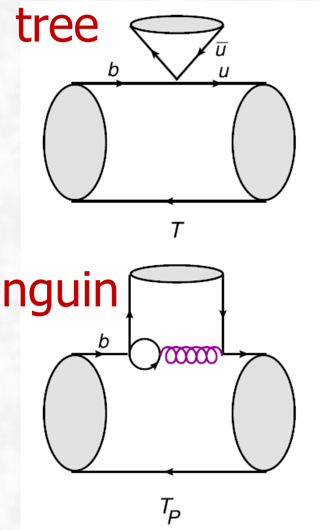
Status of NNLO calculation of T^I & T^{II}

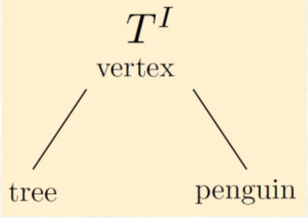
☐ **For each Q_i insertion, both tree & penguin topologies relevant for charmless decays**

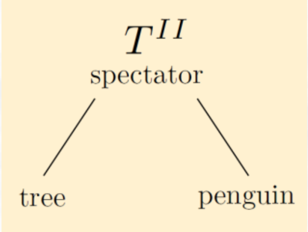
$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2}$$
$$+ T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

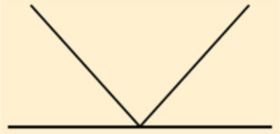
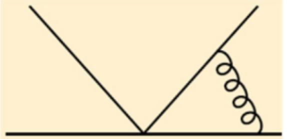
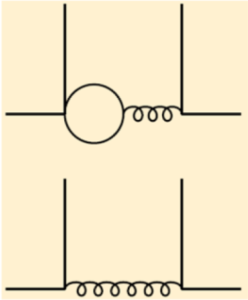
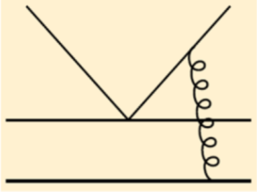
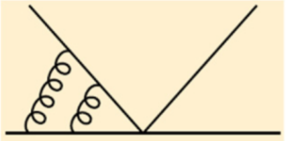
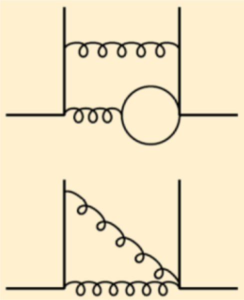
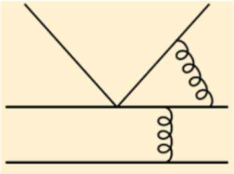
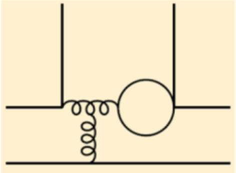
tree

penguin





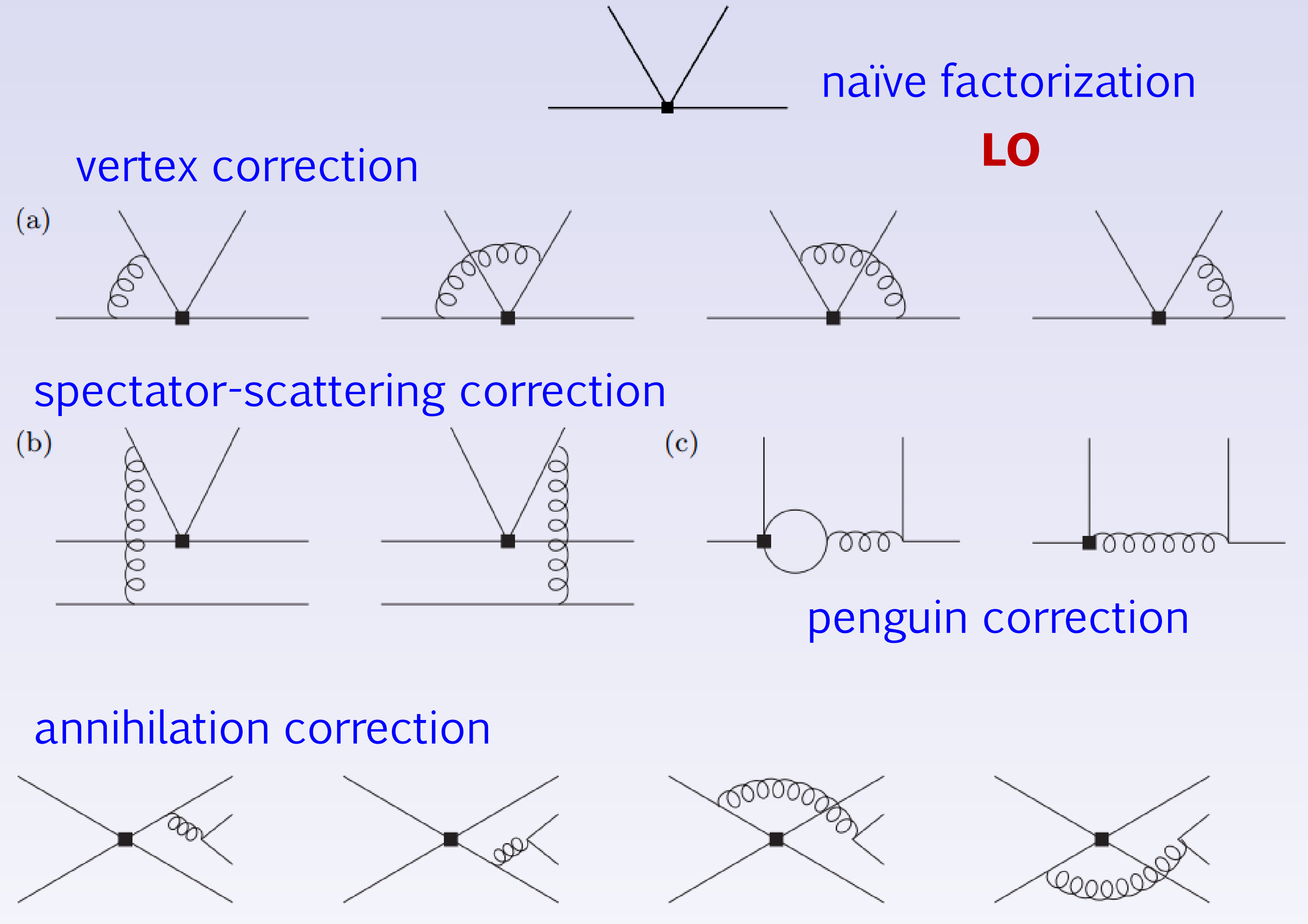


	T_i^I, tree	T_i^I, penguin	T_i^{II}, tree	T_i^{II}, penguin
LO: $\mathcal{O}(1)$		$T^I = 1 + \mathcal{O}(\alpha_s) + \cdots$		
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'03				$T^{II} = \mathcal{O}(\alpha_s) + \cdots$
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07, '09 Beneke, Huber, Li '09 Huber, Krankl, Li '16	 Kim, Yoon '11 Bell, Beneke, Huber, Li '15, '20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

☐ **For tree & penguin topologies, both contribute to T^I & T^{II}**

Phenomenological analyses based on NLO

☐ Various analyses based on NLO hard kernels



naïve factorization

LO

vertex correction

spectator-scattering correction

penguin correction

annihilation correction

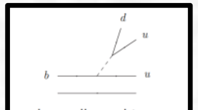
☐ complete sets of final states:

- $B \to PP, PV$: [Beneke, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237;]
- $B \to VV$: [Beneke, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237;]
- $B \to AP, AV, AA$: [Cheng, Yang, 0709.0137, 0805.0329;]
- $B \to SP, SV$: [Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403;]
- $B \to TP, TV$: [Cheng, Yang, 1010.3309;]

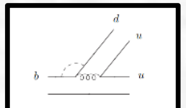
very successful but also with some problems phenomenologically. ！

Phenomenological successes based on NLO

☐ Successes at NLO:



colour-allowed tree α_1



QCD penguins α_4

- For color-allowed tree- & penguin-dominated decay modes, branching ratios usually quantitatively OK
- Dynamical explanation of intricate patterns of penguin interference seen in PP, PV, VP and VV modes

$$PP \sim a_4 + r_\chi a_6, \quad PV \sim a_4 \approx \frac{PP}{3}$$
$$VP \sim a_4 - r_\chi a_6 \sim -PV$$
$$VV \sim a_4 \sim PV$$

$$r_\chi = \frac{2m_L^2}{m_b\,(m_q + m_s)}$$

➡ $\mathrm{Br}\left(B^{\pm,0} \to \eta^{(\prime)} K^{(*)\pm,0}\right)$

- Qualitative explanation of polarization puzzle in $B \to VV$ decays, due to the large weak annihilation
- Strong phases start at $\mathcal{O}(\alpha_s)$, dynamical explanation of smallness of direct CP asymmetries

☐ Some problems encountered at NLO:

- Factorization of power corrections generally broken, due to endpoint divergence
- Could not account for some data, such as $\mathrm{Br}(B^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K)$
- How important the higher-order pert. corr.? Fact. theorem is still established for them?
- As strong phases start at $\mathcal{O}(\alpha_s)$, NNLO is only NLO to them; quite relevant for A_{CP}?

we need go beyond the LO in pert. and power corrections!

NNLO perturbative QCD corrections to hadronic matrix elements

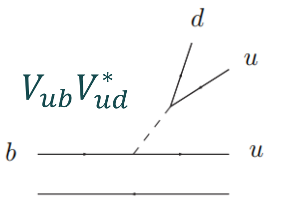
Tree-dominated B decays

☐ $B \to \pi\pi$ **decay amplitudes in QCDF:**

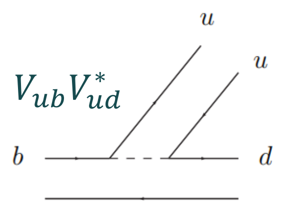
$$\sqrt{2}\,\langle \pi^- \pi^0 | \mathcal{H}_{eff} | B^- \rangle = \lambda_u \left[\alpha_1(\pi\pi) + \alpha_2(\pi\pi) \right] A_{\pi\pi}$$

$$\langle \pi^+ \pi^- | \mathcal{H}_{eff} | \bar{B}^0 \rangle = \left\{ \lambda_u \left[\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi) \right] + \lambda_c\, \alpha_4^c(\pi\pi) \right\} A_{\pi\pi}$$

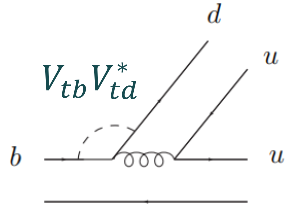
$$- \langle \pi^0 \pi^0 | \mathcal{H}_{eff} | \bar{B}^0 \rangle = \left\{ \lambda_u \left[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi) \right] - \lambda_c\, \alpha_4^c(\pi\pi) \right\} A_{\pi\pi}$$



colour-allowed tree α_1



colour-suppressed tree α_2



QCD penguins α_4

$b \to u\bar{u}d$: $\lambda_u = V_{ub}V_{ud}^* \sim \mathcal{O}(\lambda^3) \sim \lambda_c = V_{cb}V_{cd}^* \sim \mathcal{O}(\lambda^3)$ → α_4 loop-suppressed vs $\alpha_{1,2}$ → Tree-dominated!

☐ α_2 **at NLO:**

$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{\text{NLO}} + \left[\frac{r_{\text{sp}}}{0.485}\right] \left\{ [0.123]_{\text{LOsp}} + [0.072]_{\text{tw3}} \right\}$$

large cancellation between 1-loop vertex correction & LO result;

also dominated by spectator-scattering contributions;

$$r_{\text{sp}} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0) \lambda_B}$$

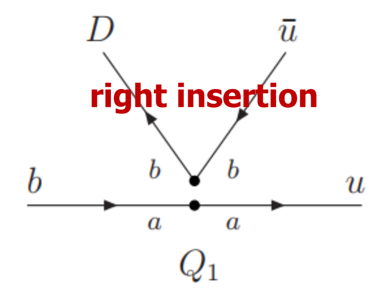
making α_2 sensitive to NNLO corrections, and large effect possible?

Hard kernel T^I at NNLO

QCD → SCETI matching calculation:

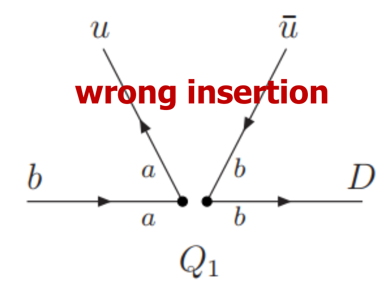
- For “right insertion”:

$$\langle Q_i \rangle = T_i \, \langle O_{\rm QCD} \rangle + \sum_{a>1} H_{ia} \langle O_a \rangle$$



- For “wrong insertion”:

$$\langle Q_i \rangle = \widetilde{T}_i \, \langle O_{\rm QCD} \rangle + \tilde{H}_{i1} \langle \tilde{O}_1 - O_1 \rangle + \sum_{a>1} \tilde{H}_{ia} \langle \tilde{O}_a \rangle$$



On-shell matrix elements at NNLO: full QCD side

$$\langle Q_i \rangle = \Big\{ A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \Big[A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \Big] + \left(\frac{\alpha_s}{4\pi}\right)^2 \Big[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} + Z_{\alpha}^{(1)} A_{ia}^{(1)} + (-i)\,\delta m^{(1)} A_{ia}'^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle O_a \rangle^{(0)}$$

On-shell matrix elements at NNLO: SCET side

$$\langle O_a \rangle = \Big\{ \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \Big[M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \Big] + \left(\frac{\hat{\alpha}_s}{4\pi}\right)^2 \Big[M_{ab}^{(2)} + Y_{ac}^{(1)} M_{cb}^{(1)} + Y_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} + Y_{ext}^{(1)} Y_{ab}^{(1)} + \hat{Z}_{\alpha}^{(1)} M_{ab}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle O_b \rangle^{(0)}$$

Master formula for T^I: right insertion

$$\begin{aligned}
T_i^{(0)} &= A_{i1}^{(0)} \,, \\
T_i^{(1)} &= A_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} A_{j1}^{(0)} \,, \\
T_i^{(2)} &= \boxed{A_{i1}^{(2)\rm nf}} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_{\alpha}^{(1)} A_{i1}^{(1)\rm nf} + (-i)\,\delta m^{(1)} A_{i1}'^{(1)\rm nf} \\
&\quad - T_i^{(1)} [C_{FF}^{(1)} + Y_{11}^{(1)} - Z_{ext}^{(1)}] - \sum_{b>1} H_{ib}^{(1)} Y_{b1}^{(1)} \,.
\end{aligned}$$

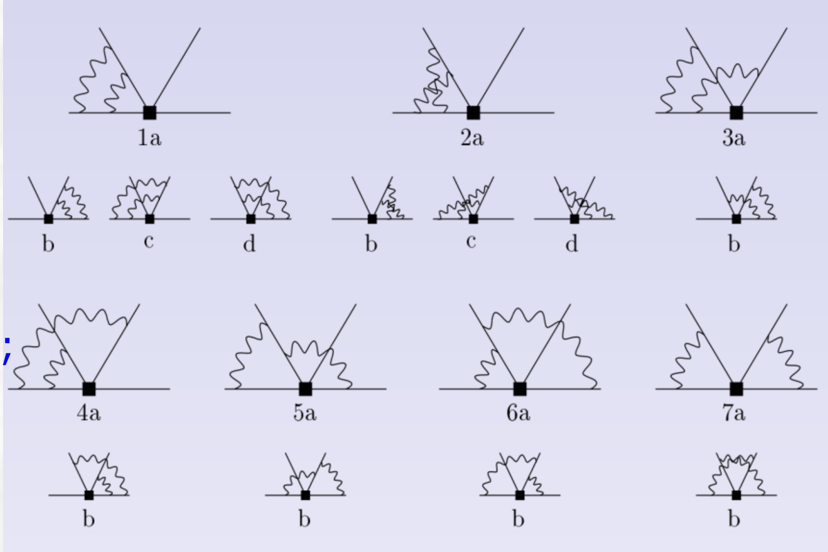
Master formula for T^I: wrong insertion

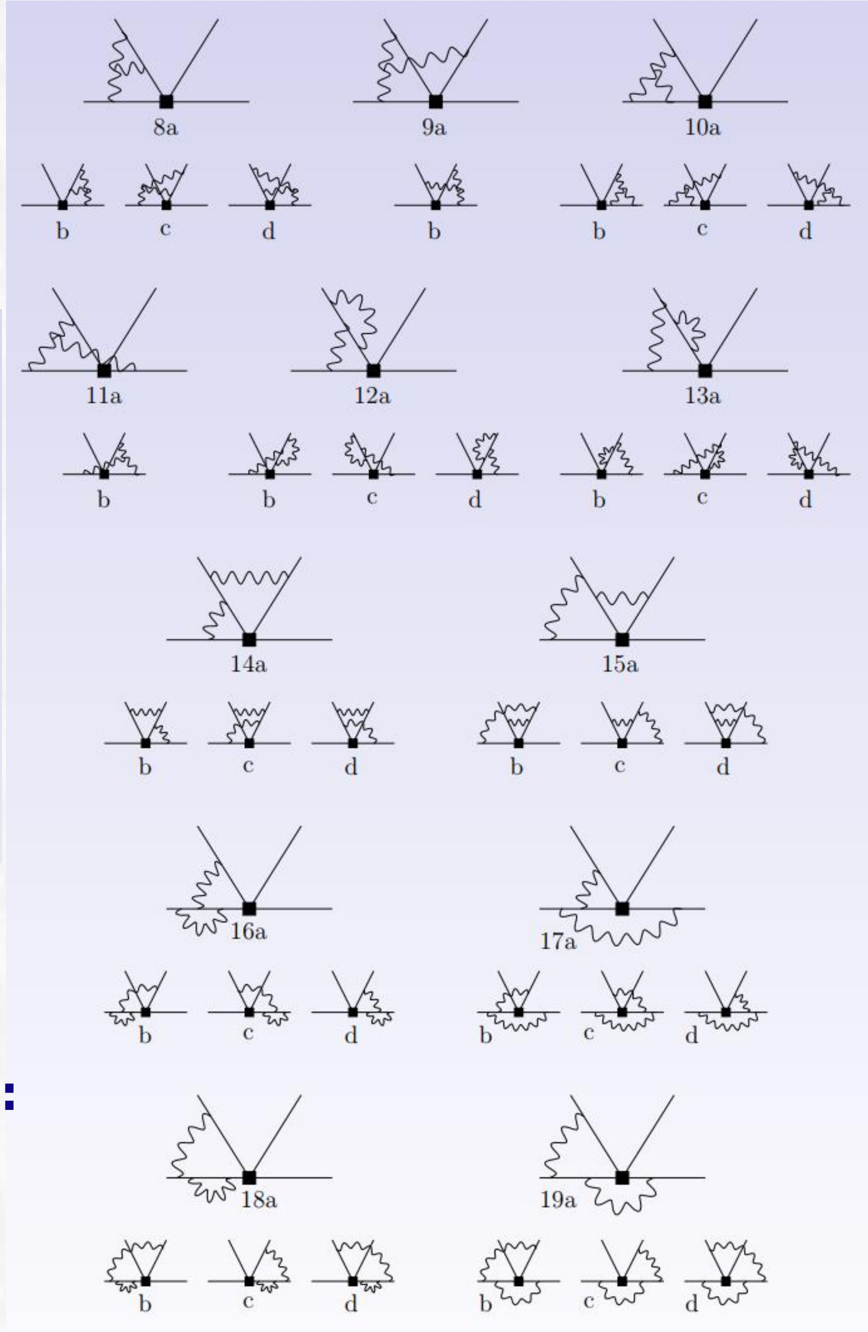
$$\begin{aligned}
\widetilde{T}_i^{(0)} &= \widetilde{A}_{i1}^{(0)} \,, \\
\widetilde{T}_i^{(1)} &= \widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} \,, \\
\widetilde{T}_i^{(2)} &= \boxed{\widetilde{A}_{i1}^{(2)\rm nf}} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)} \widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)} \widetilde{A}_{i1}^{(1)\rm nf} \\
&\quad + (-i)\,\delta m^{(1)} \widetilde{A}_{i1}'^{(1)\rm nf} + Z_{ext}^{(1)} [\widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(0)}] \\
&\quad - \widetilde{T}_i^{(1)} [C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)} \widetilde{Y}_{b1}^{(1)} \\
&\quad + [\widetilde{A}_{i1}^{(2)\rm f} - A_{21}^{(2)\rm f} \widetilde{A}_{i1}^{(0)}] + (-i)\,\delta m^{(1)} [\widetilde{A}_{i1}'^{(1)\rm f} - A_{21}'^{(1)\rm f} \widetilde{A}_{i1}^{(0)}] \\
&\quad + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)}) [\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \widetilde{A}_{i1}^{(0)}] \\
&\quad - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}] \widetilde{A}_{i1}^{(0)} \\
&\quad - (C_{FF}^{(1)} - \xi_{45}^{(1)}) [\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}] \widetilde{A}_{i1}^{(0)} \,.
\end{aligned}$$

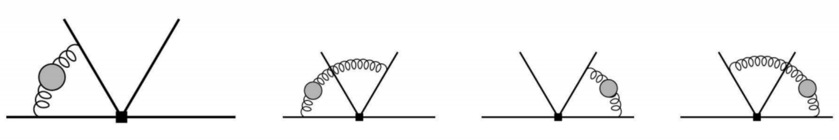
Two-loop QCD diagrams

□ $\widetilde{A}_{i1}^{(2)\mathrm{nf}}$: **relevant two-loop non-factorizable Feynman diagrams in full QCD:**

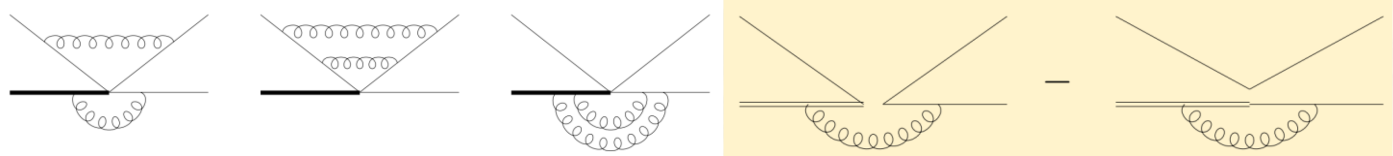
- totally ~ 70 diagrams;
- needs modern multi-loop Feynman diagrams techniques;
- IBP reduction, Mellin-Barnes representation, Differential equations, …







□ **Complicated counter-terms from QCD & SCET operators:**



Final results for $\alpha_{1,2}$

☐ **Tree amplitudes $\alpha_{1,2}$, after convolution with LCDAs:**

$$\alpha_i(M_1M_2) = \sum_j C_j V_{ij}^{(0)} + \sum_{l\geqslant 1}\left(\frac{\alpha_s}{4\pi}\right)^l \left[\frac{C_F}{2N_c}\sum_j C_j V_{ij}^{(l)} + P_i^{(l)}\right] + \cdots$$

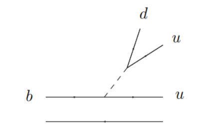
free from endpoint divergence

$$V_{1j}^{(0)} = \int_0^1 du\, T_j^{(0)}\phi_M(u), \qquad \frac{C_F}{2N_c}V_{1j}^{(l)} = \int_0^1 du\, T_j^{(l)}(u)\phi_M(u),$$

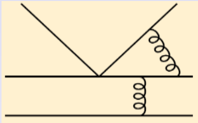
$$V_{2j}^{(0)} = \int_0^1 du\, \widetilde{T}_j^{(0)}\phi_M(u), \qquad \frac{C_F}{2N_c}V_{2j}^{(l)} = \int_0^1 du\, \widetilde{T}_j^{(l)}(u)\phi_M(u).$$

☐ **Numerical results including the NNLO corrections:**

$$\begin{aligned}\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO} \\ &\quad - \left[\frac{r_{\rm sp}}{0.445}\right]\Big\{[0.014]_{\rm LOsp} + [0.034 + 0.027i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\Big\} \\ &= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i\end{aligned}$$

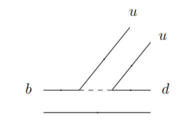


colour-allowed tree α_1



Beneke, Jager '05
Kivel '06, Pilipp '07

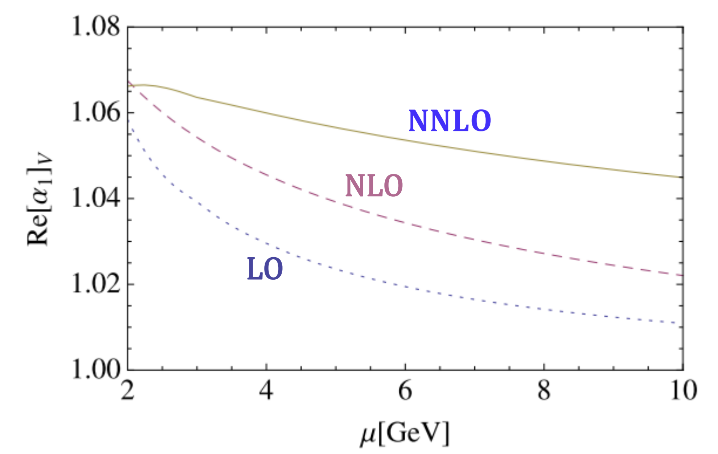
$$\begin{aligned}\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\rm NLO} - [0.031 + 0.050\,i]_{\rm NNLO} \\ &\quad + \left[\frac{r_{\rm sp}}{0.445}\right]\Big\{[0.114]_{\rm LOsp} + [0.049 + 0.051i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\Big\} \\ &= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i\end{aligned}$$



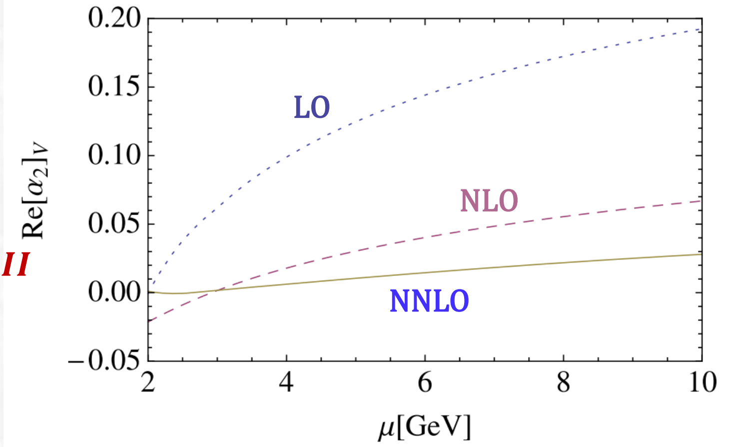
colour-suppressed tree α_2

☐ **NNLO corrections both large, but cancelled between T^I & T^{II}**

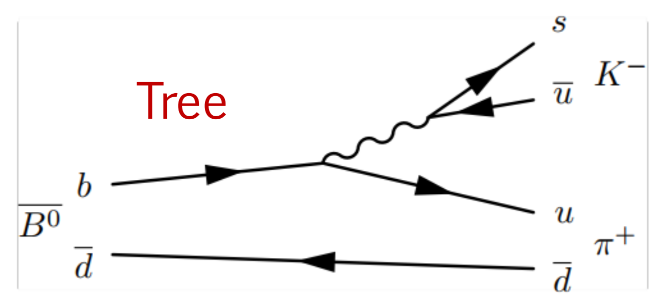
$$\langle M_1M_2|\mathcal{O}_i|\bar{B}\rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

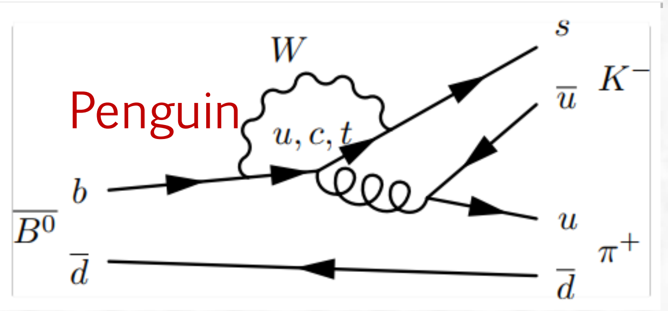


☐ **Scale-dependence much reduced!**



Penguin-dominated B decays





☐ $B \to \pi K$ **decay amplitudes:** mediated by $b \to s q\bar q$ transitions

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0 K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1 + \hat\alpha_4^p\big] + A_{\bar K\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\mathrm{EW}}^c\big],$$
$$\mathcal{A}_{\bar B^0\to\pi^+K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1 + \hat\alpha_4^p\big],$$

$\lambda_u = V_{ub}V_{us}^* \sim \mathcal{O}(\lambda^4) \ll \lambda_c = V_{cb}V_{cs}^* \sim \mathcal{O}(\lambda^2)$ → Penguin-dominated!

☐ **In QCDF, strong phases generated firstly at NLO in α_s**

$$A_{\mathrm{CP}} = [c \times \alpha_s]_{\mathrm{NLO}} + \mathcal{O}(\alpha_s^2, \Lambda/m_b)$$

→ ***NNLO is only NLO for A_{CP}, large effects still possible?***

↓

☐ **To predict accurately direct CPV, we must calculate both tree & penguin up to NNLO!**

☐ **Driven by the current exp. data on $B \to \pi K$:**

$$\Delta A_{CP}(\pi K) = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar B^0 \to \pi^+ K^-)$$
$$= (11.3 \pm 1.2)\%$$

differs from 0 by ~9σ

ΔA_{CP} puzzle

→ ***How about the situation @ NNLO?***

Penguin topologies with various insertions

- **Effective Hamiltonian including penguin operators:**

[BBL '96; CMM '98]

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pD}^* V_{pb} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

$$Q_1^p = (\bar{p}_L \gamma^\mu T^A b_L)\, (\bar{D}_L \gamma_\mu T^A p_L),$$
$$Q_2^p = (\bar{p}_L \gamma^\mu b_L)\, (\bar{D}_L \gamma_\mu p_L),$$

current-current operators

CMM operator basis

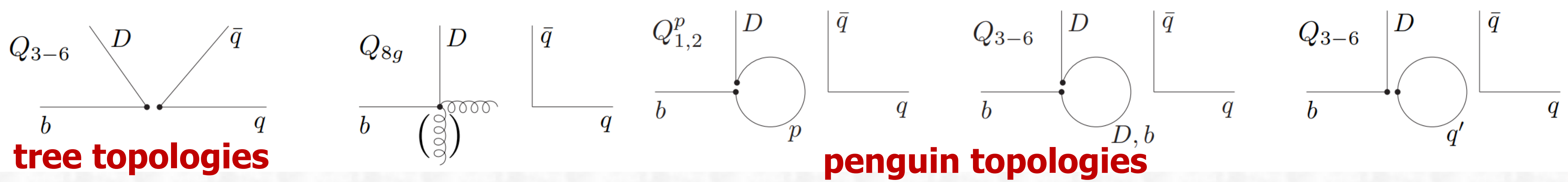
$$Q_3 = (\bar{D}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q} \gamma_\mu q),$$
$$Q_4 = (\bar{D}_L \gamma^\mu T^A b_L) \textstyle\sum_q (\bar{q} \gamma_\mu T^A q),$$
$$Q_5 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q),$$
$$Q_6 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho T^A b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^A q)\,.$$

QCD penguin operators

$$Q_{8g} = \frac{-g_s}{32\pi^2}\, \overline{m}_b\, \bar{D} \sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b,$$

chromo-magnetic dipole operators

- **Various operator insertions:**



tree topologies

penguin topologies

(i) Dirac structure of Q_i, (ii) color structure of Q_i, (iii) types of contraction, and (iv) quark masses in the fermion loop

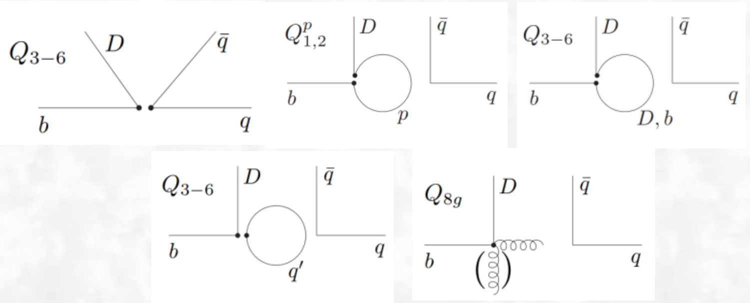
Hard kernel T^I at NNLO

☐ QCD → SCETI matching calculation:

$$\boxed{\langle Q_i \rangle = \sum_a \widetilde{H}_{ia} \, \langle \widetilde{O}_a \rangle}$$

☐ **Note: always**

wrong insertion!



☐ **Complete SCET operator basis:**

$$Q_3 = (\bar{D}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q}\gamma_\mu q),$$
$$Q_4 = (\bar{D}_L \gamma^\mu T^A b_L) \textstyle\sum_q (\bar{q}\gamma_\mu T^A q),$$
$$Q_5 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho q),$$
$$Q_6 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho T^A b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho T^A q).$$

$$O_1 = \sum_{q=u,d,s} \left[\bar{\chi}_D \frac{\not{n}_-}{2} (1-\gamma_5) \chi_q \right] \left[\bar{\xi}_q \not{n}_+ (1-\gamma_5) h_v \right],$$

the only physical operator and factorizes into FF*LCDA.

$$\tilde{O}_n = \sum_{q=u,d,s} \left[\bar{\xi}_q \gamma_\perp^\alpha \gamma_\perp^{\mu_1} \gamma_\perp^{\mu_2} \dots \gamma_\perp^{\mu_{2n-2}} \chi_q \right] \left[\bar{\chi}_q (1+\gamma_5) \gamma_{\perp\alpha} \gamma_{\perp\mu_{2n-2}} \gamma_{\perp\mu_{2n-3}} \cdots \gamma_{\perp\mu_1} h_v \right],$$

n now up to 4, with 7 gamma matrices

$\tilde{O}_1 - O_1/2$ is another evanescent operator

☐ **On-shell matrix elements at NNLO: on the full QCD side**

$$\begin{aligned}
\langle Q_i \rangle = \Big\{ & \widetilde{A}^{(0)}_{ia} + \frac{\alpha_s}{4\pi} \left[\widetilde{A}^{(1)}_{ia} + Z^{(1)}_{\text{ext}} \widetilde{A}^{(0)}_{ia} + Z^{(1)}_{ij} \widetilde{A}^{(0)}_{ja} \right] \\
& + \left(\frac{\alpha_s}{4\pi} \right)^2 \Big[\widetilde{A}^{(2)}_{ia} + Z^{(1)}_{ij} \widetilde{A}^{(1)}_{ja} + Z^{(2)}_{ij} \widetilde{A}^{(0)}_{ja} + Z^{(1)}_{\text{ext}} \widetilde{A}^{(1)}_{ia} + Z^{(2)}_{\text{ext}} \widetilde{A}^{(0)}_{ia} \\
& + Z^{(1)}_{\text{ext}} Z^{(1)}_{ij} \widetilde{A}^{(0)}_{ja} + Z^{(1)}_{\alpha} \widetilde{A}^{(1)}_{ia} + (-i)\, \delta m^{(1)} \widetilde{A}'^{(1)}_{ia} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle \widetilde{O}_a \rangle^{(0)}
\end{aligned}$$

☐ **On-shell matrix elements at NNLO: SCET side**

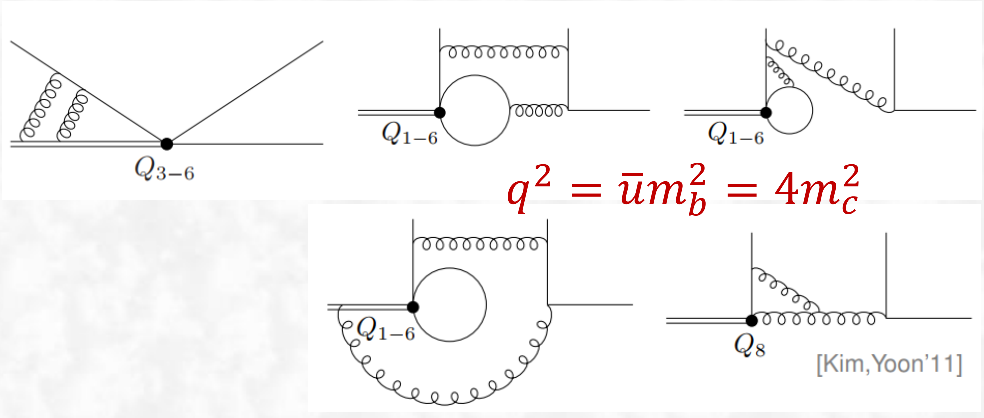
$$\begin{aligned}
\langle O_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[M^{(1)}_{ab} + Y^{(1)}_{\text{ext}} \delta_{ab} + Y^{(1)}_{ab} \right] + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \Big[M^{(2)}_{ab} + Y^{(1)}_{ac} M^{(1)}_{cb} \\
& + Y^{(2)}_{ab} + Y^{(1)}_{ext} M^{(1)}_{ab} + Y^{(2)}_{ext} \delta_{ab} + Y^{(1)}_{ext} Y^{(1)}_{ab} + \hat{Z}^{(1)}_{\alpha} M^{(1)}_{ab} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle O_b \rangle^{(0)}
\end{aligned}$$

T^I up to NNLO

☐ Master formulae for T^I:

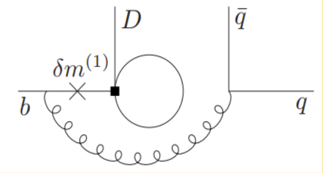
$$
\begin{aligned}
\frac{1}{2}\,\widetilde{T}_i^{(2)} = {} & \boxed{\widetilde{A}_{i1}^{(2)\mathrm{nf}}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\mathrm{nf}} \\
& + \boxed{(-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\mathrm{nf}}} + Z_{\mathrm{ext}}^{(1)}\,[\widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\
& - \frac{1}{2}\,\widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
& + [\widetilde{A}_{i1}^{(2)\mathrm{f}} - A_{31}^{(2)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1)\mathrm{f}} - A_{31}^{\prime(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)} \\
& + (Z_{\alpha}^{(1)} + Z_{\mathrm{ext}}^{(1)})\,[\widetilde{A}_{i1}^{(1)\mathrm{f}} - A_{31}^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - (C_{FF}^{(1)} - \xi_{45}^{(1)})\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - \boxed{\sum_{b>1} \widetilde{A}_{ib}^{(0)}\,\widetilde{M}_{b1}^{(2)} - \sum_{b>1} \widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(2)}}\,.
\end{aligned}
$$

~ 100 two-loop Feynman diagrams

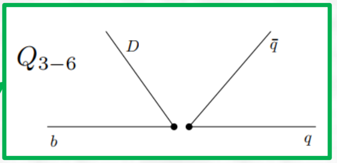


$q^2 = \bar{u}m_b^2 = 4m_c^2$

[Kim,Yoon'11]



non-vanishing fermion-tadpole contraction of QCD penguin operators



tree-level matching of Q_i involves already evanescent SCET operators

☐ Complication during calculations:

(i) fermion loop with either $m = 0, m = m_c$ or $m = m_b$

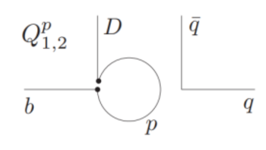
(ii) genuine 2-loop two-scale problem: $\bar{u},\ z_c = m_c^2/m_b^2$

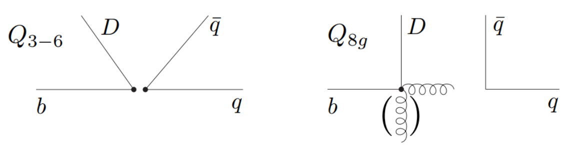
(iii) threshold at $\bar{u} = 4z_c$ introduces strong phase

Final results for a_4^p

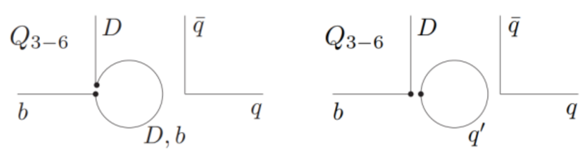
☐ Final numerical results:

$$\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$





$$\begin{aligned}
a_4^u(\pi\bar K)/10^{-2} &= -2.87 - [0.09+0.09i]_{\mathrm{V}_1} + [0.49-1.32i]_{\mathrm{P}_1} - [0.32+0.71\,i]_{\mathrm{P}_2,\,\mathrm{Q}_{1,2}} + [0.33+0.38i]_{\mathrm{P}_2,\,\mathrm{Q}_{3-6,8}} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\Big\{[0.13]_{\mathrm{LO}} + [0.14+0.12i]_{\mathrm{HV}} - [0.01-0.05i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\Big\} \\
&= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,
\end{aligned}$$

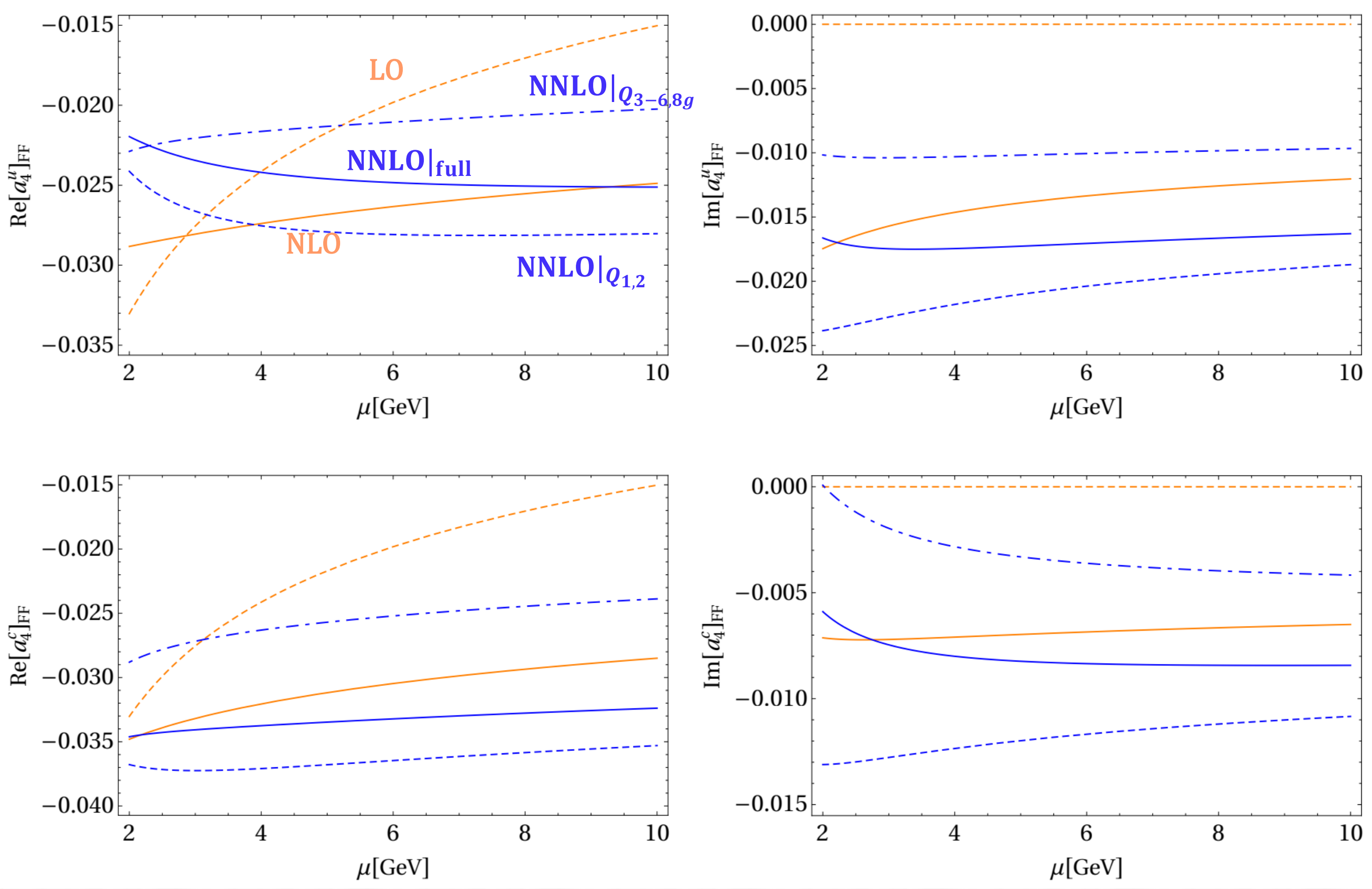


$$\begin{aligned}
a_4^c(\pi\bar K)/10^{-2} &= -2.87 - [0.09+0.09i]_{\mathrm{V}_1} + [0.05-0.62i]_{\mathrm{P}_1} - [0.77+0.50i]_{\mathrm{P}_2,\,\mathrm{Q}_{1,2}} + [0.33+0.38i]_{\mathrm{P}_2,\,\mathrm{Q}_{3-6,8}} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\Big\{[0.13]_{\mathrm{LO}} + [0.14+0.12i]_{\mathrm{HV}} + [0.01+0.03i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\Big\} \\
&= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i\,.
\end{aligned}$$

- ➢ **individual NNLO contributions from $Q^p_{1,2}$ and $Q_{3-6,8g}$ significant**
- ➢ **strong cancellation between NNLO corrections from $Q^p_{1,2}$ and $Q_{3-6,8g}$**

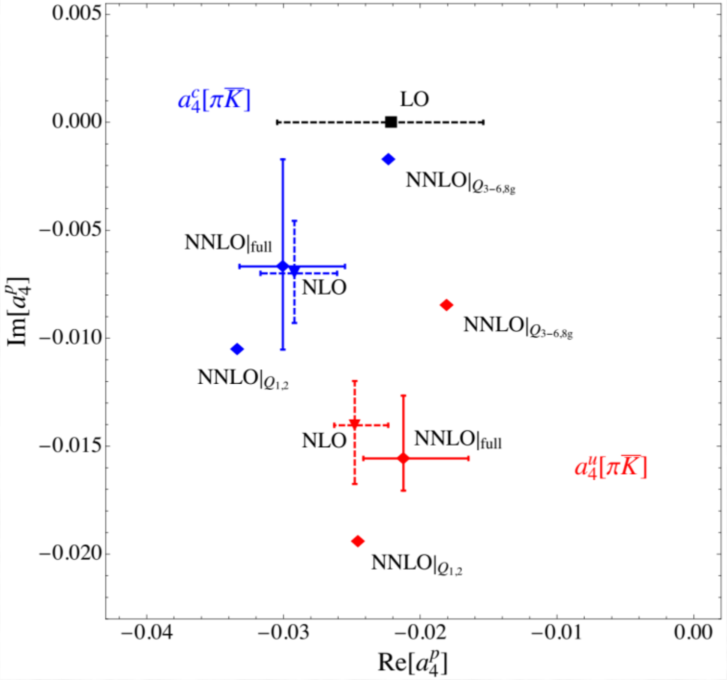
Scale dependence of a_4^p

☐ Scale dependence of a_4^p: only form-factor term



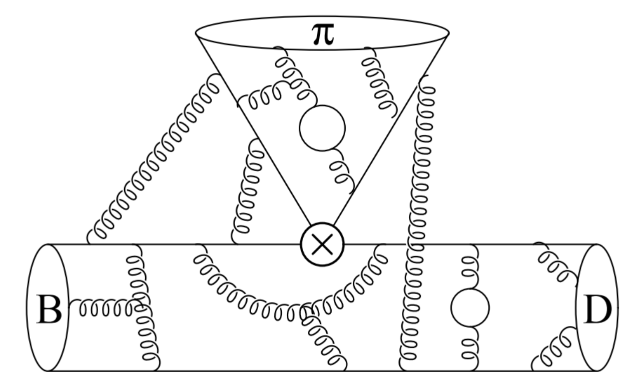
- Scale dependence negligible, especially for μ > 4 GeV.

☐ Results at different orders:



- total NNLO effects small
- uncertainty at NNLO larger than at NLO, due to non-trivial charm mass

$B_q^0 \to D_q^{(*)-} L^+$ class-I decays



☐ **At quark-level, these decays mediated by $b \to c\bar{u}d(s)$**

all four flavors different from each other,

no penguin operators & no penguin topologies!

$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

☐ **For class-I decays:** QCDF formula much simpler;

only the form-factor term at leading power

[Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\langle D_q^{(*)+} L^- |\, \mathcal{Q}_i \,| \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u)\phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

i) only color-allowed tree topology a_1

ii) spectator & annihilation power-suppressed

iii) annihilation absent in $B_{d(s)}^0 \to D_{d(s)}^- K(\pi)^+$ etc.

iv) they are theoretically simpler and cleaner

→ these decays used to test factorization theorems

☐ **Hard kernel T:** both NLO and NNLO results known;

[Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, Li '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

Calculation of T^I

□ Matching QCD onto SCET$_\mathrm{I}$: [Huber, Kränkl, Li '16]

m_c also heavy, must keep m_c/m_b fixed as $m_b \to \infty$,

thus needing two sets of SCET operator basis.

$$\boxed{\langle \mathcal{Q}_i \rangle = \hat{T}_i \langle \mathcal{Q}^{\mathrm{QCD}} \rangle + \hat{T}'_i \langle \mathcal{Q}'^{\mathrm{QCD}} \rangle + \sum_{a>1} \left[H_{ia} \langle \mathcal{O}_a \rangle + H'_{ia} \langle \mathcal{O}'_a \rangle \right]}$$

physical operators and factorizes into FF*LCDA.

$$\mathcal{O}_1 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) h_v \,,$$

$$\mathcal{O}_2 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v \,,$$

$$\mathcal{O}_3 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\delta} \gamma_{\perp,\gamma} \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v$$

$$\mathcal{O}'_1 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) h_v \,,$$

$$\mathcal{O}'_2 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} h_v \,,$$

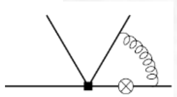
$$\mathcal{O}'_3 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} \gamma_{\perp,\gamma} \gamma_{\perp,\delta} h_v$$

evanescent operators and must be renormalized to zero.

□ Renormalized on-shell QCD amplitudes:

$$\begin{aligned}
\langle \mathcal{Q}_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \left[A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \right] \\
& + \left(\frac{\alpha_s}{4\pi} \right)^2 \Big[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} \\
& + (-i) \delta m_b^{(1)} A_{ia}^{*(1)} + (-i) \delta m_c^{(1)} A_{ia}^{**(1)} + Z_\alpha^{(1)} A_{ia}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle \mathcal{O}_a \rangle^{(0)} \\
& + (A \leftrightarrow A') \langle \mathcal{O}'_a \rangle^{(0)} \,.
\end{aligned}$$

on QCD side



□ Renormalized on-shell SCET amplitudes:

$$\begin{aligned}
\langle \mathcal{O}_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \right] \\
& + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \Big[M_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ac}^{(1)} M_{cb}^{(1)} + \hat{Z}_\alpha^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} \\
& + Y_{ext}^{(1)} Y_{ab}^{(1)} + Y_{ab}^{(2)} \Big] + \mathcal{O}(\hat{\alpha}_s^3) \Big\} \langle \mathcal{O}_b \rangle^{(0)} \,,
\end{aligned}$$

on SCET side

□ Master formulas for hard kernels:

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

$$\boxed{\begin{aligned}
\hat{T}_i^{(0)} &= A_{i1}^{(0)} \\
\hat{T}_i^{(1)} &= A_{i1}^{(1)nf} + Z_{ij}^{(1)} A_{j1}^{(0)} \\
\hat{T}_i^{(2)} &= A_{i1}^{(2)nf} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_\alpha^{(1)} A_{i1}^{(1)nf} - \hat{T}_i^{(1)} \left[C_{FF}^{\mathrm{D}(1)} + Y_{11}^{(1)} - Z_{\mathrm{ext}}^{(1)} \right] \\
& \quad - C_{FF}^{\mathrm{ND}(1)} \hat{T}_i'^{(1)} + (-i) \delta m_b^{(1)} A_{i1}^{*(1)nf} + (-i) \delta m_c^{(1)} A_{i1}^{**(1)nf} - \sum_{b \neq 1} H_{ib}^{(1)} Y_{b1}^{(1)} \,.
\end{aligned}}$$

Decay amplitudes for $B_q^0 \to D_q^- L^+$

- **Color-allowed tree amplitude a_1: collinear factorization established @ NNLO!**

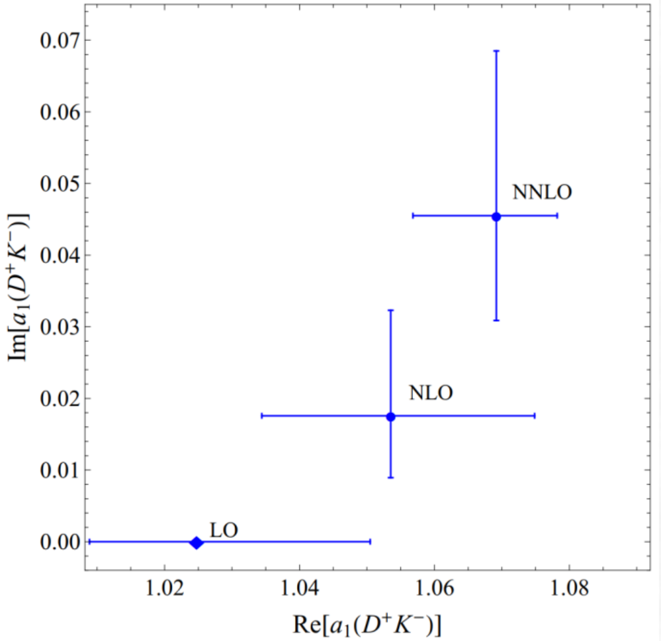
$$a_1(D^+L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) + \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu)\,,$$
$$a_1(D^{*+}L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) - \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu)\,,$$

free from the

endpoint divergence

↓

collinear factorization established



- **Numerical result:**

$$\begin{aligned} a_1(D^+K^-) &= 1.025 + [0.029 + 0.018i]_{\rm NLO} + [0.016 + 0.028i]_{\rm NNLO} \\ &= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,, \end{aligned}$$

  - both NLO and NNLO add always constructively to LO result!
  - NNLO corrections to real part quite small (2%), but rather large to imaginary part (60%).

- **For different decay modes:** *quasi-universal*, with small process dependence from *different LCDA of light mesons*.

$$\begin{aligned} a_1(D^+K^-) &= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,, \\ a_1(D^+\pi^-) &= (1.072^{+0.011}_{-0.013}) + (0.043^{+0.022}_{-0.014})i\,, \\ a_1(D^{*+}K^-) &= (1.068^{+0.010}_{-0.012}) + (0.034^{+0.017}_{-0.011})i\,, \\ a_1(D^{*+}\pi^-) &= (1.071^{+0.012}_{-0.013}) + (0.032^{+0.016}_{-0.010})i\,. \end{aligned}$$

Possible higher-order power corrections motivated by current data

Non-leptonic/semi-leptonic ratios

☐ **Non-leptonic/semi-leptonic ratios :** [Bjorken '89; Neubert, Stech '97; Beneke, Buchalla, Neubert, Sachrajda '01]

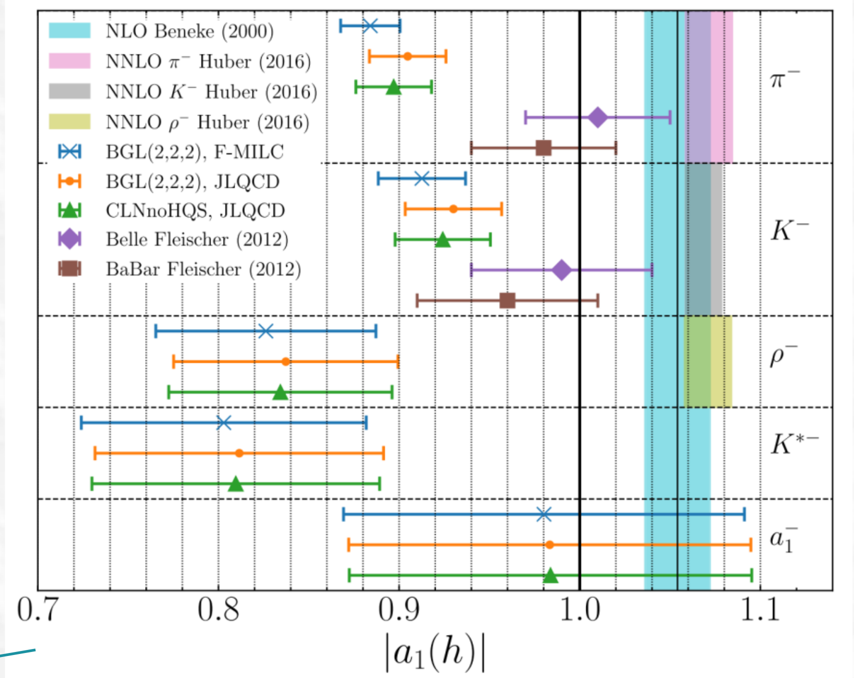
$$R^{(*)}_{(s)L} \equiv \frac{\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-)}{d\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} \ell^- \bar{\nu}_\ell)/dq^2\,|_{q^2=m_L^2}} = 6\pi^2\,|V_{uq}|^2\,f_L^2\,|a_1(D^{(*)+}_{(s)} L^-)|^2\,X^{(*)}_L$$

free from uncertainties from V_{cb} & $B_{d,s} \to D^{(*)}_{d,s}$ form factors

☐ **Updated predictions vs data:** [Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]

$R^{(*)}_{(s)L}$	LO	NLO	NNLO	Exp.	Deviation (σ)
R_π	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.74 ± 0.06	5.4
R^*_π	1.00	$1.06^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.80 ± 0.06	4.5
R_ρ	2.77	$2.94^{+0.19}_{-0.19}$	$3.02^{+0.17}_{-0.18}$	2.23 ± 0.37	1.9
R_K	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.62 ± 0.05	4.4
R^*_K	0.72	$0.76^{+0.03}_{-0.03}$	$0.79^{+0.01}_{-0.02}$	0.60 ± 0.14	1.3
R_{K^*}	1.41	$1.50^{+0.11}_{-0.11}$	$1.53^{+0.10}_{-0.10}$	1.38 ± 0.25	0.6
$R_{s\pi}$	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.72 ± 0.08	4.4
R_{sK}	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.46 ± 0.06	6.3

☐ **Latest Belle data:** 2207.00134



$|a_1(\bar{B} \to D^{*+}\pi^-)| = 0.884 \pm 0.004 \pm 0.003 \pm 0.016\ [1.071^{+0.020}_{-0.016}]$;

15% lower than SM

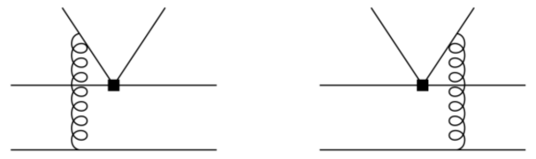
$|a_1(\bar{B} \to D^{*+}K^-)| = 0.913 \pm 0.019 \pm 0.008 \pm 0.013\ [1.069^{+0.020}_{-0.016}]$;

Power corrections

$$\langle D_q^{(*)+} L^- |\, \mathcal{Q}_i \,| \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \boxed{\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)}$$

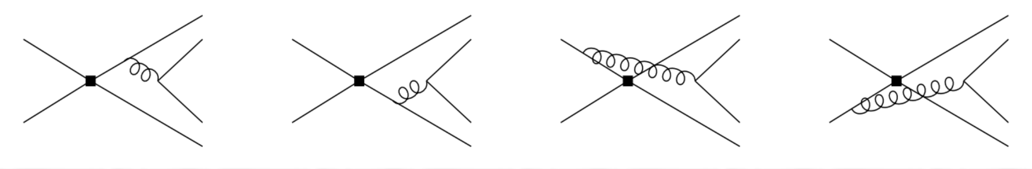
☐ **Sources of sub-leading power corrections:** [Beneke, Buchalla, Neubert, Sachrajda '01; Bordone, Gubernari, Huber, Jung, van Dyk '20]

- ➢ non-factorizable spectator interactions



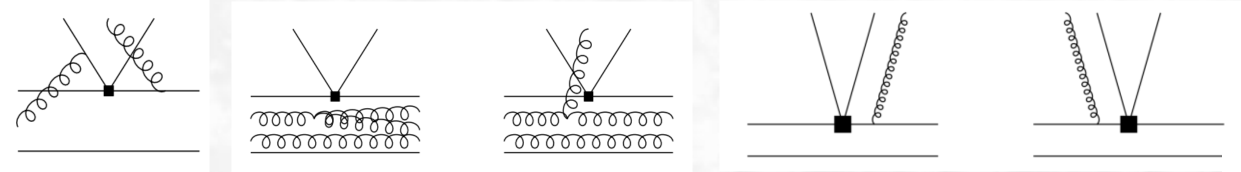
- ➢ annihilation topologies

$$\frac{\Lambda_{\rm QCD}}{m_b}$$

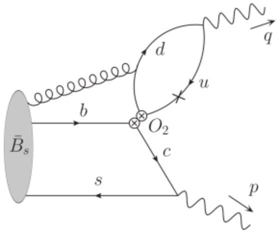


- ➢ non-leading higher Fock-state contributions

$$\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2$$



- ➢ non-factorizable soft-gluon contributions in LCSR with B-meson LCDA: [Maria Laura Piscopo, Aleksey V. Rusov '23]



$$\frac{C_2\langle O_2^d\rangle}{C_1\langle O_1^d\rangle} = 0.051^{+0.059}_{-0.052}\,, \qquad \bar{B}_s^0 \to D_s^+\pi^-\,,$$

$$\frac{C_2\langle O_2^s\rangle}{C_1\langle O_1^s\rangle} = 0.039^{+0.042}_{-0.034}\,, \qquad \bar{B}^0 \to D^+K^-\,.$$

☐ **Scaling of the leading-power contribution:** [BBNS '01]

$$\mathcal{A}(\bar{B}_d \to D^+\pi^-) \sim G_F m_b^2\, F^{B\to D}(0)\, f_\pi \sim G_F m_b^2\, \Lambda_{\rm QCD}$$

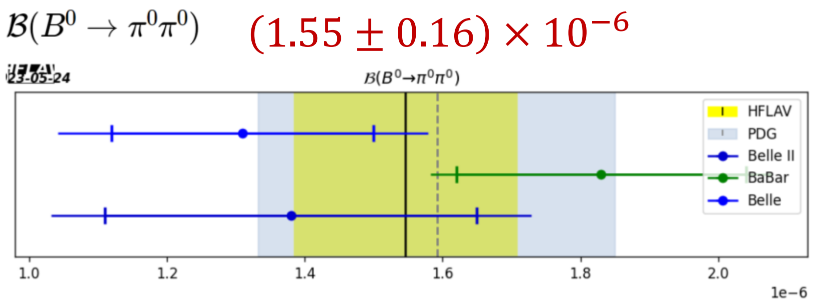
- ➢ All ESTIMATED to be power-suppressed; not even chirality-enhanced due to (V-A)(V-A)
- ➢ Difficult to explain why measured values of $|a_1(h)|$ several σ smaller than SM?
- ➢ *Must consider possible sub-leading power corrections carefully!*

Charmless two-body hadronic B decays

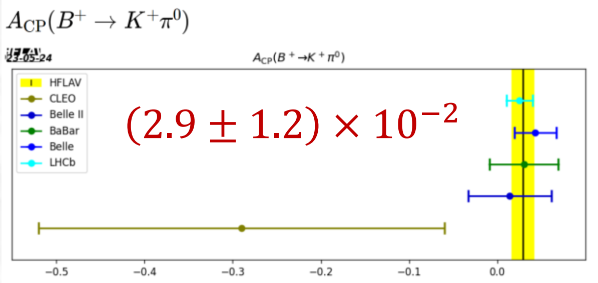
- ☐ **Long-standing puzzles in $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$:** [HFLAV '23]

$$\mathrm{Br}(B^0 \to \pi^0\pi^0) = (0.3 - 0.9) \times 10^{-6} \qquad \Delta A_{CP}(\pi K) = (11.3 \pm 1.2)\%$$

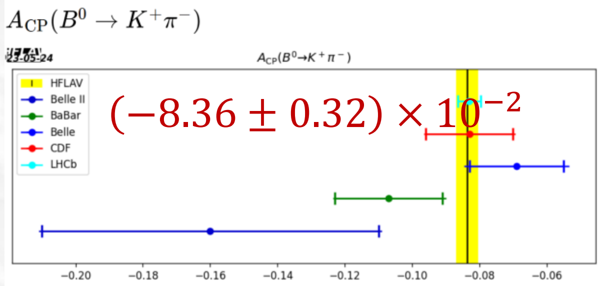
differs from 0 by ~9σ



$(1.55 \pm 0.16) \times 10^{-6}$



$(2.9 \pm 1.2) \times 10^{-2}$



$(-8.36 \pm 0.32) \times 10^{-2}$

- ☐ **Decay amplitudes in QCDF:**

$$-\mathcal{A}_{\bar{B}^0\to\pi^0\pi^0} = A_{\pi\pi}\left[\delta_{pu}(\alpha_2 - \beta_1) - \hat{\alpha}_4^p - 2\beta_4^p\right]$$

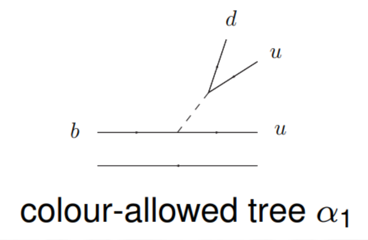
$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\right] + A_{\bar{K}\pi}\left[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\rm EW}^c\right],$$

$$\mathcal{A}_{\bar{B}^0\to\pi^+K^-} = A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\right],$$

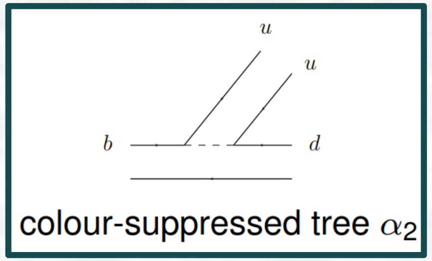
$$A_{\rm CP}(\pi^0K^\pm) - A_{\rm CP}(\pi^\mp K^\pm) = -2\sin\gamma\left(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{\rm EW})\right) + \ldots$$

α_2 **always plays a key role here!**

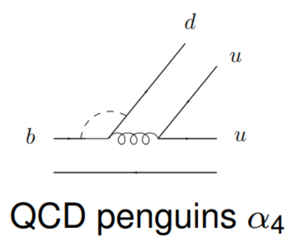
- ☐ **Dominant topologies:** **LP NNLO known**



colour-allowed tree α_1



colour-suppressed tree α_2



QCD penguins α_4

- ☐ **Find some mechanism to enhance α_2, and hence explain both puzzles!**

necessary to consider sub-leading power corrections!

Power-suppressed color-octet contribution

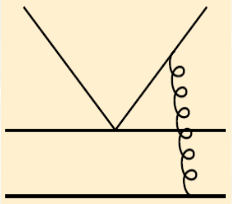
☐ **Sub-leading power corrections to a_2: spectator scattering or final-state re-scatterings**

☐ **Every four-quark operator in H_{eff} has a color-octet piece in QCD:**

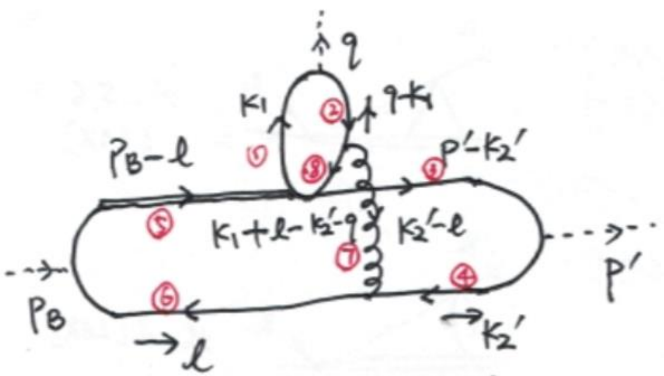
$$t^a_{ik}t^a_{jl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2N_c}\delta_{ik}\delta_{jl},$$

$$Q_1 = (\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} = \frac{1}{N_c}(\bar{s}_i b_i)_{V-A} \otimes (\bar{u}_j u_j)_{V-A} + 2(\bar{s}T^A b)_{V-A} \otimes (\bar{u}T^A u)_{V-A}$$

$$Q_2 = (\bar{u}_i b_j)_{V-A} \otimes (\bar{s}_j u_i)_{V-A} = \frac{1}{N_c}(\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} + 2(\bar{u}T^A b)_{V-A} \otimes (\bar{s}T^A u)_{V-A}$$



☐ **Soft-gluon contributions with color-octet operator insertions:**



BBNS_NPB591_313.

method of regions: 6 regions

- ➢ The gluon propagator can be in the hard-collinear region

➡ hard-spectator scattering contribution

- ➢ Can also be in the soft region; expected to be $\mathcal{O}(1/m_b)$

➡ can be non-zero at sub-leading power, numerically relevant

- ➢ Other four regions suppressed by more powers of $1/m_b$

Soft-exchange effects from emission topology

- ☐ **Real realization of the mechanism requires three-loop three-point correlators** [w.i.p.]

- ☐ **Matching from QCD to SCET$_{\mathrm{I}}$:**

$$Q_1 \to H_1(u) \otimes [\bar{u}_c h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} u_{\bar{c}}]_{\Gamma_2}(u) + H_2(u) \otimes \frac{1}{N_c} [\bar{s}_c h_v]_{\tilde{\Gamma}_1} [\bar{u}_{\bar{c}} u_{\bar{c}}]_{\tilde{\Gamma}_2}(u) + H_3(u) \otimes 2 \underline{\left[\bar{s}_c T^A h_v\right]_{\tilde{\Gamma}_1} \left[\bar{u}_{\bar{c}} T^A u_{\bar{c}}\right]_{\tilde{\Gamma}_2}(u)}$$

$$Q_2 = [\bar{u}_i b_j]_{\Gamma_1} [\bar{s}_j u_i]_{\Gamma_2} = [\bar{s} b]_{\tilde{\Gamma}_1} [\bar{u} u]_{\tilde{\Gamma}_2} \to H_1(u) \otimes [\bar{s}_c h_v]_{\tilde{\Gamma}_1} [\bar{u}_{\bar{c}} u_{\bar{c}}]_{\tilde{\Gamma}_2}(u) + H_2(u) \otimes \frac{1}{N_c} [\bar{u}_c h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} u_{\bar{c}}]_{\Gamma_2}(u) + H_3(u) \otimes 2 \underline{\left[\bar{u}_c T^A h_v\right]_{\Gamma_1} \left[\bar{s}_{\bar{c}} T^A u_{\bar{c}}\right]_{\Gamma_2}}(u) \, ,$$

colour-octet SCET$_{\mathrm{I}}$ operators

  - ➢ $H_i(u)$: hard matching coefficients; at tree-level, $H_i(u) = 1$;

- ☐ **How to implement $\langle M_1 M_2 \left| \left[\bar{u}_c T^A h_v\right]_{\Gamma_1} \left[\bar{s}_{\bar{c}} T^A u_{\bar{c}}\right]_{\Gamma_2} \right| \bar{B} \rangle$: function of u, depending on $M_{1,2}$ & $\bar{B}$**

  - ➢ For color-singlet SCET$_{\mathrm{I}}$ operators: factorization well established

$$\left\langle M_1 M_2 \middle| [\bar{u}_c h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} u_{\bar{c}}]_{\Gamma_2}(u) \middle| \bar{B} \right\rangle = c\, \hat{A}_{M_1 M_2} \phi_{M_2}(u), \text{ with } \hat{A}_{M_1 M_2} = i\, m_B^2 F^{B \to M_1}(0) f_{M_2}$$

  - ➢ For color-octet SCET$_{\mathrm{I}}$ operators: normalized to the naïve factorizable amplitude

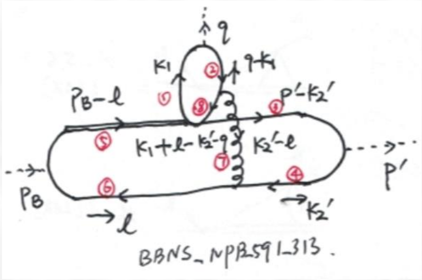
$$\left\langle M_1 M_2 \middle| [\bar{u}_c T^A h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} T^A u_{\bar{c}}]_{\Gamma_2}(u) \middle| \bar{B} \right\rangle = \hat{A}_{M_1 M_2} \mathfrak{F}_{M_2}^{BM_1}(u), \text{ with } \mathfrak{F}_{M_2}^{BM_1}(u) \text{ an arbitrary function}$$

Soft-exchange effects from emission topology

- ☐ **To have predictive power, make the following two approximations:**
  - Working to lowest order in the hard QCD → SCET$_\mathrm{I}$ matching, then $H_i(u) = 1$

$$\Rightarrow \quad \mathfrak{F}_{M_2}^{BM_1} = \int_0^1 du\, \mathfrak{F}_{M_2}^{BM_1}(u)$$

  - When the gluon propagator is soft, the propagator 8 is anti-hard-collinear;



➡ The SCET$_\mathrm{I}$ operator naively factorizes after matching to SCET$_\mathrm{II}$:

$$\mathfrak{F}_{M_2}^{BM_1}(u) = \frac{1}{\hat{A}_{M_1M_2}} \frac{f_{M_2}\phi_{M_2}(u)}{8N_c u\bar{u}} \times (-1)\int_0^\infty ds \left\langle M_1 \middle| \left[\bar{u}_c T^A h_v\right]_{\Gamma_1} \epsilon_{\mu\nu\alpha\beta} n_+^\nu g_s G^{A,\alpha\beta}(-sn_+) \middle| \bar{B} \right\rangle$$

$$= \frac{1}{\hat{A}_{M_1M_2}} \frac{f_{M_2}\phi_{M_2}(u)}{8N_c u\bar{u}} \times (-i) F^{B\to M_1}(0) g_{\Gamma_1}^{BM_1} = \frac{\phi_{M_2}(u)}{8N_c u\bar{u}} g_{\Gamma_1}^{BM_1}$$

independent of M_2

  - With the asymptotic $\phi_{M_2}(u) = 6u\bar{u}$, we have:

$$\boxed{\mathfrak{F}_{M_2}^{BM_1} = \int_0^1 du\, \mathfrak{F}_{M_2}^{BM_1}(u) = \frac{1}{4} g_{\Gamma_1}^{BM_1}}$$

- ☐ **Pheno. impacts on two-body hadronic B decays:** [Bell, Beneke, Huber, Li, w.i.p.]

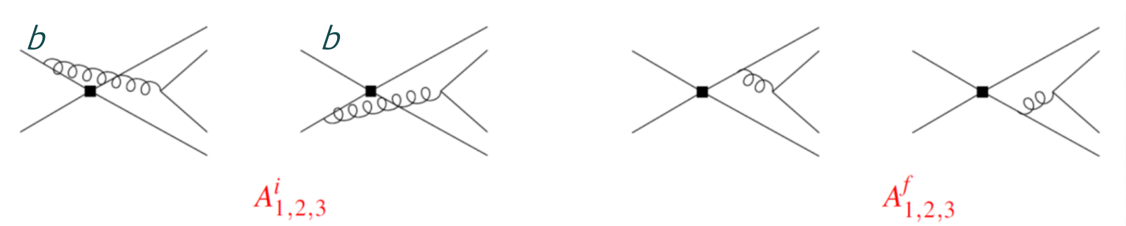
Pure annihilation B decays

- ☐ **Two typical pure annihilation decay modes:** $\bar{B}_s^0 \to \pi^+\pi^-$ vs $\bar{B}_d^0 \to K^+K^-$ related by SU(3)

$$\mathcal{A}(\bar{B}_s \to \pi^+\pi^-) = B_{\pi\pi}\left[\delta_{pu} b_1 + 2b_4^p + \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$\mathcal{A}(\bar{B}_d \to K^+K^-) = A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p + b_{4,\mathrm{EW}}^p\right] + B_{K\bar{K}}\left[b_4^p - \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$= A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p\right] + B_{K\bar{K}}\left[b_4^p\right]$$



- ☐ **Both involve $b_1 = \frac{C_F}{N_c^2} C_1 A_1^i$ & $b_4^p = \frac{C_F}{N_c^2}[C_4 A_1^i + C_6 A_2^i]$ and kernels A_1^i & A_2^i :**

A_1^i: (V − A) ⊗ (V − A)

A_2^i: (V − A) ⊗ (V + A)

$$A_1^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

- ☐ **With the asymptotic LCDAs $\Phi_M(x) = 6x\bar{x}$, we have $A_1^i = A_2^i$:**

[BBNS '99-'03]

$$A_1^i(M_1M_2) = \pi\alpha_s \left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$X_A = \left(1 + \varrho_A e^{i\varphi_A}\right)\ln\left(m_B/\Lambda_h\right),$$

$\Lambda_h = 0.5\mathrm{GeV}$, $\varrho_A \leq 1$ and an arbitrary phase φ_A

Ways to improve the modelling of annihilations

☐ With universal X_A and different scenarios, we have:

[BBNS '03]

Mode	Theory	S1 (large γ)	S2 (large a_2)	S3 ($\varphi_A = -45^{\circ}$)	S4 ($\varphi_A = -55^{\circ}$)	Exp.
$\bar{B}_s^0 \to \pi^+\pi^-$	$0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$	0.027	0.032	0.149	0.155	0.72 ± 0.11
$\bar{B}^0 \to K^-K^+$	$0.013^{+0.005+0.008+0.000+0.087}_{-0.005-0.005-0.000-0.011}$	0.007	0.014	0.079	0.070	0.080 ± 0.015

Large SU(3)-flavor symmetry breaking or flavor-dependent $A^i_{1,2}$?

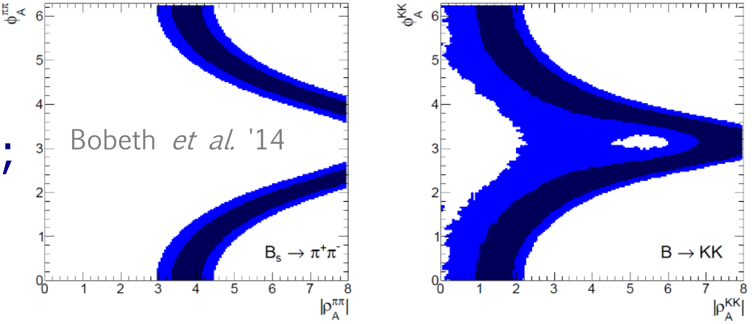
[Wang, Zhu '03; Bobeth *et al.* '14; Chang, Sun *et al.* '14-15]

☐ How to improve the situation:

- including higher Gegenbauer moments to include SU(3)-breaking effects;

$$\Phi_M(x,\mu) = 6x\bar{x}\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]$$

due to G-parity, $a_{odd}^{\pi} = 0$, but $a_{odd}^{K} \neq 0$



Bobeth *et al.* '14

FIGURE 5.8: 68% and 95% CRs for the complex parameter $\rho_A^{\pi^+\pi^-}$ and $\rho_A^{K^+K^-}$ obtained from a branching-ratio fit assuming the SM.

$$X_A = \left(1 + \varrho_A e^{i\varphi_A}\right)\ln\left(m_B / \Lambda_h\right)$$

- including the difference between the chirality factors to include SU(3)-breaking effects;

$$r_\chi^\pi(1.5\text{GeV}) = \frac{2m_\pi^2}{m_b(\mu)\big(m_u(\mu) + m_d(\mu)\big)} \simeq 0.86, \qquad r_\chi^K(1.5\text{GeV}) = \frac{2m_K^2}{m_b(\mu)\big(m_u(\mu) + m_s(\mu)\big)} \simeq 0.91$$

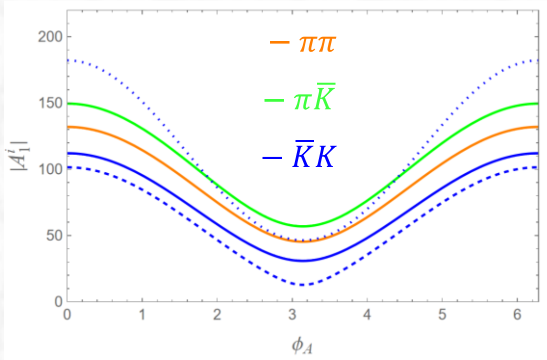
Ways to improve the modelling of annihilations

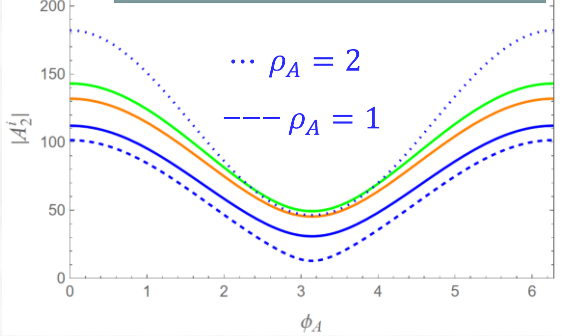
□ SU(3)-breaking effects in $A^i_{1,2}$: due to higher Gengengauber moments and quark masses

$$A_1^i(M_1M_2) = \pi\alpha_s\left\{18(1-a_1^{M_1}+a_2^{M_1})\left[(1+3a_1^{M_2}+6a_2^{M_2})X_A-(1+6a_1^{M_2}+16a_2^{M_2})\right] +54(69-7\pi^2)a_1^{M_1}a_1^{M_2}-36(385-39\pi^2)(a_1^{M_1}a_2^{M_2}-2a_2^{M_1}a_1^{M_2})\right.$$
$$\left.-6(9-\pi^2)-18(10-\pi^2)(3a_1^{M_1}-a_1^{M_2})-6(59-6\pi^2)(6a_2^{M_1}+a_2^{M_2}) -18(9593-972\pi^2)a_2^{M_1}a_2^{M_2}+r_\chi^{M_1}r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s\left\{18(1+a_1^{M_2}+a_2^{M_2})\left[(1-3a_1^{M_1}+6a_2^{M_1})X_A-(1-6a_1^{M_1}+16a_2^{M_1})\right]\right.$$
$$-6(9-\pi^2)-18(10-\pi^2)(a_1^{M_1}-3a_1^{M_2})-6(59-6\pi^2)(a_2^{M_1}+6a_2^{M_2})$$
$$+54(69-7\pi^2)a_1^{M_1}a_1^{M_2}-36(385-39\pi^2)(2a_1^{M_1}a_2^{M_2}-a_2^{M_1}a_1^{M_2})$$
$$\left.-18(9593-972\pi^2)a_2^{M_1}a_2^{M_2}+r_\chi^{M_1}r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1+\rho_A e^{i\phi_A})$$





	$\pi\pi$	$\pi\bar{K}$	$\bar{K}K$
A_1^i	$31.7X_A-51.5+6.2+1.5X_A^2$ $[18X_A-18+5.2+1.5X_A^2]$	$37.6X_A-63.4+6.5+1.6X_A^2$ $[18X_A-18+5.2+1.6X_A^2]$	$23.4X_A-36.0+5.2+1.7X_A^2$ $[18X_A-18+5.2+1.7X_A^2]$
A_2^i	$31.7X_A-51.5+6.2+1.5X_A^2$ $[18X_A-18+5.2+1.5X_A^2]$	$34.6X_A-56.2+6.9+1.6X_A^2$ $[18X_A-18+5.2+1.6X_A^2]$	$23.4X_A-36.0+5.2+1.7X_A^2$ $[18X_A-18+5.2+1.7X_A^2]$

$Br(\bar{B}_s^0 \to \pi^+\pi^-)$: $(0.72 \pm 0.11)\times 10^{-6}$
$Br(\bar{B}^0 \to K^-K^+)$: $(0.080 \pm 0.015)\times 10^{-6}$

- ➢ $|A^i_{1,2}|$ can differ by more than **20%** in the **BBNS+ model**!
- ➢ The amplitude ratios $A^i_{1,2}(\pi\pi)/A^i_{1,2}(KK)$ get **enhanced** in the **BBNS+ model**! ➡ what we need!

Ways to improve the modelling of annihilations

☐ **How to improve:**

- Making the parameter X_A to be flavour dependent & depending on its origins;

$$\begin{aligned}
\int_0^1 dy\, \frac{\Phi_{M_1}(y)}{y^2} &= \Phi'_{M_1}(0) \int_0^1 dy\, \frac{1}{y} + \int_0^1 dy\, \frac{\Phi_{M_1}(y) - y\,\Phi'_{M_1}(0)}{y^2} && \longrightarrow \quad 6X_0^{M_1} - 6\,, \\
\int_0^1 dx\, \frac{\Phi_{M_2}(x)}{\bar{x}^2} &= \Phi'_{M_2}(1) \int_0^1 dx\, \frac{1}{\bar{x}} + \int_0^1 dx\, \frac{\Phi_{M_2}(x) - \bar{x}\,\Phi'_{M_2}(1)}{\bar{x}^2} && \longrightarrow \quad 6X_1^{M_2} - 6\,, \\
\int_0^1 dy\, \frac{\Phi_{m_1}(y)}{y} &= \Phi_{m_1}(0) \int_0^1 dy\, \frac{1}{y} + \int_0^1 dy\, \frac{\Phi_{m_1}(y) - \Phi_{m_1}(0)}{y} && \longrightarrow \quad X_0^{m_1}\,, \\
\int_0^1 dx\, \frac{\Phi_{m_2}(x)}{\bar{x}} &= \Phi_{m_2}(1) \int_0^1 dx\, \frac{1}{\bar{x}} + \int_0^1 dx\, \frac{\Phi_{m_2}(x) - \Phi_{m_2}(1)}{\bar{x}} && \longrightarrow \quad X_1^{m_2}\,,
\end{aligned}$$

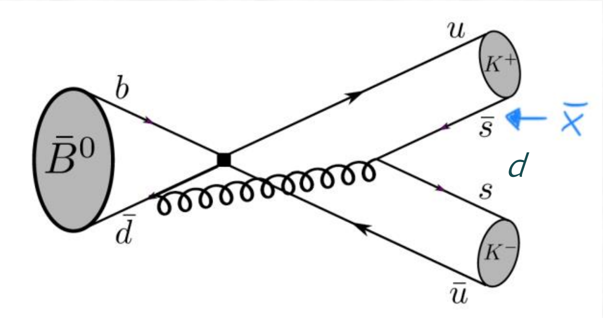
$$\Rightarrow$$

$$A_1^i(M_1M_2) = \pi\alpha_s \left\{ 18X_1^{M_2} - 18 - 6(9 - \pi^2) + r_\chi^{M_1} r_\chi^{M_2} \left(2X_0^{m_1} X_1^{m_2} \right) \right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \left\{ 18X_0^{M_1} - 18 - 6(9 - \pi^2) + r_\chi^{M_1} r_\chi^{M_2} \left(2X_0^{m_1} X_1^{m_2} \right) \right\},$$

$$\Rightarrow \boxed{A_1^i(M_1M_2) \neq A_2^i(M_1M_2)}$$

- To make it predictive, distinguish whether the endpoint configuration mediated by a soft strange quark (X_A^s) or a soft up or down quark (X_A^{ud}) .



☐ **Advantages compared to original BBNS: two free parameters!**

- **For $\pi\pi$ final states, only X_A^{ud} involved;**

↳ **easily to reproduce the data!**

- **For KK final states, both X_A^{ud} (for $M_1M_2 = K^+K^-$) and X_A^s (for $M_1M_2 = K^-K^+$) involved;**

☐ **Other interesting progress:**

Lu, Shen, Wang, Wang, Wang 2202.08073; Boer talk @ SCET2023;

Neubert talk @ Neutrinos, Flavour and Beyond 2022

Summary

- **With exp. and theor. progress, we are now entering a precision era for flavour physics**
- **Within QCDF/SCET framework, NNLO QCD corrections to color-allowed, color-suppressed tree & leading-power penguin amplitudes complete, factorization at 2-loop established**
- **Due to delicate cancellation, NNLO corrections found small; some puzzles still remain:**
  - long-standing $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar{B}^0 \to \pi^+ K^-)$;
  - for class-I $B_q^0 \to D_q^{(*)-} L^+$ decays, $\mathcal{O}(4\text{–}5\sigma)$ discrepancies observed in branching ratios;

➡ **sub-leading power corrections in QCDF/SCET need to be considered!**

  - Sub-leading color-octet matrix elements $\langle M_1 M_2 | [\bar{u}_c T^A h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} T^A u_{\bar{c}}]_{\Gamma_2}(u) | \bar{B} \rangle$ [w.i.p]
  - improved treatments of annihilation amplitudes: SU(3)-breaking effects & flavor-dependence of the building blocks $A^i_{1,2}$ [w.i.p]

Thank You for your attention!