SPT and phase/shape coexistence in nuclei

------------ Views from the boson model

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Nuclear shape (deformation) manifested by spectra

A common feature of systems that have rotational spectra is the existence of a "deformation", by which is implied a feature of anisotropy that makes it possible to specify an orientation of the system as a whole. In a molecule, as in a solid body, the deformation reflects the highly anisotropic mass distribution, as viewed from the intrinsic coordinate frame defined by the equilibrium positions of the nuclei. In the nucleus, the rotational degrees of freedom are associated with the deformations in the nuclear equilibrium shape that result from the shell structure. (Evidence for

NUCLEAR STRUCTURE
Volume II: Nuclear Deformations

AAGE BOHR
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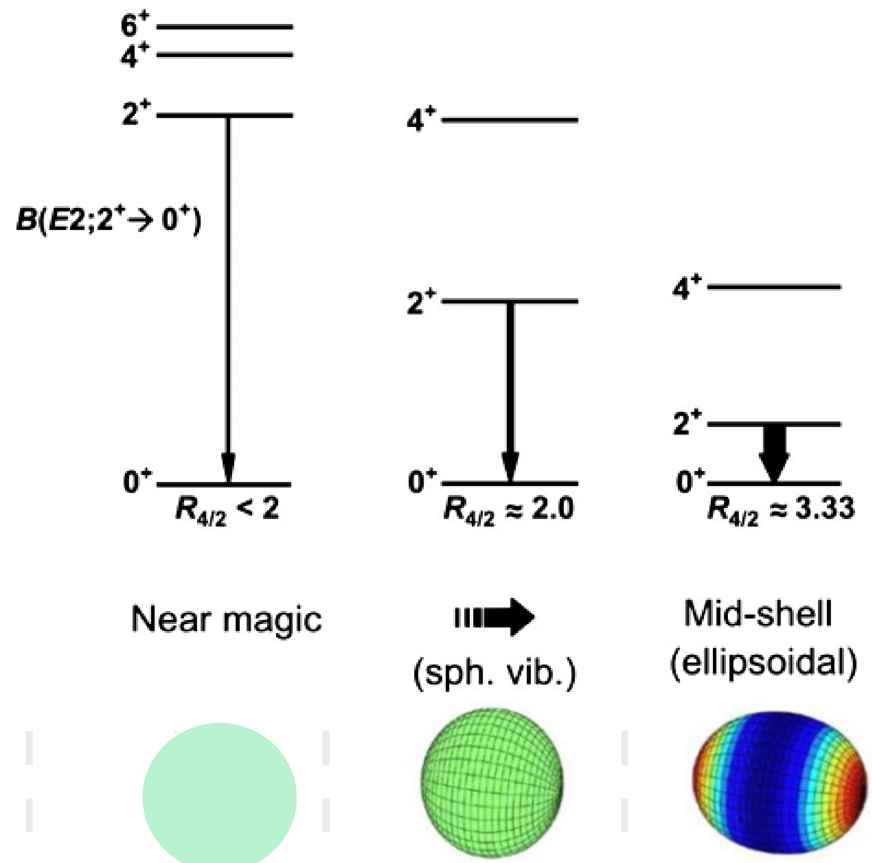
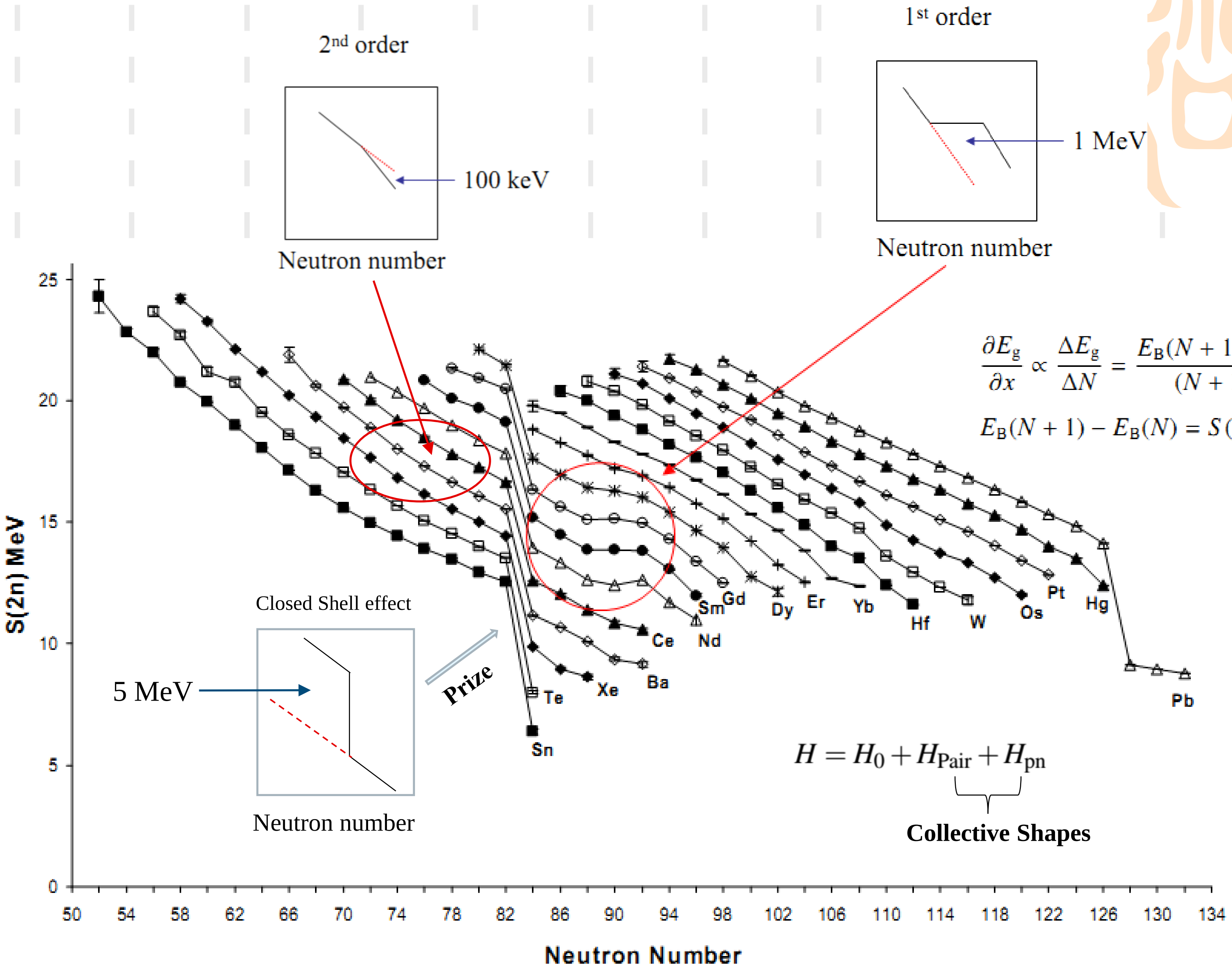


Shape Phase Transition (SPT)



$$\frac{\partial E_g}{\partial x} \propto \frac{\Delta E_g}{\Delta N} = \frac{E_B(N+1) - E_B(N)}{(N+1) - N},$$

$$E_B(N+1) - E_B(N) = S(2n),$$

$$H = H_0 + H_{Pair} + H_{pn}$$

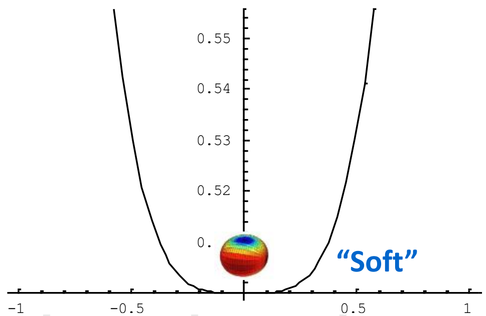
Collective Shapes

Theoretical interests

VOLUME 85, NUMBER 17 PHYSICAL REVIEW LETTERS 23 OCTOBER 2000

Dynamic Symmetries at the Critical Point

F. Iachello



VOLUME 81, NUMBER 6 PHYSICAL REVIEW LETTERS 10 AUGUST 1998

Phase Coexistence in Transitional Nuclei and the Interacting-Boson Model

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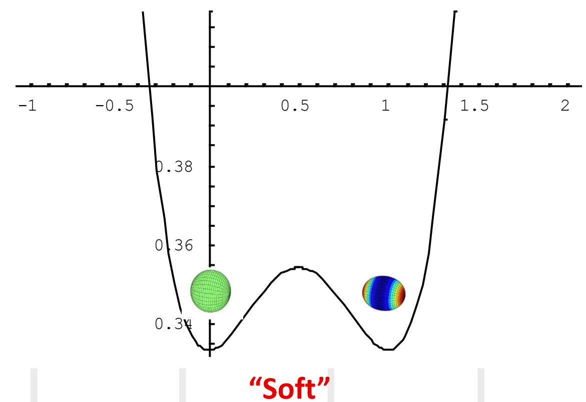
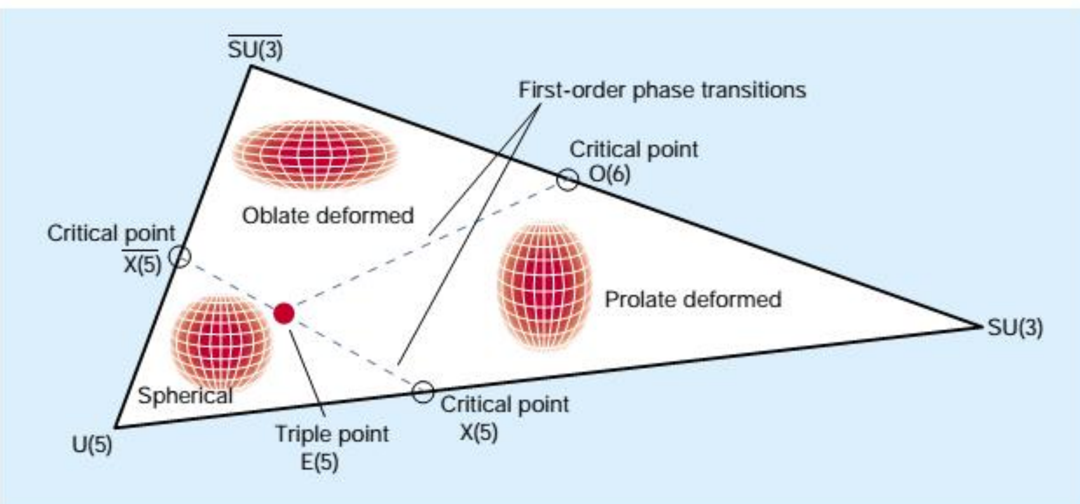
(Received 13 May 1998)





Cejnar, Jolie, Casten, RMP, 82, (2010)

Interacting Boson Model (IBM)

Arima and Iachello, 1976

$$U(6) \supset U(5) \supset SO(5) \supset SO(3),$$
$$U(6) \supset O(6) \supset SO(5) \supset SO(3),$$
$$U(6) \supset SU(3) \supset SO(3).$$

$$(d^\dagger \times \tilde{d})^{(k)}_m = \sum_{u,u'} < 2u2u'|km > d^\dagger_u \tilde{d}_{u'}, \quad k = 0,\ 1,\ 2,\ 3,\ 4$$

$$d^\dagger_u s, \quad s^\dagger \tilde{d}_u, \quad s^\dagger s$$

$$H = \varepsilon_s s^\dagger s + \varepsilon_d (d^\dagger \cdot \tilde{d}) + \tfrac{1}{2} \sum_{L=0,2,4} c_L [(d^\dagger \times d^\dagger)^{(L)} \cdot (\tilde{d} \times \tilde{d})]^{(L)} + \tfrac{1}{2} \tilde{v}_0 [(d^\dagger \times d^\dagger)^{(0)}_0 s^2 + (s^\dagger)^2 (\tilde{d} \times \tilde{d})^{(0)}_0] + \ldots$$

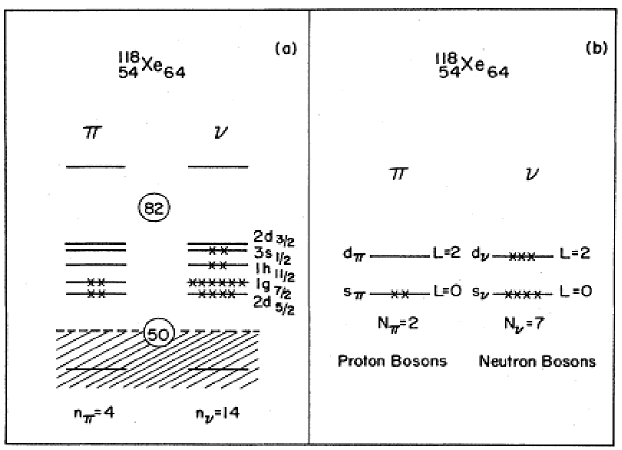
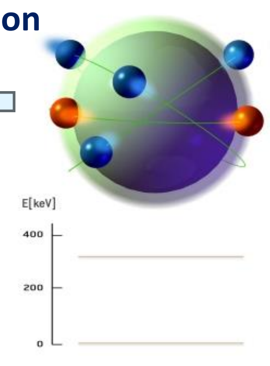


FIG. 2. (a) Schematic representation of the shell-model problem for $^{118}_{54}Xe_{64}$; (b) the boson problem, which replaces the shell-model problem for $^{118}_{54}Xe_{64}$. Both in part (a) and in part (b) the nucleons (a) or bosons (b) can be arranged in all possible ways consistent with the single-particle levels and Fermi (a) or Bose (b) statistics. Of all these possible ways only one is shown in the figure.

SM foundation



D pair → **d boson**

S pair → **s boson**

$N \to \infty$, $\hbar \sim 1/N$ → **Geometry**

QPT is strictly defined in the large-*N* limit

Arima, Otsuka, Iachello, Talmi, PLB, 66 (1977)
Otsuka, Arima, Iachello, NPA, 309, (1978)
Iachello, Talmi, RMP, 59 (1987)

Modeling (defining) nuclear SPT

$$H = (1-\eta)H_{\text{sph.}} + \eta H_{\text{def.}}$$

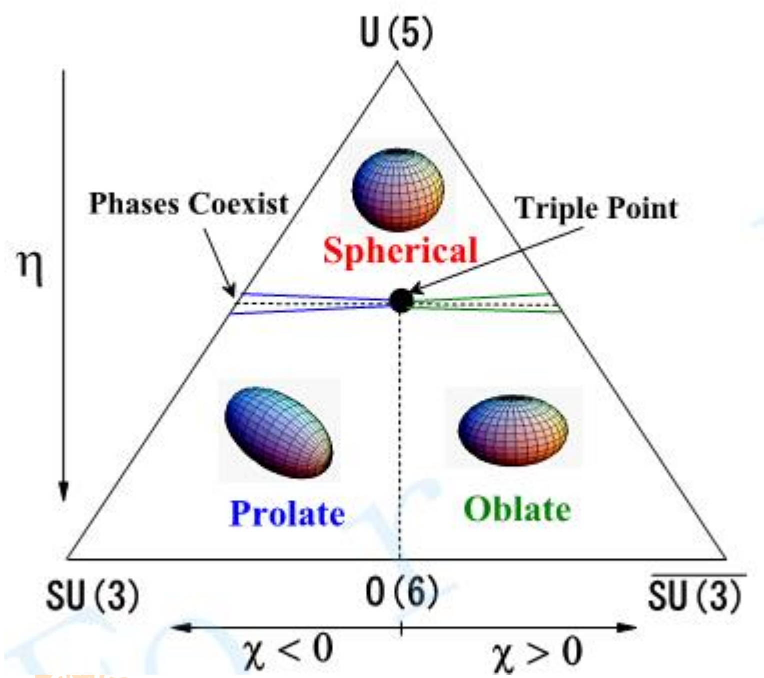
$$H_{\text{sph.}} = d^{\dagger}\cdot\tilde{d} \qquad H_{\text{def.}} = -Q\cdot Q$$

$$Q = (d^{\dagger}s + s^{\dagger}\tilde{d})^{(2)} + \chi(d^{\dagger}\tilde{d})^{(2)}$$

$$|\beta,\gamma,N\rangle = [s^{\dagger} + \beta\cos\gamma d_0^{\dagger} + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_2^{\dagger} + d_{-2}^{\dagger})]^N|0\rangle$$

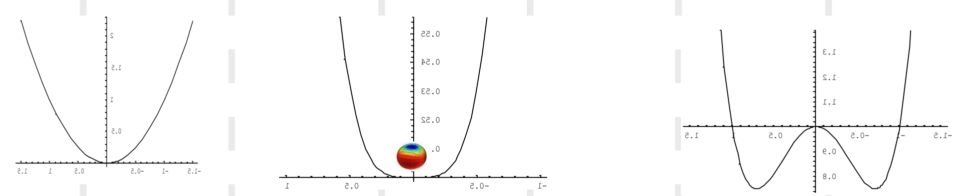
$$V(\beta,\gamma) \equiv \frac{1}{N}\frac{\langle\beta,\gamma,N|\hat{H}|\beta,\gamma,N\rangle}{\langle\beta,\gamma,N|\beta,\gamma,N\rangle}\Bigg|_{N\to\infty} = \frac{A\beta^2 + B\beta^3\cos 3\gamma + C\beta^4}{(1+\beta^2)^2}$$

$$E_g = V(\beta,\gamma)_{\min}, \quad \frac{\partial E_g}{\partial\eta}, \quad \frac{\partial^2 E_g}{\partial\eta^2}$$



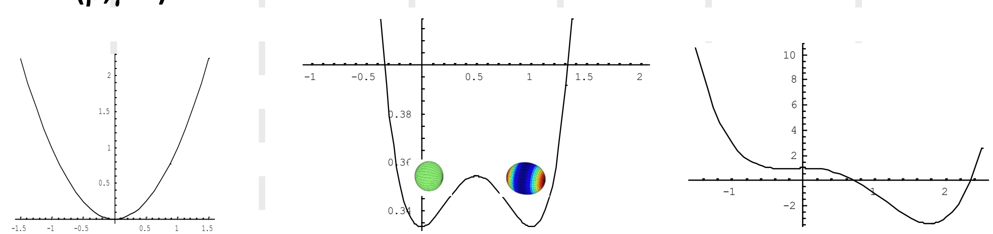
2^{nd} order QPT

***V*(β,γ=0)**

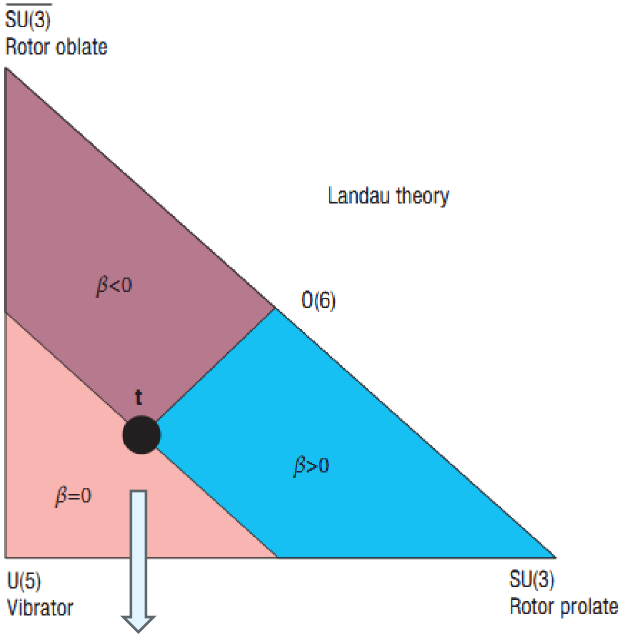


1^{st} order QPT

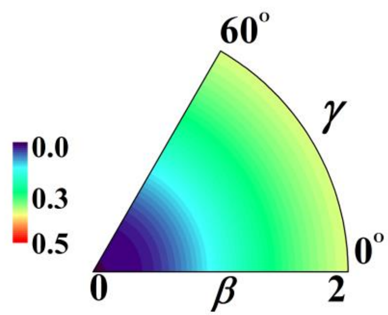
***V*(β,γ=0)**



Nuclear deformation in the 2nd order SPT

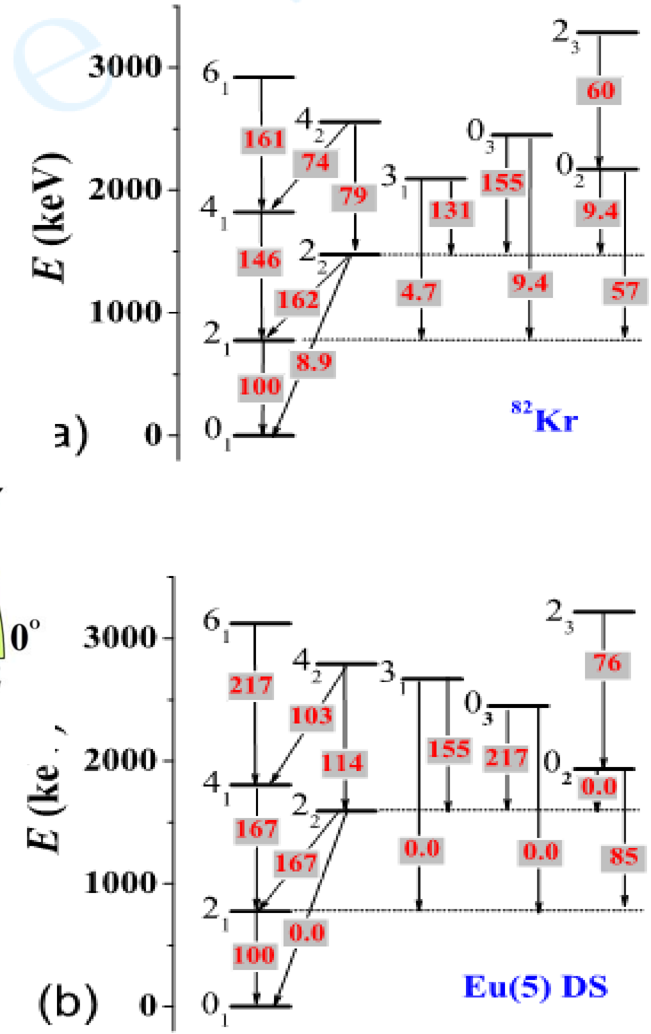


β soft and γ soft (triaxial)



Eu(5)⊃SO(5) ⊃ SO(3)

$$\hat{H}_{\mathrm{Eu(5)}} = a\, m\, \hat{C}_2[\mathrm{Eu(5)}] + b\, \hat{C}_2[\mathrm{O(5)}] + c\, \hat{C}_2[\mathrm{O(3)}]$$



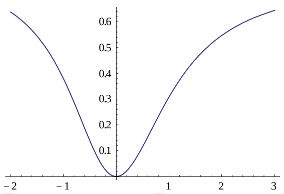
PLB, 732(2014); PRC, 90 (2014)
Symmetry, 14 (2022)

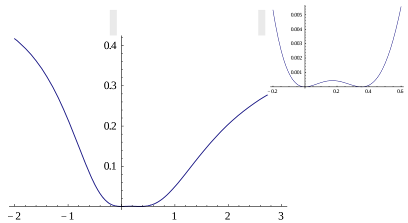
Nuclear deformation in the 1st order SPT

Type I (Sm isotopes):

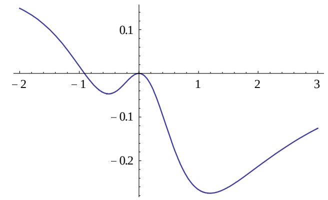
$$H = (1-\eta)H_{\text{sph.}} + \eta H_{\text{def.}}$$

$V(\beta,\gamma=0)$

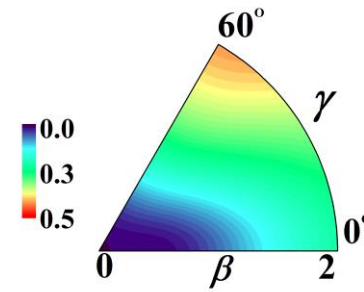




Cri.



Phase coexistence

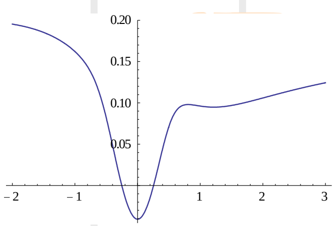


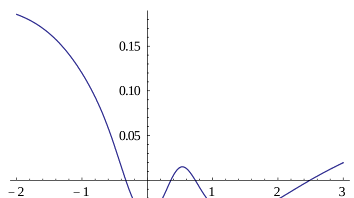
discuss later

Type II (Zr isotopes):

$$H = \begin{bmatrix} (1-\eta)H_{\text{sph.}} & W_{\text{mix.}} \\ W_{\text{mix.}} & \eta H_{\text{def.}} + \Delta \end{bmatrix}$$

$V(\beta,\gamma=0)$





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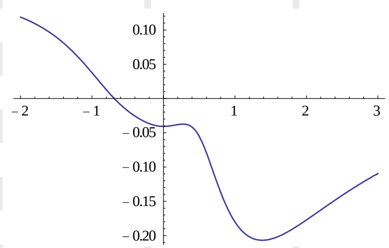
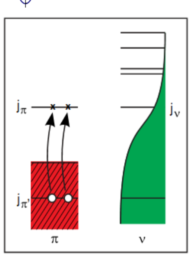
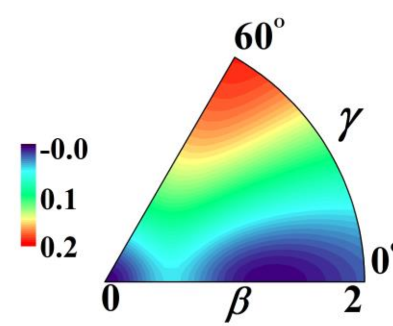




Coexistence cause instability in deformation

Shape coexistence



Togashi, et al., PRL,117(2016)172502
Gavrielov, et al., PRC,99(2019)064324
….

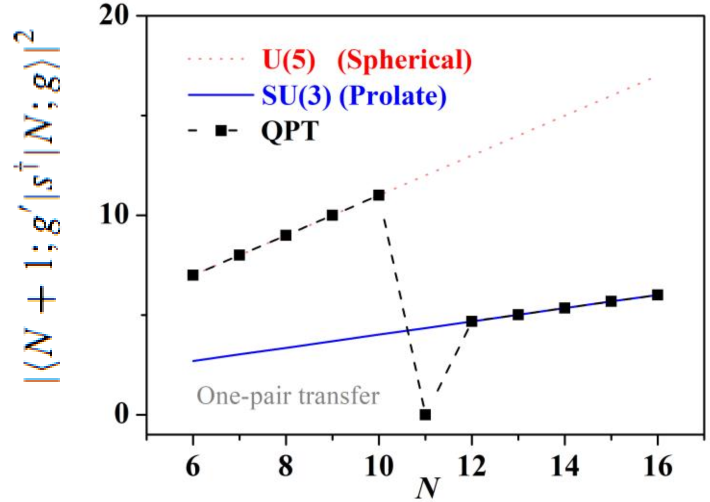
U(5)-SU(3) SPT in nucleon pair transfer intensity

$$ {}^{A}_{Z}X_{N} + 2\mathrm{n} \leftrightarrow {}^{A+2}_{Z}X_{\overline{N}} $$

$$ I^{b}(N,L \rightarrow N+1,L') = \frac{1}{2L+1} |\langle N+1,L' \| P_{+} \| N,L \rangle|^{2} $$

$$ \langle N+1,0_1 \| s^{\dagger} \| N,0_1 \rangle_{\mathrm{U(5)}} = \sqrt{N+1} $$

$$ \langle N+1,0_1 \| s^{\dagger} \| N,0_1 \rangle_{\mathrm{SU(3)}} = \sqrt{\frac{(2N+3)(N+1)}{3(2N+1)}}, $$

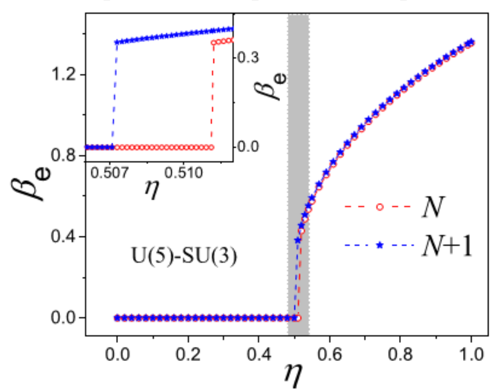


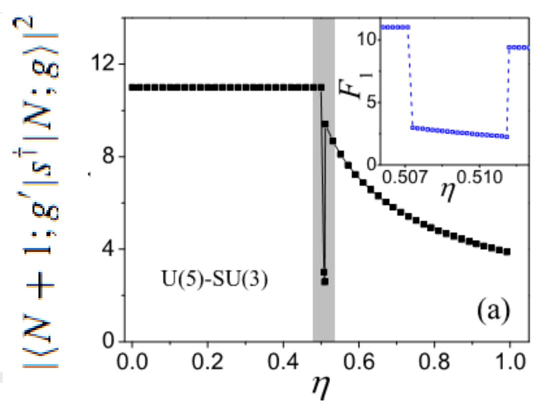
$$ \langle N+1;g'|s^{\dagger}|N;g\rangle = \frac{\sqrt{N+1}}{\sqrt{1+\beta'^2}} \left[\frac{1+\beta\beta'\cos(\gamma-\gamma')}{\sqrt{(1+\beta'^2)(1+\beta^2)}} \right]^{N} $$

PHYSICAL REVIEW C 95, 034306 (2017)

Two-nucleon transfer reactions as a test of quantum phase transitions in nuclei

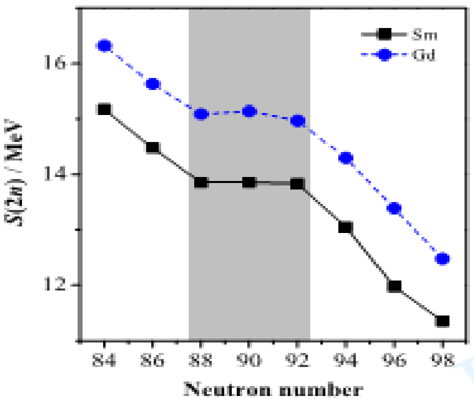
Y. Zhang[1,2] and F. Iachello[1]

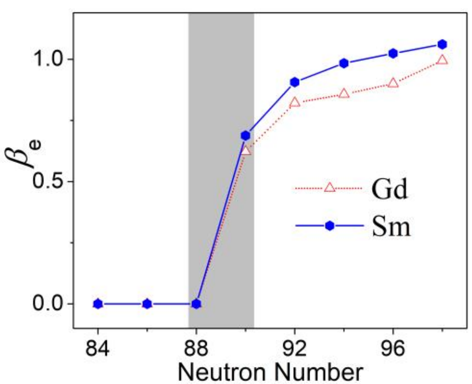


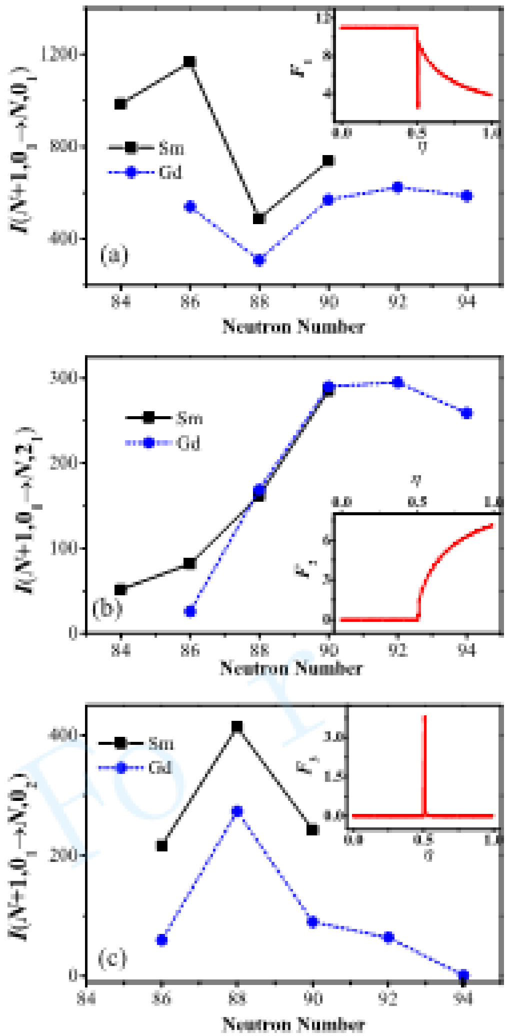


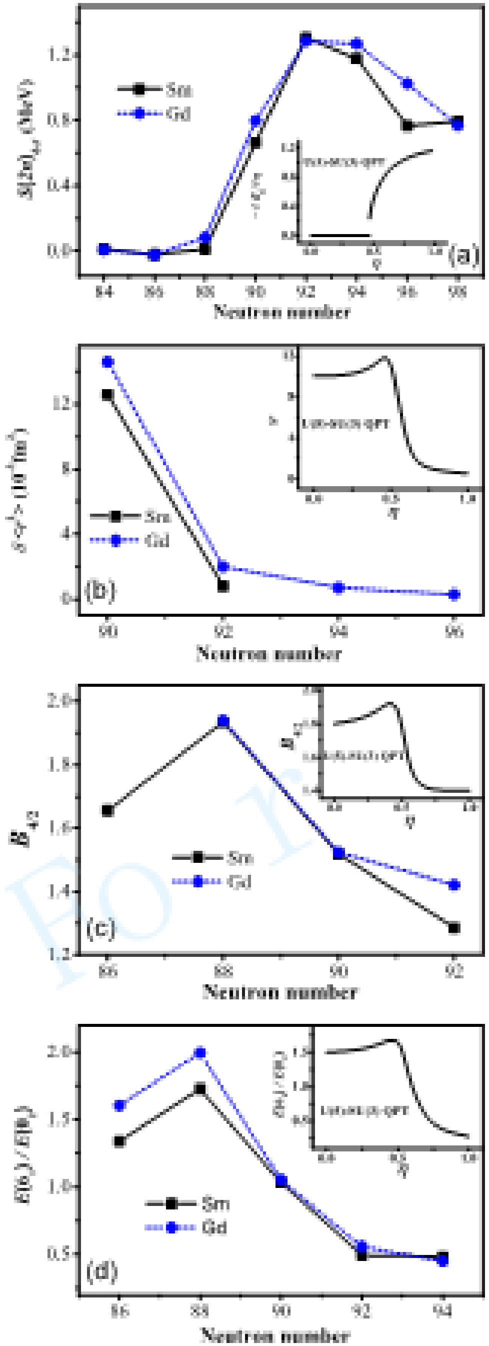
Observing the 1st order SPT in Sm, Gd

$$H = (1-\eta)H_{U(5)} + \eta H_{SU(3)}$$



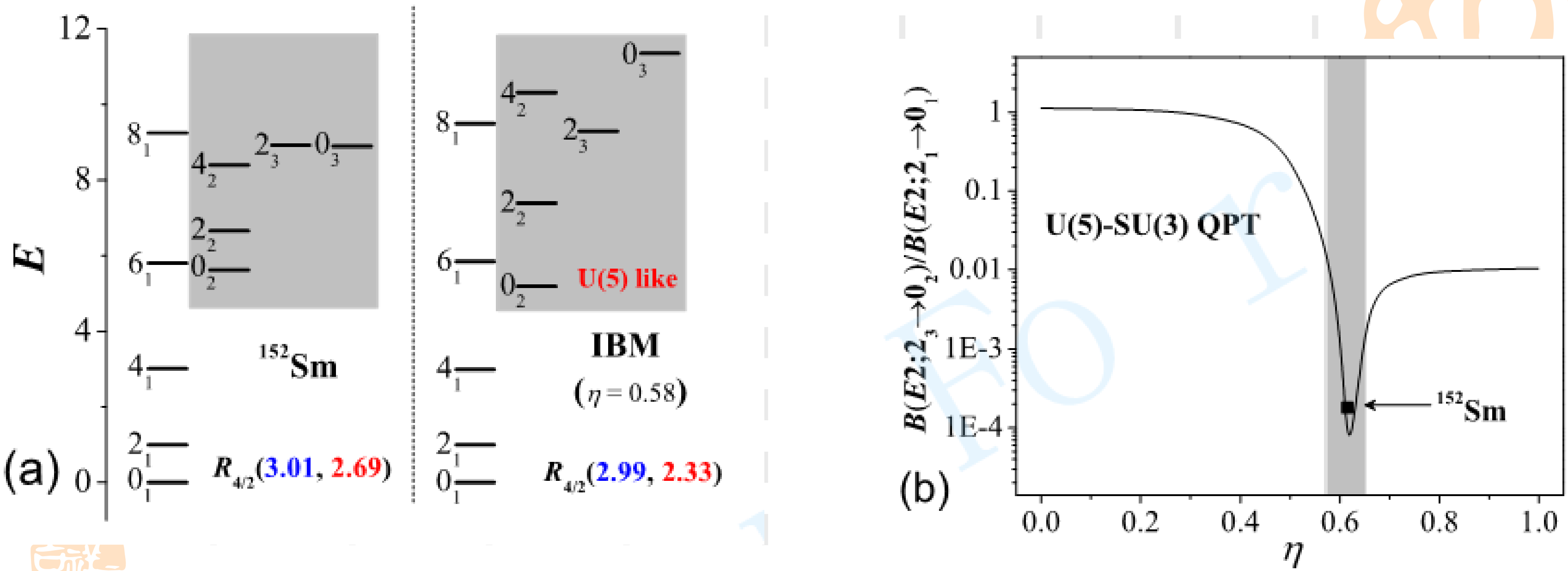






IBM1: YZ, Iachello, PRC 95 (2017) 034306
IBM2: Nomura, YZ, PRC 99 (2019) 024324

Phase/shape coexistence in Sm152



65 Iachello F, Zamfir N V, Casten R F. Phase coexistence in transitional nuclei and the interacting-boson model. Phys Rev Lett, 1998, 81: 1191-1194

66 Zhang J Y, Caprio M A, Zamfir N V, Casten R F. Phase/shape coexistence in Sm152 in the geometric collective model. Phys Rev C, 1999, 60: 061304(R)

67 Jolie J, Cejnar P, Dobeš J. Phase coexistence in the interacting boson model and Sm152. Phys Rev C, 1999, 60: 061303(R)

68 Clark R M, Deleplanque C M A, Diamond R M, et al. Reexamination of the N=90 transitional nuclei Nd150 and Sm152. Phys Rev C, 2003, 67: 041302(R)

69 Garrett P E, Kulp W D, Wood J L, et al. New features of shape coexistence in Sm152. Phys Rev Lett, 2009, 103: 062501

⋮

how to see phase coexistence

$$R(L) = \frac{E(L)-E(L-2)}{L}$$

YZ, Iachello, PRC, 95(2017)061304(R)

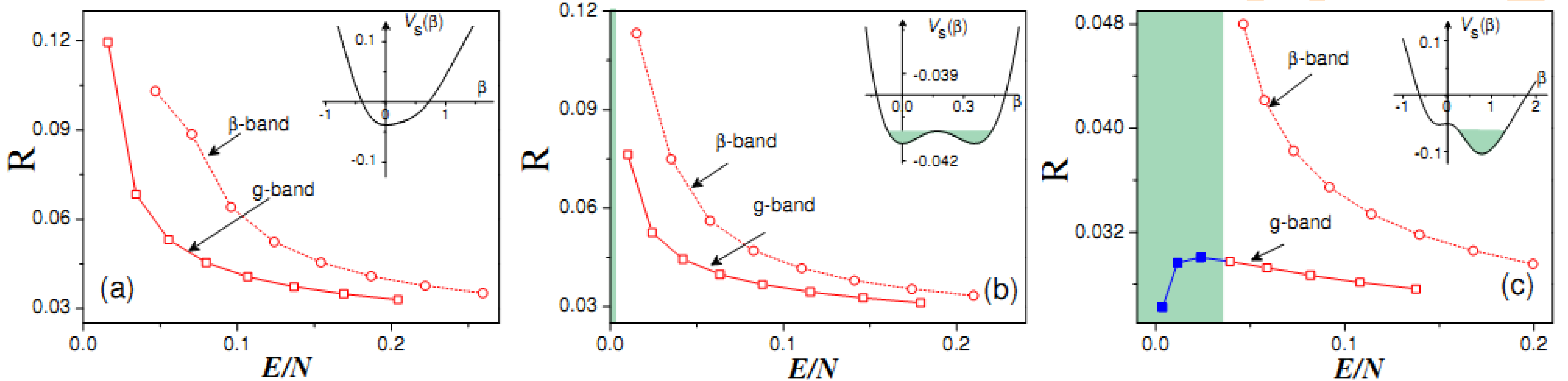
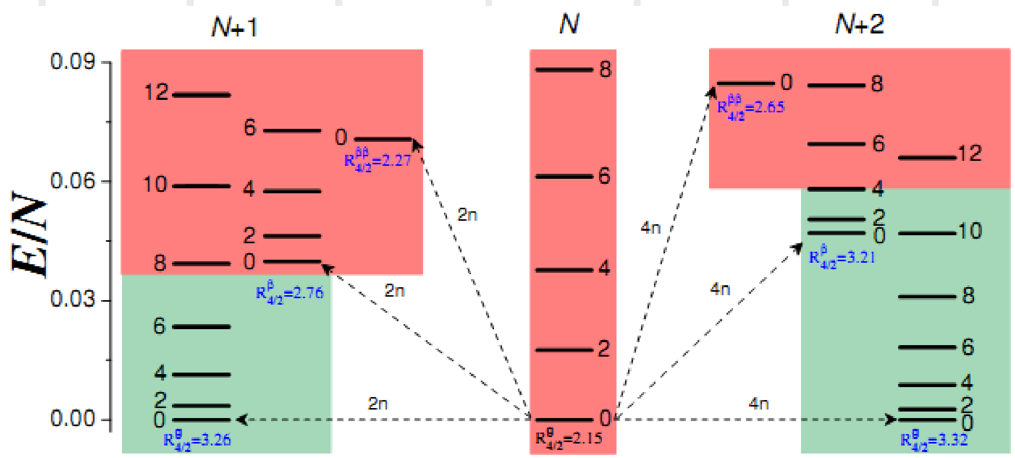


FIG. 2: The E-GOS curves and potentials ($\gamma = 0°$) calculated for the selected cases ($N = 15$) with $\eta = 0.46$, 0.497, and 0.6 representing the



(c) **(a)** **(c’)**

Phase coexistences from pair transfer

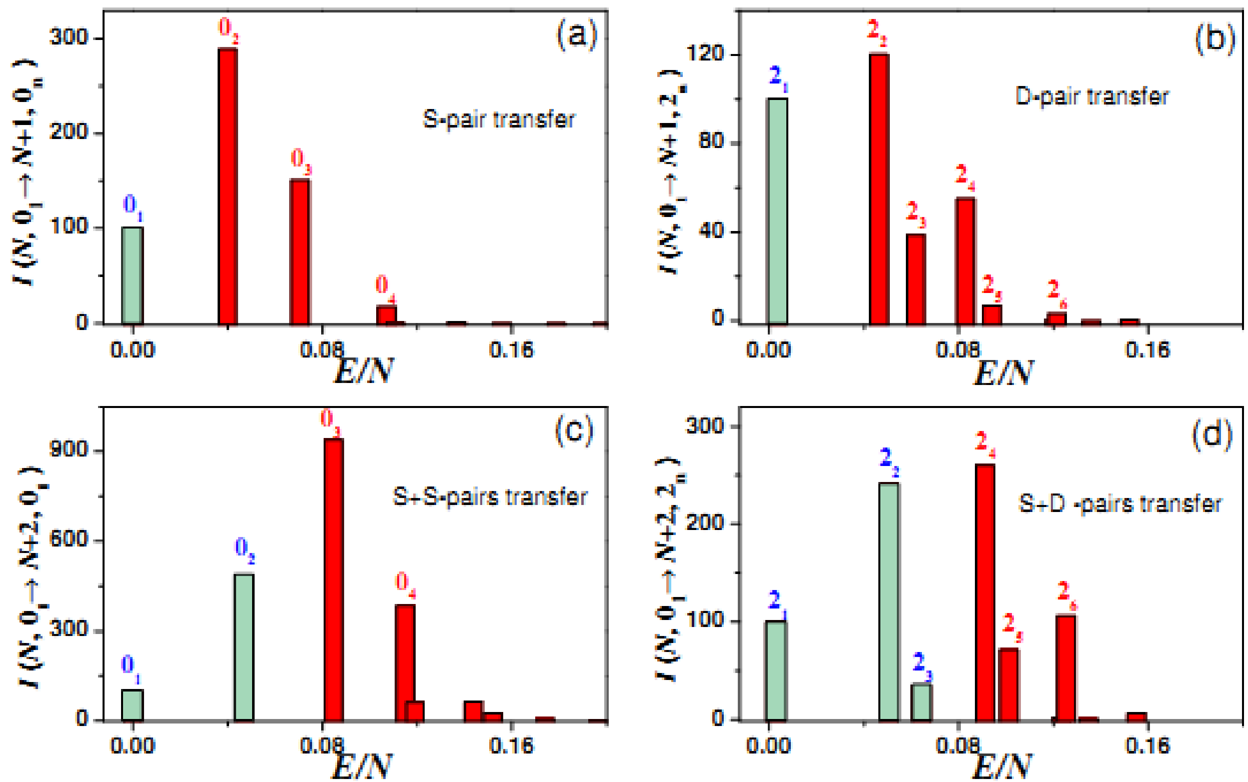


FIG. 4: The 2n- and 4n-transfer intensities (in any units) calculated for the cases correspond to those shown in Fig. 3. The results in the panels (a) and (c) are normalized to $I(N,0_1 \rightarrow N+\Delta N,0_1) = 100$ and those in (b) and (d) are normalized to $I(N,0_1 \rightarrow N+\Delta N,2_1) = 100$.

More chances to search for phase coexistence.
Data are too few to make a conclusion

Conclusion

Nuclear shape in a QPT is soft, hard to be quantitatively defined, but rich in physics!

Thanks for your attention!