$\tau^+\tau^-$ atom and τ mass

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Outline

1. Introduction
   - $\tau^+\tau^-$ atom
   - τ mass
2. The frame of Calculation
3. Reduce the uncertainties
4. Summary

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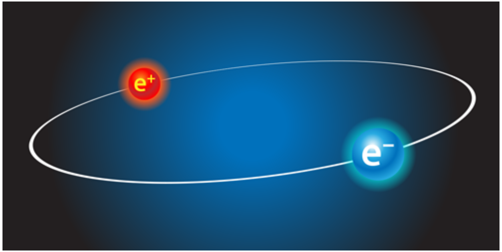
QED atom

1. QED atoms (e^+e^-, μ^+e^-, τ^+e^-, $\mu^+\mu^-$, $\tau^+\mu^-$, $\tau^+\tau^-$) are composed of unstructured and point-like lepton pairs, simple than the hydrogen formed of a proton and an electron.
2. The properties of QED atoms have been studied to test QED, fundamental symmetries, New Physics, gravity, and so on (hep-ex/0106103, 0912.0843, 1710.01833, 1802.01438, Phys.Rept. 975 (2022) 1-61).
3. Only positronium (e^+e^-) and muonium (μ^+e^-) had been discovered in 1951 and 1960 respectively.

Positronium

1. Positronium was discovered by Martin Deutsch in 1951.
2. "I'm really glad that I did not get the Nobel Prize in 1956. It would have spoiled my life." by Martin Deutsch
3. Positronium in medicine and biology: Nature Reviews Physics 1 (2019)527, Rev. Mod. Phys. 95 (2023) 021002.



True muonium

PRL **102,** 213401 (2009) PHYSICAL REVIEW LETTERS week ending 29 MAY 2009

Production of the Smallest QED Atom: True Muonium ($\mu^+\mu^-$)

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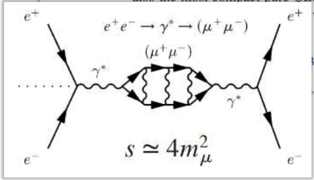
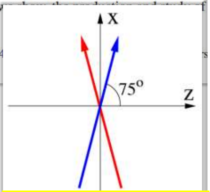
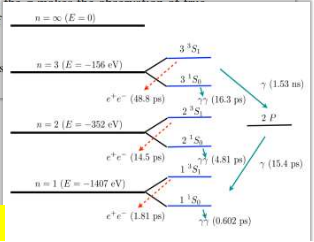
The "true muonium" ($\mu^+\mu^-$) and "true tauonium" ($\tau^+\tau^-$) bound states are not only the heaviest, but also the most compact pure QED systems. The rapid weak decay of



$$\vec{p} = \vec{p}_{e^+} + \vec{p}_{e^-} \neq \vec{0}$$

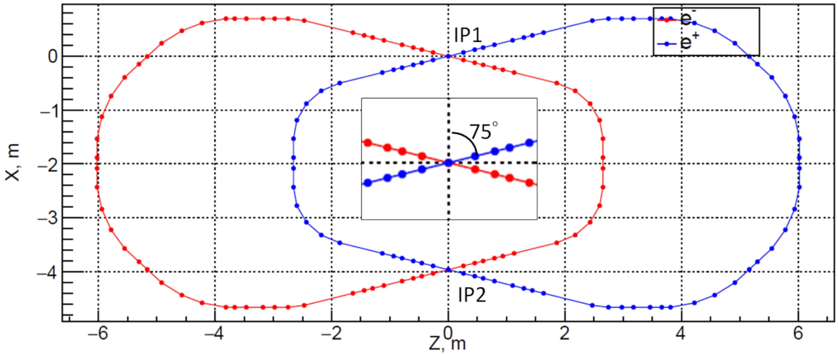


merge at 5º-15º



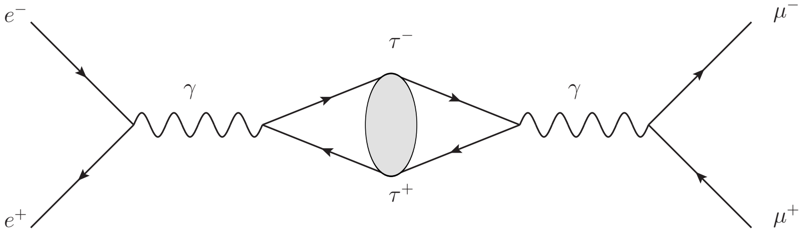
New colliders for true muonium

1. DIMUS: super-compact Dimuonium Spectroscopy collider at Fermilab, 2203.07144.
2. True muonium @ e^+e^- colliders with standard crossing angle, 2309.11683.

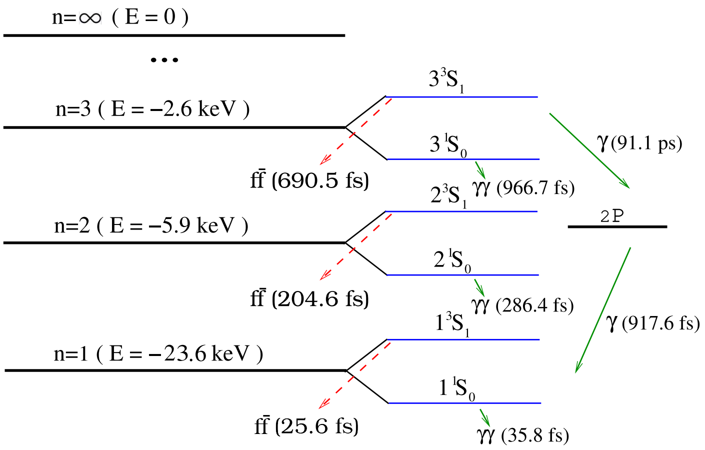


$\tau^+\tau^-$ atom

1. $\tau^+\tau^-$ atom is the smallest QED atom for Bohr radius is 30.4 fm (Moffat:1975uw)
2. $\tau^+\tau^-$ atom is is named tauonium (Avilez:1977ai,Avilez:1978sa), ditauonium (2204.07269, 2209.11439), and true tauonium (2202.02316).
3. We named them following charmonium just as $J_\tau(nS)$ for $n^{2S+1}L_J = n^3S_1$ and $J^{PC} = 1^{--}$, $\chi_{\tau J}(nP)$ for $n^{2S+1}L_J = n+1^3P_J$ and $J^{PC} = J^{++}$.
4. The production η_τ (2202.02316), and J_τ (2302.07365).



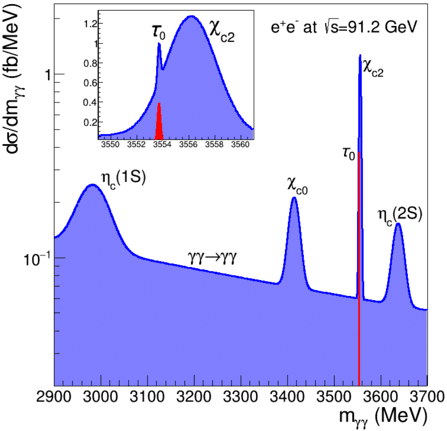
The spectroscopy of $\tau^+\tau^-$ atom, 2204.07269

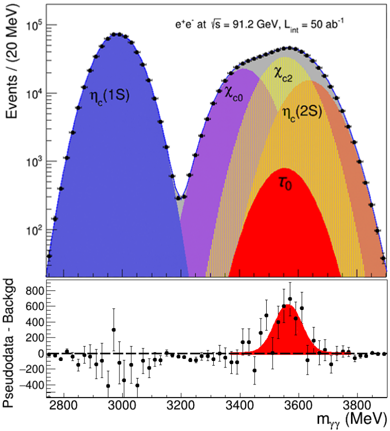


$\gamma\gamma \to \eta_\tau \to \gamma\gamma$, 2202.02316

Colliding system, c.m. energy, $\mathcal{L}_{\text{int}}$, exp.	$\sigma \times \mathcal{B}_{\gamma\gamma}$						$N \times \mathcal{B}_{\gamma\gamma}$	
	$\eta_c(1S)$	$\eta_c(2S)$	$\chi_{c,0}(1P)$	$\chi_{c,2}(1P)$	LbL	$\mathcal{T}_0$	$\mathcal{T}_0$	$\chi_{c,2}(1P)$
e^+e^- at 3.78 GeV, 20 fb^{-1}, BES III	120 fb	3.6 ab	15 ab	13 ab	30 ab	0.25 ab	–	–
e^+e^- at 10.6 GeV, 50 ab^{-1}, Belle II	1.7 fb	0.35 fb	0.52 fb	0.77 fb	1.7 fb	0.015 fb	750	38 500
e^+e^- at 91.2 GeV, 50 ab^{-1}, FCC-ee	11 fb	2.8 fb	3.9 fb	6.0 fb	12 fb	0.11 fb	5 600	$3 \cdot 10^5$
p-p at 14 TeV, 300 fb^{-1}, LHC	7.9 fb	2.0 fb	2.8 fb	4.3 fb	6.3 fb	0.08 fb	24	1290
p-Pb at 8.8 TeV, 0.6 pb^{-1}, LHC	25 pb	6.3 pb	8.7 pb	13 pb	21 pb	0.25 pb	0.15	8
Pb-Pb at 5.5 TeV, 2 nb^{-1}, LHC	61 nb	15 nb	21 nb	31 nb	62 nb	0.59 nb	1.2	62

$\gamma\gamma \to \eta_\tau \to \gamma\gamma$ at Z pole, 2202.02316





$e^+e^- \to J_\tau \to \mu^+\mu^-$ at STCF, 2302.07365

TABLE IV: Cross sections and expected number of events for the s-channel production of ortho-ditauonium ($\mathcal{T}_1$), and for the $\tau^+\tau^-$ and (background) $\mu^+\mu^-$ continua, in e^+e^- at $\sqrt{s} \approx m_\mathcal{T}$ at various facilities. The last column lists the expected signal statistical significance.

Colliding system, $\sqrt{s}$ ($\delta_{\sqrt{s}}$ spread), $\mathcal{L}_{\rm int}$, experiment	σ			N			$S/\sqrt{B}$
	$\mathcal{T}_1$	$\tau^+\tau^-$	$\mu^+\mu^-$	$\mathcal{T}_1$	$\mathcal{T}_1 \to \mu^+\mu^-$	$\mu^+\mu^-$	
e^+e^- at 3.5538 GeV (1.47 MeV), 5.57 pb^{-1}, BES III	1.9 pb	117 pb	6.88 nb	10.4	2.1	38 300	0.01σ
e^+e^- at $\sqrt{s} \approx m_\mathcal{T}$ (1.24 MeV), 140 pb^{-1}, BES III	2.2 pb	103 pb	6.88 nb	310	63	$9.63 \cdot 10^5$	0.06σ
e^+e^- at $\sqrt{s} \approx m_\mathcal{T}$ (1 MeV), 1 ab^{-1}, STCF	2.6 pb	95 pb	6.88 nb	$2.6 \cdot 10^6$	$5.3 \cdot 10^5$	$6.88 \cdot 10^9$	6.4σ
e^+e^- at $\sqrt{s} \approx m_\mathcal{T}$ (100 keV), 0.1 ab^{-1}, STCF	22 pb	46 pb	6.88 nb	$2.2 \cdot 10^6$	$4.5 \cdot 10^5$	$6.88 \cdot 10^8$	17σ

1. $S/\sqrt{B}$ is 6.4 σ (17 σ) with 1 ab^{-1} data and $\delta_W = 1(0.1)$ MeV.
2. With monochromatized beams can also provide a very precise extraction of the tau lepton mass with at least $\mathcal{O}$(25 keV) uncertainty.

Monochromatization @ $e^+e^- \to H$ @ FCC-ee, EPJP 137 (2022) 1, 31

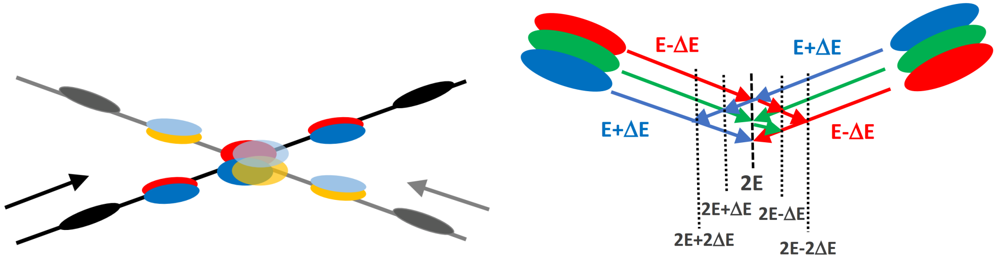


Fig. 1. FCC-ee monochromatization scheme featuring interaction-point dispersion of opposite sign for the two colliding beams, with (left) or without crab crossing and integrated resonance scan (right). Different colours schematically indicate bunch portions with slightly different energies.

Recent progress: NNNLO

1. AMFlow: 2201.11669, 2201.11636, 2201.11637
2. $e^+e^- \to t\bar{t}$ at NNNLO in QCD, 2209.14259
3. $\Upsilon \to e^+e^-$, decay constant of B_c, 2207.14259, 2208.04302

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τ

Need more precise measurements m_τ, Γ_τ, $(g-2)_\tau$ in PDG 2022

τ

$J = \frac{1}{2}$

Mass $m = 1776.86 \pm 0.12$ MeV

$(m_{\tau^+} - m_{\tau^-})/m_{\text{average}} < 2.8 \times 10^{-4}$, CL = 90%

Mean life $\tau = (290.3 \pm 0.5) \times 10^{-15}$ s

$c\tau = 87.03\ \mu$m

Magnetic moment anomaly > -0.052 and < 0.013, CL = 95%

$\text{Re}(d_\tau) = -0.220$ to 0.45×10^{-16} $e$cm, CL = 95%

$\text{Im}(d_\tau) = -0.250$ to 0.0080×10^{-16} $e$cm, CL = 95%

m_τ and lepton universality, 1405.1076

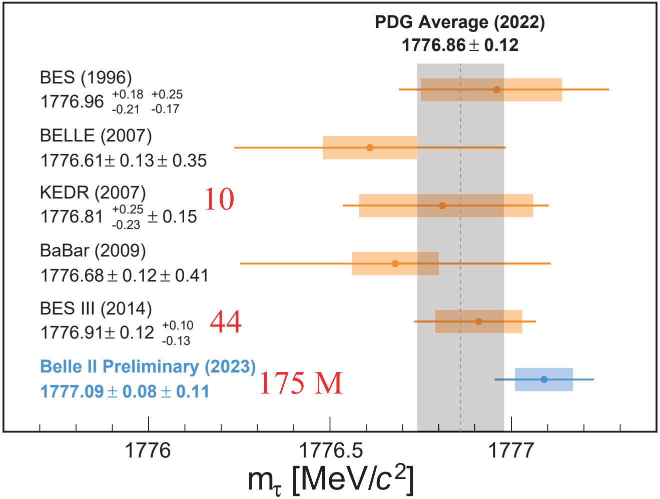
- Comparing the electronic branching fractions of τ and μ, lepton universality can be tested as

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{B(\tau \to e\nu\bar{\nu})}{B(\mu \to e\nu\bar{\nu})}(1+F_W)(1+F_\gamma), \quad (1)$$

- BESIII measurement, 1405.1076

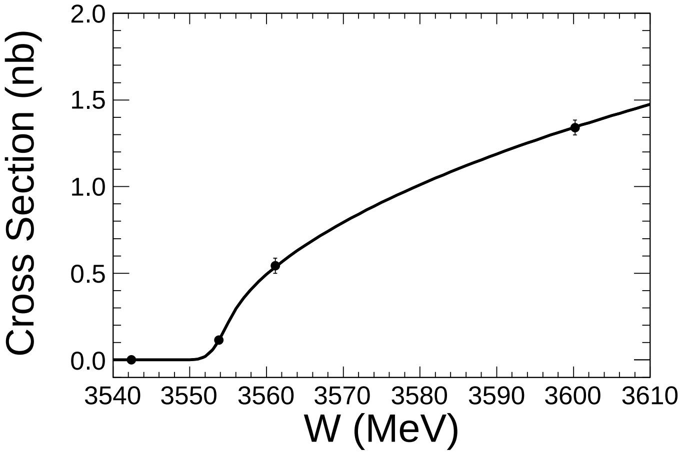
$$\left(\frac{g_\tau}{g_\mu}\right)^2 = 1.0016 \pm 0.0042, \quad (2)$$

Measured m_τ, 175 M enents with 190 fb^{-1}, Belle II 2305.19116



m_τ measurement at BESIII, 1405.1076

Scan	E_{CM} (MeV)	$\mathcal{L}$(nb^{-1})
J/ψ	3088.7	78.5 ± 1.9
	3095.3	219.3 ± 3.1
	3096.7	243.1 ± 3.3
	3097.6	206.5 ± 3.1
	3098.3	223.5 ± 3.2
	3098.8	216.9 ± 3.1
	3103.9	317.3 ± 3.8
τ	3542.4	4252.1 ± 18.9
	3553.8	5566.7 ± 22.8
	3561.1	3889.2 ± 17.9
	3600.2	9553.0 ± 33.8
ψ'	3675.9	787.0 ± 7.2
	3683.7	823.1 ± 7.4
	3685.1	832.4 ± 7.5
	3686.3	1184.3 ± 9.1
	3687.6	1660.7 ± 11.0
	3688.8	767.7 ± 7.2
	3693.5	1470.8 ± 10.3



m_τ measurement at BESIII, 1405.1076

final state	1		2		3		4		total	
	Data	MC	Data	MC	Data	MC	Data	MC	Data	MC
ee	0	0	4	3.7	13	12.2	84	76.1	101	92.0
$e\mu$	0	0	8	9.1	35	31.4	168	192.6	211	233.1
$e\pi$	0	0	8	8.6	33	29.7	202	184.4	243	222.6
eK	0	0	0	0.5	2	1.8	16	16.9	18	19.3
$\mu\mu$	0	0	2	2.9	8	9.2	49	56.3	59	68.4
$\mu\pi$	0	0	4	3.9	11	14.1	89	86.7	104	104.7
μK	0	0	0	0.2	3	0.8	7	9.0	10	10.1
$\pi\pi$	0	0	1	2.0	5	7.7	57	54.0	63	63.8
πK	0	0	1	0.3	0	0.8	10	8.2	11	9.3
KK	0	0	0	0.0	1	0.1	1	0.3	2	0.4
$e\rho$	0	0	3	6.1	19	20.6	142	132.0	164	158.7
$\mu\rho$	0	0	8	3.3	8	11.8	52	63.3	68	78.5
$\pi\rho$	0	0	5	3.4	15	10.8	97	96.0	117	110.2
Total	0	0	44	44.2	153	151.2	974	975.7	1171	1171.0

New data taking scenario at BESIII, from Zhang Jianyong TAU2018

Data comparison

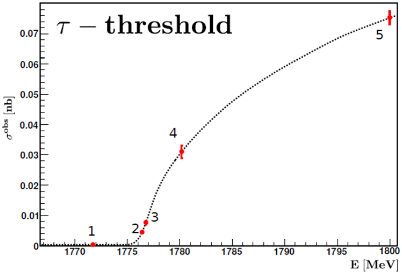
	J/ψ (pb^{-1})	ψ' (pb^{-1})	τ (pb^{-1})				
			3540 MeV	3553 MeV	3554 MeV	3560 MeV	3600 MeV
2011	1.5	7.5	4.3	0	5.6	3.9	9.6
2018	32.6	67.2	25.5	42.6	27.1	8.3	13.9

Statistical uncertainty < 45 keV, systematical uncertainty 90 keV, 1812.10056

Three energy regions:

- **Low energy region**
  Point 1, 14 pb^{-1}, to determine background
- **Near threshold**
  Point 2, 39 pb^{-1} and point 3, 26 pb^{-1}, to determine tau mass
- **High energy region**
  Point 4, 7 pb^{-1} for X^2 check
  Point 5, 14 pb^{-1} to determine detection efficiency

Total lum. ~100pb^{-1},
uncertainty: 0.1MeV



We obtain more than 130 pb^{-1} tau scan data!

2018/09/24-28 Zhang Jianyong 16

1 Introduction

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4 Summary

$e^+e^- \to \tau^+\tau^- \to \nu X^- \bar{\nu} X^+$ around the $\tau^+\tau^-$ production threshold

1. Updated cross sections

$$\sigma_{ex}(W, m_\tau, \Gamma_\tau, \delta_w) = \int_{m(J_\tau)}^{\infty} dW' \frac{e^{-\frac{(W-W')^2}{2\delta_w^2}}}{\sqrt{2\pi}\delta_w} \int_0^{1-\frac{m(J_\tau)^2}{W'^2}} dx F(x, W') \frac{\bar{\sigma}(W'\sqrt{1-x}, m_\tau, \Gamma_\tau)}{|1-\Pi(W'\sqrt{1-x})|^2}.$$

2. Cross sections in BESIII, 1405.1076

$$\sigma(E_{\rm CM}, m_\tau, \delta_w^{\rm BEMS}) = \frac{1}{\sqrt{2\pi}\delta_w^{\rm BEMS}} \int_{2m_\tau}^{\infty} dE'_{\rm CM} e^{\frac{-(E_{\rm CM}-E'_{\rm CM})^2}{2(\delta_w^{\rm BEMS})^2}} \int_0^{1-\frac{4m^2}{E'^2_{\rm CM}}} dx F(x, E'_{\rm CM}) \frac{\sigma_1(E'_{\rm CM}\sqrt{1-x}, m_\tau)}{|1-\prod(E_{\rm CM})|^2}$$

3. Difference: shift $2m_\tau$ to $m(J_\tau)$ in the range of integration and add Γ_τ as a variable of the cross sections after including $J_\tau(nS)$ atom.

$\bar{\sigma}(W, m_\tau, \Gamma_\tau)$, orthogonal perfect normalized basis, 1312.4791

1. $\bar{\sigma}(W, m_\tau, \Gamma_\tau)$

$$\bar{\sigma}(W, m_\tau, \Gamma_\tau) = \frac{4\pi\alpha^2}{3W^2}\frac{24\pi}{W^2}\mathrm{Im}\left[G_{\bar{\nu}X^+\nu X^-}(0, 0, W - 2m_\tau)\right], \quad (3)$$

2. $G_{\bar{\nu}X^+\nu X^-}(\vec{r}, \vec{r}\,', E)$ represents a Green function of $\tau^+\tau^-$ currents in the non-relativistic effective theory, where $\tau^+\tau^-$ decay to $\bar{\nu}X^+\nu X^-$

$$G_{\bar{\nu}X^+\nu X^-}(\vec{r}, \vec{r}\,', E) = \sum_n \frac{\psi_n(\vec{r})\psi_n^*(\vec{r}\,')}{E_n - E - i\epsilon} Br[n \to \bar{\nu}X^+\nu X^-] + \int \frac{d^3\vec{k}}{2\pi^3}\frac{\psi_{\vec{k}}(\vec{r})\psi_{\vec{k}}^*(\vec{r}\,')}{E_{\vec{k}} - E - i\epsilon}, \quad (4)$$

3. Then

$$\bar{\sigma}(W) = \bar{\sigma}^{J_\tau}(W) + \bar{\sigma}(W)_{con.} \quad (5)$$

Breit-Wigner formula

1. Green function approach to bound states is consistent with Breit-Wigner formula for a narrow bound states

$$\bar{\sigma}^{J_\tau}(W) = \sum_n \frac{6\pi^2}{W^2}\delta(W - m(J_\tau(nS)))\Gamma(J_\tau(nS) \to e^+e^-)Br(J_\tau(nS) \to \bar{\nu}X^+\nu X^-) \quad (6)$$

2. Ignore the binding Energy of $J_\tau(nS)$ for it much less than δ_w

$$\bar{\sigma}^{J_\tau}(W) = \frac{6\pi^2}{W^2}\delta(W - 2m_\tau)\sum_n \Gamma(J_\tau(nS) \to e^+e^-)Br(J_\tau(nS) \to \bar{\nu}X^+\nu X^-) \quad (7)$$

Decay mode of $J_\tau(nS)$

$$
\begin{aligned}
\Gamma_{total}(J_\tau(nS)) &= \Gamma_{Ani}(J_\tau(nS)) + \Gamma_{Weak}(J_\tau(nS)) + \Gamma_{E1}(J_\tau(nS)) \\
\Gamma_{Ani}(J_\tau(nS)) &= (2+R)\Gamma(J_\tau(nS) \to e^+e^-) \\
\Gamma_{Weak}(J_\tau(nS)) &= 2\Gamma(\tau \to \nu X^-)
\end{aligned} \tag{8}
$$

Parameters

1. Parameters

$$m_\tau = m_\tau^{\mathrm{PDG}} = 1776.86\,\mathrm{MeV}, \quad R = 2.342 \pm 0.0645,$$
$$\Gamma_\tau = 2.2674 \pm 0.0039\,\mathrm{meV}, \qquad \delta_W = 1\ \mathrm{MeV},$$
$$\varepsilon_{X^+Y^-\not{E}} = (8 \pm 0.2)\%, \quad \varepsilon_{\mu^+\mu^-} = 45\%,$$
$$\alpha(0) = 1/137.036, \qquad \Delta\alpha_{had}(m_{J_\tau}) = (74 \pm 7) \times 10^{-4}. \tag{9}$$

2. The resulting NLO expression for $\bar{\sigma}^{J_\tau}(W)$ is given by

$$\bar{\sigma}^{J_\tau}(W) = (3.11 \pm 0.02)\,\delta\left(\frac{W - 2m_\tau + 13.8\ \mathrm{keV}}{1\ \mathrm{MeV}}\right)\ \mathrm{pb}, \tag{10}$$

where $13.8\ \mathrm{keV} = \sum_n B_n Br^{J_\tau(nS)}_{X^+Y^-\not{E}} \Gamma^{J_\tau(nS)}_{e^+e^-} / \sum_n Br^{J_\tau(nS)}_{X^+Y^-\not{E}} \Gamma^{J_\tau(nS)}_{e^+e^-}$.

Decay mode of $J_\tau(nS)$

TABLE II: The decay data of $J_\tau(nS)$ in meV.

n	$\Gamma^{J_\tau(nS)}_{e^+e^-}$	$2\Gamma_\tau$	$\Gamma^{J_\tau(nS)}_{E1}$	$\Gamma^{J_\tau(nS)}_{\text{total}}$	$\Gamma^{J_\tau(nS)}_{e^+e^-} Br^{J_\tau(nS)}_{X^+Y^-\not{E}}$
1	6.484	4.535	0.0000	32.695	0.899
2	0.808	4.535	0.0000	8.044	0.455
3	0.239	4.535	0.0072	5.573	0.195
$\sum_{n=1}^{\infty}$					1.795 ± 0.012

Cross sections from $J_\tau(nS)$

1. Then we get the $J_\tau(nS)$ contribution the cross section

$$\bar{\sigma}^{J_\tau}(W) = (3.11 \pm 0.02)\,\delta(W - 2m_\tau)\ \mathrm{pb\ MeV} \tag{11}$$

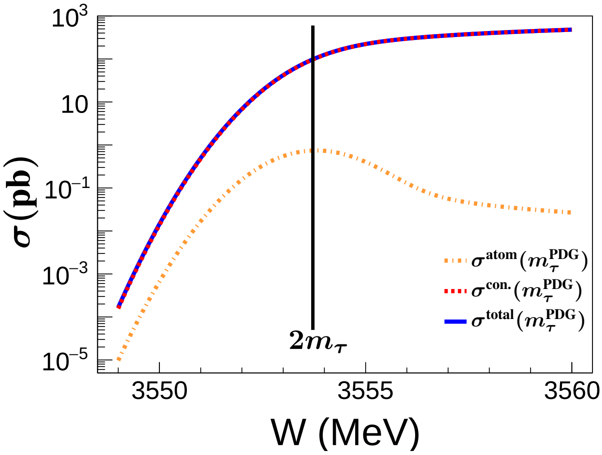
2. Updated $\bar{\sigma}(W, m_\tau, \Gamma_\tau)$

$$\bar{\sigma}(W) = (3.11 \pm 0.02)\delta\left(\frac{W - 2m_\tau + 13.8\mathrm{keV}}{\mathrm{MeV}}\right)\ \mathrm{pb} + \theta(W - 2m_\tau)\bar{\sigma}_{con.}(W) \tag{12}$$

3. Continue $\bar{\sigma}_{con.}(2m_\tau)$

$$\bar{\sigma}_{Continue}(2m_\tau) = 236\ \mathrm{pb} \tag{13}$$

Cross sections from $J_\tau(nS)$



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Reduce the uncertainties

1. The measured cross secitons

$$\sigma^{X^+Y^-\not{E}}(W) = \frac{N^{X^+Y^-\not{E}}(W)}{\mathcal{L}\varepsilon} \tag{14}$$

2. Uncertaintiy of ISR($\sim 0.5\%$ @ BESIII), the vacuum polarization factor (0.14%), and the integrated luminosity ($\sim 0.5\%$@ BESIII) are all larger than 0.1%.
3. Systematical uncertainty of cross section measurement at STCF must $> 0.2\%$.
4. The significance of 5σ require $S/\sqrt{(\Delta_{stat.}(B+S))^2 + (\Delta_{syst.}(B+S))^2} > 5$.
5. Ignore statistical uncertainty, significance of 5σ require $S/B > 1\%$ at STCF.

Uncertainty of J/ψ decay: 10 B events

$J/\psi(1S)$ $I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 3096.900 \pm 0.006$ MeV

Full width $\Gamma = 92.6 \pm 1.7$ keV (S = 1.1)

$J/\psi(1S)$ DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
hadrons		(87.7 ± 0.5) %		–
virtual $\gamma \to$ hadrons		(13.50 ± 0.30) %		–
ggg		(64.1 ± 1.0) %		–
γgg		(8.8 ± 1.1) %		–
e^+e^-		(5.971 ± 0.032) %		1548
$e^+e^-\gamma$	[hhaa]	$(8.8 \pm 1.4) \times 10^{-3}$		1548
$\mu^+\mu^-$		(5.961 ± 0.033) %		1545

Reduce the uncertainties

1. We introduce $R_{X^+Y^-\not{E}}$, ratio of the cross sections, as

$$R_{X^+Y^-\not{E}}(W,\delta_W,m_\tau)=\frac{\sigma(W,m_\tau,\Gamma_\tau,\delta_W)}{\sigma^{\mu^+\mu^-}(W,\delta_W)}. \quad (15)$$

   Here, $\sigma^{\mu^+\mu^-}(W,\delta_W)$ is calculated with $\bar{\sigma}^{\mu^+\mu^-}(W)=\frac{4\pi\alpha^2(1+3\alpha/4\pi)}{3W^2}$.

2. The measurement is

$$\mathcal{R}_{X^+Y^-\not{E}}(W,\delta_W,m_\tau)=\frac{N_{X^+Y^-\not{E}}}{N_{\mu^+\mu^-}}. \quad (16)$$

Fit approach

1. A least-square fit is applied

$$\chi^2 = \sum_{i=1} \left(\frac{\mathcal{R}_i^{\text{data}} - \hat{\mathcal{R}}_i(m_\tau)}{\Delta \mathcal{R}_i^{\text{data}}} \right)^2 . \tag{17}$$

2. $\hat{\mathcal{R}}_i(m_\tau)$ is the theoretical fit function with J_τ. The expected m_τ can be determined from the minimum value of χ^2.
3. To quantify the significance of the J_τ, another fit is performed by excluding the $\bar{\sigma}^{J_\tau}$ in $\hat{\mathcal{R}}_i$. This leads to a new minimum value $\chi^2_{\text{without } J_\tau}$ at a new τ mass.
4. The significance of the J_τ atom can be calculated from $\Delta\chi^2_{J_\tau} = \chi^2_{\text{without } J_\tau} - \chi^2$.

Determine energy points

1. A least-square fit is applied

$$\chi^2 = \sum_{i=1}^{3} \chi_i^2 = \sum_{i=1}^{3} \left(\frac{\mathcal{R}_i^{\mathrm{data}} - \hat{\mathcal{R}}_i(m_\tau)}{\Delta \mathcal{R}_i^{\mathrm{data}}} \right)^2 , \tag{18}$$

2. Where $\mathcal{R}_i^{\mathrm{data}} = \frac{N^{\mathrm{data}}_{X^+Y^-\not{E},\, i}}{N^{\mathrm{data}}_{\mu^+\mu^-,\, i}}$ and $\Delta\mathcal{R}_i^{\mathrm{data}}$ is its statistical uncertainty (the systematic uncertainty is discussed below).
3. The values of $\frac{\chi_i^2}{\mathcal{L}_i}$ are relatively large at $W = 3552.56$ and 3555.83 MeV.
4. An additional energy point of 3549.00 MeV is needed to obtain the whole lineshape of the $e^+e^- \to X^+Y^-\not{E}$ cross section.

Numbers of the events

TABLE III: Numbers of $e^+e^- \to X^+Y^-\not{E}$ and $\mu^+\mu^-$ events and their statistical uncertainties in the pseudoexperiments with $m_\tau = m_\tau^{\rm PDG}$.

i	$\mathcal{L}_i/\mathrm{fb}^{-1}$	W_i/MeV	$N^{\rm data}_{X^+Y^-\not{E},\,i}$	$N^{\rm data}_{\mu^+\mu^-,\,i}$
1	5	3549.00	$0.1^{+1.2}_{-0.1}$	$(1.1764 \pm 0.0003) \times 10^7$
2	500	3552.56	$(8.772 \pm 0.009) \times 10^5$	$(1.17394 \pm 0.00003) \times 10^9$
3	1000	3555.83	$(2.4052 \pm 0.0005) \times 10^7$	$(2.34331 \pm 0.00005) \times 10^9$

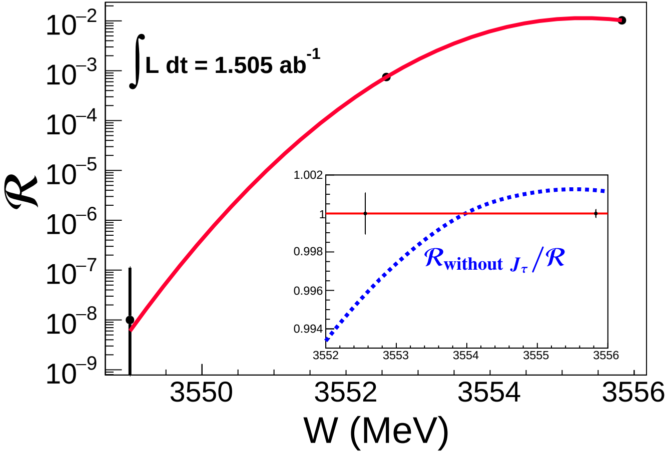
Determine χ^2 and m_τ

1. A least-square fit is applied

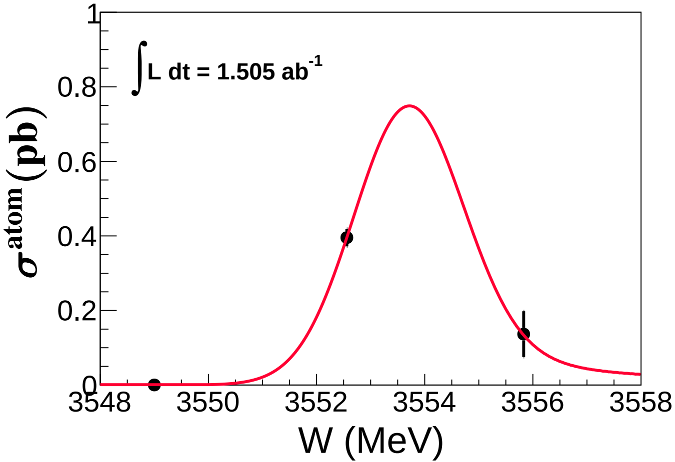
$$\chi^2 = \sum_{i=1}^{3} \left(\frac{\mathcal{R}_i^{\rm data} - \hat{\mathcal{R}}_i(m_\tau)}{\Delta \mathcal{R}_i^{\rm data}} \right)^2 , \tag{19}$$

2. Where $\mathcal{R}_i^{\rm data} = \frac{N^{\rm data}_{X^+Y^- \not{E},\, i}}{N^{\rm data}_{\mu^+\mu^-,\, i}}$ and $\Delta \mathcal{R}_i^{\rm data}$ is its statistical uncertainty.
3. And $\hat{\mathcal{R}}_i(m_\tau)$ is the expected ratio at the τ mass m_τ to be determined from the fit.

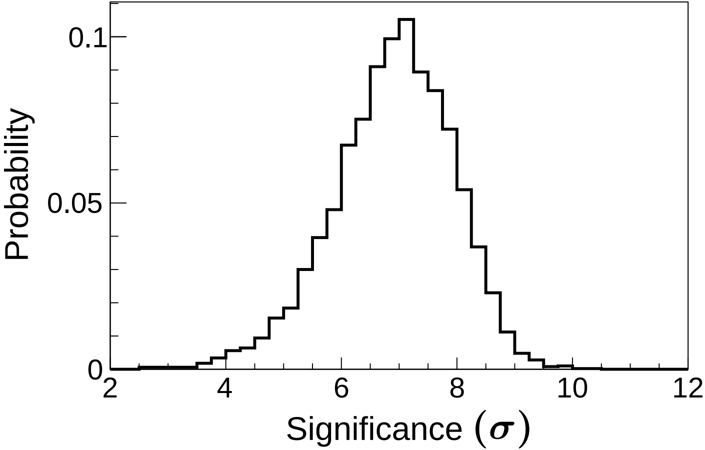
Ratio of the events



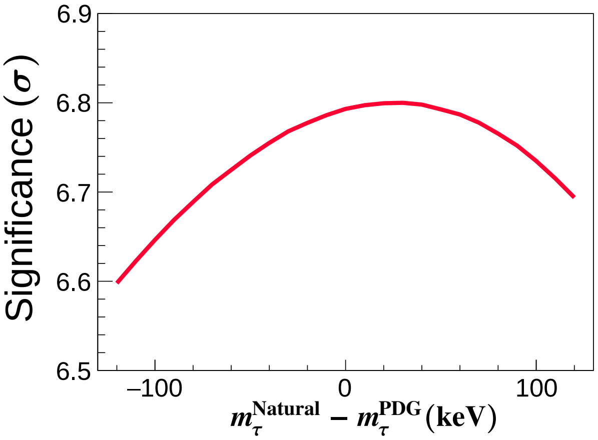
The cross section of J_τ



The statistical significance distribution in 10^5 sets pseudoexperiments



The significance of $J_\tau(nS)$ as a function of $m_\tau^{\mathrm{Natural}} - m_\tau^{\mathrm{PDG}}$.



The significance of $J_\tau(nS)$ in 10^5 sets pseudoexperiments

1. The average value of χ^2/ndf is 0.7/2 with $J_\tau(nS)$, and 51/2 without $J_\tau(nS)$.
2. Taking into account the systematic uncertainties, the average signal significance of J_τ is 6.7σ, which is 6.8σ without systematic uncertainties.
3. We conclude that in the scenario of taking 5 fb^{-1} data at 3549.00 MeV, 500 fb^{-1} at 3552.56 MeV, and 1000 fb^{-1} at 3555.83 MeV, we have a 96% chance of discovering the $J_\tau(nS)$ with a statistical significance larger than 5σ and an almost 100% chance of observing it with a significance larger than 3σ.
4. These data samples correspond to 350 (175) days' runtime at the STCF(SCT).
5. If the δ_W is reduced to $0.1\mathrm{MeV}$, the required integrated luminosity is only 66 fb^{-1}.

m_τ

1. With these data samples, a high precision τ mass is obtained

$$m_\tau = (1\,776\,860.00 \pm 0.25\,(\mathrm{stat.}) \pm 0.99\,(\mathrm{syst.}))\ \mathrm{keV}.$$

2. The fit with the $J_\tau(nS)$ contribution removed gives a shift of -4 keV relative to the nominal fit with both the bound state and continuum contributions.

The systematic uncertainties σ_{m_τ}

1. The uncertainty of the energy scale W is estimated according to the VEPP-4M, which had a characteristic uncertainty of 1.5 keV in the beam energy in the $\psi(2S)$ mass scan (hep-ex/0306050). The uncertainty of W_2 (W_3) is estimated to be $1.5\sqrt{2} = 2.12$ keV, leading to 0.72 (0.35) keV in σ_{m_τ}.
2. σ_{m_τ} from energy spread and energy scale are 16 keV and $^{+22}_{-86}$ keV from BESIII (1405.1076), and 25 keV and 40 keV from KEDR (JETP Lett. 85 (2007) 347-352). Take the maximum ratio of $16/22 \sim 0.73$, leading to $0.73 \times \sqrt{0.72^2 + 0.35^2}$ =0.59 keV in σ_{m_τ}.
3. $\varepsilon_{X^+Y^-\not{E}} = (8.0 \pm 0.2)\%$ lead to 0.04 keV in σ_{m_τ}.
4. By exchanging the NLO correction with the NNLO correction in the calculation of the $e^+e^- \to X^+Y^-\not{E}$ cross sections, which is included in 0.07 keV in σ_{m_τ} due to the theoretical accuracy.

The systematic uncertainties σ_{m_τ}

TABLE IV: The systematic uncertainties of the m_τ (σ_{m_τ}) in keV.

Sources	σ_{m_τ}/keV
Energy scale of W_2	0.72
Energy scale of W_3	0.35
Energy spread δ_W	0.59
Efficiency	0.04
Theory	0.07
Systematic uncertainties	0.99

The systematic uncertainties σ_{m_τ}, this work VS BESIII 1405.1076

Sources	σ_{m_τ}/keV
Energy scale of W_2	0.72
Energy scale of W_3	0.35
Energy spread δ_W	0.59
Efficiency	0.04
Theory	0.07
Systematic uncertainties	0.99

Source	Δm_τ (MeV/c^2)
Theoretical accuracy	0.010
Energy scale	$^{+0.022}_{-0.086}$
Energy spread	0.016
Luminosity	0.006
Cut on number of good photons	0.002
Cuts on PTEM and acoplanarity angle	0.05
mis-ID efficiency	0.048
Background shape	0.04
Fitted efficiency parameter	$^{+0.038}_{-0.034}$
Total	$^{+0.094}_{-0.124}$

1 Introduction

2 The frame of Calculation

3 Reduce the uncertainties

4 Summary

Summary

1. We show that the $\tau^+\tau^-$ atom can be observed with a significance larger than 5σ with a $1.5\ \mathrm{ab}^{-1}$ data sample at STCF or SCTF, by measuring the cross section ratio of the processes $e^+e^- \to X^+Y^-\not{E}$ and $e^+e^- \to \mu^+\mu^-$.
2. With the same data sample, the τ lepton mass can be measured with a precision of 1 keV, a factor of 100 improvement over the existing world best measurements.
3. We propose to measure the relative rate $\mathcal{R} = \frac{N_{X^+Y^-\not{E}}}{N_{\mu^+\mu^-}}$ rather than the absolute cross section so that the uncertainties are controlled at a low level since those in VP, ISR, and luminosity determinations are canceled.