

**第九届"⼿征有效场论研讨会" 2024年10月18-22日,长沙**

# 手征核力与高密核物质 **的状态方程**



# **南开大学物理科学学院**

**11/20/13 19/10/2024 Jinniu Hu <sup>1</sup>**

Rutherford scattering NN scattering

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J. Bystricky, F. Lehar, and P. Winternitz, J. Phys. 39(1978)1.  $\mathcal{I}$  the context of  $\mathcal{I}$  one particle (the beam), one particle (the beam), one particle (the beam),  $\mathcal{I}$ 

**03/04/2024 Jinniu Hu** with kinetic energy *E*lab, is incident on a stationary particle  $\bullet$  the rest frame of the initial proton. In the initial problem of the initial problem of the initial pr

where *d*σ*/d*( is the (unpolarized) differential cross section,  $J$ *inniu* Hu

# **NN scattering data**

**R. Navarro Pérez, J. E. Amaro, and E. Ruiz Arriola, Phys. Rev. C 89(2014)064006** 

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## ➢**N2LO chiral potential**

 **S. Weinberg, Nuclear Phys. B 363 (1991) 3**

- **C. Ordonez, L. Ray, U. van Kolck, Phys. Rev. Lett. 72 (1994) 1982**
- **C. Ordonez, L. Ray, U. van Kolck, Phys. Rev. C 53 (1996) 2086**

**J.X. Lu, C.X. Wang, Y. Xiao, L.S. Geng, J. Meng, and P. Ring, Phys. Rev. Lett. 128 (2022) 142002 (Relativistic version)** 

### ➢**N3LO chiral potential**

 **D. R. Entem, R. Machleidt, Phys. Rev. C 66 (2002) 014002**

 **E. Epelbaum, W. Gloeckle, U.-G. Meissner, Nucl. Phys. A 747 (2005) 362** 

# ➢**N4LO chiral potential and N5LO (partially) D. R. Entem, N. Kaiser, R. Machleidt, and Y. Nosyk, Phys. Rev. C 91 (2015) 014002**

**E. Epelbaum, H. Krebs, U.-G. Meissner, Phys. Rev. Lett. 115 (2015) 122301 (EKM) D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004 (EMN) P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54 (2018) 86 (RKE) S. K. Saha, D. R. Entem, R. Machleidt, and Y. Nosy, Phys. Rev. C107(2023)034002**

# **Chiral NN interaction 网络剧看剧大学**





**19/10/2024 Jinniu Hu** refer to the vertices of dimension ∆*<sup>i</sup>* = 0, ∆*<sup>i</sup>* = 1, ∆*<sup>i</sup>* = 2, ∆*<sup>i</sup>* = 3 and ∆*<sup>i</sup>* = 4, respectively.

Jinniu Hu — A similar program is being pursued for in chiral EFT with explicit Δ(1232) DOF

#### **Nuclear force in Chiral EFT Nu Force in Chiral EFT** 《翻》 新 2



The pion-exchange part of the NN potential The NN potential with contac<u>t</u> term D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004  $V_{\text{LO}} \equiv V^{(0)} = V_{1\pi}^{(0)}$  $\frac{1}{1\pi}$ ,  $V_{\rm NLO} \equiv V^{(2)} = V_{\rm LO} + V_{1\pi}^{(2)} + V_{2\pi}^{(2)}$ ,  $\frac{1}{2\pi}$ ,  $V_{\text{NNLO}} \equiv V^{(3)} = V_{\text{NLO}} + V_{1\pi}^{(3)} + V_{2\pi}^{(3)}$ ,  $V_{\text{N}^3\text{LO}} \equiv V^{(4)} = V_{\text{NNLO}} + V_{1\pi}^{(4)} + V_{2\pi}^{(4)} + V_{3\pi}^{(4)}$  $V_{\rm N^4LO} \equiv V^{(5)} = V_{\rm N^3LO} + V_{1\pi}^{(5)} + V_{2\pi}^{(5)} + V_{3\pi}^{(5)}$  $V_{\text{LO}} \equiv V^{(0)} = V_{1\pi} + V_{\text{ct}}^{(0)}$ ,  $V_{\text{NLO}} \equiv V^{(2)} = V_{\text{LO}} + V_{2\pi}^{(2)} + V_{\text{ct}}^{(2)}$ ,  $V_{\rm NNLO} \equiv V^{(3)} = V_{\rm NLO} + V^{(3)}_{2\pi}$ ,  $\frac{1}{\sqrt{t}}$  range-independent 1PE is  $\frac{1}{\sqrt{t}}$  is  $\frac{1}{\sqrt{t}}$  is  $\frac{1}{\sqrt{t}}$  is  $\frac{1}{\sqrt{t}}$  $V_{\rm N^3LO} \equiv V^{(4)} = V_{\rm NNLO} + V_{2\pi}^{(4)} + V_{3\pi}^{(4)} + V_{\rm ct}^{(4)}$  $V_{\rm N^4LO} \equiv V^{(5)} = V_{\rm N^3}$  $N_{2i}^3$ LO +  $V_{2i}^3$ *f* 2  $V_{\rm N^4LO} \equiv V^{(5)} = V_{\rm N^3LO} + V_{2\pi}^{(5)} + V_{3\pi}^{(5)}$ , *q*<sub>2</sub> + *m*<sup>2</sup> π WE WOO NOT = (σ "  $\overline{\phantom{a}}$ and *W*<sup>α</sup> (α = *C,S,LS,T* ) can be expressed as functions of  $\frac{1\pi}{2}$  comprehensive discussion of  $\frac{1\pi}{2}$ **F. The full potential**  $T_n$   $\sum_{i=1}^{n}$   $\sum_{i=1}$  $V_{\rm N^4LO} \equiv V^{(3)} = V_{\rm N^3LO} + V_{1\pi}^{(3)} +$  $\frac{1}{2\pi}$ ,

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*p*2

where we left out the higher order corrections to the 1PE

# Parameters in in chiral EFT **《翻》有图大**.



#### shift analysis and by the high-precision CD-Bonn potential *versus* the total number of D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004



# Nuclear force in chiral EFT



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 $\triangle V$ 

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E. Epelbaum, H. Krebs, U.-G. Meissner, Phys. Rev. Lett. 115 (2015) 122301 (EKM)

19/10/2024 **Jinniu Hu** fully local potentials up to N2LO **[Gezerlis et al. '14]**; minimally nonlocal N3LO potential including

results at LO, NLO, N<sup>2</sup>LO, N<sup>3</sup>LO and N<sup>4</sup>LO, respectively, calculated using the cuto↵ *R* = 0*.*9 fm. Only those partial wave are 1st generation χ N3LO forces (nonlocal) **[Epelbaum-Glöckle-Meißner '04, Entem-Machleidt '03]**

#### **Nuclear force in chiral EFT** with the fact that  $\mathbf{r}$  is a low-momentum expansion which  $\mathbf{r}$  is a low-momentum expansion which which which which  $\mathbf{r}$ **indicipal force in chiral CF!**  $\overline{r}$



 $\sum_{k=1}^{\infty}$ 

 $\overline{\text{u}_i}$  is much the regulator function  $\overline{\text{u}_i}$ 

#) *f* (*p*!

"(*p* 1 *,p*

#) '−→ *V*

 $\mathbb{C}^n$  is during the past  $\mathbb{Z}$ 

*,p*) (2.42)

# **Nuclear force in chiral EFT**



P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54 (2018) 86 (RKE) FIG. 8: (Color online) Chiral expansion of the np phase shifts in comparison with the Nijmegen [20] (solid dots) and the GWU

in contemporary many-body nuclear methods), only Weinberg

 $\mathbb{R}$  has been used the past 21 years of the past 25 years of the past

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**19/10/2024 Jinniu Hu** dotted and violet long-dashed-dotted lines show the results at LO, NLO, N<sup>2</sup>LO, N<sup>3</sup>LO and N<sup>4</sup>LO, respectively, calculated using

10

5

**Jinniu Hu** [100] (open triangles) np partial wave analysis. Black dotted, orange dashed, green short-dashed-dotted, blue dashed-double-

# **Nuclear force in chiral EFT**



#### **Description of the np and pp phase shifts Description of the nation of the new pp process chiral order** 0 200 MeV 1780 90 37 2*.*00 1*.*42 0*.*95 escription of the np and pp phase shifts **the same is a set of the set of the s**



**P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88**

#### **Deuteron properties a** :  $\frac{1}{1}$ **2 LECs + 7 + 1 IB LECs + 12 LECs + 1 LEC (np) + 4 LEC** calculated without taking into account meson-exchange current contributions and relativistic corrections. The deuteron binding energy is calculated by solving the relativistic Schr¨odinger equation.

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 $\frac{1}{\sqrt{2}}$ l, I H. Krebs, and E. Epelt<br>———————————————————— *M*<sup>2</sup> ⇡ ) 1 +  $\ddot{\phantom{0}}$ I Phys. J. A 54 (2018) 86 (I *M*<sup>2</sup> I P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54 (2018) 86 (RKE)

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## **Total cross sections**



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19/10/2024 Jinniu Hu  $T = 10/1$ 

### **The many-body ab initio calculations**

➢ **Brueckner-Hartree-Fock method M. Hjorth-Jensen, T.T.S. Kuo, and E. Osens, Phys. Rep. 261(1995)125** ➢ **Relativistic Brueckner-Hartree-Fock method S. Shen, H. Liang, J. Meng, P. Ring, and S. Zhang, Phys. Rev. C 96(2017)014316**  ➢ **Self-consistent Green's function method W. H. Dickhoff and C. Barbieri, Prog. Part. Nucl. Phys. 52(2004)377**  ➢ **Many-body perturbation theory J.W. Holt and N. Kaiser, Phys. Rev. C, 95(2017)034326**  ➢ **In-medium Similarity Renormalization Group (IMSRG) H. Hergert, S.K. Bogner, T.D. Morris, A. Schwenk, K. Tsukiyama, Phys. Rep. 621(2016)165**  ➢ **No core shell model B. R. Barrett, P. Navraetil, and J. P. Vary, Prog. Part. Nucl. Phys. 69(2013)131**  ➢ **Lattice effective field theory D. Lee, Prog. Part. Nucl. Phys. 63(2009)117** ➢ **Quantum Monte Carlo methods** 

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 **J. Carlson, S. Gandolfi, F. Pederiva, Steven C. Pieper, R. Schiavilla, K. E. Schmidt, and R. B. Wiringa, Rev. Mod. Phys.87(2015)1067** 

➢ **Coupled Clusters** 

 **R. J. Bartlett and M. Musiał. Rev. Mod. Phys.79(2007)291**

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 **...**

# The BHF method the 《剛》有 31 -

In BHF theory of nuclear matter, the realistic *NNI* potential in the realistic *NNI* pote effective **NN** is a construction of the solution of the soluti energies becomes smaller and smaller by including higher perturbation order in chiral *NN* potential. It becomes saturation up  $\mathcal I$ **《編》右 31 e e e**<sub>(**a**)</sub> + *i* **j j** *j j*  $$ *<sup>V</sup>* <sup>|</sup>*kakb*!"*kakb*<sup>|</sup> ω − *e*(*ka*) − *e*(*kb*) + *i*# **G**<sub>*ω*</sub>, (1)

 $\overline{\phantom{a}}$ 

**Bethe-Goldstone equation**  og Goldstone equation **USTONE Equation** where  $V$  is a realistic  $\mathcal{N}$  is a realistic  $\mathcal{N}$  is a realistic  $\mathcal{N}$  of  $\mathcal{N}$  is a realistic  $\mathcal{N}$  is a realistic  $\mathcal{N}$ 

$$
G[\omega,\rho] = V + \sum_{k_a,k_b > k_F} V \frac{|k_a k_b\rangle\langle k_a k_b|}{\omega - e(k_a) - e(k_b) + i\epsilon} G[\omega,\rho],
$$

*ka*,*kb*>*kF*

**Single-particle energy**   $k$  energy

$$
e(k) = e(k; \rho) = \frac{k^2}{2m} + U(k, \rho)
$$

**Single-particle potential**   $\epsilon$  potential  $\epsilon$ *U*<br>United in the present BHF theory is used in the present BHF theory is used in the present BHF theory is used in

$$
U(k; \rho) = \Re \sum_{k' < k_F} \langle kk'|G[e(k) + e(k'); \rho]|kk'\rangle_a,
$$

**Energy per nucleon Energy per nucleon** Later, this state-of-the-art chiral *NN* potentials were applied on study of few-nucleon systems [28], such as the nucleonw ner nucleon

$$
\frac{E}{A} = \frac{3}{5} \frac{k_F^2}{2m} + \frac{1}{2\rho} \mathfrak{R} \sum_{k,k' < k_F} \langle kk' | G[e(k) + e(k'); \rho] | kk' \rangle_a.
$$

**JH, Y. Zhang, E. Epebaum, U.-G. Meissner, and J. Meng, Phys. Rev. C 96(2017)034307**   $\overline{a}$ trix elements. These equations are coupled together and solved  $JH$ ,  $Y$ , E. Epebaum, U.-G. Meissner, and J. Meng, Phys. Rev. C 96(2017)034

**19/10/2024 Jinniu Hu** isted, which should be described by the chiral *NN* potentials. Up to oxygen isotope, the properties of light nuclei were

where the subscript *a* indicates antisymmetrization of the matrix elements. These equations are controlled to the coupled to  $\mathsf{I}_\mathsf{S}$  and solved to  $\mathsf{I}_\mathsf{S}$  and solved to  $\mathsf{I}_\mathsf{S}$  are controlled to  $\mathsf{I}_\mathsf{S}$  and solved to  $\mathsf{I}_\mathsf{S}$  and solved to  $\mathsf{I}_\math$ plotted for chiral *NN* potentials from LO to N4LO with different coordinate-space cutoffs, *R* = 0.8, 0.9, 1.0, 1.1 and 1.2 fm. **The nuclear matter from chiral potentials** 

### The EOSs and symmetry energy from N<sup>4</sup>LO chiral potentials

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**19/10/2024 Jinniu Hu** symmetry energy *a*symm (lower row) based on chiral *NN* potentials of [35,36] for all available cutoff values in the range of *R* = 0*.*8–1*.*2 fm.

FIG. 1. Density dependence of the energy per particle of SNM (*E/A*)SNM (upper row), of PNM (*E/A*)PNM (middle row), and of the

**Coester Line**  $\overline{\phantom{a}}$ 

and yields results slightly less repulsive than the DBHF ones

 $\frac{1}{\sqrt{2}}$ 

 $\overline{\phantom{a}}$ 



### **The EOSs from different chiral potentials**



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order EFT coefficients renders explicit in the calculation

**An observable X in EFT can be expanded as**  heenv

$$
X=X_0\sum_{n=1}^{\infty}c_n Q^n,
$$

where  $X_0$  is the natural size of the observable X, and  $c_n$  are **dimensionless coefficients.**  where *X*<sup>0</sup> is the natural size of the observable *X*, and {*cn*}  $n=0$ are dimensionless coefficients, some of which may be zero.

**In nuclear matter, with suitable additional, with suitable additional, with suitable additional, with suitable a** In most EFTs the expansion (1) is inherited directly from the expansion (1) is inherited directly from the expansion  $\mathcal{L}_\mathcal{F}$ 

$$
Q=\frac{k_F}{\Lambda_b}
$$

**and**

$$
X_0 = (E_{\rm LO}/A)
$$

**J. A. Melendez, S. Wesolowski, and R. J. Furnstahl, Phys. Rev. C 96(2017)024003** molondoz, o: wooduwer

## **The Bayesian analysis**



**The truncated error at order k is** 

 $\Delta X = X_0 \Delta_k$ 

**where the scaled, dimensionless parameter is** 

$$
\Delta_k = \sum_{n=k+1}^{\infty} c_n Q^n
$$

**In Bayesian framework, one believes with (100\*p)% certainty the true value of the observable X lies within**   $\pm X_0 d_k^{(p)}$ 

of the (k+1)<sup>th</sup> order (N<sup>k</sup>LO), where d is related to the **degree-of-belief intervals p,** 

$$
p = \int_{-d_k^{(p)}}^{d_k^{(p)}} d\Delta \mathrm{pr}_h(\Delta | \mathbf{c_k})
$$

prh **is the posterior probability distribution functions** 



# The pairing gap in nuclear matter **《 3**





**We calculated the properties of nuclear matter with different state-of-the-art chiral NN potentials.** 

**The equations of state of nuclear matter from different chiral potentials have different behaviors due to regularized factor.** 

**The chiral truncation errors in nuclear matter were discussed with Bayesian analysis. The chiral potentials are good convergence at higher order.** 

**They pairing gaps at different spin channels are calculated with chiral force in symmetric matter.** 



### Time-ordered perturbation theory

$$
\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi \rangle \\ |\psi \rangle \end{pmatrix} = E \begin{pmatrix} |\phi \rangle \\ |\psi \rangle \end{pmatrix} \implies |\psi \rangle = \frac{1}{E - \lambda H \lambda} H |\phi \rangle
$$
  

$$
\implies (H_0 + V_{\text{eff}}^{\text{TD}}) |\phi \rangle = E |\phi \rangle
$$

**with effective potential**  with the control potential *V* TD is positive potential

$$
V_{\text{eff}}^{\text{TD}} = \eta H_I \eta + \eta H_I \lambda \frac{1}{E - \lambda H \lambda} \lambda H_I \eta
$$

and

$$
|\phi\rangle \equiv |N\rangle + |NN\rangle + |NNN\rangle + \dots
$$

$$
|\psi\rangle \equiv |N\pi\rangle + |N\pi\pi\rangle + \dots + |NN\pi\rangle + \dots
$$
  
Nuclear force

*V* TD  $\frac{H}{\sqrt{H}}$   $\lambda$   $\frac{H}{\sqrt{H}}$  +  $\frac{H}{\sqrt{H}}$   $\lambda$   $\frac{H}{\sqrt{H}}$  + *HI*⌘ + ⌘*H<sup>I</sup>*  $V_{\text{eff}}^{\text{TD}}=\eta H_I\eta+\eta H_I\frac{\lambda}{E-I}$  $E - H_0$  $H_I \eta + \eta H_I \frac{\lambda}{E - \lambda}$  $E - H_0$  $H_I\frac{\lambda}{E-1}$  $E - H_0$  $H_I\eta + \dots$  Nuclear force in chiral EFT

**Unitary transformation**   $\frac{1}{1919}$  (a)  $\frac{1}{1919}$  (b)  $\frac{1}{1919}$  (b)

$$
H = \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \Longrightarrow \tilde{H} \equiv U^{\dagger} H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}
$$

 $\overline{\mathbf{D}}$ 

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**and** 

$$
U = \begin{pmatrix} \eta (1 + A^\dagger A)^{-1/2} & - A^\dagger (1 + A^\dagger A)^{-1/2} \\ A (1 + A^\dagger A)^{-1/2} & \lambda (1 + A^\dagger A)^{-1/2} \end{pmatrix}
$$

### A should be solved by **A** should be solved by

$$
\lambda(H-[A,H]-AHA)\eta=0
$$

Nuclear force<br>  $V_{\text{eff}} = -\eta H_I \frac{\lambda}{E_{\pi}} H_I \eta - \eta H_I \frac{\lambda}{E_{\pi}} H_I \frac{\lambda}{E_{\pi}} H_I \eta + \frac{1}{E_{\pi}} \eta H_I \frac{\lambda}{E_{\pi}} H_I \eta H_I \frac{\lambda}{E_{\pi}^2} H_I \eta + \cdots$ 



A-nucleon interactions receives contributions  $\left(\bigcap A\right)$ <sup>V</sup> power of an involvement  $\mathsf{\small{I}}$ **Paractions receives contributions**<br> $(\bigcirc/\Lambda_{\mathsf{\tiny S}})^\nu$ D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004<br>In interactions receives contributions

 $(Q/\Lambda_\chi)^\nu$  $\langle \begin{array}{cc} \mathbf{0} & \mathbf{1} \end{array} \rangle$ 

#### Weinberg power counting for N-nucleon  $\alpha$  convertise for  $\mathbb{R}$  must be  $\alpha$ Power counting for it neered

state (hadronic scale). Determining the power  $n_i$  $\nu = -2 + 2A - 2C + 2L + \sum_{i} \Delta_i, \ \ \Delta_i \equiv d_i + \frac{n_i}{2} - 2$  $-2C+2L+\sum \Delta_i$ ,  $\Delta_i = d_i + \frac{n_i}{2} - 2$  $\frac{a}{2}$  - 2

- A: number of nucleon fields (in and out-states) *i* of nucleon fields (*in and our-sidies)*<br>(*chinally individual* connected multi-nucleon forces. Consider a  $\frac{7}{10}$  connected diagram is  $\frac{2}{10}$
- **L: number of pion loops**  imber of pion loops<br>*n*
- C: number of connected pieces connected parts of the diagram while *L* is the number of loops; *d<sup>i</sup>* indicates how
- $V_i$ : **: number of vertices with vertex dimension**  nucleon connected pieces.<br>The Feynman rules of covariant person rules of covariant person theory, a nucleon theory, a nucleon theory, a **V**<sub>i</sub>: number of vertices with vertex dimension<br>In the two equations above: for each vertex **i**, *c* represents the number of individual matrices with vertex dimension **number of vertices** with vertex dimension
- $d_i$ : **: number of derivatives or pion mass at the vertex i**  interaction is *Q*, and each four-momentum integration *Q*<sup>4</sup>. This is also known as naive d: number of derivatives or pion mass at the vertex i n of denivatives on nion mass at the ventey i For later purposes, we note that for an irreducible *NN* diagram (*A* = 2, *C* = 1),
- $n_i$ : **: number of nucleon fields at the vertex i** dimpersional dimensional and the verties. The topological identities, one obtains for the theory of the theory  $n_i$ : number of nucleon fields at the vertex i

 $p = 2L + \sum_i \Delta_i$ 

**NN interaction:** 
$$
v = 2L + \sum_{i} \Delta_i
$$

**19/10/2024 Jinniu Hu** (*d*<sub>19</sub>/10/2024 **between pions and a nucleon pions and a nucleon pions and a nucleon pions and a nucleon have one or more or** 

### Power counting  $P_{\text{oucon}}$  counting:  $P_{\text{oucon}}$



