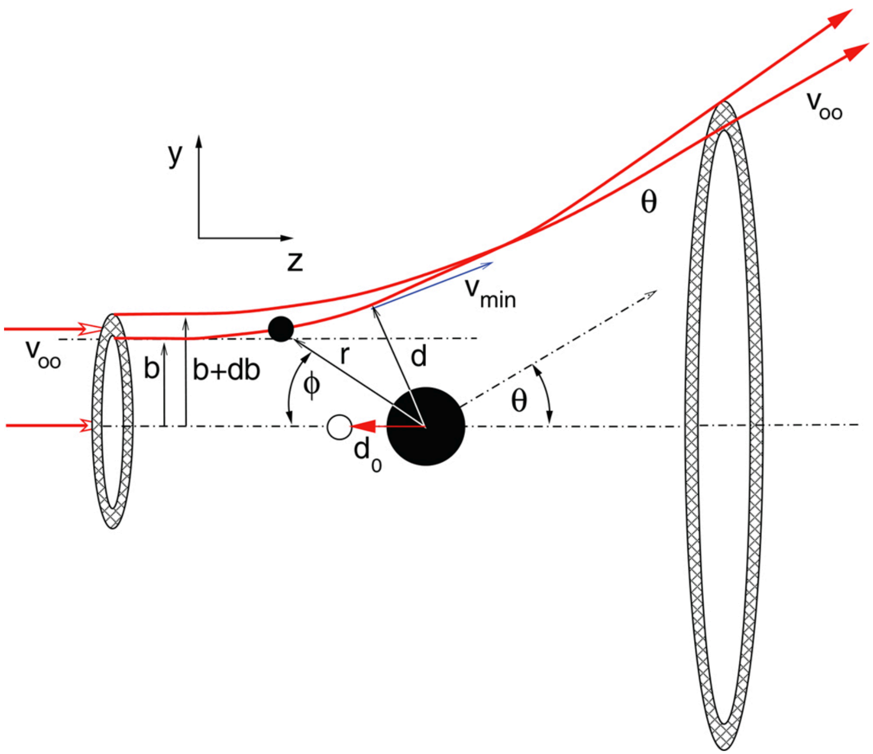
手征核力与高密核物质的状态方程

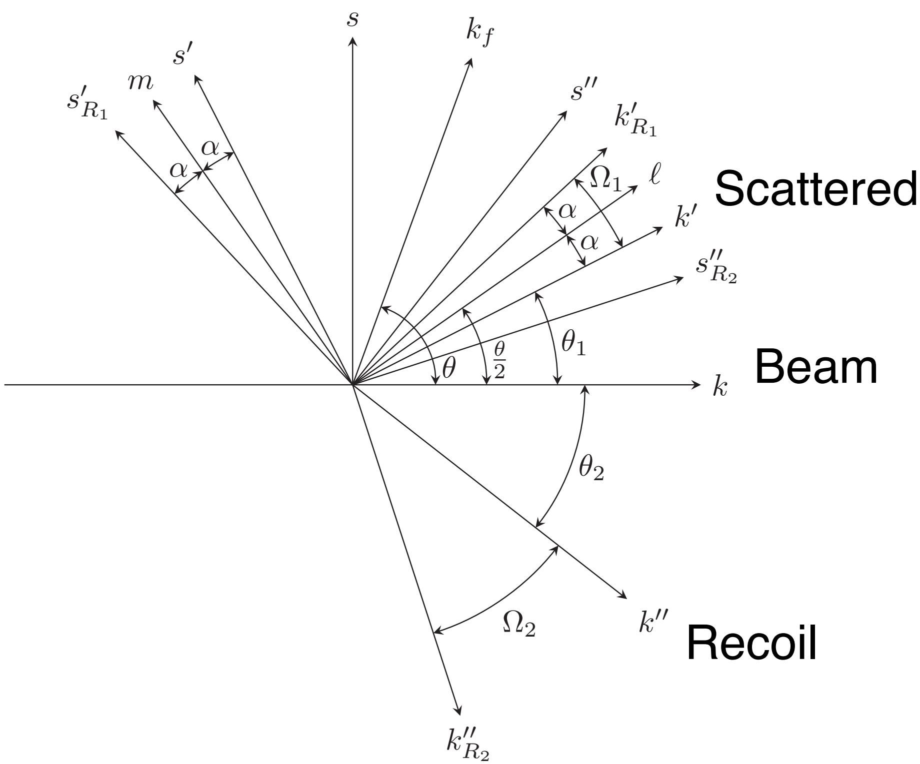
胡 金 牛

南开大学物理科学学院

Rutherford scattering



NN scattering



J. Bystricky, F. Lehar, and P. Winternitz, J. Phys. 39(1978)1.

NN scattering data

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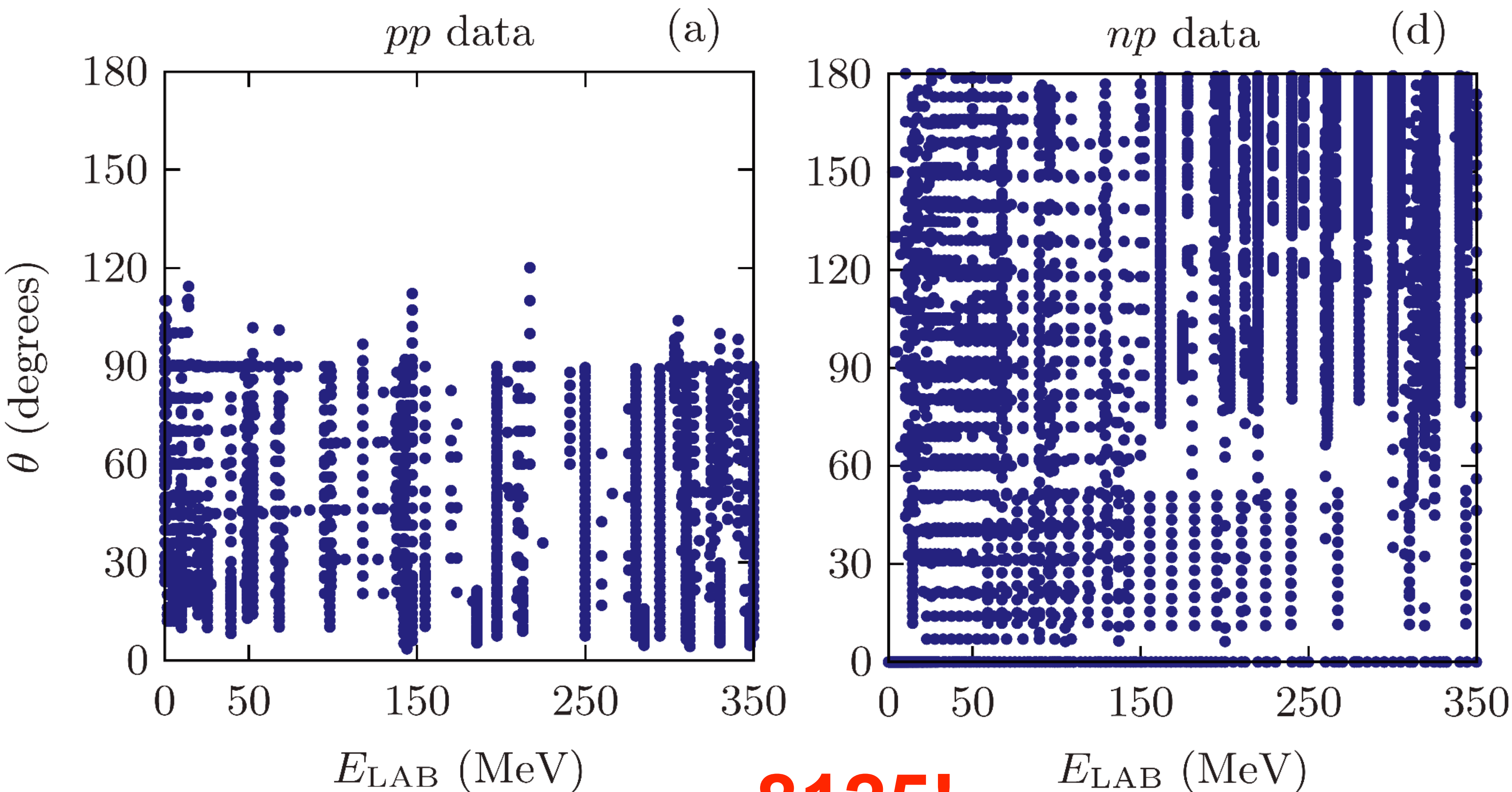
R. Navarro Pérez, J. E. Amaro, and E. Ruiz Arriola, Phys. Rev. C 89(2014)064006



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Chiral NN interaction

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➢ N²LO chiral potential

S. Weinberg, Nuclear Phys. B 363 (1991) 3

C. Ordonez, L. Ray, U. van Kolck, Phys. Rev. Lett. 72 (1994) 1982

C. Ordonez, L. Ray, U. van Kolck, Phys. Rev. C 53 (1996) 2086

J.X. Lu, C.X. Wang, Y. Xiao, L.S. Geng, J. Meng, and P. Ring, Phys. Rev. Lett. 128 (2022) 142002 (Relativistic version)

➢ N³LO chiral potential

D. R. Entem, R. Machleidt, Phys. Rev. C 66 (2002) 014002

E. Epelbaum, W. Gloeckle, U.-G. Meissner, Nucl. Phys. A 747 (2005) 362

➢ N⁴LO chiral potential and N⁵LO (partially)

D. R. Entem, N. Kaiser, R. Machleidt, and Y. Nosyk, Phys. Rev. C 91 (2015) 014002

E. Epelbaum, H. Krebs, U.-G. Meissner, Phys. Rev. Lett. 115 (2015) 122301 (EKM)

D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004 (EMN)

P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54 (2018) 86 (RKE)

S. K. Saha, D. R. Entem, R. Machleidt, and Y. Nosy, Phys. Rev. C107(2023)034002

Chiral NN interaction

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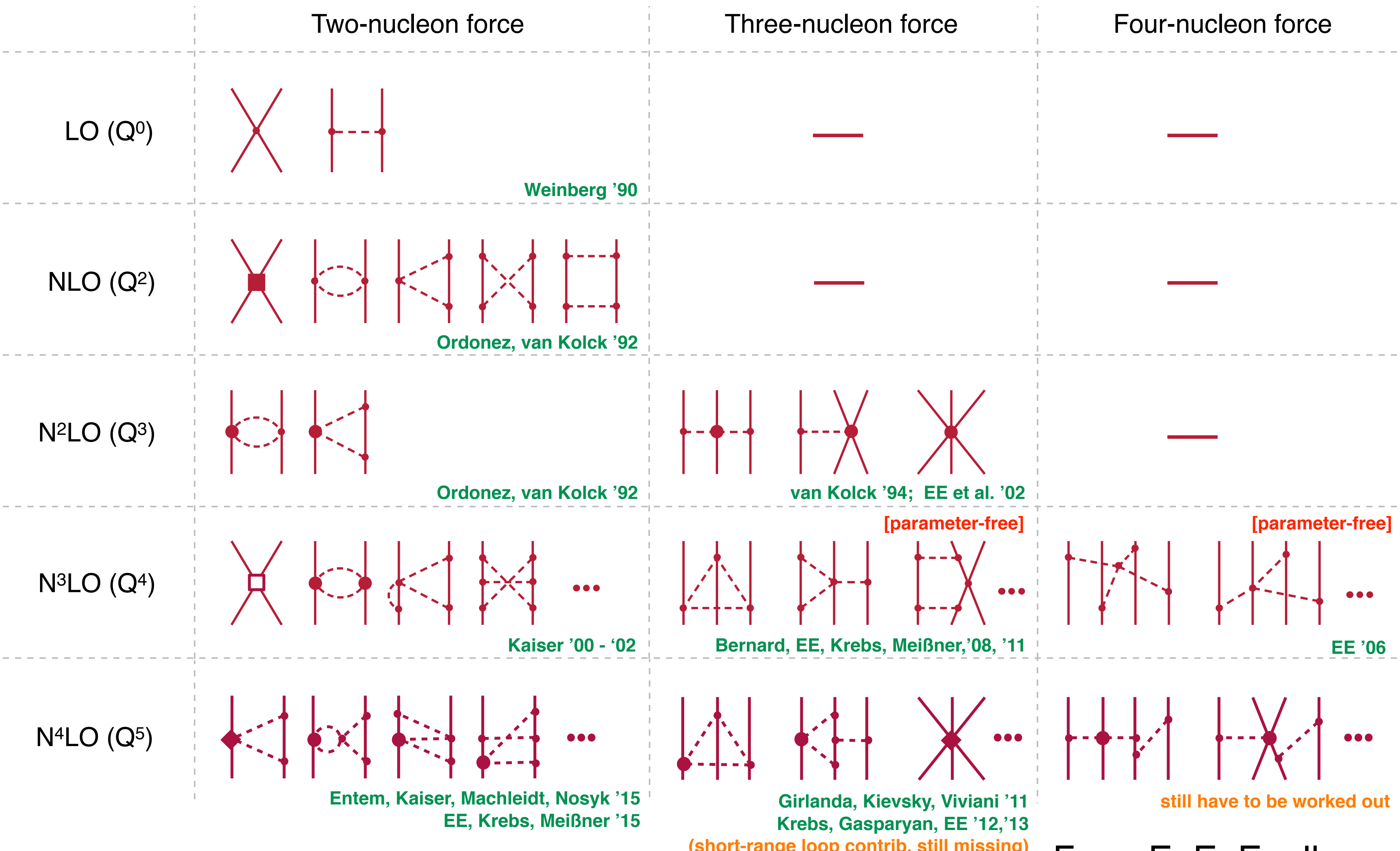

From E. E. Epelbaum

D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004

The pion-exchange part of the NN potential

$$V_{\mathrm{LO}} \equiv V^{(0)} = V_{1\pi}^{(0)},$$

$$V_{\mathrm{NLO}} \equiv V^{(2)} = V_{\mathrm{LO}} + V_{1\pi}^{(2)} + V_{2\pi}^{(2)},$$

$$V_{\mathrm{NNLO}} \equiv V^{(3)} = V_{\mathrm{NLO}} + V_{1\pi}^{(3)} + V_{2\pi}^{(3)},$$

$$V_{\mathrm{N^3LO}} \equiv V^{(4)} = V_{\mathrm{NNLO}} + V_{1\pi}^{(4)} + V_{2\pi}^{(4)} + V_{3\pi}^{(4)},$$

$$V_{\mathrm{N^4LO}} \equiv V^{(5)} = V_{\mathrm{N^3LO}} + V_{1\pi}^{(5)} + V_{2\pi}^{(5)} + V_{3\pi}^{(5)},$$

The NN potential with contact term

$$V_{\mathrm{LO}} \equiv V^{(0)} = V_{1\pi} + V_{\mathrm{ct}}^{(0)},$$

$$V_{\mathrm{NLO}} \equiv V^{(2)} = V_{\mathrm{LO}} + V_{2\pi}^{(2)} + V_{\mathrm{ct}}^{(2)},$$

$$V_{\mathrm{NNLO}} \equiv V^{(3)} = V_{\mathrm{NLO}} + V_{2\pi}^{(3)},$$

$$V_{\mathrm{N^3LO}} \equiv V^{(4)} = V_{\mathrm{NNLO}} + V_{2\pi}^{(4)} + V_{3\pi}^{(4)} + V_{\mathrm{ct}}^{(4)},$$

$$V_{\mathrm{N^4LO}} \equiv V^{(5)} = V_{\mathrm{N^3LO}} + V_{2\pi}^{(5)} + V_{3\pi}^{(5)},$$

Parameters in in chiral EFT

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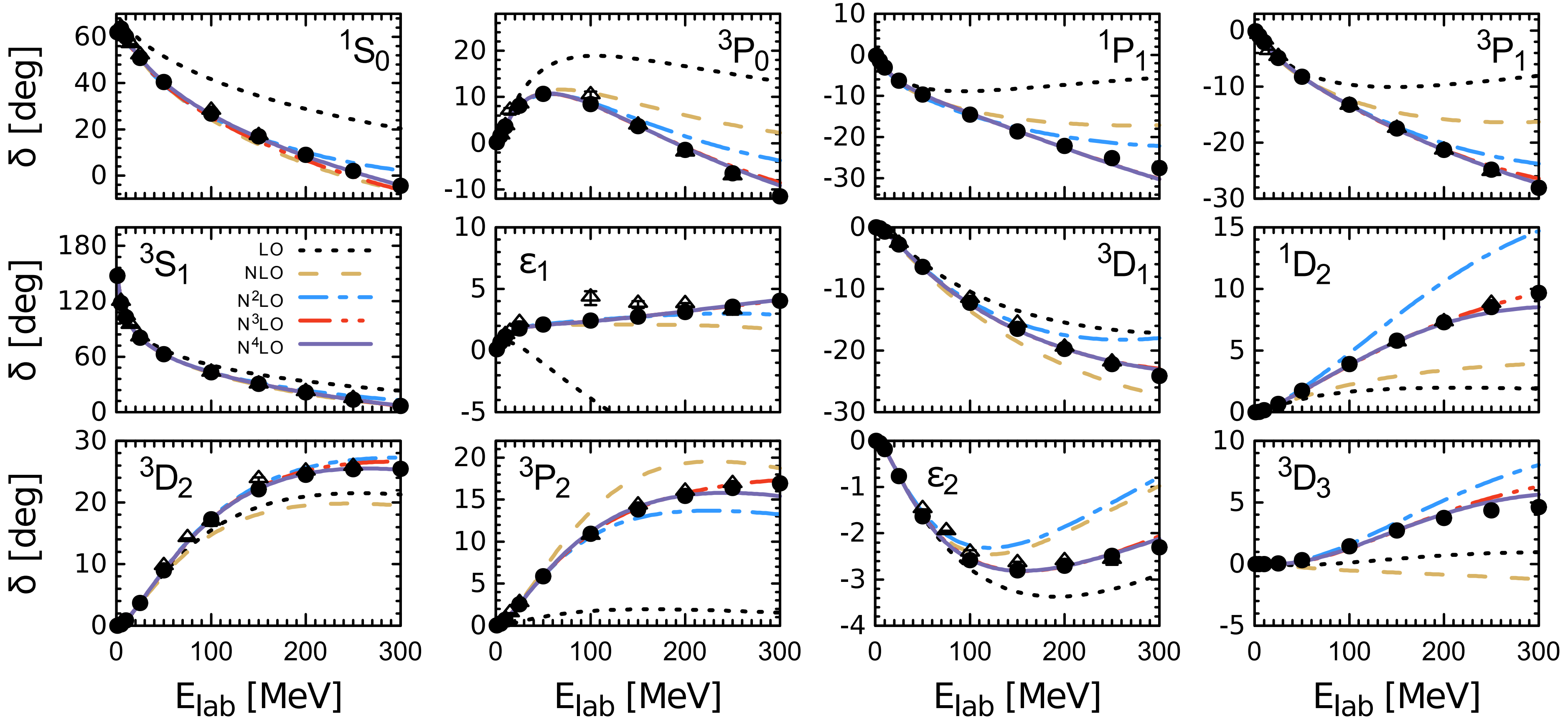
D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004

	Nijmegen PWA93 Ref [47]	CD-Bonn pot. Ref. [44]	— EFT contact potentials [33] — Q^0	Q^2	Q^4	Q^6
$^1S_0$	3	4	1	2	4	6
$^3S_1$	3	4	1	2	4	6
$^3S_1$-$^3D_1$	2	2	0	1	3	6
$^1P_1$	3	3	0	1	2	4
$^3P_0$	3	2	0	1	2	4
$^3P_1$	2	2	0	1	2	4
$^3P_2$	3	3	0	1	2	4
$^3P_2$-$^3F_2$	2	1	0	0	1	3
$^1D_2$	2	3	0	0	1	2
$^3D_1$	2	1	0	0	1	2
$^3D_2$	2	2	0	0	1	2
$^3D_3$	1	2	0	0	1	2
$^3D_3$-$^3G_3$	1	0	0	0	0	1
$^1F_3$	1	1	0	0	0	1
$^3F_2$	1	2	0	0	0	1
$^3F_3$	1	2	0	0	0	1
$^3F_4$	2	1	0	0	0	1
$^3F_4$-$^3H_4$	0	0	0	0	0	0
$^1G_4$	1	0	0	0	0	0
$^3G_3$	0	1	0	0	0	0
$^3G_4$	0	1	0	0	0	0
$^3G_5$	0	1	0	0	0	0
Total	35	38	2	9	24	50

Nuclear force in chiral EFT

$$V^{\rm reg}_{\rm long-range}(\vec{r}) = V_{\rm long-range}(\vec{r}) f_{\rm reg}\left(\frac{r}{R}\right), \quad f_{\rm reg} = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6$$



E. Epelbaum, H. Krebs, U.-G. Meissner, Phys. Rev. Lett. 115 (2015) 122301 (EKM)

Nuclear force in chiral EFT

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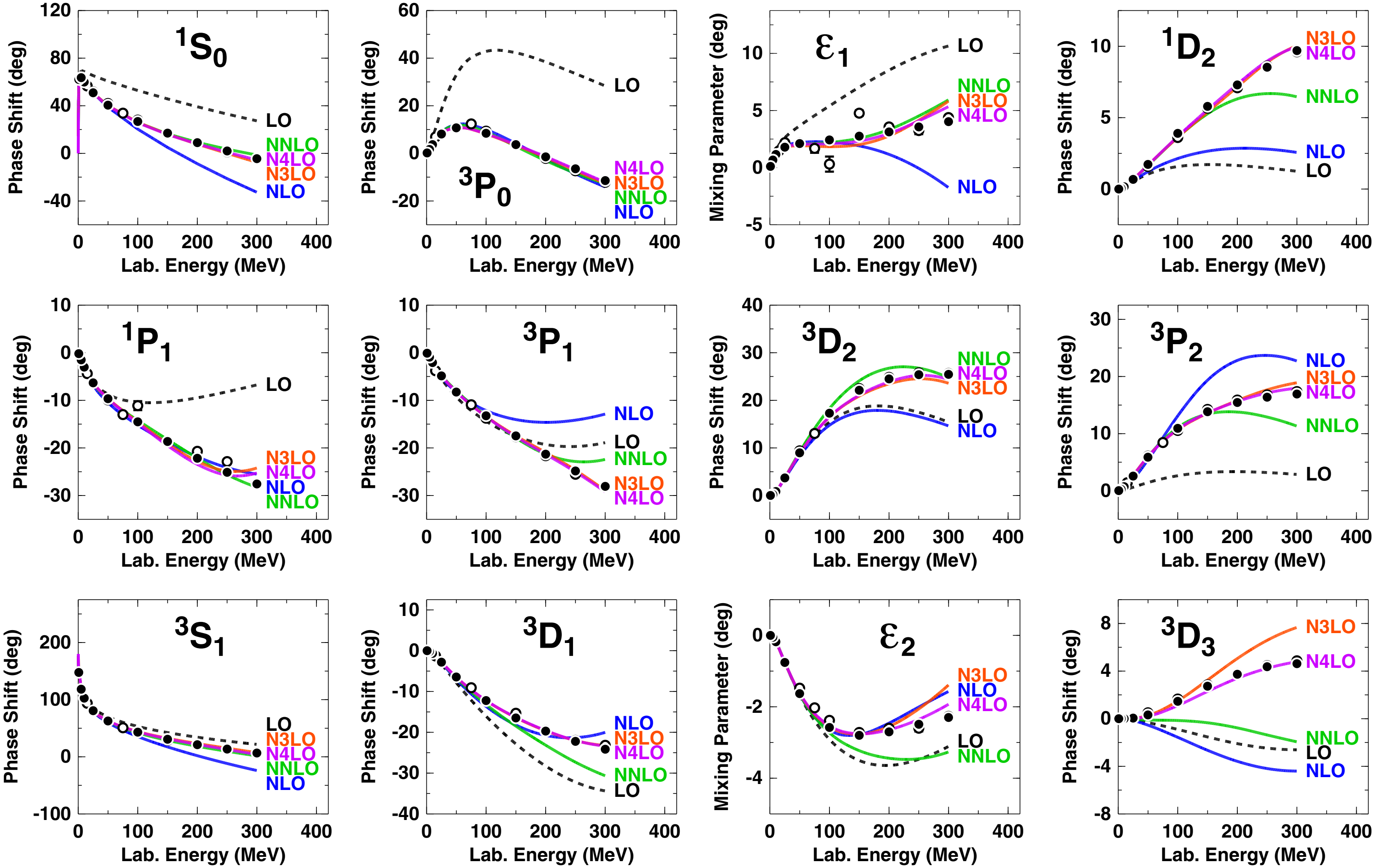
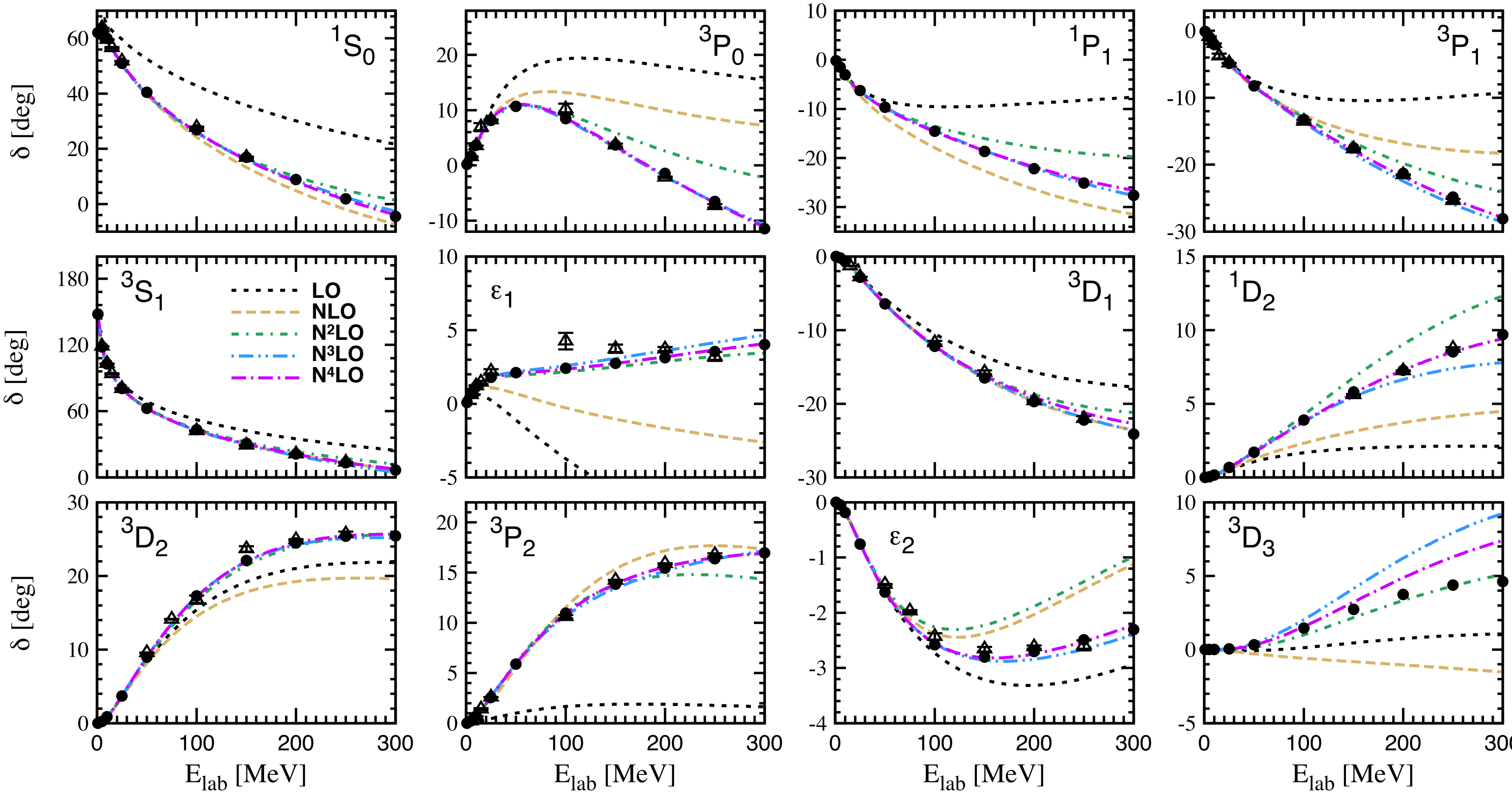
D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004 (EMN)

$$\widehat{V}(\vec{p}\,',\vec{p}) \longmapsto \widehat{V}(\vec{p}\,',\vec{p})\, f(p',p) \qquad f(p',p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}]$$



Nuclear force in chiral EFT

$$\widehat{V}(\vec{p}\,',\vec{p}) \longmapsto \widehat{V}(\vec{p}\,',\vec{p})\, f(p',p) \qquad f(p',p) = \exp[-((\vec{p}\,'-\vec{p})^2 + m_\pi^2)/\Lambda^2]$$



P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54 (2018) 86 (RKE)

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Description of the np and pp phase shifts

E_{lab} bin	LO (Q^0)	NLO (Q^2)	N^2LO (Q^3)	N^3LO (Q^4)	N^4LO (Q^5)	N^4LO^+
neutron-proton scattering data						
0 – 100	73	2.2	1.2	1.08	1.08	1.07
0 – 200	62	5.4	1.8	1.09	1.08	1.07
0 – 300	75	14	4.4	1.99	1.18	1.06
proton-proton scattering data						
0 – 100	2300	10	2.1	0.91	0.88	0.86
0 – 200	1780	91	33	2.00	1.42	0.95
0 – 300	1380	89	38	3.42	1.67	1.00

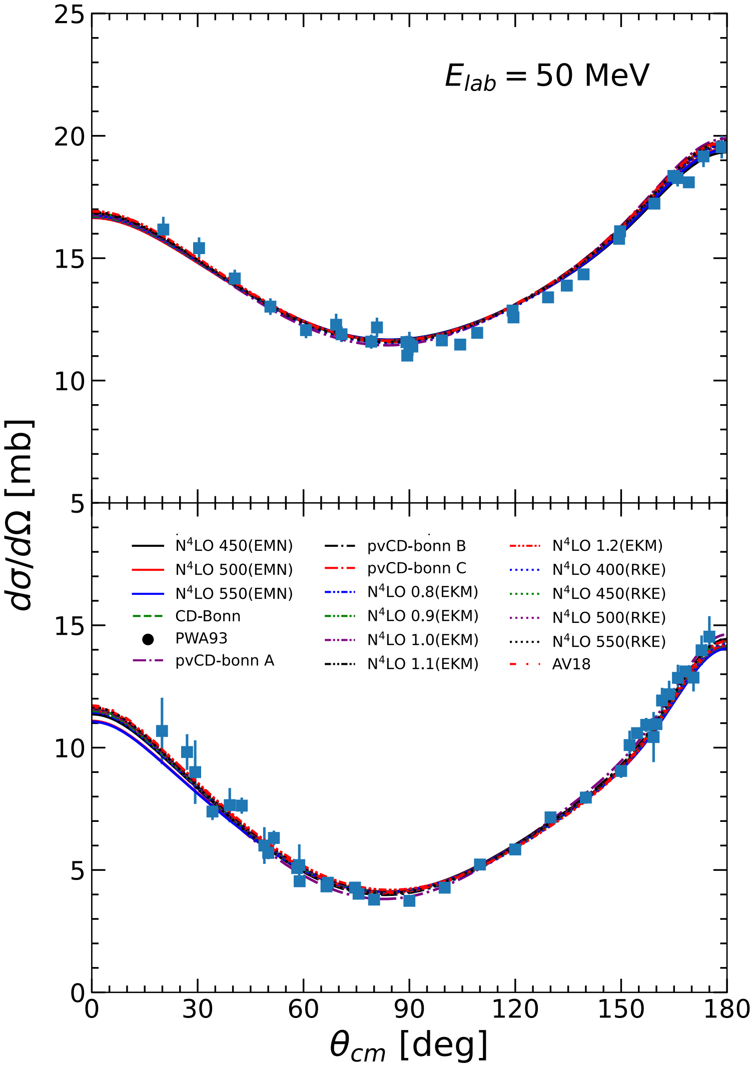
Deuteron properties

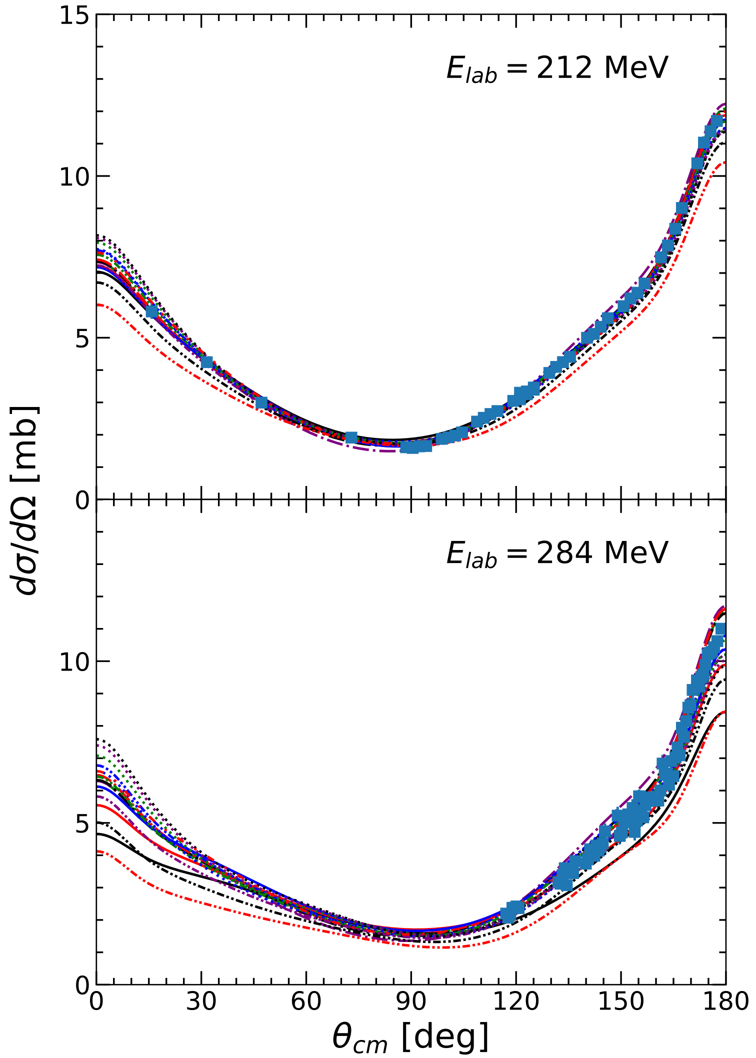
	LO	NLO	N^2LO	N^3LO	N^4LO	N^4LO^+	Empirical
B_d (MeV)	2.1201	2.1843	2.2012	$2.2246^{(a)}$	$2.2246^{(a)}$	$2.2246^{(a)}$	2.224575(9) [119]
$\langle T_{\text{kin}} \rangle$ (MeV)	14.24	13.47	14.44	14.35	14.16	14.22	–
A_S ($\text{fm}^{-1/2}$)	0.8436	0.8727	0.8786	0.8844	0.8847	0.8847	0.8846(8) [140]
η	0.0220	0.0236	0.0251	0.0257	0.0255	0.0255	0.0256(4) [141]
r_d (fm)	1.946	1.967	1.970	1.966	1.966	1.966	$1.97535(85)^{(b)}$ [142]
Q (fm^2)	0.227	0.249	0.268	0.272	0.269	0.270	0.2859(3) [143]
P_D (%)	2.77	3.59	4.63	4.70	4.54	4.59	–

P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54 (2018) 86 (RKE)

Total cross sections

K. Nan, J. Hu, H. Shen, and Y. Zhang, arXiv: 2410.00679





The many-body ab initio calculations

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- **Brueckner-Hartree-Fock method**
  M. Hjorth-Jensen, T.T.S. Kuo, and E. Osens, Phys. Rep. 261(1995)125
- **Relativistic Brueckner-Hartree-Fock method**
  S. Shen, H. Liang, J. Meng, P. Ring, and S. Zhang, Phys. Rev. C 96(2017)014316
- **Self-consistent Green's function method**
  W. H. Dickhoff and C. Barbieri, Prog. Part. Nucl. Phys. 52(2004)377
- **Many-body perturbation theory**
  J.W. Holt and N. Kaiser, Phys. Rev. C, 95(2017)034326
- **In-medium Similarity Renormalization Group (IMSRG)**
  H. Hergert, S.K. Bogner, T.D. Morris, A. Schwenk, K. Tsukiyama, Phys. Rep. 621(2016)165
- **No core shell model**
  B. R. Barrett, P. Navraetil, and J. P. Vary, Prog. Part. Nucl. Phys. 69(2013)131
- **Lattice effective field theory**
  D. Lee, Prog. Part. Nucl. Phys. 63(2009)117
- **Quantum Monte Carlo methods**
  J. Carlson, S. Gandolfi, F. Pederiva, Steven C. Pieper, R. Schiavilla, K.E. Schmidt, and R.B. Wiringa, Rev. Mod. Phys.87(2015)1067
- **Coupled Clusters**
  R. J. Bartlett and M. Musiał. Rev. Mod. Phys.79(2007)291

The BHF method

Bethe-Goldstone equation

$$G[\omega,\rho] = V + \sum_{k_a,k_b>k_F} V \frac{|k_a k_b\rangle\langle k_a k_b|}{\omega - e(k_a) - e(k_b) + i\epsilon} G[\omega,\rho],$$

Single-particle energy

$$e(k) = e(k;\rho) = \frac{k^2}{2m} + U(k,\rho).$$

Single-particle potential

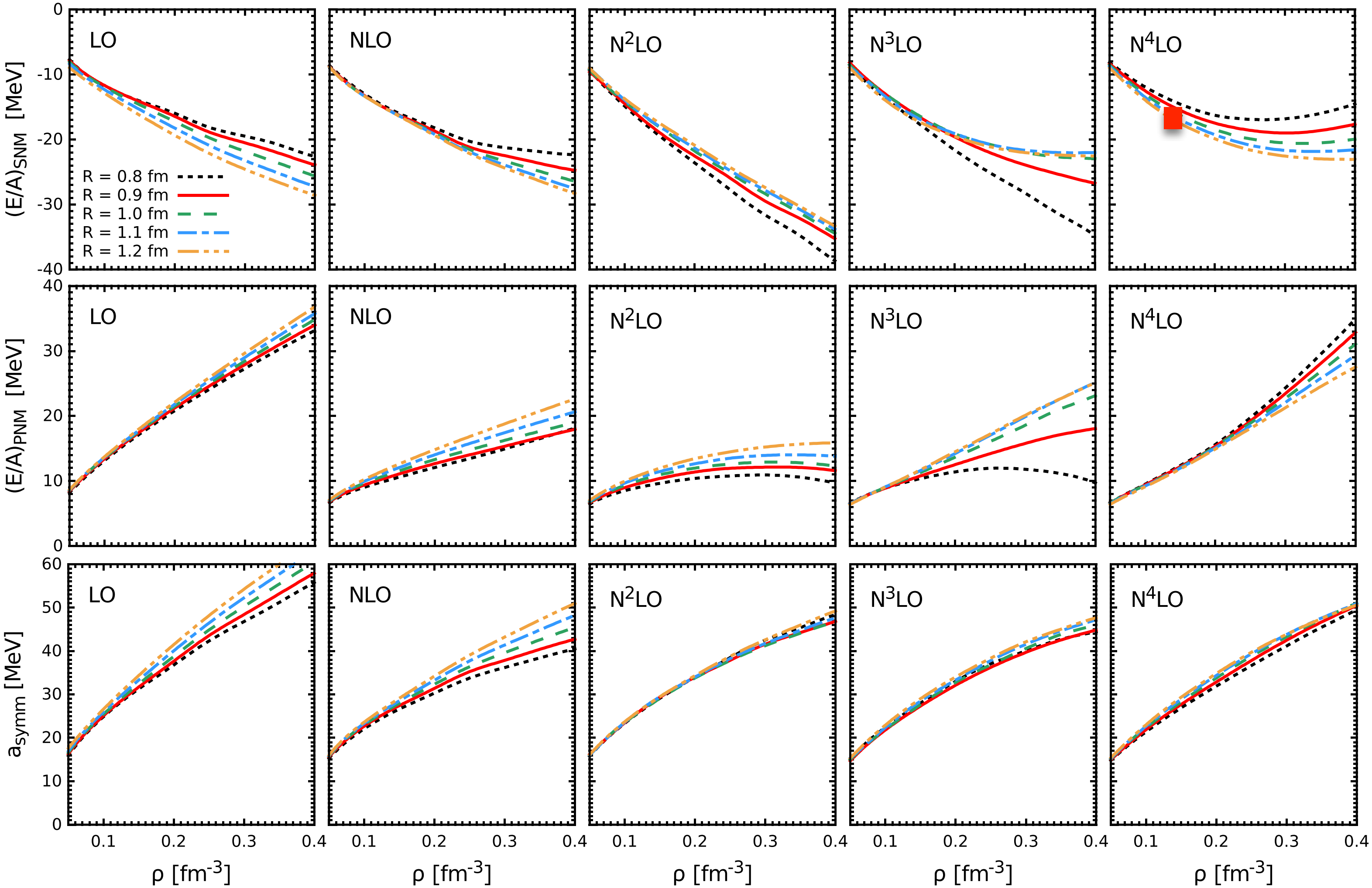
$$U(k;\rho) = \Re \sum_{k'<k_F} \langle kk'|G[e(k)+e(k');\rho]|kk'\rangle_a,$$

Energy per nucleon

$$\frac{E}{A} = \frac{3}{5}\frac{k_F^2}{2m} + \frac{1}{2\rho}\Re \sum_{k,k'<k_F} \langle kk'|G[e(k)+e(k');\rho]|kk'\rangle_a.$$

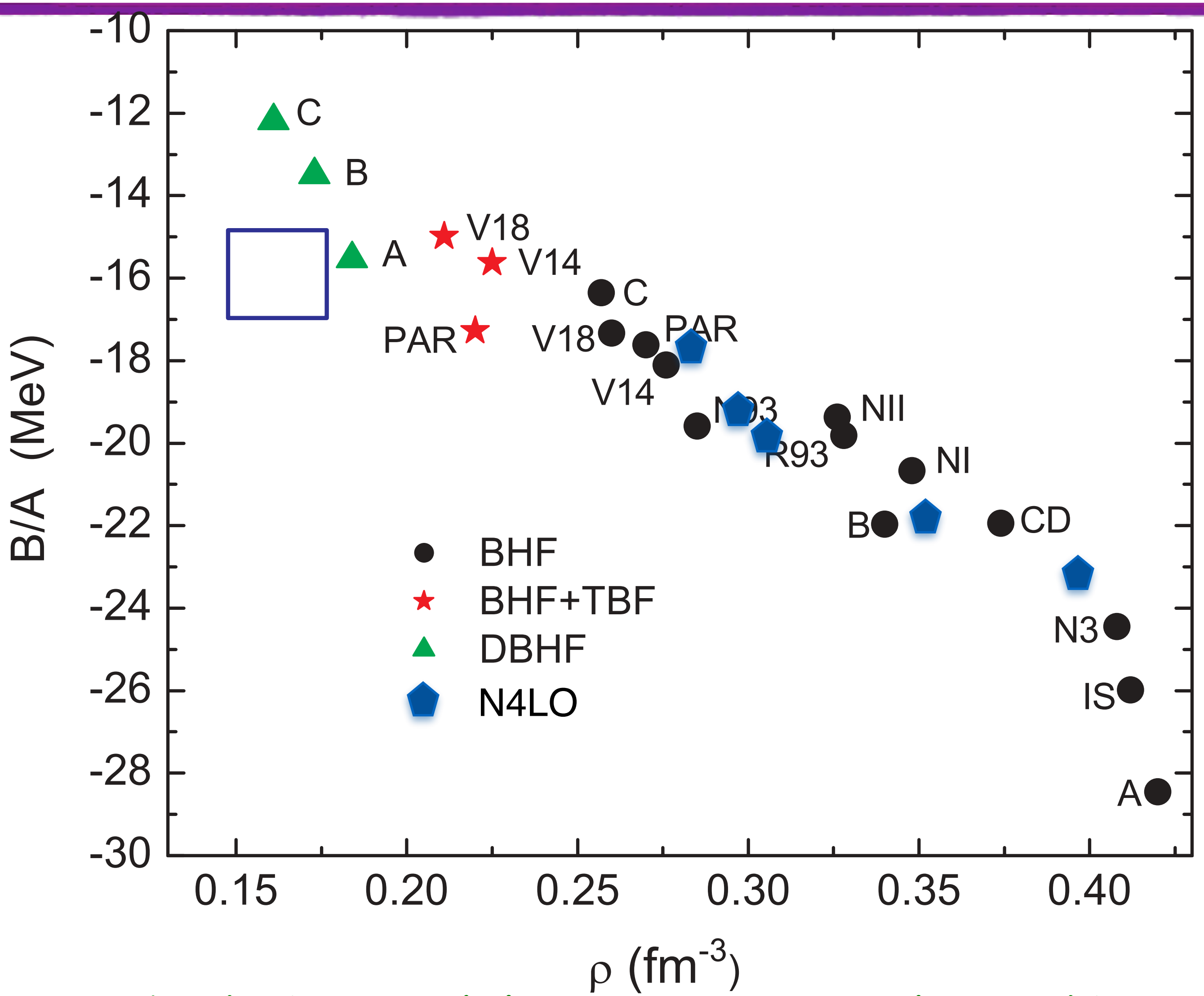
JH, Y. Zhang, E. Epebaum, U.-G. Meissner, and J. Meng, Phys. Rev. C 96(2017)034307

The EOSs and symmetry energy from N⁴LO chiral potentials



J. Hu, Y. Zhang, E. Epelbaum, Ulf-G. Meissner, J. Meng, Phys. Rev C 96(2017)034307

Coester Line



Z. H. Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, and H. R. Ma, Phys. Rev. C 74 (2006)047304

The EOSs from different chiral potentials

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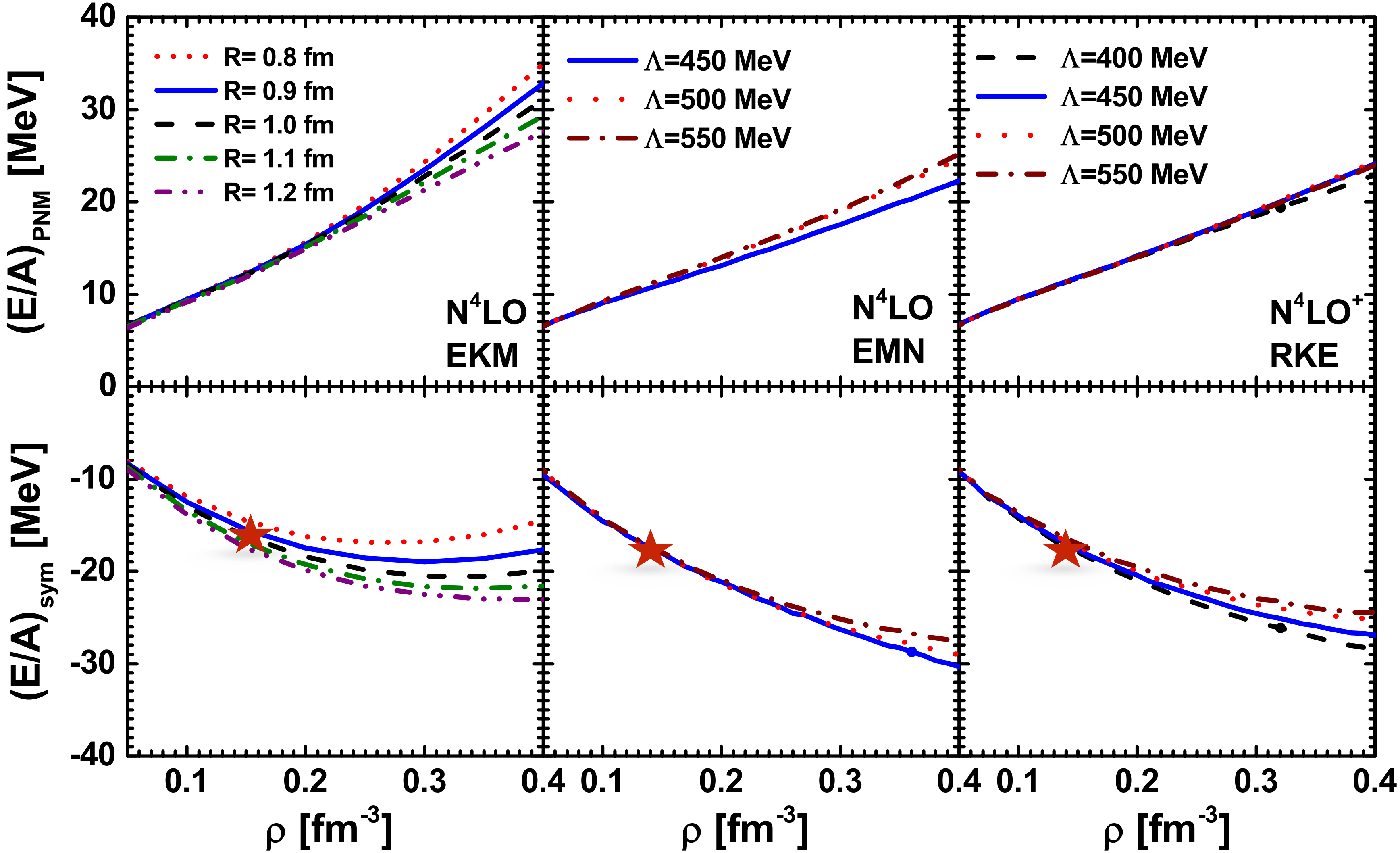

The Bayesian analysis

An observable X in EFT can be expanded as

$$X = X_0 \sum_{n=0}^{\infty} c_n Q^n,$$

where X_0 is the natural size of the observable X, and c_n are dimensionless coefficients.

In nuclear matter,

$$Q = \frac{k_F}{\Lambda_b}$$

and

$$X_0 = (E_{\mathrm{LO}}/A)$$

J. A. Melendez, S. Wesolowski, and R. J. Furnstahl, Phys. Rev. C 96(2017)024003

The truncated error at order k is

$$\Delta X = X_0 \Delta_k$$

where the scaled, dimensionless parameter is

$$\Delta_k = \sum_{n=k+1}^{\infty} c_n Q^n$$

In Bayesian framework, one believes with (100*p)% certainty the true value of the observable X lies within

$$\pm X_0 d_k^{(p)}$$

of the $(k+1)^{th}$ order (N^kLO), where d is related to the degree-of-belief intervals p,

$$p = \int_{-d_k^{(p)}}^{d_k^{(p)}} d\Delta \mathrm{pr}_h(\Delta|\mathbf{c_k})$$

pr_h is the posterior probability distribution functions

The truncation error

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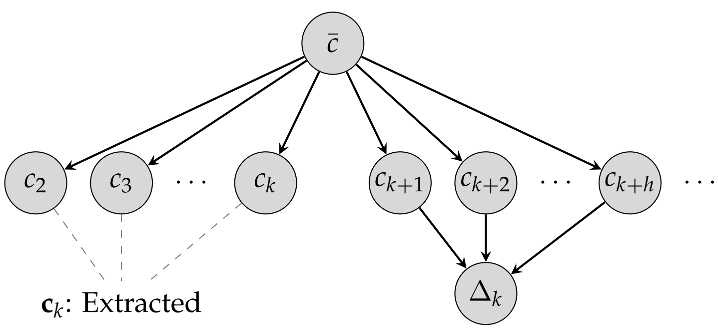
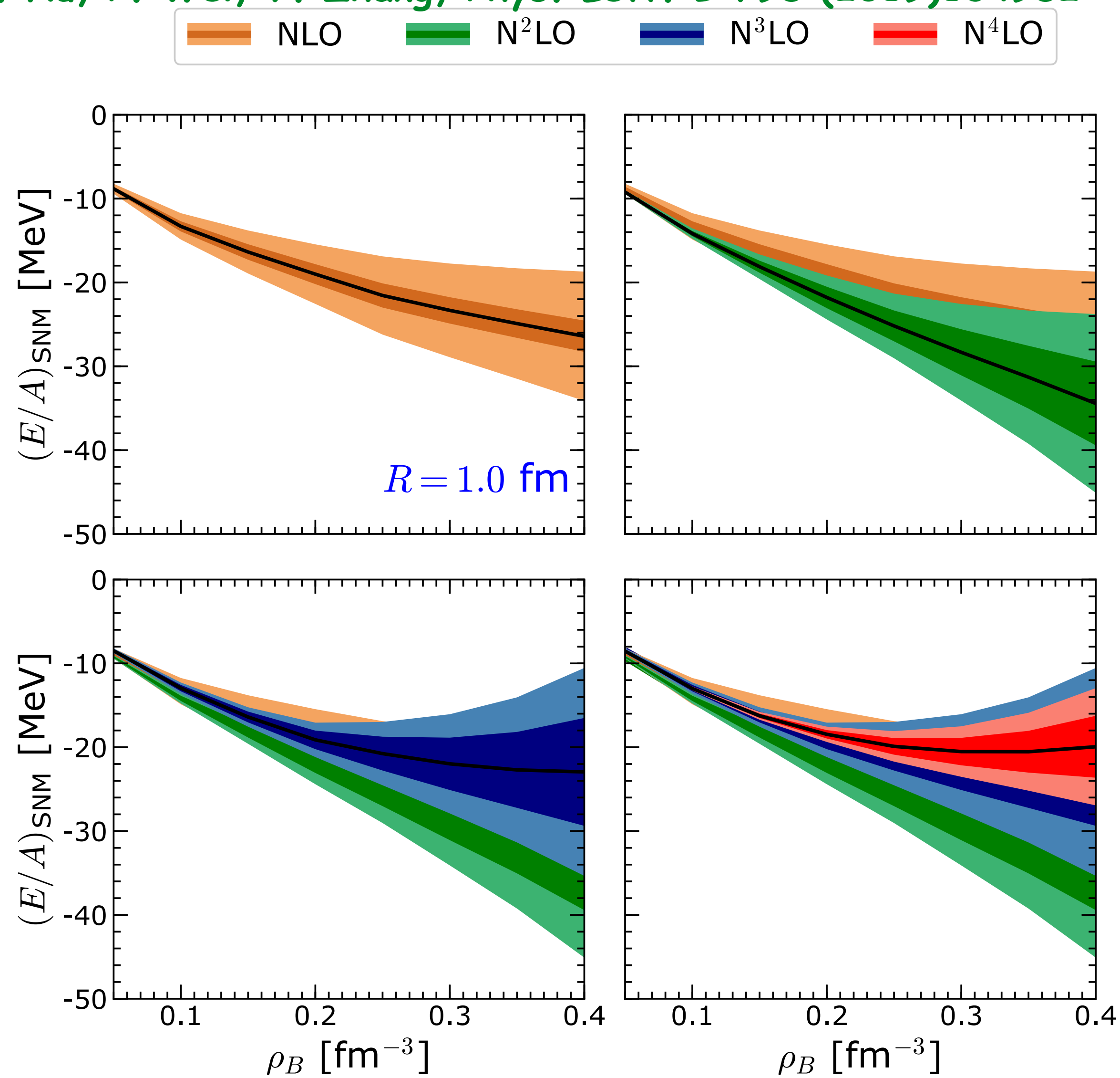
J. Hu, P. Wei, Y. Zhang, Phys. Lett. B 798 (2019)134982

$$X(p,\theta) = X_{\mathrm{ref}}\left(c_0 + c_1 Q + \cdots + c_k Q^k + \Delta_k\right)$$

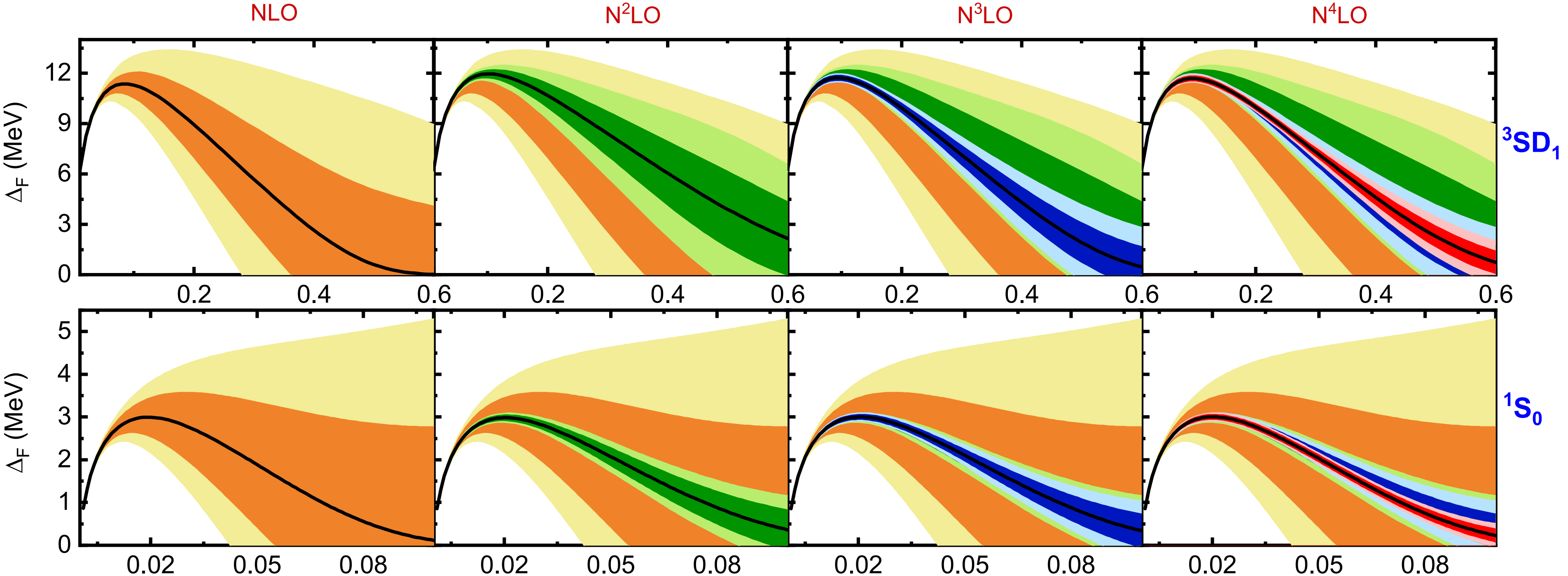
- X_{ref} : Dimensionful scale
- k : Truncation order
- Q : Expansion parameter
- c_n : *Natural* coefficients
- Δ_k : Truncation error
- Λ_b: Breakdown scale

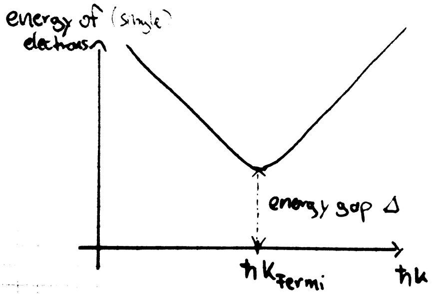
$$\Delta_k = \sum_{n=k+1}^{\infty} c_n Q^n$$





The pairing gap in nuclear matter





P. Yin, X. Shang, J. Hu, J. Fu, E. Epelbaum, and W. Zuo, Phys. Rev. C 108(2023)034002

The summary and outlook

We calculated the properties of nuclear matter with different state-of-the-art chiral NN potentials.

The equations of state of nuclear matter from different chiral potentials have different behaviors due to regularized factor.

The chiral truncation errors in nuclear matter were discussed with Bayesian analysis. The chiral potentials are good convergence at higher order.

They pairing gaps at different spin channels are calculated with chiral force in symmetric matter.

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Time-ordered perturbation theory

$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} \Longrightarrow |\psi\rangle = \frac{1}{E - \lambda H \lambda} H |\phi\rangle$$

$$\Longrightarrow \quad (H_0 + V_{\rm eff}^{\rm TD})|\phi\rangle = E|\phi\rangle$$

with effective potential

$$V_{\rm eff}^{\rm TD} = \eta H_I \eta + \eta H_I \lambda \frac{1}{E - \lambda H \lambda} \lambda H_I \eta$$

and

$$|\phi\rangle \equiv |N\rangle + |NN\rangle + |NNN\rangle + \ldots$$

$$|\psi\rangle \equiv |N\pi\rangle + |N\pi\pi\rangle + \ldots + |NN\pi\rangle + \ldots$$

Nuclear force

$$V_{\rm eff}^{\rm TD} = \eta H_I \eta + \eta H_I \frac{\lambda}{E - H_0} H_I \eta + \eta H_I \frac{\lambda}{E - H_0} H_I \frac{\lambda}{E - H_0} H_I \eta + \ldots$$

Nuclear force in chiral EFT

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Unitary transformation

$$H = \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \Longrightarrow \tilde{H} \equiv U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$$

and

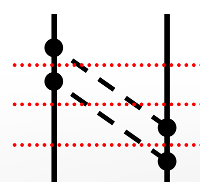
$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + A^\dagger A)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + A^\dagger A)^{-1/2} \end{pmatrix}$$

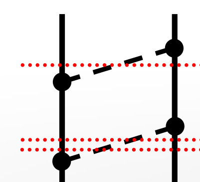
A should be solved by

$$\lambda(H - [A, H] - AHA)\eta = 0$$

Nuclear force

$$V_{\text{eff}} = -\eta H_I \frac{\lambda}{E_\pi} H_I \eta - \eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta + \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta + \ldots$$





Power counting

D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004

A-nucleon interactions receives contributions

$$(Q/\Lambda_\chi)^\nu$$

Weinberg power counting for N-nucleon

$$\nu = -2 + 2A - 2C + 2L + \sum_i \Delta_i\,, \quad \Delta_i \equiv d_i + \frac{n_i}{2} - 2$$

A: number of nucleon fields (in and out-states)

L: number of pion loops

C: number of connected pieces

V_i: number of vertices with vertex dimension

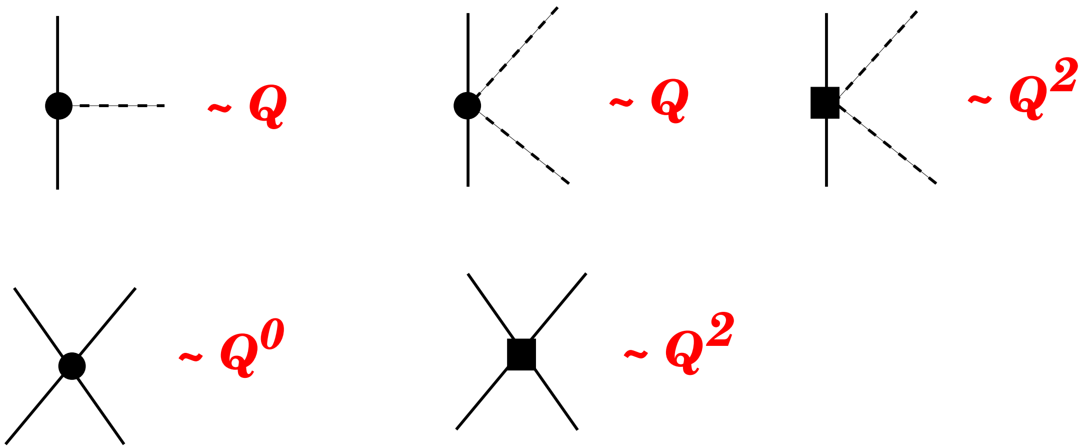
d_i: number of derivatives or pion mass at the vertex i

n_i: number of nucleon fields at the vertex i

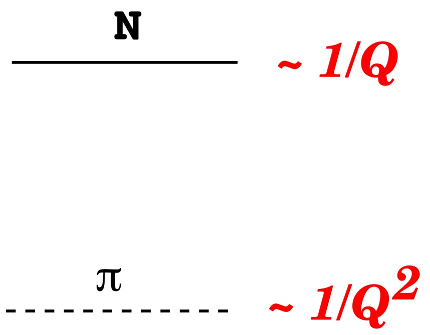
NN interaction: $$\nu = 2L + \sum_i \Delta_i$$

Power counting

Vertices



Propagators



Interactions