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Axion-light hadron interactions in chiral effective field theory

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Introduction

Strong CP problem

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q e^{i\theta_q})q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \to e^{i\gamma_5\alpha} q \quad \longrightarrow \quad \theta_q \to \theta_q + 2\alpha\,, \qquad \theta \to \theta - 2\alpha$$

implying the invariant quantity: $\bar{\theta} = \theta + \theta_q$

$$\Rightarrow \quad \mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \boxed{\bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Constraints from neutron electric dipole moment (nEDM)

$$|\bar{\theta}| \lesssim 10^{-10}$$

- **Strong CP problem: why is $\bar{\theta}$ so unnaturally tiny ?**

Axion: an elegant solution to strong CP

[Peccei,Quinn,PRL'77]
[Weinberg,PRL'78] [Wilzeck,PRL'78]

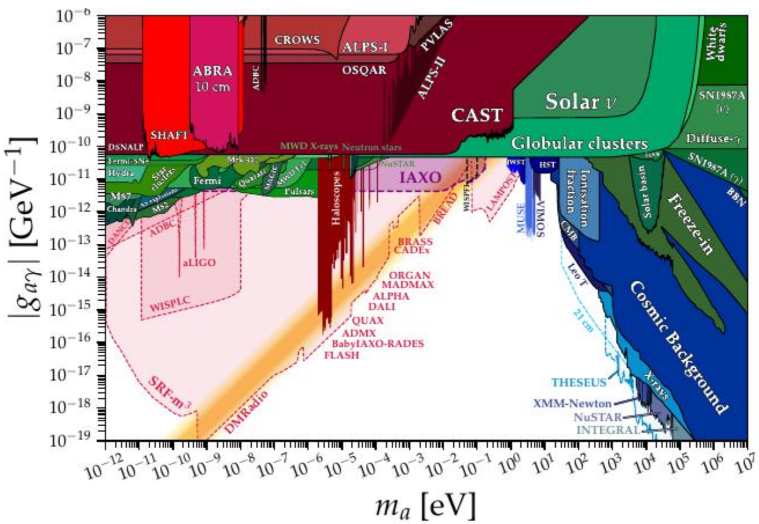
$$\mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \cdots$$

Spontaneous breaking of $U(1)_{PQ}$ symmetry & anomalous term: axion (pseudo-NGB)

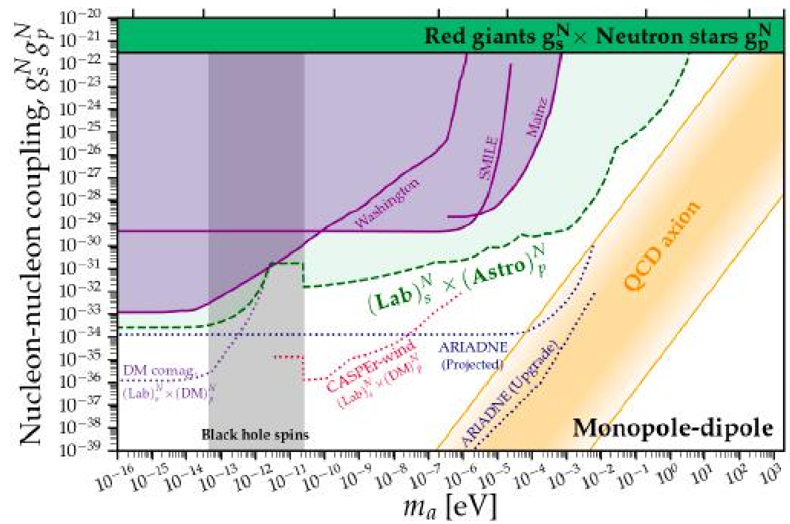
- **f_a: the axion decay constant. Invisible axion: $f_a >> v_{EW}$.**
- **Diverse constraints from different experiments**

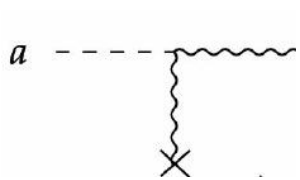
[Di Luzio, et al., Phy.Rep'20] [Sikivie, RMP'21] [Irastorza, Redondo,PPNP'18] ... ...

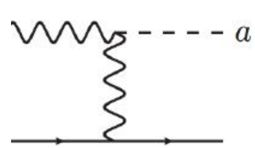
Cosmology, Astronomy, Colliders, Quantum precision measurements , Cavity Haloscope, ...

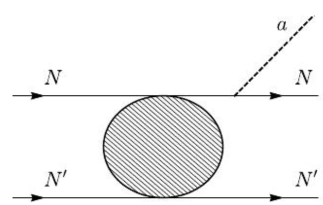


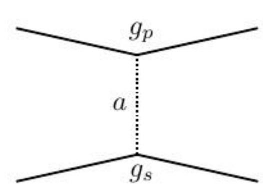
[O'Hare, Github]











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Axion chiral perturbation theory (AχPT)

- We will focus on the QCD-like axion: $m_{a,0}\,(\neq 0) \ll f_a$ with model-independent $aG\tilde{G}$ interaction, i.e., the **MODEL INDEPENDENT QCD axion** interactions.

- Axion-hadron interactions are relevant at low energies.

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the $aG\tilde{G}$ term via the quark axial transformation

$$q \to e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$$

$\mathrm{Tr}(Q_a) = 1$

$$-\frac{a\alpha_s}{8\pi f_a}G\tilde{G} - \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q \qquad M_q \to M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a}$$

Mapping to χPT

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger\rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\rm LO}$$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a(\partial^\mu U U^\dagger + U^\dagger\partial^\mu U)\rangle$$

- $Q_a = M_q^{-1}/\mathrm{Tr}(M_q^{-1})$ [Georgi,Kaplan,Randall, PLB'86]

- $J_A^\mu\,\partial_\mu a$ [Bauer, et al., PRL'21]

(2) Explicitly keep the $aG\widetilde{G}$ term and match it to χPT

Reminiscent:

QCD $U(1)_A$ anomaly that is caused by topological charge density $\omega(x) = \alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}/(8\pi)$ **is responsible for the massive singlet η_0.**

Axion could be similarly included as the η_0 via the U(3) χPT:

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12}M_0^2 X^2$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}\,, \qquad \chi = 2B(s+ip)\,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u\,,$$

$$u_\mu = i u^\dagger D_\mu U u^\dagger\,, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

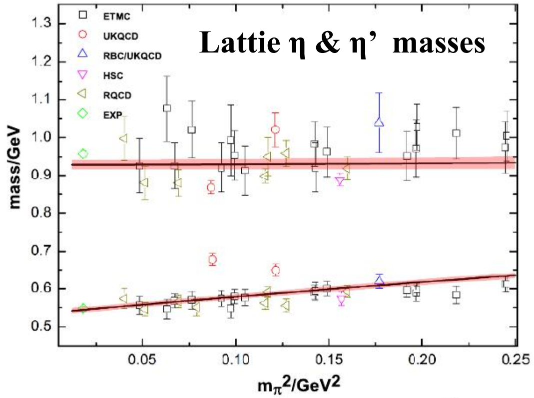
$$X = \log(\det U) + i\frac{a}{f_a} \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

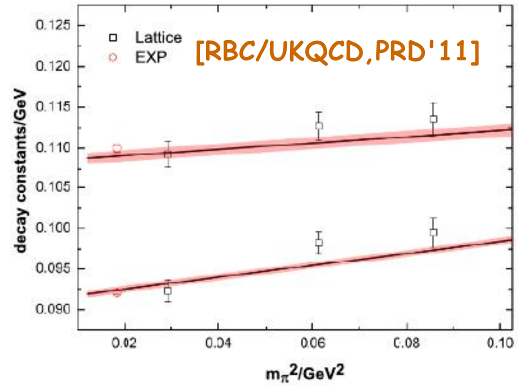
- **Q_a is not needed in U(3) χPT.**
- **$M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim O(1/N_c)$.**
- **δ expansion scheme: $\delta \sim O(p^2) \sim O(m_q) \sim O(1/N_c)$.**
- **Axion interactions enter via the axion-meson mixing terms.**

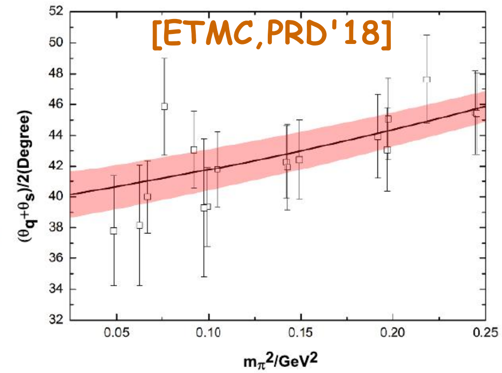
π-η-η'-a mixing in U(3) AχPT

[Gao,ZHG,Oller,Zhou,JHEP'23]

Parameters	NLO Fit
F(MeV)	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
Λ_1	$-0.17^{+0.05}_{-0.05}$
Λ_2	$0.06^{+0.08}_{-0.09}$
$\chi^2/(\text{d.o.f.})$	$219.9/(111-5)$







$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\text{LO+NLO}} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

$$M^{\text{LO+NLO}} = \begin{pmatrix} 1+(0.015\pm0.001) & 0.017+(-0.010\pm0.001) & 0.009+(-0.007\pm0.001) & \frac{12.1+(0.48\pm0.08)}{f_a} \\ -0.019+(0.007\pm0.001) & 0.94+(0.21\pm0.01) & 0.33+(-0.22\pm0.03) & \frac{34.3+(0.9\pm0.2)}{f_a} \\ -0.003+(-0.003\pm0.000) & -0.33+(-0.18\pm0.03) & 0.94+(0.13\pm0.02) & \frac{25.9+(-0.5\pm0.1)}{f_a} \\ \frac{-12.1+(-0.20\pm0.03)}{f_a} & \frac{-23.8+(1.6^{+0.8}_{-0.8})}{f_a} & \frac{-35.7+(-5.7^{+1.6}_{-1.7})}{f_a} & 1+\frac{27.6\pm1.0}{f_a^2} \end{pmatrix}$$

Two-photon couplings

$$\mathcal{L}^{\text{LO}}_{\text{WZW}} = -\frac{3\sqrt{2}e^2}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\Phi\rangle \qquad \mathcal{L}^{\text{NLO}}_{\text{WZW}} = it_1\varepsilon_{\mu\nu\rho\sigma}\langle f_+^{\mu\nu} f_+^{\rho\sigma}\chi_-\rangle + k_3\varepsilon_{\mu\nu\rho\sigma}\langle f_+^{\mu\nu} f_+^{\rho\sigma}\rangle X$$

Note: one needs the π-η-η'-*a* mixing as input to calculate $g_{a\gamma\gamma}$.

$$F^{\text{Exp}}_{\pi^0\gamma\gamma} = 0.274 \pm 0.002\,\text{GeV}^{-1}\,,$$
$$F^{\text{Exp}}_{\eta\gamma\gamma} = 0.274 \pm 0.006\,\text{GeV}^{-1}\,,$$
$$F^{\text{Exp}}_{\eta'\gamma\gamma} = 0.344 \pm 0.008\,\text{GeV}^{-1}\,.$$

$$t_1 = -(4.4\pm2.3)\times10^{-4}\,\text{GeV}^{-2}\,,$$

$$k_3 = (1.25\pm0.23)\times10^{-4}$$

$$g_{a\gamma\gamma} = 4\pi\alpha_{em}F_{a\gamma\gamma} = -\frac{\alpha_{em}}{2\pi f_a}(1.63\pm0.01)$$

which can be compared to: 1.92±0.04 [Grilli de Cortona, et al., JHEP'16] **and 2.05±0.03** [Lu, et al., JHEP'20]

- **IB corrections could cause visible effects. See Poster of Jin Hao !**

Cosmology constraints on axion thermalization rate

Axion thermal production in the early Universe : Extra radiation (ΔN_{eff})

Extra effective number of relativisitc d.o.f :

$$\Delta N_{\text{eff}} \simeq \frac{4}{7}\left(\frac{43}{4 g_{\star s}(T_D)}\right)^{\frac{4}{3}}$$

$g_{\star s}(T)$: effective number of entropy d.o.f at temperature T

T_D: axion decoupling temperature from the thermal medium

- **CMB constraint (Plank'18)** [Aghanim et al., 2020] **:** $\Delta N_{\text{eff}} \leq 0.28$
- **T_D : Instantaneous decoupling approximation**

$$\Gamma_a(T_D) = H(T_D)$$

Axion thermalization rate

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int d\tilde{\Gamma} \sum |\mathcal{M}_{a-\text{SM}}|^2 n_B(E_1) n_B(E_2) [1 + n_B(E_3)][1 + n_B(E_4)]$$

Axion-SM particle scattering amplitudes

Hubble expansion parameter

$$H(T) = T^2 \sqrt{4\pi^3 g_*(T)/45} / m_{\text{Pl}}$$

Key thermal channels of axion-SM scatterings at different temperatures

- $T_D \gtrsim 1$ GeV: $ag \leftrightarrow gg$.
  [Masso et al., 2002, Graf and Steffen, 2011]
- $T_D \lesssim 1$ GeV: Hadrons need to be included.
- $T_D \lesssim 200$ MeV: $a\pi \leftrightarrow \pi\pi$.
  [Chang and Choi, 1993, Hannestad et al., 2005, Giusarma et al., 2014, D'Eramo et al., 2022]

❑ **Reliable $a\pi$ interaction is crucial to determine Γ_a for $T_D < T_c \approx 155$ MeV**

- **For a long time, only the LO $a\pi \leftrightarrow \pi\pi$ amplitude is employed to calculate Γ_a, e.g.,**
  [Chang, Choi, PLB'93] [Hannestad, et al., JCAP'05] [Hannestad, et al., JCAP'05] [D'Eramo, et al., PRL'22] ... ...

- **Recent NLO calculation of Γ_a : χPT invalid for $T_\chi > 70$ MeV [Di Luzio, et al., PRL'21]**

- **Chiral unitarization approach: [Di Luzio, et al., PRD'23]**

- **However, to our knowledge, all the previous works have ignored the thermal corrections to the $a\pi \leftrightarrow \pi\pi$ amplitudes. We give the first estimation of such effects on the determination of axion parameters. [Wang, ZHG, Zhou, PRD'24]**

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int d\tilde{\Gamma} \sum |\mathcal{M}_{a\pi;\pi\pi}|^2 n_B(E_1) n_B(E_2) \, [1 + n_B(E_3)][1 + n_B(E_4)]$$

Calculation of thermal *aπ* ↔*ππ* amplitudes at one-loop level

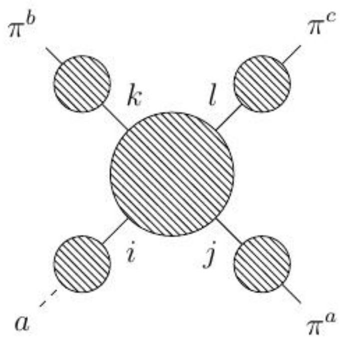
- Finite-temperature effects are included by imaginary time formalism (ITF), where [Kapusta and Gale, 2011, Bellac, 2011, Laine and Vuorinen, 2016]

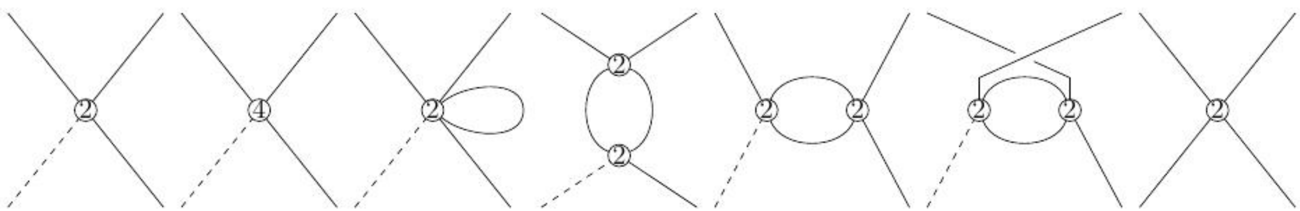
$$p^0 \to i\omega_n , \quad \text{with}\, \omega_n = 2\pi n T\,, n \in \mathbb{Z}\,,$$

$$-i\int \frac{\mathrm{d}^d q}{(2\pi)^d} \to -i\int_\beta \frac{\mathrm{d}^d q}{(2\pi)^d} \equiv T\sum_n \int \frac{\mathrm{d}^{d-1} q}{(2\pi)^{d-1}}\,.$$

- Compute the thermal Green functions in ITF

$$G^T_{a\pi^a;\pi^b\pi^c}(p_1,p_2;p_3,p_4) = \sum_{i,j,k,l} G_{ai}(p_1^2)G_{\pi^a j}(p_2^2)G_{k\pi^b}(p_3^2)G_{l\pi^c}(p_4^2)A_{ij;kl}(p_1,p_2;p_3,p_4)\,.$$





Feynman diagrams for amputated functions up to NLO.

- The effective Lagrangian at $\mathcal{O}(p^4)$

$$\mathcal{L}_4 \supset \frac{l_3+l_4}{16}\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle + \frac{l_4}{8}\left\langle \partial_\mu U \partial^\mu U^\dagger\right\rangle\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle$$
$$-\frac{l_7}{16}\left\langle \chi_a U^\dagger - U\chi_a^\dagger\right\rangle\left\langle \chi_a U^\dagger - U\chi_a^\dagger\right\rangle + \frac{h_1-h_3-l_4}{16}\left[\left(\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle\right)^2\right.$$
$$\left.+\left(\left\langle \chi_a U^\dagger - U\chi_a^\dagger\right\rangle\right)^2 - 2\left\langle \chi_a U^\dagger \chi_a U^\dagger + U\chi_a^\dagger U\chi_a^\dagger\right\rangle\right] + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\mathrm{NLO}}\,,$$

$$J_A^\mu|_{\mathrm{NLO}} \supset -il_1\left\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\right\rangle\left\langle \partial_\nu U\partial^\nu U^\dagger\right\rangle$$
$$-i\frac{l_2}{2}\left\langle Q_a\left\{\partial_\nu U, U^\dagger\right\}\right\rangle\left\langle \partial^\mu U\partial^\nu U^\dagger + \partial^\nu U\partial^\mu U^\dagger\right\rangle$$
$$-i\frac{l_4}{4}\left\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\right\rangle\left\langle \chi_a U^\dagger + U\chi_a^\dagger\right\rangle\,.$$

For details, see Poster of Jin-Bao Wang !

Unitarization of the partial-wave $a\pi \leftrightarrow \pi\pi$ amplitude

Inverse amplitude method (IAM)

$$\mathcal{M}^{\rm IAM}_{a\pi;IJ} = \frac{\left(\mathcal{M}^{(2)}_{a\pi;IJ}\right)^2}{\mathcal{M}^{(2)}_{a\pi;IJ} - \mathcal{M}^{(4)}_{a\pi;IJ}}$$

$$\mathcal{M}_{a\pi;IJ}(E_{cm}) = \frac{1}{2}\int_{-1}^{+1} {\rm d}\cos\theta\, \mathcal{M}_{a\pi;I}(E_{cm}, \cos\theta) P_J(\cos\theta)$$

$${\rm Im}\mathcal{M}_{a\pi;IJ}(E_{cm}) \stackrel{s}{=} \frac{1}{2}\rho^T_{\pi\pi}(E_{cm})\mathcal{M}^{IJ*}_{\pi\pi;\pi\pi}\mathcal{M}_{a\pi;IJ}\,, \quad (E_{cm} > 2m_\pi)$$

$$\rho^T_{\pi\pi}(E_{cm}) = \frac{\sigma_\pi(E^2_{cm})}{16\pi}\left[1 + 2n_B(\frac{E_{cm}}{2})\right]\,, \qquad \sigma_\pi(s) = \sqrt{1 - \frac{4m^2_\pi}{s}}\,, \quad n_B(E) = \frac{1}{e^{E/T} - 1}$$

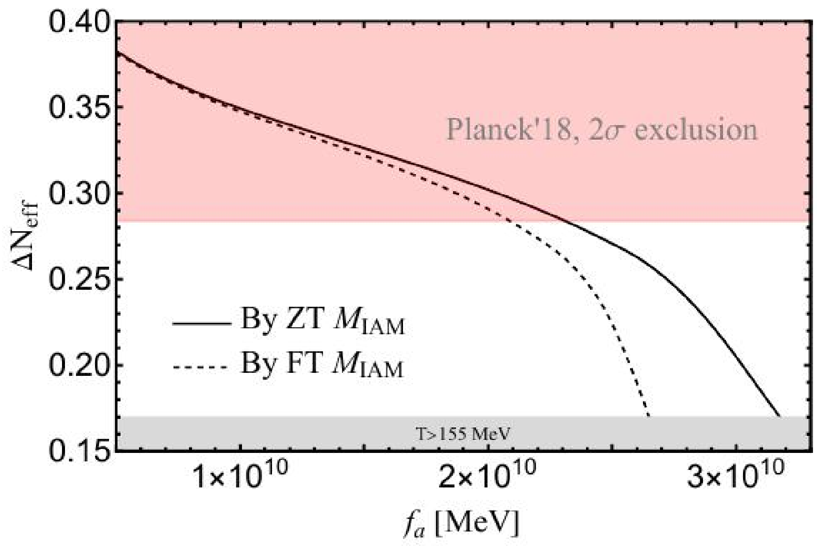
- Resonances poles on the second Riemann sheet

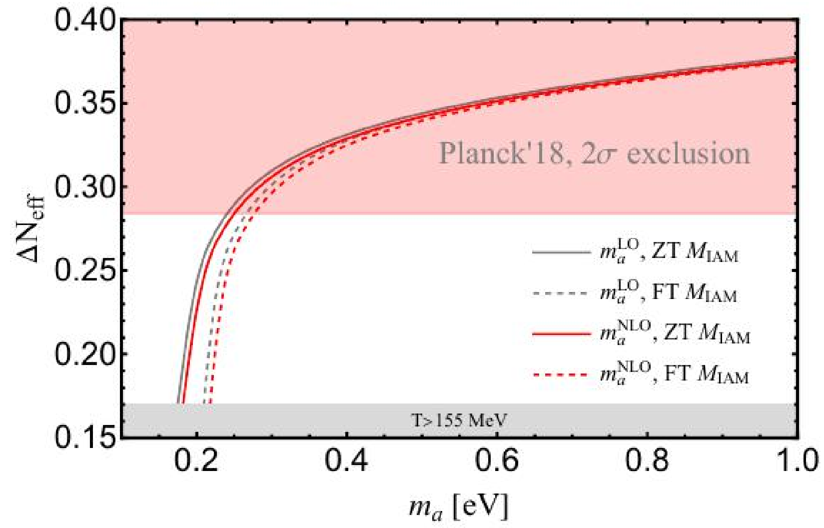
	$f_0(500)/\sigma$		$\rho(770)$	
	$M_\sigma \pm i\frac{\Gamma_\sigma}{2}$	$\lvert f_a g_{\sigma a\pi}\rvert$	$M_\rho \pm i\frac{\Gamma_\rho}{2}$	$\lvert f_a g_{\rho a\pi}\rvert$
$T = 0$ MeV	$422 \pm i240$ MeV	$0.032\ {\rm GeV}^2$	$739 \pm i72$ MeV	$0.035\ {\rm GeV}^2$
$T = 100$ MeV*	$368 \pm i310$ MeV	$0.037\ {\rm GeV}^2$	$744 \pm i77$ MeV	$0.036\ {\rm GeV}^2$

*Only include s-channel unitary thermal correction.

Updated bounds on the axion parameters

[Wang,ZHG,Zhou,PRD'24]





- The constrians **10% corrections are observed**

	lower limit of f_a	upper limit of m_a by $m_a^{\rm LO}$	upper limit of m_a by $m_a^{\rm NLO}$
ZT	2.3×10^7 GeV	0.24 eV	0.25 eV
FT	2.1×10^7 GeV	0.27 eV	0.28 eV

- The QCD axion mass up to LO & NLO

$$m_a^2|_{\rm LO} = \gamma_{ud}\, m_\pi^2 \frac{F^2}{f_a^2}\,, \qquad \text{where} \quad \gamma_{ud} = \frac{m_u m_d}{(m_u+m_d)^2}\,,$$

$$m_a^2|_{\rm NLO} = \gamma_{ud}\, m_\pi^2 \frac{F^2}{f_a^2}\left\{1 - 2\frac{m_\pi^2}{(4\pi F)^2}\log\frac{m_\pi^2}{\mu^2} + 2\left[h_1^r(\mu^2) - h_3\right]\frac{m_\pi^2}{F^2} - 8 l_7 \gamma_{ud}\frac{m_\pi^2}{F^2}\right\}$$

Axion production from η→ππa decay in SU(3) χPT

Why focus on axion in η decay:

- ✓ **Valuable channel to search axion @colliders: many available experiments with large data samples of η/η' [BESIII, STCF, JLab, REDTOP, … …]**
- ✓ **η→πππ (IB suppressed), η→ππa (no IB suppression)**
- ✓ **η→ππa: theoretically easier to handel than η'→ππa (next step)**

Previous works:

- ❖ **Most of them rely on leading-order χPT**
- ❖ **Possible issue: bulk contributions@LO χPT are constant terms, and potential large corrections from higher orders may result.**
- ❖ **Hadron resonance effects may lead to enhancements.**

Advances in our work :

- ➢ **Study of renormalization of η→ππa @1-loop level in SU(3) χPT**
- ➢ **To implement unitarization to the η→ππa χPT amplitude**
- ➢ **Uncertainty analyes in the phenomenological discussions**

LO χPT Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger\rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu\big|_{\rm LO} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2$$

$$\chi_a = 2B_0 M(a) \qquad M(a) \equiv \exp\left(-i\frac{a}{2f_a}Q_a\right) M \exp\left(-i\frac{a}{2f_a}Q_a\right) \qquad J_A^\mu\big|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\rangle$$

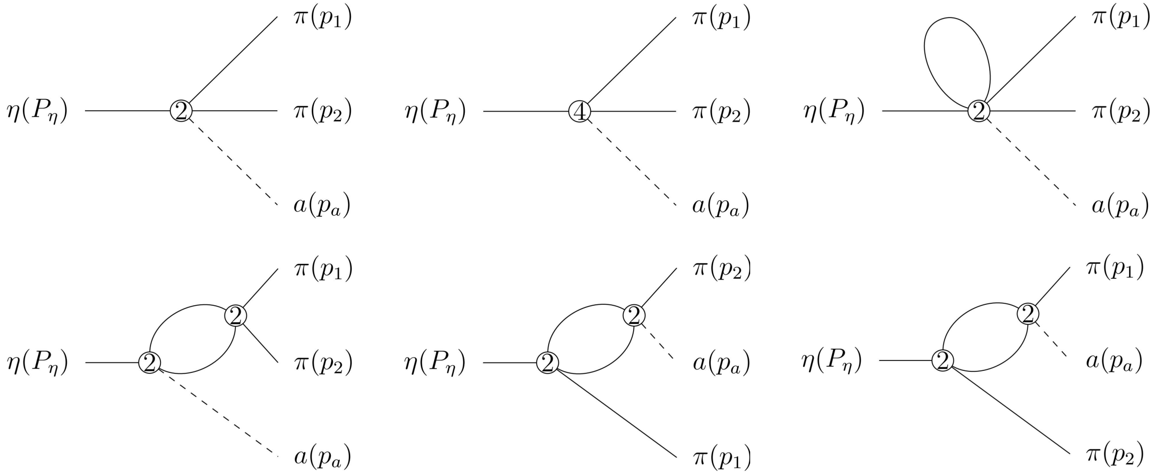
Note: we consider the octet part ($\bar{Q}_a$) of Q_a in SU(3) χPT

NLO χPT Lagrangian

$$\mathcal{L}_4 = L_1\langle \partial_\mu U \partial^\mu U^\dagger\rangle\langle \partial_\nu U \partial^\nu U^\dagger\rangle + \cdots + \frac{\partial_\mu a}{2f_a} J_A^\mu\big|_{\rm NLO},$$

$$J_A^\mu\big|_{\rm NLO} = -4iL_1\langle \bar{Q}_a\{U^\dagger, \partial^\mu U\}\rangle\langle \partial_\nu U \partial^\nu U^\dagger\rangle + \cdots .$$

Feynman diagrams up to NLO



Parameters

Masses and F_π [MeV]				LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of 10^{-3})							
m_π	m_K	m_η	F_π	L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
137	496	548	92.1	1.0(1)	1.6(2)	−3.8(3)	0.0(3)	1.2(1)	0.0(4)	−0.3(2)	0.5(2)

[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

✓ **Renomarlization condition is verified to be consistent with conventional ChPT.**

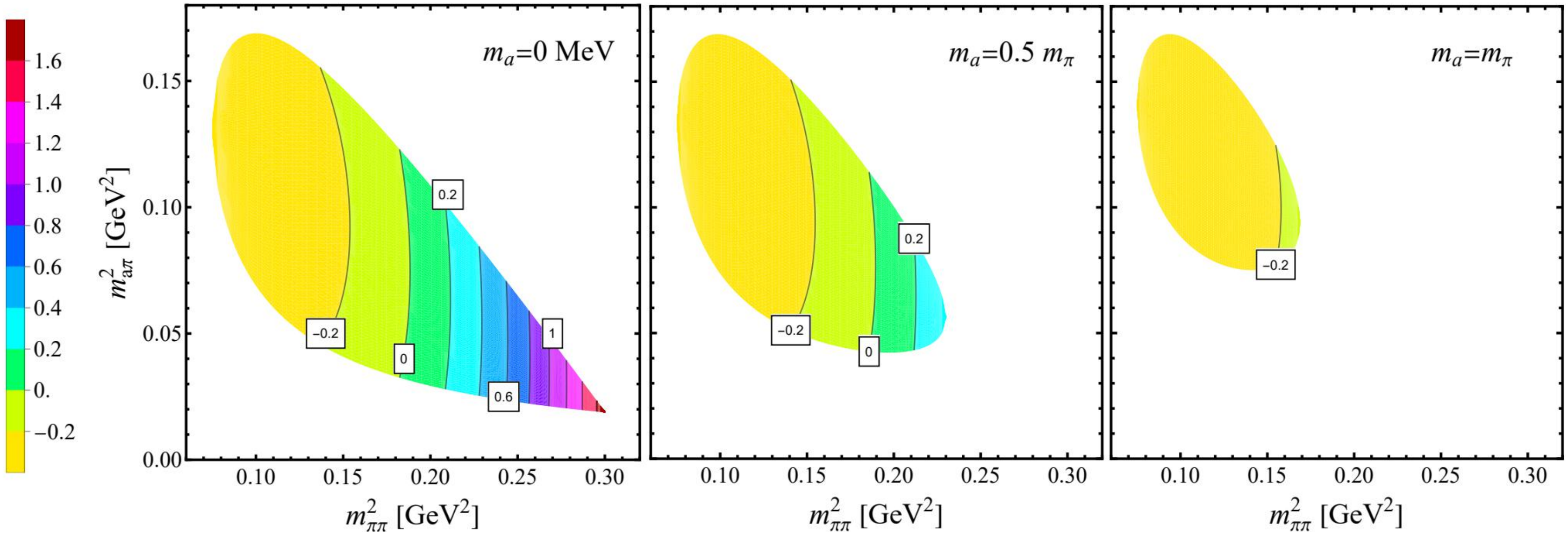
Observations:

- Strong isospin breaking effects enter the $\eta\to\pi\pi a$ amplitudes at the order of $(m_u-m_d)^2$
- In the isospin limit ($m_u=m_d$), the amplitudes with $\pi^+\pi^-$ and $\pi^0\pi^0$ in $\eta\to\pi\pi a$ processes are identical.

- **Dalitz plots to show the NLO/LO convergence**

$$\left(2\mathcal{M}^{(2)}_{\eta;\pi\pi a}\mathrm{Re}\left(\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right)+\left|\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right|^2\right)/\left|\mathcal{M}^{(2)}_{\eta;\pi\pi a}\right|^2$$

[Wang,ZHG,Lu,Zhou, 2403.16064, To appear in JHEP]



Important lessons:

- Non-perturbative effect in the $\pi\pi$ subsystem can be important.
- Perturbative treatment of the $a\pi$ subsystem is justified.

- **Unitarization of the partial-wave η→ππa amplitude**

$$\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)} \,,$$

$$G_{\pi\pi}(s) = -\frac{1}{(4\pi)^2}\left(\log\frac{m_\pi^2}{\mu^2} - \sigma_\pi(s)\log\frac{\sigma_\pi(s)-1}{\sigma_\pi(s)+1} - 1\right)\,,$$

$$\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s) = \mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s) + \mathcal{M}^{00,(4)}_{\eta;\pi\pi a}(s) - G_{\pi\pi}(s)\,\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)\,T^{00,(2)}_{\pi\pi\to\pi\pi}(s)\,.$$

The unitarized amplitude satisfies the relation

$$\mathrm{Im}\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \rho_{\pi\pi}(s)\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s)\left(T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s)\right)^* \,, \qquad (2m_\pi < \sqrt{s} < 2m_K)$$

with the unitarized PW ππ amplitude $T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \dfrac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$

- **Unitarized PW amplitude based on LO η→ππa amplitude**

$$\mathcal{M}^{00,\mathrm{Uni-LO}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}\,.$$

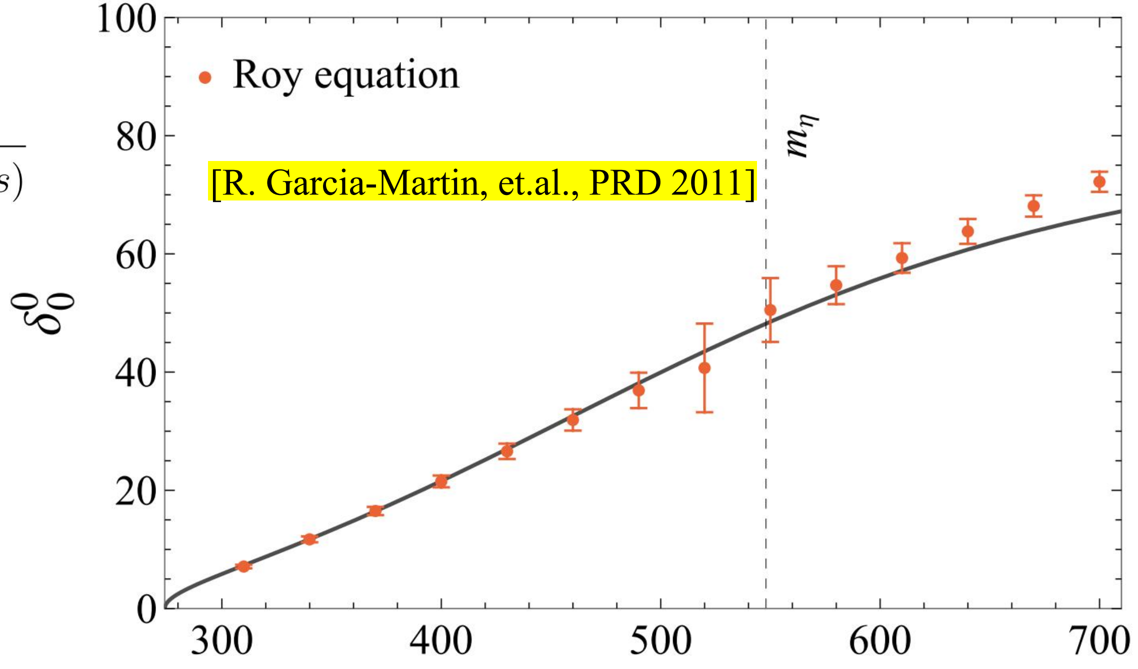
Resemble the method:

Alves and Sergi, arXiv:2402.02993 [hep-ph].

$$M_0(s) = P(s)\Omega_0^0(s)$$

Phase shifts from the unitarized PW $\pi\pi$ amplitude

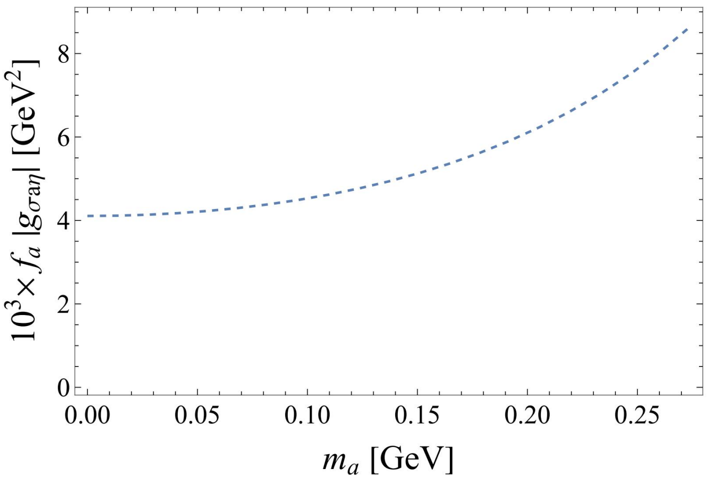
$$T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \frac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s)T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$$



- **Pole position of $f_0(500)/\sigma$:**

$$\sqrt{s_\sigma} = 457 \pm i251 \text{ MeV}$$

$$\mathcal{M}^{00,\mathrm{Uni,II}}_{\eta;\pi\pi a}(s)\big|_{s\to s_\sigma} \sim -\frac{g_{\sigma\pi\pi}\, g_{\sigma a\eta}}{s - s_\sigma}$$



Predictions of the η→ππa branching ratios by varying m_a

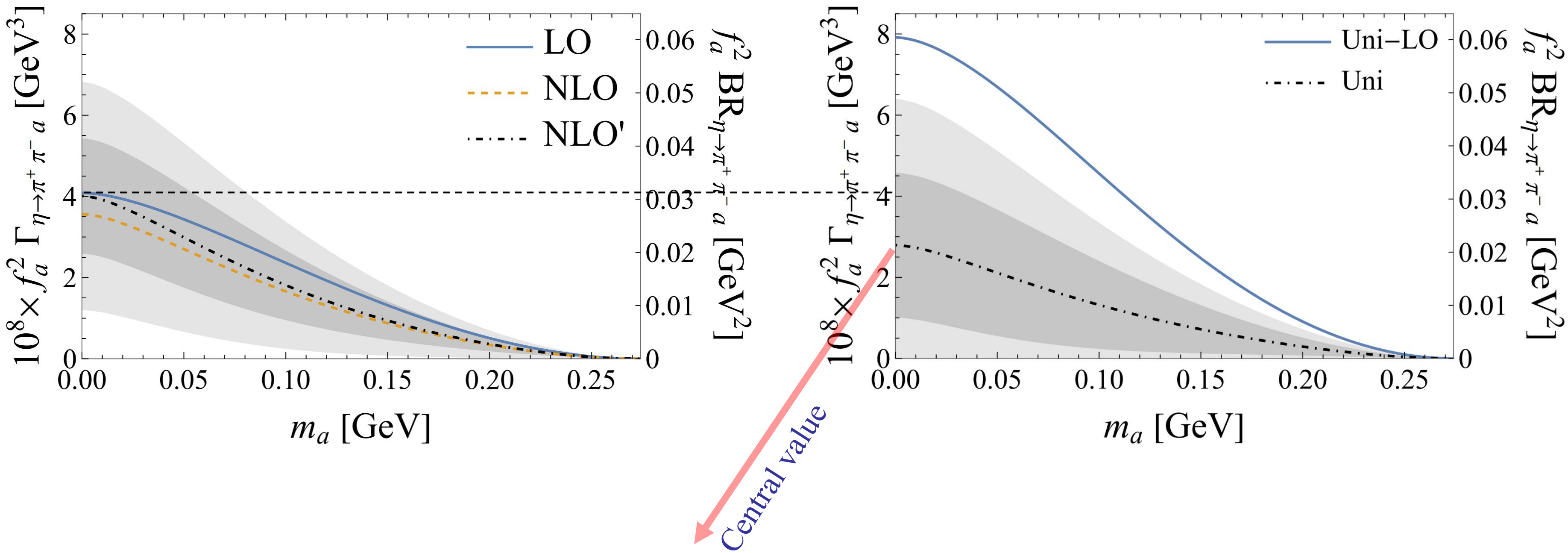
Uncertainty bands:

- **Lighter regions:**

L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
1.0(1)	1.6(2)	−3.8(3)	0.0(3)	1.2(1)	0.0(4)	−0.3(2)	0.5(2)

- **Darker regions: freeze the 1/Nc suppressed ones (L_4,L_6,L_7)**

[Wang,ZHG,Lu,Zhou, 2403.16064, To appear in JHEP]



$$\mathrm{BR}_{\eta\to\pi^+\pi^- a}\Big|_{m_a\to 0} = 2.1\times 10^{-2}\left(\frac{\mathrm{GeV}^2}{f_a^2}\right)$$

Possible detection channels: a→γγ, a→e^+e^-, a→$\mu^+\mu^-$

Summary

- Chiral perturbation theory provides a systematical and useful framework to study the axion-meson interactions .
- π-η-η'-a mixing and $g_{a\gamma\gamma}$ are predicted in U(3) AχPT by taking the various hadronic and lattice inputs.
- Thermal $a\pi \leftrightarrow \pi\pi$ amplitudes in SU(2) AχPT are worked out. Thermal corrections to amplitudes cause around 10% shift of the axion parameters.
- Axion production from the $\eta \to \pi\pi a$ decay is calculated. Large uncertainties from higher-order LECs are found.
- More promising studies of axion-hadron interactions are on the way.

谢谢！