Modeling $0\nu\beta\beta$ decay based on nuclear forces and transition operators from chiral effective field theory

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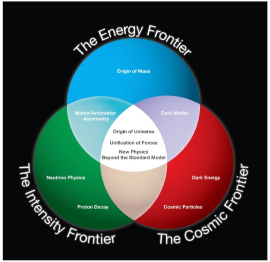
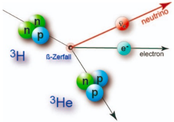
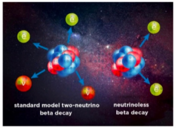
Probe new physics at the nuclear energy scale

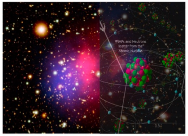




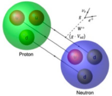
Single-beta decay



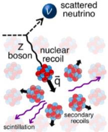
Neutrinoless double beta decay



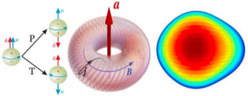
Dark matter direct detection



Superallowed Fermi transitions



Neutrino scattering

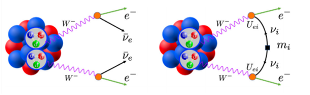


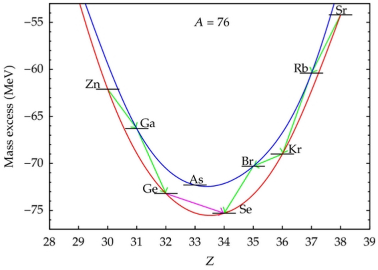
Symmetry-violating moments

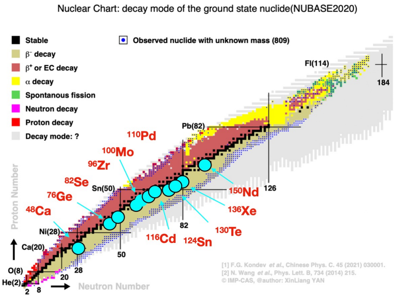
- **High-intensity frontiers**: searching for $0\nu\beta\beta$ decay, dark matter detector, atomic EDM, etc.
- **Accurate nuclear matrix elements**: crucial for testing fundamental symmetries and interactions with low-energy probes.

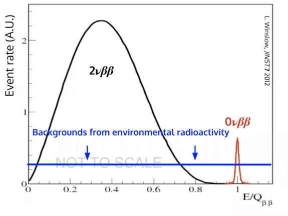
- The two modes of $\beta^-\beta^-$ decay:

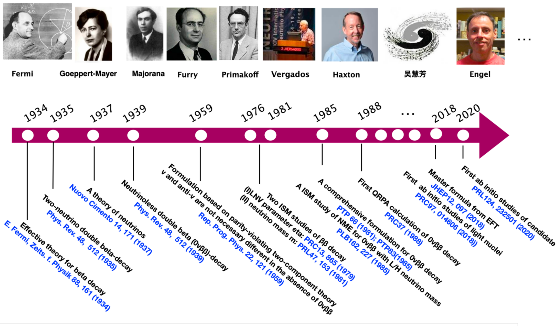
$$(A, Z) \to (A, Z+2) + 2e^- + (2\bar{\nu}_e)$$

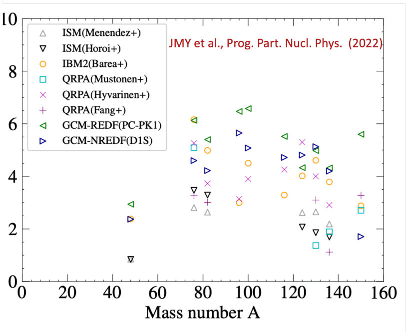








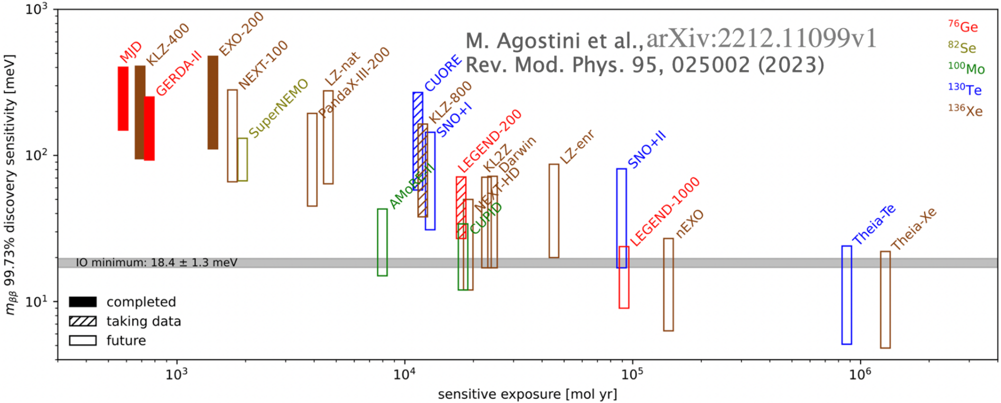




If $0\nu\beta\beta$ decay is driven by exchanging light massive Majorana neutrinos,

$$\langle m_{\beta\beta}\rangle \equiv \left|\sum_{j=1}^{3} U_{ej}^2 m_j\right| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T_{1/2}^{0\nu} |M^{0\nu}|^2}\right]^{1/2}$$

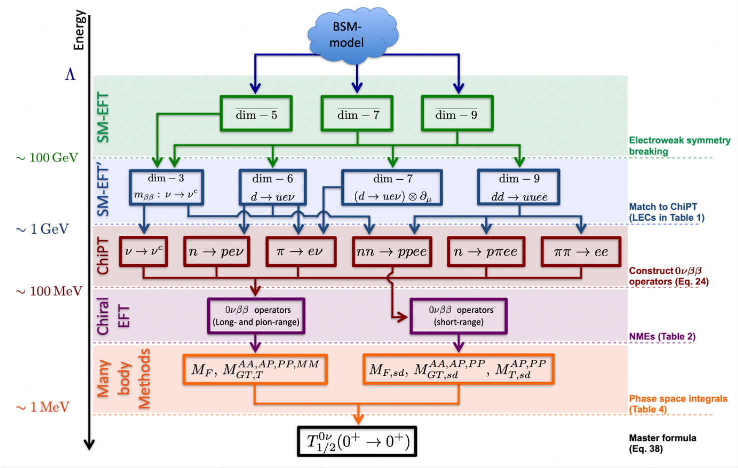
Accurate values of the NMEs $M^{0\nu}$ are crucial for designing and interpreting those experiments, as they link the observed decay rate to the neutrino mass scale.

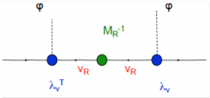


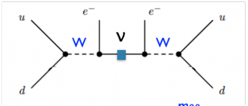
- Lifetime sensitivity of the ton-scale experiments: $T^{0\nu}_{1/2} > 10^{28}$yr.
- Whether or not the ton-scale experiments are able to cover the entire parameter space for the IO case depends strongly on the employed NME.

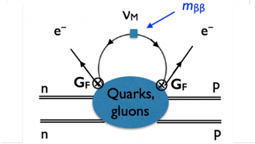
The EFT provides a model-independent framework for describing $0\nu\beta\beta$ decay.

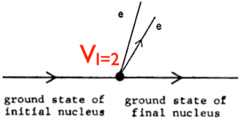
Cirigliano (2018)

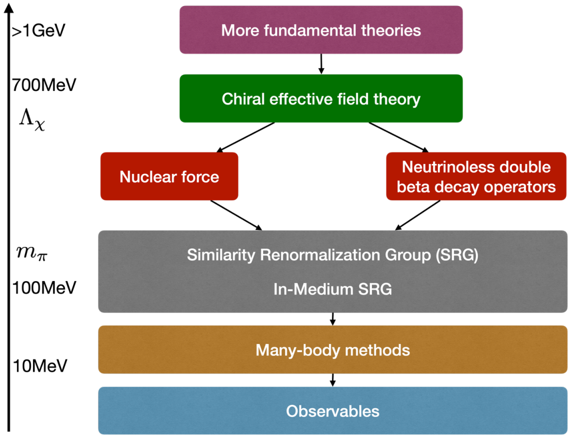












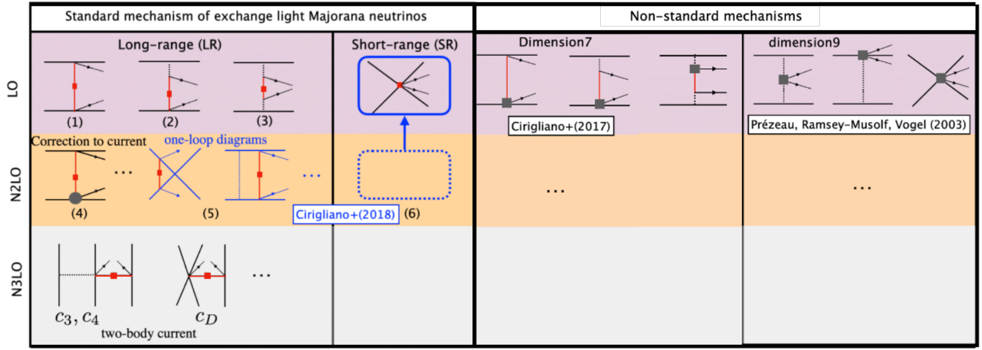
The basic idea of current efforts:

- Indentify the active dof at the nuclear energy scale: $N, \pi, (e, \nu)$
- Write down all possible contributions to both nuclear force and transition operators according to a power counting rule, $(Q, m_\pi)/\Lambda_\chi$.
- Carry out a quantum many-body calculation and compute the NME.

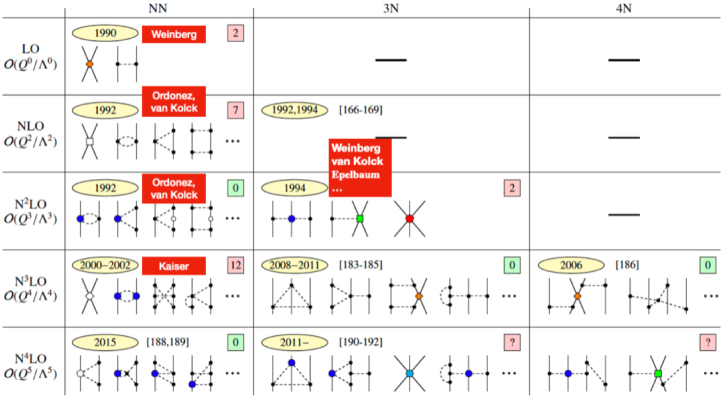
The $0\nu\beta\beta$ decay operators in the chiral EFT

- At $E \sim$100 MeV: operators are expressed in terms of nucleons, pions, and leptons, arranged in the order $(Q, m_\pi/\Lambda_\chi)^\nu$,

$$\nu = 2A + 2L - 2 + \sum_i \left(\frac{n_f}{2} + d - 2 + n_e\right)_i$$



- Non-relativistic chiral 2N+3N interactions (Weinberg power counting and others)

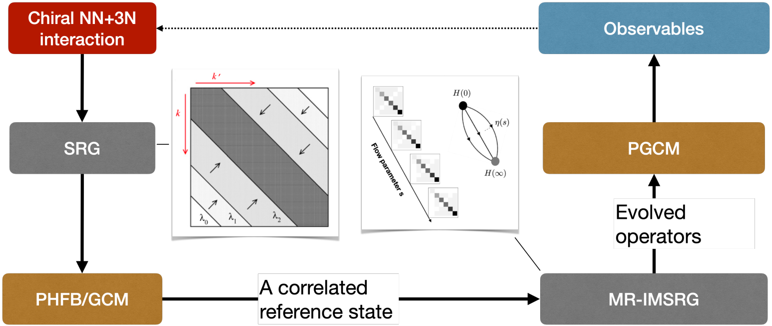


K. Hebeler, Phys. Rep. 890, 1 (2020)

- Relativistic chiral 2N interaction (up to N^2LO, different PC from the NR case)

J.-X. Lu et al., PRL128, 142002 (2022)

The Framework of IM-GCM



- **In-medium similarity renormalization group (IMSRG)**: capture dynamic correlations associated with high-energy few-particle, few- hole excitations
- **Projected generator coordinate method (PGCM)**: include the collective (static) correlations associated with pairing and deformation.

- Multi-reference in-medium generator coordinate method (IM-GCM)

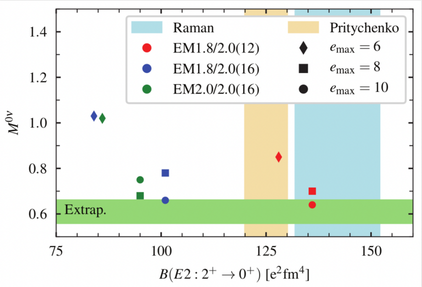
  JMY et al., PRL124, 232501 (2020)

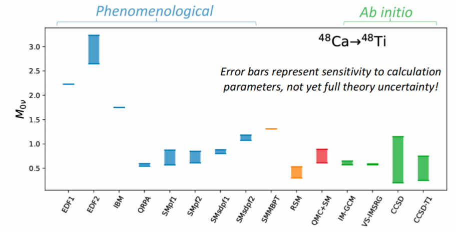
- Valence-space shell model+IMSRG (VS-IMSRG)

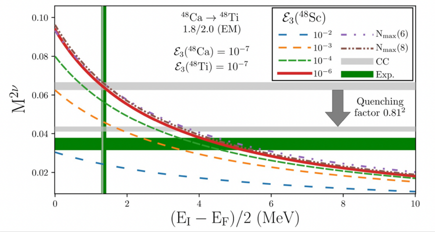
  A. Belley et al., PRL126, 042502 (2021)

- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1) .

  S. Novario et al., PRL126, 182502 (2021)



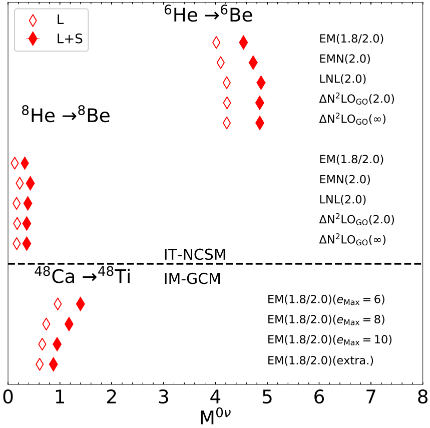




Contribution of the contact transition operator

- The contact transition operator could either enhance or quench the $0\nu\beta\beta$ decay of cadidate nuclei.
- The LEC g_ν^{NN} consistent with the employed chiral interaction (EM1.8/2.0) is determined based on the synthetic data.
- The contact term turns out to enhance the NME for ^{48}Ca by 43(7)%, thus reducing the half-life $T_{1/2}^{0\nu}$ significantly.

R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)



The true value of the NME can be written as

$$M^{0\nu} = M_k^{0\nu} + \epsilon_{\chi\text{EFT}} + \epsilon_{\text{MBT}} + \epsilon_{\text{OP}} + \epsilon_{\text{EM}},$$

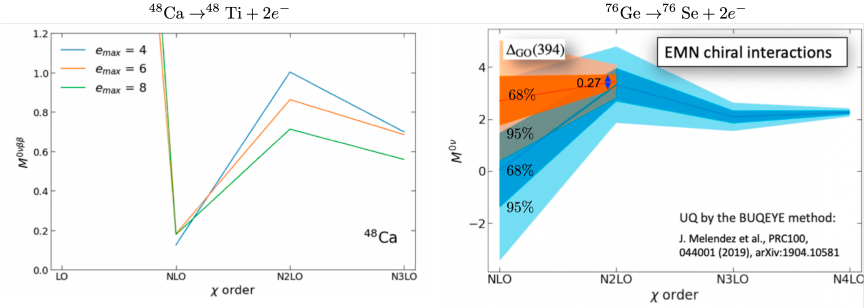
where the posterior probability distribution (PPD) of an LEC sample to yield results for a set of calibration observables that match experimental data

$$\text{PPD} = \{M_k^{0\nu}(\mathbf{c}) : \mathbf{c} \sim \mathcal{P}(\mathbf{c}|\text{calibration})\}.$$

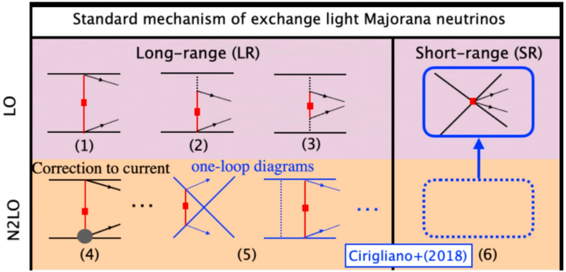
from which one finds the statistical error χ_{LEC},

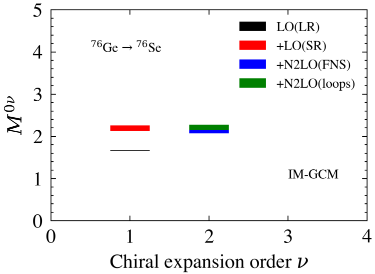
- $\epsilon_{\chi\text{EFT}}$: chiral expansion truncation on nuclear forces
- ϵ_{MBT}: approximation in many-body methods
- ϵ_{OP}: chiral expansion truncation on transition operators
- ϵ_{EM}: the error of the emulator

All the errors ϵ are assumed to be normally distributed and mutually independent.



- The NME converges with respect to the chiral expansion order χ of nuclear forces for candidate nuclei ^{48}Ca and ^{76}Ge.
- The EFT truncation error (evaluated using the BUQEYE method) is shrinking with χ.





- In the N2LO, we choose $\mu_{us} = m_\pi$ to eliminate all the terms depending on $\ln \frac{m_\pi^2}{\mu_{us}^2}$, and the LECs of the counterterms as $g_\nu^{\pi\pi} = -7.6 - (36/5)\ln(\mu/m_\rho)$, and $g_\nu^{\pi N} = 0$. V. Cirigliano, et al., PRC97, 065501 (2018)
- The NME for ^{76}Ge converges with respect to the chiral expansion order of transition operators.

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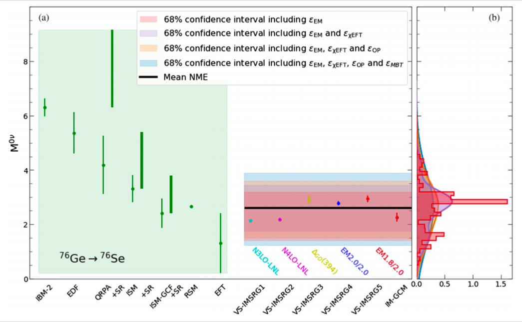
truncation of transition operators — Emulator error

$$M^{0\nu} = M^{0\nu}_k + \epsilon_{\chi\mathrm{EFT}} + \epsilon_{\mathrm{MBT}} + \epsilon_{\mathrm{OP}} + \epsilon_{\mathrm{EM}},$$

truncation of nuclear forces — many-body truncation

TABLE I. The recommended value for the total NME of $0\nu\beta\beta$ decay in ^{76}Ge, together with the uncertainties from different sources.

$M^{0\nu}$	ϵ_{LEC}	$\epsilon_{\chi\mathrm{EFT}}$	ϵ_{MBT}	ϵ_{OP}	ϵ_{EM}
$2.60^{+1.28}_{-1.36}$	0.75	0.3	0.88	0.47	< 0.06



- Our recommended value $M^{0\nu} = 2.60^{+1.28}_{-1.36}$.
- Together with the best half-life limit: $> 1.8 \times 10^{26}$ yr, it sets the upper limit $\langle m_{\beta\beta}\rangle = 187^{+205}_{-62}$ meV, and the sensitivity of the next-generation experiment $\langle m_{\beta\beta}\rangle = 22^{+24}_{-7}$ meV, covering almost the entire range of IO hierarchy.

A. Belley, JMY et al., PRL132, 182502 (2024)

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- $0\nu\beta\beta$ decay: only way to determine the nature of neutrinos, a complementary way to determine the absolute mass scale of neutrinos. **The NMEs of candidate nuclei are crucial for designing and interpreting those experiments.**
- Large uncertainty in NMEs: major systematical uncertainty, impacting the interpretation of the measurements.
- Remarkable progress in ab initio studies of NMEs: **development of a novel ab initio method for candidate nuclei, disclosing non-trivial contributions from high-energy light neutrinos, and rapid convergence w.r.t. the chiral expansion order, a comprehensive uncertainty quantification.**

What's next?

- The NMEs of heavier candidates ^{82}Se, ^{100}Mo, ^{130}Te, ^{136}Xe, with reduced uncertainty by considering higher-order nuclear interactions, reducing many-body truncation errors, and finding more constraints to shrink the uncertainty.
- Contributions from non-standard mechanisms.

Collaborators

- SYSU
  C.R. Ding, Q.Y. Luo, C.F. Jiao, C.C. Wang, G. Li, X. Zhang, E.F. Zhou
- PKU
  L. S. Song, J. Meng, P. Ring, Y. K. Wang, P. W. Zhao
- LZU: Y.F. Niu
- CAEP: B.N. Lv
- SWU: L.J. Wang
- MSU: S. Bogner, H. Hergert, R. Wirth
- UNC: J. Engel, A. M. Romero
- TRIUMF: A. Belly, J. Holt
- TU Darmstadt: T. Miyagi
- Notre-Dame U: R. Stroberg
- UAM: B. Bally, T.Rodriguez

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Thank you for your attention!

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- Leading order (LO)

A. Belley, JMY et al., PRL132, 182502 (2024)

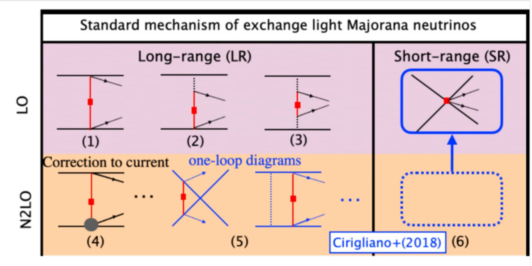
$$O^{\rm LO}_{\alpha,L}(r_{12}) = \frac{2R}{\pi g_A^2(0)}\int_0^\infty {\rm d}q\, q^2 \frac{h^{\rm LO}_{\alpha,L}(q^2)}{q(q+E_d)} j_L(qr_{12}),$$

$$O^{0\nu}_{\rm LO,SR} = -2g_\nu^{NN}\left(\frac{m_N g_A^2}{4f_\pi^2}\right) f_\Lambda^{n_{\rm exp}}(r_{12}) f_\Lambda^{n_{\rm exp}}(r'_{12})$$

$$h^{\rm LO}_{\rm F,0}(q^2) = -g_V^2(0),$$

$$h^{\rm LO}_{\rm GT,0}(q^2) = g_A^2(0) - \frac{2}{3}\frac{q^2}{2m_p} g_A(0) g_P(0) + \frac{1}{3}\frac{q^4}{4m_p^2} g_P^2(0),$$

$$h^{\rm LO}_{\rm T,2}(q^2) = \frac{2}{3}\frac{q^2}{2m_p} g_A(0) g_P(0) - \frac{1}{3}\frac{q^4}{4m_p^2} g_P^2(0).$$



- N2LO

correction to current and induced weak-magnetism

$$g_V(q^2) = g_V(0)\left(1 + q^2/\Lambda_V^2\right)^{-2},$$

$$g_A(q^2) = g_A(0)\left(1 + q^2/\Lambda_A^2\right)^{-2}.$$

$$h^{\rm N2LO}_{\rm GT,0}(q^2) = g_M^2(q^2)\frac{1}{6}\frac{q^2}{m_p^2},$$

$$h^{\rm N2LO}_{\rm T,2}(q^2) = g_M^2(q^2)\frac{1}{12}\frac{q^2}{m_p^2},$$

loop diagrams

$$O^{0\nu}_{\rm N2LO,GT} = \frac{4R}{3\pi m_\pi^2 T^2}\int q^4 dq\, j_0(qr)\Big(K_{VV}(\tilde q) - K_{AA}(\tilde q) - \frac{2g_A^2 m_\pi^2}{q^2+m_\pi^2}\ln\frac{m_\pi^2}{\mu_{us}^2} - K_{CT}(\tilde q)\Big)\sigma_1\cdot\sigma_2,$$

$$O^{0\nu}_{\rm N2LO,F} = \frac{4R}{\pi T^2}\int q^2 dq\, j_0(qr)\Big(\Big[\frac{3C_T T^2}{\pi^2} - \frac{2g_A^2 q^2}{q^2+m_\pi^2}\Big]\ln\frac{m_\pi^2}{\mu_{us}^2} - K'_{AA}(\tilde q)\Big)\mathbf{1}_1\times\mathbf{1}_2,$$

$$O^{0\nu}_{\rm N2LO,T} = -\frac{4R}{3m_\pi^2\pi T^2}\int q^4 dq\, j_2(qr)\Big(K_{VV}(\tilde q) - K_{AA}(\tilde q) - \frac{2g_A^2 m_\pi^2}{q^2+m_\pi^2}\ln\frac{m_\pi^2}{\mu_{us}^2} - K_{CT}(\tilde q)\Big)S^r_{12}(\hat r),$$

with $\tilde q = q^2/m_\pi^2$, $T = 4\pi f_\pi$, and $L_\pi = \ln\frac{\mu^2}{m_\pi^2}$,

$$K_{VV}(\tilde q) = \frac{2(1-\tilde q)^2}{\tilde q^2(1+\tilde q)}\ln(1+\tilde q) - \frac{2}{\tilde q} + \frac{7-3\tilde q L_\pi}{(1+\tilde q)^2} + \frac{L_\pi}{1+\tilde q},$$

$$K_{AA}(\tilde q) = \frac{g_A^2}{1+\tilde q}(L_\pi - 4) + \frac{1}{(1+\tilde q)^2},$$

$$K'_{AA}(\tilde q) = \frac{1}{g_A^2}\Big[-\frac{3}{4}\left(1-g_A^2\right)^2 L_\pi + g_A^4 f_4(\tilde q) + g_A^2 f_2(\tilde q) + f_0(\tilde q) + 24 g_A^2 f_\pi^2 C_T (L_\pi + 1)\Big],$$

$$K_{CT}(\tilde q) = \frac{5}{6}g_\nu^{\pi\pi}\frac{\tilde q}{(1+\tilde q)^2} - g_\nu^{\pi N}\frac{1}{1+\tilde q}.$$

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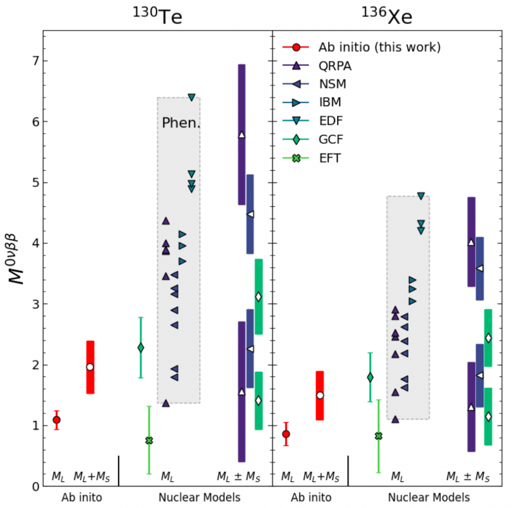
The ab initio VS-IMSRG method is applied to study the NMEs of heavier candidates:

- For ^{130}Te, $M^{0\nu}_{L+S} \in [1.52, 2.40]$
- For ^{136}Xe, $M^{0\nu}_{L+S} \in [1.08, 1.90]$

The uncertainty is composed of different sources: nuclear interaction, reference-state, basis extrapolation, closure approximation, and the LEC for the short-range transition operators.

The values are generally smaller than those from phenomenological nuclear models.

A. Belley et al, arXiv:2307.15156 (2023)



A more comprehensive quantification analysis

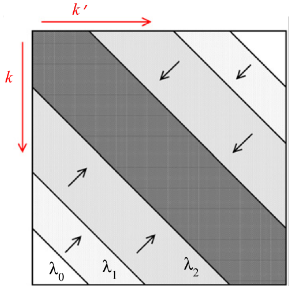
different nuclear many-body solvers, convergence of NMEs with chiral expansion orders, etc.

- Apply unitary transformations to decouple high and low-momentum states

$$H_s = U_s H U_s^\dagger \equiv T_{\rm rel} + V_s$$

from which one finds the flow equation

$$\frac{dH_s}{ds} = [\eta_s, H_s], \quad \eta_s = [T_{\rm rel}, H_s]$$

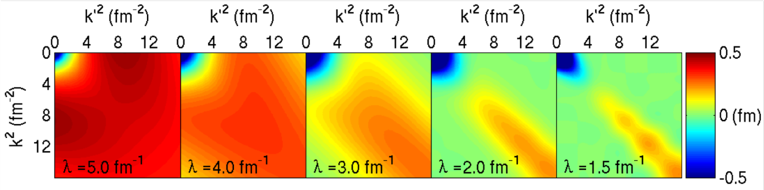


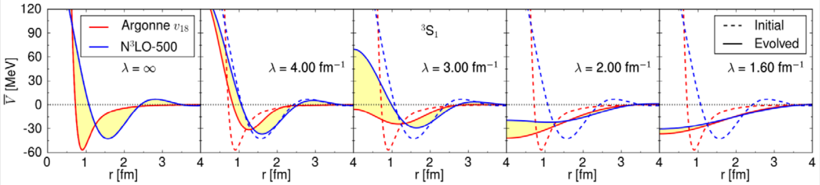
The flow parameter s is usually replaced with $\lambda = s^{-1/4}$ in units of fm^{-1} (a measure of the spread of off-diagonal strength).

Evolution of the potential

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

Preprocessing the nuclear potential with SRG





- The hard core "disappears" in the SRG softened interactions
- Induced higher-body interactions: $3N, \cdots$

S. K. Bogner et al. PPNP (2010); Wendt et al. PRC (2012)

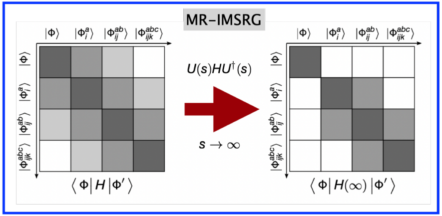
- Apply unitary transformations to H in the configuration space

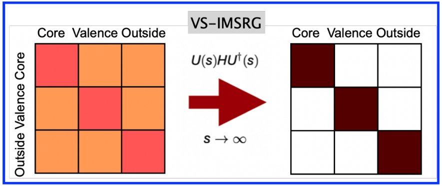
$$\hat{H}(s) = \hat{U}(s)\hat{H}_0\hat{U}^\dagger(s)$$

Flow equation

$$\frac{d\hat{H}(s)}{ds} = [\hat{\eta}(s), \hat{H}(s)]$$

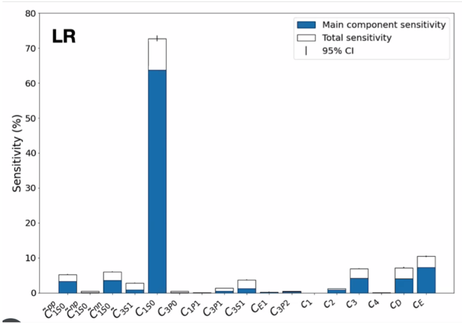
- Generator $\eta(s)$: chosen either to decouple a given reference state from its excitations or to decouple the valence space from the excluded spaces.
- Not necessary to construct the whole H matrix in the config. space.

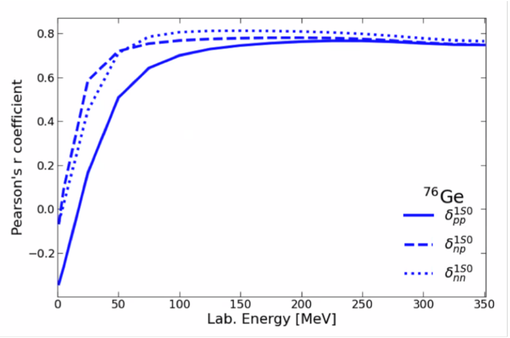




H. Hergert et al., Phys. Rep. 621, 165 (2016); S. R. Stroberg et al., Annu. Rev. Nucl. Part. Sci. 69, 307 (2019)

Quantification of statistic uncertainty in the NME of ^{76}Ge





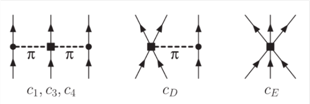
- The long-range part of the NME is sensitive to the LEC $C_{^1S_0}$.
- The phase shift of the $^1S_0$ channel is linearly correlated to the NME.
- The neutron-proton phase-shift δ_{np}^{1S0} at 50 MeV is used to weight the samples.

A. Belley et al., arXiv:2408.02169 [nucl-th]

Isotope	$G_{0\nu}$ $[10^{-14}\ \mathrm{yr}^{-1}]$	$M^{0\nu}(\chi\mathrm{EFT})$ [min, max]	$T^{0\nu}_{1/2}$ [yr]	$\langle m_{\beta\beta}\rangle$ [meV]	Worldwide Exps current best limits	Inside China
76Ge	0.24	$2.60^{+1.27}_{-1.36}$	$> 1.8 \cdot 10^{26}$	187^{+205}_{-62}	GERDA: PRL125, 252502(2020)	CDEX
82Se	1.01		$> 4.6 \cdot 10^{24}$	.	CUPID-0: PRL129, 111801 (2023)	NvDEx
^{100}Mo	1.59		$> 3.0 \cdot 10^{24}$		AMoRE: arXiv:2407.05618 [nucl-ex] (2024)	CPUID-China
^{130}Te	1.42	[1.52, 2.40]	$> 2.2 \cdot 10^{25}$	[236, 373]	CUORE: Nature 604, 53(2022)	JUNO
^{136}Xe	1.46	[1.08, 1.90]	$> 2.3 \cdot 10^{26}$	[91, 160]	KamLAND-Zen: PRL130, 051801(2023)	PANDAX

The magic chiral NN+3N interaction

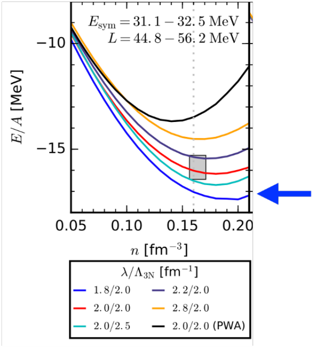
The "magic" interaction EM1.8/2.0: The NN (N^3LO: D.R. Entem, R. Machleidt, PRC68 041001 (2003)) and local 3N interactions (N^2LO: K. Hebeler et al., PRC**83**, 031301(R) (2011)).



The LECs of the 3N are fitted on top of the SRG evolved NN interaction.

TABLE I. Results for the c_D and c_E couplings fit to $E_{^3\text{H}} = -8.482$ MeV and to the point charge radius $r_{^4\text{He}} = 1.464$ fm (based on Ref. [26]) for the NN/3N cutoffs and different EM/EGM/PWA c_i values used. For $V_{\text{low}\,k}$ (SRG) interactions, the 3NF fits lead to $E_{^4\text{He}} = -28.22 \ldots -28.45$ MeV ($-28.53 \ldots -28.71$ MeV).

Λ or λ/Λ_{3NF} (fm)	$V_{\text{low}\,k}$ c_D	$V_{\text{low}\,k}$ c_E	SRG c_D	SRG c_E
1.8/2.0 (EM c_i's)	+1.621	−0.143	+1.264	−0.120
2.0/2.0 (EM c_i's)	+1.705	−0.109	+1.271	−0.131
2.0/2.5 (EM c_i's)	+0.230	−0.538	−0.292	−0.592
2.2/2.0 (EM c_i's)	+1.575	−0.102	+1.214	−0.137
2.8/2.0 (EM c_i's)	+1.463	−0.029	+1.278	−0.078
2.0/2.0 (EGM c_i's)	−4.381	−1.126	−4.828	−1.152
2.0/2.0 (PWA c_i's)	−2.632	−0.677	−3.007	−0.686



C. Drischler et al., PRL122, 042501 (2019)

The saturation properties are not well reproduced.