Baryon chiral perturbation theory in the presence of gravitational fields

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Outline

- Effective action of chiral EFT in curved spacetime;
- Energy-momentum-tensor;
- Gravitational form factors of the nucleon;
- Problem of defining spatial densities;
- Spatial densities corresponding to EMT;
- Summary;

Effective action of chiral EFT in curved spacetame

Gravitational Form Factors (GFFs) of hadrons characterize their internal structure.

They parameterize one-particle matrix elements of EMT operator.

GFFs cannot be directly measured in experiments.

However they can be accessed indirectly in processes like DVCS.

GFFs can be calculated in lattice QCD.

For small momentum transfers Chiral EFT can be used.

I will talk about nucleon, although the delta resonances and vector mesons have been also studied.

J. F. Donoghue and H. Leutwyler, Z. Phys. C **52**, 343 (1991),
H. Alharazin, *at al.* Phys. Rev. D **102**, 076023 (2020).

Action of pions and nucleons in curved spacetime:

$$
\begin{aligned}
S_\pi &= \int d^4x\sqrt{-g}\left\{\frac{F^2}{4}\, g^{\mu\nu}\,\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger) + \frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\} \\
S_{\pi\mathrm{N}} &= \int d^4x\sqrt{-g}\left\{\frac{1}{2}\,\bar\Psi\, i e^\mu_a \gamma^a \nabla_\mu \Psi - \frac{1}{2}\,\nabla_\mu\bar\Psi\, i e^\mu_a\gamma^a\Psi - m\bar\Psi\Psi\right. \\
&+ \frac{g_A}{2}\,\bar\Psi e^\mu_a \gamma^a\gamma_5 u_\mu\Psi + c_1\langle\chi_+\rangle\bar\Psi\Psi \\
&- \frac{c_2}{8m^2} g^{\mu\alpha} g^{\nu\beta}\langle u_\mu u_\nu\rangle\left(\bar\Psi\left\{\nabla_\alpha,\nabla_\beta\right\}\Psi + \left\{\nabla_\alpha,\nabla_\beta\right\}\bar\Psi\Psi\right) \\
&+ \frac{c_3}{2}\, g^{\mu\nu}\langle u_\mu u_\nu\rangle\bar\Psi\Psi + \frac{ic_4}{4}\,\bar\Psi\, e^\mu_a e^\nu_b\sigma^{ab}\,[u_\mu, u_\nu]\,\Psi + c_5\bar\Psi\hat\chi_+\Psi \\
&+ \frac{c_6}{8m}\bar\Psi\, e^\mu_a e^\nu_b \sigma^{ab} F^+_{\mu\nu}\Psi + \frac{c_7}{8m}\bar\Psi\, e^\mu_a e^\nu_b\sigma^{ab}\langle F^+_{\mu\nu}\rangle\Psi \\
&\left.+ \frac{c_8}{8}\, R\bar\Psi\Psi + \frac{ic_9}{m}\, R^{\mu\nu}\left(\bar\Psi e^a_\mu\gamma_a\nabla_\nu\Psi - \nabla_\nu\bar\Psi e^a_\mu\gamma_a\Psi\right)\right\}.
\end{aligned}
$$

$g^{\mu\nu}$ and e_a^μ are the metric and vielbein fields.

The building blocks:

$$
\begin{aligned}
\nabla_\mu \Psi &= \partial_\mu \Psi + \frac{i}{2}\,\omega_\mu^{ab}\sigma_{ab}\Psi + \left(\Gamma_\mu - i v_\mu^{(s)}\right)\Psi, \\
\nabla_\mu \bar{\Psi} &= \partial_\mu \bar{\Psi} - \frac{i}{2}\,\bar{\Psi}\,\sigma_{ab}\,\omega_\mu^{ab} - \bar{\Psi}\left(\Gamma_\mu - i v_\mu^{(s)}\right), \\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger - i(u^\dagger v_\mu u + u v_\mu u^\dagger)\right], \\
\omega_\mu^{ab} &= -g^{\nu\lambda} e_\lambda^a \left(\partial_\mu e_\nu^b - e_\sigma^b \Gamma_{\mu\nu}^\sigma\right), \\
\Gamma_{\alpha\beta}^\lambda &= \frac{1}{2}\, g^{\lambda\sigma}\left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right), \\
R^\rho{}_{\sigma\mu\nu} &= \partial_\mu \Gamma_{\nu\sigma}^\rho - \partial_\nu \Gamma_{\mu\sigma}^\rho + \Gamma_{\mu\lambda}^\rho \Gamma_{\nu\sigma}^\lambda - \Gamma_{\nu\lambda}^\rho \Gamma_{\mu\sigma}^\lambda, \\
R &= g^{\mu\nu} R^\lambda{}_{\mu\lambda\nu},
\end{aligned}
$$

$$
\begin{aligned}
u_\mu &= i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger - i(u^\dagger v_\mu u - u v_\mu u^\dagger)\right] , \\
F^+_{\mu\nu} &= u^\dagger F_{R\mu\nu} u + u F_{L\mu\nu} u^\dagger , \\
F_{R\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu] , \\
F_{L\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu] , \\
\chi_+ &= u^\dagger \chi u^\dagger + u\chi^\dagger u , \\
\hat{\chi}_+ &= \chi_+ - \frac{1}{2}\langle\chi_+\rangle ,
\end{aligned}
$$

$\chi = 2B_0(s + ip)$, $D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu$,
$U = u^2$ represents the pion fields,
B_0 is related to the vacuum condensate of quark fields,
s, p, l_μ, r_μ and $v_\mu^{(s)}$ are external sources.

Energy-momentum-tensor

Using the definition of the EMT for bosonic matter fields

$$T_{\mu\nu}(g,\psi) = \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{m}}}{\delta g^{\mu\nu}} \,,$$

we obtain in flat spacetime

$$\begin{aligned} T^{(\pi)}_{\mu\nu} &= \frac{F^2}{4}\,\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger + D_\nu U (D_\mu U)^\dagger) \\ &- \eta_{\mu\nu} \left\{ \frac{F^2}{4}\,\mathrm{Tr}(D^\alpha U (D_\alpha U)^\dagger) + \frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U \chi^\dagger) \right\}, \end{aligned}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor.

For the fermion fields we use

$$T_{\mu\nu}(g,\psi) = \frac{1}{2e}\left[\frac{\delta S}{\delta e^{a\mu}}\, e^a_\nu + \frac{\delta S}{\delta e^{a\nu}}\, e^a_\mu\right],$$

where e is the determinant of e^a_μ, and obtain in flat spacetime:

$$\begin{aligned}
T^{(\pi N)}_{\mu\nu} &= \frac{i}{4}\left(\bar\Psi\,\gamma_\mu D_\nu\Psi + \bar\Psi\,\gamma_\nu D_\mu\Psi - D_\mu\bar\Psi\,\gamma_\nu\Psi - D_\nu\bar\Psi\,\gamma_\mu\Psi\right)\\
&+ \frac{g_A}{4}\left(\bar\Psi\,\gamma_\mu\gamma_5 u_\nu\Psi + \bar\Psi\,\gamma_\nu\gamma_5 u_\mu\Psi\right)\\
\ldots\\
&+ \frac{c_8}{4}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\bar\Psi\Psi\\
&+ \frac{ic_9}{2m}(\eta_{\mu\alpha}\eta_{\nu\beta}\partial^2 + \eta_{\mu\nu}\partial_\alpha\partial_\beta - \eta_{\mu\alpha}\partial_\nu\partial_\beta - \eta_{\nu\alpha}\partial_\mu\partial_\beta)\\
&\times\left(\bar\Psi\gamma^\alpha D^\beta\Psi - D^\beta\bar\Psi\gamma^\alpha\Psi + \bar\Psi\gamma^\beta D^\alpha\Psi - D^\alpha\bar\Psi\gamma^\beta\Psi\right),
\end{aligned}$$

where

$$\begin{aligned}
D_\mu\Psi &= \partial_\mu\Psi + \left(\Gamma_\mu - iv^{(s)}_\mu\right)\Psi,\\
D_\mu\bar\Psi &= \partial_\mu\bar\Psi - \bar\Psi\left(\Gamma_\mu - iv^{(s)}_\mu\right).
\end{aligned}$$

Gravitational form factors of the nucleon

At chiral order four there are tree and one-loop contributions to the nucleon matrix element of the EMT.

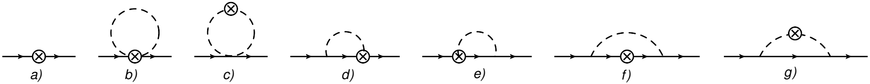

Figure: Diagrams contributing to GFFs of the nucleon. Solid and dashed lines correspond to nucleons and pions. Crosses stand for EMT insertions.

Standard power counting rules apply to these diagrams:

- The pion lines count as of chiral order minus two;
- The nucleon lines have order minus one;
- Interaction vertices originating from the effective Lagrangian of order N count also as of chiral order N;
- Vertices generated by EMT have orders corresponding to the number of quark mass factors and derivatives acting on the pion fields.
- The momentum transfer between the initial and final nucleons counts as of chiral order one.
- Integration over loop momenta is counted as of chiral order four.

Power counting is realized only after an appropriate renormalization.

M. V. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A **33**, 26, 1830025 (2018).

The one-nucleon matrix element of the EMT is parameterised as

$$\langle p_f, s_f | T_{\mu\nu} | p_i, s_i \rangle = \bar{u}(p_f, s_f) \left[A(t) \frac{P_\mu P_\nu}{m_N} + iJ(t) \frac{P_\mu \sigma_{\nu\alpha} \Delta^\alpha + P_\nu \sigma_{\mu\alpha} \Delta^\alpha}{2 m_N} + D(t) \frac{\Delta_\mu \Delta_\nu - \eta_{\mu\nu} \Delta^2}{4 m_N} \right] u(p_i, s_i) ,$$

(p_i, s_i) and (p_f, s_f) are momentum and polarization of incoming and outgoing nucleons, and $P = (p_i + p_f)/2$, $\Delta = p_f - p_i$, $t = \Delta^2$.

Tree-order diagrams up to chiral order four lead to

$$\begin{aligned} A_{\rm tree}(t) &= 1 - \frac{2 c_9}{m_N} t + x_1 M_\pi^2 t + x_2 t^2 , \\ J_{\rm tree}(t) &= \frac{1}{2} - \frac{c_9}{m_N} t , \\ D_{\rm tree}(t) &= c_8 m_N + y_1 t + y_2 M_\pi^2 . \end{aligned} \tag{1}$$

y_i and x_i are contributions of the third and fourth order Lagrangians.

We renormalize loop diagrams by applying the EOMS scheme
J. G. and G. Japaridze, Phys. Rev. D **60**, 114038 (1999),
T. Fuchs, J. G., G. Japaridze, S. Scherer, Phys. Rev. D **68**, 056005 (2003).

Power counting breaking (PCB) part of $A(t)$ is absorbed into c_9.

c_8 cancels the divergent part and the PCB piece of $D(t)$.

$D(0)$ expanded in powers of the pion mass:

$$\frac{D(0)}{m_N} = c_8 + \frac{g_A^2}{16\pi F^2} M_\pi + \frac{2\left(c_2 + 2c_3 - 4c_1\right) - \frac{3g_A^2}{m_N}}{8\pi^2 F^2} M_\pi^2 \ln\left(\frac{M_\pi}{m_N}\right) + \frac{(8c_3 - 16c_1) - g_A^2\left(3c_8 + \frac{14}{m_N}\right)}{32\pi^2 F^2} M_\pi^2 + \frac{y_2}{m_N} M_\pi^2 + \mathcal{O}(M_\pi^3) .$$

GFFs $A(t), J(t)$ and $D(t)$ are related spatial densities.

M. V. Polyakov, Phys. Lett. B **555**, 57 (2003),

M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A **33**, 1830025 (2018),

Behavior of densities in the region $1/\Lambda_{\rm strong} \ll r \ll 1/M_\pi$ can be obtained from GFFs for small t in chiral limit:

$$\begin{aligned}
\rho_E(r) &= \frac{9g_A^2}{64\pi^2 F^2}\frac{1}{r^6} - \frac{3\left(10g_A^2/m_N + (c_2 + 10c_3)\right)}{16\pi^3 F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right), \\
\rho_J(r) &= \frac{5g_A^2}{64\pi^3 F^2}\frac{1}{r^5} - \frac{9g_A^2}{64\pi^2 F^2 m_N}\frac{1}{r^6} + O\left(\frac{1}{r^7}\right), \\
p(r) &= -\frac{3g_A^2}{64\pi^2 F^2}\frac{1}{r^6} + \frac{\left(5g_A^2/m_N + 4\left(c_2 + 5c_3\right)\right)}{16\pi^3 F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right), \\
s(r) &= \frac{9g_A^2}{64\pi^2 F^2}\frac{1}{r^6} - \frac{21\left(5g_A^2/m_N + 4\left(c_2 + 5c_3\right)\right)}{128\pi^3 F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right).
\end{aligned}$$

Problem of defining spatial densities

Charge density of a nucleon is traditionally defined as Fourier transform of the electric FF in the Breit frame.

R. Hofstadter, F. Bumiller, and M. R. Yearian, Rev. Mod. Phys. **30**, 482 (1958).
F. J. Ernst, R. G. Sachs and K. C. Wali, Phys. Rev. **119**, 1105-1114 (1960).
R. G. Sachs, Phys. Rev. **126**, 2256-2260 (1962).

Similar relations have been suggested for Fourier transforms of GFFs and various local distributions in

M. V. Polyakov and A. G. Shuvaev, [arXiv:hep-ph/0207153 [hep-ph]].
M. V. Polyakov, Phys. Lett. B **555**, 57 (2003).
M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A **33** (2018) no.26, 1830025.

This definition of spatial densities was criticized in

M. Burkardt, Phys. Rev. D **62** (2000), 071503(R), [erratum: Phys. Rev. D **66** (2002), 119903(E)].
G. A. Miller, Phys. Rev. Lett. **99**, 112001 (2007).
G. A. Miller, Phys. Rev. C **79**, 055204 (2009).
G. A. Miller, Ann. Rev. Nucl. Part. Sci. **60** (2010), 1-25.
R. L. Jaffe, Phys. Rev. D **103** (2021) no.1, 016017.
G. A. Miller, Phys. Rev. C **99**, no.3, 035202 (2019).
A. Freese and G. A. Miller, Phys. Rev. D **103**, 094023 (2021).

One-sentence summary:
Interpretation of the Fourier transformed FFs as charge densities is not valid for systems with $\Delta \sim 1/m$.

Spatial densities defined via sharply localized states:

E.Epelbaum, J.G., N.Lange., U.-G.Meißner., M.V.Polyakov,
Phys. Rev. Lett. **129**, 012001 (2022).

J.Y.Panteleeva, E.Epelbaum, J.G., U.-G.Meißner,
Phys. Rev. D **106**, no.5, 056019 (2022).

H. Alharazin, B.-D. Sun, E. Epelbaum, J. G., U.-G. Meißner,
JHEP **02**, 163 (2023).

J.Y.Panteleeva, E.Epelbaum, J.G., U.-G.Meißner,
Eur. Phys. J. C **83**, no.7, 617 (2023).

We use spherically symmetric wave packets, and consider ZAMF.

Localized states

We use normalizable Heisenberg-picture states:

$$|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}} \phi(s, \mathbf{p})\, e^{-i\mathbf{p}\cdot\mathbf{X}} |p, s\rangle, \tag{2}$$

where $\mathbf{X}$ is the position of the system, and $|p, s\rangle$ are normalized as

$$\langle p', s'|p, s\rangle = 2E(2\pi)^3 \delta_{s's}\, \delta^{(3)}(\mathbf{p}' - \mathbf{p}), \qquad p = (E, \mathbf{p}). \tag{3}$$

Profile function $\phi(s, \mathbf{p}) = \phi(\mathbf{p}) = \phi(|\mathbf{p}|)$ corresponds to ZAMF and:

$$\int d^3p\, |\phi(s, \mathbf{p})|^2 = 1. \tag{4}$$

It is convenient to define dimensionless profile functions

$$\phi(\mathbf{p}) = R^{3/2}\, \tilde{\phi}(R\mathbf{p}), \tag{5}$$

Sharp localizations of the system correspond to small R.

EMT spatial densities

EMT matrix element of a spin-1/2 system:

$$
\begin{aligned}
t_\phi^{\mu\nu}(\mathbf{r}) &= \langle \Phi, \mathbf{0} | \hat{T}^{\mu\nu}(\mathbf{r}, 0) | \Phi, \mathbf{0} \rangle \\
&= \int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\, \bar{u}(p', s') \left[A(q^2) \frac{P^\mu P^\nu}{m} \right. \\
&\quad \left. + iJ(q^2) \frac{P^\mu \sigma^{\nu\alpha} q_\alpha + P^\nu \sigma^{\mu\alpha} q_\alpha}{2m} + D(q^2) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4m} \right] u(p, s) \\
&\quad \times \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot(\mathbf{r})} \, .
\end{aligned} \tag{6}
$$

where $q = p' - p$, $P = (p' + p)/2$, and the Dirac spinors are normalized as $\bar{u}(p, s')u(p, s) = 2m\,\delta_{s's}$.

Static approximation is obtained by expanding the integrand in $1/m$ and taking $R \to 0$ limit.

This can be done without specifying $F\left(q^2\right)$ and $\phi(\mathbf{p})$ using the method of dimensional counting

J.G., G.Japaridze, K.Turashvili, Theor. Math. Phys. **101**, 1313 (1994).

$$
\begin{aligned}
t^{00}_{\rm naive}(\mathbf{r}) &= m \int \frac{d^3q}{(2\pi)^3} A\left(-\mathbf{q}^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}, \\
t^{0i}_{\rm naive}(\mathbf{r}) &= -\frac{i}{2}\epsilon^{ijk}\sigma^k \int \frac{d^3q}{(2\pi)^3} q^j J\left(-\mathbf{q}^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}, \\
t^{ij}_{\phi,\rm naive}(\mathbf{r}) &= \frac{1}{R^2}\int d\tilde{P}\tilde{P}^4|\tilde{\phi}(\tilde{\mathbf{P}})|^2 \frac{4\pi\,\delta^{ij}}{3m}\int \frac{d^3q}{(2\pi)^3} A\left(-\mathbf{q}^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}} \\
&+ \frac{1}{4m}\int \frac{d^3q}{(2\pi)^3} D\left(-\mathbf{q}^2\right)\left(-\mathbf{q}^2\delta^{ij} + q^i q^j\right) e^{-i\mathbf{q}\cdot\mathbf{r}},
\end{aligned}
$$

The $t^{00}_{\rm naive}$, $t^{0i}_{\rm naive}$ and the second term of $t^{ij}_{\phi,\rm naive}$ coincide with spatial densities in the Breit frame.

EMT matrix element for sharply localized states, i.e. in $R \to 0$ limit:

$$t_\phi^{\mu\nu}(\mathbf{r}) = N_{\phi,R} \int \frac{d^2\hat{\tilde{P}}\, d^3q}{(2\pi)^3} \left[\frac{i}{2m} \left(\hat{\tilde{P}}^\mu (\boldsymbol{\sigma}_\perp \times \mathbf{q})^\nu + \hat{\tilde{P}}^\nu (\boldsymbol{\sigma}_\perp \times \mathbf{q})^\mu \right.\right.$$
$$\left. + \, \hat{\tilde{\mathbf{P}}} \cdot (\boldsymbol{\sigma}_\perp \times \mathbf{q}) (\delta^{\mu 0} \hat{\tilde{P}}^\nu + \delta^{\nu 0} \hat{\tilde{P}}^\mu) \right) J\left(-\mathbf{q}_\perp^2\right)$$
$$\left. + \hat{\tilde{P}}^\mu \hat{\tilde{P}}^\nu A\left(-\mathbf{q}_\perp^2\right) \right] e^{-i\mathbf{q}\cdot\mathbf{r}}$$
$$+ \frac{1}{2} N_{\phi,R,2} \int \frac{d^2\hat{\tilde{P}}\, d^3q}{(2\pi)^3} \left(\tilde{q}^\mu \tilde{q}^\nu + g^{\mu\nu} \mathbf{q}_\perp^2 \right) D\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}},$$

where $(\boldsymbol{\sigma}_\perp \times \mathbf{q})^0 = 0$, $\tilde{q}^\mu = (\hat{\tilde{\mathbf{P}}} \cdot \mathbf{q}, \mathbf{q})$, $\tilde{P}^\mu = (\tilde{P}, \tilde{\mathbf{P}})$, $\hat{\tilde{P}}^\mu = \left(1, \frac{\tilde{\mathbf{P}}}{\tilde{P}}\right)$, $\tilde{P} = |\tilde{\mathbf{P}}|$, $\mathbf{q}_\perp = \hat{\tilde{\mathbf{P}}} \times \left(\mathbf{q} \times \hat{\tilde{\mathbf{P}}}\right)$, $\mathbf{q}_\perp^2 \equiv -\tilde{q}^2$ and

$$N_{\phi,R} = \frac{1}{R} \int d\tilde{P} \tilde{P}^3 |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2,$$
$$N_{\phi,R,2} = \frac{R}{2} \int d\tilde{P} \tilde{P} |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2.$$

Interpretation

In sharply localized states $t^{00}(\mathbf{r})$ and $t^{0i}(\mathbf{r})$ can be interpreted as energy and momentum spatial distributions, respectively.

Breit frame expressions correspond to systems in ZAMF in states with packets much larger than $1/m$.

Such packet is dominated by states with $E \approx m$, and therefore $t^{00}(\mathbf{r})$ can be interpreted in this case as the spatial distribution of the mass.

$$\bar{t}_2^{ij}(s', s, \mathbf{r}) = \frac{1}{2} N_{\phi,R,2}\, \delta_{s's} \int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} \left(q^i q^j - \delta^{ij}\mathbf{q}_\perp^2\right) D\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$t_{2,naive}^{ij}(\mathbf{r}) = \frac{1}{4m} \int \frac{d^3q}{(2\pi)^3} \left(q^i q^j - \delta^{ij}\mathbf{q}^2\right) D\left(-\mathbf{q}^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}},$$

t_2^{ij} is interpreted as characterizing the distribution of internal forces.

$$t_2^{ij}(\mathbf{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij} p(r),$$

$s(r)$ and $p(r)$ are shear force and the pressure, respectively.

M. V. Polyakov, Phys. Lett. B **555**, 57 (2003). [hep-ph/0210165].
M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A **33** (2018) no.26, 1830025. [arXiv:1805.06596 [hep-ph]].

The dependence of spatial density on $F(-\mathbf{q}_\perp^2)$ rather than on $F(-\mathbf{q}^2)$ affects the radial profile of the charge density.

Demonstration

We compare $\rho(r)$ and $\rho_{\rm naive}(r)$ for a charged and a neutral particles.

We employ form factors

$F_{\rm p}(q^2) = (1 - q^2/\Lambda^2)^{-2}$

with $\Lambda^2 = 0.71$ GeV^2,

and

$F_{\rm n}(q^2) = A\tau/(1 + B\tau)\,(1 - q^2/\Lambda^2)^{-2}$,

where $\tau = -q^2/(4m_p^2)$, $A = 1.70$, $B = 3.30$.

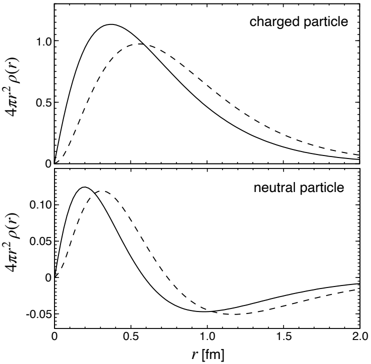

Figure: Radial charge density distributions $4\pi r^2 \rho(r)$ (solid lines) and $4\pi r^2 \rho_{\rm naive}(r)$ (dashed lines) for a charged and a neutral particles.

Summary

- Presented the effective chiral Lagrangian of pions and nucleons up to the second chiral order in curved spacetime.
- Derived the corresponding EMT of pions and nucleons in flat spacetime.
- Calculated the the one-nucleon matrix element of the EMT at fourth chiral order and extracted the GFFs.
- We introduced an unambiguous definition of spatial distributions of expectation values of local operators via localized states.
- New definition also applies to systems independently on the Compton wavelength.
- In case of EMT and gravitational FFs the static approximation leads to spatial densities obtained from FFs in Breit frame.

  Sharply localized states lead to analogous but different results.
- Our results suggest $\langle r^2 \rangle = 4A'(0)$ in contrast to the usual Breit frame expression $\langle r^2 \rangle_{\rm naive} = 6A'(0)$.