Exotic hadrons with heavy quarks in EFT approach

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Long-term fruitful collaboration with colleagues from China, Germany, India, Slovenia, Spain is gratefully acknowledged!

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- V. Baru, X. K. Dong, M. L. Du, A. Filin, F. K. Guo, C. Hanhart, AN, J. Nieves, Q. Wang, "Effective range expansion for narrow near-threshold resonances," Phys. Lett. B **833**, 137290 (2022)
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Hadronic physics before and after 2003

Consensus before 2003:

- Quark model provides a decent description of low-lying hadrons
- Quark model works surprisingly well even for light flavours
- Heavy flavours (c and b) comply with nonrelativistic theory
- Relativistic corrections somewhat improve the description
- Experiment gradually fills "missing states"
- Lattice provides additional/alternative source of information

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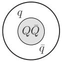
Situation after 2003:

- $X(3872)$ observed by Belle with properties at odds with quark model
- Number of such unconventional hadrons with heavy quarks grows fast
- New branch of hadrons spectroscopy — exotic XYZ states

“Exotic” versus “ordinary”

- “Ordinary” hadron = quark-antiquark mesons or 3-quark baryons
- “Exotic” hadron = not ordinary hadron
- Simplest exotic hadron = tetraquark ($Q\bar{Q}q\bar{q}$)

Compact tetraquarks (bound by confinement)



Hadro-Quarkonium (compact $\bar{Q}Q$ core plus light-quark cloud)

Hadronic molecule (extended object)

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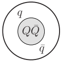

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Hadronic molecule (extended object)

Molecule = large probability to observe physical state in hadron-hadron channel

“Exotic” versus “ordinary”

- “Ordinary” hadron = quark-antiquark mesons or 3-quark baryons
- “Exotic” hadron = not ordinary hadron

- ${}^3S_1$ NN system with $I = 0$:

  Pole on RS-I with $E_B = 2.23$ MeV $\Longrightarrow$ deuteron

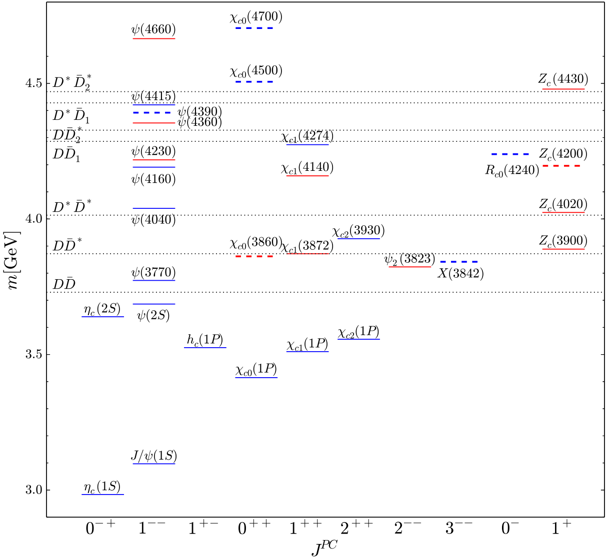
- ${}^1S_0$ NN system with $I = 1$:

  Pole on RS-II with $E_B = 0.067$ MeV $\Longrightarrow$ virtual state

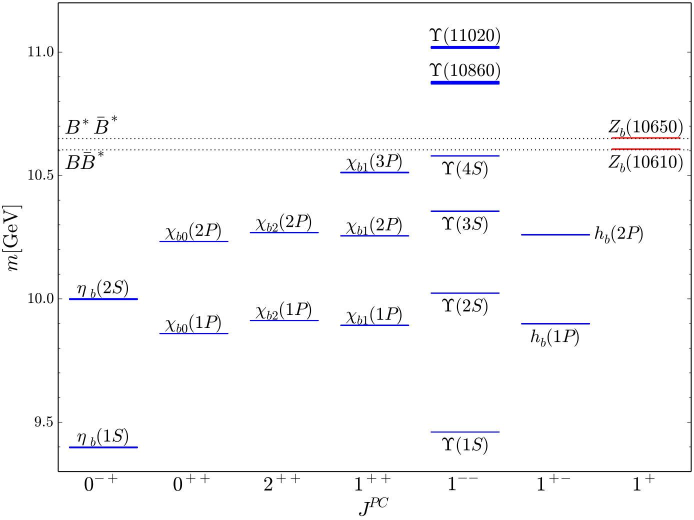
Hadronic molecule (extended object)

Molecule = large probability to observe physical state in hadron-hadron channel

Spectrum of charmonium

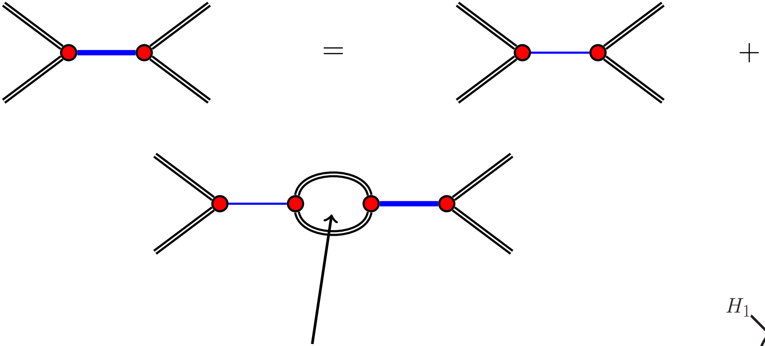


Spectrum of bottomonium

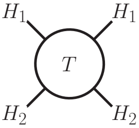


Effect of hadronic loops

$$|X\rangle = \begin{pmatrix} \lambda|\psi\rangle \\ \chi(\boldsymbol{p})|H_1H_2\rangle \end{pmatrix}$$



Flatté: $\dfrac{1}{E - E_f + \frac{i}{2}(gk + \Gamma_0)}$ $\Longrightarrow$ parameterised

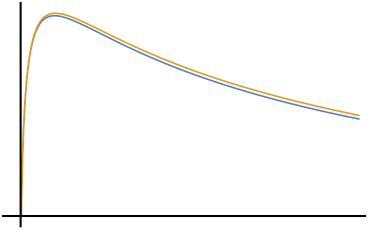


Γ_0 parameterises decay modes not related to H_1H_2 channel

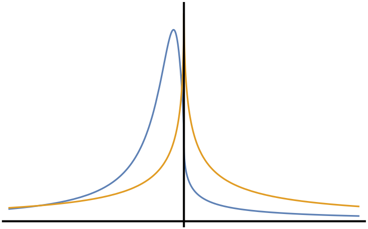
Examples of line shapes

$$\left.\begin{array}{l} k(E) = \sqrt{2\mu E}\,\Theta(E) \\ \kappa(E) = \sqrt{-2\mu E}\,\Theta(-E) \end{array}\right\} \Longrightarrow$$ Threshold phenomena

$$\frac{gk(E)}{\left(E-E_f-\frac{1}{2}g\kappa(E)\right)^2+\frac{1}{4}(\Gamma_0+gk(E))^2}$$



$$\frac{\Gamma_0}{\left(E-E_f-\frac{1}{2}g\kappa(E)\right)^2+\frac{1}{4}(\Gamma_0+gk(E))^2}$$



- Blue curve — bound state (pole on RS-I)
- Yellow curve — virtual state (pole on RS-II)

Examples of line shapes

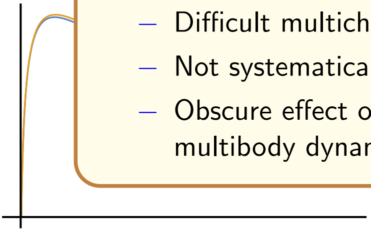
$k(E) = \sqrt{2\mu E}\,\Theta(E)$

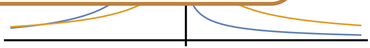
$\kappa($

ena

$(E-E$

$k(E))^2$



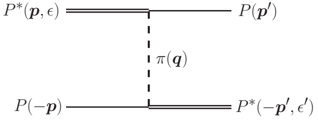


Flatté parametrisation:

- \+ Simple and physically transparent
- \+ Accounts for threshold phenomena
- – Difficult multichannel generalisation
- – Not systematically improvable
- – Obscure effect of particle exchanges and multibody dynamics

- Blue curve — bound state (pole on RS-I)
- Yellow curve — virtual state (pole on RS-II)

Pion exchange

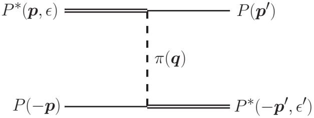


$$V_\pi(\boldsymbol{p},\boldsymbol{p}') = \left(\frac{g}{2f_\pi}\right)^2 \langle \boldsymbol{\tau}\cdot\boldsymbol{\tau}\rangle \frac{(\boldsymbol{\epsilon}\cdot\boldsymbol{q})(\boldsymbol{q}\cdot\boldsymbol{\epsilon}'^*)}{u-m_\pi^2}$$

$$\Longrightarrow \quad \left(\frac{g}{2f_\pi}\right)^2 \langle \boldsymbol{\tau}\cdot\boldsymbol{\tau}\rangle \left(-1+\overbrace{\frac{\mu_\pi^2}{\boldsymbol{q}^2+\underbrace{[m_\pi^2-(m_{P^*}-m_P)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\right)$$

central ~~recoil~~

Pion exchange



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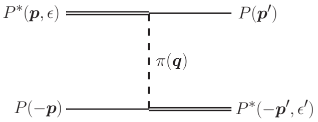
$\Longrightarrow$ central ~~recoil~~

$$\left(\frac{g}{2f_\pi}\right)^2 \langle \boldsymbol{\tau}\cdot\boldsymbol{\tau}\rangle \Bigg(-1+\overbrace{\frac{\mu_\pi^2}{\boldsymbol{q}^2+\underbrace{[m_\pi^2-(m_{P^*}-m_P)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\Bigg)$$

Bottomonium system ($m_\pi > m_{B^*} - m_B \Longrightarrow \mu_\pi^2 > 0$):

$\Longrightarrow$ Qualitatively similar to deuteron but $\mu_\pi < m_\pi$

Pion exchange



$$V_\pi(\boldsymbol{p}, \boldsymbol{p}') = \left(\frac{g}{2f_\pi}\right)^2 \langle \boldsymbol{\tau} \cdot \boldsymbol{\tau} \rangle \frac{(\boldsymbol{\epsilon} \cdot \boldsymbol{q})(\boldsymbol{q} \cdot \boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

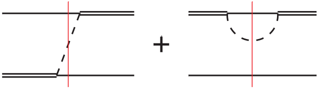
$$\underset{\text{central, ~~recoil~~}}{\Longrightarrow} \left(\frac{g}{2f_\pi}\right)^2 \langle \boldsymbol{\tau} \cdot \boldsymbol{\tau} \rangle \left(-1 + \overbrace{\frac{\mu_\pi^2}{\boldsymbol{q}^2 + \underbrace{[m_\pi^2 - (m_{P^*} - m_P)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\right)$$

Bottomonium system ($m_\pi > m_{B^*} - m_B \Longrightarrow \mu_\pi^2 > 0$):

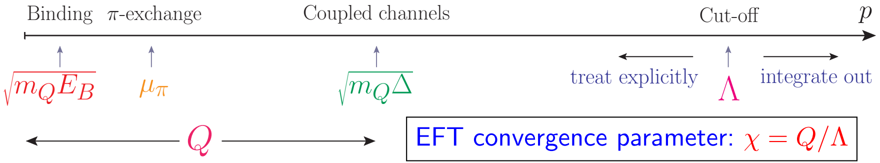
$\Longrightarrow$ Qualitatively similar to deuteron but $\mu_\pi < m_\pi$

Charmonium system ($m_\pi < m_{D^*} - m_D \Longrightarrow \mu_\pi^2 < 0$ & $|\mu_\pi| \ll m_\pi$):

$\Longrightarrow$ 3-body effects:

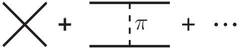


Effective field theory for hadronic molecules



Interaction potential between heavy hadrons:

- Includes all relevant interactions



- Complies with relevant symmetries
- Incorporates coupled-channel dynamics
- Expanded in powers of p^2/Λ^2 and truncated at necessary order (LO, NLO...)
- Iterated to all orders via (multichannel) Lippmann-Schwinger equation

$$T = V - VGT$$

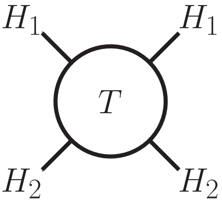
Effective field theory for hadronic molecules

Binding π-exchang… Cut-off p

$\sqrt{m_Q E_B}$ μ_π …tly Λ integrate out

…neter: $\chi = Q/\Lambda$

EFT provides approach to



- model-independent
- systematically improvable
- compatible with symmetries
- unitary

Interaction potentia…

- Includes all rel…
- Complies with…
- Incorporates c…
- Expanded in p… …order (LO, NLO...)
- Iterated to all … …ger equation

$$T = V - VGT$$

Effective field theory for hadronic molecules

Free parameters:

- Low-energy constants
- Couplings to hadronic channels

Input (combined analysis):

- Line shapes (Dalitz plots)
- Partial branchings

Output:

- Pole position M_0 (“mass”$=\text{Re}(M_0)$, “width”$=2\times\text{Im}(M_0)$)
- Nature of state (compositeness as a cross check)

Predictions:

- New properties of “old” state: line shapes, partial widths,...
- Properties of “new” states: poles, line shapes, partial widths,...
- Chiral extrapolations (lattice data interpretation)

Heavy quark symmetry

- Exotic XYZ states contain heavy quarks (HQ)
- In the limit $m_Q \to \infty$ $(m_Q \gg \Lambda_{\mathrm{QCD}})$ spin of HQ decouples

  $\Longrightarrow$ Heavy Quark Spin Symmetry (HQSS)
- For realistic m_Q's HQSS is approximate but accurate symmetry of QCD
- HQSS relates properties of states with different HQ spin orientation

  $\Longrightarrow$ Spin partners

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Prediction!

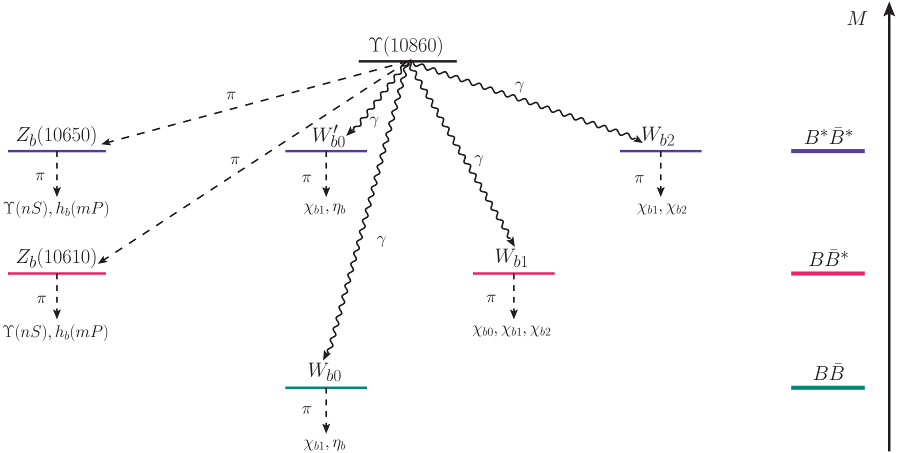
Twins $Z_b(10610)$ & $Z_b(10650)$

$$I = 1 \quad J^{PC} = 1^{+-}$$

Minimal quark content: $\bar{b}b\bar{q}q$

$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[B\bar{B}^{(*)}\right]$$
$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[\pi h_b(1,2P)\right]$$
$$\Upsilon(10860) \to \pi Z_b^{(\prime)} \to \pi\left[\pi \Upsilon(1,2,3S)\right]$$

Z_b's ($J^{PC}=1^{+-}$) and W_{bJ}'s ($J^{PC}=J^{++}$) in decays of $\Upsilon(10860)$



$$Z_b(10610) \sim B\bar{B}^* \sim 0^-_{\bar{q}b}\otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b}\otimes 0^-_{\bar{q}q} + 0^-_{\bar{b}b}\otimes 1^-_{\bar{q}q}$$

$$Z_b'(10650) \sim B^*\bar{B}^* \sim 1^-_{\bar{q}b}\otimes 1^-_{\bar{b}q} \sim 1^-_{\bar{b}b}\otimes 0^-_{\bar{q}q} - 0^-_{\bar{b}b}\otimes 1^-_{\bar{q}q}$$

(Bondar et al'2011,Voloshin'2011,...)

Coupled-channel problem

Elastic potential:

$$V_{\text{el-el}} = V_{\text{CT}}(\text{to order } O(p^0))$$

Coupled channels:

$$1^{+-} : B\bar{B}^*(^3S_1, -), B^*\bar{B}^*(^3S_1)$$
$$0^{++} : B\bar{B}(^1S_0), B^*\bar{B}^*(^1S_0)$$
$$1^{++} : B\bar{B}^*(^3S_1, +)$$
$$2^{++} : B^*\bar{B}^*(^5S_2)$$

Coupled-channel problem

Elastic potential:

$$V_{\text{el-el}} = V_{\text{CT}}(\text{to order } O(p^2)) + V_\pi$$

Coupled channels:

$$1^{+-} : B\bar{B}^*({}^3S_1,-), B^*\bar{B}^*({}^3S_1), B\bar{B}^*({}^3D_1,-), B^*\bar{B}^*({}^3D_1)$$
$$0^{++} : B\bar{B}({}^1S_0), B^*\bar{B}^*({}^1S_0), B^*\bar{B}^*({}^5D_0)$$
$$1^{++} : B\bar{B}^*({}^3S_1,+), B\bar{B}^*({}^3D_1,+), B^*\bar{B}^*({}^5D_1)$$
$$2^{++} : B^*\bar{B}^*({}^5S_2), B\bar{B}({}^1D_2), B\bar{B}^*({}^3D_2), B^*\bar{B}^*({}^1D_2), B^*\bar{B}^*({}^5D_2), B^*\bar{B}^*({}^5G_2)$$

Lippmann-Schwinger equation ($V^{\text{eff}} = V_{\text{el-el}} + \sum_{\text{inel}} V_{\text{el-inel-el}}$):

$$T_{\alpha\beta}(M,\boldsymbol{p},\boldsymbol{p}') = V^{\text{eff}}_{\alpha\beta}(\boldsymbol{p},\boldsymbol{p}') - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\text{eff}}_{\alpha\gamma}(\boldsymbol{p},\boldsymbol{q}) G_\gamma(M,\boldsymbol{q}) T_{\gamma\beta}(M,\boldsymbol{q},\boldsymbol{p}')$$

Coupled-channel problem

Elastic potential:

$$V_{\text{el-el}} = V_{\text{CT}}(\text{to order } O(p^2)) + V_\pi$$

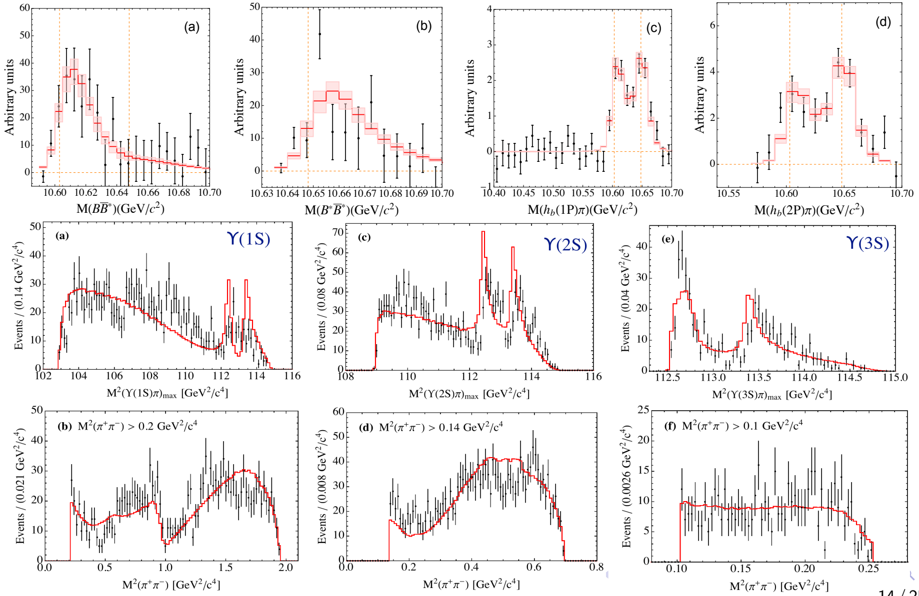
$$\left.\begin{aligned} \gamma_B &= \sqrt{m_B E_B} \simeq 100\ \text{MeV} \\ |\mu_\pi| &= \sqrt{m_\pi^2 - (m_{B^*} - m_B)^2} \simeq 100\ \text{MeV} \\ p_{\text{coupl.ch.}} &= \sqrt{m_B (m_{B^*} - m_B)} \simeq 500\ \text{MeV} \\ p_{\text{data}}^{\text{max}} &= \sqrt{m_B\, \Delta E_{\text{data}}} \simeq 500\ \text{MeV} \end{aligned}\right\} \Longrightarrow$$

$\Lambda \simeq 1$ GeV
Potential at NLO
OPE included
Couple channels

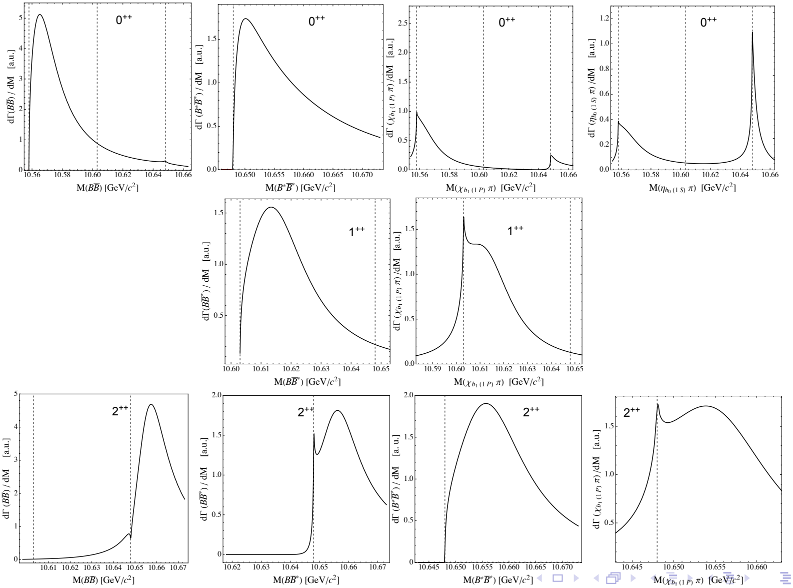
Lippmann-Schwinger equation ($V^{\text{eff}} = V_{\text{el-el}} + \sum_{\text{inel}} V_{\text{el-inel-el}}$):

$$T_{\alpha\beta}(M, \boldsymbol{p}, \boldsymbol{p}') = V^{\text{eff}}_{\alpha\beta}(\boldsymbol{p}, \boldsymbol{p}') - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\text{eff}}_{\alpha\gamma}(\boldsymbol{p}, \boldsymbol{q}) G_\gamma(M, \boldsymbol{q}) T_{\gamma\beta}(M, \boldsymbol{q}, \boldsymbol{p}')$$

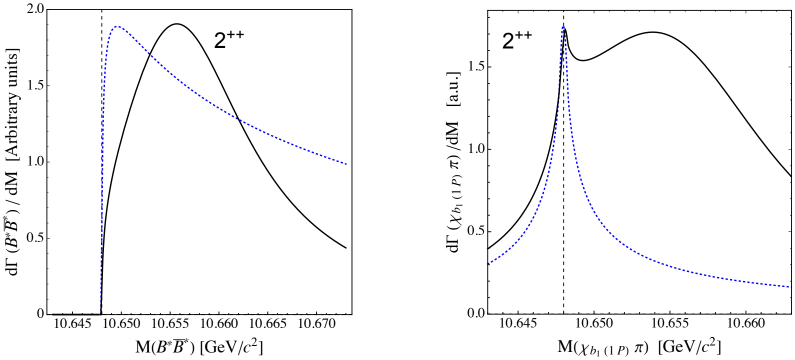
Fitted line shapes for Z_b's



Predicted line shapes for W_{bJ}'s



Role of pions



- Blue dashed line — prediction of the pionless theory
- Black solid line — prediction of the full theory with pions

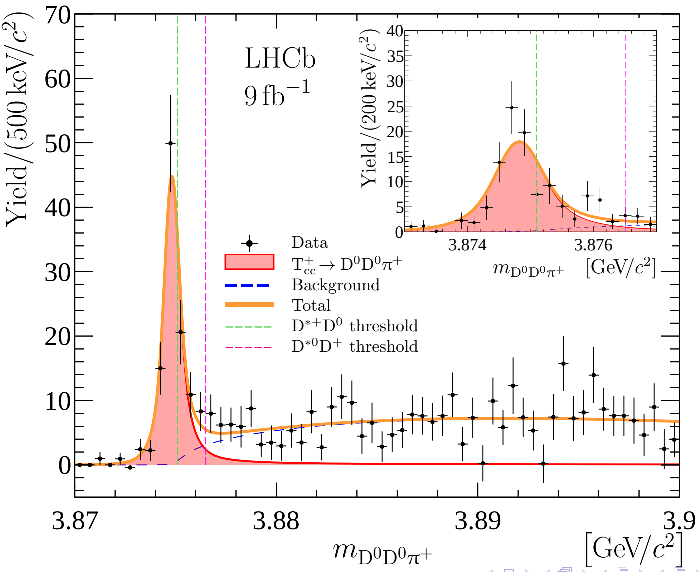
Double-charm state T_{cc}^+

$$I = 0 \quad J^P = 1^+$$

Minimal quark content: $cc\bar{u}\bar{d}$

$$T_{cc}^+ \to D^0 D^{*+} \to D^0 D^0 \pi^+$$

$T_{cc}^+(cc\bar{u}\bar{d})$ @ LHCb (Nature Phys. 18 (2022) 7, 751)



EFT approach to T_{cc}^+

$$\left.\begin{aligned}
\gamma_B &= \sqrt{m_D E_B} \simeq 25 \text{ MeV} \\
|\mu_\pi| &= \sqrt{(m_{D^*} - m_D)^2 - m_\pi^2} \simeq 40 \text{ MeV} \\
p_{\text{coupl.ch.}} &= \sqrt{m_D (m_{D^*} - m_D)} \simeq 500 \text{ MeV} \\
p_{\text{data}}^{\text{max}} &= \sqrt{m_D\, \Delta E_{\text{data}}} \simeq 100 \text{ MeV}
\end{aligned}\right\} \Longrightarrow$$

$\Lambda = 500$ MeV

Potential at LO

OPE included

No couple channels

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$$\left.\begin{aligned} \gamma_B &= \sqrt{m_D E_B} \simeq 25\ \text{MeV} \\ |\mu_\pi| &= \sqrt{(m_{D^*}-m_D)^2 - m_\pi^2} \simeq 40\ \text{MeV} \\ p_{\text{coupl.ch.}} &= \sqrt{m_D(m_{D^*}-m_D)} \simeq 500\ \text{MeV} \\ p_{\text{data}}^{\text{max}} &= \sqrt{m_D\,\Delta E_{\text{data}}} \simeq 100\ \text{MeV} \end{aligned}\right\} \Longrightarrow$$

$\Lambda = 500$ MeV
Potential at LO
OPE included
No couple channels

- Lippmann-Schwinger equation for scattering amplitude (v_0 — free parameter)

$$T = V - VGT$$

$$V = v_0 + V_\pi$$

- Production amplitude (P — free parameter = overall normalisation)

$$U = P - PGT$$

EFT approach to T_{cc}^+

$\gamma_B = \sqrt{m_D E_B} \simeq 25$ MeV

$|\mu_\pi| = \sqrt{(m_{D^*} - m_D)^2 - m_\pi^2} \simeq 40$ MeV

$p_{\text{coupl.ch.}} = \sqrt{m_D(m_{D^*} - m_D)} \simeq 500$ MeV

$p^{\text{max}}_{\text{data}} = \sqrt{m_{\dots}}$ …

$\Longrightarrow$

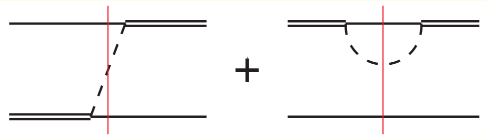
$\Lambda = 500$ MeV

Potential at LO

OPE included

No couple channels

3-body effects:



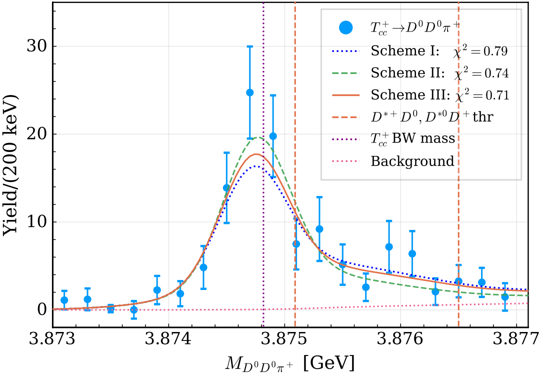
- Lippmann-S … free parameter)

- Production amplitude (P — free parameter = overall normalisation)

$$U = P - PGT$$

Fitting schemes, results, and conclusions

	Γ_{D^*}=const, ~~OPE~~	$\Gamma_{D^*}(p,M)$, ~~OPE~~	$\Gamma_{D^*}(p,M)$, OPE
χ^2/d.o.f.	0.79	0.74	0.71
v_0 [GeV^{-2}]	-23.34 ± 0.08	$-22.88^{+0.08}_{-0.06}$	$-5.04^{+0.10}_{-0.08}$
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$



- (Quasi)bound state just below $D^{*+}D^0$ threshold
- Compositeness: 70% & 30%

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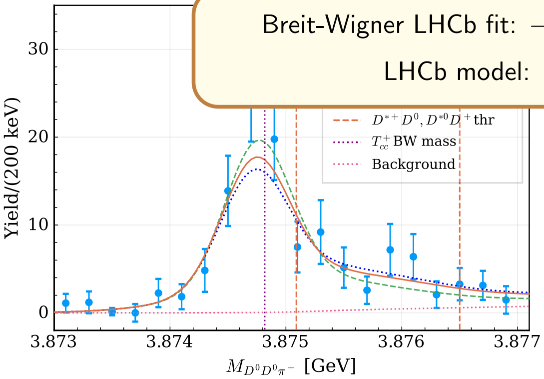
Breit-Wigner LHCb fit: $-273 - i410$ keV

LHCb model: $-360 - i24$ keV



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Lattice studies of T_{cc}^+

- "Signature of a Doubly Charm Tetraquark Pole in DD^* Scattering on Lattice," M. Padmanath and S. Prelovsek, Phys. Rev. Lett. **129**, 032002 (2022)
  "Towards the quark mass dependence of T_{cc}^+ from lattice QCD, S. Collins, A. Nefediev, M. Padmanath and S. Prelovsek, Phys. Rev. D **109**, 094509 (2024)

  $m_\pi = 280$ MeV 5 points in m_c

- "$T_{cc}^+(3875)$ relevant DD^* scattering from $N_f = 2$ lattice QCD," S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun and R. Zhang, Phys. Lett. B **833**, 137391 (2022)

  $m_\pi = 348$ MeV

- "Doubly Charmed Tetraquark T_{cc}^+ from Lattice QCD near Physical Point," Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda and J. Meng, Phys. Rev. Lett. **131**, 161901 (2023)

  $m_\pi = 146$ MeV HALQCD technique

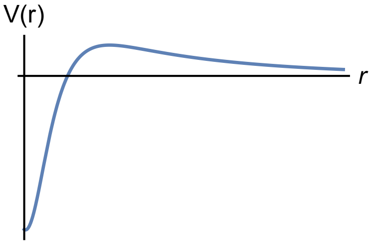
EFT analysis "lattice" T_{cc}^+

Lippmann–Schwinger equation

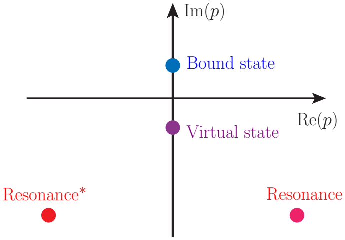
$$T(\boldsymbol{p},\boldsymbol{p}';E) = V(\boldsymbol{p},\boldsymbol{p}') - \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p},\boldsymbol{k}) G(\boldsymbol{k};E) T(\boldsymbol{k},\boldsymbol{p}';E)$$

$$V(\boldsymbol{p},\boldsymbol{p}') = \underbrace{\left[2c_0 + 2c_2(p^2+p'^2)\right]}_{\text{Contact interactions}} + \underbrace{V_\pi^S(p,p')}_{S\text{-wave OPE}}$$

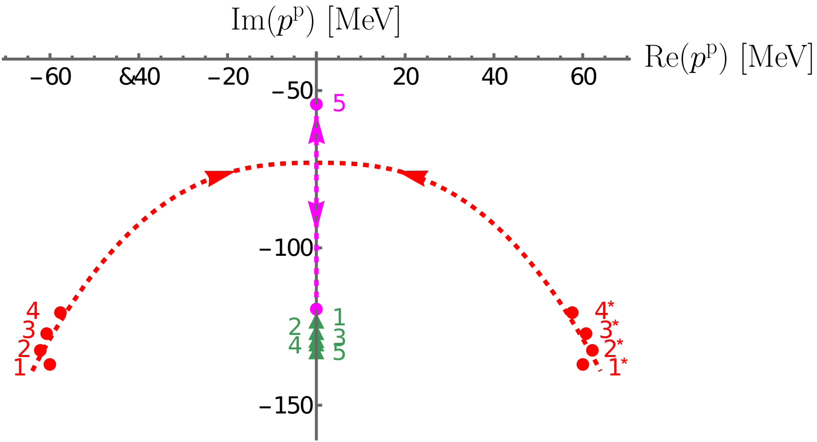
Sketch of full potential ($c_2 = 0$)



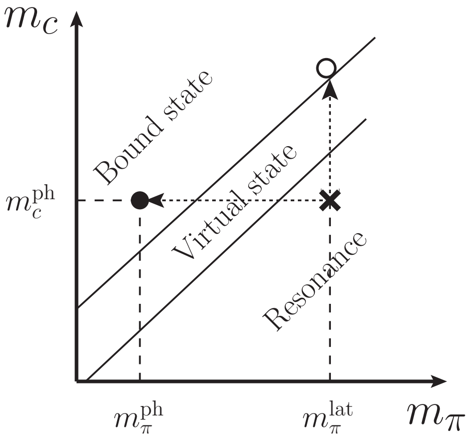
Types of supported poles



Lattice T_{cc}^+ pole dependence on m_c

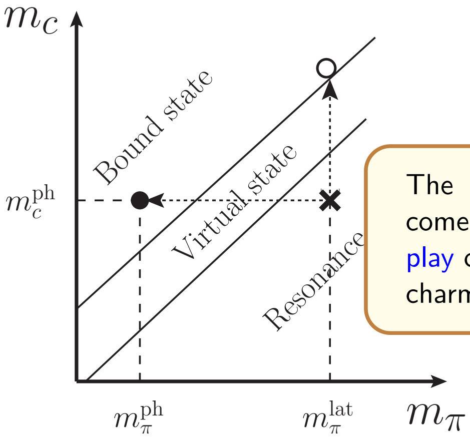


T_{cc}^+ pole motion across (m_c, m_π) plane



- Filled circle — physical T_{cc}^+
- Cross — starting lattice point
- Open circle — lattice T_{cc}^+ as shallow bound state

T_{cc}^+ pole motion across (m_c, m_π) plane



- Filled circle — physical T_{cc}^+

The pole position of the physics T_{cc}^+ comes as a result of a delicate interplay of the light quark mass m_q and the charmed quark mass m_c

Conclusions

- Collider experiments at energies above open-flavour thresholds started new era in hadronic physics
- Threshold phenomena, coupled channels, pion exchange are important
- Multibody unitarity and analyticity of amplitude need to be preserved
- Line shapes of non-Breit-Wigner form is current reality
- From "mass" and "width" to pole position and residues (couplings)
- Lattice simulations fill gaps in experimental data and provide information on "parallel" Universe
- EFT is model-independent, systematically improvable tool
- Results of EFT analysis are input for QCD-inspired models