Hadron-Hadron Interaction from Lattice QCD: HAL QCD Method

Yan Lyu (吕岩)

iTHEMS, RIKEN

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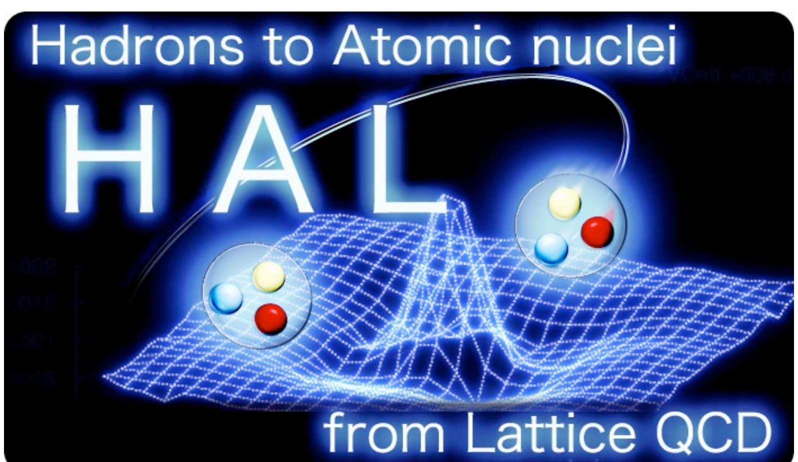
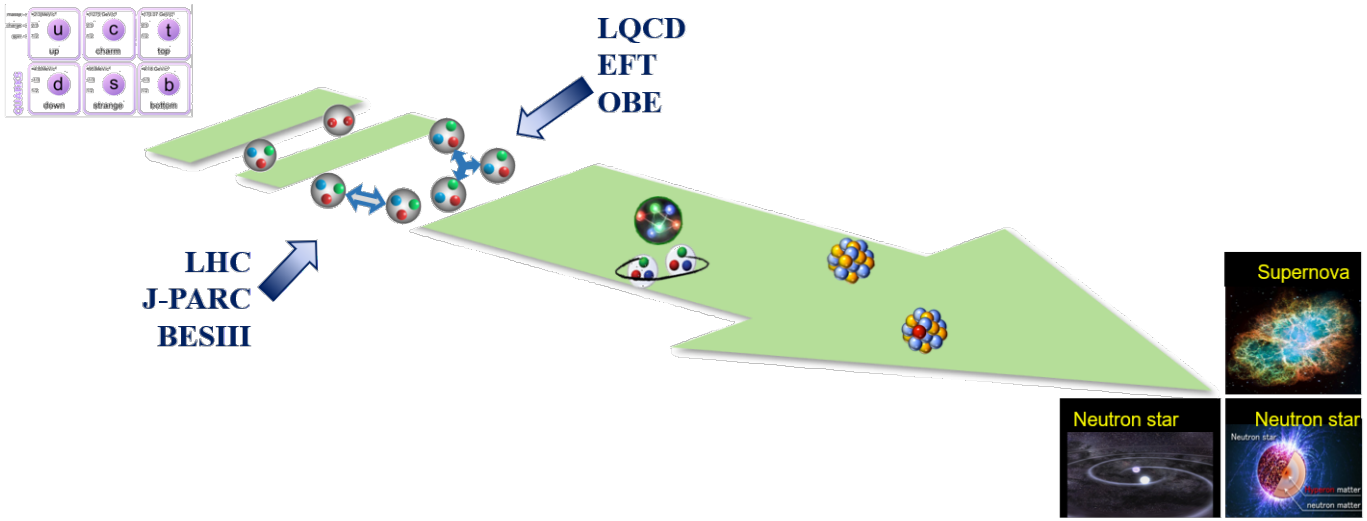


Hadron-hadron interaction

- Key ingredients of the path from quarks to universe



- Phenomenological models and EFT are successful in the light quark sector; many parameters need to determined through large experimental data
  - NN interaction constrained by thousand of data

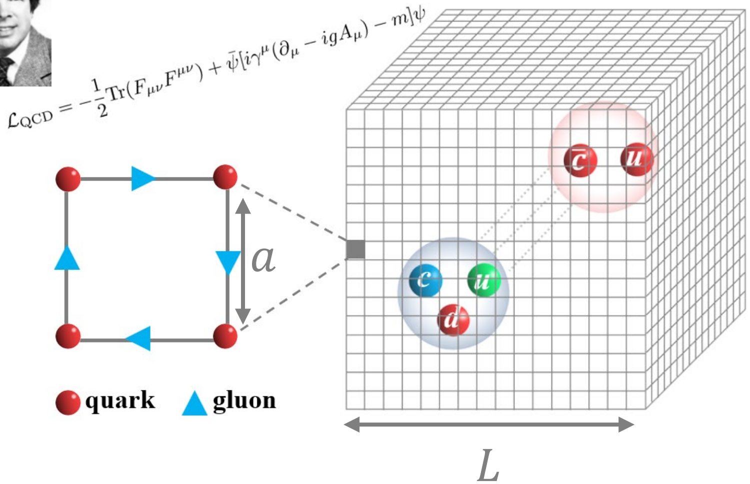
R. Machleidt and D.R. Entem, Phys. Rep. 503, 1 (2011)
J-X Lu et al, Phys. Rev. Lett. 128 142002 (2022)

	AV18	CD Bonn	EFT	
# parameters	40	38	24 (NR-N3LO)	19(R-N2LO)

- Hadron-hadron interaction in the strange and charm quark sector
  - experiemental data are scarce
  - LQCD plays important role in practice

Lattice QCD

K.G. Wilson, Phys. Rev. D 10, 2445 (1974)



2 parameters: m_q, g

➢ Lattice regularization

- UV cutoff $\sim \frac{1}{a}$; IR cutoff $\sim \frac{1}{L}$
- Quark field $q(x)$ on lattice sites
- Gauge field $U_\mu(x) = e^{iagA_\mu(x)}$ on links

➢ Path integral

- dof: $\sim 100^4 \times 8 \times 4$

$$\langle \hat{O} \rangle = \frac{1}{Z_E} \int [\mathcal{D}\bar{q}][\mathcal{D}q][\mathcal{D}U] O e^{-S_E}$$

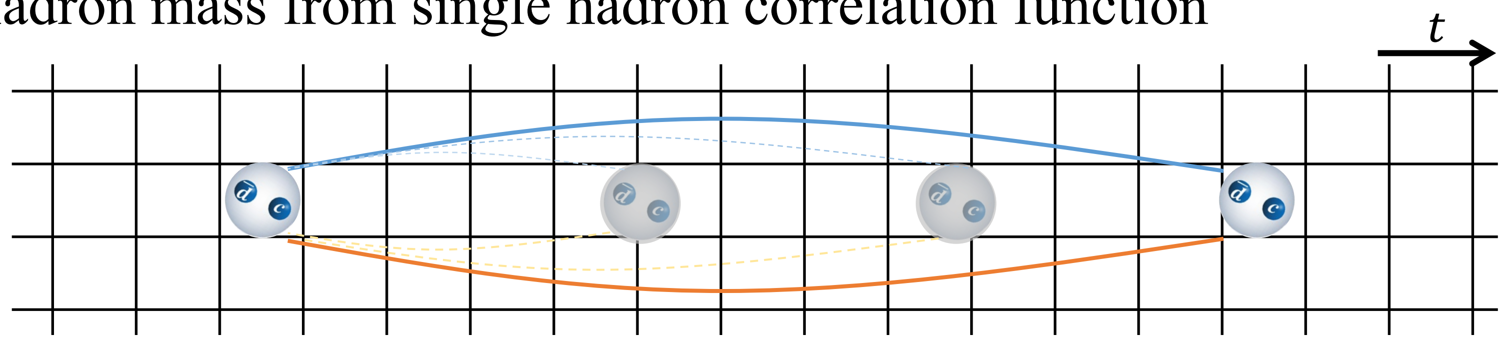
➢ Monte Carlo Simulation

- Importance sampling: a series of QCD configurations with probability e^{-S_E}
- Multiple measurements $O_1, \cdots, O_N$

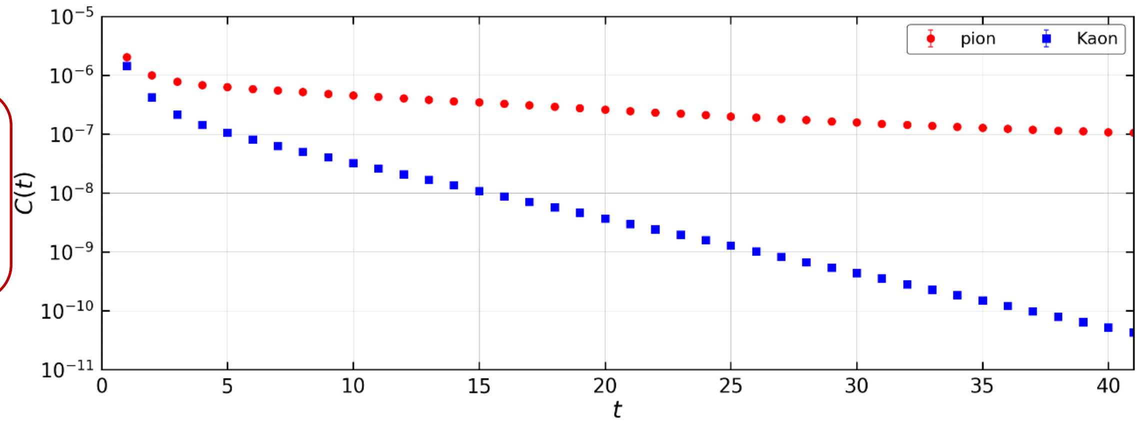
$$\langle \hat{O} \rangle = \bar{O} \pm \frac{\sigma}{\sqrt{N}}$$

Examples

- Hadron mass from single hadron correlation function



$$C(t) = \langle \hat{O}(t)\hat{O}^\dagger(0)\rangle = \sum_n |\langle 0|\hat{O}|n\rangle|^2 e^{-E_n t}$$



- Two-hadron eigen- energies/states from two-hadron correlation function

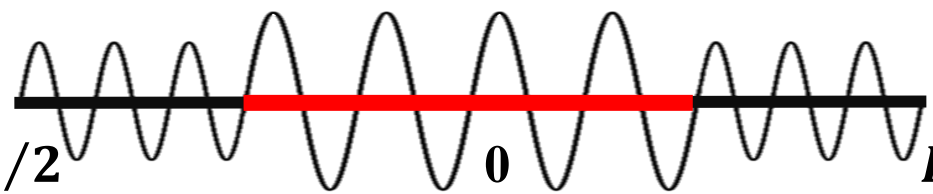
$$F(\boldsymbol{r},t) = \langle \hat{O}(\boldsymbol{r},t)\hat{O}^\dagger(0)\rangle = \sum_n \langle 0|\hat{O}(\boldsymbol{r},t)|n\rangle \langle n|\hat{O}^\dagger(0)|0\rangle$$

$$= \sum_n a_n \psi_n(\boldsymbol{r}) e^{-E_n t} \xrightarrow{t \gg \frac{1}{E_1 - E_0}} a_0 \psi_0(\boldsymbol{r}) e^{-E_0 t}$$

Finite volume method

E_{FV}
FV energy
⟷
$\delta(E_{\mathrm{FV}})$
Phase shift

- 1d scattering



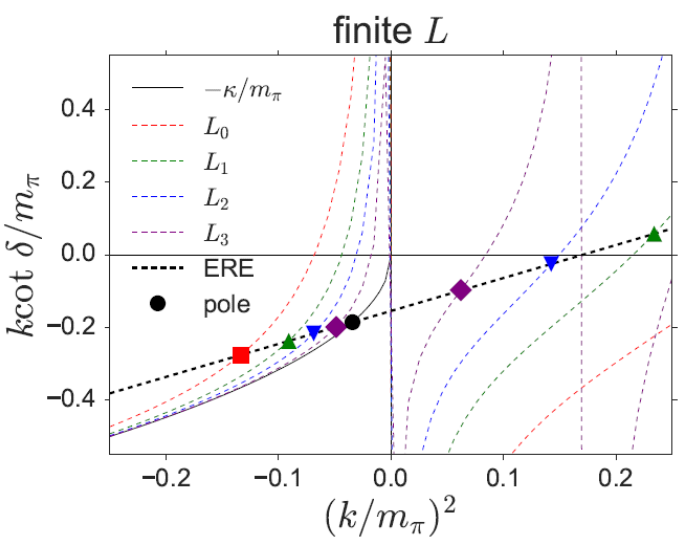
$$\psi(x) \sim \cos(k|x| + \delta(k)), \quad k + \frac{2}{L}\delta(k) = \frac{2n\pi}{L}$$

PBC: $\psi(x) = \psi(x+L)$

- 3d scattering

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi}L} Z_{00}(1, q^2)$$

$$Z_{00}(s, q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n}\in\mathbb{Z}^3} \frac{1}{(\vec{n}^2 - q^2)^s}, \quad q = \frac{kL}{2\pi}$$



M. Lüscher, Nucl. Phys. B 354, 531 (1991)

HAL QCD method

- Nambu-Bethe-Salpeter (NBS) amplitude

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)
N. Ishii, *et al.* [HAL QCD Coll.], Phys. Lett. B 712, 437 (2012)

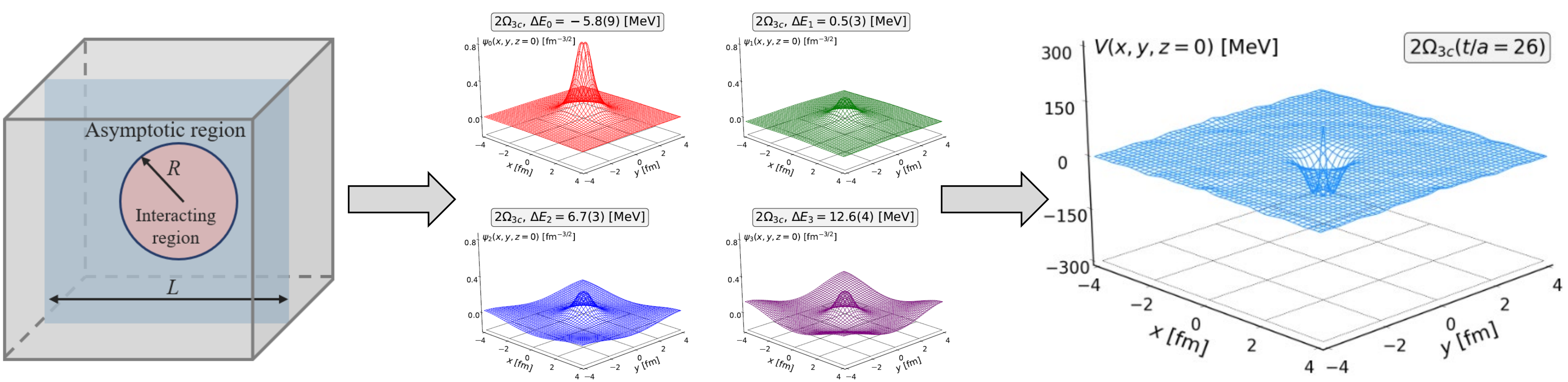
$$\psi^k(\boldsymbol{r})e^{-Et} = \langle 0|\hat{D}^*(\boldsymbol{r},t)\hat{D}(\boldsymbol{0},t)|D^*(\boldsymbol{k})D(-\boldsymbol{k});E\rangle$$

- Asymptotic region: $\psi^k(r) \simeq A\,\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$
- Interacting region: define a nonlocal E-independent potential

$$(\nabla^2 + k^2)\psi^k(\boldsymbol{r}) = 2\mu\int d\boldsymbol{r}'\,U(\boldsymbol{r},\boldsymbol{r}')\psi^k(\boldsymbol{r}')$$

✓ derivative expansion $U(\boldsymbol{r},\boldsymbol{r}') = V(\boldsymbol{r})\delta(\boldsymbol{r}-\boldsymbol{r}') + \sum V_i(\boldsymbol{r})\,\nabla^i\delta(\boldsymbol{r}-\boldsymbol{r}')$

Check on convergence: Iritani *et al.*, JHEP 03, 007 (2019); YL *et al.*, PRD 105, 074512 (2022)



Lattice simulation | NBS amplitude | Potential

Lattice configurations near physical point

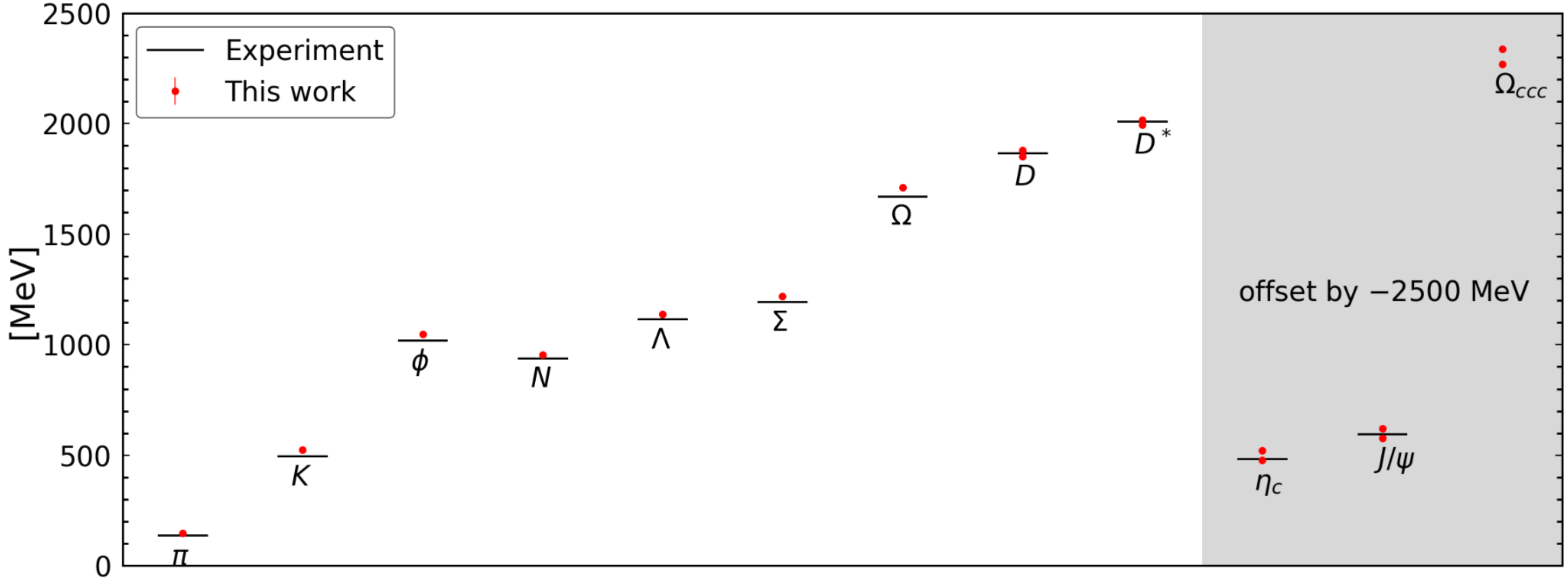
- (2+1)-flavor configuration
  - Iwasaki gauge action
  - $O(a)$-improved Wilson quark action for uds quark
  - Relativistic heavy quark action for c quark

K.-I. Ishikawa *et al.* [PACS Coll.], *Proc. Sci.*, LATTICE2015 075 (2016)
Y. Namekawa *et al.* [PACS Coll.], *Proc. Sci.*, LATTICE2016 125 (2017)

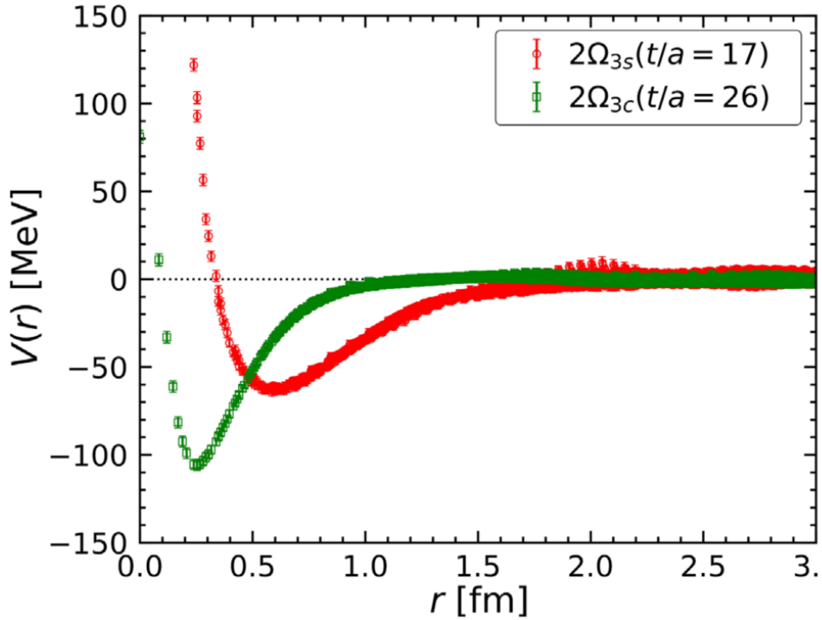
Fugaku supercomputer (440 PFlops)

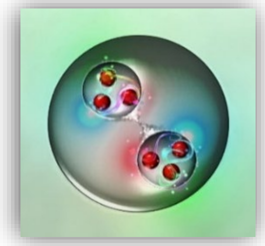
$L^3 \times T$	a [fm]	La [fm]	m_π [MeV]	m_K [MeV]
$96^3 \times 96$	0.0846	8.1	146	525

- Hadron mass

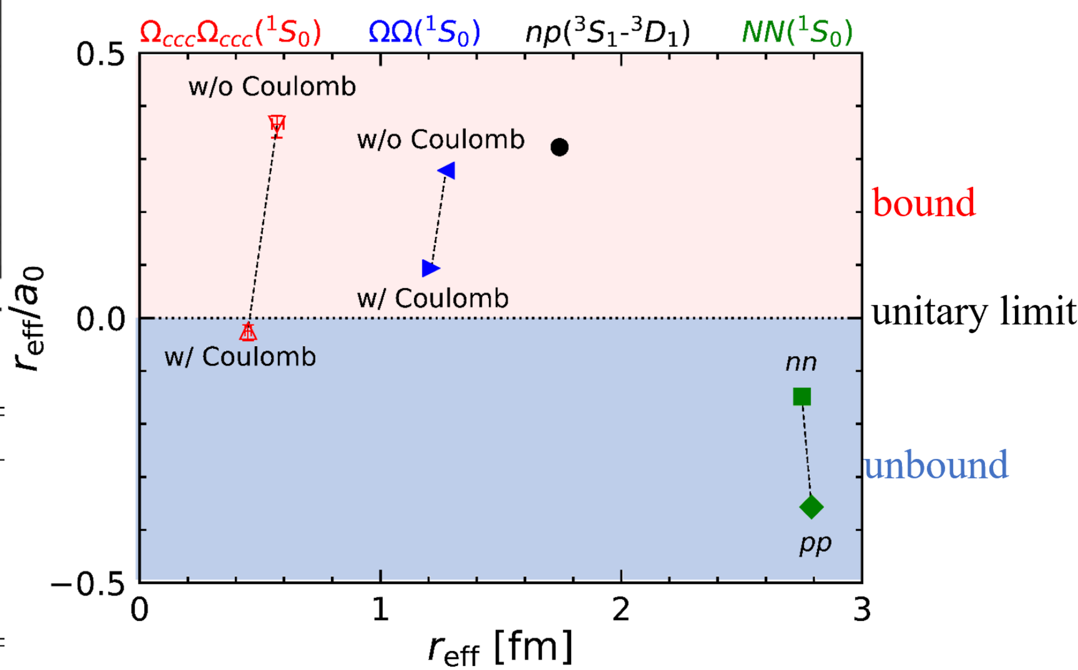


$\Omega\Omega$ and $\Omega_{ccc}\Omega_{ccc}$



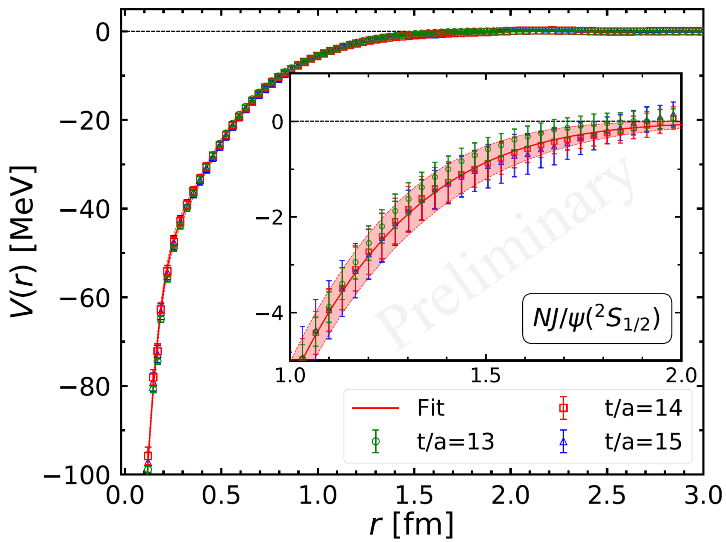


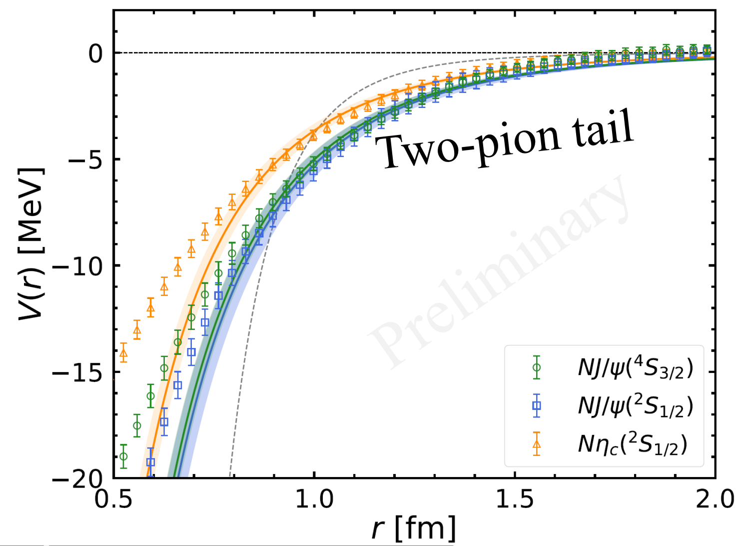
	$2\Omega_{3s}$	$2\Omega_{3c}$
E_b [MeV]	1.6(6)	5.7(8)
$\sqrt{\langle r^2 \rangle}$ [fm]	3.3(5)	1.1(1)
a_0 [fm]	−4.6(6)	−1.6(1)
$r_{\rm eff}$ [fm]	1.27(3)	0.57(2)



YL, Hui Tong *et al*, Phys. Rev. Lett. 127, 072003 (2021)
S. Gongyo *et al.*, Phys. Rev. Lett. 120, 212001 (2018)

N-$c\bar{c}$





channel	a_0 [fm]	$r_{\rm eff}$ [fm]
$NJ/\psi(^4S_{3/2})$	$0.30(2)\binom{+0}{-2}$	$3.25(12)\binom{+6}{-9}$
$NJ/\psi(^2S_{1/2})$	$0.38(4)\binom{+0}{-3}$	$2.66(21)\binom{+0}{-10}$
$N\eta_c(^2S_{1/2})$	$0.21(2)\binom{+0}{-1}$	$3.65(20)\binom{+0}{-6}$

Preliminary

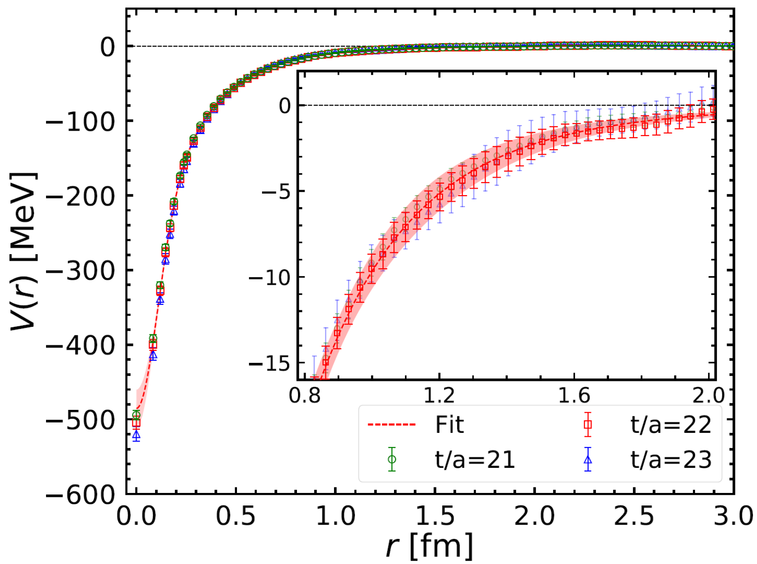
- A direct phenomenological application
  - The J/ψ mass modification in nuclear medium is related to the spin-averaged scattering length of N-J/ψ scattering

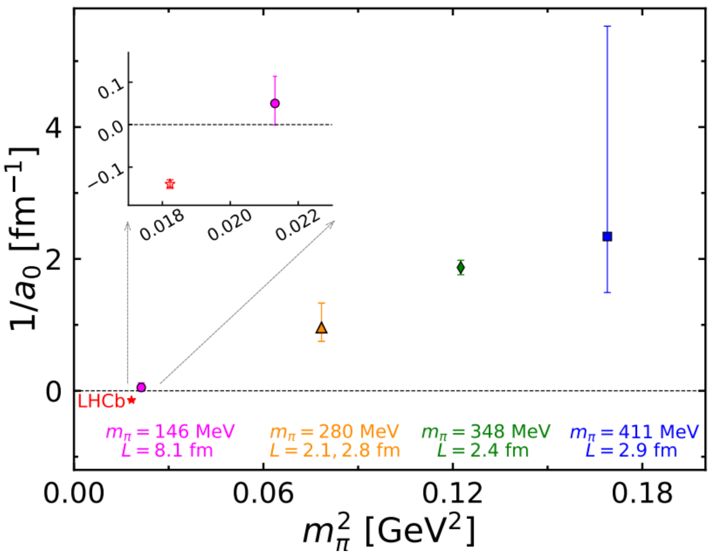
A. Hayashigaki, Prog. Theor. Phys. 101 (1999)

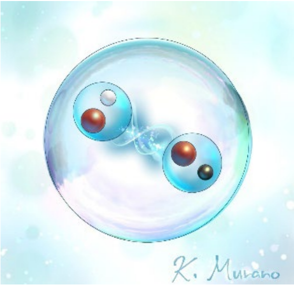
$$\delta m_{J/\psi} \simeq -\frac{2\pi(m_N + m_{J/\psi})}{m_N m_{J/\psi}} a_{J/\psi}^{\text{spin-av}} \rho_{\rm nm} = -19(3)\ \text{MeV}$$

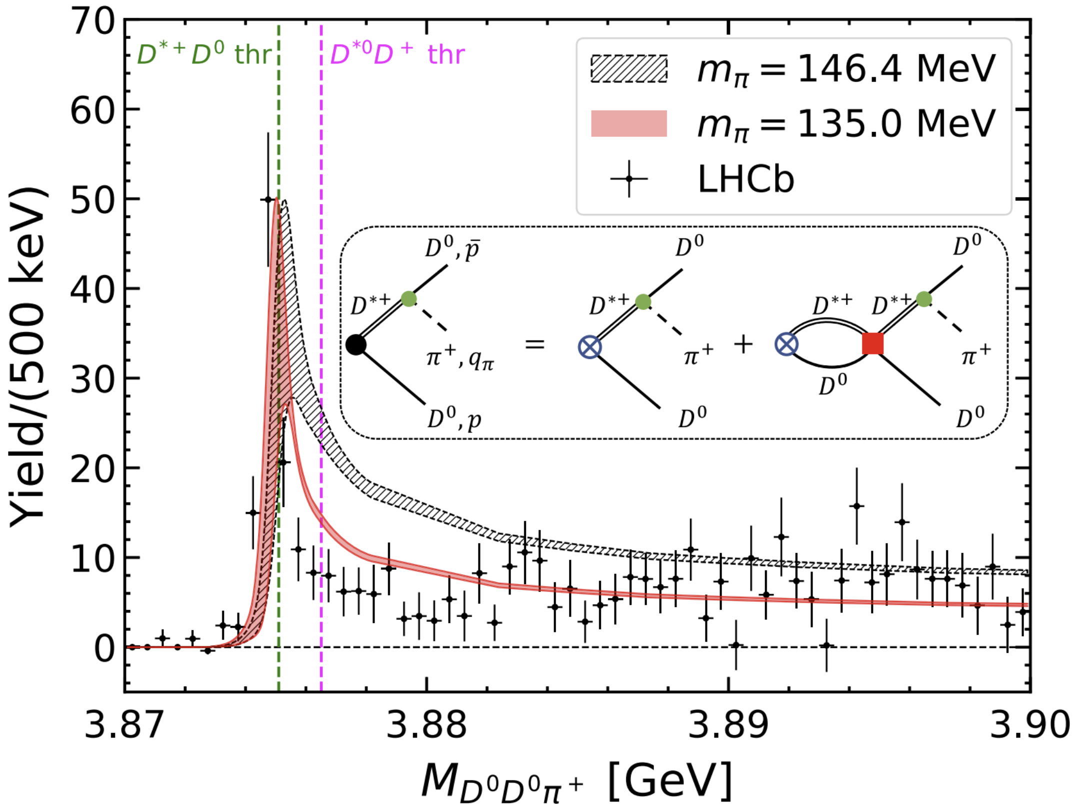
YL, T. Doi, T. Hatsuda, T. Sugiura, arXiv: 2410.xxxxx

D^*D in the T_{cc}^+ channel









YL, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, PRL 131, 161901 (2023)

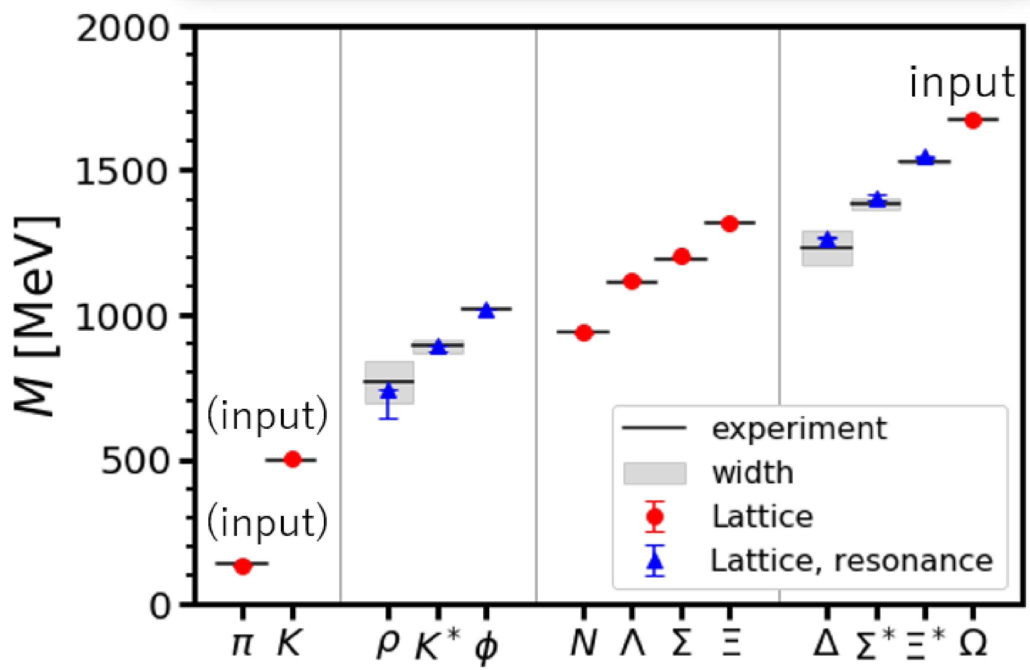
Physical point simulations

- Configurations with physical quark masses by HAL QCD Coll. are ready

RIKEN-iTHEMS-Report-24, YITP-24-73

Scale setting and hadronic properties in light quark sector with $(2+1)$-flavor Wilson fermions at the physical point

Tatsumi Aoyama,[1] Takahiro M. Doi,[2,3] Takumi Doi,[3] Etsuko Itou,[4,3,*]
Yan Lyu,[3] Kotaro Murakami,[5,3] and Takuya Sugiura[6,3]
(HAL QCD Collaboration)



- Stay tuned for physical point studies

PRD in press, arXiv:2406.16665

Summary & Outlook

- Hadron interactions from LQCD
  - Finite volume formalism
  - HAL QCD method
- Selected results from HAL QCD at $m_\pi = 146$ MeV
  - $\Omega\Omega$ and $\Omega_{ccc}\Omega_{ccc}$ are bound under strong interaction
  - N-$c\bar{c}$ interaction is consistent with two-pion exchange at long distance
  - D^*D scattering length and $DD\pi$ spectrum are close to the LHCb data
- Lattice calculations at physical point are under way

Thanks for your attention!