Recent developments and applications of nuclear lattice EFT

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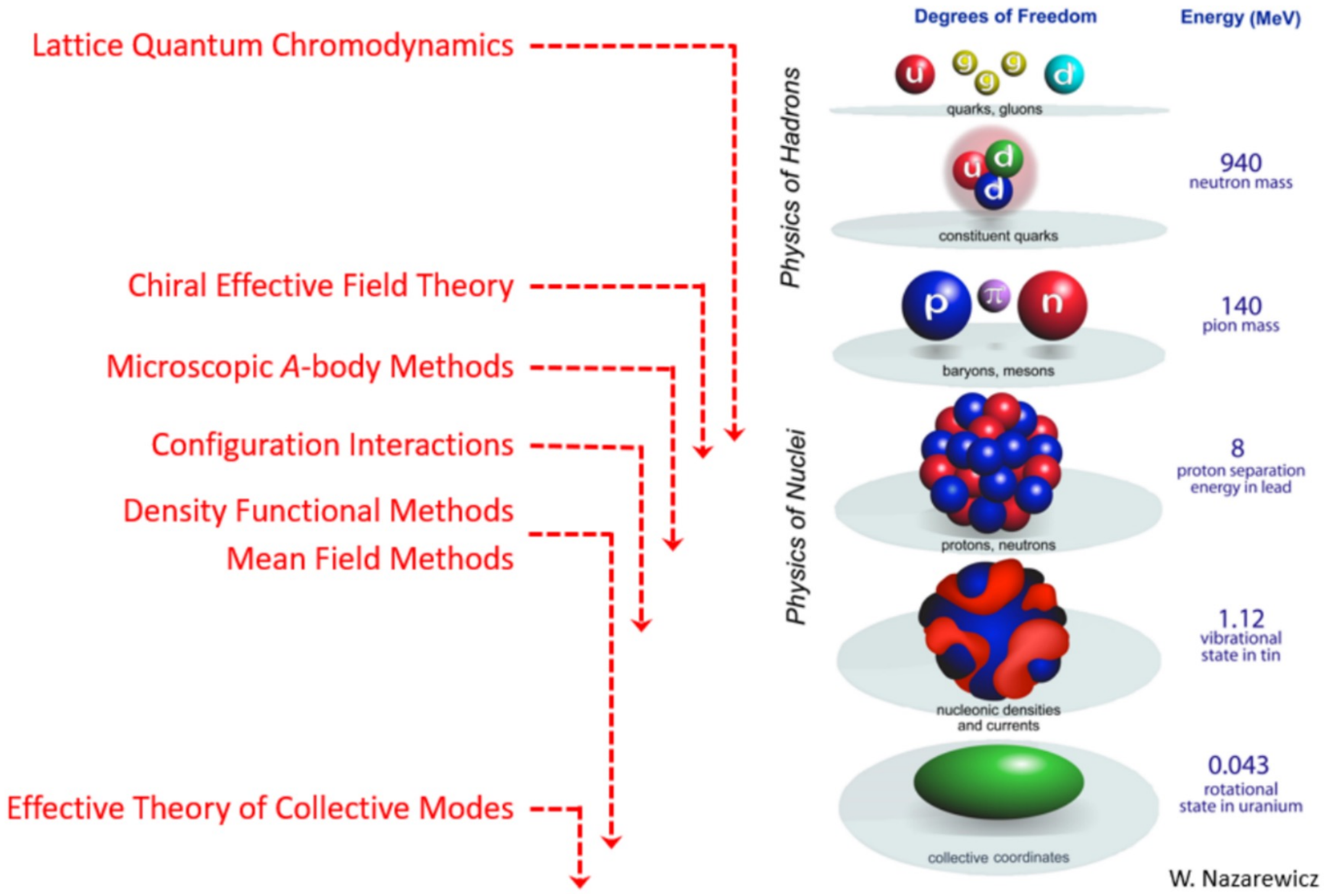
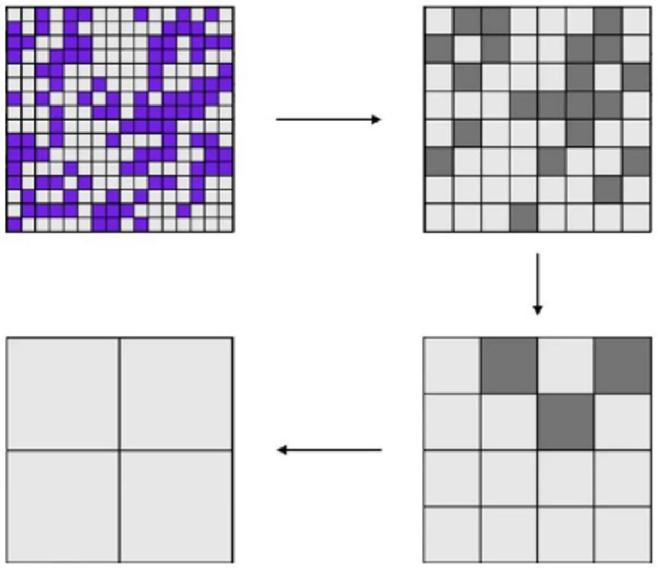
Table of contents

- **Lattice effective field theory**
- First principles cal. I: Minimal effective interactions
- First principles cal. II: High-precision chiral forces
- Summary & Perspective

What is a nuclear EFT?



- **Modern nuclear force constructions are based on the Effective Field Theory**
- **Theoretical foundation of EFT is the Wilsonian renormalization group**:
  - **High-momentum** details can be integrated out & hidden in LECs
  - **Low-momentum** physics kept invariant under ren. group transformations

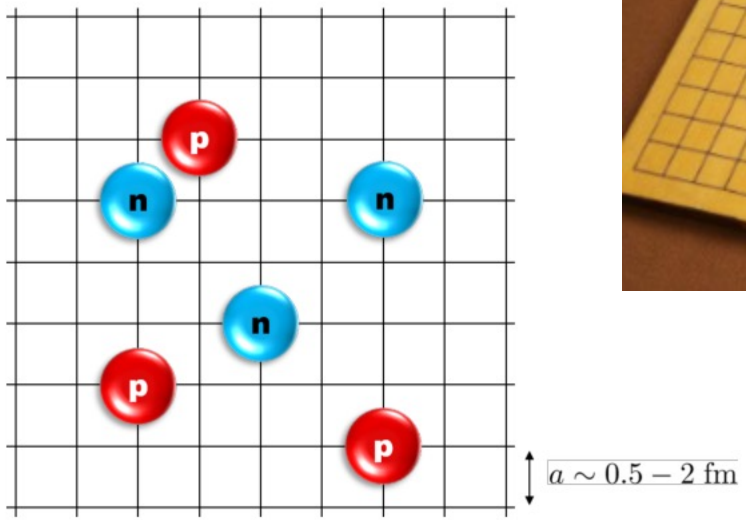


Lattice EFT: A many-body EFT solver

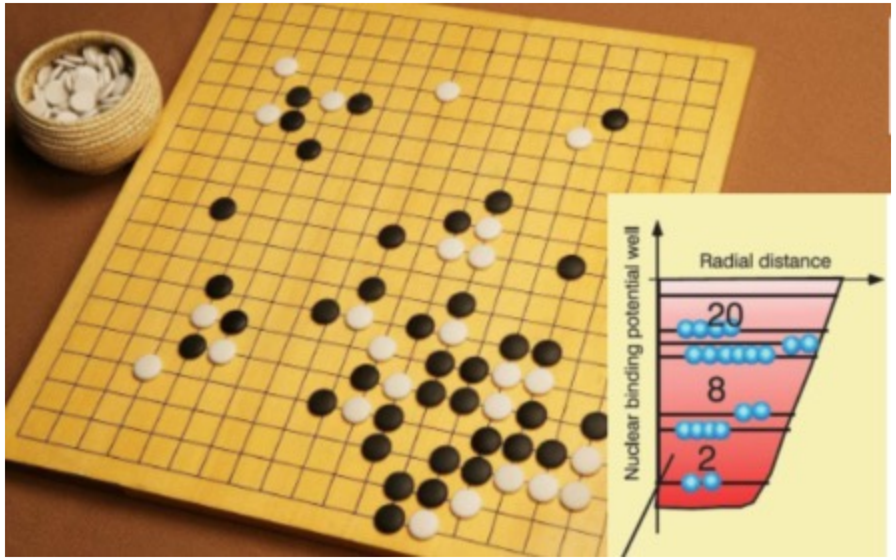
Lattice EFT = **Chiral EFT** + **Lattice** + **Monte Carlo**

Review: Dean Lee, Prog. Part. Nucl. Phys. 63, 117 (2009),
Lähde, Meißner, "Nuclear Lattice Effective Field Theory", Springer (2019)

- Discretized chiral nuclear force
- Lattice spacing $a \approx 1$ fm $= 620$ MeV (~chiral symmetry breaking scale)
- Protons & neutrons interacting via **short-range, δ-like** and **long-range, pion-exchange** interactions
- Exact method, polynomial scaling ($\sim A^2$)



Lattice adapted for nucleus



- 非微扰量子多体问题是计算物理的核心挑战
- 核物理：2~300粒子系统，可能找到严格解

Lattice EFT: A many-body EFT solver

- Get *interacting g. s.* from imaginary time projection:

$$|\Psi_{g.s.}\rangle \propto \lim_{\tau\to\infty} \exp(-\tau H)|\Psi_A\rangle$$

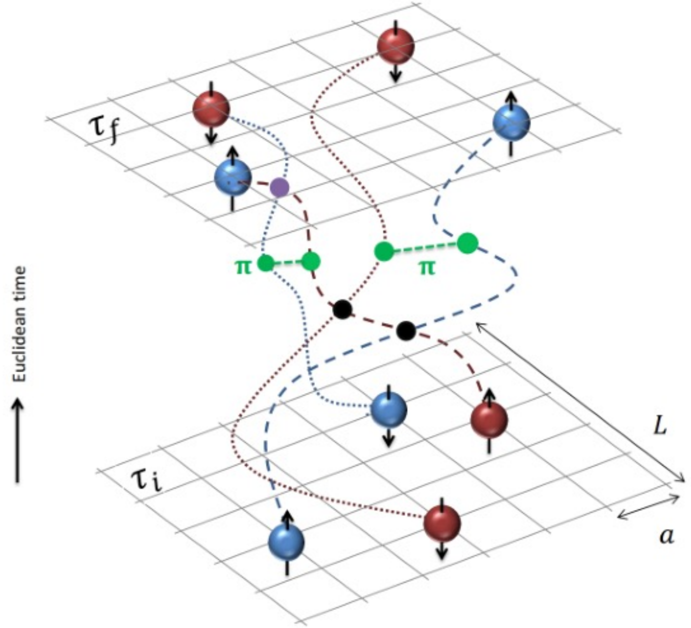
with $|\Psi_A\rangle$ representing A *free* nucleons.

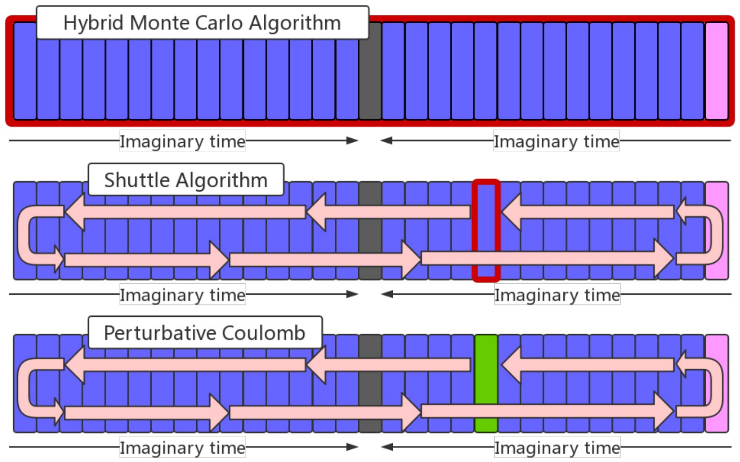
- Expectation value of any operator $\mathscr{O}$:

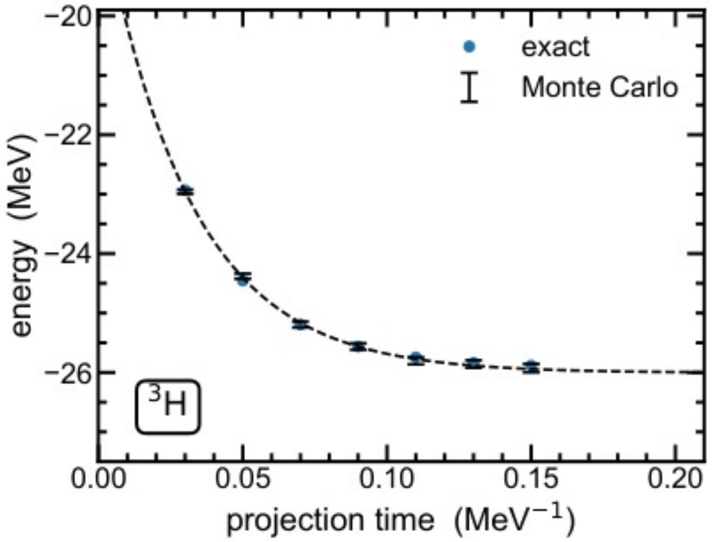
$$\langle O\rangle = \lim_{\tau\to\infty} \frac{\langle\Psi_A|\exp(-\tau H/2)\mathscr{O}\exp(-\tau H/2)|\Psi_A\rangle}{\langle\Psi_A|\exp(-\tau H)|\Psi_A\rangle}$$

- τ is discretized into time slices:

$$\exp(-\tau H) \simeq \left[:\exp(-\frac{\tau}{L_t}H):\right]^{L_t}$$







All possible configurations in $\tau \in [\tau_i, \quad \tau_f]$ are sampled.
Complex structures like nucleon clustering emerges naturally.

Nuclear Force Problem

Nuclear Force Problem: Can the nuclear force calibrated with the **N-N scattering and few-body data** uniquely and correctly predict the **structures of finite nuclei**?

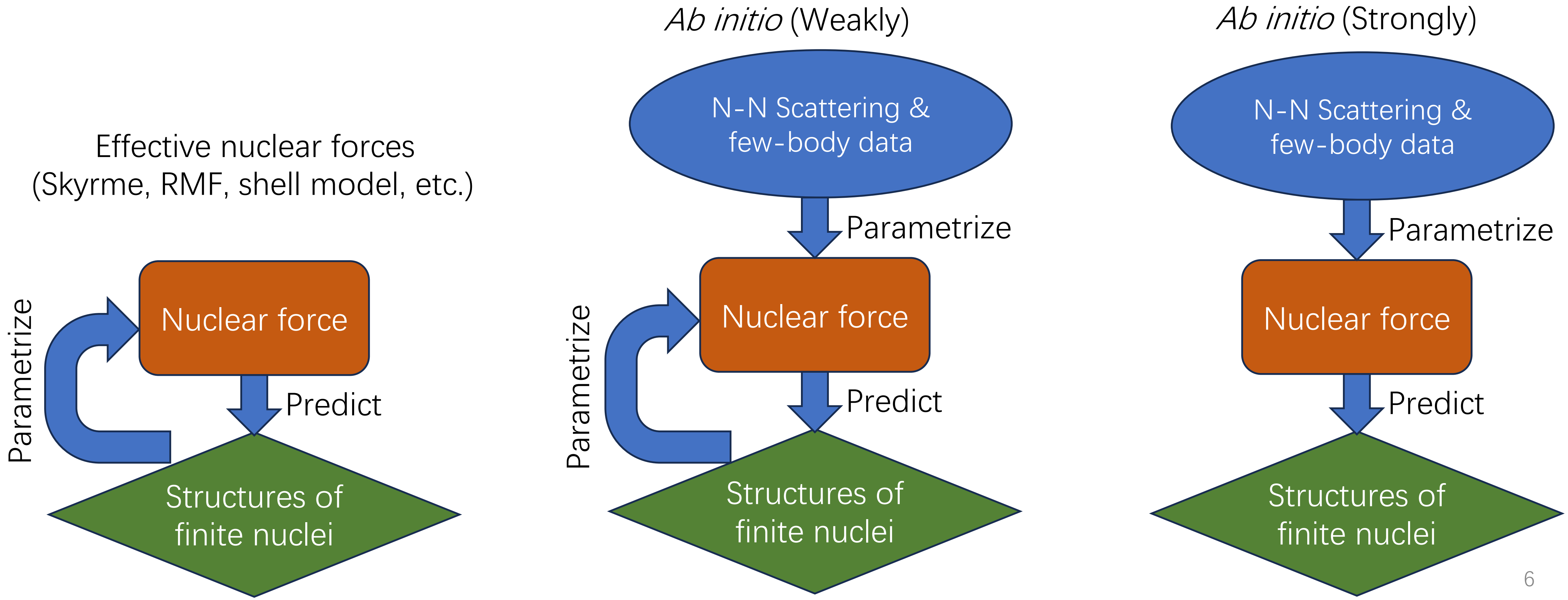


Table of contents

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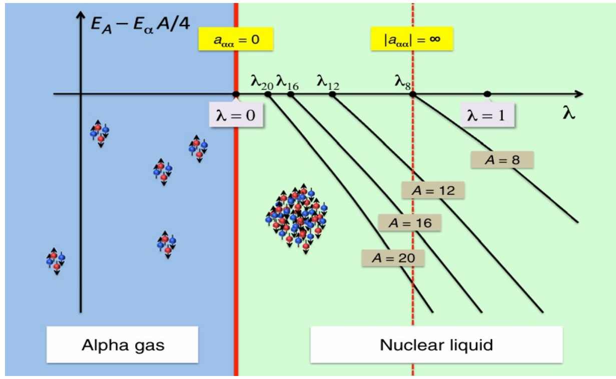
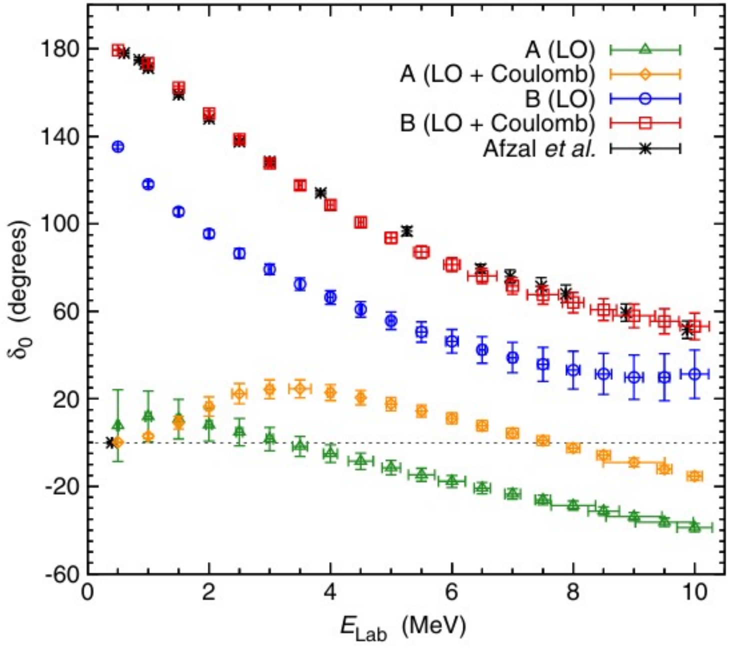
Nuclear binding near a quantum phase transition

Selected for a Viewpoint in *Physics*
PRL **117,** 132501 (2016) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2016

Nuclear Binding Near a Quantum Phase Transition

Serdar Elhatisari,[1] Ning Li,[2] Alexander Rokash,[3] Jose Manuel Alarcón,[1] Dechuan Du,[2] Nico Klein,[1] Bing-nan Lu,[2] Ulf-G. Meißner,[1,2,4] Evgeny Epelbaum,[3] Hermann Krebs,[3] Timo A. Lähde,[2] Dean Lee,[5] and Gautam Rupak[6]





Nucleus	*A* (LO)	*B* (LO)	*A* (LO + Coulomb)	*B* (LO + Coulomb)	Experiment
^{3}H	−7.82(5)	−7.78(12)	−7.82(5)	−7.78(12)	−8.482
^{3}He	−7.82(5)	−7.78(12)	−7.08(5)	−7.09(12)	−7.718
^{4}He	−29.36(4)	−29.19(6)	−28.62(4)	−28.45(6)	−28.296
^{8}Be	−58.61(14)	−59.73(6)	−56.51(14)	−57.29(7)	−56.591
^{12}C	−88.2(3)	−95.0(5)	−84.0(3)	−89.9(5)	−92.162
^{16}O	−117.5(6)	−135.4(7)	−110.5(6)	−126.0(7)	−127.619
^{20}Ne	−148(1)	−178(1)	−137(1)	−164(1)	−160.645

- The nuclear force can be either local (position-dependent) or non-local (velocity-dependent).
- Locality is an essential element for nuclear binding.

Zeroth order Hamiltonian (perturbative order)

We use a zeroth order lattice Hamiltonian that respects the Wigner-SU(4) symmetry

$$H_0 = K + \frac{1}{2} C_{\mathrm{SU4}} \sum_{\boldsymbol{n}} : \tilde{\rho}^2(\boldsymbol{n}) :$$

The smeared density operator $\tilde{\rho}(\boldsymbol{n})$ is defined as

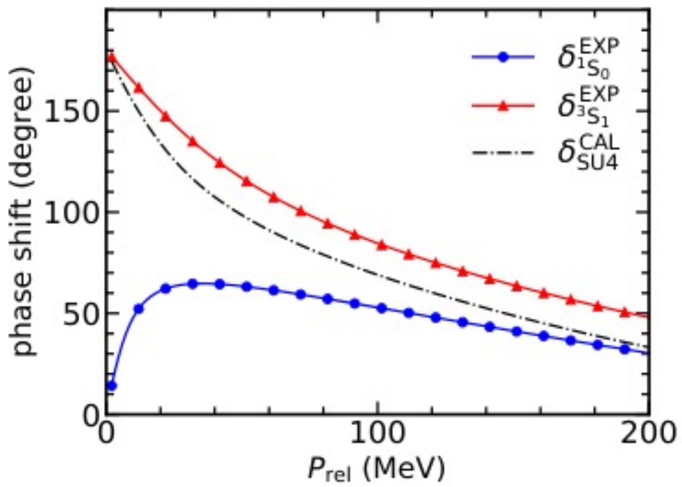
$$\tilde{\rho}(\boldsymbol{n}) = \sum_i \tilde{a}_i^\dagger(\boldsymbol{n}) \tilde{a}_i(\boldsymbol{n}) + s_L \sum_{|\boldsymbol{n}'-\boldsymbol{n}|=1} \sum_i \tilde{a}_i^\dagger(\boldsymbol{n}') \tilde{a}_i(\boldsymbol{n}'), \tag{1}$$

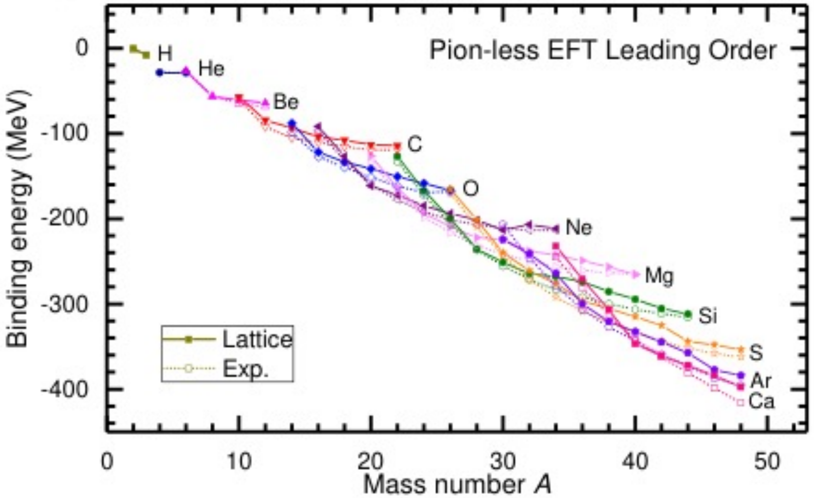
where i is the joint spin-isospin index

$$\tilde{a}_i(\boldsymbol{n}) = a_i(\boldsymbol{n}) + s_{NL} \sum_{|\boldsymbol{n}'-\boldsymbol{n}|=1} a_i(\boldsymbol{n}'). \tag{2}$$

In this work we use a lattice spacing $a = 1.32$ fm and the parameter set

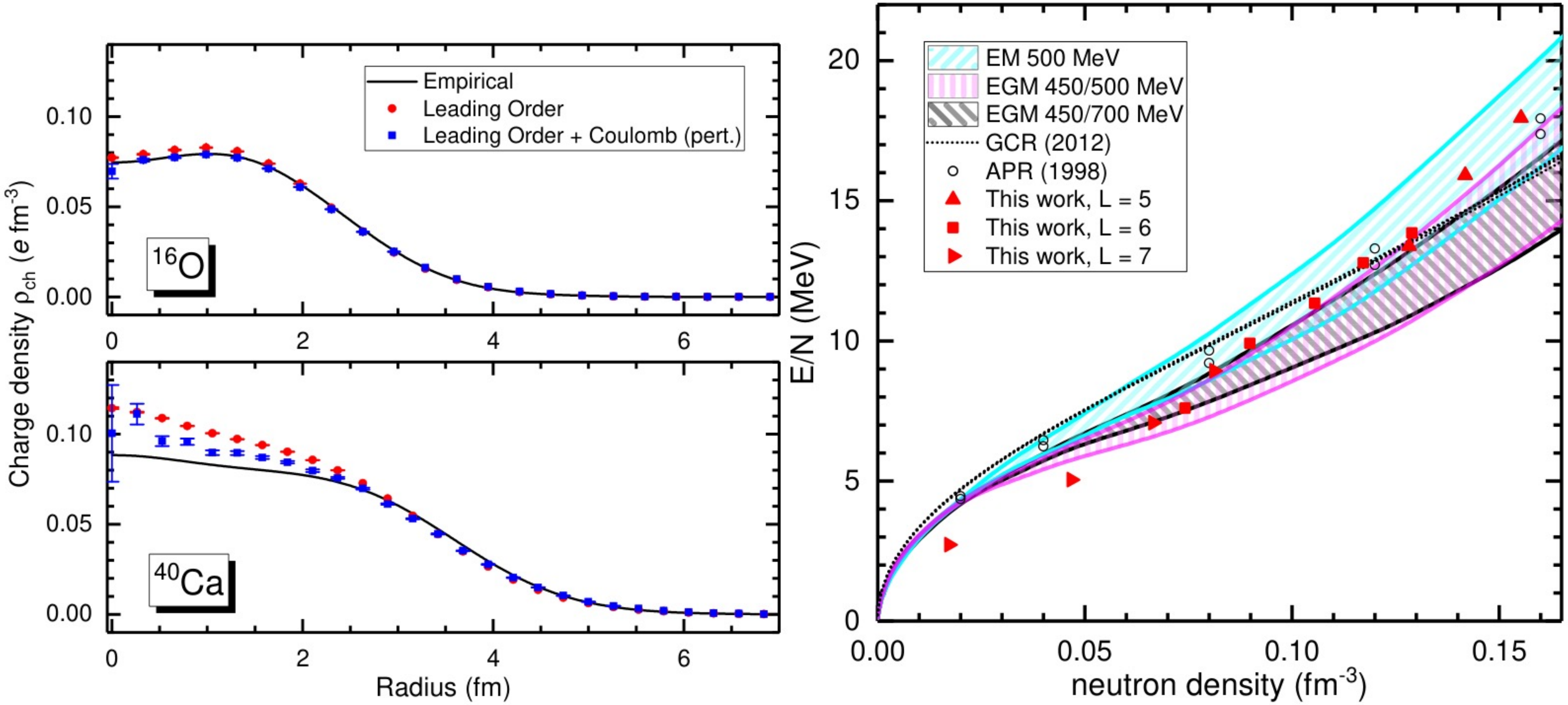
$C_{\mathrm{SU4}} = -3.41 \times 10^{-7}$ MeV^{-2}, $s_L = 0.061$ and $s_{NL} = 0.5$.





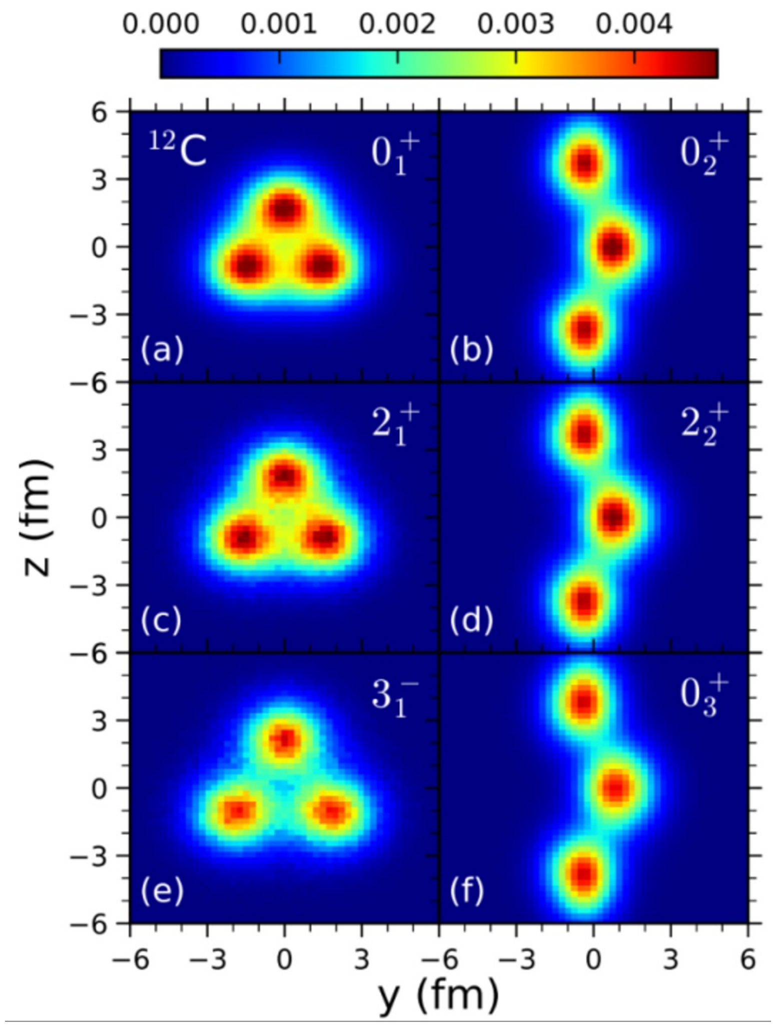
Essential elements for nuclear binding

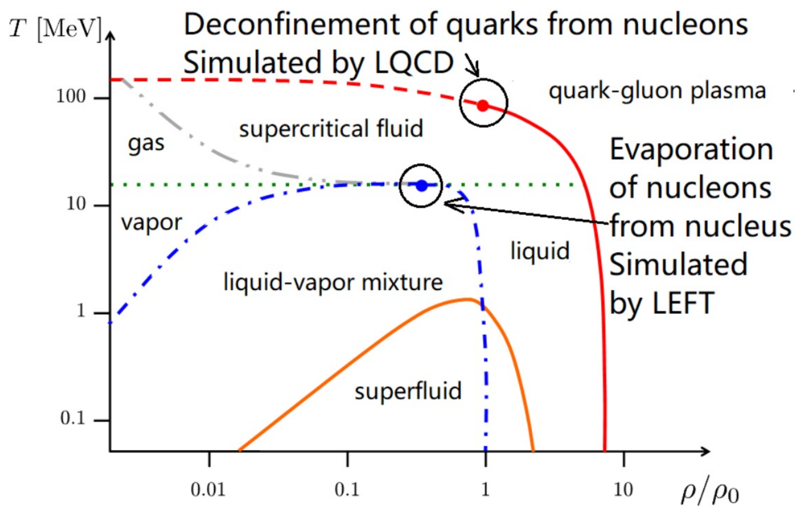
Charge density and neutron matter equation of state are impotant in element creation, neutron star merger, etc.

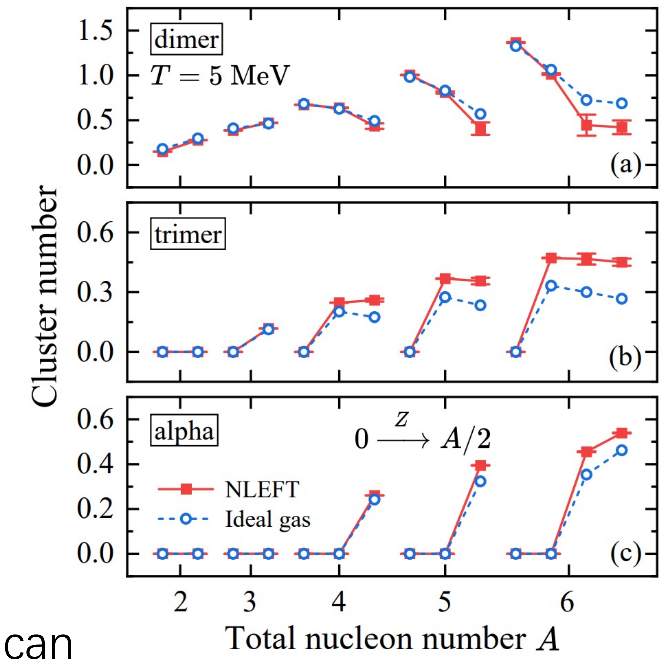


Lu, Li, Elhatisari, Lee, Epelbaum, Meissner, Phys. Lett. B 797 (2019) 134863

Applications of SU(4) lattice interaction



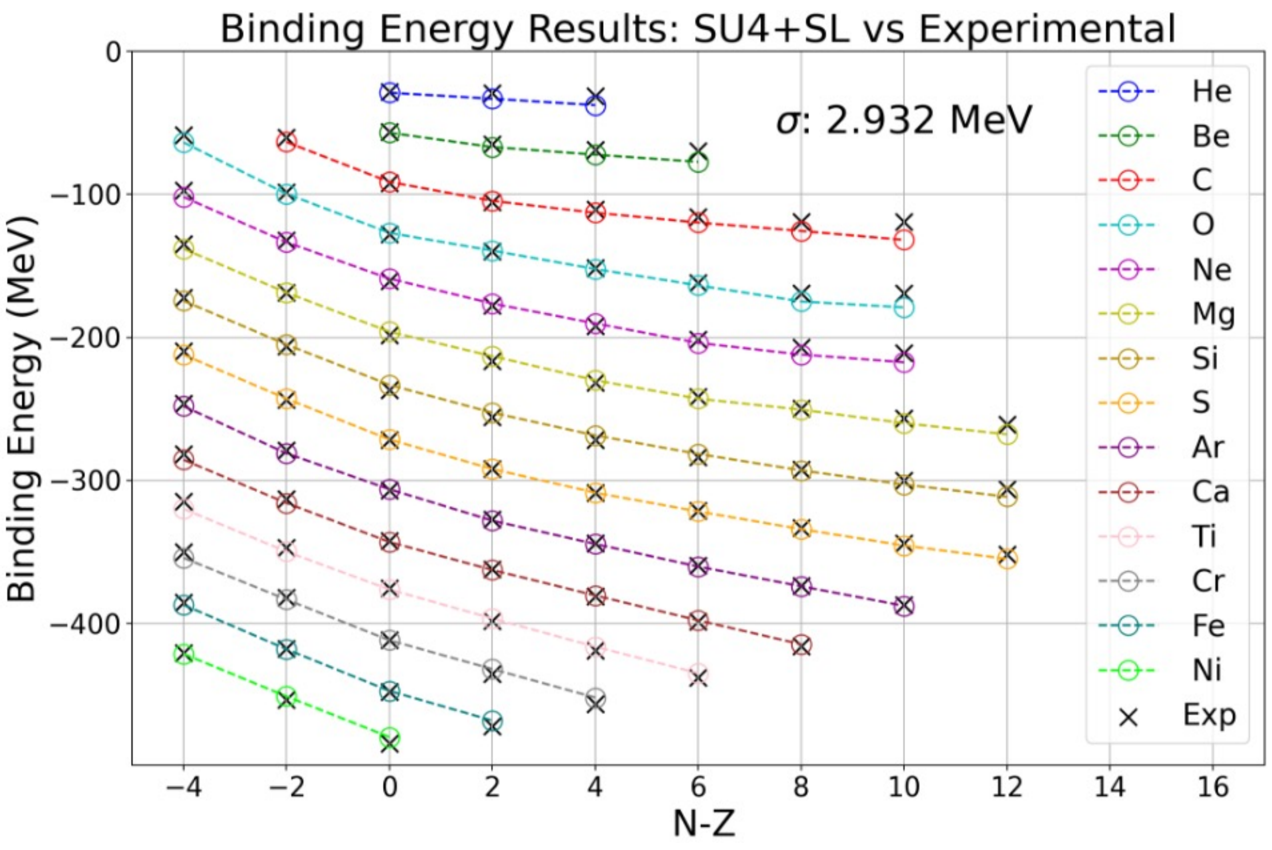




- SU(4) interaction is sign-problem-free and can reproduce the bulk properties.

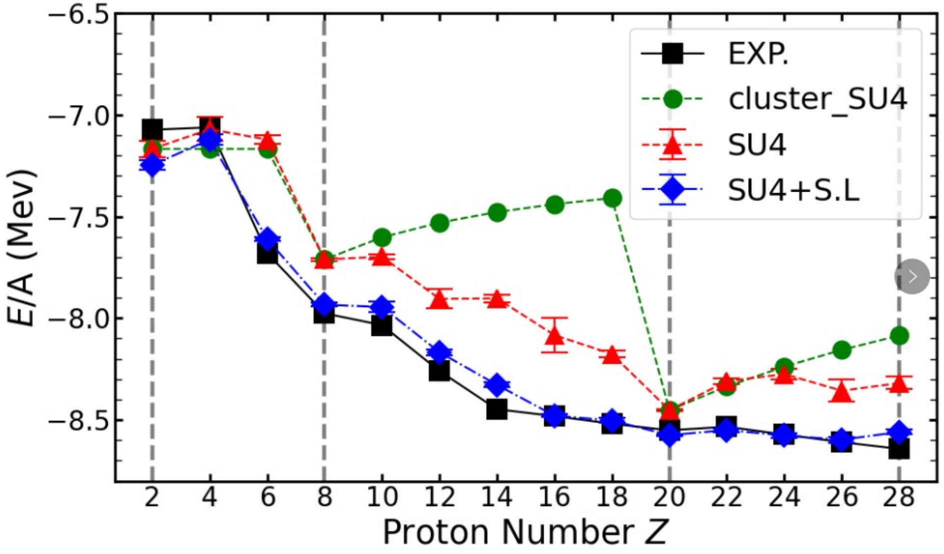
  - Tomography of nuclear clustering
    Shi-Hang Shen et al., Nat. Commun. 14, 2777 (2023) ← 申时行’ s talk
  - Ab initio nuclear thermodynamics
    Lu et al., Phys. Rev. Lett. 125, 192502 (2020)
  - Ab initio study of nuclear clustering in hot medium
    Zheng-Xue Ren et al., Phys. Lett. B 850, 138463 (2024)

Nuclear binding energies with spin-orbit term (preliminary)



- Average error for **76** even-even nuclei: **2.932 MeV**
  Applicable to light/medium mass nuclei
  Zhong-Wang Niu et al., in preparation
  Can be viewed as an *ab initio* nuclear mass model

- Errors in other models
  - Relativistic mean field (PC-PK1): **2.258 MeV**
    Peng-Wei Zhao et al., PRC82, 054319 (2010)
  - Non-rel. mean field (UNDEF1): **3.380 MeV**
    Kortelainen et al., PRC 85, 024304 (2012).
  - Finite range droplet model: **1.142 MeV**
    P. Moller et al., **Atom. Data Nucl. Data Tables 109, 1 (2016)**



- **Spin-orbit term** is essential for **shell evolutions**.
  (proper SL term **do not induce sign problem**)
- SU(4) + SL Hamiltonian, **5 parameters** optimized with masses of ^{4}He, ^{16}O, ^{24}Mg, ^{28}Si, ^{40}Ca, etc.

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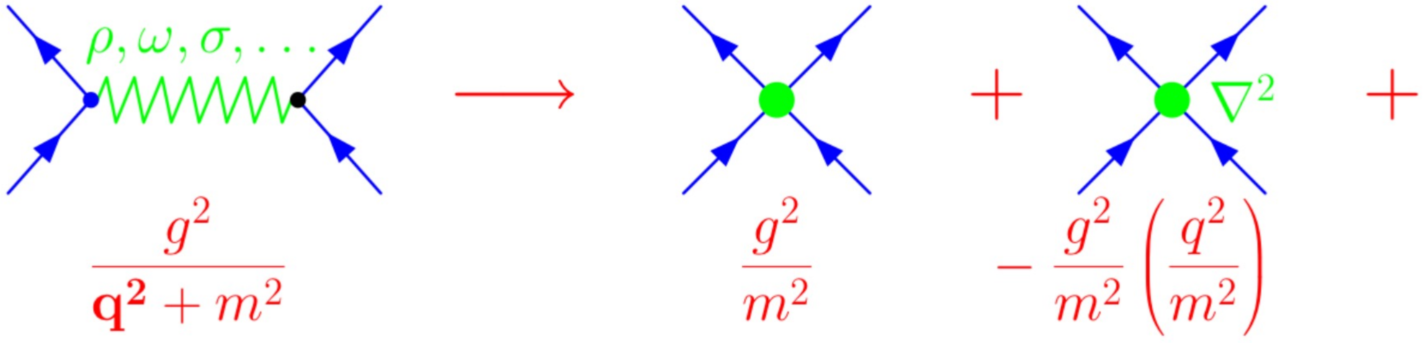
Contact term regulators

Boson exchange $\Longrightarrow$ model of short-distance physics

$\Longrightarrow$ unresolved in chiral EFT (except for pion)

$\Longrightarrow$ encoded in coefficients of contact terms



- Contact terms originate from heavy-meson exchanges
- Mesons with mass $m_H \gg \Lambda$ can be absorbed in δ-functions
- Choose an appropriate Λ satisfying $Q \ll \Lambda \ll m_H$

In momentum space, contact terms become **polynomials** of in & out momenta

- Incoming momenta: $\boldsymbol{p}_1, \boldsymbol{p}_2$
- Outgoing momenta: $\boldsymbol{p}_1', \boldsymbol{p}_2'$

Galilean invariance: V only depends on

$\boldsymbol{p} = (\boldsymbol{p}_1 - \boldsymbol{p}_2)/2$, $\boldsymbol{p}' = (\boldsymbol{p}_1' - \boldsymbol{p}_2')/2$

Equivalently, V can also be expressed using

$\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}, \boldsymbol{k} = (\boldsymbol{p}' + \boldsymbol{p})/2$

Local regulator:

$$V(\boldsymbol{p}', \boldsymbol{p}) \to V(\boldsymbol{p}', \boldsymbol{p}) f_\Lambda(q)$$

Non-local regulator:

$$V(\boldsymbol{p}', \boldsymbol{p}) \to V(\boldsymbol{p}', \boldsymbol{p}) f_\Lambda(p') f_\Lambda(p)$$

Single-particle regulator:

$$V(\boldsymbol{p}', \boldsymbol{p}) \to V(\boldsymbol{p}', \boldsymbol{p}) f_\Lambda(p_1') f_\Lambda(p_1) f_\Lambda(p_2') f_\Lambda(p_2)$$

$$\boldsymbol{f_\Lambda(p) = \exp\left[-(p/\Lambda)^{2n}\right]}$$

Λ is an **arbitrarily** introduced parameter corresponding to **no** physical reality.

Ren. Group vs. Similarity Ren. Group

- To answer the question on the self-consistency, we begin by building an inherently RG-invariant (Λ-independent) EFT
- We may use **Wilsonian RG approach**. However, integrating out the high-momentum modes result in time-dependent interactions that are difficult to handle in Hamiltonian formalism
- A more suitable choice is the **Similarity Renormalization Group (SRG)** method which involves unitary transformations of the Hamiltonian

Wisonian RG: Integrating out a momentum shell

$$Z = \int D\phi \exp[-S_0(\phi)] = \int \prod_{p<\Lambda} d\phi_< \prod_{\Lambda<p<\Lambda_0} d\phi_> \exp[-S_0(\phi_< + \phi_>)] = \int \prod_{p<\Lambda} d\phi_< \exp[-S_{eff}(\phi_<)]$$

Similarity RG: Decoupling the low- and high-momentum subspaces via unitary transformations

$$H' = U^{-1} H U = U^{-1} \begin{pmatrix} H_{11} & H_{10} \\ H_{01} & H_{00} \end{pmatrix} U = \begin{pmatrix} H'_{11} & 0 \\ 0 & H'_{00} \end{pmatrix}$$

In Wilsonian RG the high-momentum modes are integrated into running coupling constants

Question: how to apply the SRG to generate the Wilsonian flow for a non-rel. Hamiltonian?

Lu & Deng, arXiv:2308.14559

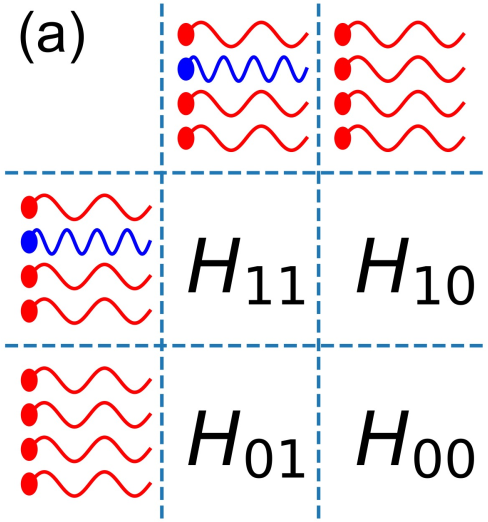
Decoupling the low- and high-momenta

- We search for a unitary transformation that decouples the two subspaces space-0' and space-1'
- This can be achieved by applying a SRG transformation

$$i\partial_t H(t) = [\eta(t), H(t)] \qquad \eta(t) \;=\; i[H_0(t), H_1(t)]$$

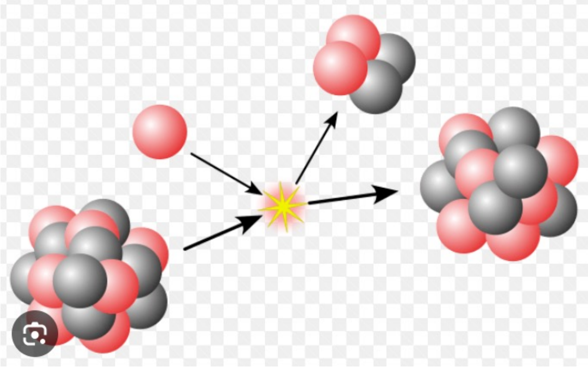
- For sufficiently large *t*, the non-diagonal blocks contained in H1 are suppressed → H1 vanishes
- Remember that H0 is block-diagonal in the Fock space, we can safely drop all terms containing $\delta\Phi$

$$\begin{aligned} H_0 \;=\; \int d\tau : & -\frac{\Phi^\dagger \nabla^2 \Phi}{2m} + \frac{C_2}{2}(\Phi^\dagger_{\Lambda'}\Phi_{\Lambda'})^2 + \frac{C_3}{6}(\Phi^\dagger_{\Lambda'}\Phi_{\Lambda'})^3 \\ & + 2C_2(\Phi^\dagger_{\Lambda'}\Phi_{\Lambda'})(\delta\Phi^\dagger\delta\Phi) + C_2(\Phi^\dagger_{\Lambda'}\delta\Phi)(\delta\Phi^\dagger\delta\Phi) \\ & + \frac{3C_3}{2}(\Phi^\dagger_{\Lambda'}\Phi_{\Lambda'})^2\delta\Phi^\dagger\delta\Phi + \frac{C_3}{2}(\Phi^\dagger_{\Lambda'}\delta\Phi)^2\delta\Phi^\dagger\delta\Phi + \ldots :, \end{aligned}$$



- Now we have a transformed Hamiltonian, $\Lambda \to \Lambda'$ and all coupling constants updated to new values
- As we used unitary transformations, new Hamiltonian has the same spectrum

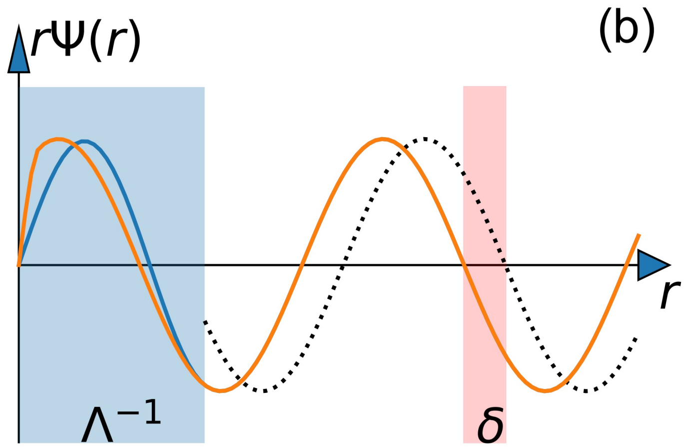
Nuclear scattering and reaction



- The transformation is unitary and localized:

$$U(\infty)(|\phi_1\rangle \otimes |\phi_2\rangle) = U(\infty)|\phi_1\rangle \otimes U(\infty)|\phi_2\rangle$$

  with $\phi_{1,2}$ single- or many-particle wave packets separated by a distance $\gg \Lambda^{-1}$
- For a typical reaction, the incoming (outgoing) state long before (after) the collision consists of well seperated clusters, each of which corresponds to a bound state of the Hamiltonian.
- Applying the transformation U to both the Hamiltonian and the incoming (outgoing) state, we transform H to H′ and the cluster wave functions from eigenvectors of H to that of H′ with the same binding energies
- the unitarity of U ensures the invariance of the S-matrix element

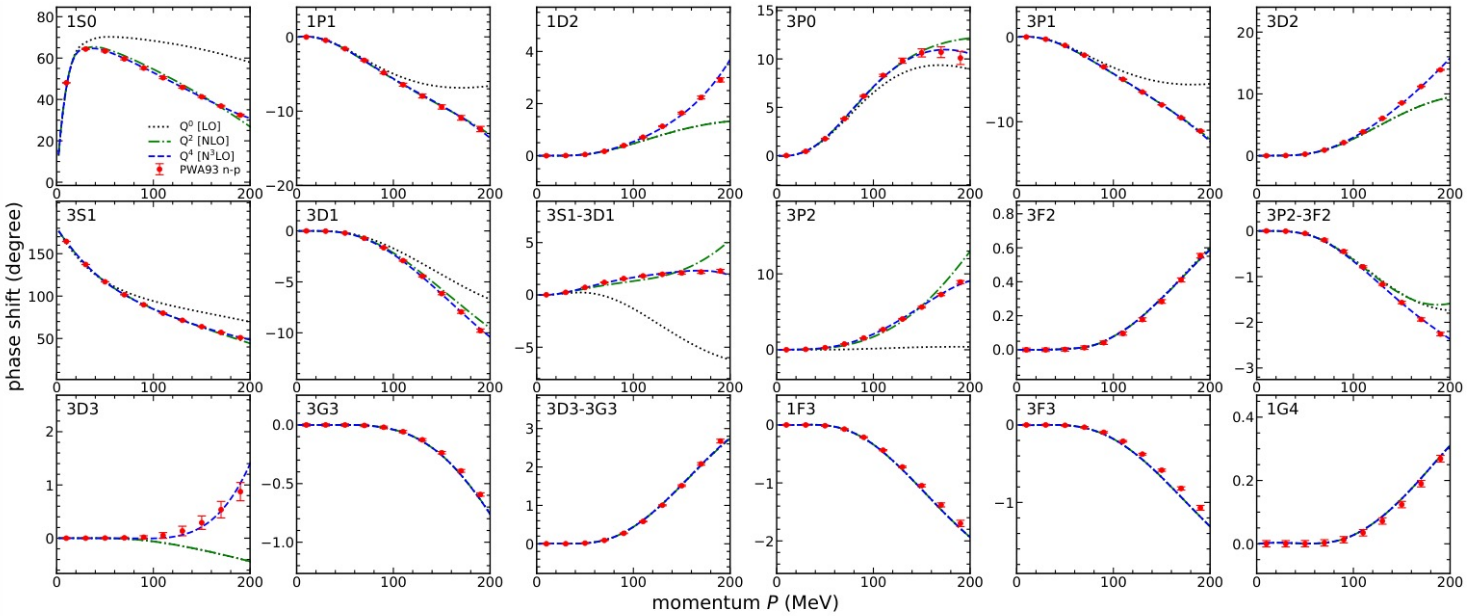


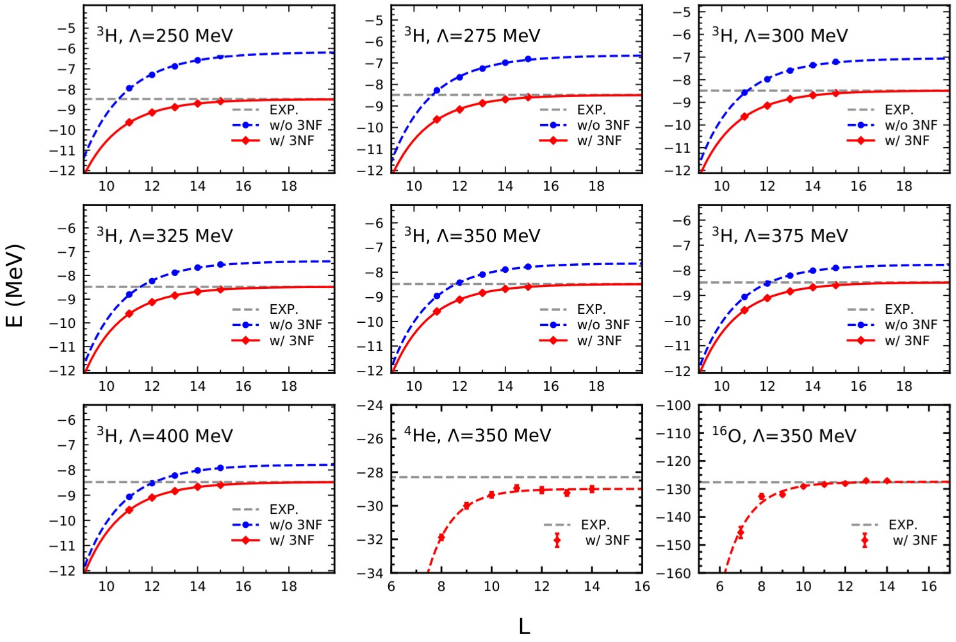
- Similar to the wave-function-matching method
  Nature 630, 59-63 (2024)

All low-momentum physics are invariant under $\Lambda \to \Lambda'$ and calculating with new coupling constants (Renormalization Group Invariance)

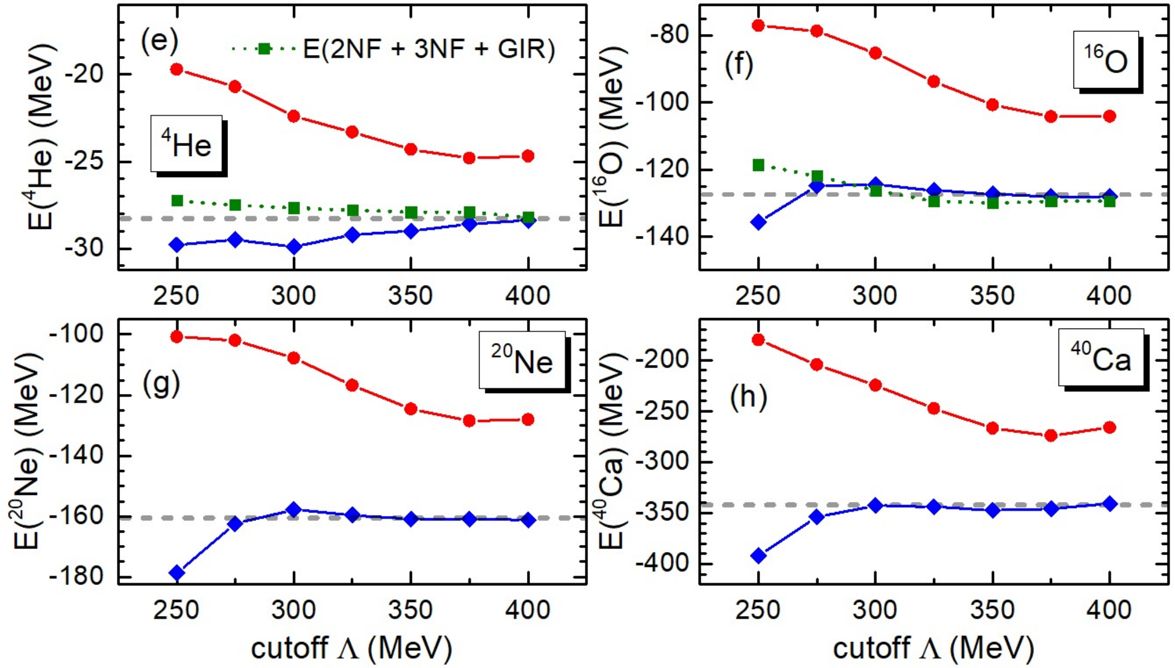
Determination of the N2LO LECs

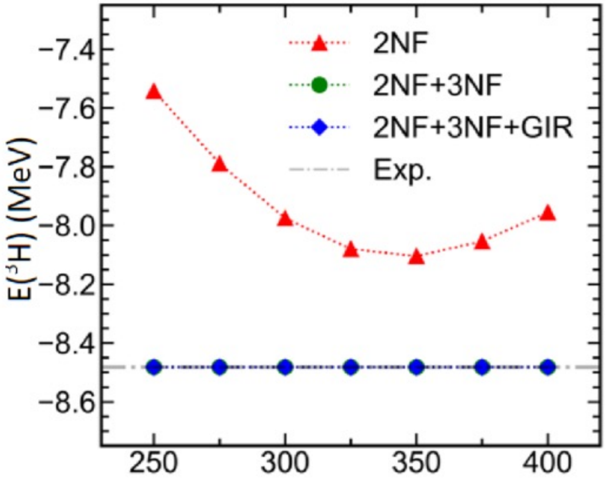
- For each value of Λ, We determine the coupling constants (LECs) by fitting to the N-N scattering phase shifts up to 200 MeV and ^{3}H binding energy E3=-8.482 MeV
- We extrapolate to infinite box size to eliminate the finite volume effects
- All LECs are completely fixed in A<=3 systems

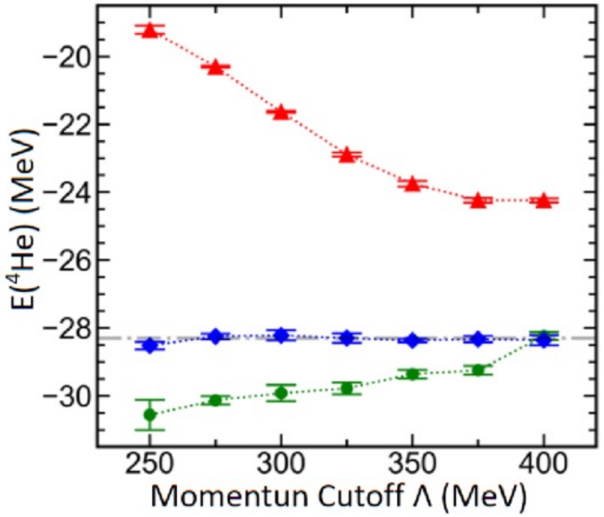




Role of Galilean invariance restoration terms (preliminary)







- The lattice itself is stationary and breaks the GR.
- Varying cutoff induces Galilean invariance breaking contact terms in the Hamiltonian.
- Such terms are irrelevant and disappear for large cutoffs.
- Inclusion of these terms improves the RG-invariance.

Jia-Ai Shi et al., in preparation

Summary & Perspective

- Lattice Effective Field Theory is an efficient tool for solving the nuclear many-body problem. Nuclear force → Binding energies
- Two groups of lattice nuclear forces
  - **Minimal nuclear forces with essential elements (locality, three-body, spin-orbit, etc.)**, easy to handle, no/weak sign problem, reproduce bulk properties, extensively applied
  - **High-precision chiral forces fixed by few-body data (NN scattering, triton mass, etc.)** difficult to solve, severe sign problem, wave-function-matching / perturbative-quantum-MC, many-body forces, RG invariance
- Nuclear binding energies are fundamental constraints to nuclear forces. Still a long way to understand the binding mechanism in *ab initio* calculations (clustering, shell evolution, shapes, etc.).