手征有效场论研讨会

Study of the $f_0(1710)$ and $a_0(1710)$ states with chiral unitary approach

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Outline

1. Introduction
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§1. Introduction

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

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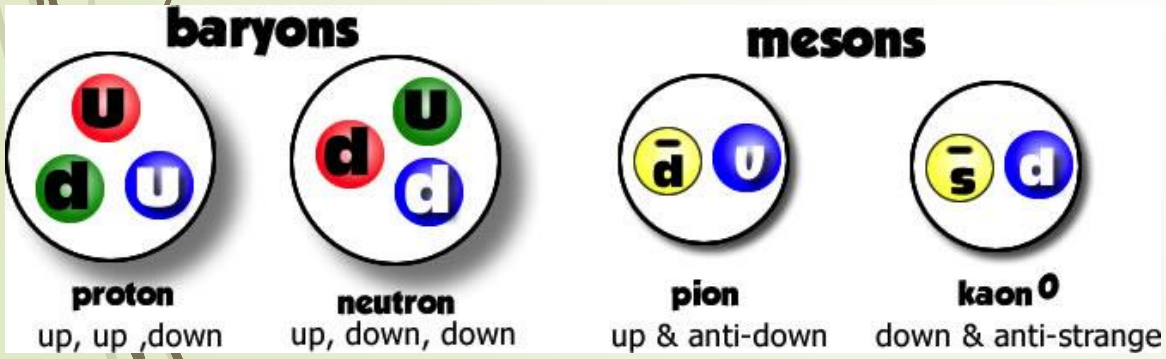
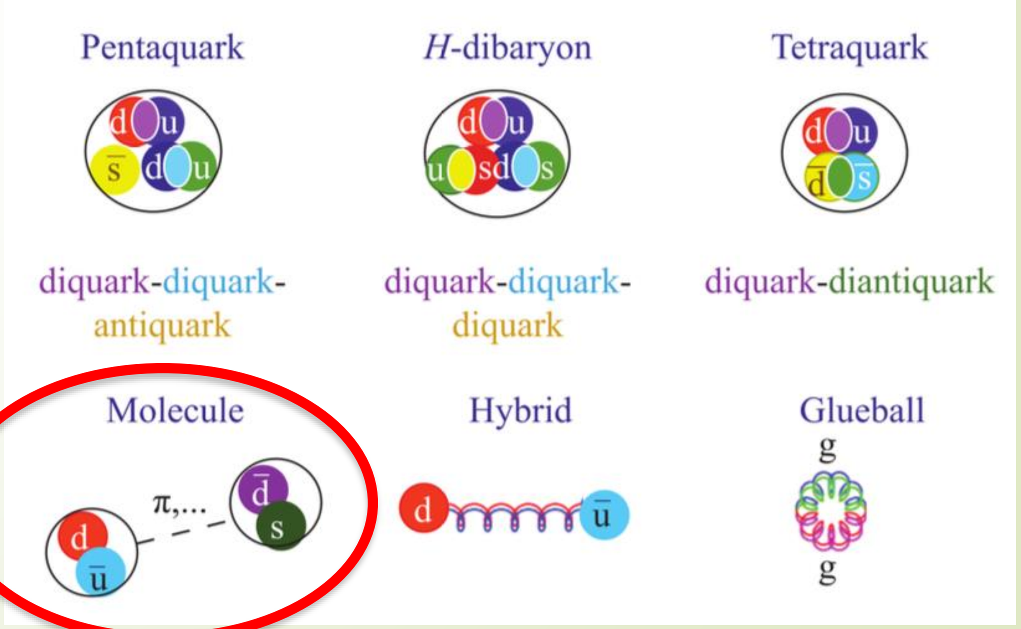
G. Zweig *)
CERN - Geneva

In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".

Quark model

Exotic States





Molecular nature

PHYSICAL REVIEW LETTERS **126**, 152001 (2021)

Explaining the Many Threshold Structures in the Heavy-Quark Hadron Spectrum

Xiang-Kun Dong[1,2] Feng-Kun Guo[1,2,*] and Bing-Song Zou[1,2,3]

F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)

f_0(1710) was **discovered** about **40 years ago**:

A. Etkin, et al., Phys. Rev. D 25, 1786 (1982)

C. Edwards, et al., Phys. Rev. Lett. 48, 458 (1982)

ELSEVIER Nuclear Physics A 620 (1997) 438–456

Chiral symmetry amplitudes in the S-wave isoscalar and isovector channels and the σ, $f_0(980)$, $a_0(980)$ scalar mesons

J.A. Oller, E. Oset

$K\bar{K}$ **molecular state**

$K^*\bar{K}^*$ **molecular state:** Coupled channel approach

L. S. Geng and E. Oset, Phys. Rev. D 79, 074009 (2009)

But, its isovector partner a_0(1710) were **NOT** found for a long time……..

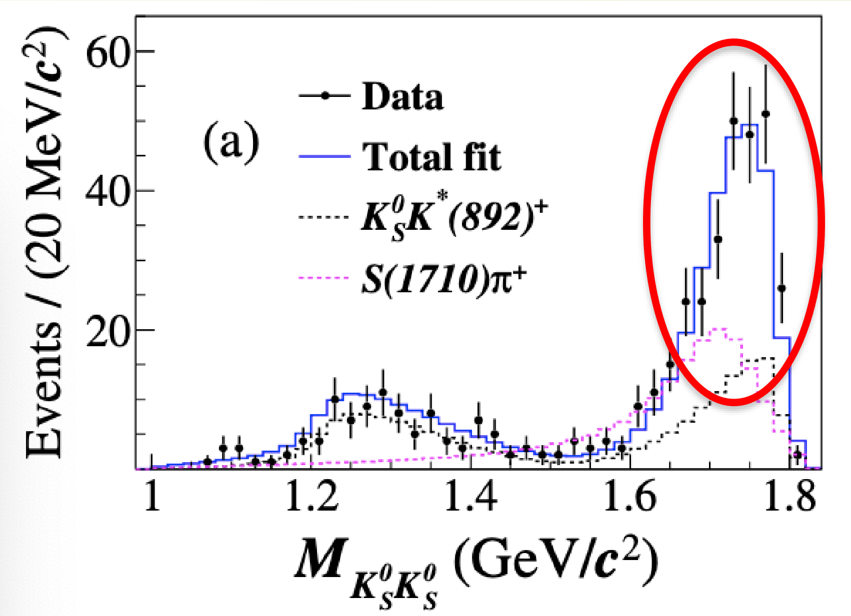
Recent Findings from BESIII

PHYSICAL REVIEW D **105**, L051103 (2022)

Letter

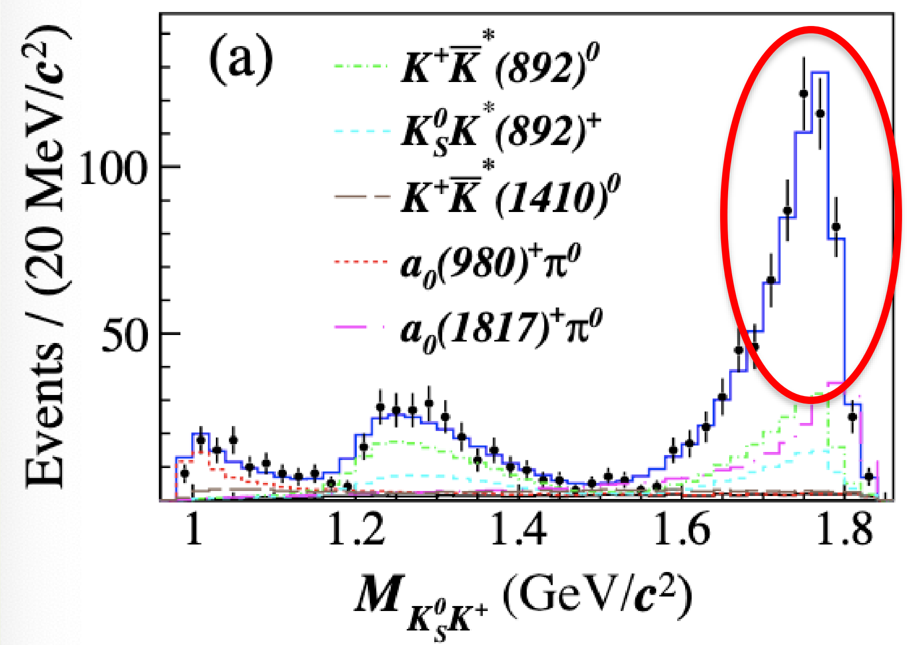
Study of the decay $D_s^+ \to K_S^0 K_S^0 \pi^+$ and observation of an isovector partner to $f_0(1710)$

existence of an isospin one partner of the $f_0(1710)$.



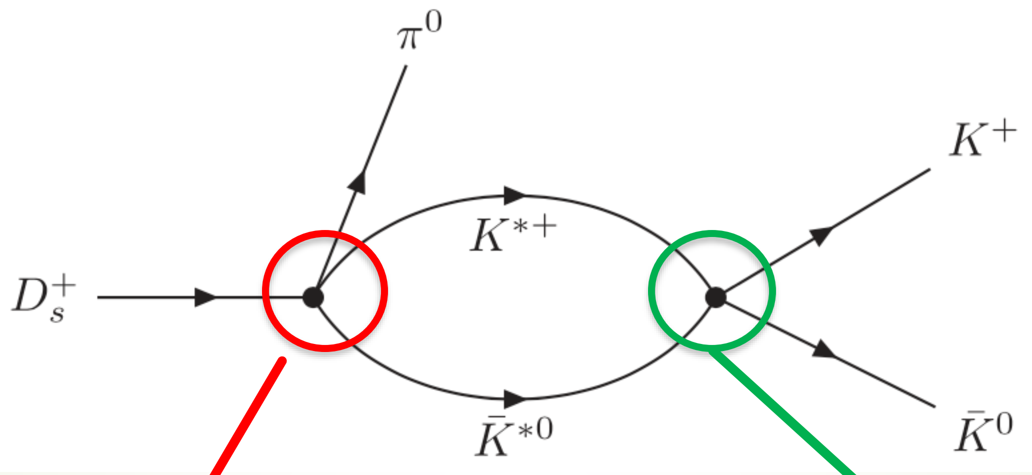
PHYSICAL REVIEW LETTERS **129**, 182001 (2022)

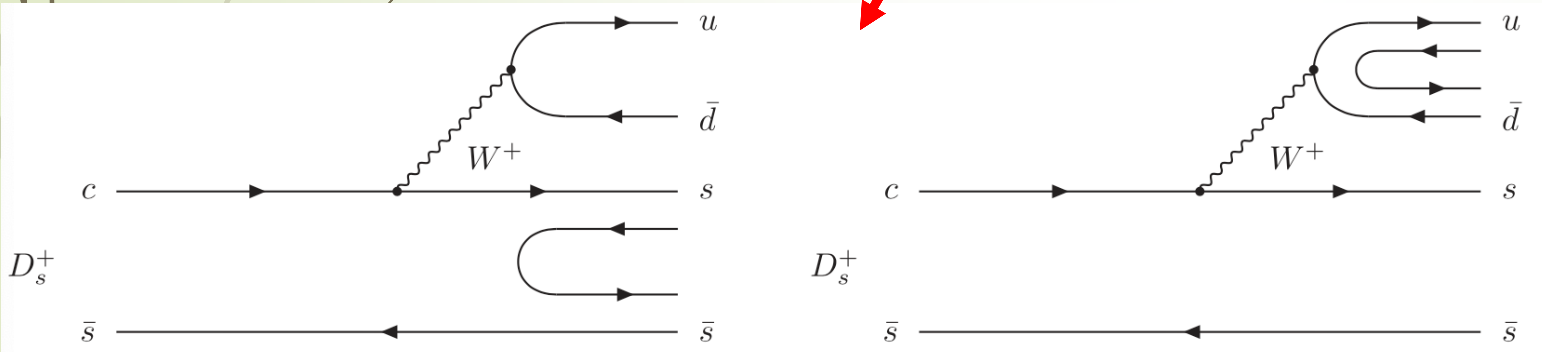
Observation of an a_0-like State with Mass of 1.817 GeV in the Study of $D_s^+ \to K_S^0 K^+ \pi^0$ Decays

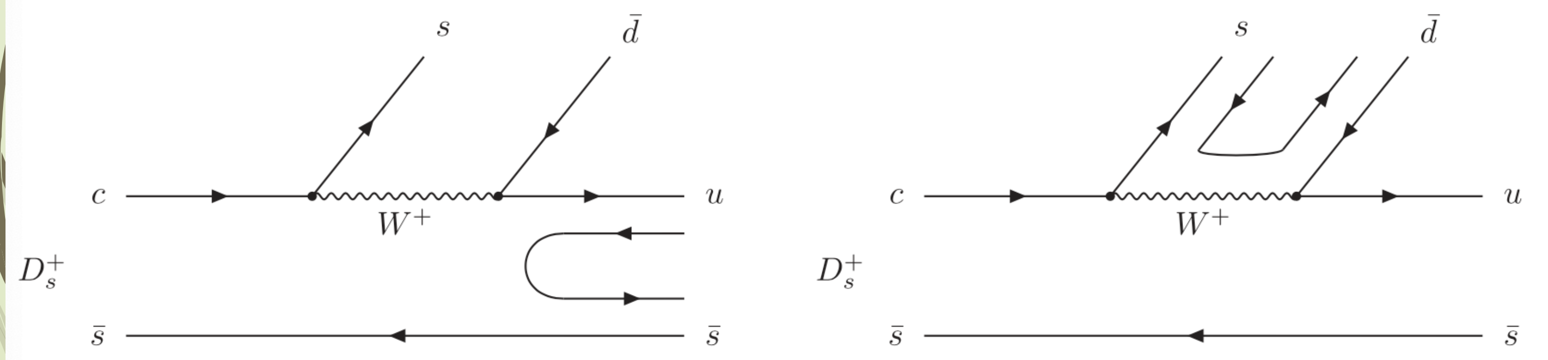


§2. Formalism

Final state interaction





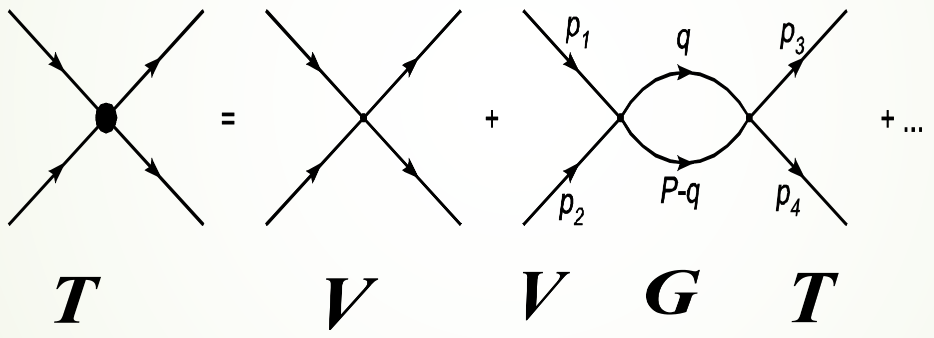


S-wave

$$T = [1 - VG]^{-1} V$$

Coupled Channel Unitary Approach: solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V\,G\,T,\; T = [1 - V\,G]^{-1}\,V$$



D. L. Yao, L. Y. Dai, H. Q. Zheng and Z. Y. Zhou, Rept. Prog. Phys. 84, 076201 (2021)

where V matrix (potentials) can be evaluated from the interaction Lagrangians.

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438
E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99
J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** ：

$$\mathrm{Im}\, T_{ij} = T_{in}\, \sigma_{nn}\, T_{nj}^*$$

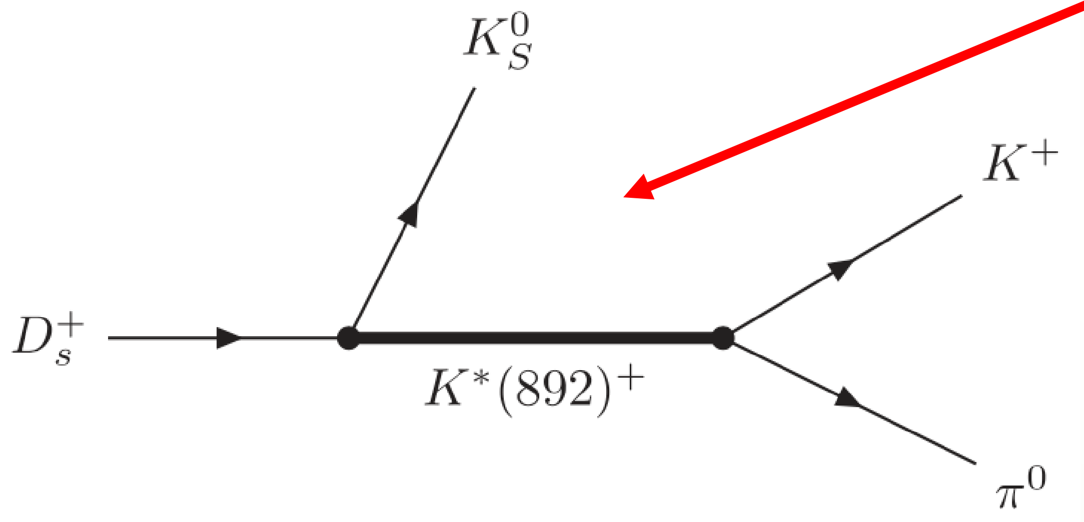
$$\sigma_{nn} \equiv \mathrm{Im}\, G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}}\theta(s - (m_1 + m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^{I}(s) + i\,\frac{q_{cm}}{4\pi\sqrt{s}}$$

P wave contribution

$$\frac{d^2\Gamma}{dM_{12}dM_{13}} = \frac{1}{(2\pi)^3}\frac{M_{12}M_{13}}{8m^3_{D_s^+}} \times \left(|t_{S\text{-wave}} + t_{\bar{K}^*(892)^0} + t_{K^*(892)^+}|^2\right)$$



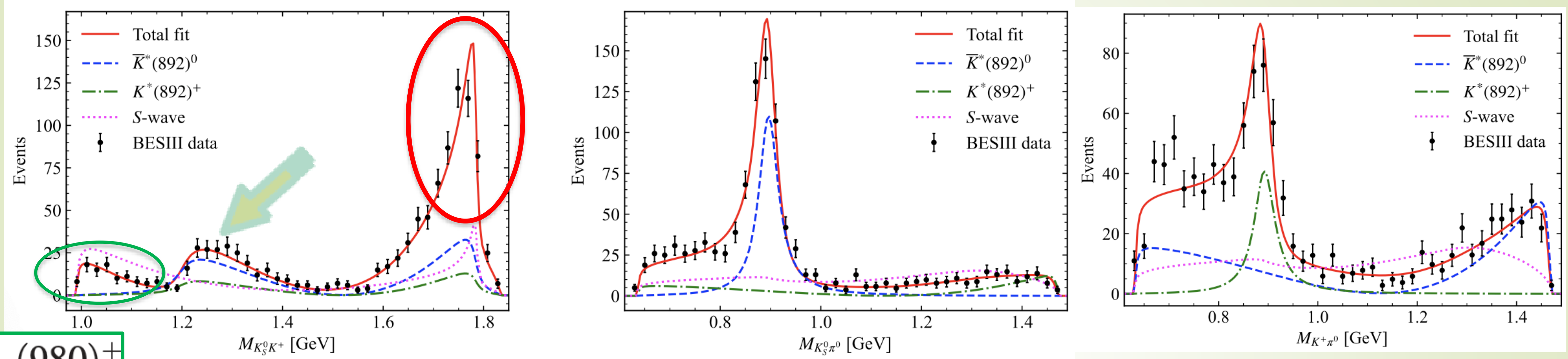
$$t_{K^*(892)^+}(M_{12}, M_{13}) = \frac{\mathcal{D}_2 e^{i\phi_{K^*(892)^+}}}{M_{23}^2 - m^2_{K^*(892)^+} + i m_{K^*(892)^+}\Gamma_{K^*(892)^+}} \times \left[(m^2_{K^+} - m^2_{\pi^0})\frac{m^2_{D_s^+} - m^2_{K_S^0}}{m^2_{K^*(892)^+}} - M_{12}^2 + M_{13}^2\right]$$

Z. Y. Wang, Y. W. Peng, J. Y. Yi, W. C. Luo and ***CWX****, Phys. Rev. D 107 (2023) 116018.*

§3. Results

$D_s^+ \to K_S^0 K^+ \pi^0$

$a_0(1710)^+$



$a_0(980)^+$

Parameter	μ	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$
Fit	0.716 ± 0.013 GeV	47518.79 ± 7523.18	1595.34 ± 138.51	46454.25 ± 3868.04
	$\mathcal{D}_1$	$\mathcal{D}_2$	$\phi_{\bar{K}^*(892)^0}$	$\phi_{K^*(892)^+}$
	61.65 ± 2.33	40.43 ± 2.95	1.46 ± 0.12	1.67 ± 0.15

$$a_{K^{*+}\bar{K}^{*0}} = -1.91, \quad a_{\rho^+\omega} = -1.82, \quad a_{\rho^+\phi} = -2.02, \quad a_{K^+\bar{K}^0} = -1.59, \quad a_{\pi^+\eta} = -1.63.$$

	This work	Reference [63]	Reference [26]	Reference [29]	Reference [27]
Parameter	$\mu = 0.716$	$q_{\max} = 0.931$, $q_{\max} = 1.08$	$\mu = 1.00$	$q_{\max} = 1.00$	$q_{\max} = 1.00$, $g_1 = 4.596$
$a_0(980)$	$1.0419 + 0.0345i$	$1.0029 + 0.0567i$, $0.9745 + 0.0573i$	$\cdots$	$\cdots$	$\cdots$
$a_0(1710)$	$1.7936 + 0.0094i$	$\cdots$	$1.780 - 0.066i$	$1.72 - 0.10i$	$1.76 \pm 0.03i$

Partial decay widths

$\Gamma_{a_0(980)^+\to K^+\bar{K}^0}$	$\Gamma_{a_0(980)^+\to \pi^+\eta}$
28.38 MeV	43.60 MeV

$\Gamma_{a_0(1710)^+\to \rho^+\omega}$	$\Gamma_{a_0(1710)^+\to K^+\bar{K}^0}$	$\Gamma_{a_0(1710)^+\to \pi^+\eta}$
19.65 MeV	0.54 MeV	0.05 MeV

$$\Gamma_{R\to i} = -\frac{1}{16\pi^2}\int_{E_{\min}}^{E_{\max}} dE\,\frac{q_{cmi}}{E^2}4M_R\mathrm{Im}T_{ii}$$

$$\Gamma_{R\to j} = -\frac{1}{16\pi^2}\int_{E_{\min}}^{E_{\max}} dE\,\frac{q_{cmj}}{E^2}4M_R\frac{(\mathrm{Im}T_{ji})^2}{\mathrm{Im}T_{ii}}$$

Eur. Phys. J. C (2022) 82:509
https://doi.org/10.1140/epjc/s10052-022-10460-4
THE EUROPEAN PHYSICAL JOURNAL C
Regular Article - Theoretical Physics

Two dynamical generated a_0 resonances by interactions between vector mesons

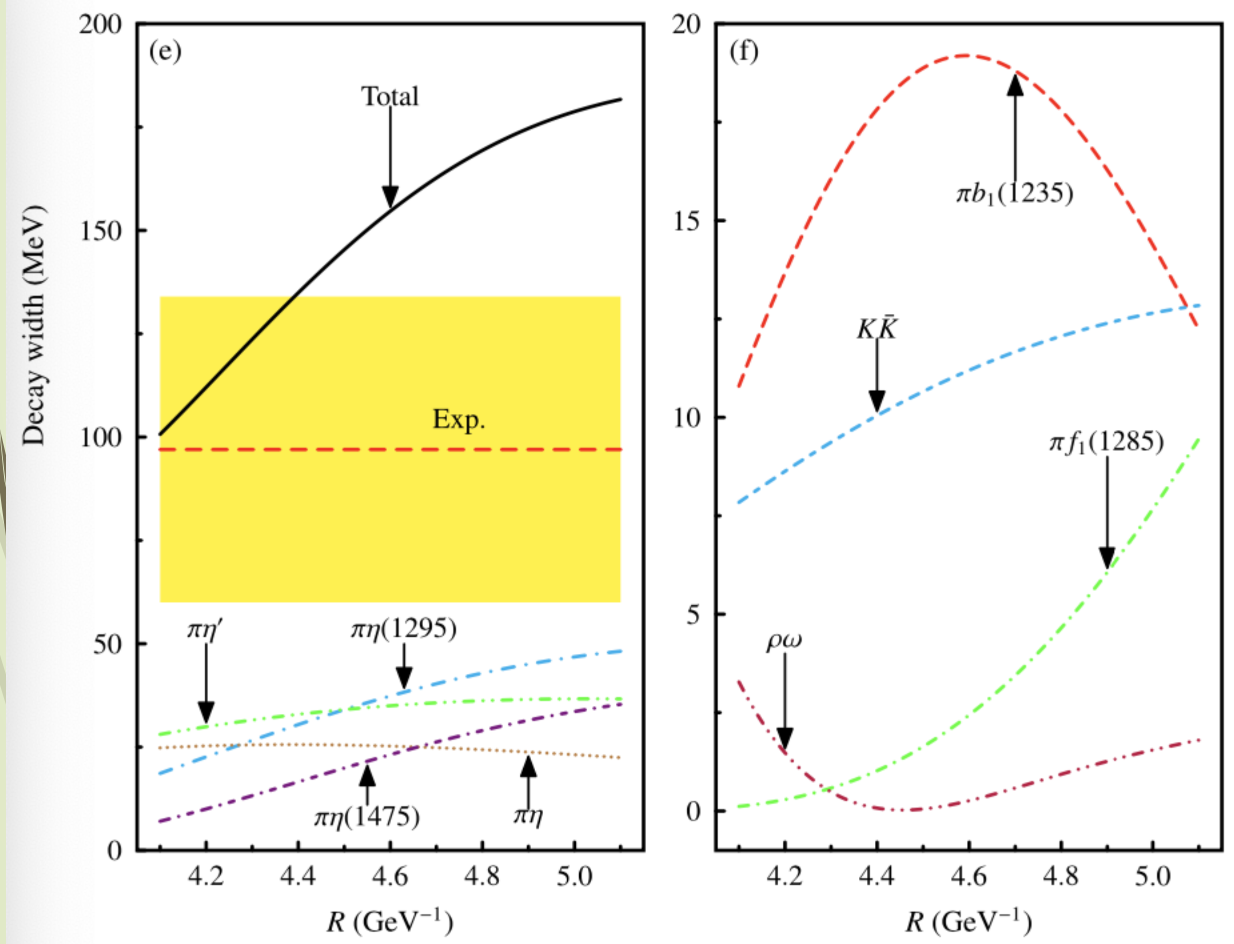
Zheng-Li Wang[1,2,a], Bing-Song Zou[1,2,3,b]

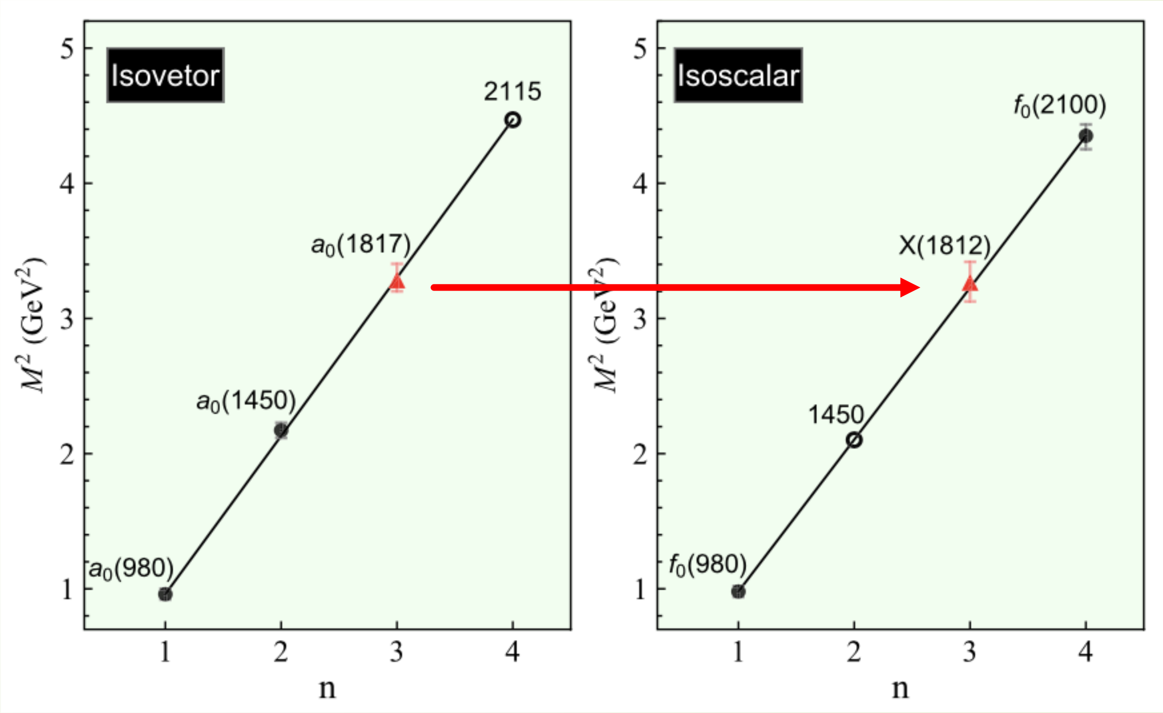
$\Gamma(K^*\bar{K}^*\to\rho\omega)$	$\Gamma(K^*\bar{K}^*\to K\bar{K})$	$\Gamma(K^*\bar{K}^*\to\pi\eta)$
61.0 MeV	74.4 MeV	66.9 MeV
$\Gamma(\rho\phi\to\rho\omega)$	$\Gamma(\rho\phi\to K\bar{K})$	$\Gamma(\rho\phi\to\pi\eta)$
60.8 MeV	74.2 MeV	66.6 MeV

PHYSICAL REVIEW D **105,** 114014 (2022)

Newly observed $a_0(1817)$ as the scaling point of constructing the scalar meson spectroscopy

Dan Guo,[1,2,*] Wei Chen,[4,†] Hua-Xing Chen,[5,‡] Xiang Liu,[1,2,3,7,§] and Shi-Lin Zhu[6,∥]





Regge trajectory

E. Oset, L. R. Dai and L. S. Geng, Sci. Bull. 68 (2023) 243-246.

Branching ratios

$$\frac{\mathcal{B}(D_s^+ \to K^*(892)^+ K_S^0, K^*(892)^+ \to K^+\pi^0)}{\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+, \bar{K}^*(892)^0 \to K_S^0\pi^0)} = 0.40^{+0.002}_{-0.003}$$

$$\frac{\mathcal{B}(D_s^+ \to a_0(980)^+ \pi^0, a_0(980)^+ \to K_S^0 K^+)}{\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+, \bar{K}^*(892)^0 \to K_S^0\pi^0)} = 0.53^{+0.06}_{-0.08},$$

$$\frac{\mathcal{B}(D_s^+ \to a_0(1710)^+ \pi^0, a_0(1710)^+ \to K_S^0 K^+)}{\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+, \bar{K}^*(892)^0 \to K_S^0\pi^0)} = 0.41^{+0.04}_{-0.05}.$$

$$\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+, \bar{K}^*(892)^0 \to K_S^0\pi^0) = (4.77 \pm 0.38 \pm 0.32) \times 10^{-3}$$

Our predictions

$$\mathcal{B}(D_s^+ \to K^*(892)^+ K_S^0, K^*(892)^+ \to K^+\pi^0) = (1.91 \pm 0.20^{+0.01}_{-0.01}) \times 10^{-3}$$

$$\mathcal{B}(D_s^+ \to a_0(980)^+ \pi^0, a_0(980)^+ \to K_S^0 K^+) = (2.53 \pm 0.26^{+0.27}_{-0.38}) \times 10^{-3}$$

$$\mathcal{B}(D_s^+ \to a_0(1710)^+ \pi^0, a_0(1710)^+ \to K_S^0 K^+) = (1.94 \pm 0.20^{+0.18}_{-0.24}) \times 10^{-3}$$

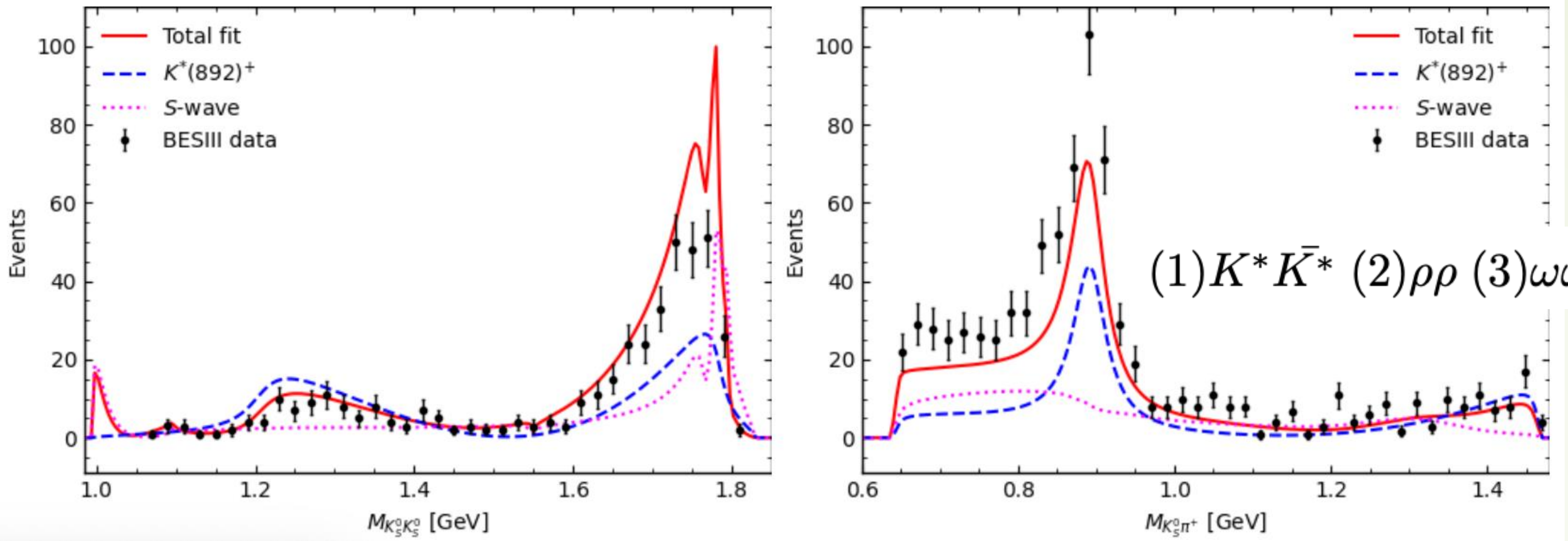
BESIII measurements

$$\mathcal{B}(D_s^+ \to K^*(892)^+ K_S^0, K^*(892)^+ \to K^+\pi^0) = (2.03 \pm 0.26 \pm 0.20) \times 10^{-3}$$

$$\mathcal{B}(D_s^+ \to a_0(980)^+ \pi^0, a_0(980)^+ \to K_S^0 K^+) = (1.12 \pm 0.25 \pm 0.27) \times 10^{-3}$$

$$\mathcal{B}(D_s^+ \to a_0(1710)^+ \pi^0, a_0(1710)^+ \to K_S^0 K^+) = (3.44 \pm 0.52 \pm 0.32) \times 10^{-3}$$

$D_s^+ \to K_S^0 K_S^0 \pi^+$



eleven coupled channels

$(1) K^* \bar{K}^*$ $(2) \rho\rho$ $(3) \omega\omega$ $(4) \omega\phi$ $(5) \phi\phi$ $(6) \pi\pi$ $(7) K\bar{K}$ $(8) \eta\eta$

$I = 0$

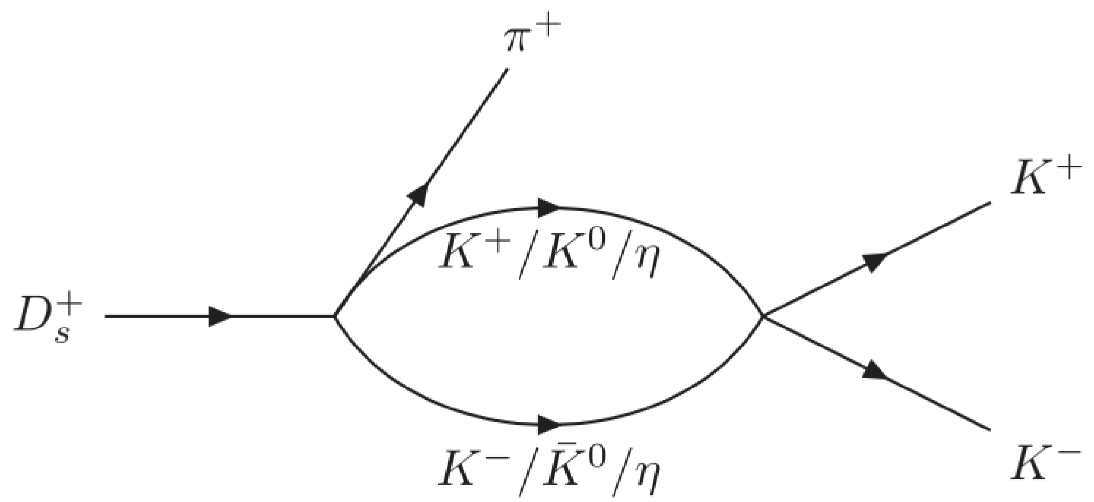
	This work	Ref. [64]	Ref. [96]	Ref. [43]	Ref. [62]	Ref. [44]
Parameters	$\mu = 0.648$	$\mu = 0.716$	$q_{max} = 0.931$	$\mu = 1.0$	$q_{max} = 1.0$	$q_{max} = 1.0$
$a_0(980)$	$1.0598 + 0.024i$	$1.0419 + 0.0345i$	$1.0029 + 0.0567i$	$\cdots$	$\cdots$	$\cdots$
$f_0(980)$	$0.9912 + 0.003i$	$\cdots$	$0.9912 + 0.0135i$	$\cdots$	$\cdots$	$\cdots$
$a_0(1710)$	$1.7981 + 0.0018i$	$1.7936 + 0.0094i$	$\cdots$	$1.780 - 0.066i$	$1.72 - 0.010i$	$1.76 \pm 0.03i$
$f_0(1710)$	$1.7676 + 0.0093i$	$\cdots$	$\cdots$	$1.726 - 0.014i$	$\cdots$	$\cdots$

$$\mathcal{B}(D_s^+ \to S(980)\pi^+, S(980) \to K_S^0 K_S^0) = (0.36 \pm 0.04^{+0.10}_{-0.06}) \times 10^{-3}$$
$$\mathcal{B}(D_s^+ \to S(1710)\pi^+, S(1710) \to K_S^0 K_S^0) = (1.66 \pm 0.17^{+1.38}_{-0.89}) \times 10^{-3}$$

$$B(D_s^+ \to K^*(892) K_S^0 \to K_S^0 K_S^0 \pi^+) = (3.0 \pm 0.3 \pm 0.1) \times 10^{-3};$$
$$B(D_s^+ \to S(1710)\pi^+ \to K_S^0 K_S^0 \pi^+) = (3.1 \pm 0.3 \pm 0.1) \times 10^{-3}.$$

Y. W. Peng, W. Liang, X. Xiong and ***CWX****, Phys. Rev. D 110 (2024) 036013.*

$D_s^+ \to K^+K^-\pi^+$



$$T_{K^+K^-\to K^+K^-} = \frac{1}{2}(T^{I=0}_{K\bar{K}\to K\bar{K}} + T^{I=1}_{K\bar{K}\to K\bar{K}})$$

$$T_{K^0\bar{K}^0\to K^+K^-} = \frac{1}{2}(T^{I=0}_{K\bar{K}\to K\bar{K}} - T^{I=1}_{K\bar{K}\to K\bar{K}})$$

of Refs. [4,42]. Therefore, there should be only the resonance $f_0(980)$ contribution in the K^+K^- invariant mass distribution, and without the one of $a_0(980)$.

$$\mathcal{B}[D_s^+ \to f_0(980)\pi^+, f_0(980) \to K^+K^-] = (0.61 \pm 0.02^{+0.06}_{-0.17})\%, \quad \text{[Theo]}$$

$$\mathcal{B}[D_s^+ \to \bar{K}^*(892)^0K^+, \bar{K}^*(892)^0 \to K^-\pi^+] = (2.61 \pm 0.10^{+0.05}_{-0.12})\%,$$

$$\mathcal{B}[D_s^+ \to S(980)\pi^+, S(980) \to K^+K^-] = (1.05 \pm 0.04 \pm 0.06)\%, \quad \text{[BESIII]}$$

$$\mathcal{B}[D_s^+ \to \bar{K}^*(892)^0K^+, \bar{K}^*(892)^0 \to K^-\pi^+] = (2.64 \pm 0.06 \pm 0.07)\%,$$

Z. Y. Wang, J. Y. Yi, Z. F. Sun and ***CWX****, Phys. Rev. D 105 (2022) 016025.*

§4. Summary

- We use the final state interaction formalism to investigate the Ds three-body weak decays
- In the final state interaction, $f_0/a_0(1710)$ and/or $f_0/a_0(980)$ generated (molecular nature)
- Related branching ratios are evaluated, some of which are consistent with the experiments.

Hope future experiments and theories bring more clarifications on these issues…….

Thanks for your attention!

感谢大家的聆听！