



The pole structures of the $X(1840)/X(1835)$ and the $X(1880)$

Niu Pengyu, Zhang Zhenyu, Li Yiyao, Wang Qian, Zhao Qiang

South China Normal University, Institute of Quantum Matter

arXiv: 2408.14876 (accepted by PRD)

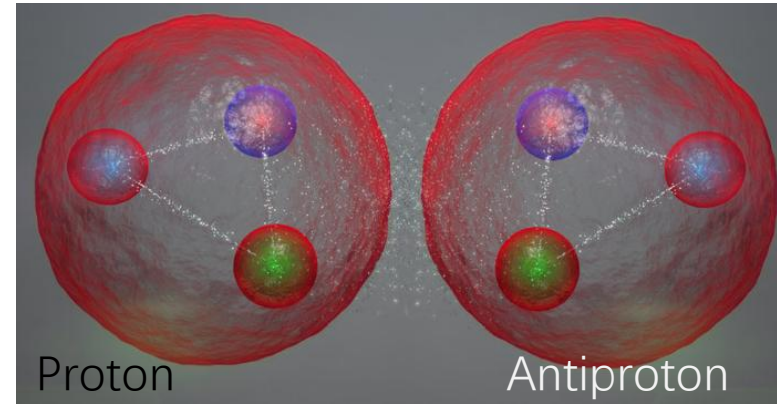
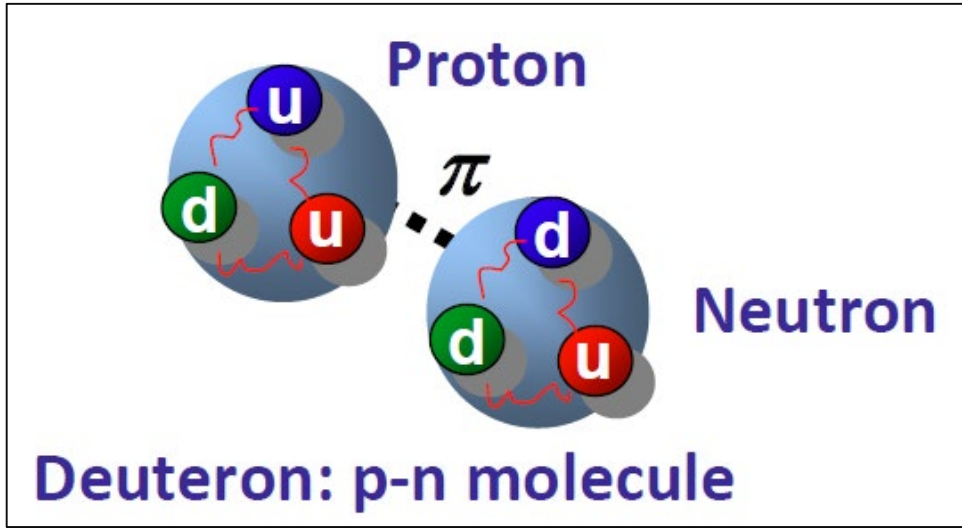
The 9th Workshop on Chiral Effective Field Theory

Chang Sha, Hu Nan, 2024.10.18-2024.10.22

Outline

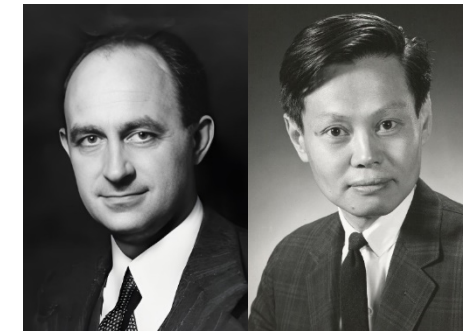
- 1. Background**
- 2. Framework**
- 3. Results and Discussion**
- 4. Summary**





Bound state?

- Its mass locates just below the neutron proton threshold.
- It has a sizeable spatial extension.
- ✓ These two features can be used for defining a hadronic molecule.



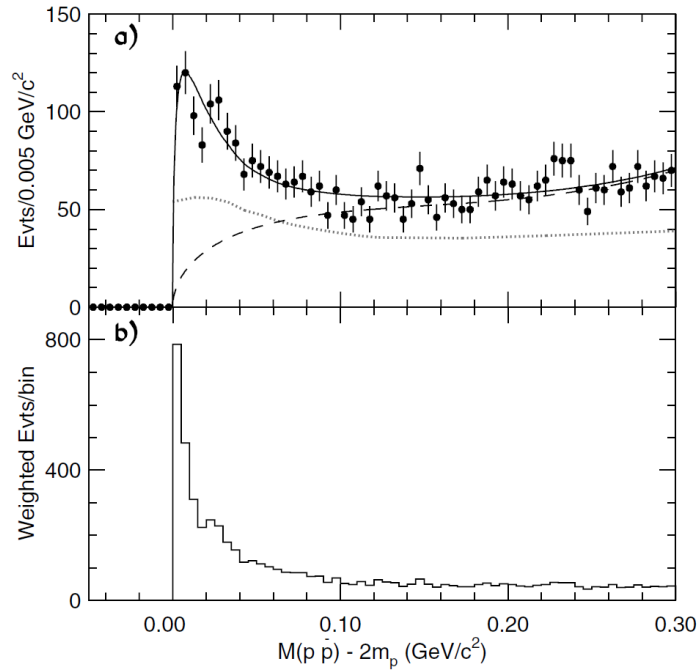
Are Mesons Elementary Particles?

E. FERMI AND C. N. YANG*
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received August 24, 1949)

The hypothesis that π -mesons may be composite particles formed by the association of a nucleon with an anti-nucleon is discussed. From an extremely crude discussion of the model it appears that such a meson would have in most respects properties similar to those of the meson of the Yukawa theory.

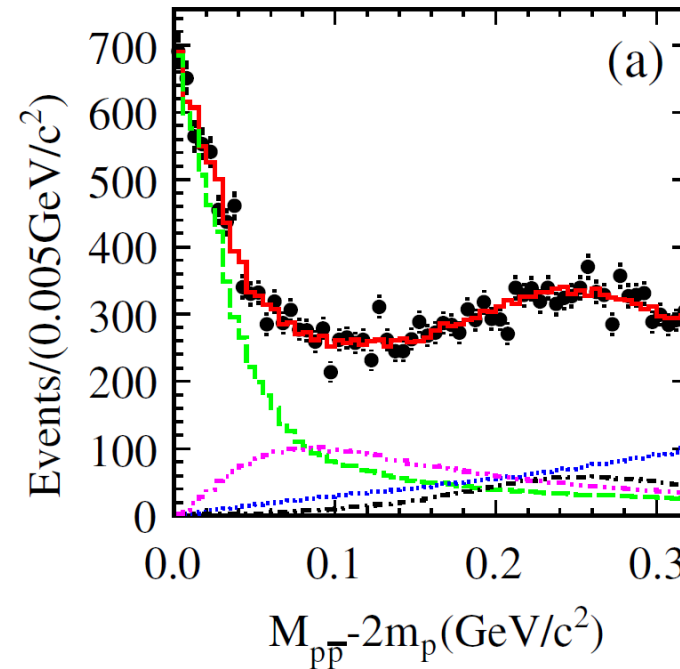
[1]. Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner, Qian Wang, Qiang Zhao, and Bing-Song Zou, Rev.Mod.Phys. 90, 015004

[2]. B. Q. Ma, arXiv:2406.19180



BES, Phys. Rev. Lett. 91, 022001 (2003)

- A narrow enhancement was observed near $2m_p$ for the first time.
- Using the S-wave fit:
 $M = 1859_{-10}^{+3}(\text{stat})_{-25}^{+5}(\text{syst}) \text{ MeV}$
 $\Gamma < 30 \text{ MeV}$



BESIII, Phys. Rev. Lett. 108, 112003 (2012)

- The quantum number: 0^{-+}
- With the inclusion of Julich-FSI effects:
 $M = 1832_{-5}^{+19}(\text{stat})_{-17}^{+18}(\text{syst}) \pm 19 \text{ MeV}$
 $\Gamma < 76 \text{ MeV}$

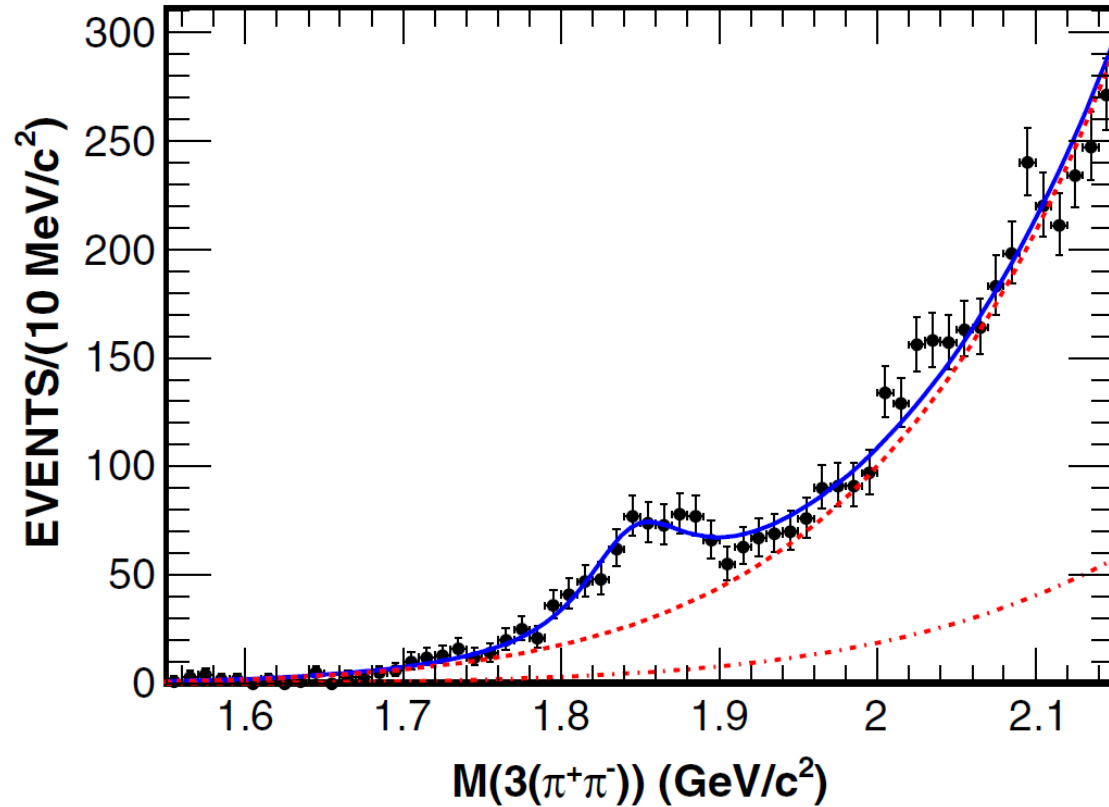
The $p\bar{p}$ bound state?

processes	(Mass width) (MeV)	J^{PC}
$J/\psi \rightarrow \gamma p\bar{p}$	(1859,<30)[1]; (1861,<38)[2]; (1837, ≈ 0)[3]; (1832,<76)[5]; (1831,<153)[6]	X(1835) superposition of two states? 0^{-+} [4]
$J/\psi \rightarrow \pi^0 p\bar{p}$	No similar structure [1,5]	-
$J/\psi \rightarrow \omega p\bar{p}$	No similar structure [7,8]	-
$J/\psi \rightarrow \eta p\bar{p}$	No similar structure [9]	-
$\psi(2S) \rightarrow X p\bar{p}$ ($X = \gamma, \pi^0, \eta$)	No similar structure [3]	-
$e^+ e^- \rightarrow p\bar{p}$	near-threshold enhancement is observed [10]	-
$B \rightarrow X p\bar{p}$ ($X = \pi^+, K, K_S, D^{(*)}$)	near-threshold enhancement is observed [11-17]	-

Refs.

- [1]: BES, Phys. Rev. Lett. 91, 022001
- [2]: BESIII, Chin. Phys. C 34, 421
- [3]: CLEO, Phys. Rev. D 82, 092002
- [4]: BES, Phys. Rev. D 80, 052004
- [5]: BESIII, Phys. Rev. Lett. 108, 112003
- [6]: BES, Phys.Rev.Lett. 95, 262001
- [7]: BES, Eur.Phys.J.C 53, 15
- [8]: BESIII, Phys. Rev. D 87,, 112004
- [9]: BES, Phys.Lett.B 510, 75
- [10]: BaBar, Phys. Rev. D 73, 012005
- [11]: Belle, Phys. Rev. Lett. 88, 181803
- [12]: Belle, Phys. Lett. B 659, 80
- [13]: BaBar, Phys.Rev.D 72, 051101
- [14]: Belle, Phys.Lett.B 617, 141-149
- [15]: Belle, Phys.Rev.Lett. 89, 151802
- [16]: BaBar, Phys.Rev.D 85, 092017
- [17]: CLEO, Phys.Rev.D 82, 092002

...



➤ A structure at 1.84 GeV is observed in the $3(\pi^+ \pi^-)$ mass spectrum in $J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)$ with a statistical significance of 7.6σ .

➤ Modified Breit-Wigner function:

$$M = 1842.2 \pm 4.2^{+7.6}_{-2.6} \text{ MeV}$$

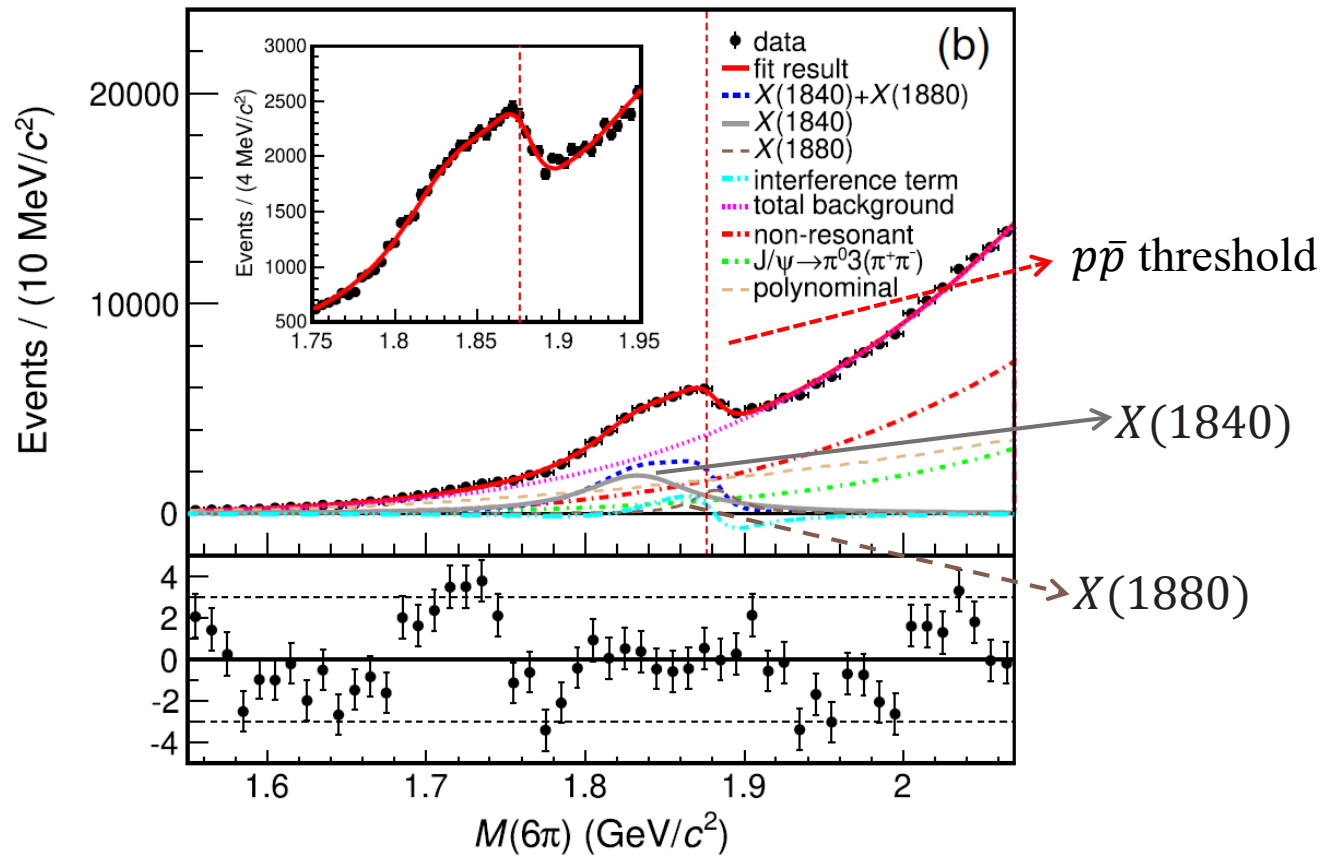
$$\Gamma = 83 \pm 14 \pm 11 \text{ MeV}$$

$X(1840)$

BESIII, Phys. Rev. D 88, 091502(R) (2013)

1. Background

The observation of $X(1880)$ in the process of $J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)$



BESIII, Phys. Rev. Lett. 132, 151901 (2024)

A simple resonant structure (Breit-Wigner) fails to describe the $M(6\pi)$ spectrum ($\chi^2/N_{\text{dof}}=399.0/45$).

Model-I: one resonant structure ($\chi^2/N_{\text{dof}}=317.9/44$):

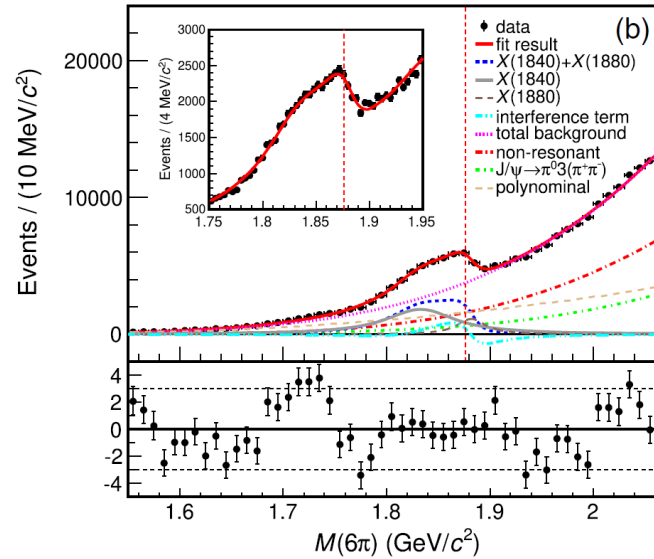
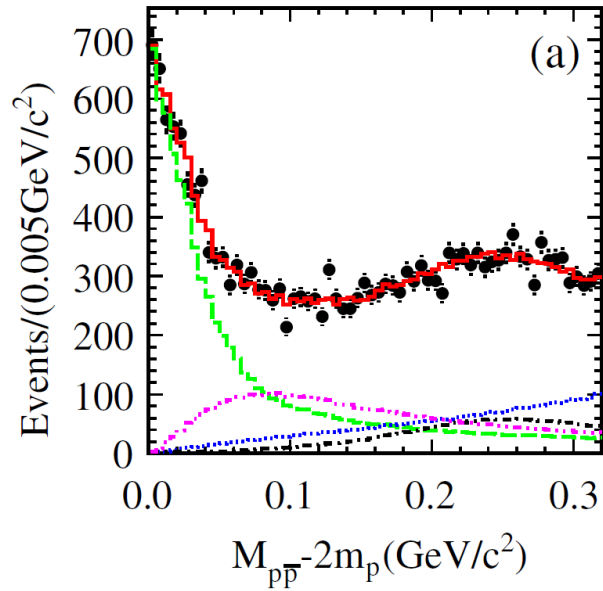
$$A = \left| \frac{1}{M^2 - s - i \sum_j g_j^2 \rho_j} \right|^2$$

Model-II: two resonant structures ($\chi^2/N_{\text{dof}}=155.6/41$):

$$A = \left| \frac{1}{M_1^2 - s - i M_1 \Gamma_1} + \beta \frac{1}{M_2^2 - s - i M_2 \Gamma_2} \right|^2 \text{ 🧐}$$

Particle	X(1840)	X(1880)
J^{PC}	0^{-+}	0^{-+}
Mass (MeV)	$1832.5 \pm 3.1 \pm 2.5$	$1882.1 \pm 1.7 \pm 0.7$
Width (MeV)	$80.7 \pm 5.2 \pm 7.7$	$30.7 \pm 5.5 \pm 2.4$

This result further supports the existence of a $p\bar{p}$ bound state.



Although a great effort has been put forward, the properties of the resonances in the mass region of $[1.8, 1.9]$ GeV is still remain controversial.

➤ Final states interactions effects

- L. Y. Dai, J. Haidenbauer and U. G. Meißner, Phys. Rev. D 98, 014005
- Q. H. Yang, D. Guo and L. Y. Dai, Phys. Rev. D 107, 034030
- G. Y. Chen, H. R. Dong and J. P. Ma, Phys. Rev. D 78, 054022
- X. W. Kang, J. Haidenbauer and U. G. Meißner, Phys. Rev. D 91, 074003
- A. I. Milstein and S. G. Salnikov, Nucl. Phys. A 966, 54-63
- S. G. Salnikov and A. I. Milstein, Nucl. Phys. B 1002, 116539
- ...

➤ Pseudoscalar glueball

- B. A. Li, Phys. Rev. D 74, 034019
- N. Kochelev and D. P. Min, Phys. Rev. D 72, 097502
- N. Kochelev and D. P. Min, Phys. Lett. B 633, 283-288
- X. G. He, X. Q. Li, X. Liu and J. P. Ma, Eur. Phys. J. C 49, 731
- G. Hao, C. F. Qiao and A. L. Zhang, Phys. Lett. B 642, 53-61
- L. C. Gui, J. M. Dong, Y. Chen and Y. B. Yang, Phys. Rev. D 100, 054511
- ...

➤ Radial excitations of η'

- T. Huang and S. L. Zhu, Phys. Rev. D 73, 014023
- J. S. Yu, Z. F. Sun, X. Liu and Q. Zhao, Phys. Rev. D 83, 114007
- L. M. Wang, Q. S. Zhou, C. Q. Pang and X. Liu, Phys. Rev. D 102, 114034
- ...

➤ $3^1S_0 q\bar{q}$ state

- D. M. Li and B. Ma, Phys. Rev. D 77, 074004

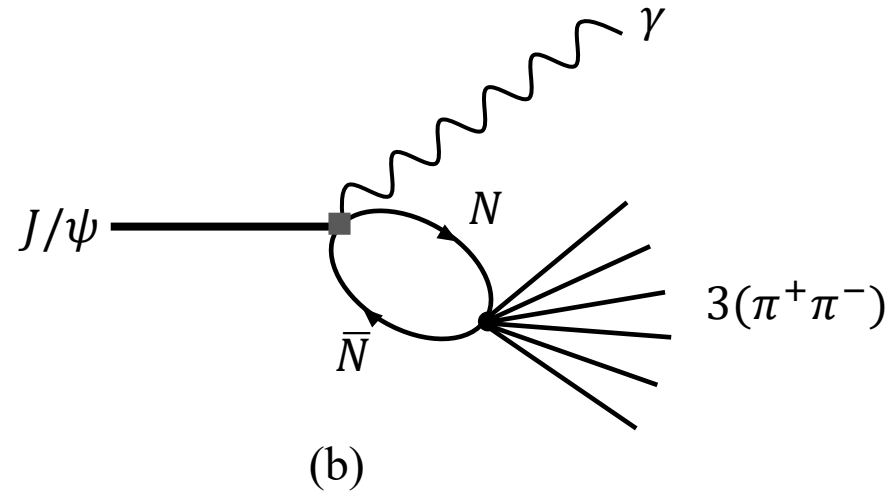
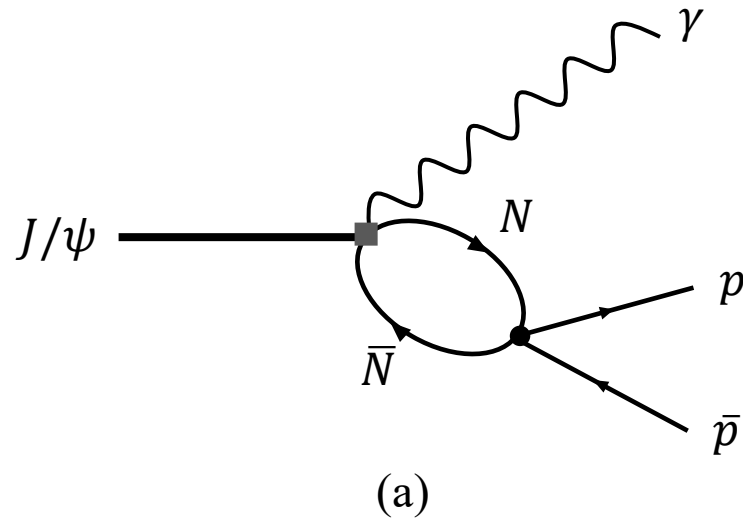
➤ Reviews

- Y. F. Liu and X. W. Kang, Symmetry 8, 14
- B. Q. Ma, arXiv:2406.19180
- ...

➤ ...

1. Background

Can we describe $J/\psi \rightarrow \gamma p \bar{p}$ and $J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)$ simultaneously ?



- The $N\bar{N}$ and $3(\pi^+ \pi^-)$ channels are considered dynamically and non-dynamically, respectively.
- We would not include the contribution of resonances but focus on the rescattering effect of the dynamic $N\bar{N}$ channel.
- Whether the dynamic generated states in the $N\bar{N}$ channel can describe the experimental data.

The leading and next-leading order $N\bar{N}$ contact interactions in the chiral effective field theory:

$$L_{N\bar{N}}^{(0)} = C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,$$

$$L_{N\bar{N}}^{(2)} = C_1 \mathbf{q}^2 + C_2 \mathbf{k}^2 + (C_3 \mathbf{q}^2 + C_4 \mathbf{k}^2) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ + \frac{i}{2} C_5 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}) + C_6 (\mathbf{q} \cdot \boldsymbol{\sigma}_1) (\mathbf{q} \cdot \boldsymbol{\sigma}_2) \\ + C_7 (\mathbf{k} \cdot \boldsymbol{\sigma}_1) (\mathbf{k} \cdot \boldsymbol{\sigma}_2).$$

The isospin basis and the particle basis:

$$|I = 1, I_3 = 0\rangle = \frac{1}{\sqrt{2}} (|p\bar{p}\rangle + |n\bar{n}\rangle) \\ |I = 0, I_3 = 0\rangle = \frac{1}{\sqrt{2}} (|p\bar{p}\rangle - |n\bar{n}\rangle)$$



$$V_{p\bar{p}\rightarrow p\bar{p}} = V_{n\bar{n}\rightarrow n\bar{n}} = \frac{1}{2} (V_{1S_0}^{I=1} + V_{1S_0}^{I=0}) \\ V_{p\bar{p}\leftrightarrow n\bar{n}} = \frac{1}{2} (V_{1S_0}^{I=1} - V_{1S_0}^{I=0})$$

The partial wave interaction:

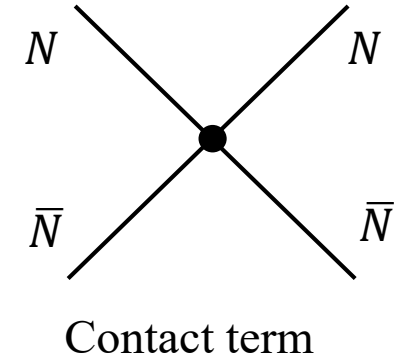
$$L(^1S_0) = C'_{01} + C'_{02} (\mathbf{p}^2 + \mathbf{p}'^2)$$

$$L(^3S_0) = C'_{11} + C'_{12} (\mathbf{p}^2 + \mathbf{p}'^2)$$

The $N\bar{N}$ interaction in the isospin basis

$$V_{1S_0}^I = C_{I1} + C_{I2} (\mathbf{p}^2 + \mathbf{p}'^2), I = 0, 1$$

C_{I1} : complex number to take into account the annihilation contribution

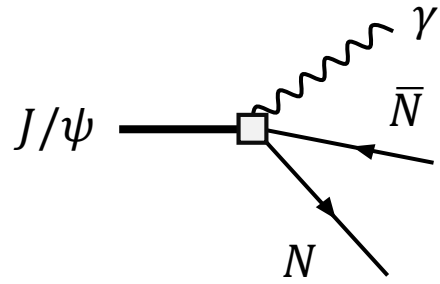


The transition matrix of $N\bar{N} \rightarrow N\bar{N}$ is obtained with the LS equation:

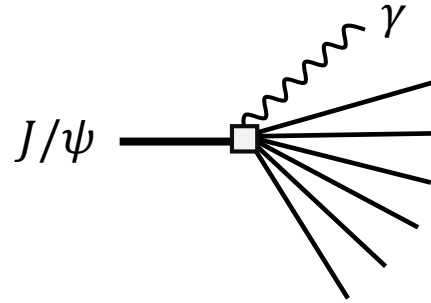
$$T_{N\bar{N}\rightarrow N\bar{N}}(p, p') = V_{N\bar{N}\rightarrow N\bar{N}}(p, p') + \int \frac{d^3 \mathbf{p}''}{(2\pi)^3} V_{N\bar{N}\rightarrow N\bar{N}}(p, \mathbf{p}'') G^+(E, \mathbf{p}'') T_{N\bar{N}\rightarrow N\bar{N}}(\mathbf{p}'', p')$$

[1]. X. W. Kang, J. Haidenbauer and U. G. Meisner, JHEP 02, 113

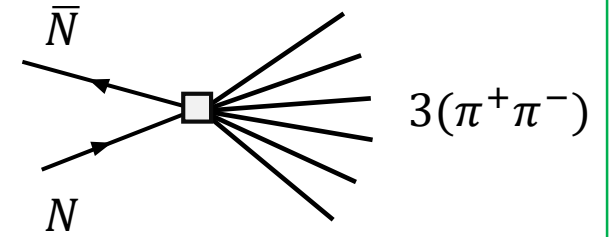
[2]. L. Y. Dai, J. Haidenbauer and U. G. Meißner, JHEP 07, 078



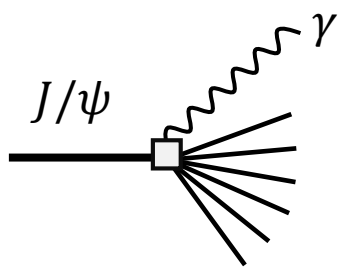
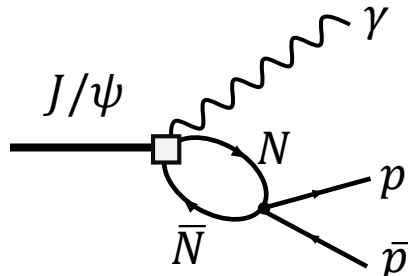
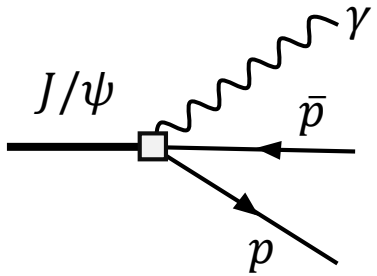
$$A_{J/\psi \rightarrow \gamma N \bar{N}}^0 = C_{J/\psi \rightarrow \gamma N \bar{N}}$$



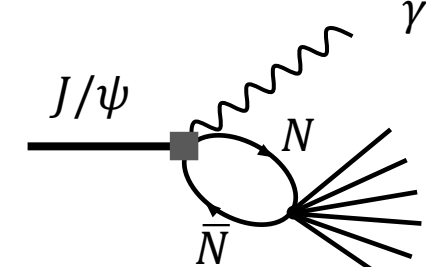
$$A_{J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)}^0 = C_{J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)}$$



$$A_{N \bar{N} \rightarrow 3(\pi^+ \pi^-)} = C_{N \bar{N} \rightarrow \gamma 6\pi}$$



$3(\pi^+ \pi^-)$



$3(\pi^+ \pi^-)$

The physical decay amplitudes:

$$\tilde{M}_{J/\psi \rightarrow \gamma p \bar{p}} = 8\pi^2 \sqrt{E_{J/\psi} E_\gamma E_p E_{\bar{p}}} M_{J/\psi \rightarrow \gamma p \bar{p}},$$

$$\tilde{M}_{J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)} = 32\pi^{7/2} \sqrt{E_{J/\psi} E_\gamma E_2 E_3 E_4} M_{J/\psi \rightarrow 3(\pi^+ \pi^-)},$$

where

$$M_{J/\psi \rightarrow \gamma N \bar{N}} = \underbrace{A_{J/\psi \rightarrow \gamma N \bar{N}}^0}_{\text{green}} + \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \underbrace{A_{J/\psi \rightarrow \gamma N \bar{N}}^0 \cdot G^+ \cdot T_{N \bar{N} \rightarrow N \bar{N}}}_{\text{red}},$$

$$M_{J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)} = \underbrace{A_{J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)}^0}_{\text{green}} + \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \underbrace{M_{J/\psi \rightarrow \gamma N \bar{N}} \cdot G^+ \cdot A_{N \bar{N} \rightarrow 3(\pi^+ \pi^-)}}_{\text{red}}.$$

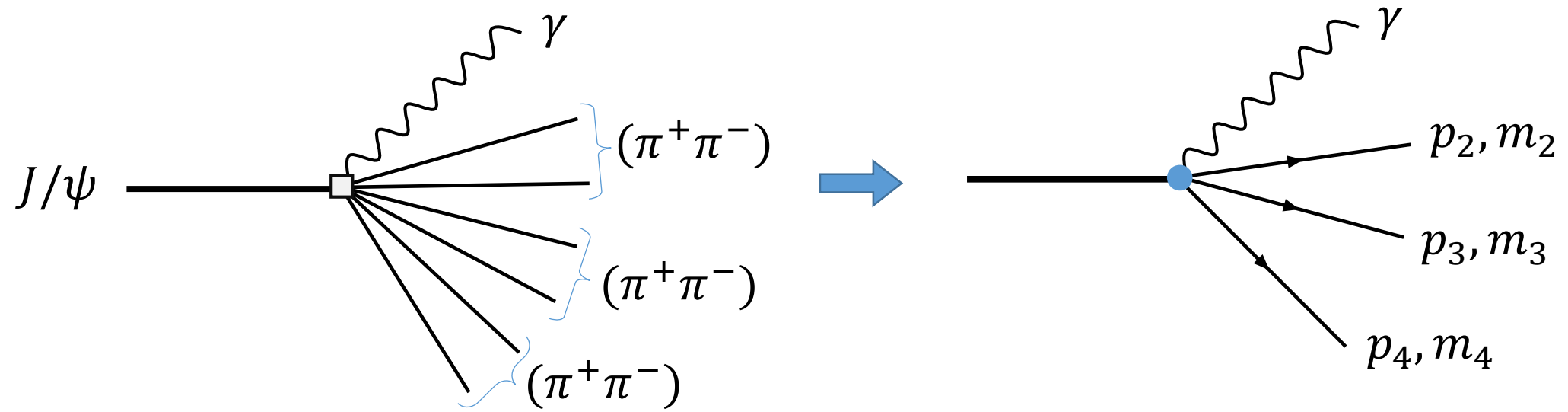
[1]. Q. H. Yang, D. Guo and L. Y. Dai, Phys. Rev. D 107, 034030

[2]. L. Y. Dai, J. Haidenbauer and U. G. Meißner, Phys. Rev. D 98, 014005

1. Data 2003: $\text{Events}(m_{p\bar{p}}) = \text{fac1} \times \frac{d\Gamma_{J/\psi \rightarrow \gamma p\bar{p}}}{dm_{p\bar{p}}}$,
2. Data 2012: $\text{Events}(m_{p\bar{p}}) = \text{fac2} \times \frac{d\Gamma_{J/\psi \rightarrow \gamma p\bar{p}}}{dm_{p\bar{p}}}$,
3. Data 2024: $\text{Events}(m_{6\pi}) = \text{fac3} \times \frac{d\Gamma_{J/\psi \rightarrow \gamma 3(\pi^+\pi^-)}}{dm_{6\pi}} + \frac{d\Gamma_{J/\psi \rightarrow \gamma 3(\pi^+\pi^-)}^{\text{bg}}}{dm_{6\pi}}$.

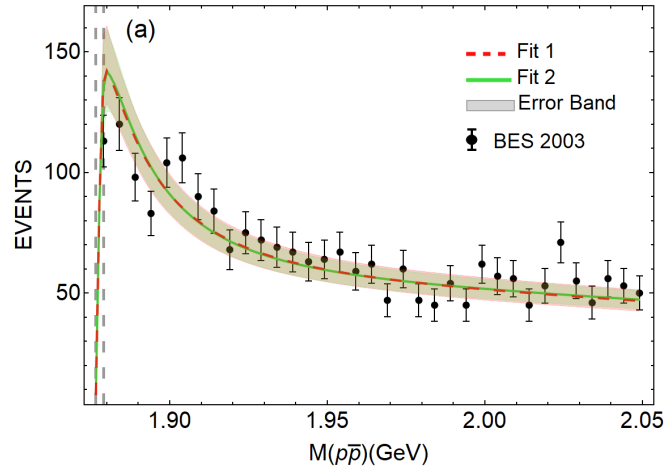
Background for the $J/\psi \rightarrow \gamma 3(\pi^+\pi^-)$:
 $bg_{J/\psi \rightarrow \gamma 3(\pi^+\pi^-)} = a + bQ + cQ^2$.

Phase space: $d\Phi_7 \rightarrow d\Phi_4$:

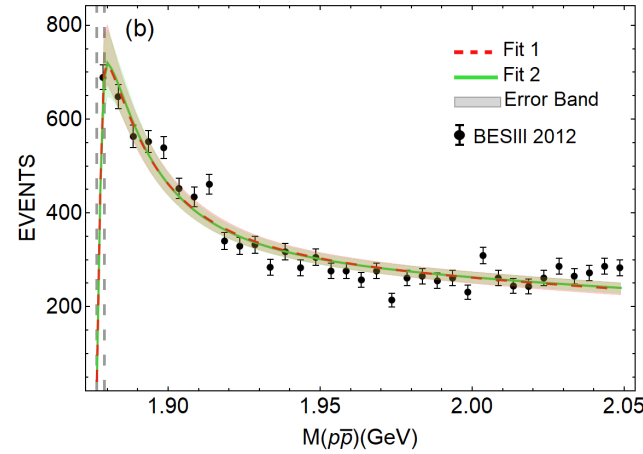


3. Results

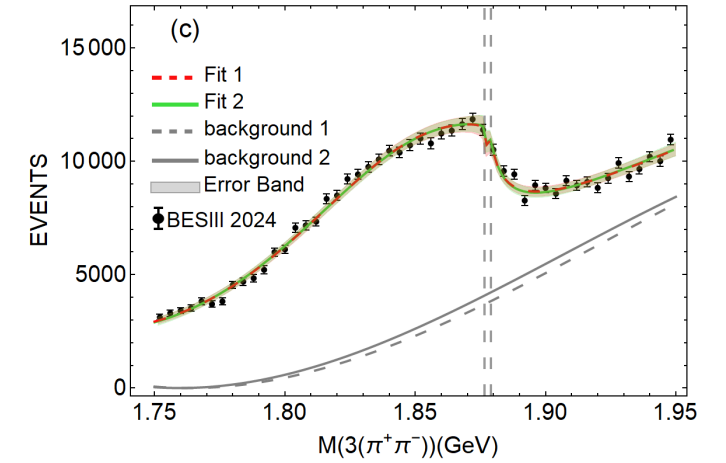
The fitting results



BES, Phys. Rev. Lett. 91, 022001 (2003)



BESIII, Phys. Rev. Lett. 108, 112003 (2012)



BESIII, Phys. Rev. Lett. 132, no.15, 151901 (2024)

Parameters	Solution-I	Solution-II
C_{01} (GeV^{-2})	$(87.41 \pm 0.32) - (6.08 \pm 0.10)i$	$(97.24 \pm 0.66) - (-6.72 \pm 0.16)i$
C_{02} (GeV^{-4})	-102.36 ± 0.21	-109.26 ± 0.47
C_{11} (GeV^{-2})	$(-33.47 \pm 0.31) + (0.56 \pm 0.76)i$	$(153.79 \pm 3.98) + (12.56 \pm 8.26)i$
C_{12} (GeV^{-4})	57.56 ± 16.44	247.88 ± 2.01
$C_{J/\psi \rightarrow \gamma p \bar{p}}$ (GeV^{-2})	168.06 ± 9.08	-88.22 ± 23.26
$C_{J/\psi \rightarrow \gamma n \bar{n}}$ (GeV^{-2})	-372.26 ± 8.23	428.47 ± 8.39
$C_{p \bar{p} \rightarrow 6\pi}$ ($\text{GeV}^{-7/2}$)	-330.66 ± 10.63	-111.08 ± 2.12
$C_{n \bar{n} \rightarrow 6\pi}$ ($\text{GeV}^{-7/2}$)	263.04 ± 10.75	81.65 ± 5.83
fac1 (10^{-3})	1.42 ± 0.10	0.85 ± 0.048
fac2 (10^{-3})	7.19 ± 0.50	4.32 ± 0.23
fac3 (10^{-3})	0.71 ± 0.04	3.14 ± 0.11
a (GeV^{-1})	$-2.58 \times 10^7 \pm 1.61 \times 10^4$	$-2.84 \times 10^7 \pm 9.47 \times 10^5$
b (GeV^{-2})	$2.54 \times 10^7 \pm 8.50 \times 10^3$	$2.84 \times 10^7 \pm 1.12 \times 10^3$
c (GeV^{-3})	$-6.17 \times 10^7 \pm 4.38 \times 10^3$	$-6.96 \times 10^6 \pm 2.62 \times 10^4$
$\chi^2/\text{d.o.f}$	2.32(2.24)	2.33(2.31)

- Fitting program: Minuit2
- The two solutions can describe the experimental data almost equally well.

Two channels

RS-I: $\text{Im } k_1 > 0, \text{Im } k_2 > 0,$

RS-II: $\text{Im } k_1 < 0, \text{Im } k_2 > 0,$

RS-III: $\text{Im } k_1 < 0, \text{Im } k_2 < 0,$

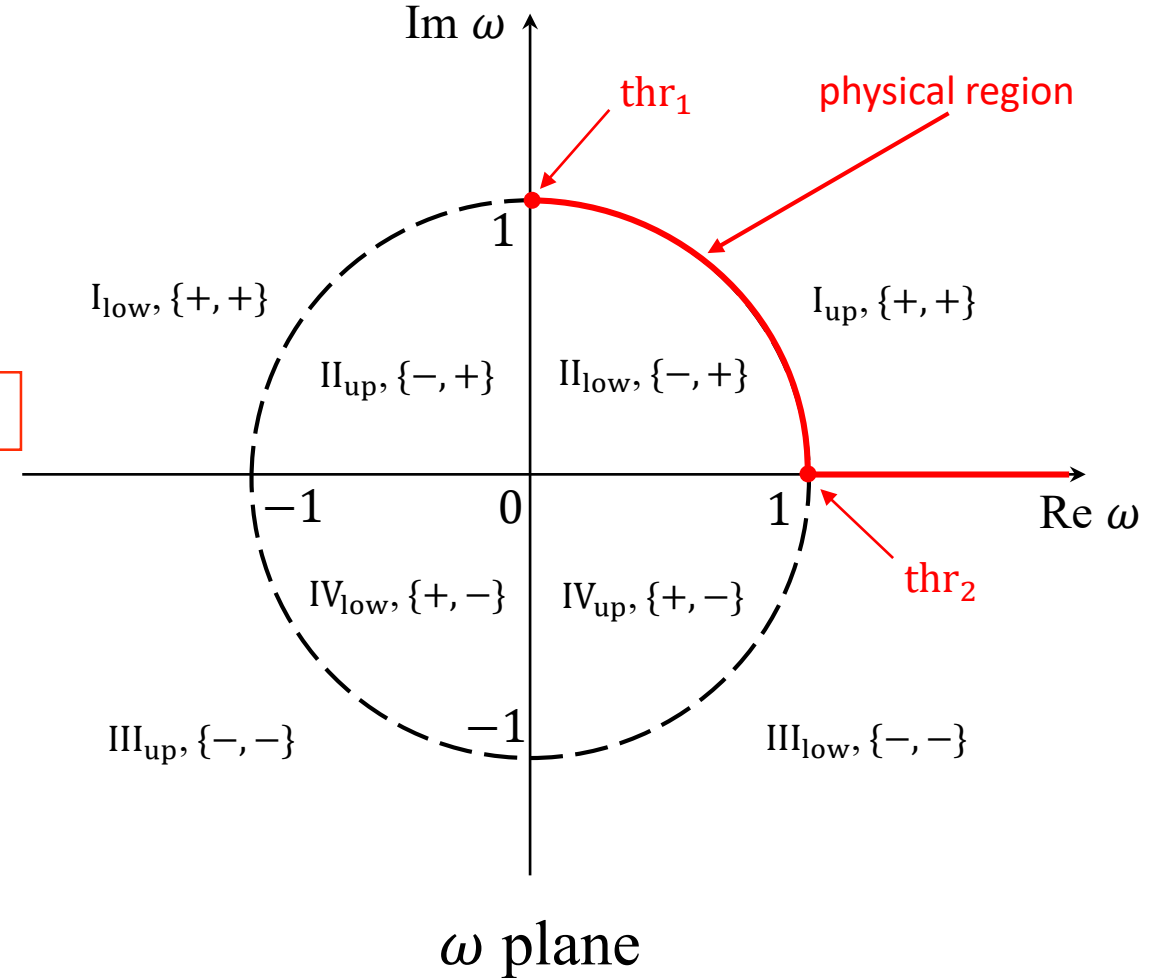
RS-IV: $\text{Im } k_1 > 0, \text{Im } k_2 < 0.$

$$k_1 = \sqrt{\frac{\mu_1 \delta}{2}} \left(\omega + \frac{1}{\omega} \right),$$

$$k_2 = \sqrt{\frac{\mu_2 \delta}{2}} \left(\omega - \frac{1}{\omega} \right).$$

$$\delta = \text{thr}_1 - \text{thr}_2 = 2.89 \text{ MeV}$$

- **Bound state:** pole below threshold on real axis of the first Riemann sheet of complex energy plane
- **Virtual state:** pole below threshold on real axis of the second Riemann sheet
- **Resonance:** pole in the complex plane on the second Riemann sheet



[1]. M. Kato, Annals Phys. 31, 130

[2]. V. Baru, E. Epelbaum, A. A. Filin, C. Hanhart, A. V. Nefediev and Q. Wang, Phys. Rev. D 99, 094013

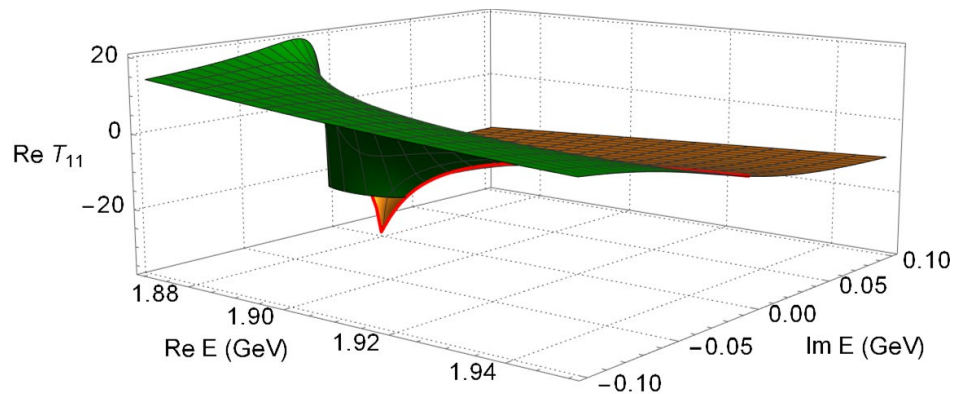
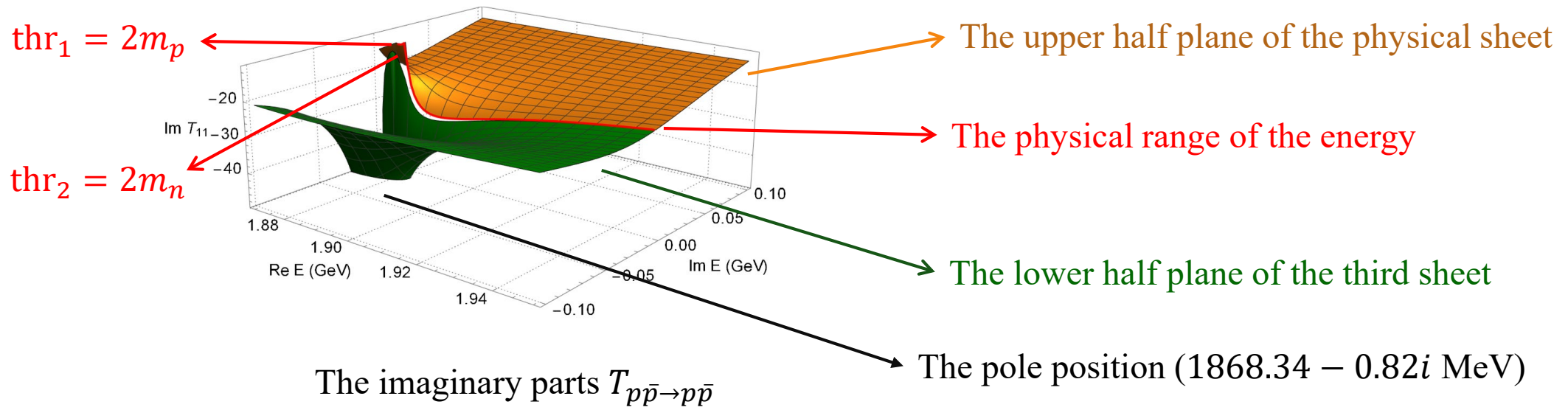
R. S.	I	IV	II	III
Solution-I (MeV)	$1851.90^{+3.02}_{-2.71}$ $- 80.49^{+1.68}_{-1.63}i$	$1866.07^{+22.41}_{-7.20}$ $+ 86.34^{+8.65}_{-12.76}i$	$1875.46^{+20.61}_{-6.60}$ $+ 87.20^{+9.45}_{-12.92}i$	$1868.34^{+1.66}_{-0.55}$ $- 0.82^{+1.17}_{-1.41}i$
$(g_{pp\bar{p}}, g_{nn\bar{n}})(\text{GeV}^{-1/2})$	(1.61, 1.64)	(3.42, 2.22)	(2.16, 3.42)	(0.98, 0.94)
$(g_1, g_0)(\text{GeV}^{-1/2})$	(0.017, 2.37)	(3.32, 2.36)	(3.31, 2.32)	(1.35, 0.032)
Solution-II (MeV)	$1852.90^{+3.57}_{-3.31}$ $- 82.35^{+2.45}_{-2.80}i$	$1860.31^{+8.77}_{-8.34}$ $+ 61.23^{+6.18}_{-5.38}i$	$1855.23^{+8.49}_{-8.07}$ $+ 62.01^{+6.02}_{-5.29}i$	$1868.92^{+1.13}_{-1.48}$ $- 2.58^{+1.70}_{-1.86}i$
$(g_{pp\bar{p}}, g_{nn\bar{n}})(\text{GeV}^{-1/2})$	(1.85, 1.88)	(2.82, 1.80)	(1.76, 2.82)	(0.94, 0.90)
$(g_1, g_0)(\text{GeV}^{-1/2})$	(0.013, 2.89)	(2.68, 1.99)	(2.67, 1.98)	(1.30, 0.033)

$$2m_p = 1876.54 \text{ MeV}$$

- Similar results
- All the poles positions are below the lowest threshold. Since the LECs C_{01} and C_{11} are set to be complex numbers.
- Only the pole on the physical sheet strongly couples to isospin singlet.

3. Results

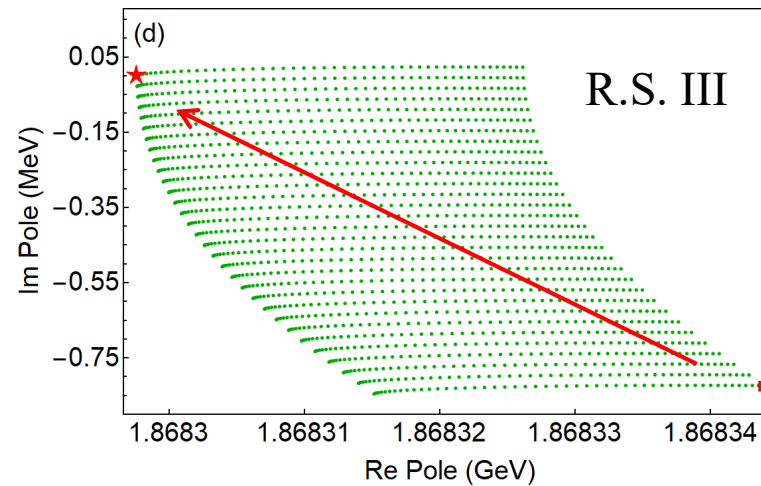
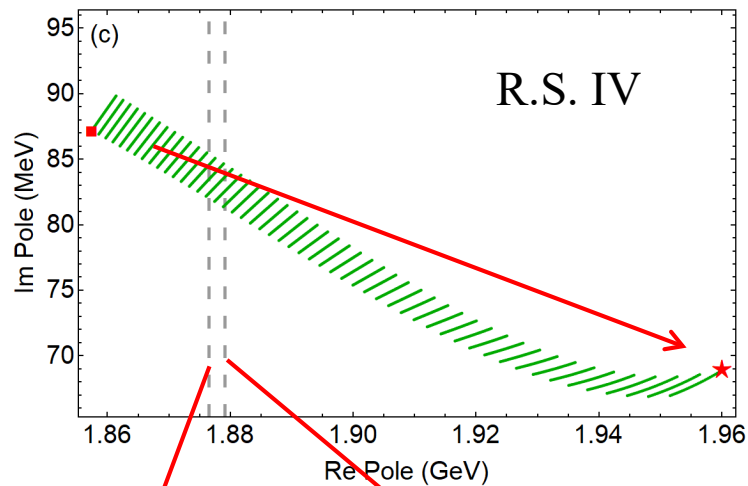
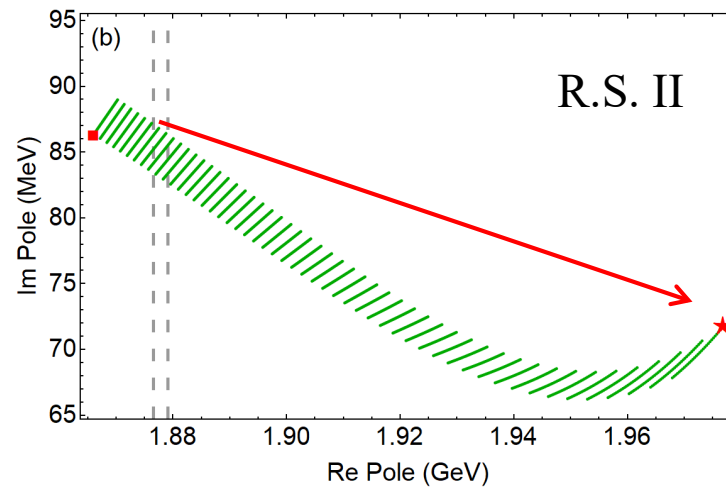
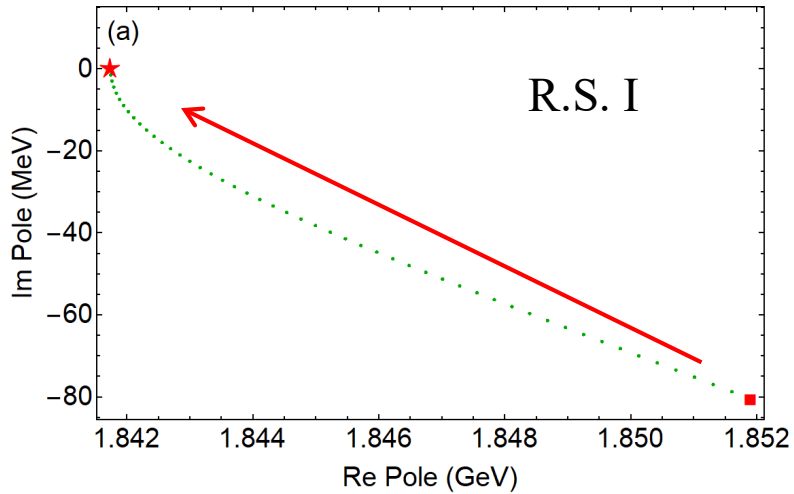
$T_{p\bar{p} \rightarrow p\bar{p}}$ on the complex energy plane



- We can clearly see how the pole on the third sheet results in the enhancement near $N\bar{N}$ threshold.
- Blow and far from the thr_1 , only the pole on the first R.S. has significant affects on the line shape.

3. Results

Poles trajectories on the complex energy plane

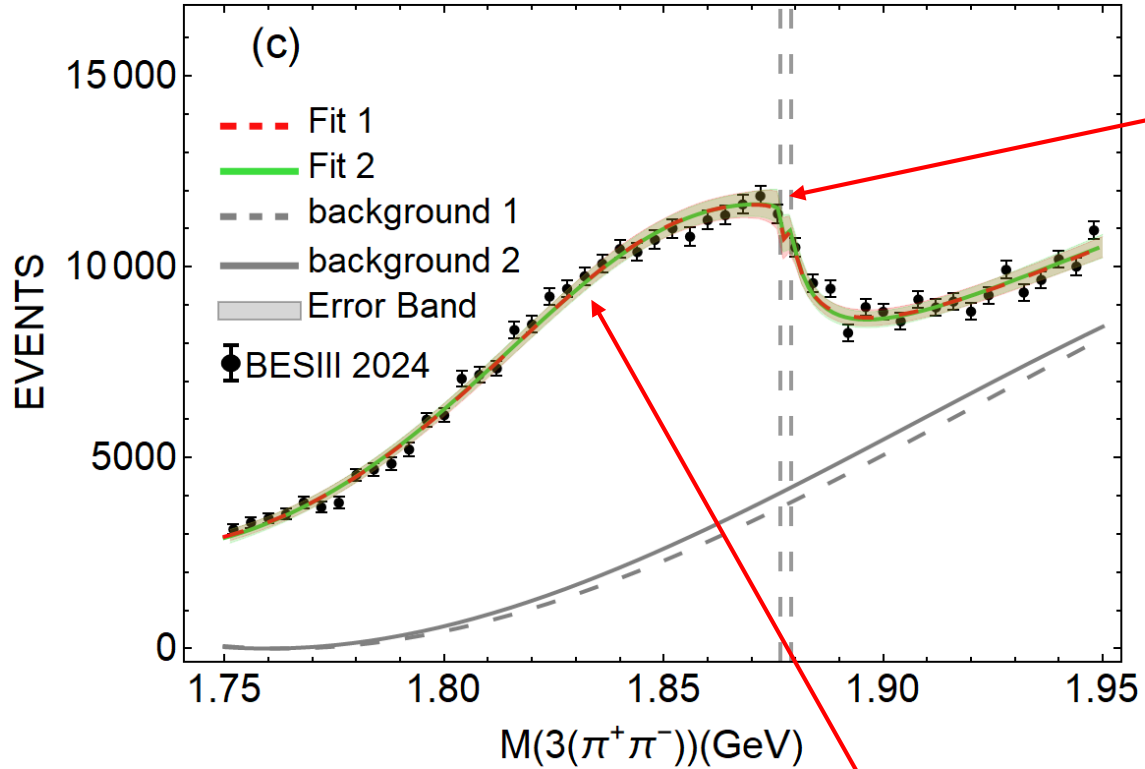


- The poles trajectories on the complex energy plane with the variation of $\text{Im } C_{01}$ and $\text{Im } C_{11}$ from the fitted values to zero in Solution-I.
- For Solution-II, the behaviors are similar to those of Solution-I.

- The pole on the first R.S. can be treated as a bound state.
- The pole on the third R.S. can be treated as a virtual state.
- The other two poles are resonance.

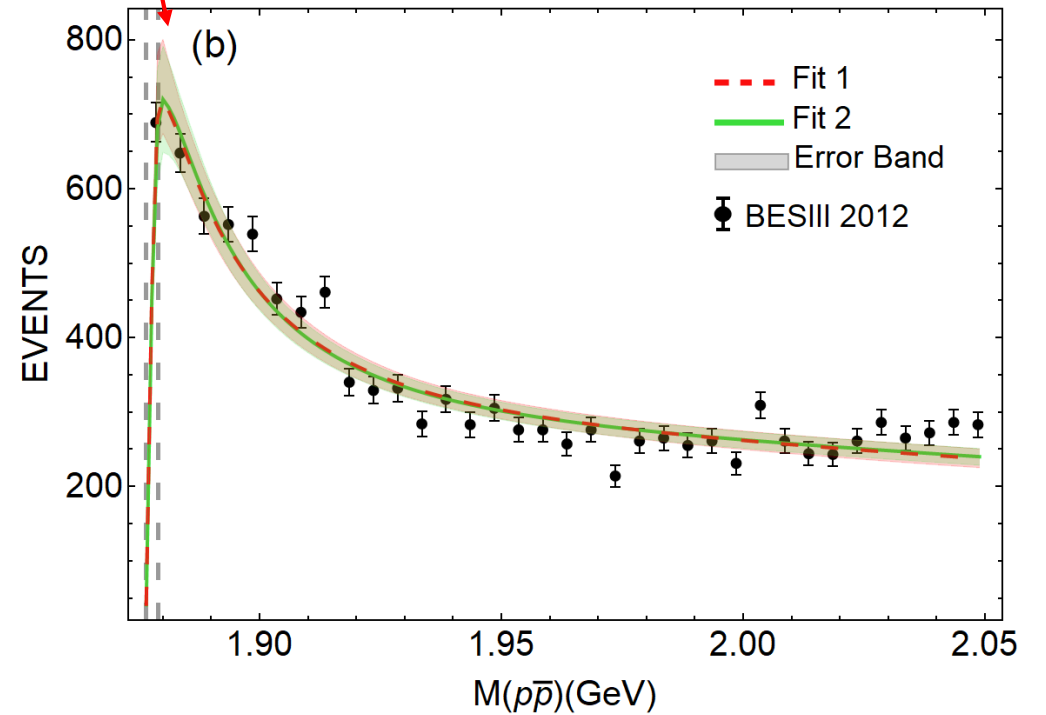
$\text{thr}_1 = 1876.54 \text{ MeV}$

$\text{thr}_2 = 1879.13 \text{ MeV}$



$1851.90^{+3.02}_{-2.71} - 80.49^{+1.68}_{-1.63}i$
 On the first R. S. of $T_{N\bar{N}}$ (bound state)

$1868.34^{+1.66}_{-0.55} - 0.82^{+1.17}_{-1.41}i$
 On the third R. S. of $T_{N\bar{N}}$ (virtual state)



Effective-Range-Expansion:

$$T^{-1}(k) = -\frac{\mu}{2\pi} \left[-\frac{1}{a_0} + \frac{1}{2}r_0k^2 - ik + \mathcal{O}(k^4) \right]$$

⇒

scattering length: $a_0 = \frac{2\pi}{\mu} T(k)|_{k \rightarrow 0}$,

effective range: $r_0 = -\frac{2\pi}{\mu^2} \text{Re} \left[\frac{dT^{-1}(E)}{dE} \right]$,

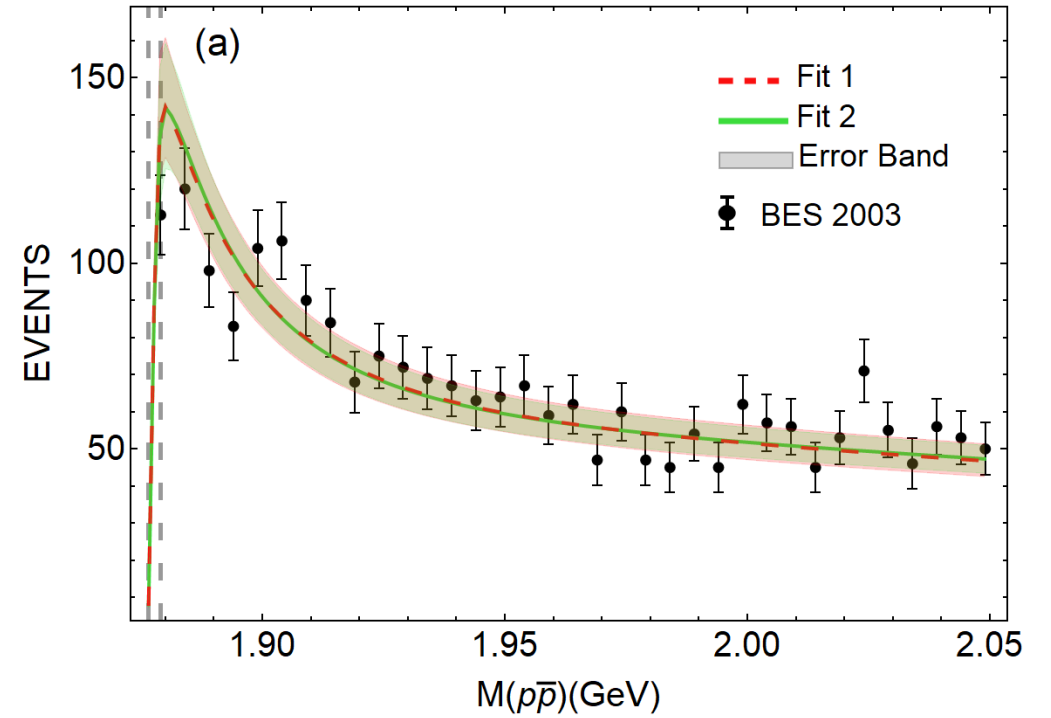
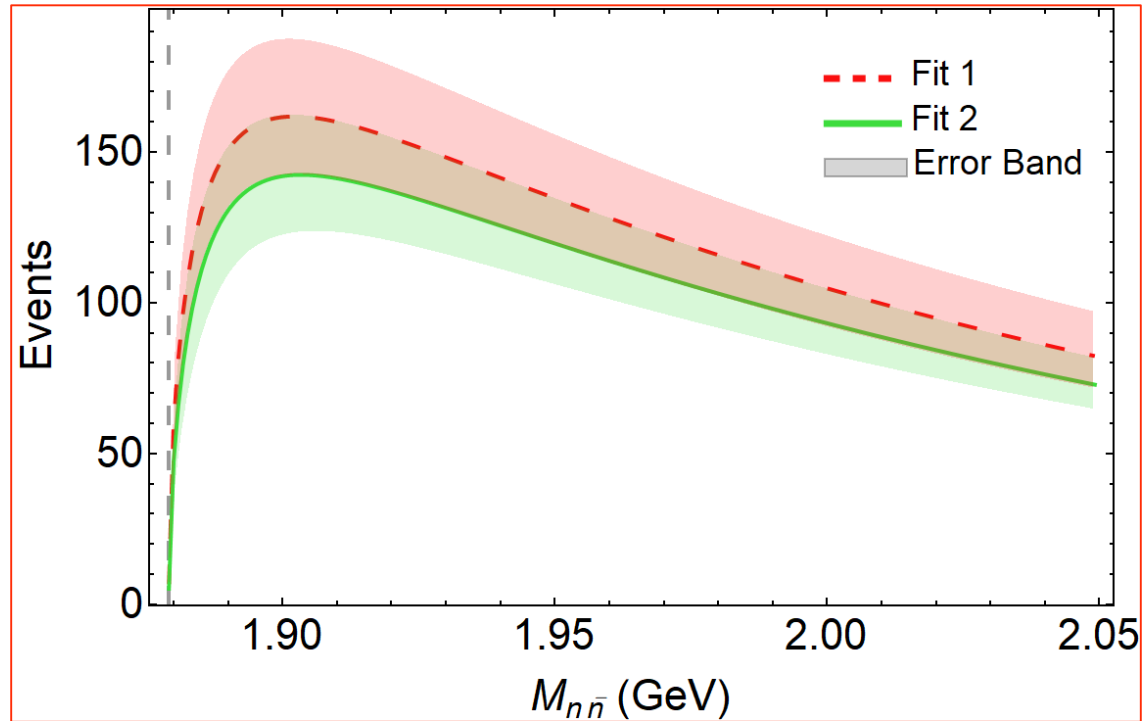
correction from isospin breaking effect:

$$r'_0 = r_0 + \sqrt{\frac{1}{2\mu\Delta}}$$

Compositeness: $\bar{X} = \frac{1}{\sqrt{1+2\left|\frac{r'_0}{\text{Re}[a_0]}\right|}}$

	r_0 (fm)	r'_0 (fm)	a_0 (fm)	\bar{X}
Solution-I	-2.30	1.70	-65.46-31.94i	0.98
Solution-II	1.50	5.50	-56.52-27.16i	0.91

There exists $p\bar{p}$ dynamical generated states.



- We can see a clear threshold enhancement in the $n\bar{n}$ channel, but not as significant as that in the $p\bar{p}$ channel.
- This result can be used to compare with the measurement of the future experiment.

4. Summary

- The $p\bar{p}$ threshold enhancement are both observed in $J/\psi \rightarrow \gamma p\bar{p}$ and $J/\psi \rightarrow \gamma 3(\pi^+ \pi^-)$.
- By constructing the $N\bar{N}$ interaction respecting chiral symmetry, we extract the pole positions by fitting the experimental data.
- There exists $p\bar{p}$ dynamical generated states:
 - RS-I: $m = 1852$ MeV, $\Gamma = 160$ MeV (bound state)
 - RS-III: $m = 1868$ MeV, $\Gamma = 2$ MeV (virtual state)
- We can see a clear threshold enhancement in the $n\bar{n}$ channel.

Thank you !