

Neutrino Opacity with Supernova Matter using chiral effective field theory potential

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Guo & Martinez-Pinedo, ApJ 887 (2019) 58

Guo, Martinez-Pinedo, & Wu, PRC 110 (2024) 015504

Fischer, Guo, Langanke, Martinez-Pinedo, Qian, & Wu, “neutrinos and nucleosynthesis of elements”, PPNP 137 (2024) 104107

Outline

Introduction to neutrino-driven supernova explosion

Supernova neutrino opacity calculations (χ EFT potential)

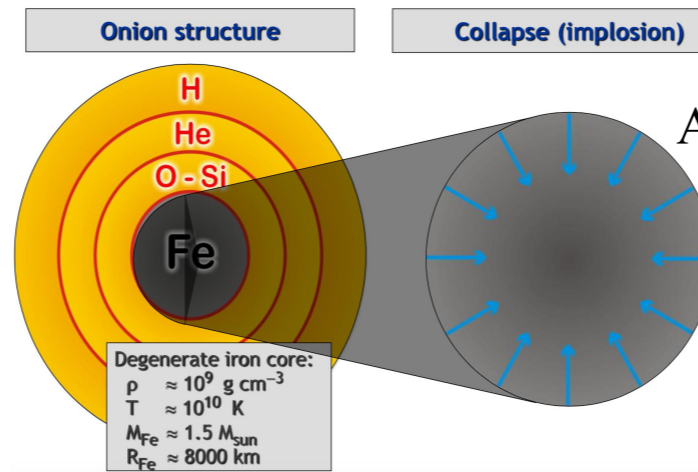
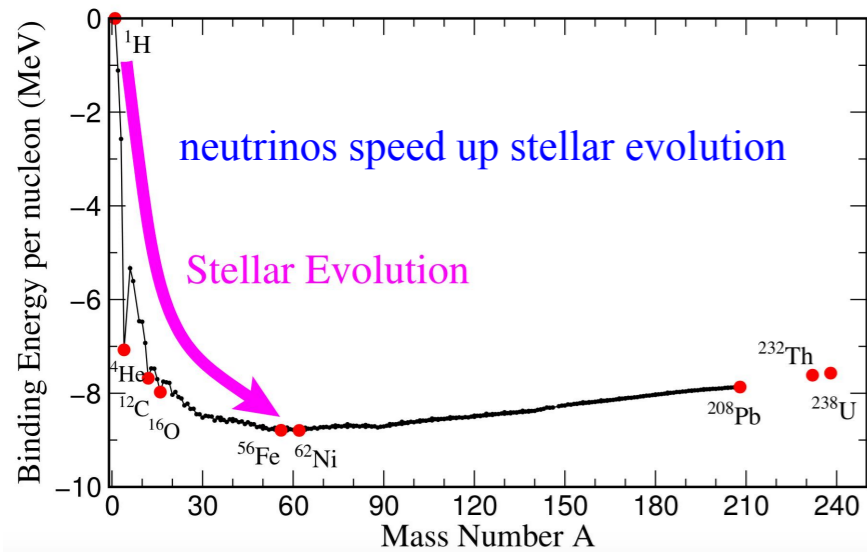
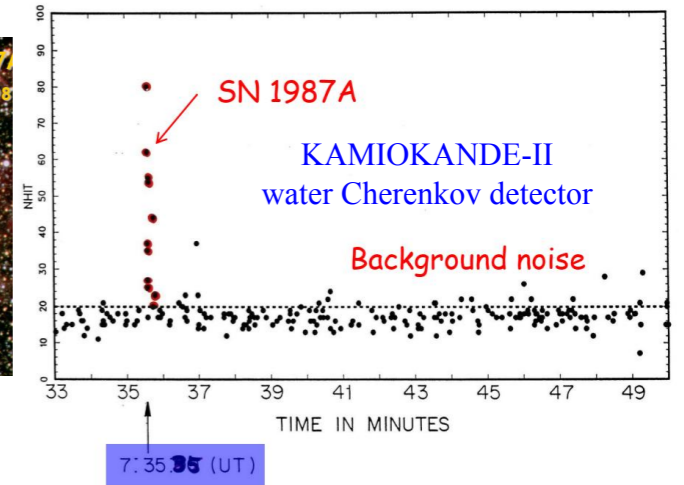
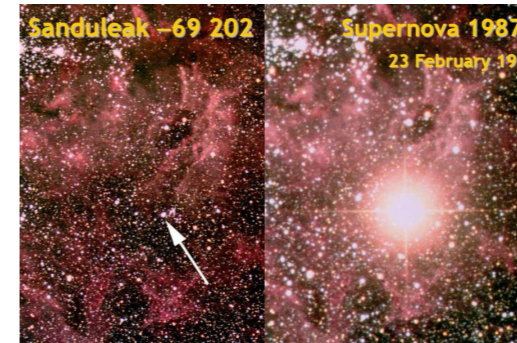
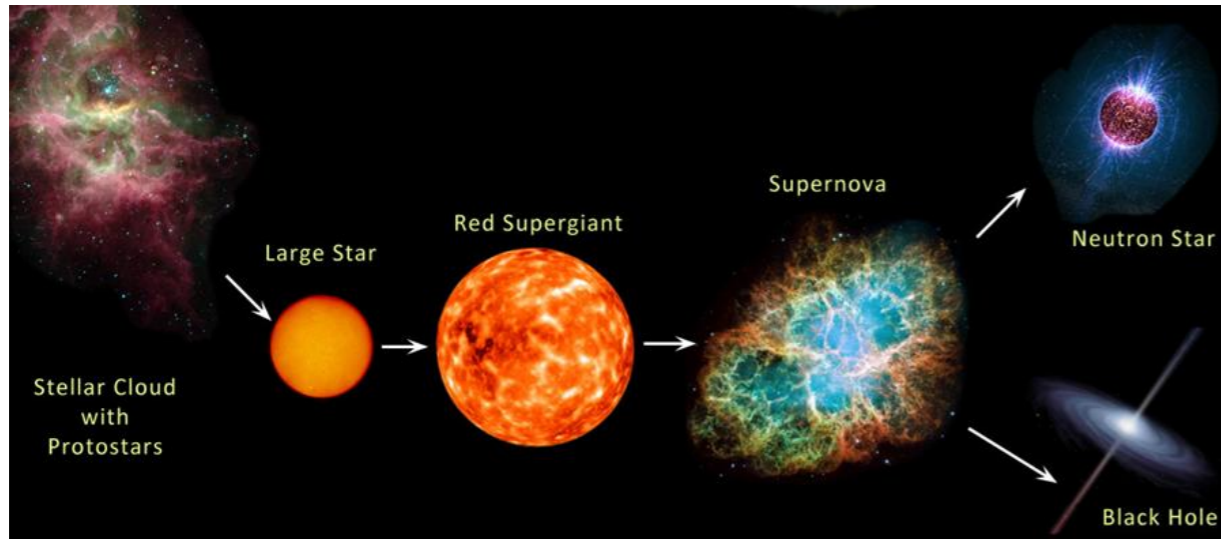
neutrino-nucleon reaction at mean field level

neutrino nucleon-nucleon bremsstrahlung

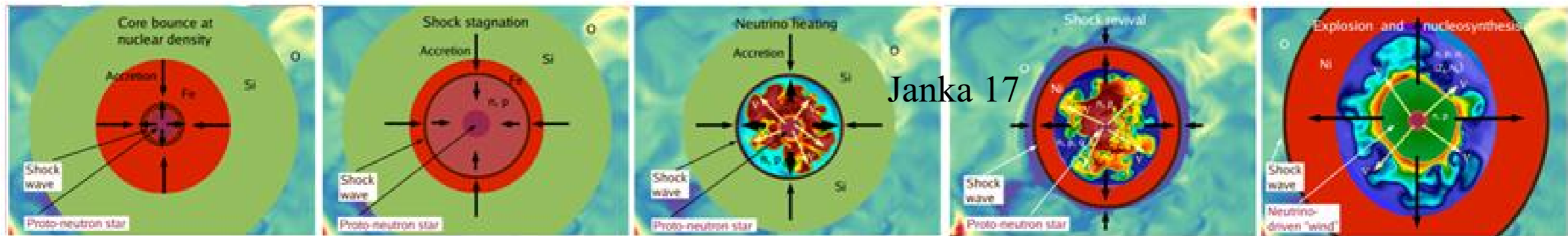
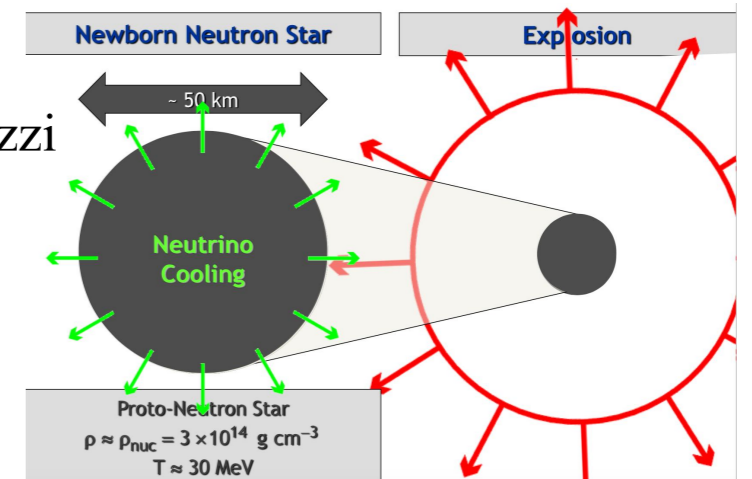
neutrino-nucleon considering nucleon-nucleon correlation effects

Summary

nuclear & neutrino reactions driving evolution and core-collapse explosion of massive stars



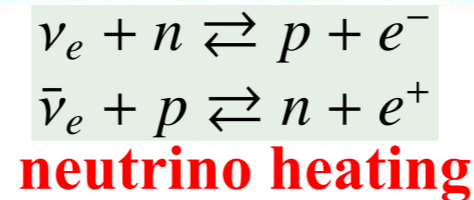
A. Mirizzi



Janka 17

core-collapse & bounce shock

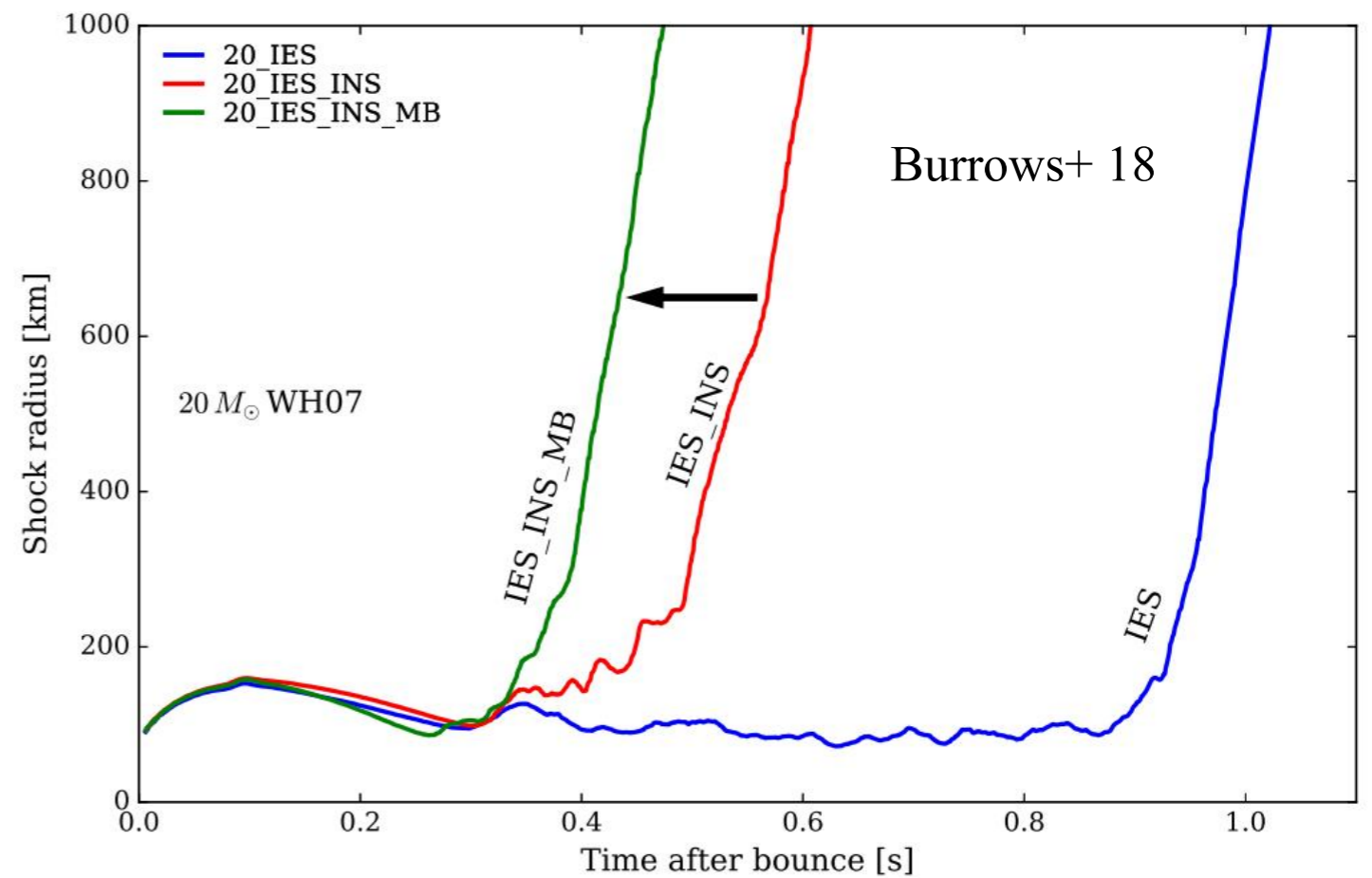
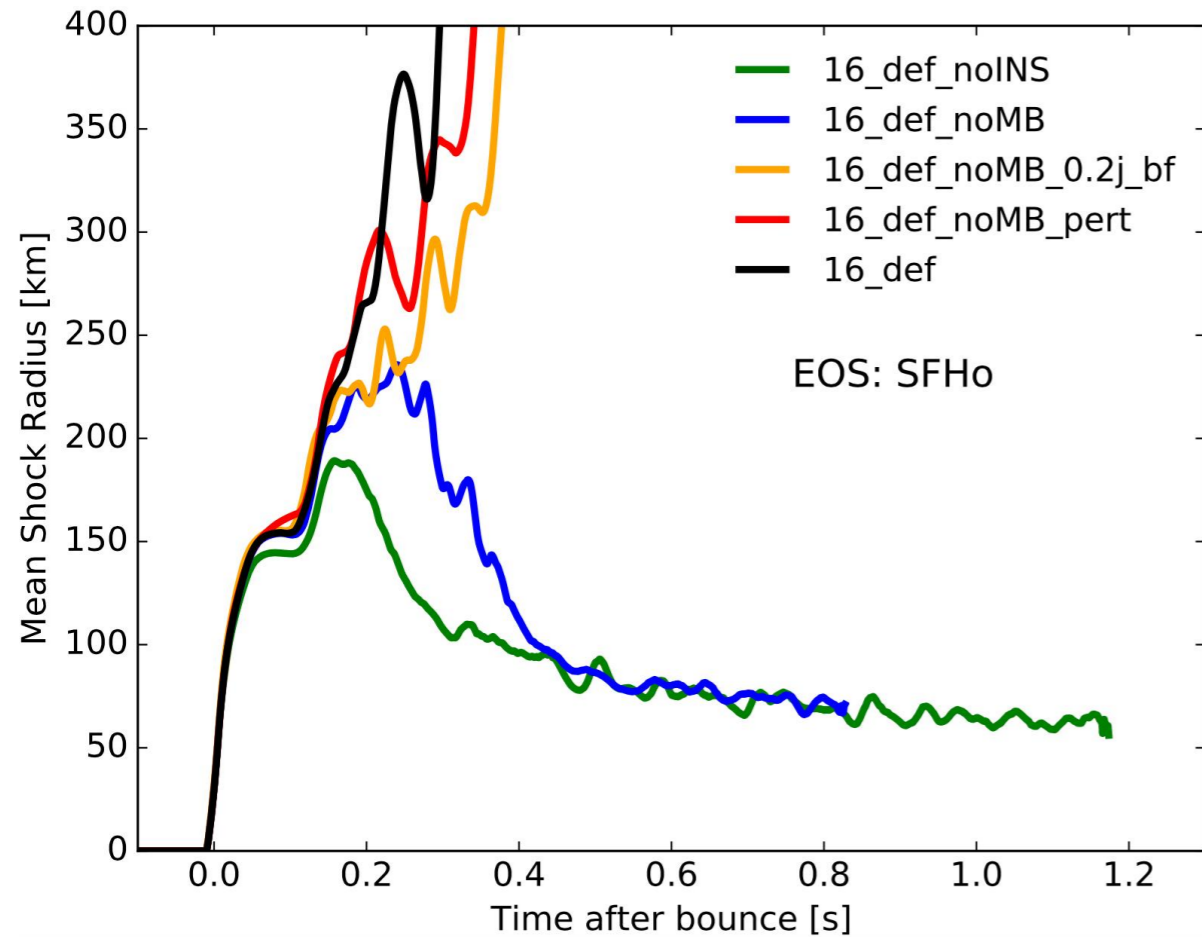
shock stagnation



shock revival

explosion

stellar explosion or not is very sensitive to neutrino



- Neutrino signals from core-collapse supernova (CCSN)
- Nucleosynthesis depends sensitively on the proton fraction Y_p

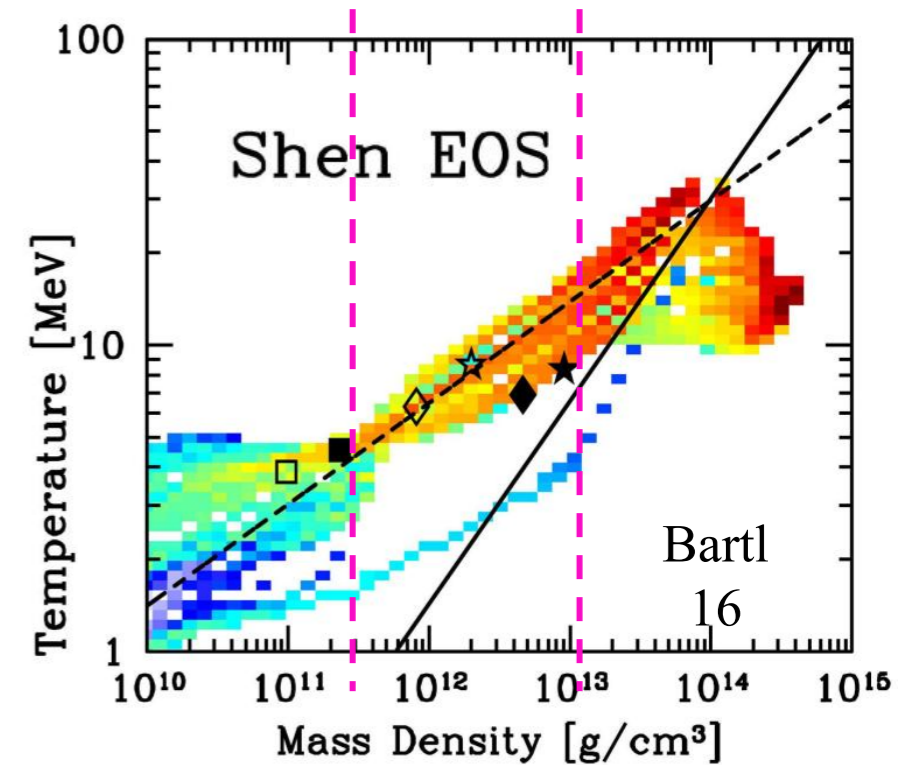
Accurate description of neutrino rates is highly demanded !!!

The nuclear Equation of State (EoS) matters too

Complicated Neutrinos Processes in CCSN

Process	Reaction ^a
Beta-processes (direct URCA processes)	
electron and ν_e absorption by nuclei	$e^- + (A, Z) \longleftrightarrow (A, Z - 1) + \nu_e$
electron and ν_e captures by nucleons	$e^- + p \longleftrightarrow n + \nu_e$
positron and $\bar{\nu}_e$ captures by nucleons	$e^+ + n \longleftrightarrow p + \bar{\nu}_e$
“Thermal” pair production and annihilation processes	
Nucleon-nucleon bremsstrahlung	$N + N \longleftrightarrow N + N + \nu + \bar{\nu}$
Electron-positron pair process	$e^- + e^+ \longleftrightarrow \nu + \bar{\nu}$
Plasmon pair-neutrino process	$\tilde{\gamma} \longleftrightarrow \nu + \bar{\nu}$
Reactions between neutrinos	
Neutrino-pair annihilation	$\nu_e + \bar{\nu}_e \longleftrightarrow \nu_x + \bar{\nu}_x$
Neutrino scattering	$\nu_x + \{\nu_e, \bar{\nu}_e\} \longleftrightarrow \nu_x + \{\nu_e, \bar{\nu}_e\}$
Scattering processes with medium particles	
Neutrino scattering with nuclei	$\nu + (A, Z) \longleftrightarrow \nu + (A, Z)$
Neutrino scattering with nucleons	$\nu + N \longleftrightarrow \nu + N$
Neutrino scattering with electrons and positrons	$\nu + e^\pm \longleftrightarrow \nu + e^\pm$

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neutrinosphere
(below ρ_0)

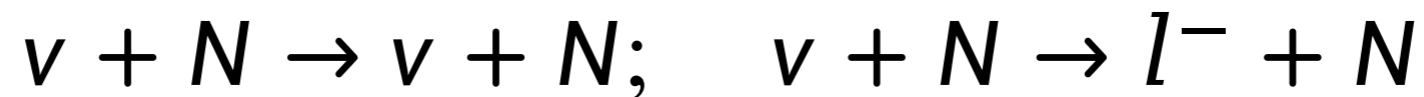
EoS (above ρ_0)

Neutrino-nucleon rates are largely affected by the nuclear medium effects:
mean field effects, nucleon correlation beyond mean field

hard to accurately compute

towards accurate description of neutrino-nucleon rates in CCSNe

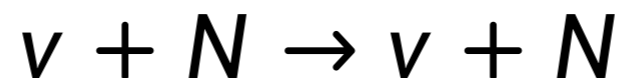
- Neutrino-nucleon scattering or absorption rates at mean field level



- Neutrino pair process from nucleon-nucleon bremsstrahlung



- Neutrino-nucleon scattering rates taking into nucleon-nucleon correlation



neutrino-nucleon reactions at mean field level

Each nucleon feels **an averaged potential** from all the other nucleons and nucleon in medium can be viewed as **quasiparticles** with **modified energy-momentum relation** as

$$E_{n,p} = \sqrt{m_{n,p}^{*2} + p^2} + U_{n,p} = E_{n,p}^* + U_{n,p} \quad \text{depending on EoS}$$

Reaction rate $\nu(K_1) + N(P_2) \rightarrow \nu(K_3) + N(P_4)$ or $\nu(K_1) + N(P_2) \rightarrow l(K_3) + N(P_4)$
from **Fermi's golden rule**:

$$\frac{d^2\Gamma}{d\omega d\cos\theta_{13}} = \frac{(1-f_3)k_3}{8\pi^2 E_1} \int \frac{d^3\mathbf{p}_2}{(2\pi)^3} \int \frac{d^3\mathbf{p}_4}{(2\pi)^3} \frac{|\mathcal{M}|^2}{4E_2^* E_4^*} \longrightarrow \text{amplitude squared}$$
$$\times (2\pi)^4 \delta^{(4)}(K_1 + P_2 - K_3 - P_4) f_2 (1 - f_4)$$

Hadronic current:

$$\bar{\psi}_4 \left\{ \underline{\gamma^\mu [G_V(q^2) - G_A(q^2)\gamma^5]} + \underline{\frac{iF_2(q^2)}{2M_N} \sigma^{\mu\nu} q_\nu^*} - \underline{\frac{G_P(q^2)}{M_N} \gamma^5 q^{*\mu}} \right\} \psi_2$$

Kinematics:

non-relativistic kinematics, elastic approximation, neglect final-state blocking, assume initial nucleons at rest, etc.

'full' treatments of neutrino–nucleon rates at mean field level are now available

Charged-current muonic reactions in core-collapse supernovae

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³*Institut für Kernphysik (Theoriezentrum), Technische Universität Darmstadt, Schlossgartenstraße 2, 64298 Darmstadt, Germany*

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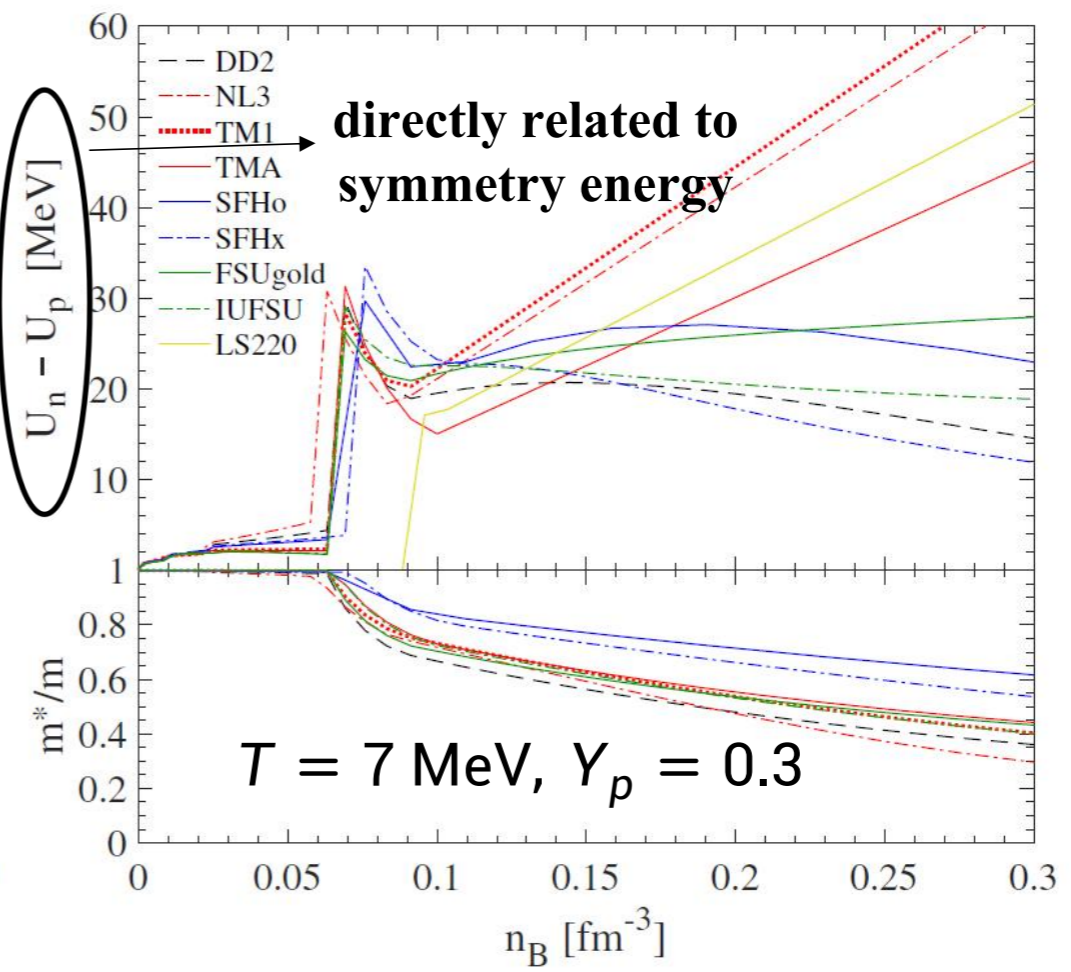
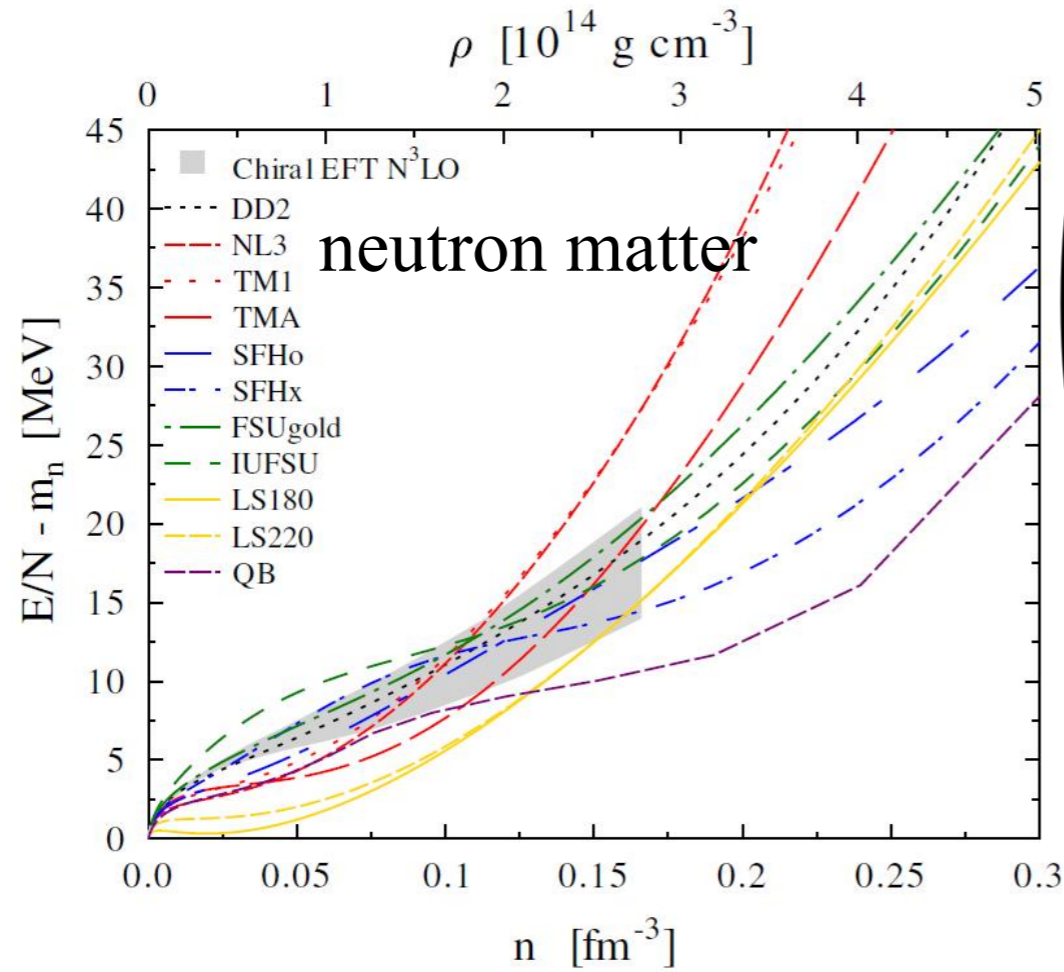


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The steady advance in core-collapse supernova simulations requires a more precise description of neutrino processes in hot and dense matter. In this work, we study the rates of charged-current (CC) weak processes with (anti)muons in supernova matter. At the relativistic mean field level, we derive results for the rates of CC neutrino-nucleon reactions, taking into account full kinematics, weak magnetism and pseudoscalar terms, and q^2 -dependent nucleon form factors in the hadronic current. In addition to muonic semileptonic processes we also consider purely leptonic processes. In particular, we show that inverse muon decay can dominate the opacities for low energy ν_μ and $\bar{\nu}_e$ at densities $\gtrsim 10^{13}$ g cm⁻³.

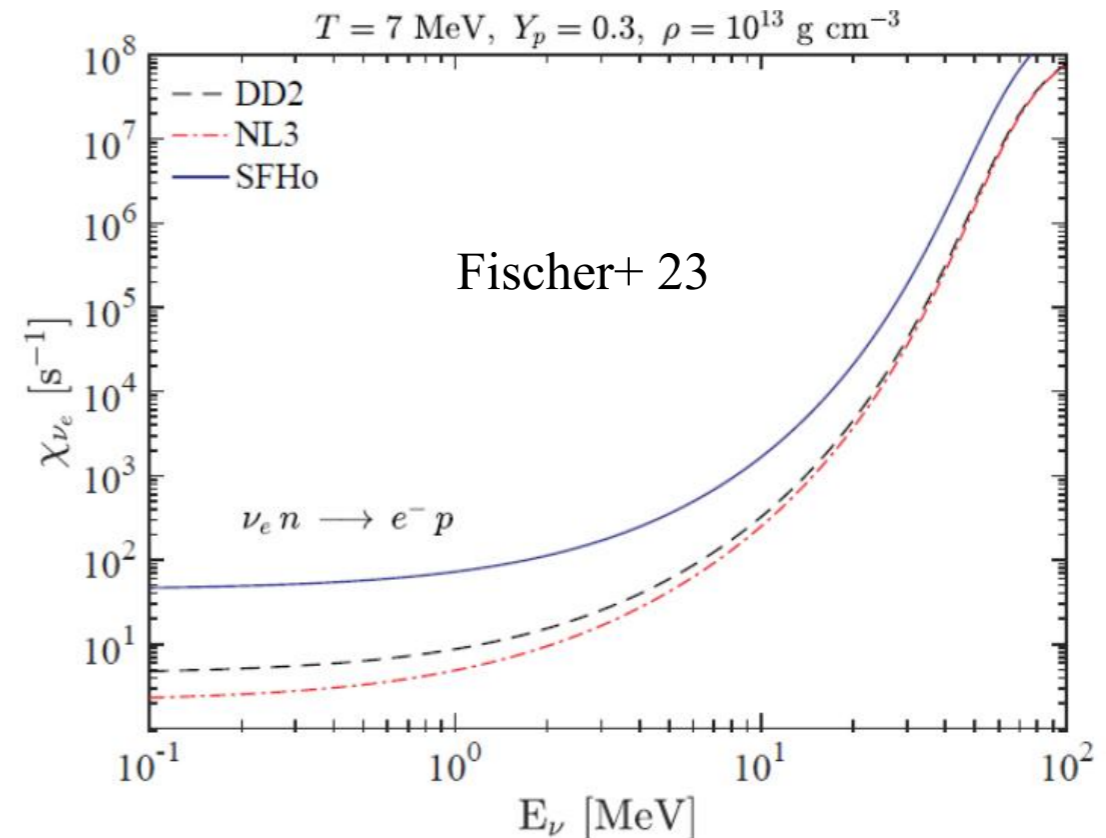
DOI: 10.1103/PhysRevD.102.023037

nontrivial dependence of neutrino rates on EoS



used EoS are mostly based on relativistic mean field theory

EoS (below ρ_0) taking into account cluster formation using χ EFT potential is highly required !!!



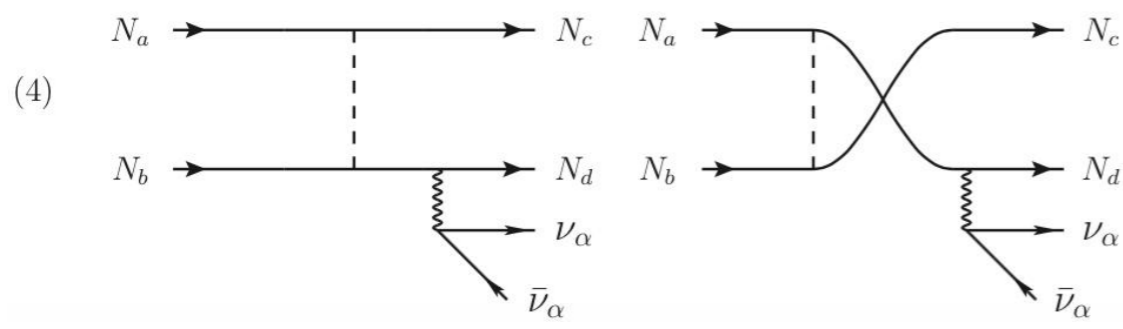
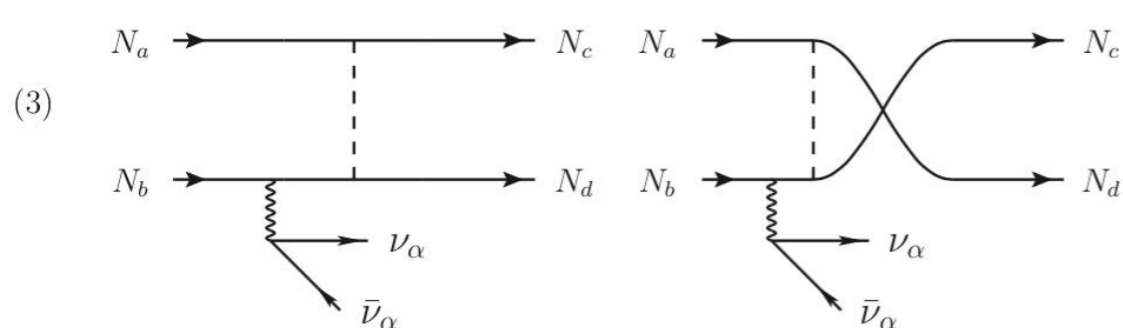
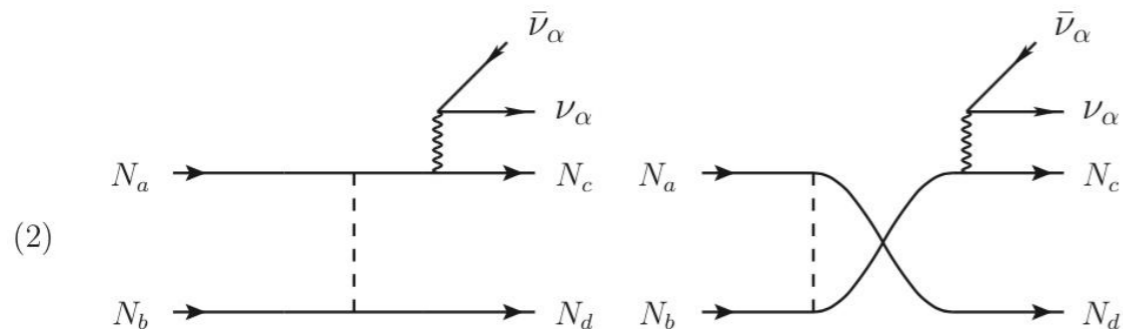
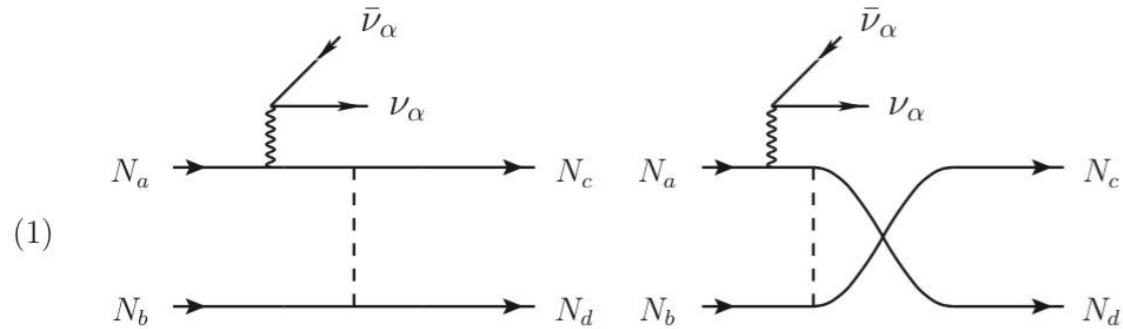
Neutrino pair process from n-n bremsstrahlung

pair emission

$$N_a + N_b \rightarrow N_c + N_d + \nu_\alpha + \bar{\nu}_\alpha$$

pair absorption

$$N_a + N_b + \nu_\alpha + \bar{\nu}_\alpha \rightarrow N_c + N_d$$



direct exchanged

$$\mathcal{M}^{(1)} = \frac{G_F g_A^a}{2\sqrt{2}} \langle cd | V \sigma_i^{(a)} | ab \rangle \frac{l_i}{\omega}$$

$$\mathcal{M}^{(2)} = -\frac{G_F g_A^c}{2\sqrt{2}} \langle cd | \sigma_i^{(c)} V | ab \rangle \frac{l_i}{\omega},$$

$$\mathcal{M}^{(3)} = \frac{G_F g_A^b}{2\sqrt{2}} \langle cd | V \sigma_i^{(b)} | ab \rangle \frac{l_i}{\omega},$$

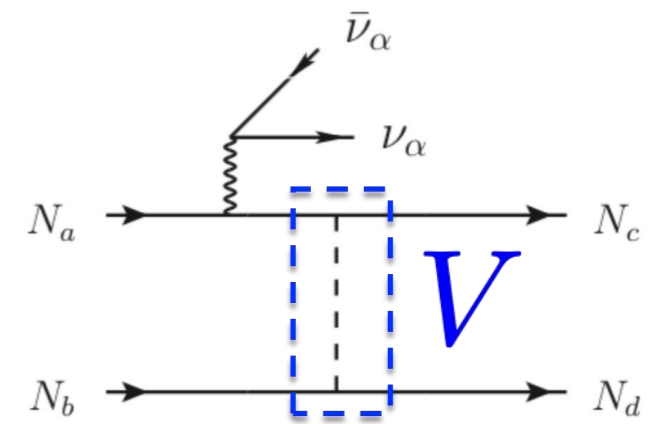
$$\mathcal{M}^{(4)} = -\frac{G_F g_A^d}{2\sqrt{2}} \langle cd | \sigma_i^{(d)} V | ab \rangle \frac{l_i}{\omega},$$

$$\mathcal{M}_{\text{tot}} = \sum_{j=1}^4 \mathcal{M}^{(j)} = \frac{G_F g_A}{2\sqrt{2}} \langle cd | [V, \sum_{r=1,2} \sigma_i^{(r)} \tau_z^{(r)}] | ab \rangle \frac{l_i}{\omega},$$

$$g_A^p = -g_A^n = g_A \simeq 1.27$$

- ◆ only axial-vector term contributes in non-relativistic limit
- ◆ ignoring many-body interactions
- ◆ what to take for V ?

nucleon-nucleon potentials for



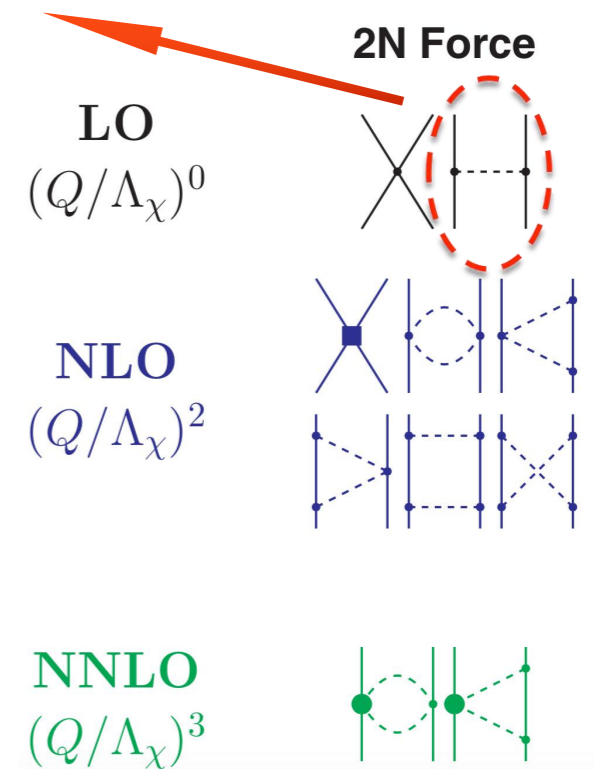
V could be (at Born level):

One-Pion Exchange (OPE) potential (Hannestad & Raffelt 1998)

$$V_{\text{OPE}}(\vec{p}', \vec{p}) = -\frac{g_A^2}{4f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}$$

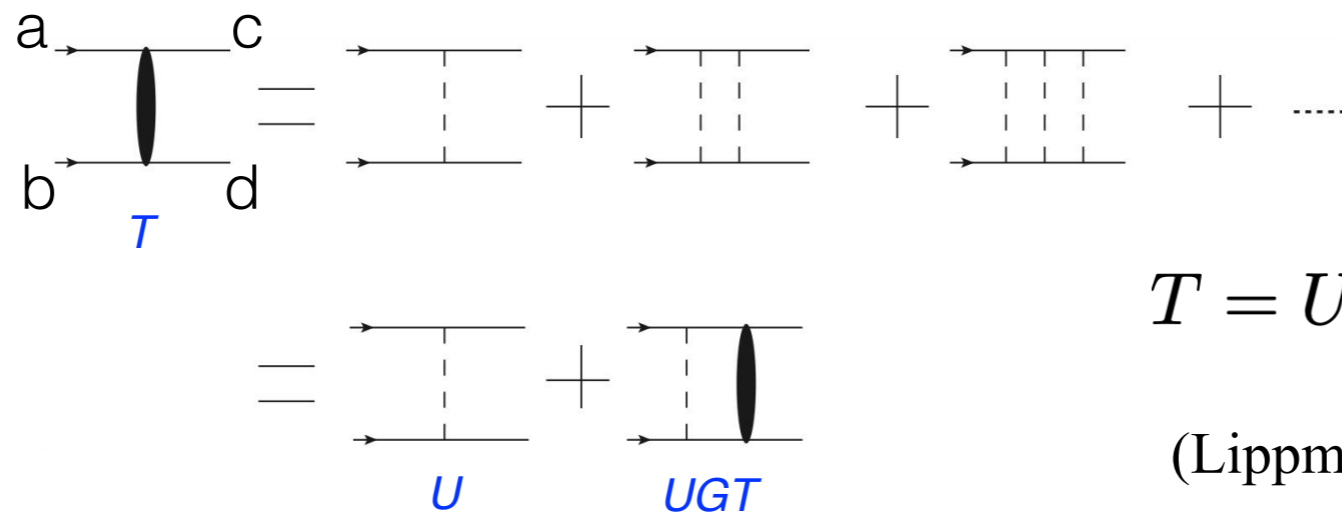
Chiral effective field theory potential

- ◆ hamiltonian built from spontaneously broken chiral symmetry;
- ◆ an expansion in terms of Q/Λ_χ , coefficients fitted to scattering data, reach desired accuracy



D. R. Entem *et al.* 2017

T-matrix formalism

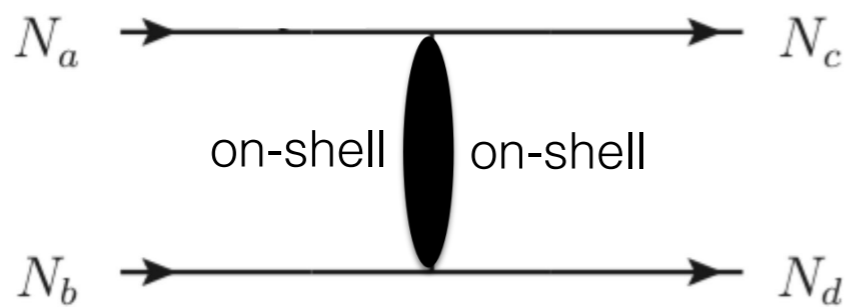


$$T = U + U \frac{1}{E - H_0 + i\epsilon} T$$

(Lippmann-Schwinger Equation)

scattering amplitude is proportional to the T-matrix

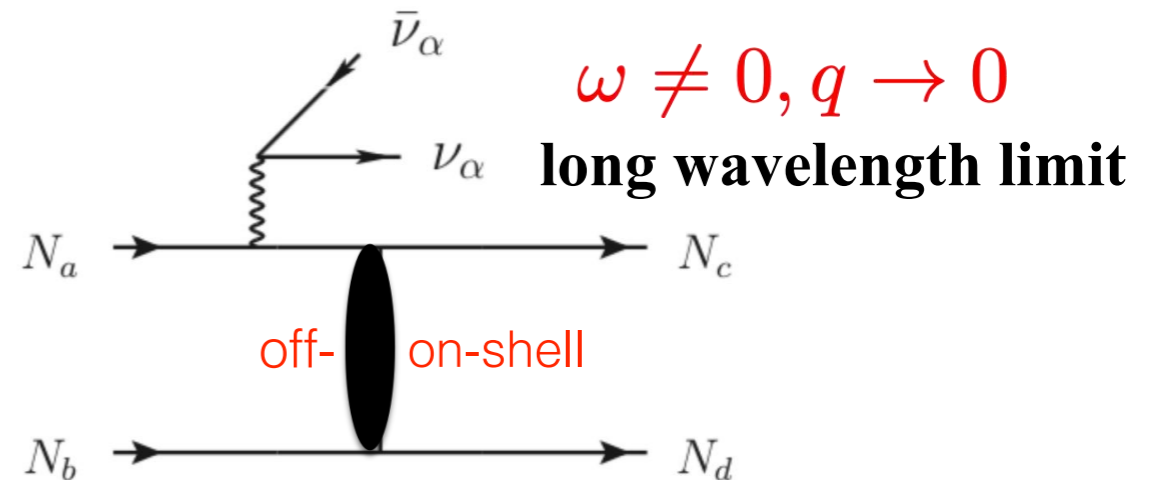
nucleon-nucleon scattering



$$E = k^2/m_N = k'^2/m_N$$

on-shell T-matrix (by Bartl+14)

nucleon-nucleon bremsstrahlung



$$\omega \neq 0, q \rightarrow 0$$

long wavelength limit

$$\frac{k^2}{m_N} - \omega = E = \frac{k'^2}{m_N}$$

half-off-shell T-matrix (by Guo+19)

T -matrix elements in partial wave basis

$$\mathcal{T}_{LL'}^{JST}(k', k; E) = U_{LL'}^{JST}(k', k) + \sum_{L''} \int \frac{k''^2 dk''}{(2\pi)^3} U_{LL''}^{JST}(k', k'') \frac{1}{E - \frac{k''^2}{m_N} + i\epsilon} \mathcal{T}_{LL''}^{JST}(k'', k; E),$$

discretization of k , then

$$A_{ij} T_{jk} = U_{ik} \xrightarrow{\text{matrix inversion}} T = A^{-1} U$$

Bethe-Goldstone equation (in-medium version of LS)

$$\mathcal{T}_{LL'}^{JST}(k', k; K, \Omega) = V_{LL'}^{JST}(k', k) + \sum_{L''} \int \frac{k''^2 dk''}{(2\pi)^3} V_{LL''}^{JST}(k', k'') \bar{g}_{II}(K, \Omega, k'') \mathcal{T}_{L''L}^{JST}(k'', k; K, \Omega)$$

in-medium T -matrix

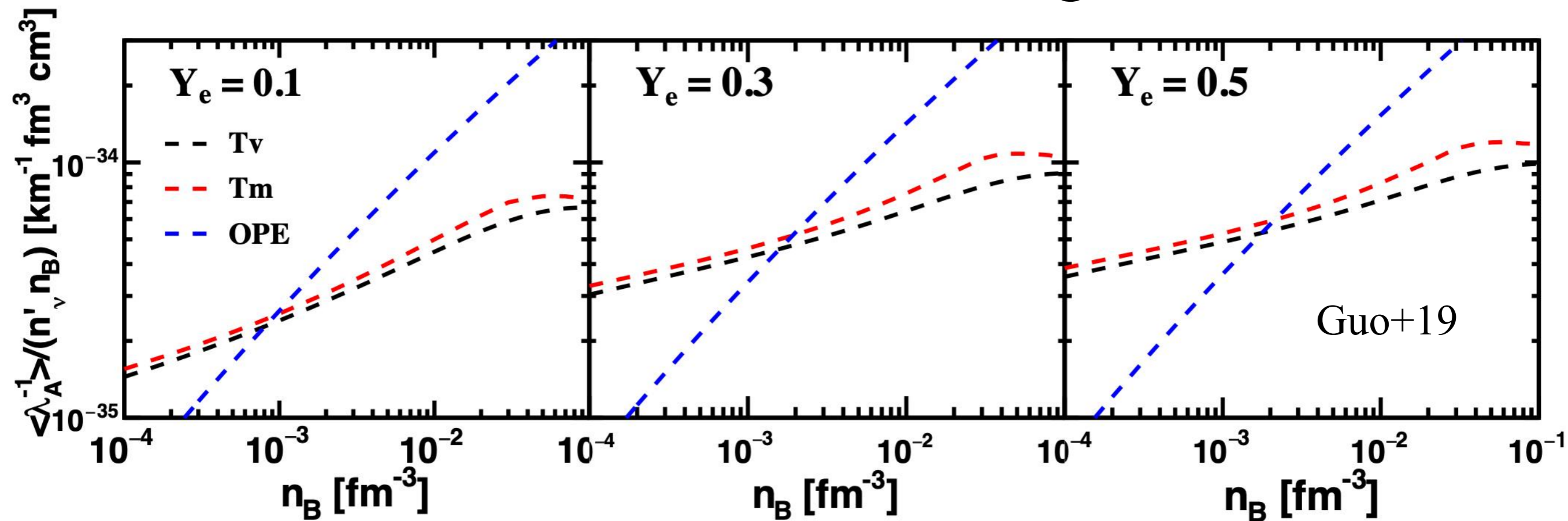
$$\bar{g}_{II}(K, \Omega, k) = \left\langle \frac{1 - f(\varepsilon(k_1)) - f(\varepsilon(k_2))}{\Omega - \varepsilon(k_1) - \varepsilon(k_2) + i\eta} \right\rangle_{\theta}$$

Including the blocking for the intermediate two-nucleon states

Take U to be the chiral potential of [D. R. Entem et al. 2017, N4LO](#)

half-off-shell vacuum/in-medium T -matrix is computed

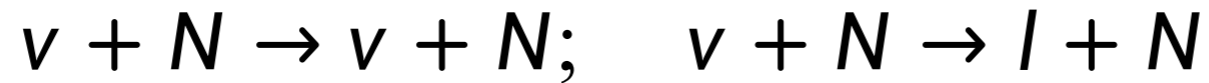
Born level is not enough



For T -matrix studies:

- ◆ enhanced rates at low density due to resonant nuclear force at low energy (**Bartl *et al.* 2014**)
- ◆ suppressed rates at high density due to repulsive short-range forces & non-perturbative effects
- ◆ medium effects on T -matrix (in-medium T -matrix) enhance rates by $\sim 10\%$; RPA correlation effect is typically minor

- Neutrino-nucleon scattering or absorption rates at mean field level



- Neutrino pair process from nucleon-nucleon bremsstrahlung



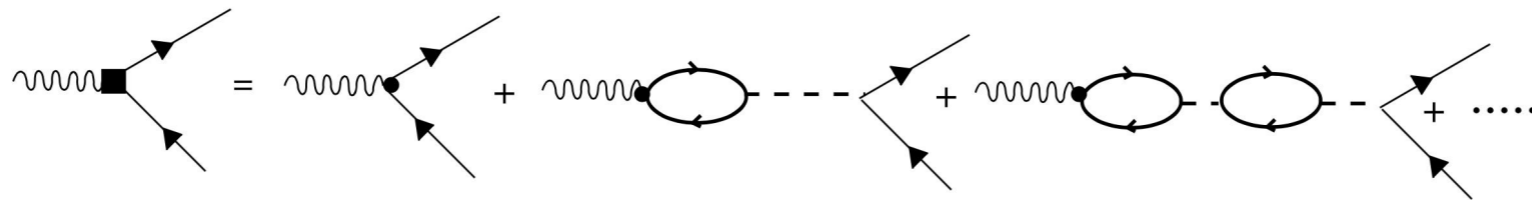
- **Neutrino-nucleon scattering rates considering nucleon-nucleon correlation**



one of the keys to robust supernova explosion in simulation

Non-perturbative problem, no perfect way, approx. needed:

- Random Phase Approx. (RPA) (e.g., Burrows & Sawyer 98, 99)



- Virial expansion at low density (Horowitz+06, 17; Bedaque+18)
- Lattice Formulation (Alexandru et al. 20, 21; Ma et al. 24 with N3LO)
- Molecular dynamics simulation (Horowitz et al. 04, Li et al. 17)

neutrino-nucleon scattering rates from EoS calculations

Neutrino response (used to calculate the neutrino-nucleon scattering rate) is exactly given by the derivative

$$S_v(q=0) = \frac{z}{n} \left(\frac{\partial n}{\partial z} \right)_T = \frac{1 + 4b_n z}{1 + 2b_n z} \quad \begin{array}{l} z = e^{\mu/T} : \text{the fugacity parameter} \\ b_n : \text{2nd virial coefficients} \end{array}$$

virial expansion at low densities ($z < 0.5$)

Horowitz et al. 06, 17; Lin et al 17

Limitations:

- Virial calculation of EoS only works at low density regions
- From the thermodynamic derivatives, one can only obtain the so-called **static neutrino response** in the **long wavelength limit** (i.e., neutrino-nucleon scattering with zero momentum & energy transfer). See also Bedaque+18

Brueckner-Hartree-Fock calculation of EoS

$$G_{ll'}^{JSS_z}(k', k; P, \Omega) = V_{ll'}^{JS}(k', k) + \sum_{l''} \frac{k''^2 dk''}{(2\pi)^3} V_{ll''}^{JS}(k', k'') \times \bar{g}_{ll}^{S_z}(P, \Omega, k'') G_{l''l'}^{JSS_z}(k'', k),$$

G-matrix calculated based on chiral EFT potential

Neutron self-energy
(**pure neutron matter**)

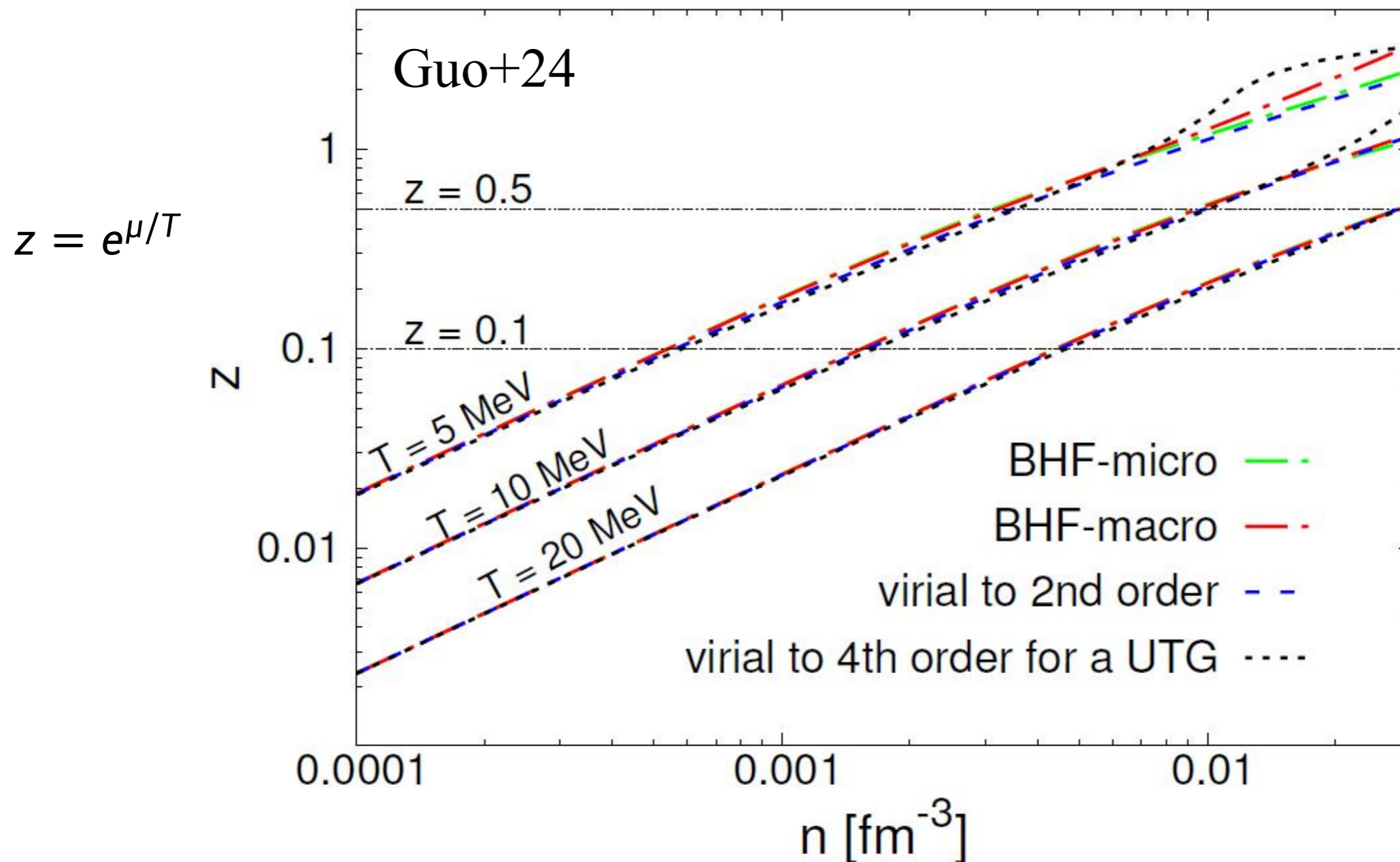
$$\Sigma_\sigma(p, \varepsilon_\sigma(p)) = \frac{1}{2\pi} \sum_{\sigma'=-\frac{1}{2}}^{\frac{1}{2}} \int \frac{d^3 p'}{(2\pi)^3} \sum_{Jl'l'SS_z}^{S+l \text{ is even}} \left(C_{\frac{1}{2}\sigma \frac{1}{2}\sigma'}^{SS_z} \right)^2 \times \sum_{Lm} (-1)^m C_{lmSS_z}^{J(m+S_z)} C_{l'mSS_z}^{J(m+S_z)} C_{lml'(-m)}^{L0} C_{l0l'0}^{L0} [ll'] P_L(\hat{\mathbf{k}}) \times f_{\sigma'}(p') G_{ll'}^{JSS_z}(k, k; P, \Omega = \varepsilon_\sigma(p) + \varepsilon_{\sigma'}(p')),$$

Thermodynamic quantities

$$f = \epsilon - Ts,$$

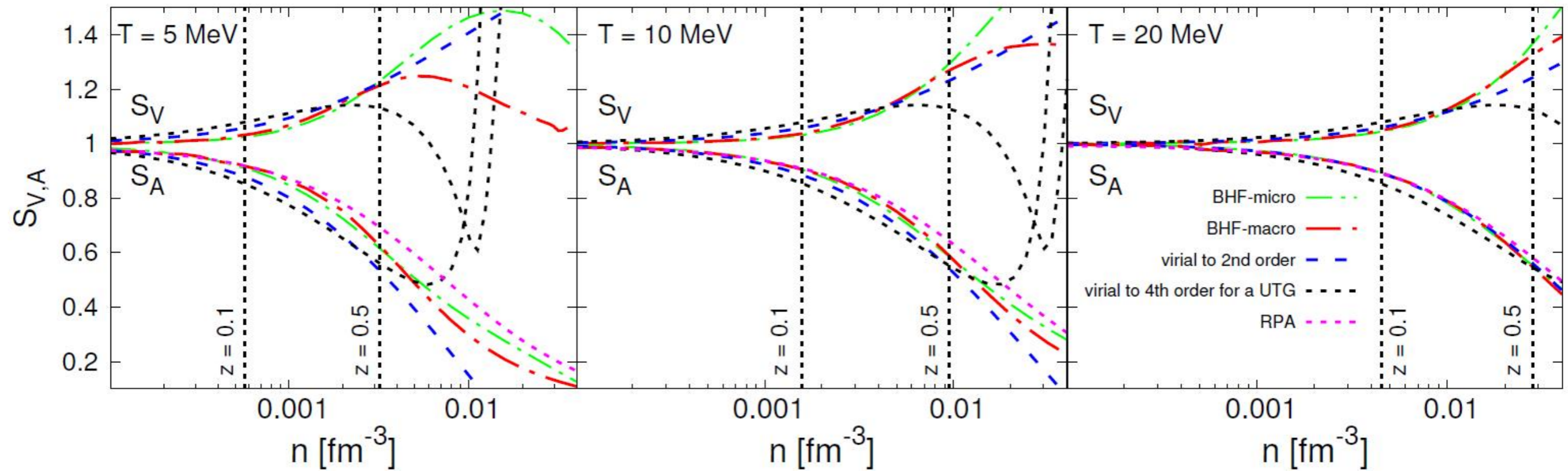
$$\epsilon = \sum_\sigma \int \frac{d^3 p}{(2\pi)^3} \left[\frac{p^2}{2m_n} + \frac{1}{2} \text{Re} \Sigma_\sigma(p, \varepsilon_\sigma(p)) \right] f_\sigma(p),$$

$$s = - \sum_\sigma \int \frac{d^3 p}{(2\pi)^3} \left[f_\sigma(p) \ln(f_\sigma(p)) + (1 - f_\sigma(p)) \ln(1 - f_\sigma(p)) \right] \quad \tilde{\mu} = T \ln(\tilde{z}) = \left(\frac{\partial f}{\partial n} \right)_T,$$



BHF reproduces the virial results at low densities

static responses from virial expansion & BHF



S_V : neutrinos couple to nucleon density

S_A : neutrinos couple to nucleon spin density

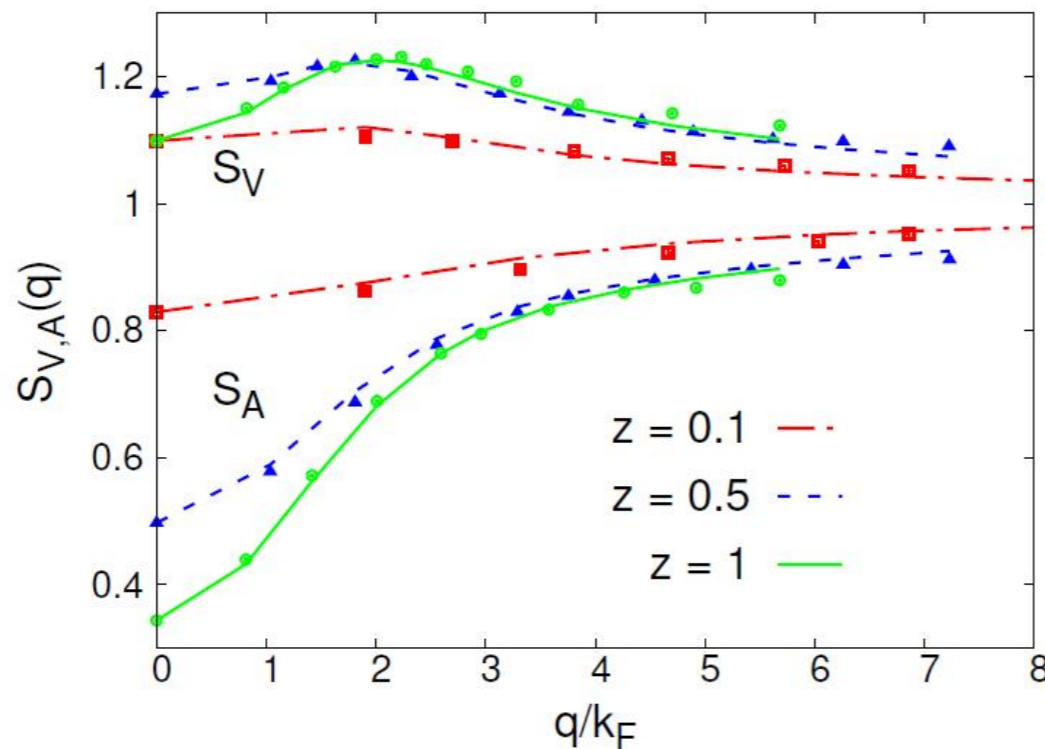
Guo+24

Virial results are ill-behaved at high densities ($z \gtrsim 0.5$), while the BHF calculations are more reliable

from static responses to dynamic ones

To recover the momentum transfer-dependence

SN neutrinosphere ~ dilute neutron gas ~ a unitary gas



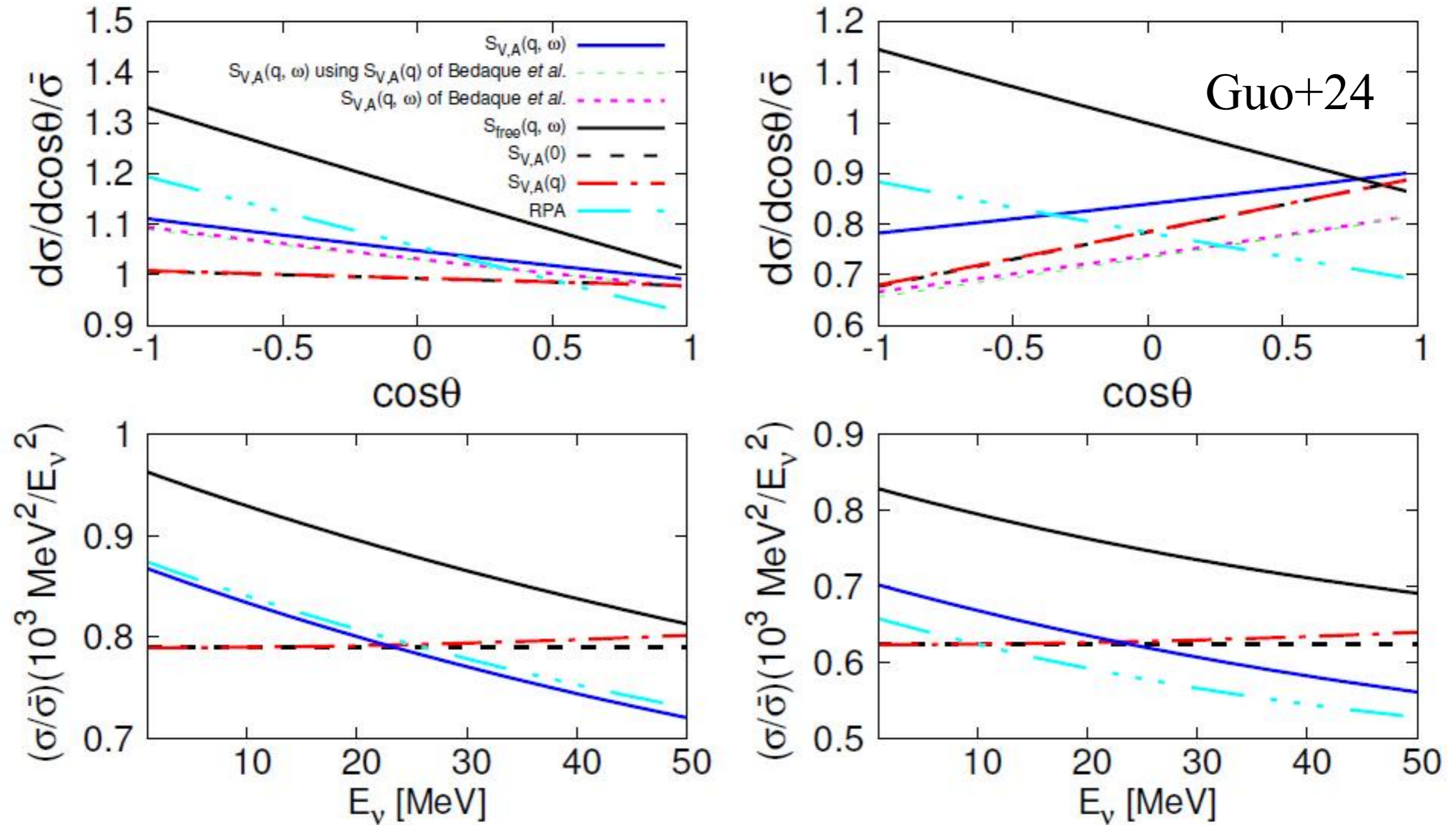
Lattice calculation for a unitary gas at SN conditions (Alexandru+2020)

To recover the energy transfer-dependence using sum rules

$$S_{V,A}(q, \omega) = A_{V,A} e^{-\frac{(\omega - \omega_q)^2}{4\omega_q T}} + B_{V,A} e^{-\frac{(\omega - \frac{1}{2}\omega_q)^2}{2\omega_q T}},$$

$$\begin{aligned} & \int_{-\infty}^{+\infty} d\omega \omega S_{V,A}(q, \omega) & \int_{-\infty}^{+\infty} d\omega S_{V,A}(q, \omega) \\ = & \int_0^{+\infty} d\omega (1 + e^{-\omega/T}) \omega S_{V,A}(q, \omega) = \frac{q^2}{2m_n}, & = \int_0^{+\infty} d\omega (1 + e^{-\omega/T}) S_{V,A}(q, \omega) = S_{V,A}(q). \end{aligned}$$

Neutrino-nucleon rates with static & dynamic responses



Correlation reduces the scattering rates by 10%-15%;
 Dynamic response enhances/suppresses the rates for low/high-energy neutrinos.

Summary

- Full weak interaction terms of nucleons at mean field level and relativistic kinematics (Robert+17; Fischer+20; Guo+20)

$$j^\mu = \bar{\psi}_4 \left\{ \gamma^\mu [G_V(q^2) - G_A(q^2)\gamma^5] + \frac{iF_2(q^2)}{2M_N} \sigma^{\mu\nu} q_\nu^* - \frac{G_P(q^2)}{M_N} \gamma^5 q^{*\mu} \right\} \psi_2$$

EoS of nuclear matter considering light cluster at subsaturation densities based on χ EFT potential is highly desired !!!

- Neutrino-nucleon bremsstrahlung revisited with full T-matrix using χ EFT potential (Guo & Martinez-Pinedo 19); **3-body force, mean field effects**

- Many progresses on nucleon correlation effects, especially the ab initio studies using χ EFT potential

Guo et al. 24: neutrino rates from BHF calculation of EoS with χ EFT potential (N4LO)

Duan & Urban 23: the 1st mean field (HF level) + RPA study using χ EFT potential (N3LO-414)

Ma et al. 24: the 1st ab initio lattice calculations in hot neutron matter using χ EFT potential (N3LO-414)

Thanks

backup

Adopt typical conditions in SN

$$T_{SN}(\rho) = 3 \text{ MeV} \left(\frac{\rho}{10^{11} \text{ g cm}^{-3}} \right)^{1/3}$$

$$\rho : 1.7 \times 10^{11-14} \text{ g/cm}^3$$

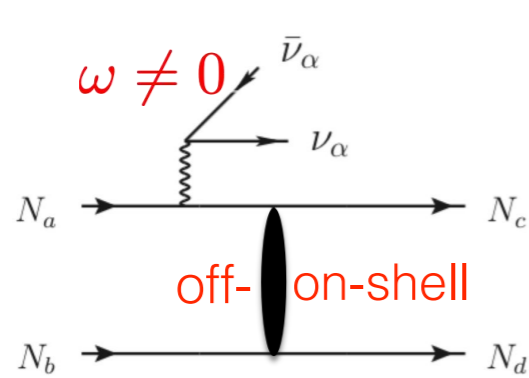
$$n_B : 10^{-4} - 10^{-1} \text{ fm}^{-3}$$

$$Y_e = \frac{n_p}{n_B} = \frac{n_p}{n_n + n_p} < 0.5$$

Opacity due to pair absorption $N + N + \nu + \bar{\nu} \rightarrow N + N$

$$\frac{\langle \lambda_A^{-1} \rangle}{n'_\nu} \equiv \frac{1}{n'_\nu} \frac{\int \lambda_A^{-1}(E_\nu) f(E_\nu) E_\nu^2 dE_\nu}{\int f(E_\nu) E_\nu^2 dE_\nu}$$

using effective on-shell T -matrix is not enough

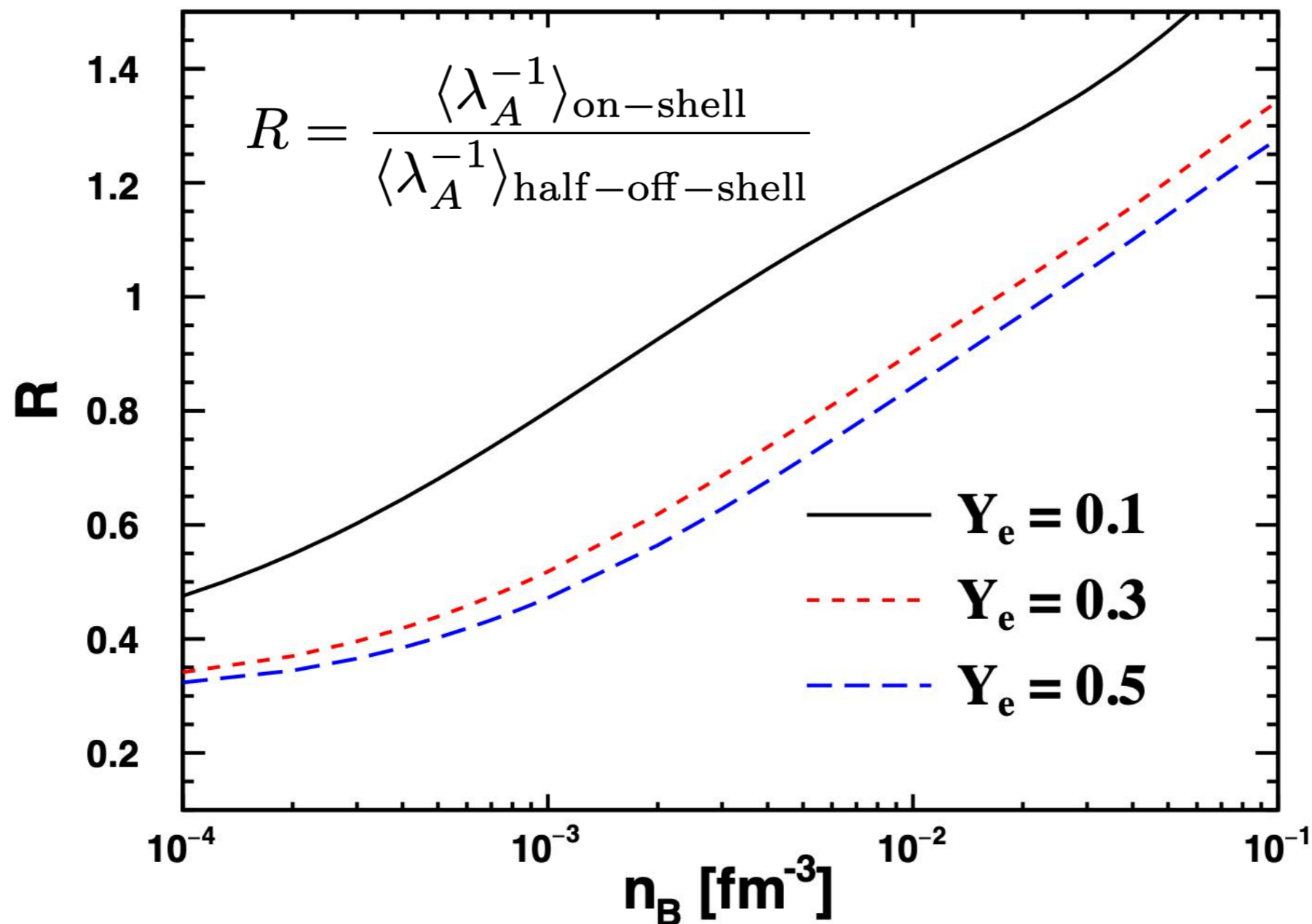


$$\langle k_f | \mathcal{T} | k_i \rangle \longrightarrow \langle \bar{k} | \mathcal{T} | \bar{k} \rangle, \quad \bar{k} = \sqrt{\frac{1}{2}(k_i^2 + k_f^2)}$$

half-off-shell 'on-shell'

Bartl *et al.* 2014

(can be simply extracted from NN scattering data)



virial expansion at low densities

SN neutrinosphere: $T \sim 5\text{-}10$ MeV; density $\sim 1\text{E}12\text{-}1\text{E}13$ g/cm³
neutron rich (with proton fraction $< 10\%$)

SN neutrinosphere \sim dilute neutron gas \sim a unitary gas

(with a very large scattering length &
small interaction range)

virial expansion of the EoS for the dilute neutron gas

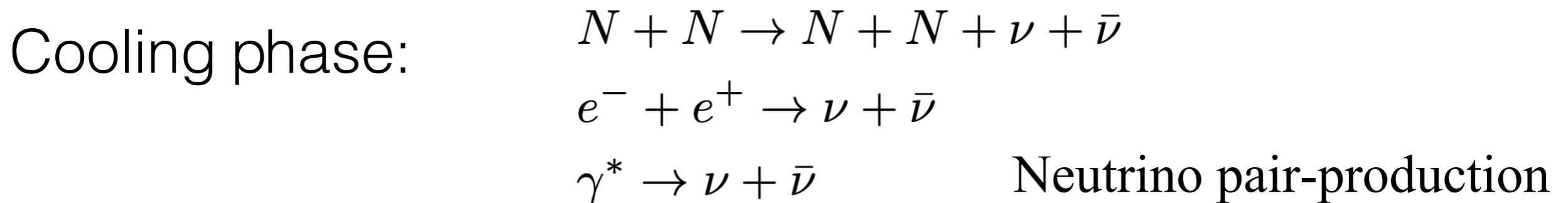
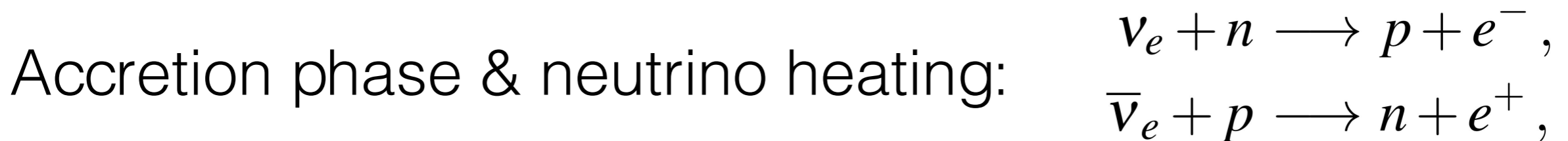
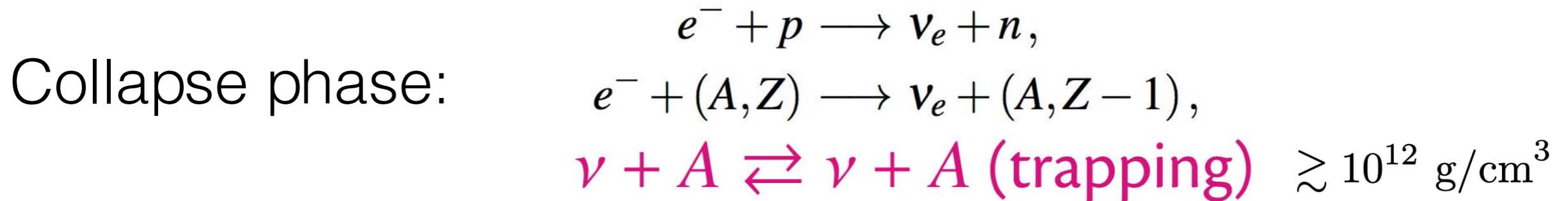
$$P = \frac{2T}{\lambda^3} (z + z^2 b_n + z^3 b_n^{(3)} + \mathcal{O}(z^4))$$

$$n = \frac{2}{\lambda^3} (z + 2z^2 b_n + 3z^3 b_n^{(3)} + \mathcal{O}(z^4))$$

$z = e^{\mu/T}$: the fugacity parameter, with μ the nucleon chemical potential;
 b_n : the 2nd virial coefficients, related to neutron scattering phase shifts.

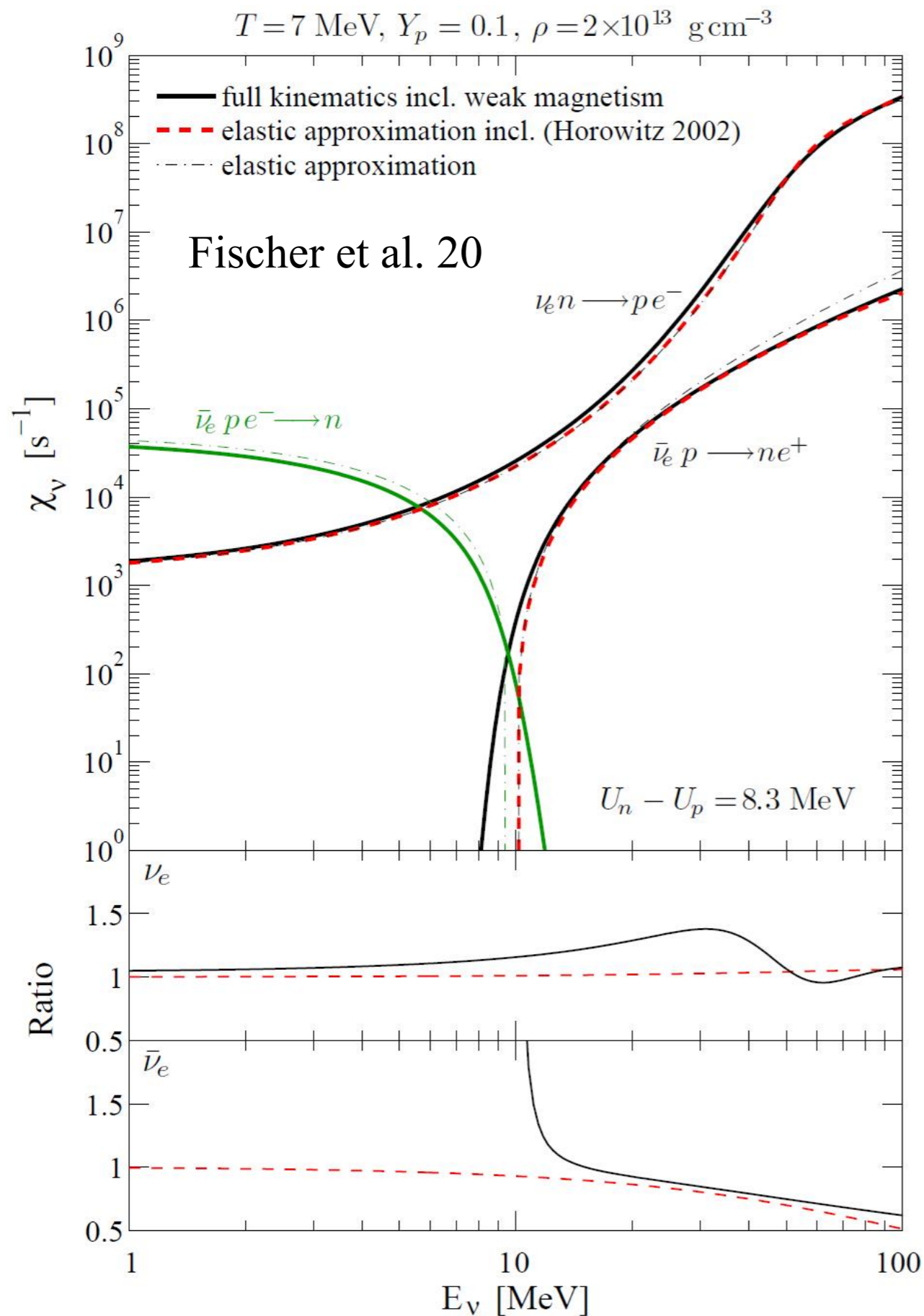
virial results are reliable & model-independent

Complicated Neutrinos Processes in CCSNe



High-Dimension Detailed Neutrino Transport is Required

very time-consuming!

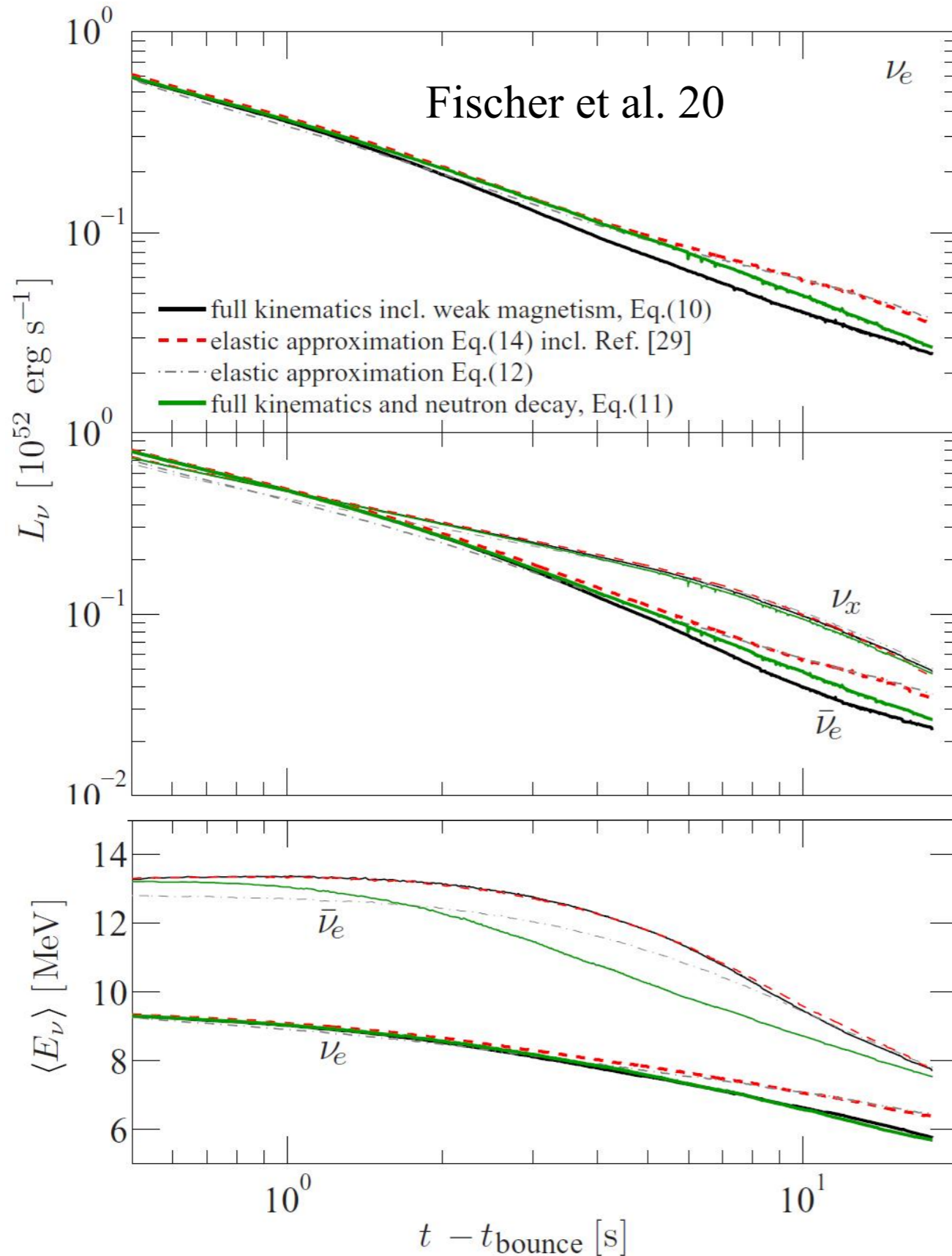


charged-current opacities for $\nu_e (\bar{\nu}_e)$

weak magnetism + full
kinematics **enhances**
opacities for $\nu_e/\bar{\nu}_e$ at low &
intermediate energies

inverse neutron decay
contributes significantly to
 $\bar{\nu}_e$ opacity at low energies

effects of CC ν_e rates on SN neutrino signals



larger opacity



lower luminosity

larger (inelastic) opacity

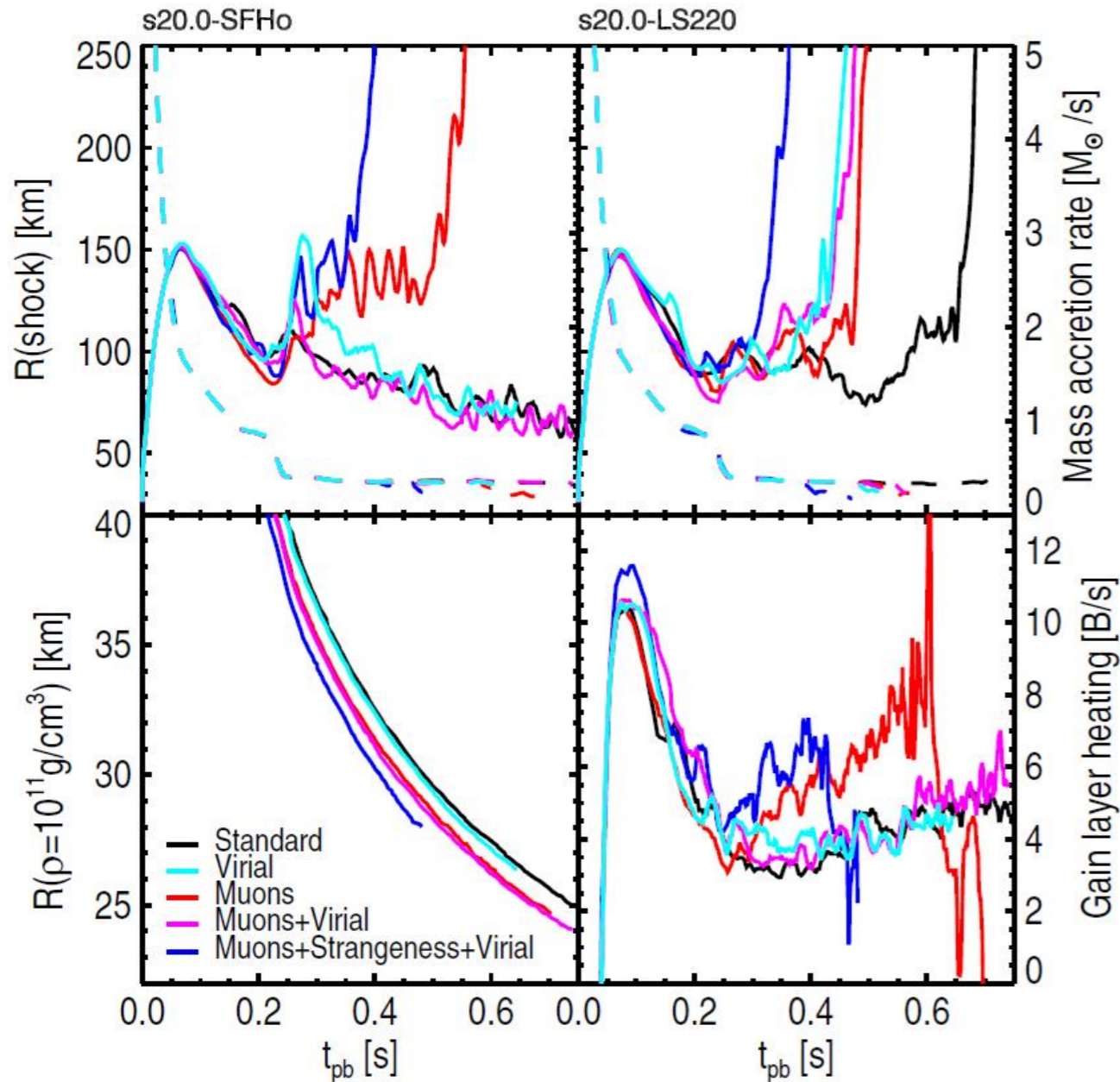


decouple at lower density region with lower temperatures

inclusion of inverse neutron decay

- larger $\bar{\nu}_e$ opacity \rightarrow lower E_ν ;
- less spectral difference between ν_e and $\bar{\nu}_e \rightarrow$ higher $Y_e \sim 0.5$ & lower neutron abundance
- \rightarrow lower scattering opacity on neutron
- \rightarrow higher luminosity for both ν_e and $\bar{\nu}_e$

muons facilitates supernova explosion



large muon rest mass, CC for muonic rates were previously ignored



creation of μ^{-} and μ^{+} softens the EoS by conversion of thermal and degeneracy energy of e^{-} into rest-mass energy of μ^{\pm}

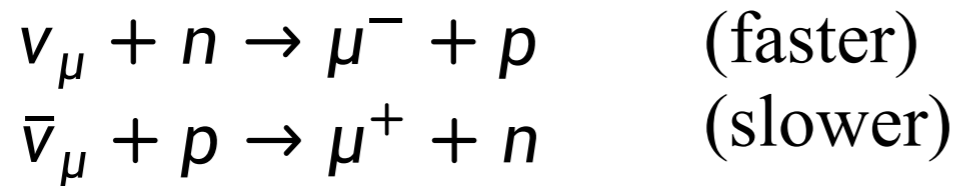
neutrino luminosities and average energy increase

enhanced neutrino heating & easier explosion

Bollig et al. 17

muonization and neutrino rates

muon-neutrinos are pair produced
around core bounce

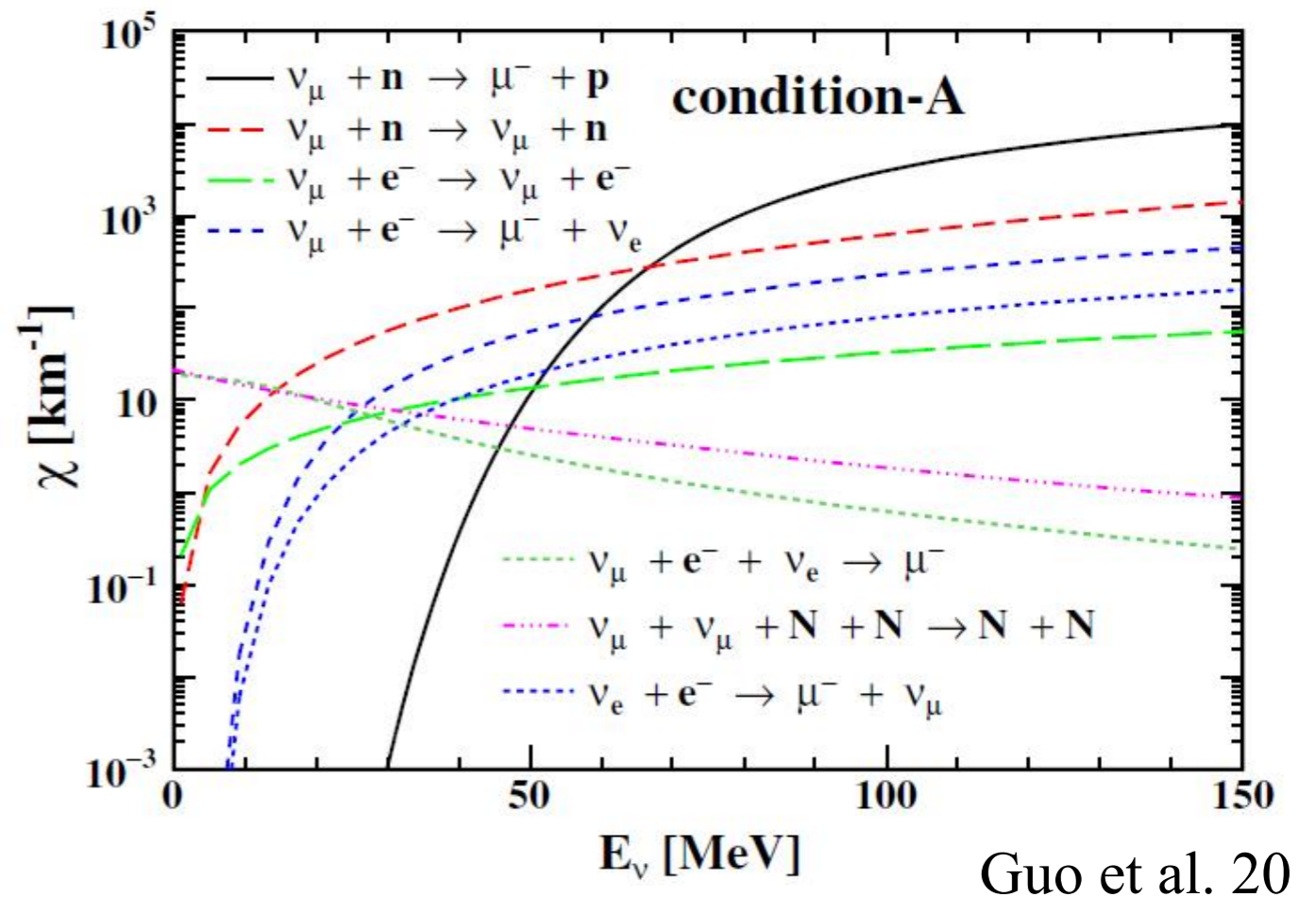
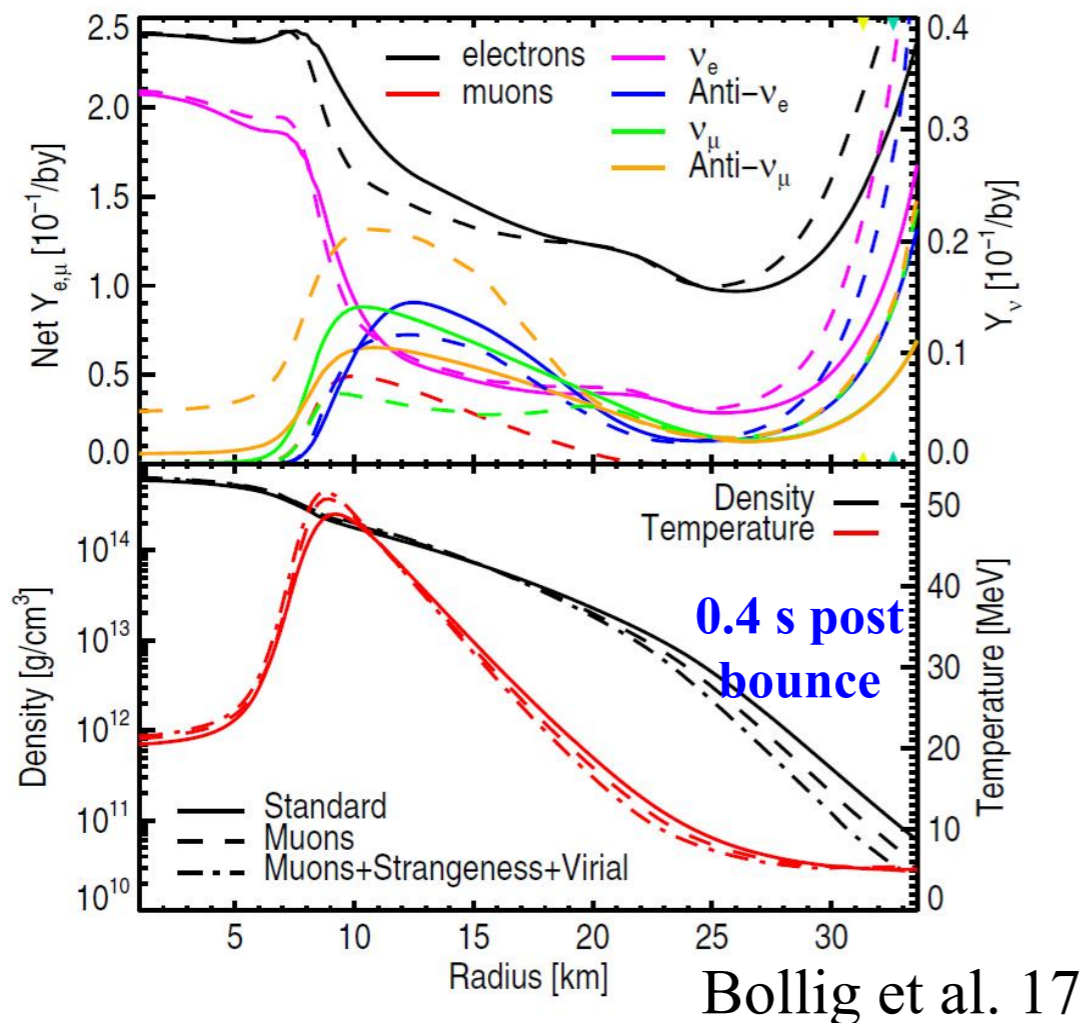


$\bar{\nu}_\mu$ escape more easily with lower opacities, which also aids muonization
(an excess of μ^- over μ^+)

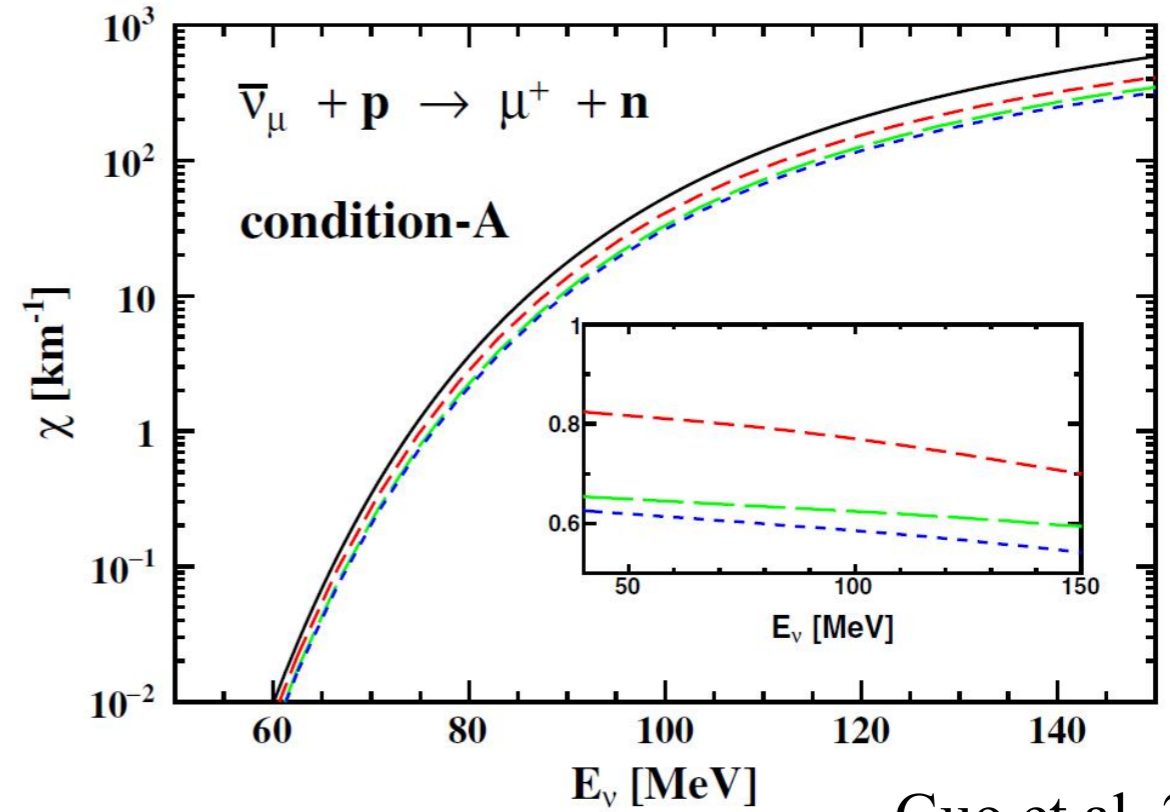
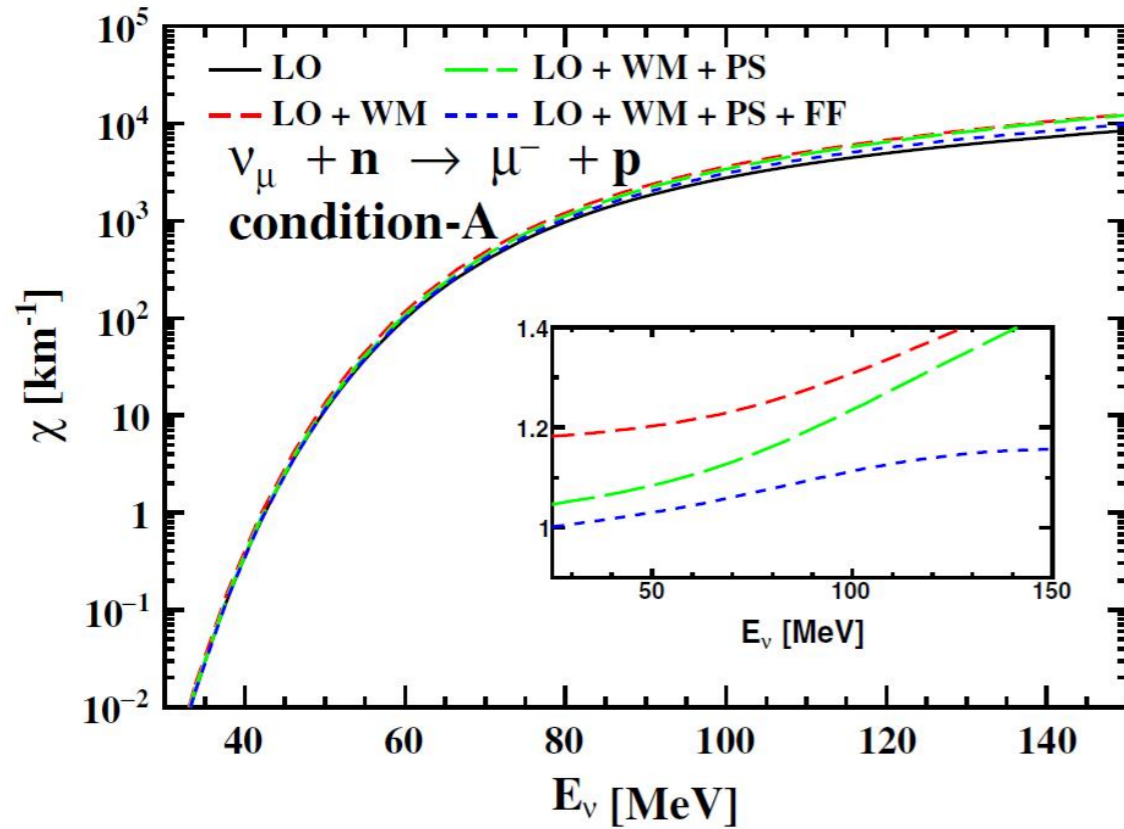
reactions involving muons



CC $\nu_\mu(\bar{\nu}_\mu)$ -nucleon dominates muonization



accurate calculation of CC muonic rates with nucleons at mean field level

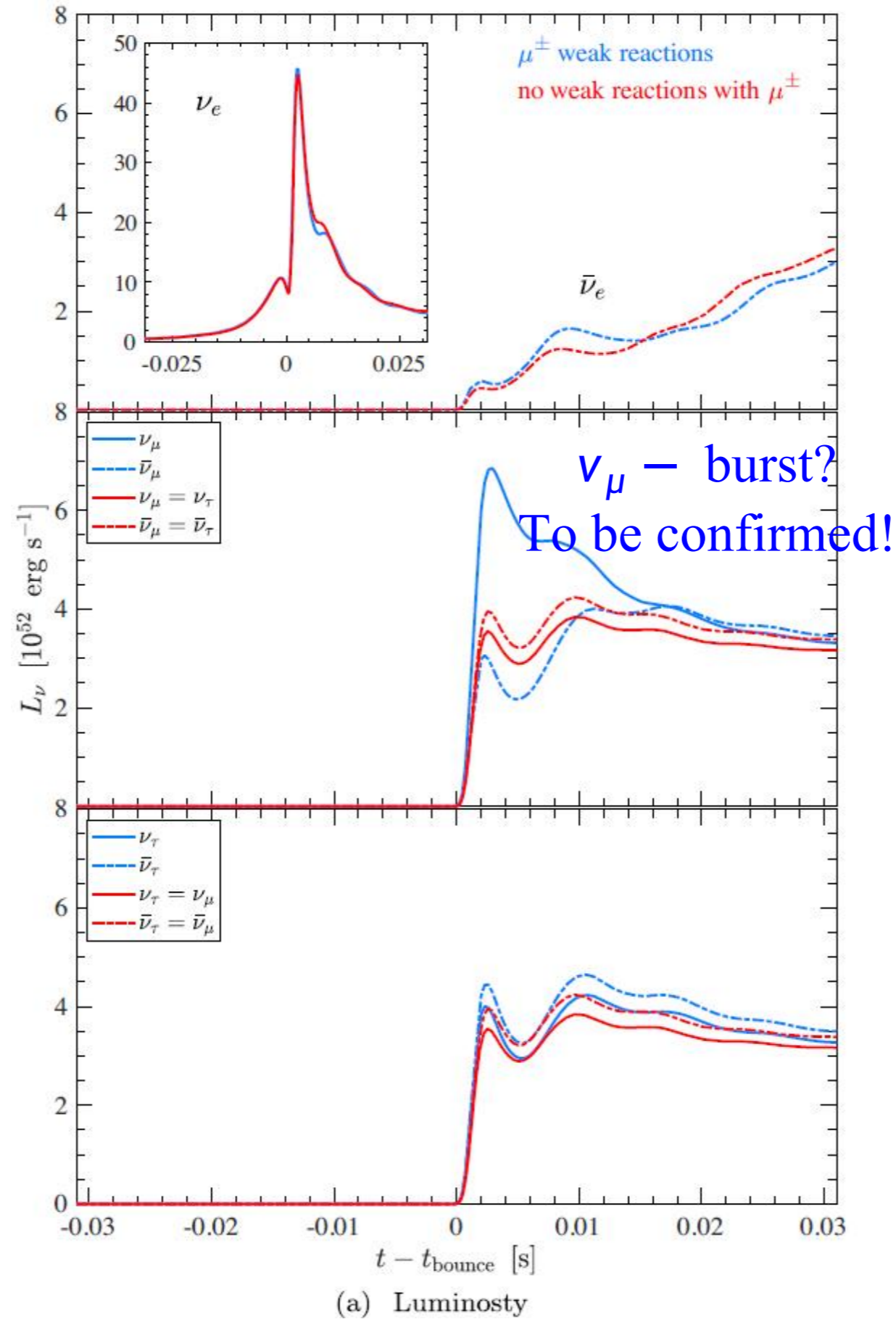


Guo et al. 20

$$\bar{\psi}_4 \left\{ \gamma^{\mu} [G_V(q^2) - G_A(q^2)\gamma^5] + \frac{iF_2(q^2)}{2M_N} \sigma^{\mu\nu} q_{\nu}^* - \frac{G_P(q^2)}{M_N} \gamma^5 q^{*\mu} \right\} \psi_2$$

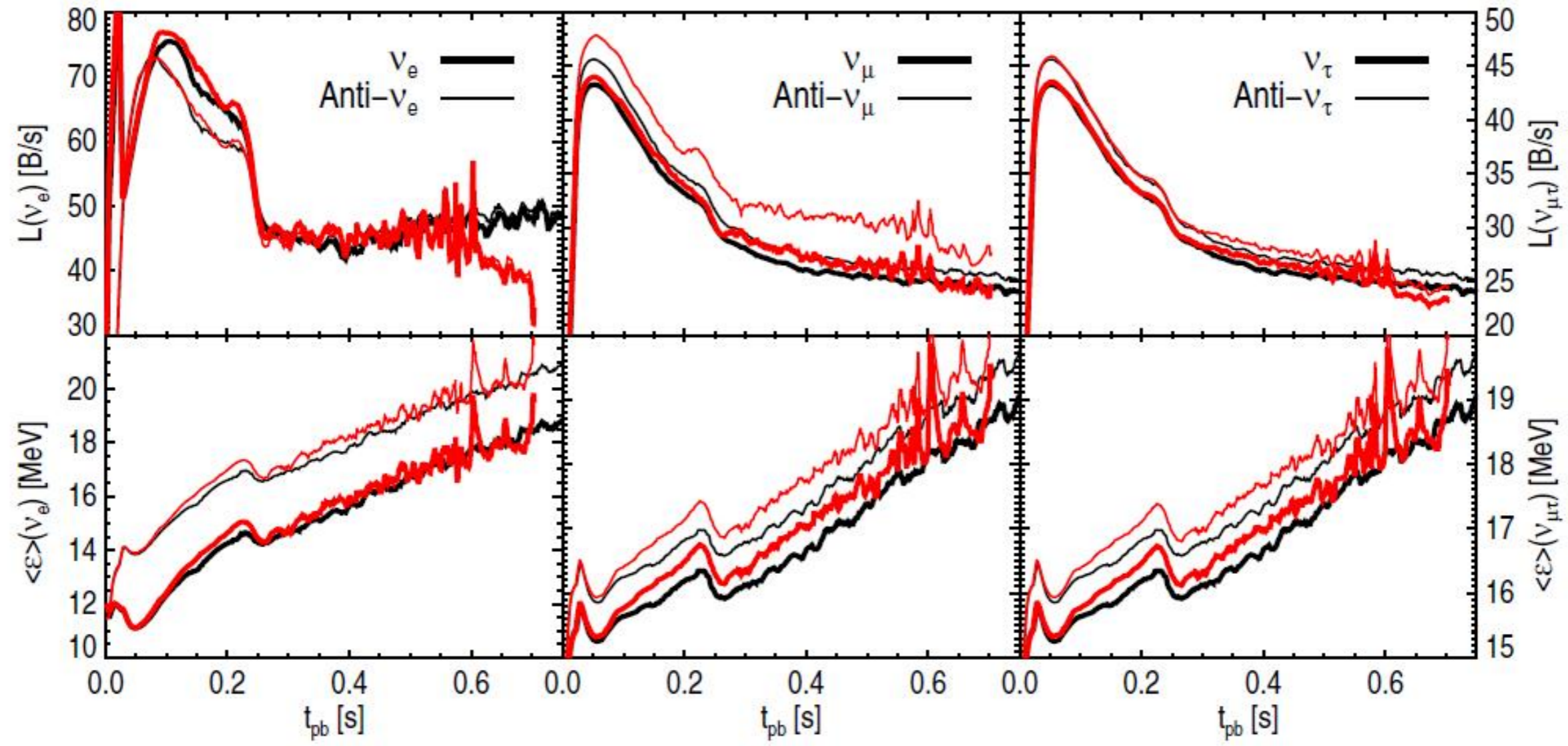
- pseudoscalar term and form factor dependence matter for muonic reaction;
- For ν_{μ} , the enhancement due to weak magnetism is largely cancelled;
- The rates for $\bar{\nu}_{\mu}$ are highly suppressed as all corrections reduce the rate;
- The new muonic rates to be tested in simulations.

effects on neutrino signals



Fischer et al. 20

muons rates on neutrino signals



Bollig+17

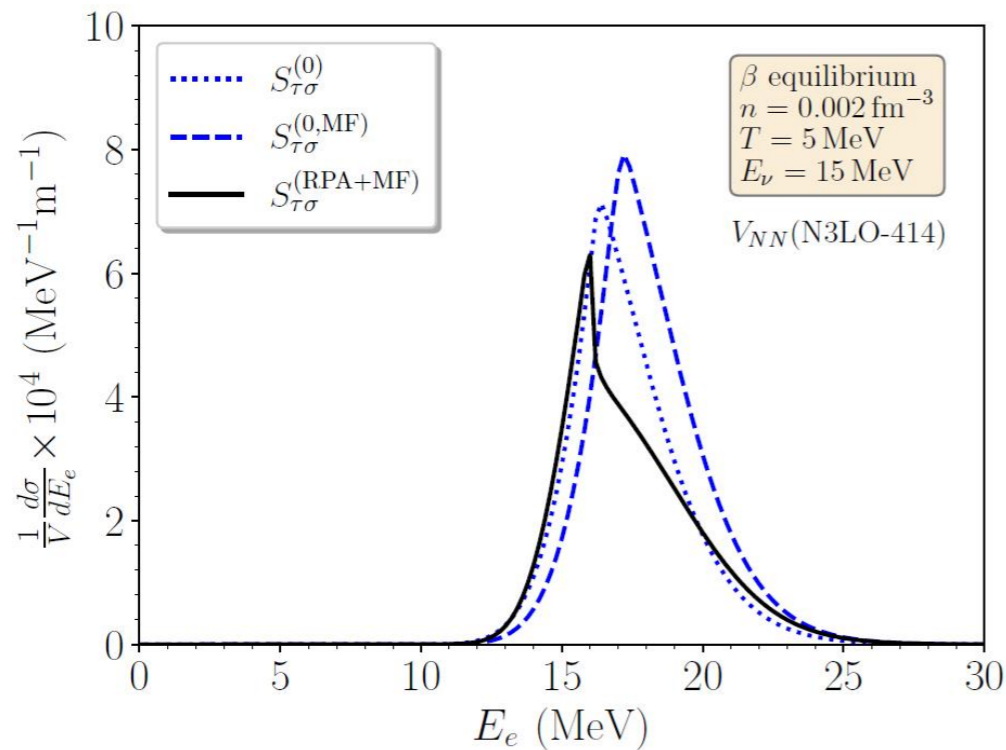
RPA corrections on neutrino opacity

ph interaction in terms of simple Landau parameters (Burrows & Sawyer 98, 99)

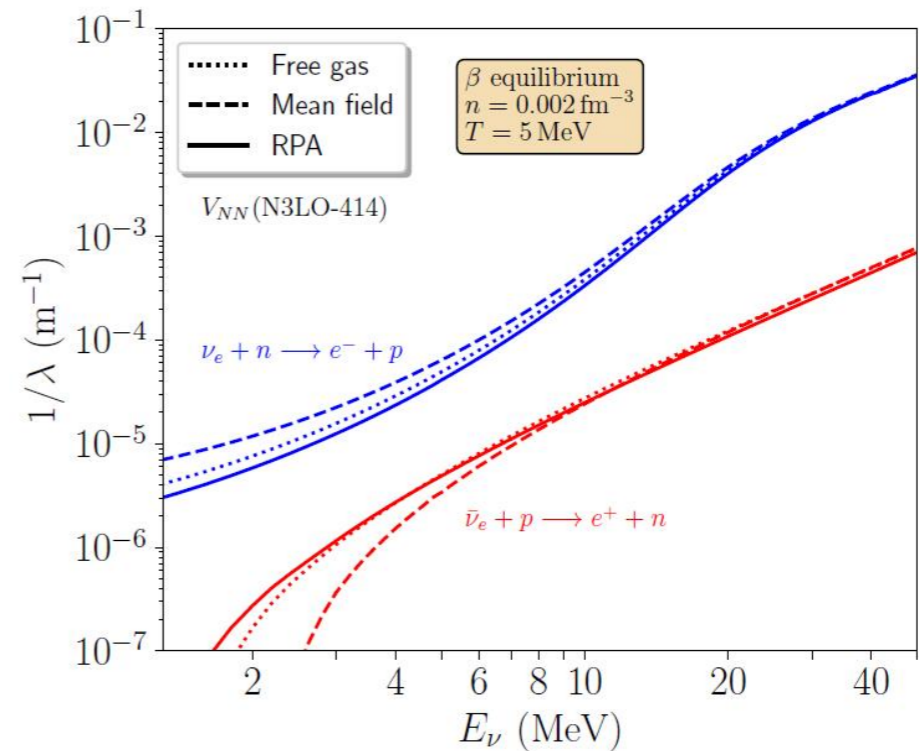
Full ph interaction from Skyrme interactions (Duan & Urban 23)

$$\mathcal{V}_{21}^0 = \frac{\delta^2 E_s}{\delta\rho_{2'2}\delta\rho_{11'}}$$

the 1st mean field (HF level) + RPA study using χ EFT potential (N3LO-414)



Shin+23



SN project

