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Effects of scale symmetry under relativistic mean field scheme

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Outline

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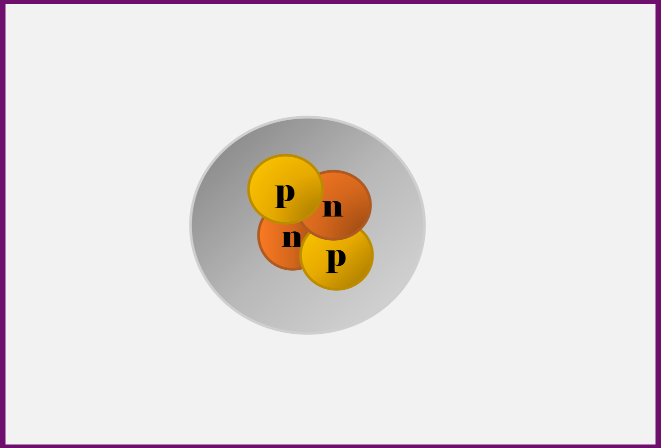
01

Motivation

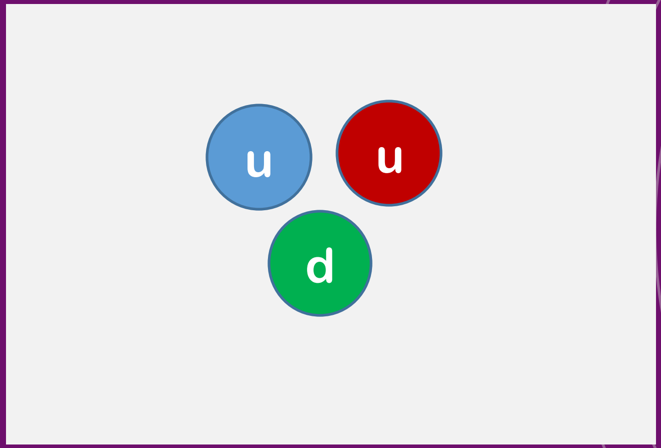
01 Motivation

nuclear matter



$\sim n_0$

quark matter



$\gg n_0$

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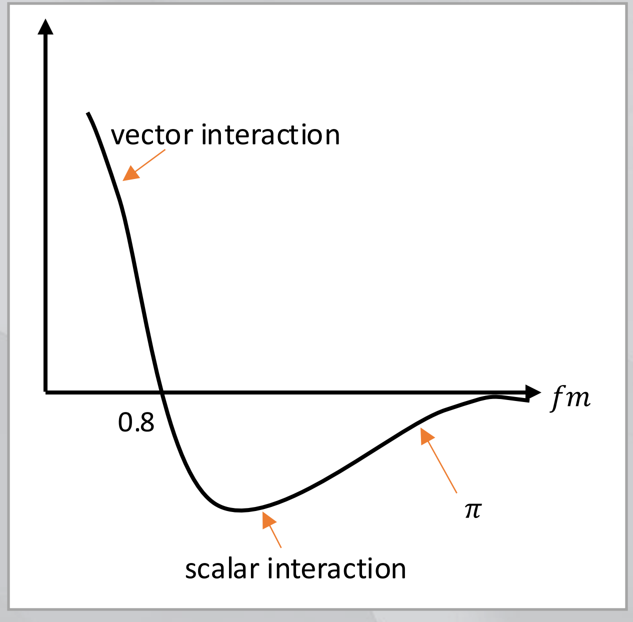
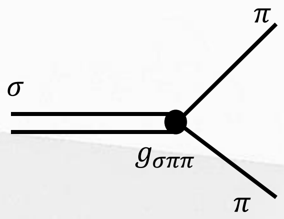


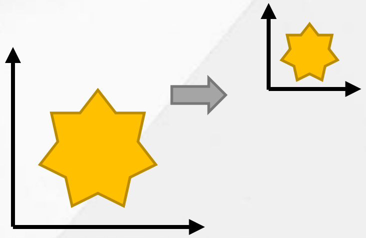
- The attractive potential is mainly provided by scalar meson when goes to dense environment .e. g. the nuclear saturation point, $n_0 \approx 0.16fm^{-3}$~$r_0 \approx 1.14fm$ is far from region (r $> 3fm$) where one-pion-exchange dominates.

- $f_0(500)$(denote as σ) as a lowest-lying scalar make significant contribution for attractive interaction. However, the pole (400–550)−i(200–350) MeV remains large uncertainty on its components.



- In our work, we treat the scalar as the Nambu-Goldstone(NG) boson of scale symmetry which is also found in QCD with chiral limit.

$$\begin{cases} x \rightarrow x' = \lambda^{-1}x, \\ \phi \rightarrow \phi' = \lambda\phi. \end{cases}$$



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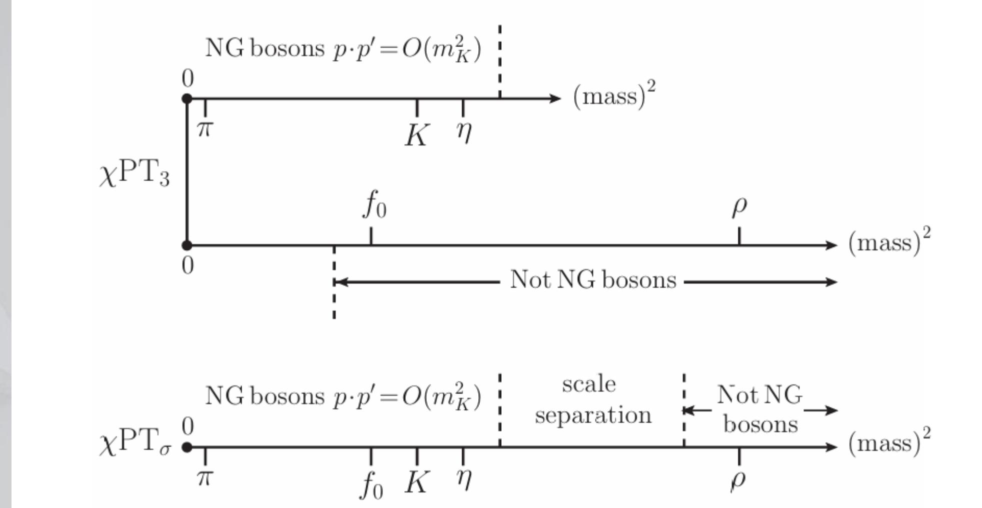
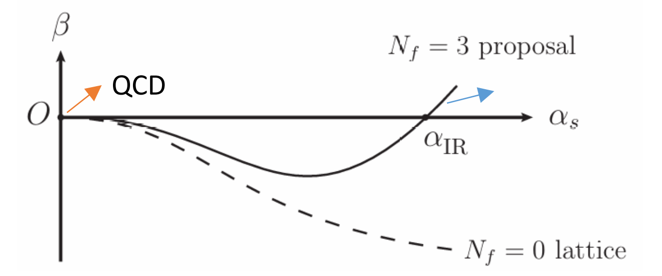
Reason for treatment σ as a N-G boson



- $m_\sigma^2 = O(m_K^2)$

Phys.Rev.D 91 (2015) 3, 034016

Spontaneous symmetry breaking



The scale symmetry emerges at an assumed Infrared(IR) point, then the scalar meson can be identified with Nambu-Goldstone(NG) boson.

Explicit symmetry breaking

- $$\partial_\mu D^\mu = \theta_\mu^\mu = \frac{\beta}{4\alpha_s} G_{\mu\nu}^a G^{a\mu\nu} + (1+\gamma_m) \sum_q m_q \bar{q} q$$

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02

Construction

02 Construction

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Compensator field

$$\chi = f_\chi \Phi = f_\chi e^{\frac{\sigma}{f_\chi}},$$

$$\chi(x) \to \chi'(x') = \lambda \chi(x)$$

(scale invariant terms) →

$$\int d^4x \left(\frac{1}{2}\partial^\mu \Phi \partial_\mu \Phi + \lambda \Phi^4 - m_N \Phi \bar{N} N + \cdots \right)$$

$$\frac{m_N}{f_\chi}\sigma \bar{N} N + \cdots$$

Hidden local symmetry

(a way to introduce vector meson)

$$\hat{\alpha}^\mu_{\perp,\parallel} = \frac{1}{2i}\left(D^\mu \xi \cdot \xi^\dagger \mp D^\mu \xi^\dagger \cdot \xi\right), \xi = \sqrt{U} = e^{i\frac{\pi}{f_\pi}}$$

$$D^\mu = \partial^\mu - i\frac{1}{2}\left(g_\omega \omega^\mu + g_\rho \rho^{a\mu} \tau^a\right)$$

02 Construction

Baryonic Lagrangian

$$\mathcal{L}_B = \bar{N} i\gamma_\mu D^\mu N - m_N \Phi \bar{N} N$$
$$+ g_{\omega NN}\omega^\mu \bar{N}\gamma_\mu N + g_{\rho NN}\rho^{a\mu}\bar{N}\tau^a\gamma_\mu N$$
$$+ g_{\omega NN}^{SSB}\left(\Phi^{\beta'} - 1\right)\omega^\mu \bar{N}\gamma_\mu N$$
$$+ g_{\rho NN}^{SSB}\left(\Phi^{\beta'} - 1\right)\rho^{a\mu}\bar{N}\tau^a\gamma_\mu N$$
$$+ \cdots,$$

weak decay in nuclei-g_A

Phys.Rev.Lett. 125 (2020) 14, 142501

Mesonic Lagrangian

$$\mathcal{L}_M = \frac{m_\rho^2}{g_\rho^2}\Phi^2 \mathrm{Tr}\left(\hat{\alpha}_\parallel^\mu \hat{\alpha}_{\mu\parallel}\right) + \frac{1}{2}\left(\frac{m_\omega^2}{g_\omega^2} - \frac{m_\rho^2}{g_\rho^2}\right)\Phi^2\, \mathrm{Tr}(\hat{\alpha}_\parallel^\mu)\, \mathrm{Tr}(\hat{\alpha}_{\mu\parallel})$$
$$+ h_5\Phi^4 + h_6\Phi^{4+\beta'} + \cdots,$$

leading order scale symmetry(LOSS)

Prog.Part.Nucl.Phys. 113 (2020), 103791

02 Construction

extension to dense medium

- relativistic mean field (RMF) approximation:

  (static, homogeneous, classical)

$$\rho^{ia} = 0, i = 1,2,3,$$
$$\rho^{\mu a} = \rho^{0a}\delta^{\mu 0},$$
$$\partial_\mu \rho^{\nu a} = 0.$$

- Brown-Rho (B-R) scaling:

  (The information of dense vacuum has been encoded in coupling constants)

$$\langle 0^* | \bar{q}q | 0^* \rangle / \langle 0 | \bar{q}q | 0 \rangle = (f_\pi^* / f_\pi)^3$$

$$\frac{f^*_{\pi(\chi)}}{f_{\pi(\chi)}} \approx \frac{m^*_{\rho(\omega,N)}}{m_{\rho(\omega,N)}} \approx \Phi^*,$$

Phys.Rev.Lett. 66 (1991), 2720-2723

$$\frac{m^*_\sigma}{m_\sigma} \approx \Phi^{*1+\frac{\beta'}{2}},$$

where Φ^* is parameterized as $\frac{1}{1+r\frac{n}{n_0}}$.

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03

Results

03 Results

nuclear matter (NM) properties

	Empirical	bsHLS-L	bsHLS-H
n_0	0.155 ± 0.050	0.159	0.159
$b.e.$	-15.0 ± 1.0	-16.0	-16.0
K_0	230 ± 30	232	284
$E_{sym}(n_c)$	26.7 ± 0.2	20.8	20.9
$E_{sym}(n_0)$	30.9 ± 1.9	30.5	29.2
$E_{sym}(2n_0)$	46.9 ± 10.1	51.5	50.2
$L(n_c)$	43.7 ± 7.8	53.2	54.2
$L(n_0)$	52.5 ± 17.5	85.9	68.3
J_0	-700 ± 500	-767	-599

TABLE I . NM properties from bsHLS-L, bsHLS-H.
The unit of physical quantities except $n_{0/c}$ are MeV while the latter is fm^{-3}.

	bsHLS-L	bsHLS-H
M_σ	1.05	2.30
β'	0.395	1.15
r	0.161	0.191
$g_{\omega NN}$	11.5	11.0
$g_{\rho NN}$	3.78	4.17
$g_{\omega NN}^{SSB}$	16.3	8.85
$g_{\rho NN}^{SSB}$	9.45	4.85

TABLE II . Two sets of parameters. $M_\sigma = f_\chi m_\sigma$ is in unit of $10^5\mathrm{MeV}^2$.

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response of density

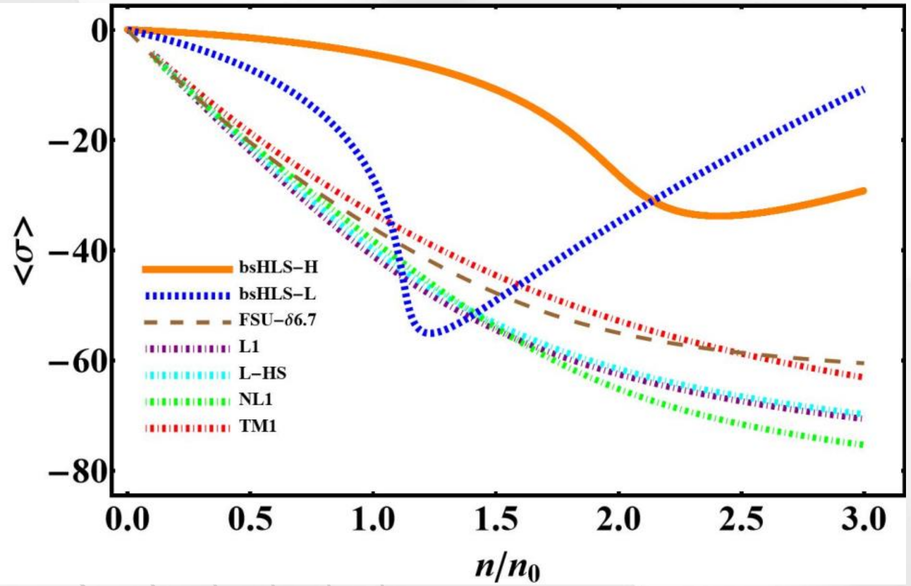

Fig.1 The expectation of σ behaves with respect to density in different models.

$$
\begin{aligned}
\mathcal{L} = \bar{\psi}\big[& i\gamma_\mu\partial^\mu - m_N - g_\sigma\sigma - g_{\omega NN}\gamma_\mu\omega^\mu - g_{\rho NN}\gamma_\mu\rho^{a\mu}\tau^a - g_\delta\delta^a\tau^a\big]\psi \\
& + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
& - \frac{1}{2g^2}\mathrm{Tr}\left(V_{\mu\nu}V^{\mu\nu}\right) \\
& + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 + \frac{1}{2}m_\rho^2\rho^{a\mu}\rho_\mu^a \\
& + \frac{1}{2}\Lambda_V\rho_\mu^a\rho^{a\mu}\omega_\nu\omega^\nu + \frac{1}{2}\left(\partial_\mu\delta^a\partial^\mu\delta^a - m_\delta^2\delta^a\delta^a\right) + \frac{1}{2}C_{\delta\sigma}\sigma^2(\delta^a)^2
\end{aligned}
$$

Lagrangian for Walecka-type model

03 Results

equations of motions(EOMs)

$$m_\omega^2\Phi^2\omega = \left[g_{\omega NN} + g_{\omega NN}^{SSB}(\Phi^{\beta'} - 1)\right]\left[\langle p^\dagger p\rangle + \langle n^\dagger n\rangle\right],$$
$$m_\rho^2\Phi^2\rho = \left[g_{\rho NN} + g_{\rho NN}^{SSB}(\Phi^{\beta'} - 1)\right]\left[\langle p^\dagger p\rangle - \langle n^\dagger n\rangle\right].$$

EOMs of nonlinear models

$$m_\omega^2\omega = g_{\omega NN}\left[\langle p^\dagger p\rangle + \langle n^\dagger n\rangle\right] - c_3\omega^3 - \Lambda_V\rho^2\omega,$$
$$m_\rho^2\rho = g_{\rho NN}\left[\langle p^\dagger p\rangle - \langle n^\dagger n\rangle\right] - \Lambda_V\omega^2\rho.$$

EOMs of Walecka-type models

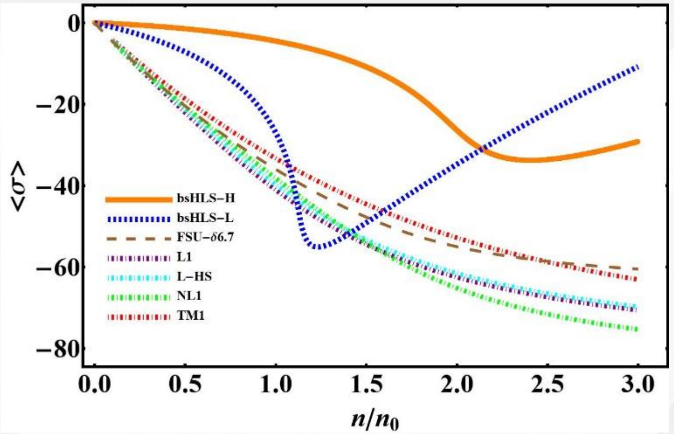

Fig.1 The expectation of σ behaves with respect to density in different models.

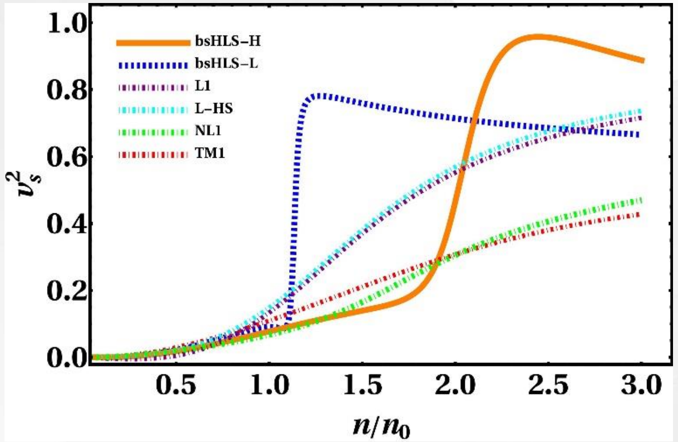

Fig.2 Speed of sound from Walecka (L1, L-HS, NL1, TM1) and nonlinear (bsHLS-H, bsHLS-L) models.

mass-radius (M-R) relation

Tolman-Oppenheimer-Volkoff (TOV) equation:

$$\frac{dp}{dr} = -\frac{G\left[m(r) + \frac{4\pi r^3 p}{c^2}\right](\epsilon + p)}{c^2 r^2 \left[1 - \frac{2Gm(r)}{c^2 r}\right]},$$

$$\frac{dm}{dr} = \frac{4\pi r^2}{c^2}\epsilon.$$

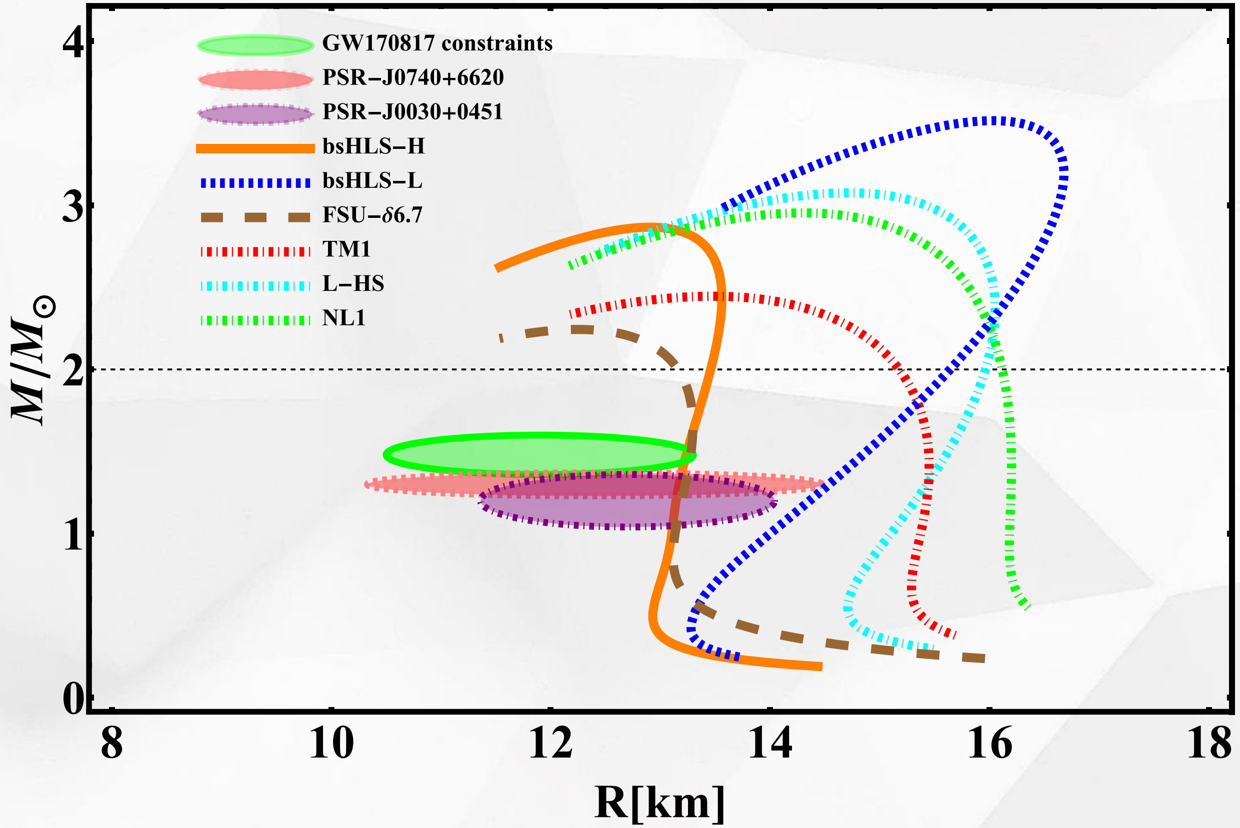

Fig.3 M-R relation of neutron stars given by different models.

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symmetry energy and tidal deformation

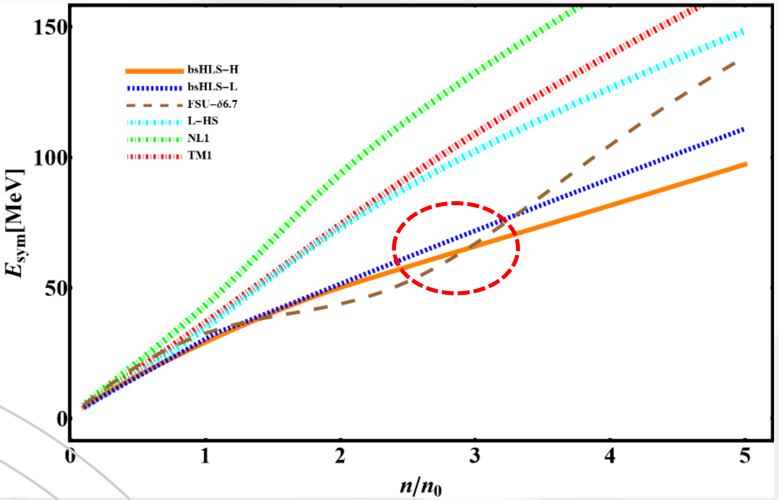

Fig.4 Symmetry energies of different models

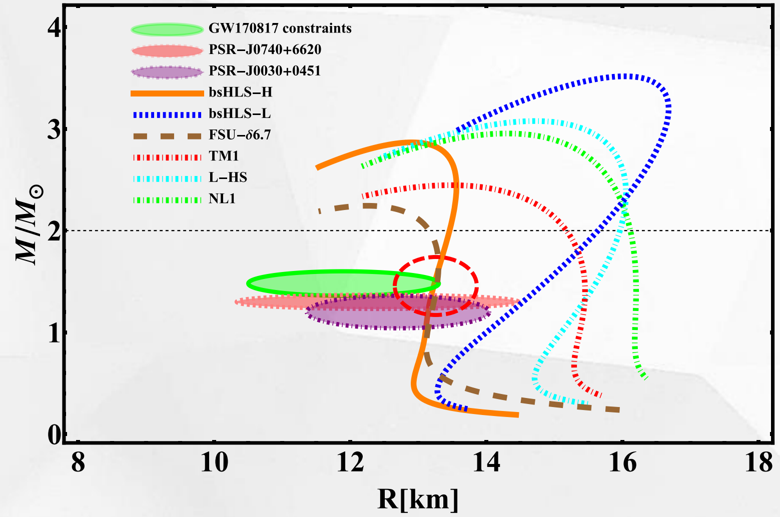

Fig.3 M-R relation of neutron stars given by different models.

	bsHLS-L	bsHLS-H	TM1	L-HS	NL1	FSU-δ6.7
$\Lambda_{1.4}$	2120	910	2240	2780	2620	878

TABLE Ⅲ. Tidal deformations with mass of 1.4 $M_{\odot}$ from bsHLS and linear sigma models.

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04

Summary

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- We employed scale-symmetry and its explicit symmetry breaking in construction the Lagrangian by introducing scalar field σ, which can be treated as the scale compensator.

- Once considering density effects (B-R scaling and RMF approximation), the NM properties can be well reproduced at low density region.

- A kink behavior of the σ field has been investigated in bsHLS compared to other linear sigma models, owing to the universal coupling between scalar and vector mesons stem from the nonlinear parameterization.

- The M-R relation derived from bsHLS-H is within the constraints of astronomical observation without introducing additional freedom like δ meson.

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Thank you