Light meson decays at BESIII

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第九届手征有效场论研讨会

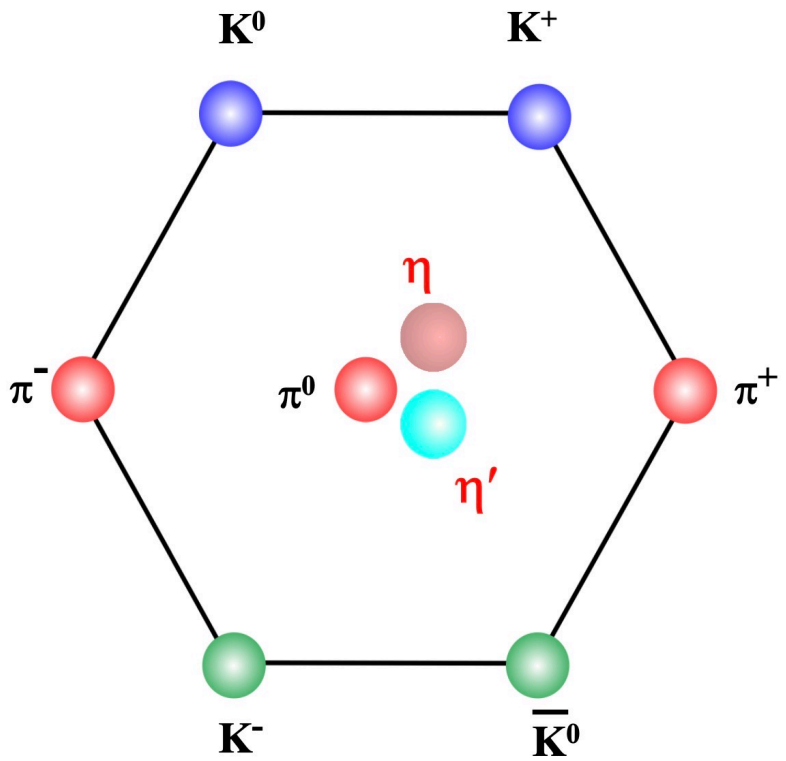
2024年10月19日　湖南·长沙

Outline

- Light meson physics
- BESIII: a light meson factory
- Recent η/η' decays at BESIII
- Summary

Light Meson Physics

- Important roles in particle physics
  - ✓Strong interactions, Quark Model...
- Rich physics
  - ✓Test ChPT predictions
  - ✓EM Form Factors
  - ✓Test fundamental symmetries
  - ✓Probe new physics beyond the SM

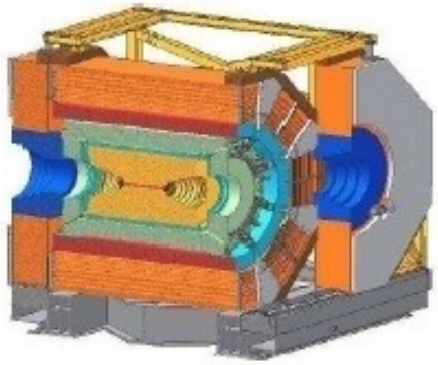


Source of η/η' events

New Proposals

e^+e^- Collider

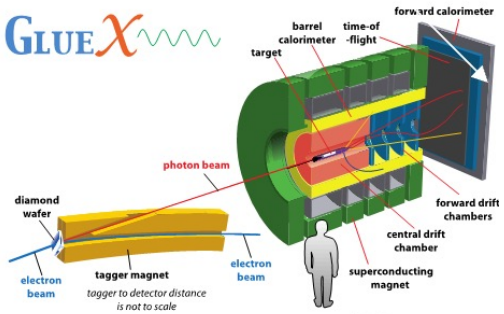
BESIII at BEPCII



KLOE-2

Fixed-target

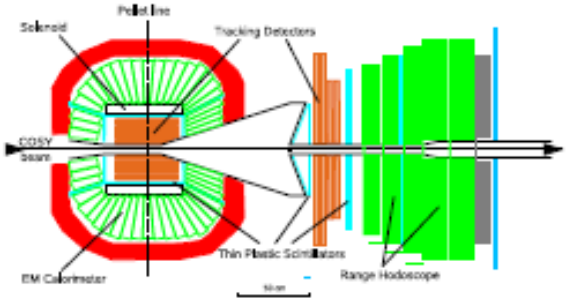
JEF at JLab



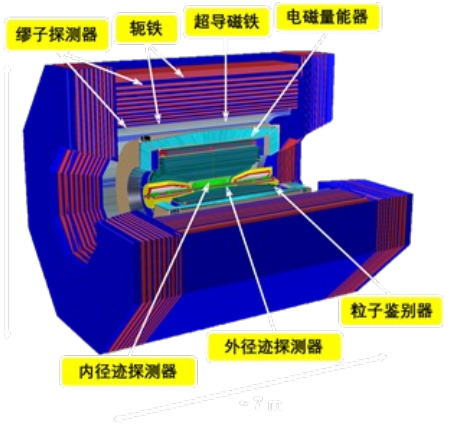
CLAS(12)

Crystal Ball

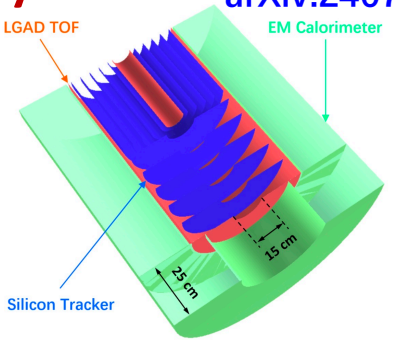
WASA at COSY



STCF

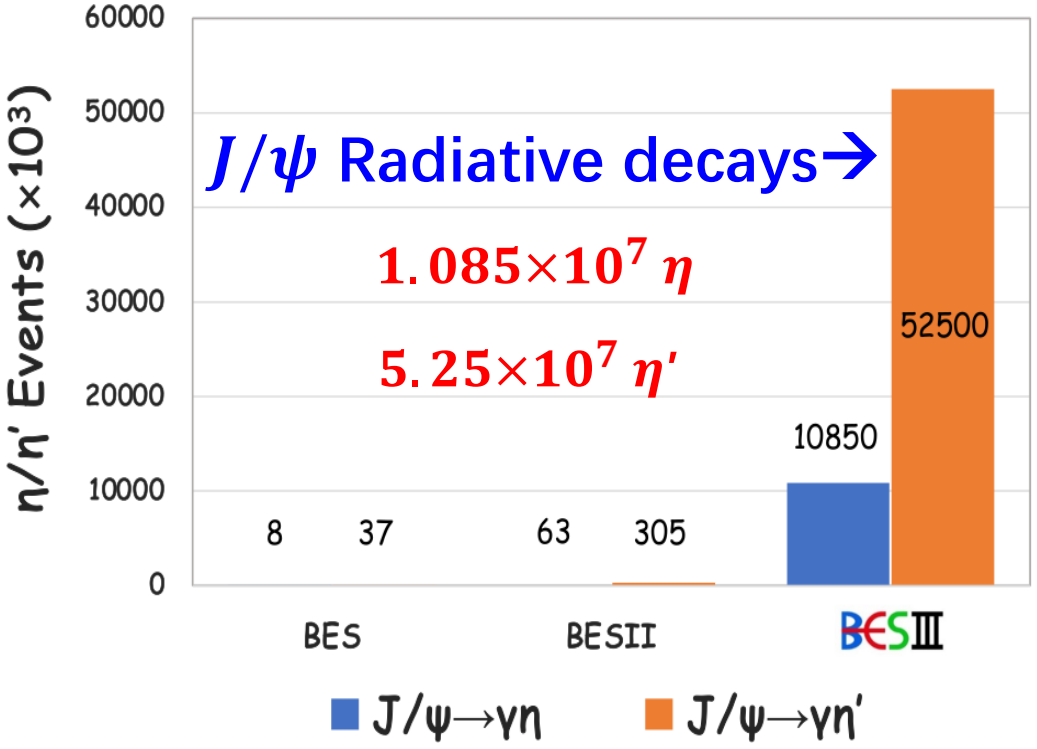


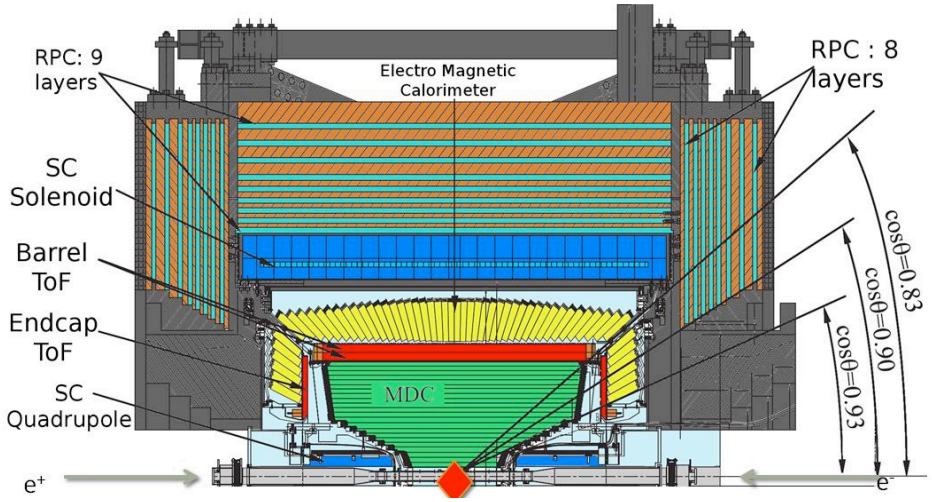
η factory at HIAF arXiv:2407.00874



η/η′ sample at BESIII

- The BESIII detector records symmetric e+e- collisions provided by the BEPCII at 2.0-4.95 GeV
- Collected 10 billion J/ψ events, $J/\psi \to \gamma P, VP$





A light meson factory !

BESIII: an important role in η/η′ decays

- Decay mechanisms
- Form factor

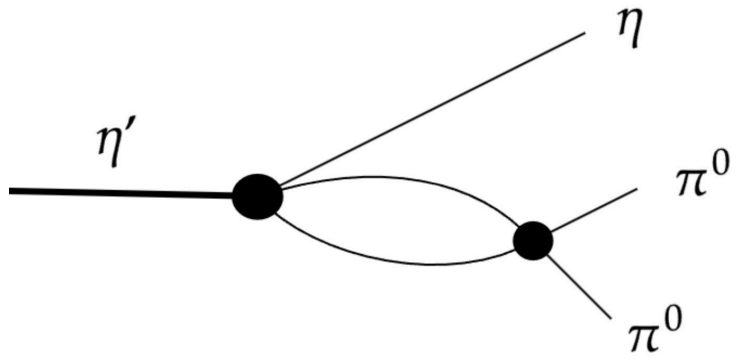
Decay channel	Physics	Publication
$\eta'\to 2(\pi^+\pi^-)$, $\pi^+\pi^-\pi^0\pi^0$	First observation, BR	PRL112, 251801(2014)
$\eta'\to\gamma e^+e^-$	First observation, BR, TFF	PRD92, 012001(2015)
$\eta\to\pi^+\pi^-\pi^0$, $\eta/\eta'\to\pi^0\pi^0\pi^0$	Matrix elements, m_u-m_d, C-inv	PRD92, 012014(2015)
$\eta'\to\omega e^+e^-$	First observation, BR	PRD92, 051101(2015)
$\eta'\to K\pi$	Weak decay, UL	PRD93, 072008 (2016)
$\eta'\to\rho\pi$	First observation, BR	PRL118, 012001(2017)
$\eta'\to\gamma\gamma\pi^0$	BR, B boson	PRD96, 012005(2017)
$\eta'\to\gamma\pi^+\pi^-$	BR, decay dynamic (box anomaly)	PRL120, 242003(2018)
$\eta'\to\pi^+\pi^-\eta$, $\eta'\to\pi^0\pi^0\eta$	Matrix elements, cusp effect	PRD97, 012003(2018)
$\omega\to\pi^+\pi^-\pi^0$	Dalitz plot analysis	PRD98, 112007(2018)
$P\to\gamma\gamma$	BRs, chiral anomaly	PRD97, 072014(2018)
$\eta'\to\gamma\gamma\eta$	UL	PRD100, 052015(2019)
Absolute BF of η′ decays	BRs	PRL122, 142002(2019)
$\eta'\to\pi^0\pi^0\pi^0\pi^0$	CP-Vio, UL	PRD101, 032001(2020)
$\eta'\to\pi^+\pi^-e^+e^-$	BR, CP-viol assymm	PRD103, 092005(2021)
$\eta'\to\pi^+\pi^-u^+u^-$	BR, decay dynamic	PRD103, 072006(2021)
Absolute BF of η decays	BRs	PRD104, 092004(2021)
$\eta'\to e^+e^-e^+e^-$	BR, TFF	PRD105, 112010(2022)
$\eta'\to\eta\pi^0\pi^0$	Cusp effect	PRL130, 081901(2023)
$\eta\to\pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$	Matrix elements, cusp effect	PRD107,092007(2023)
$\eta'\to 2(\pi^+\pi^-)$,$\pi^+\pi^-\pi^0\pi^0$, $2(\pi^0\pi^0)$	VMD, CP-Vio	PRD 109, 032006 (2024)
$\eta'\to\pi^+\pi^-e^+e^-$, $\pi^+\pi^-u^+u^-$	BR, decay dynamic, CP-Vio	JHEP07, 135 (2024)
$\eta/\eta'\to\gamma e^+e^-$	TFF	PRD109, 072001 (2024)

Decay mechanism

- Evidence of the cusp effect in $\eta' \to \pi^0\pi^0\eta$ PRL 130, 081901 (2023)
- Dalitz plot of $\eta \to 3\pi$ PRD 107, 092007 (2023)
- Improved study of decays $\eta' \to 4\pi$ PRD 109, 032006 (2024)

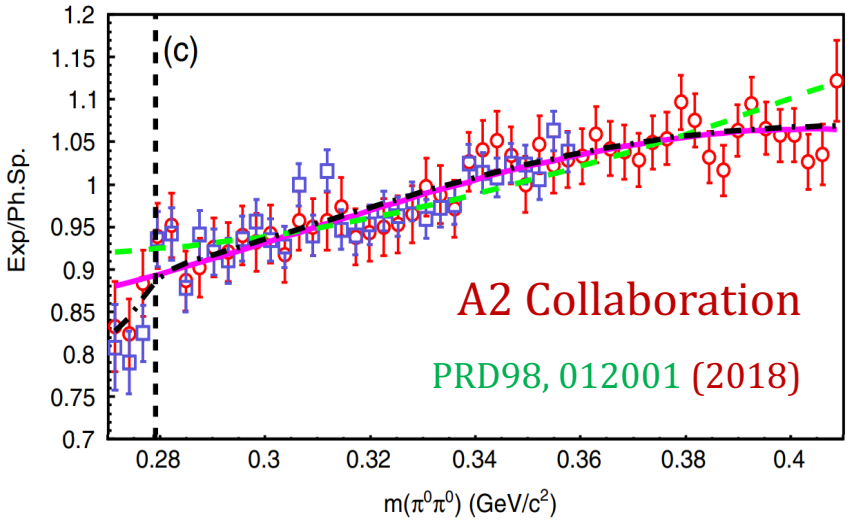
Evidence of cusp effect in $\eta' \to \pi^0\pi^0\eta$

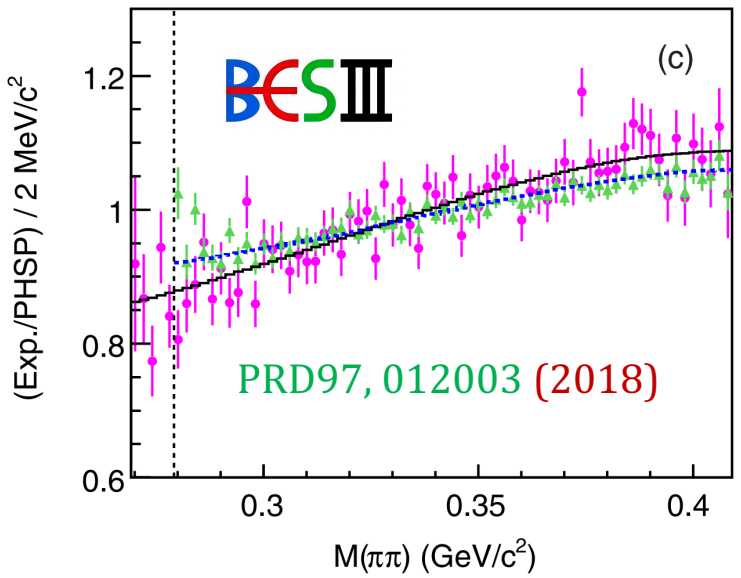
- Investigation on ππ and πη final interactions
- Charge-exchange rescattering: $\pi^+\pi^- \to \pi^0\pi^0$
- Sizable effect ~6% in this decay within NREFT

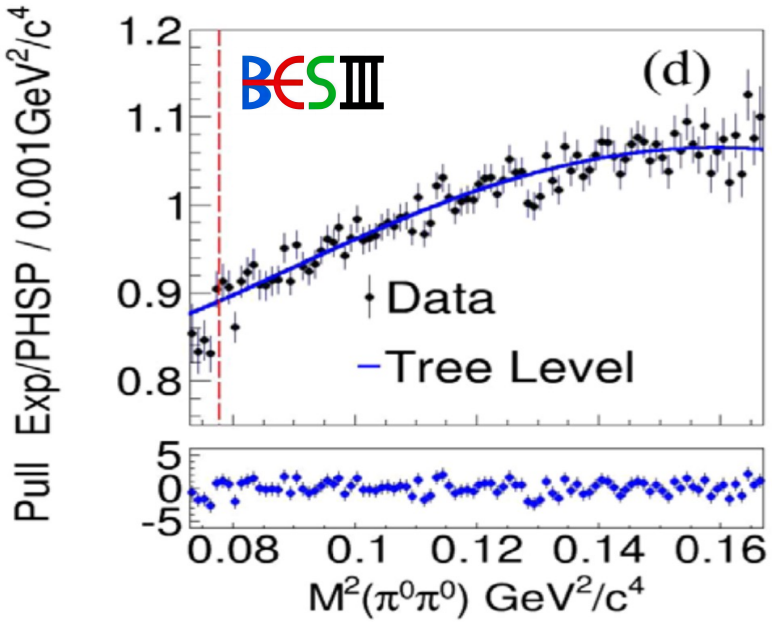


B. Kubis and S. P. Schneider, EPJC 62, 511 (2009)

BESIII: PRL130, 081901(2023)







BESIII: PRL130, 081901(2023)

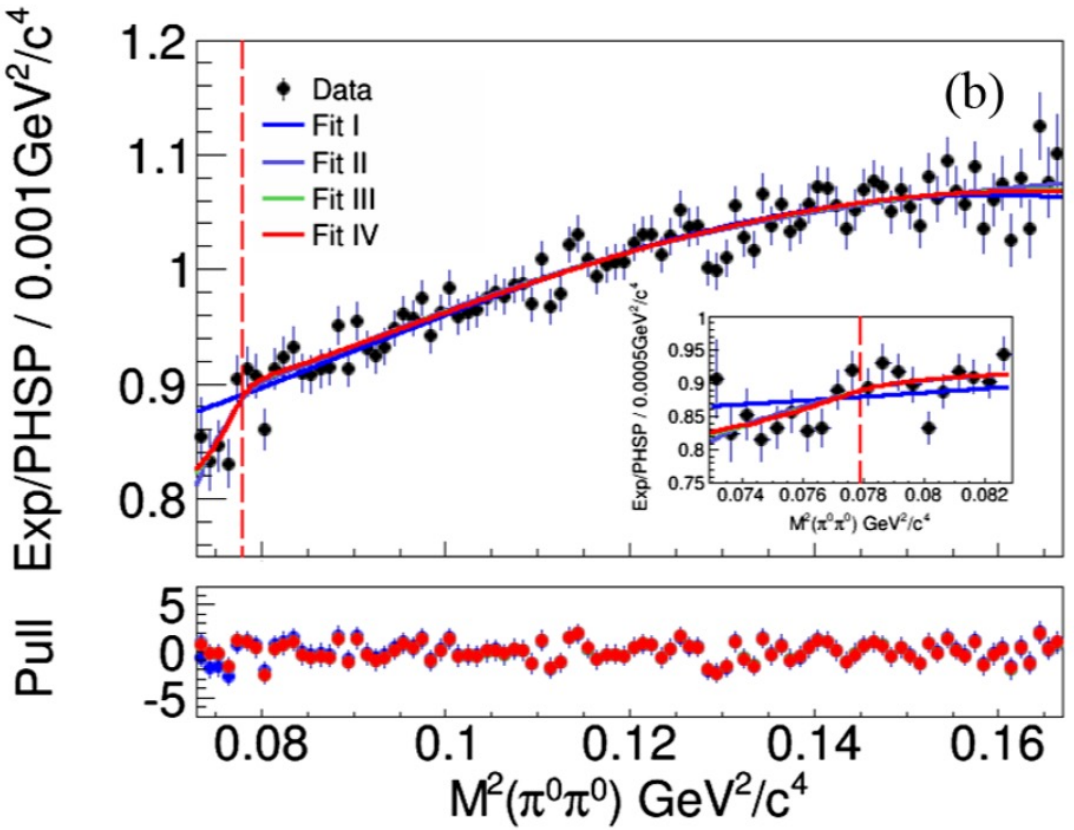
- One and two-loop level contributions based on NREFT are introduced

B. Kubis, S. P. Schneider, EPJC 62, 511 (2009)

Cusp effect with ~3.5 σ!

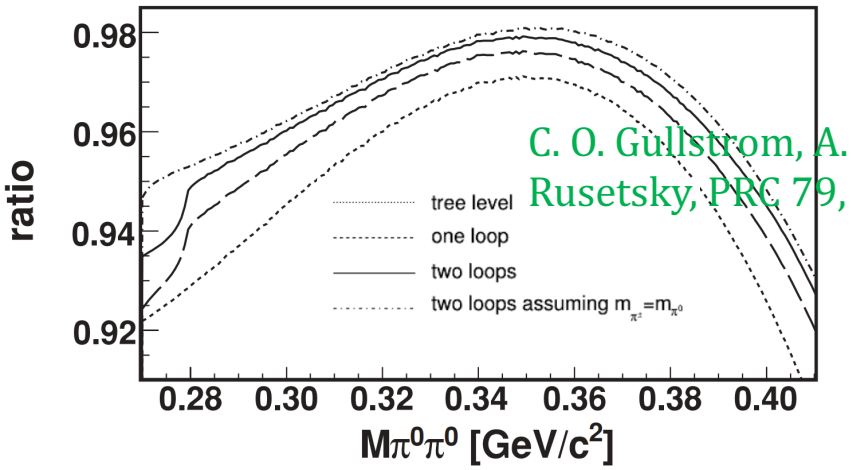
- $\pi - \pi$ scattering parameters:

$$a_0 - a_2 = 0.226 \pm 0.060 \pm 0.013$$



- Amplitude analysis of $\eta' \to \pi^+\pi^-\eta$ within NREFT is forthcoming, sizeble contribution from final state interactions

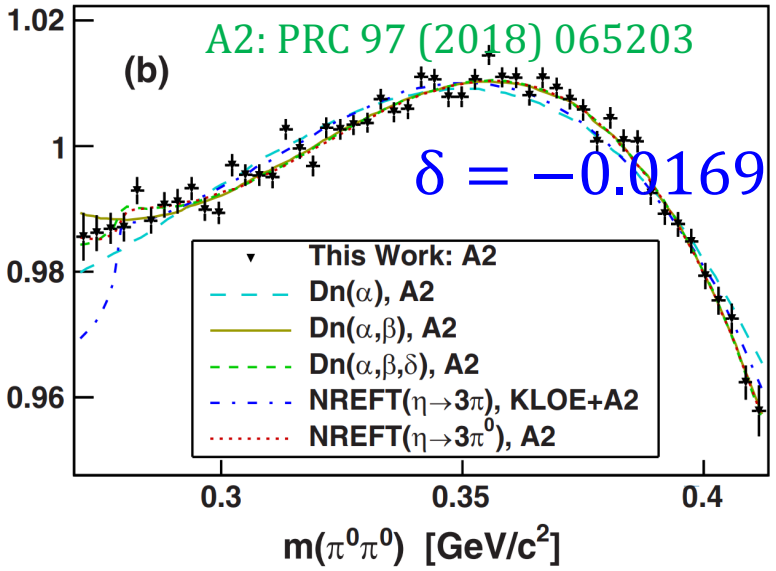
Cusp structure in $\eta \to \pi^0\pi^0\pi^0$



C. O. Gullstrom, A. Kupsc, A. Rusetsky, PRC 79, 028201 (2009)

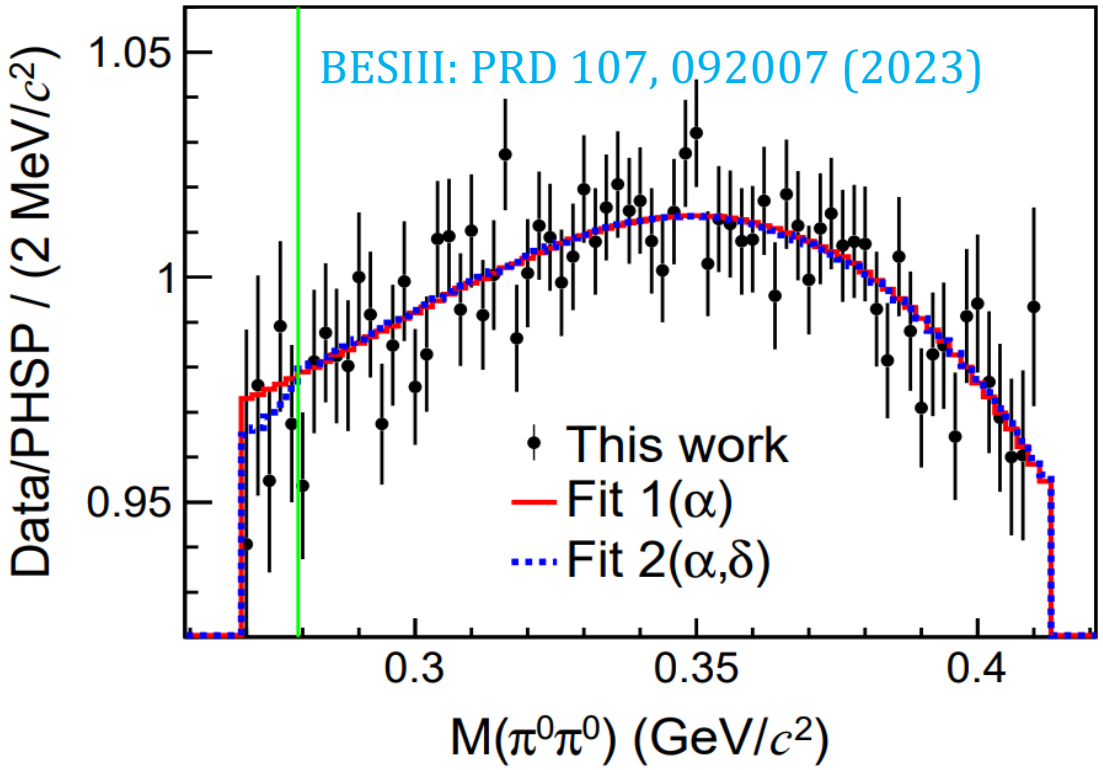
$$|A(X,Y)|^2 \propto 1 + 2\alpha Z + 2\delta \sum_{i=1}^{3} \Re\sqrt{1 - s_i/4m^2_{\pi^\pm}}$$

$\delta = -0.018 \pm 0.022_{stat.}$



A2: PRC 97 (2018) 065203

$\delta = -0.0169 \pm 0.0067$

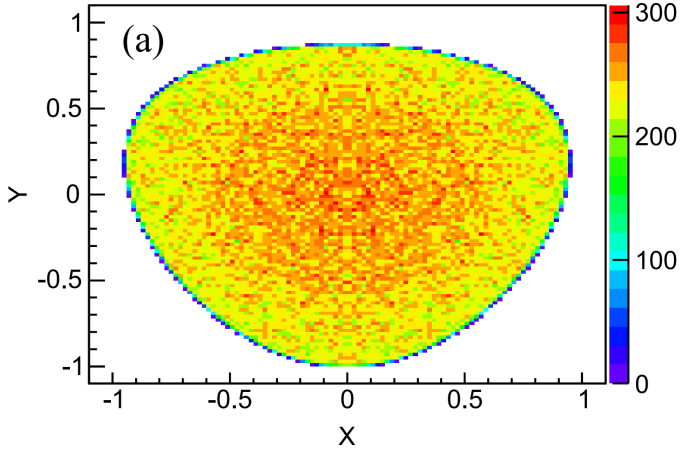


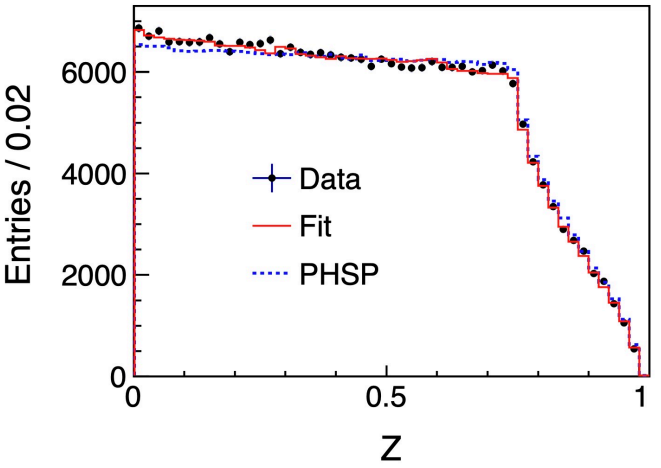
Matrix elements for $\eta \to \pi^0\pi^0\pi^0$

BESIII: PRD 107, 092007 (2023)
https://www.hepdata.net/record/141642

$$Z = X^2 + Y^2 = \frac{2}{3}\sum_{i=1}^{3}(\frac{3T_i}{Q} - 1)^2$$

$$|A(X,Y)|^2 \propto 1 + 2\alpha Z + 2\beta(3X^2Y - Y^3) + 2\gamma Z^2 + \cdots$$





$$\alpha = -0.0406 \pm 0.0035 \pm 0.0008$$

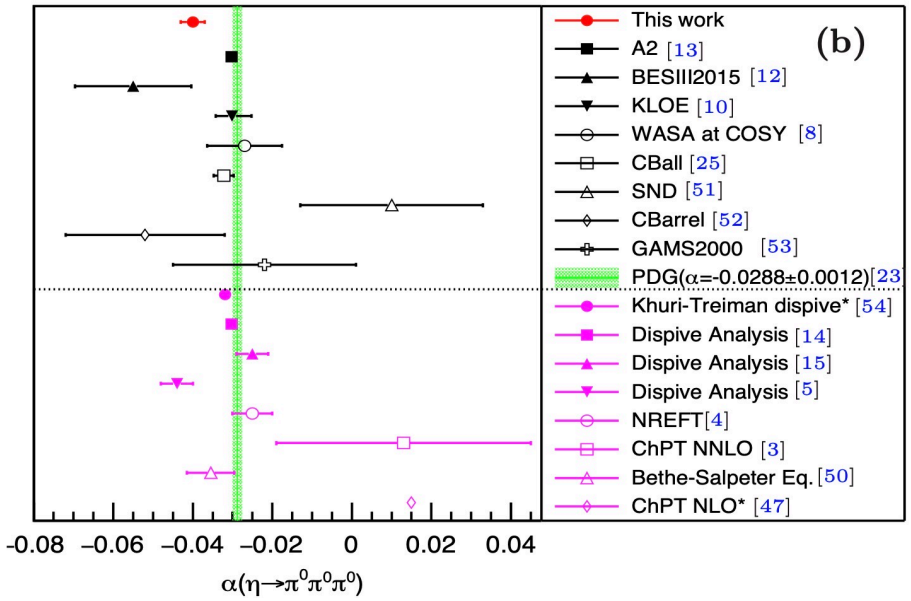
$$\beta = 0.0038 \pm 0.0033_{stat.}$$

$$\gamma = -0.018 \pm 0.014_{stat.}$$

α is consistent with A2 result ($-0.0302 \pm 0.0008_{stat.}$) in 2.8σ

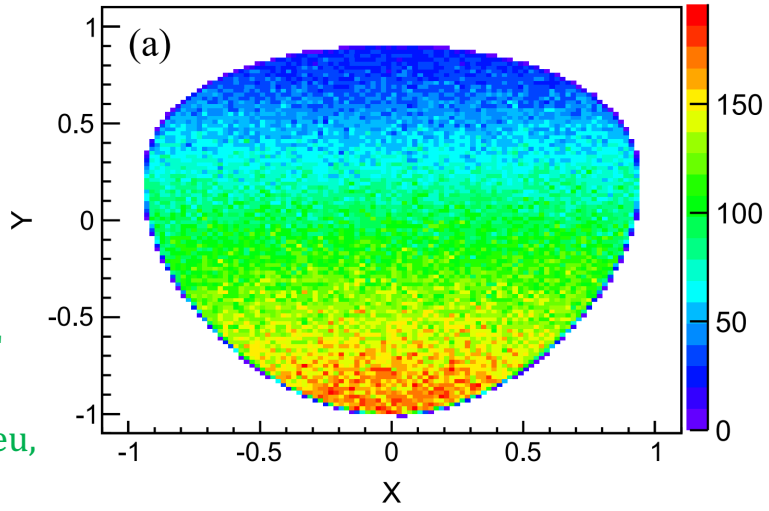
$$\beta(A_2) = -0.0070 \pm 0.0010_{stat.}$$

$$\gamma(A_2) = -0.0023 \pm 0.0040_{stat.}$$



Matrix elements for $\eta \to \pi^+\pi^-\pi^0$

BESIII: PRD 107, 092007 (2023)
https://www.hepdata.net/record/141642



➢SM: C conserved, isospin broken, EM effects suppressed

⇒ ideal process to extract $m_u - m_d$

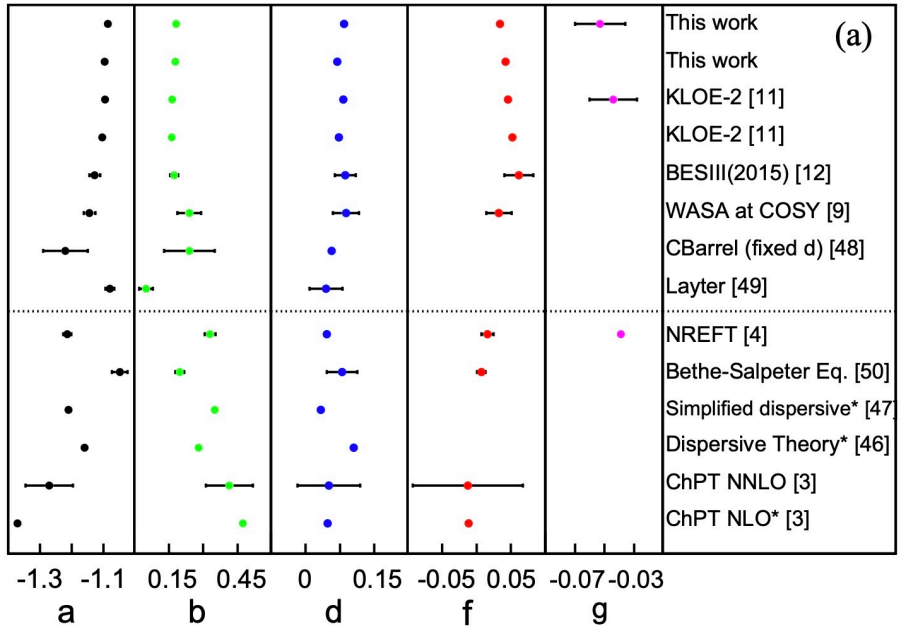
G. Colangelo, S. Lanz, H. Leutwyler, E. Passemar, PRL 118, 022001 (2017)

P. Guo, I. V. Danilkin, C. Fernández-Ramírez, V. Mathieu, A. P. Szczepaniak, PLB 771, 497 (2017)

$$X = \frac{\sqrt{3}}{Q}(T_{\pi^+} - T_{\pi^-}), Y = \frac{3T_{\pi^0}}{Q} - 1,$$

$$|A(X,Y)|^2 \propto 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + gX^2Y + \cdots$$

$$\begin{aligned} a &= -1.086 \pm 0.006 \pm 0.001, \\ b &= 0.162 \pm 0.006 \pm 0.003, \\ d &= 0.083 \pm 0.007 \pm 0.001, \\ f &= 0.118 \pm 0.011 \pm 0.003, \\ g &= -0.053 \pm 0.017 \pm 0.003. \end{aligned}$$



$c = (-0.086 \pm 2.986)\times10^{-3}$, $e = -0.001 \pm 0.007$ ⇒ no C symmetry breaking

Dalitz plot Asymmetries in $\eta \to \pi^+\pi^-\pi^0$

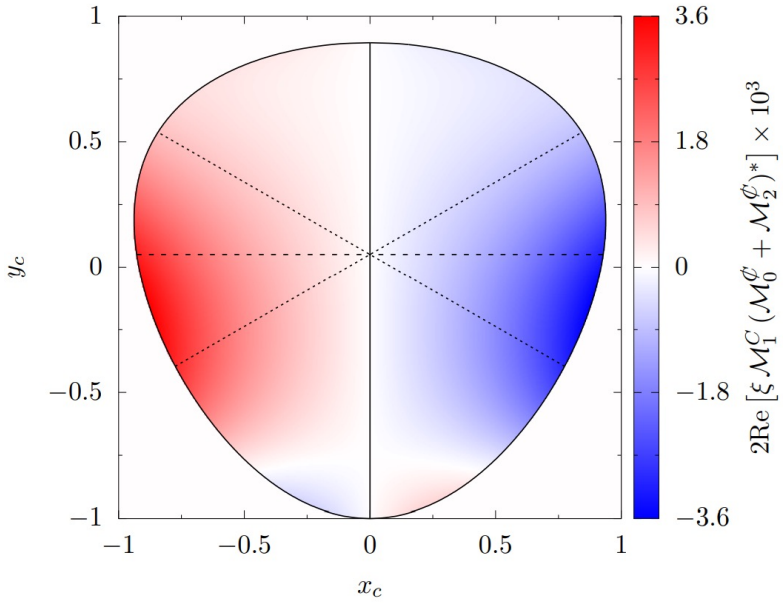
BESIII: PRD 107, 092007 (2023)

S. Gardner, J. Shi, PRD 101 (2020) 115038
H. Akdag, T. Isken, B. Kubis, JHEP 02 (2022)137
J. Shi, J. Liang, S. Gardner PR 110 (2024) 055039

➢ BSM: C broken, isospin either conserved or broken

$$\mathcal{M}(s,t,u) = \mathcal{M}_1^C(s,t,u) + \mathcal{M}_0^{\not C}(s,t,u) + \mathcal{M}_2^{\not C}(s,t,u)$$

➢ The interferences give rise to mirror symmetry breaking (permille level) in the Dalitz plot

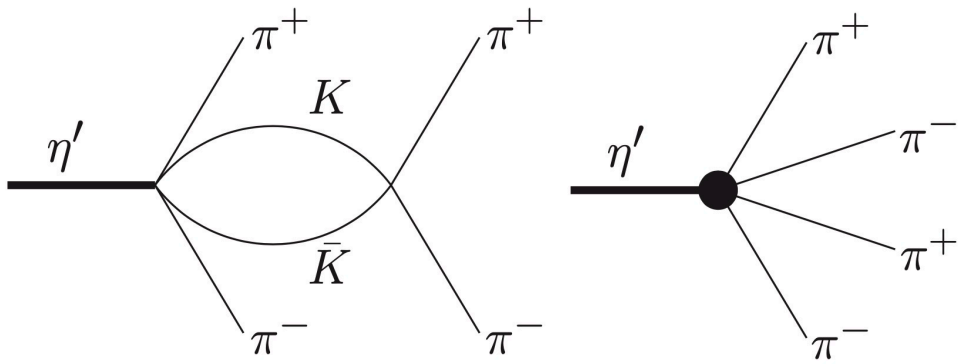


overall C/CP-violation | ΔI = 2 | ΔI = 0

Experiment	$A_{LR}(\%)$	$A_Q(\%)$	$A_S(\%)$
This work	$0.114 \pm 0.131 \pm 0.001$	$-0.035 \pm 0.131 \pm 0.011$	$-0.070 \pm 0.131 \pm 0.009$
KLOE-2 [11]	$-0.050 \pm 0.045^{+0.050}_{-0.110}$	$0.018 \pm 0.045^{+0.048}_{-0.023}$	$0.004 \pm 0.045^{+0.031}_{-0.035}$
Jane [40]	0.28 ± 0.26	-0.30 ± 0.25	0.20 ± 0.25
Layter [24]	-0.05 ± 0.22	-0.07 ± 0.22	0.10 ± 0.22
Gormley [41]	1.5 ± 0.5	-	0.5 ± 0.5

Amplitude analysis for $\eta' \to 4\pi$

BESIII: PRD 109, 032006 (2024)



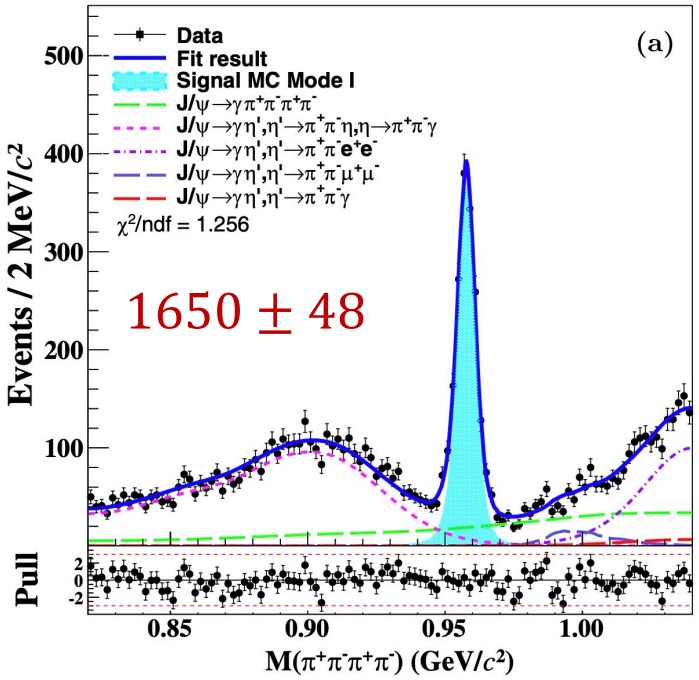
Loop and counter term at O(p⁶)

F. K. Guo, B. Kubis, A. Wirzba, PRD 85,014014 (2012)

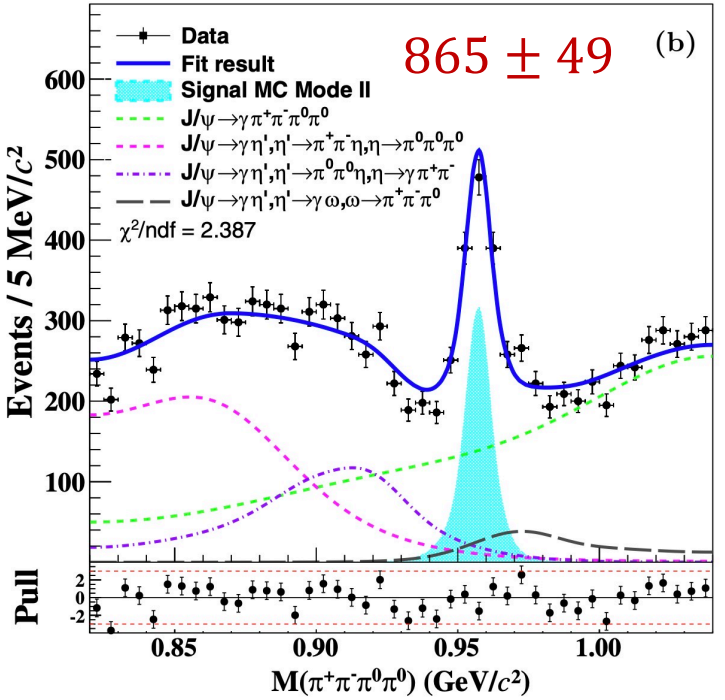
$Br(\eta' \to 2(\pi^+\pi^-)) = (1.0 \pm 0.3)\times10^{-4}$

$Br(\eta' \to \pi^+\pi^-2\pi^0) = (2.4 \pm 0.7)\times10^{-4}$

$\eta' \to \pi^+\pi^-\pi^+\pi^-$



$\eta' \to \pi^+\pi^-\pi^0\pi^0$



$Br(\eta' \to \pi^+\pi^-\pi^+\pi^-) = (8.56 \pm 0.25 \pm 0.23)\times10^{-5}$

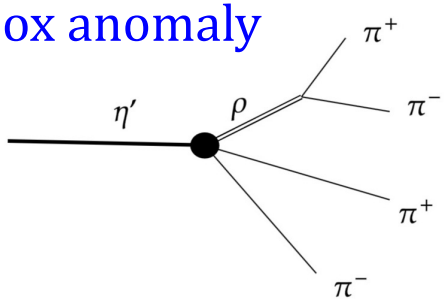
$Br(\eta' \to \pi^+\pi^-\pi^0\pi^0) = (2.12 \pm 0.12 \pm 0.10)\times10^{-4}$

Amplitude analysis for $\eta' \to 2(\pi^+\pi^-)$

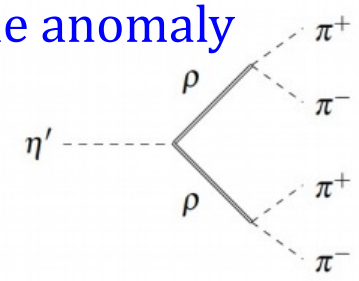
BESIII: PRD 109, 032006 (2024)

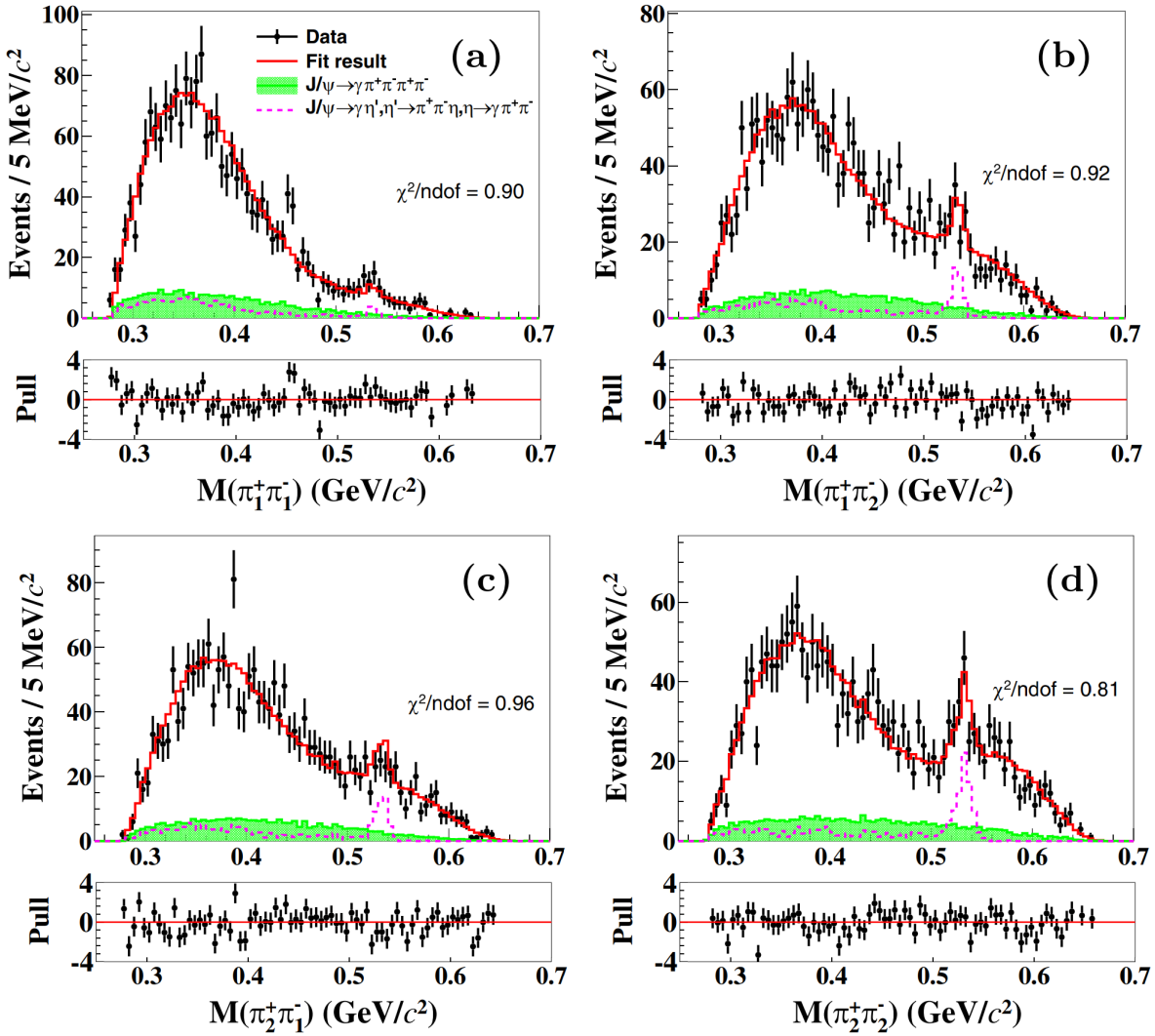
$$\mathcal{A}(\eta' \to \pi^+\pi^-\pi^+\pi^-) = \epsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu p_3^\alpha p_4^\beta \times \left\{ \left[\frac{s_{12}}{D_\rho(s_{12})} + \frac{s_{34}}{D_\rho(s_{34})} - \frac{s_{14}}{D_\rho(s_{14})} - \frac{s_{23}}{D_\rho(s_{23})} \right] + \alpha \left[\frac{M_\rho^2(s_{12}+s_{34})}{D_\rho(s_{12})D_\rho(s_{34})} - \frac{M_\rho^2(s_{14}+s_{23})}{D_\rho(s_{14})D_\rho(s_{23})} \right] \right\},$$

Box anomaly



Triangle anomaly





- First measurement of the doubly virtual isovector form factor

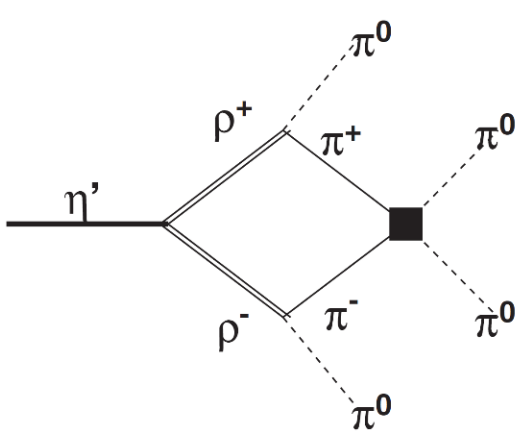
$$\alpha = \frac{c_3}{c_1 - c_2} = 1.22 \pm 0.33 \pm 0.04$$

⇒ If $\alpha \simeq 1$, triangle anomaly would be dominated

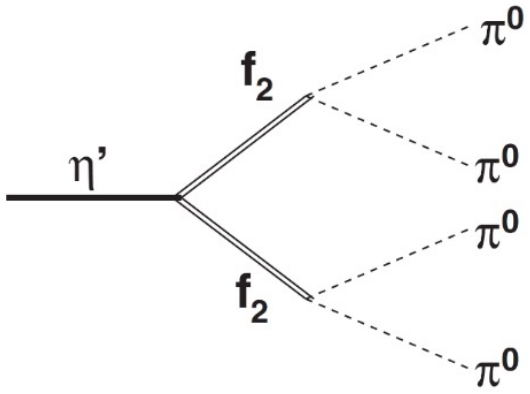
Search for rare decay $\eta' \to \pi^0\pi^0\pi^0\pi^0$

BESIII: PRD 109, 032006 (2024)

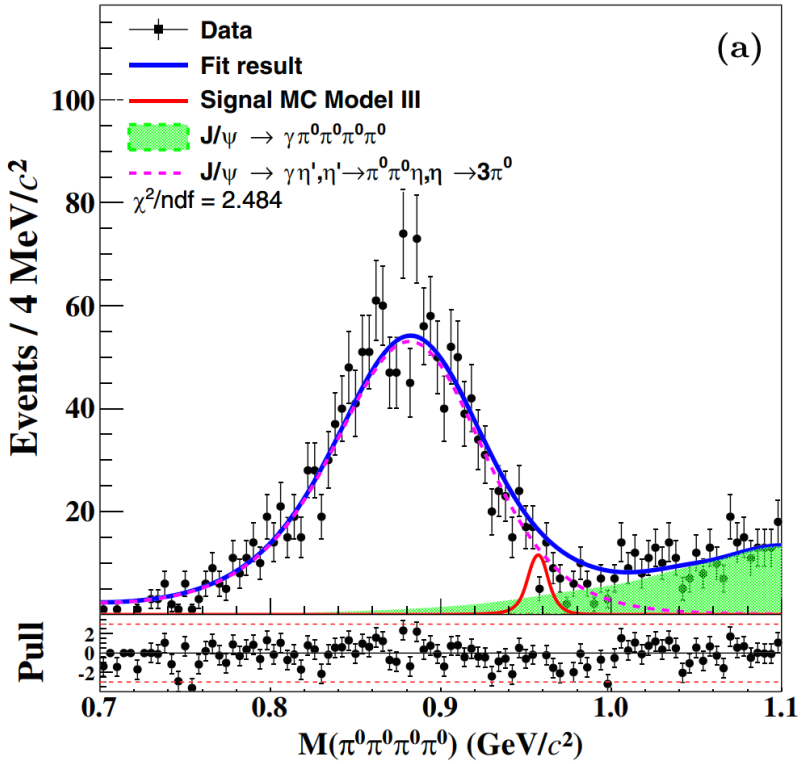
- CP-violation S-wave, induced by the QCD Lagrangian θ-term ⇒ Br~10^{-23}
- CP-conserving higher order ⇒ Br~10^{-8} F. K. Guo, B. Kubis, A. Wirzba, PRD 85,014014 (2012)



D-wave pion loop



Small contribution from two f_2 tensor mesons

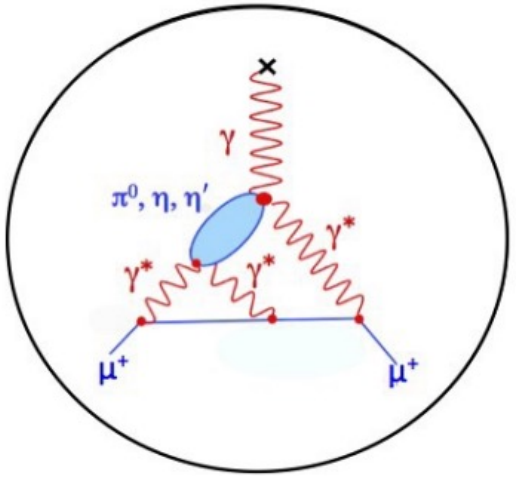


- With 10 billion J/ψ, the UL at 90% CL is set as 1.24×10^{-5}

Transition form factors

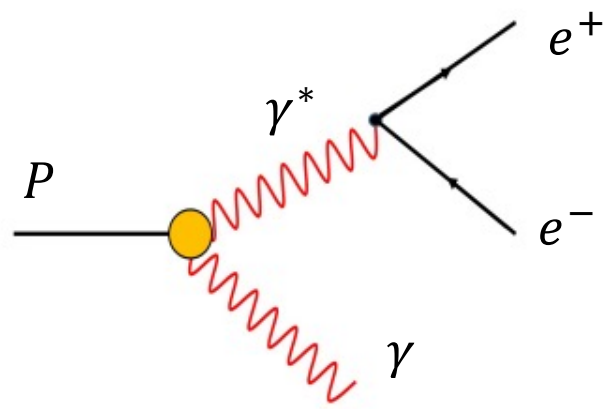
- Improved measurements of $\eta/\eta' \to \gamma e^+ e^-$ PRD 109, 072001 (2024)
- Measurement of the EM TFF in $\eta' \to \pi^+\pi^- l^+ l^-$ JHEP 07, 135 (2024)

Transition form factor at BESIII

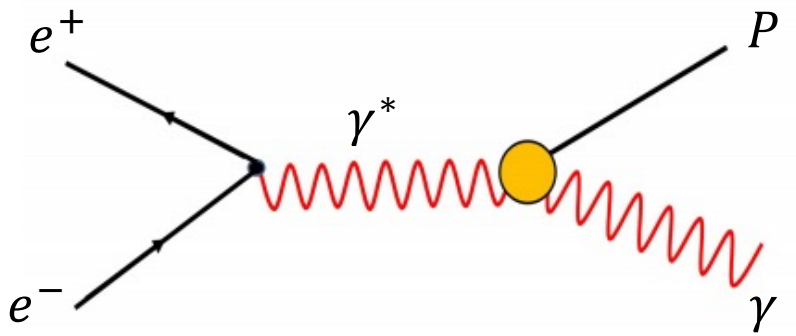


- Important input for HLbL contributions

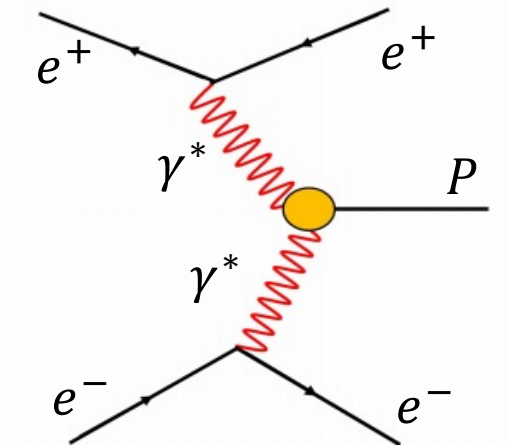
Pseudoscalar TFFs are experimentally accessible in three different processes



Dalitz decays $0<q^2<M^2$



Annihilation process $q^2>M^2$



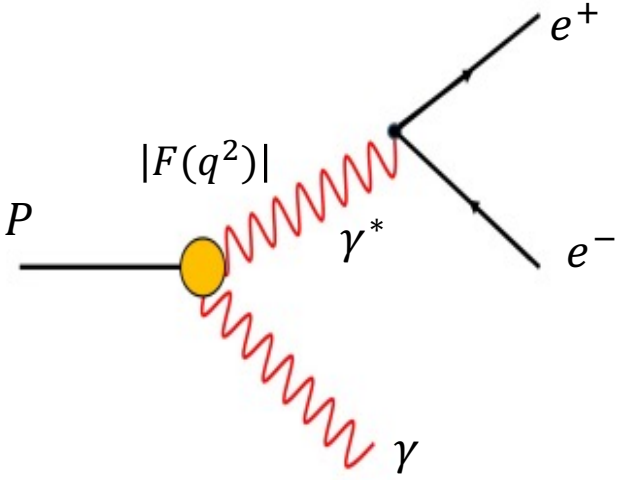
Two photon process

Transition form factor of $\eta/\eta' \to \gamma e^+ e^-$

BESIII: PRD 109, 072001 (2024)

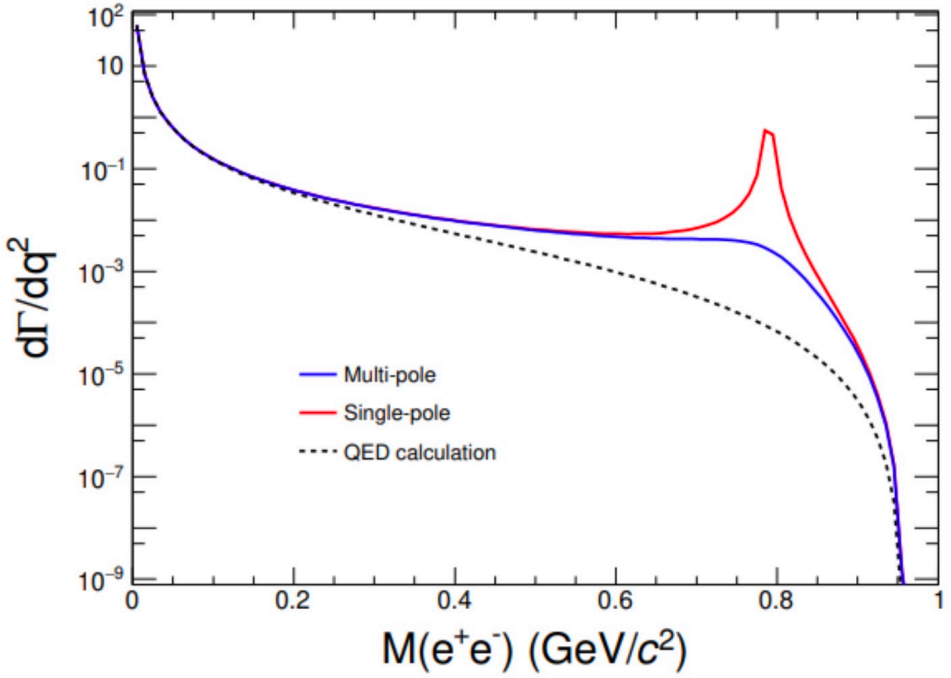
$$\frac{d\Gamma(P \to \gamma l^+ l^-)}{dq^2 \Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi}\frac{1}{q^2}\sqrt{1-\frac{4m_l^2}{q^2}}\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{q^2}{M_P^2}\right)^3 |F_P(q^2,0)|^2$$

$$= QED(q^2) \times |F_P(q^2,0)|^2$$



❖ **Single-pole model:** $F(q^2) = \dfrac{1}{1 - q^2/\Lambda^2}$

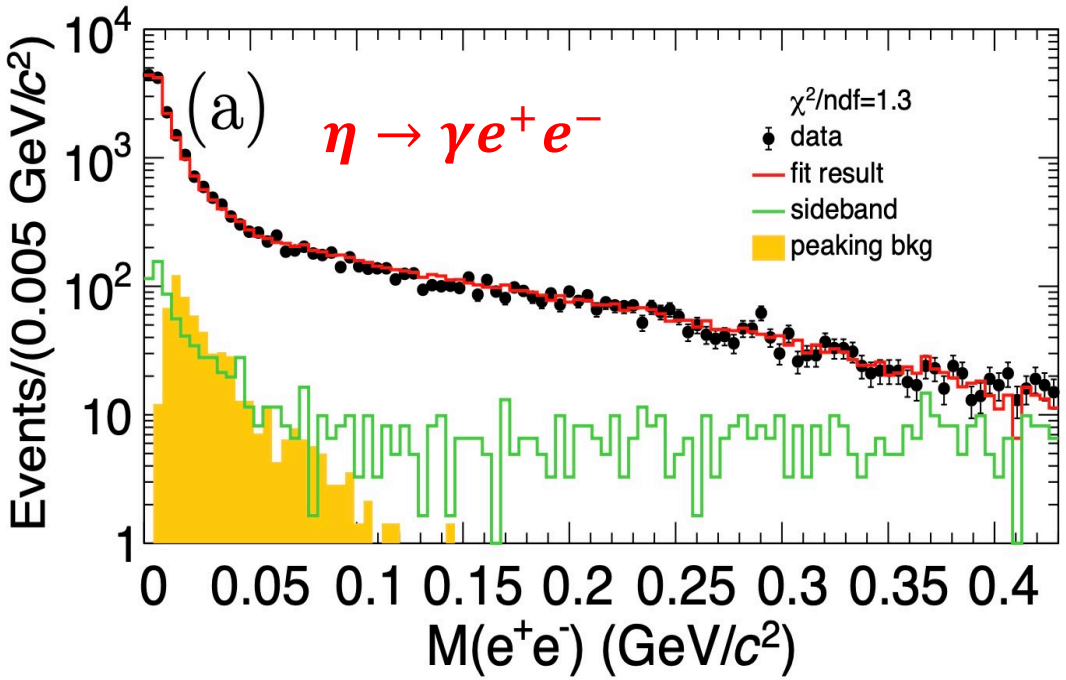
❖ **Multi-pole model:** $|F(q^2)|^2 = \dfrac{\Lambda^2(\Lambda^2+\gamma^2)}{(\Lambda^2-q^2)^2+\Lambda^2\gamma^2}$



Transition form factor of $\eta/\eta' \to \gamma e^+ e^-$

BESIII: PRD 109, 072001 (2024)

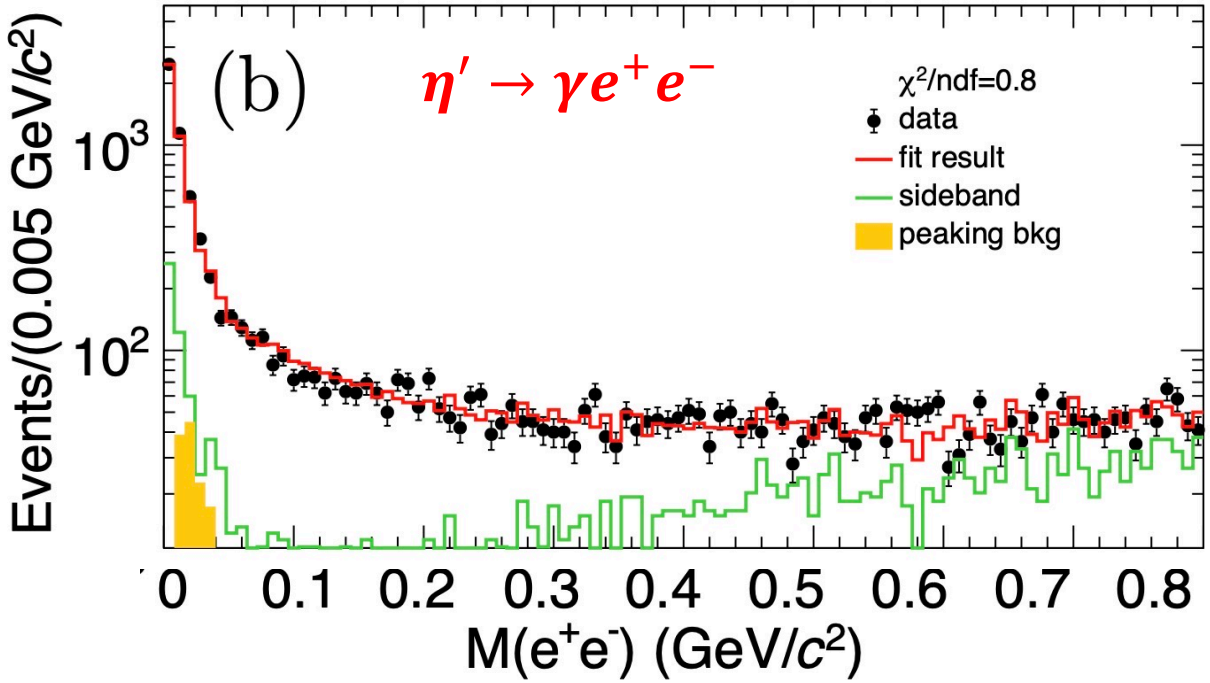
$\Lambda_\eta = (0.749 \pm 0.026 \pm 0.008)\ GeV/c^2$



$R_\eta = (0.645 \pm 0.023 \pm 0.007)\ fm$

$\Lambda_{\eta'} = (0.749 \pm 0.026 \pm 0.008)\ GeV/c^2$

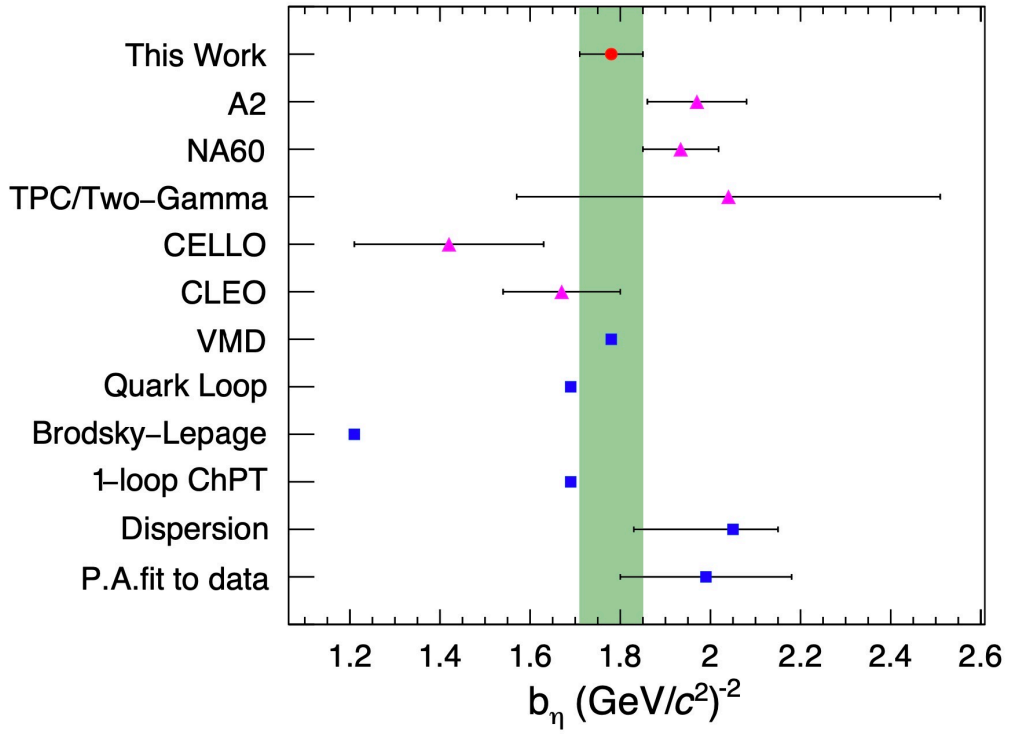
$\gamma_{\eta'} = (0.113 \pm 0.009 \pm 0.002)\ GeV/c^2$

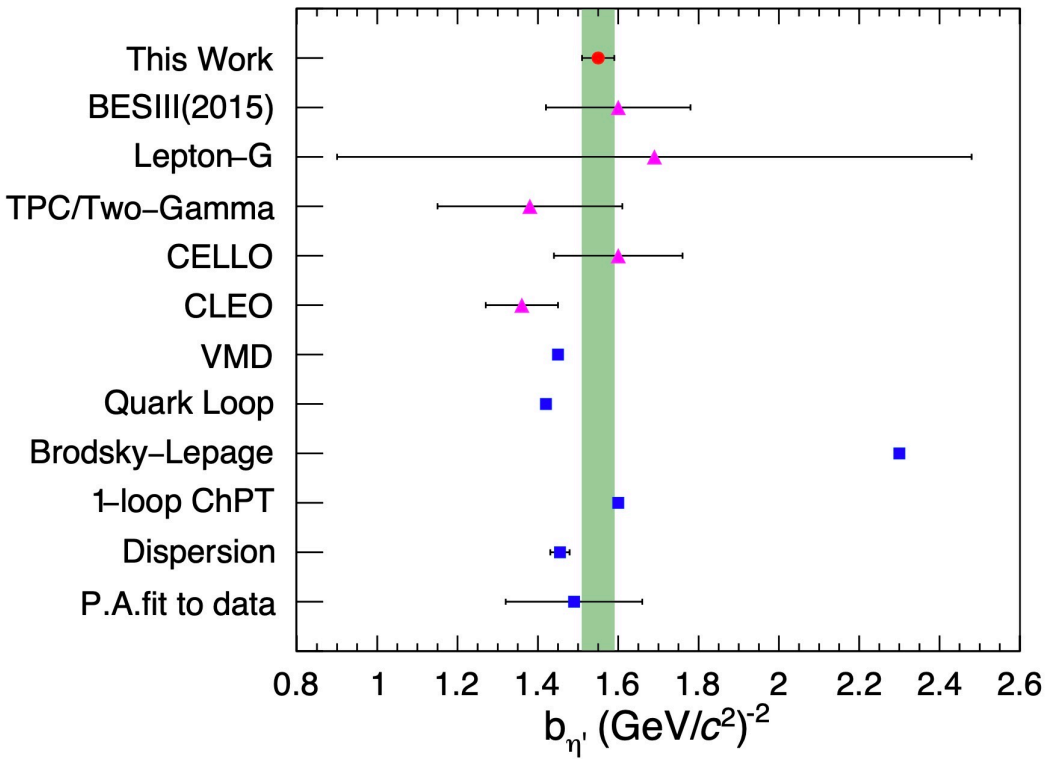


$R_{\eta'} = (0.596 \pm 0.005 \pm 0.006)\ fm$

Slope parameter: $b_{\eta/\eta'} = \frac{d|F(q^2)|}{dq^2}\Big|_{q^2=0}$

BESIII: PRD 109, 072001 (2024)



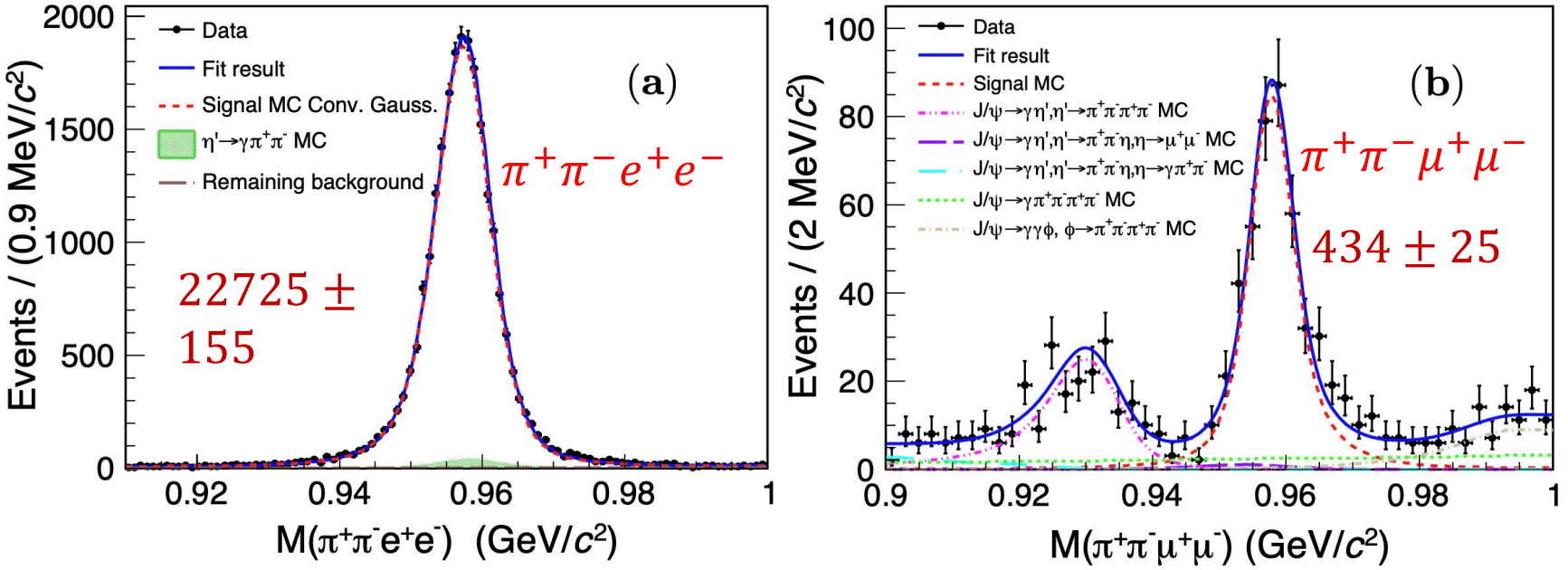


$b_\eta = 1.781 \pm 0.123 \pm 0.033\ (\mathrm{GeV/c^2})^{-2}$

$b_{\eta'} = 1.574 \pm 0.048 \pm 0.016\ (\mathrm{GeV/c^2})^{-2}$

Precision study of $\eta' \to \pi^+\pi^- l^+ l^-$

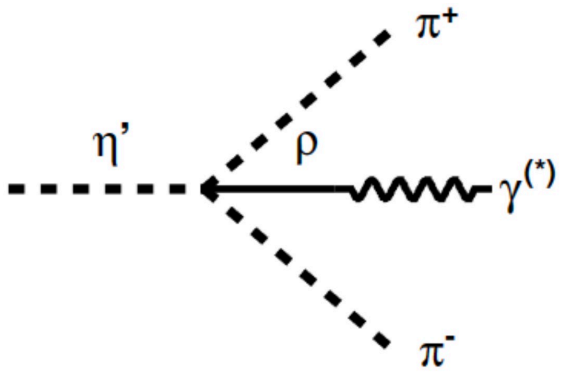
BESIII: JHEP 07, 135 (2024)

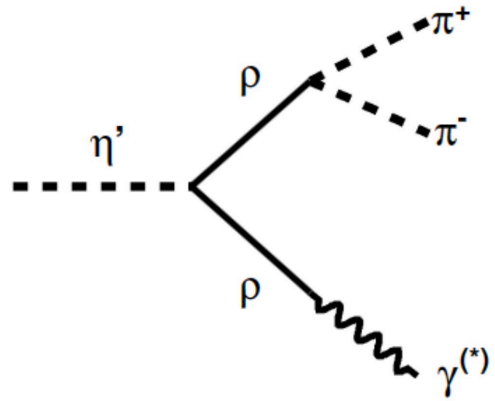


BESIII24 $2.45 \pm 0.02 \pm 0.08$ $2.16 \pm 0.12 \pm 0.06$

	$\mathcal{B}(\eta' \to \pi^+\pi^- e^+ e^-)$ (10^{-3})	$\mathcal{B}(\eta' \to \pi^+\pi^- \mu^+ \mu^-)$ (10^{-5})
Hidden gauge*	2.17 ± 0.21	2.20 ± 0.30
Unitary χPT*	$2.13^{+0.17}_{-0.31}$	$1.57^{+0.96}_{-0.75}$
VMD*	2.27 ± 0.13	2.41 ± 0.25
BESIII (2013)◇	$2.11 \pm 0.12 \pm 0.15$	< 2.9
BESIII (2021)◇	$2.42 \pm 0.05 \pm 0.08$	$1.97 \pm 0.33 \pm 0.19$
CLEO◇	$2.50^{+1.2}_{-0.9} \pm 0.5$	< 24

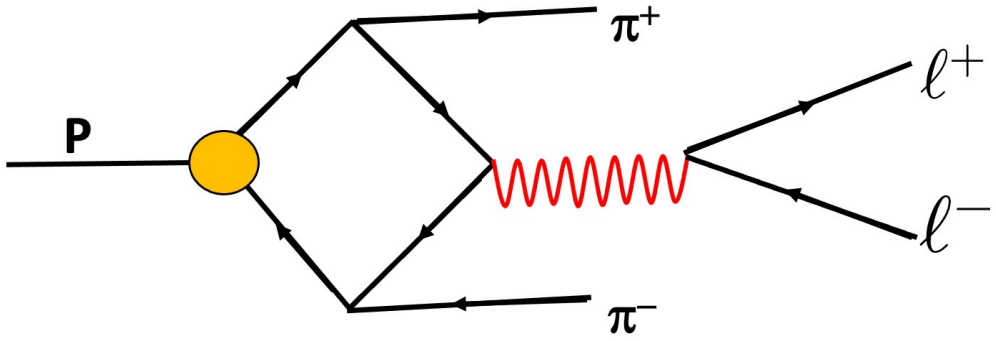
VMD Contribution

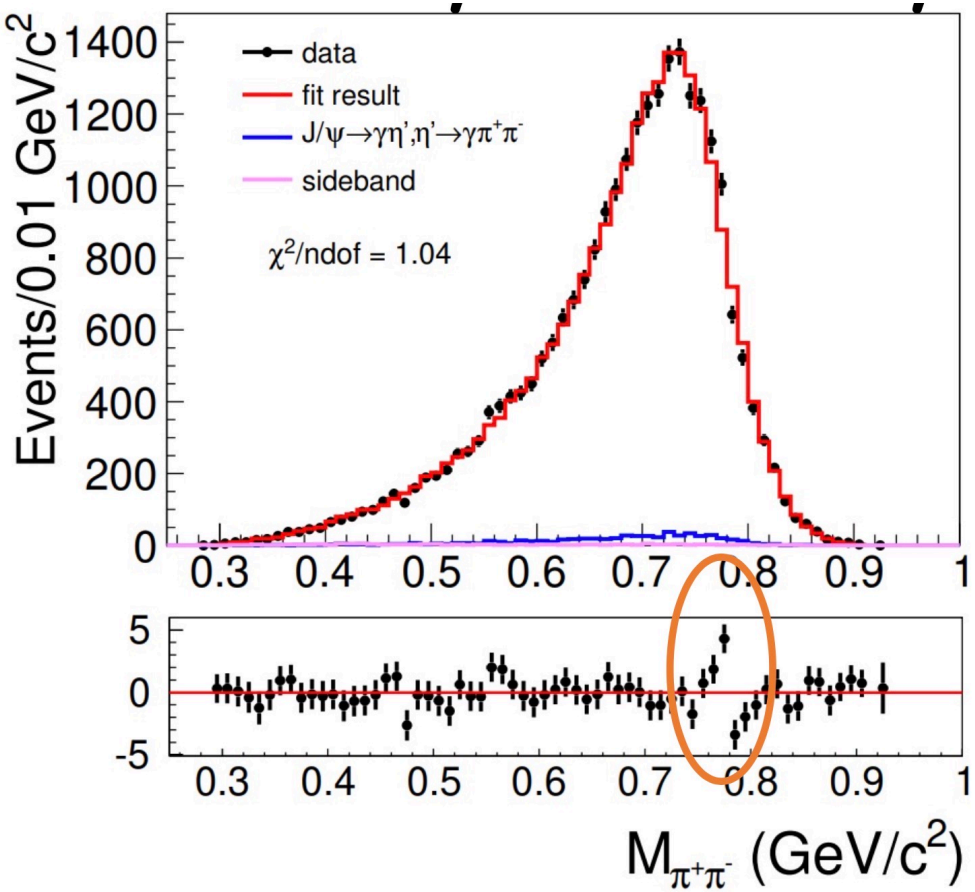




Precision study of $\eta' \to \pi^+\pi^- l^+ l^-$

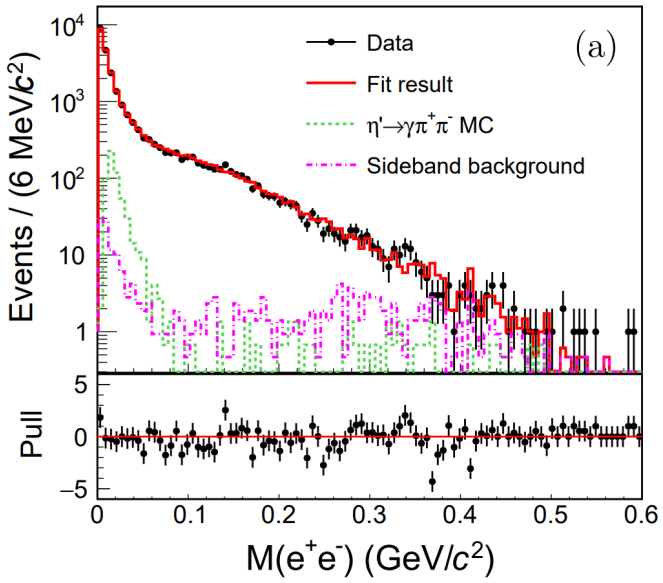
BESIII: JHEP 07, 135 (2024)

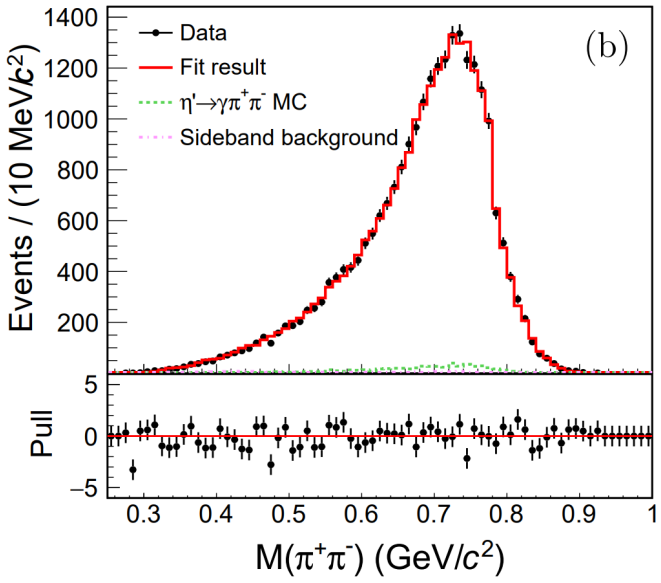


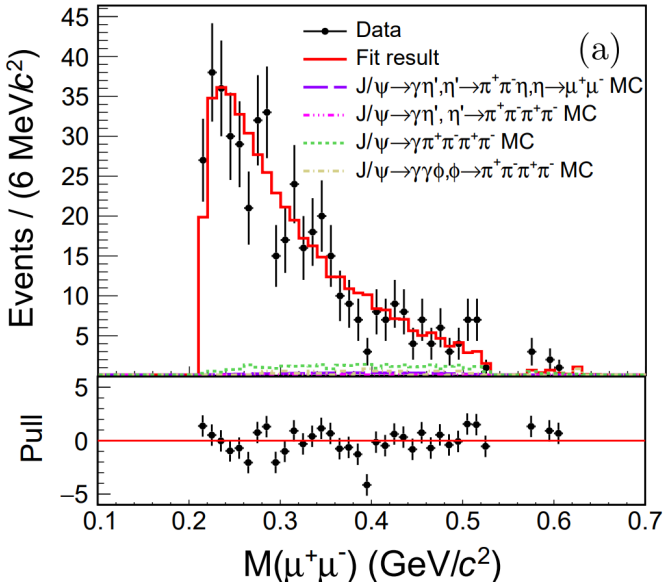


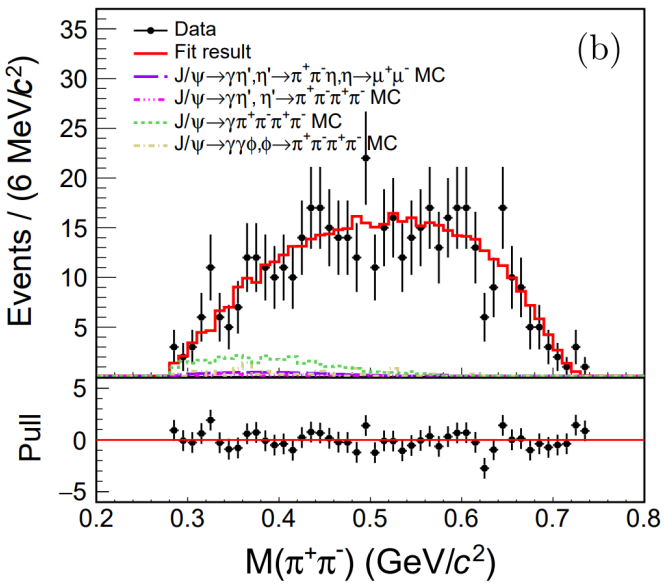
- **Box-anomaly** is needed to describe data
  - ✓ Similar structure as $\eta' \to \gamma\pi^+\pi^-$, replacing the γ with an off-shell one
- $\omega \to \pi^+\pi^-$ is also necessary

Amplitude analysis result of $\eta' \to \pi^+\pi^- l^+ l^-$ BESIII: JHEP 07, 135 (2024)



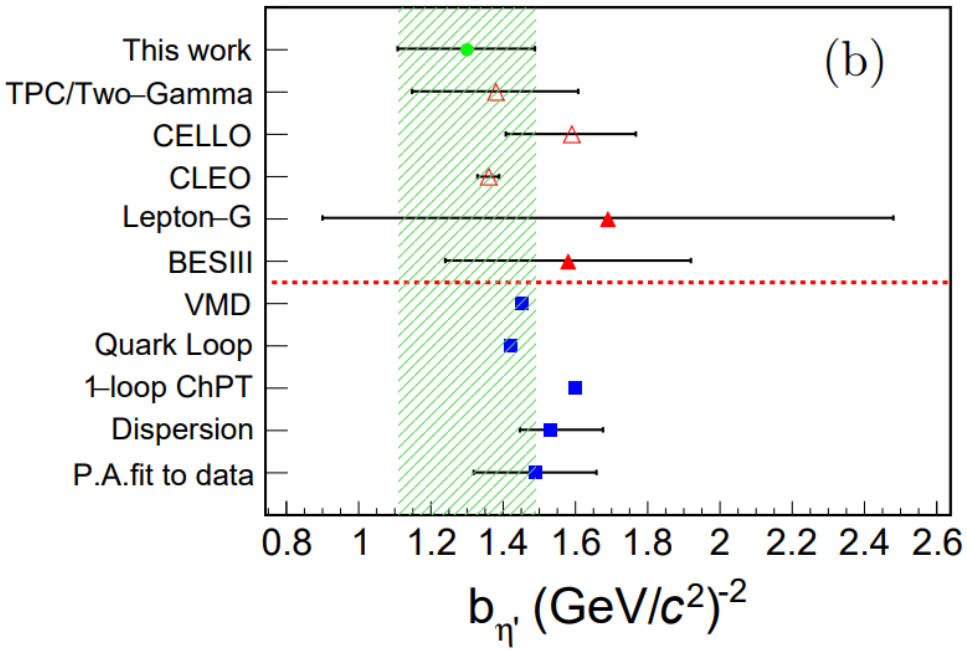






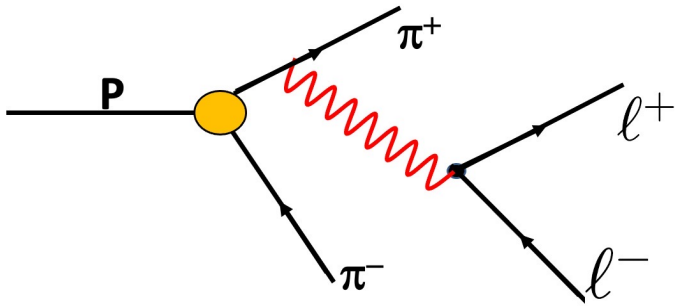
- ✓ First time to assess the form factors with $\eta' \to \pi^+\pi^- l^+ l^-$

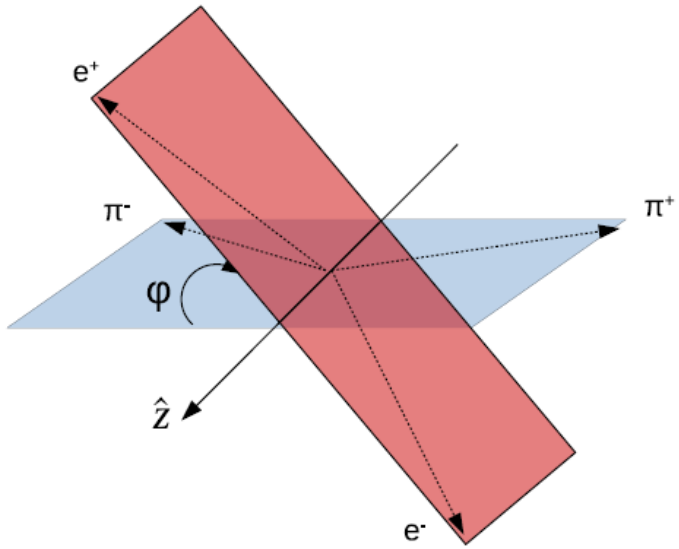
$$b_{\eta'} = 1.30 \pm 0.19 \left(\mathrm{GeV/c^2}\right)^{-2}$$



Asymmetry in $\eta' \to \pi^+\pi^- l^+ l^-$

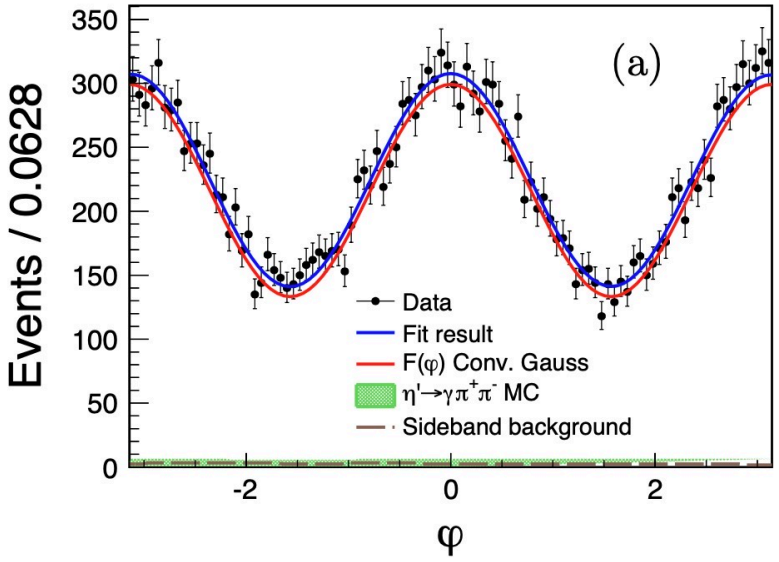
BESIII: JHEP 07, 135 (2024)





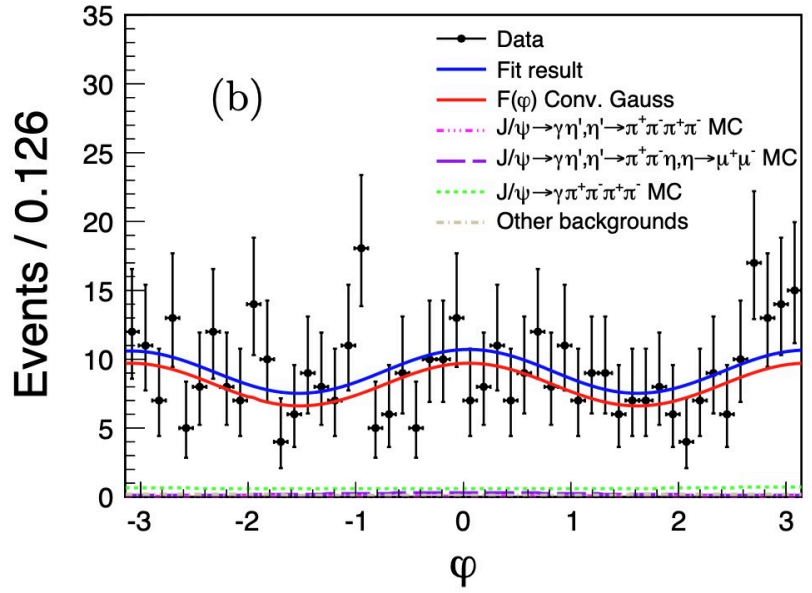
D. N. Gao, Mod Phys Lett A17 (2002) 1583
M. Zillinger, B. Kubis, P. Sánchez-Puertas, JHEP 12 (2022) 001

$\eta' \to \pi^+\pi^- e^+ e^-$



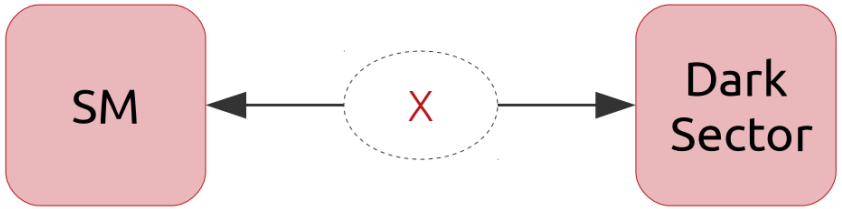
$A_{CP} = (-0.21 \pm 0.73 \pm 0.01)\%$

$\eta' \to \pi^+\pi^- \mu^+ \mu^-$



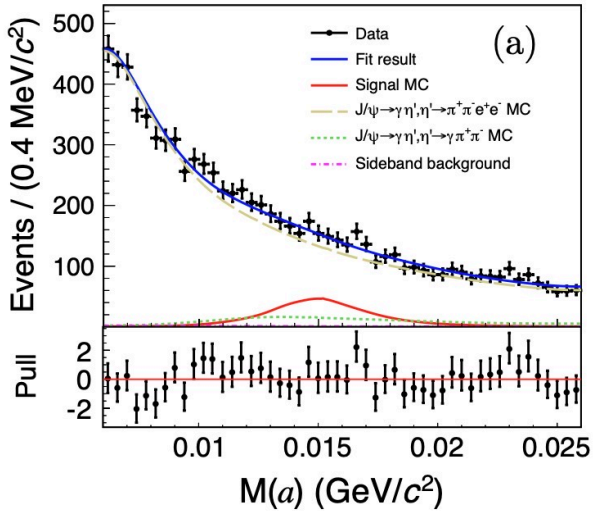
$A_{CP} = (0.62 \pm 4.71 \pm 0.08)\%$

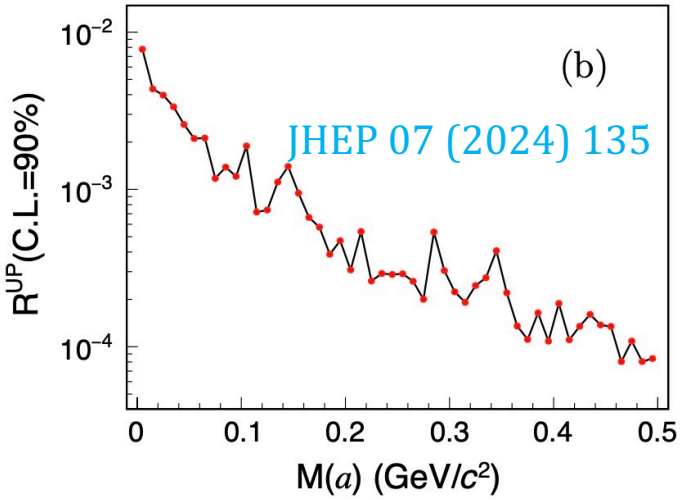
BSM Physics in Dark Sector



- ALPs in $\eta' \to \pi^+\pi^- a,\ a \to e^+e^-$
- Dark photon in $\eta/\eta' \to \gamma A', A' \to e^+e^-$

$\eta' \to \pi^+\pi^- a, a \to e^+e^-$

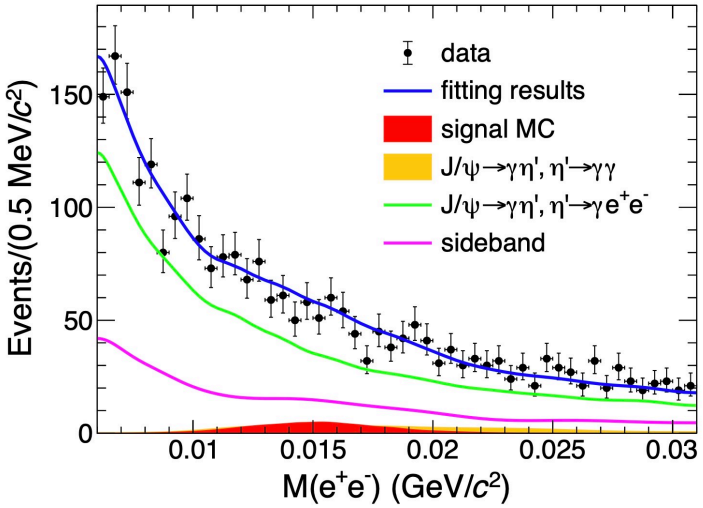


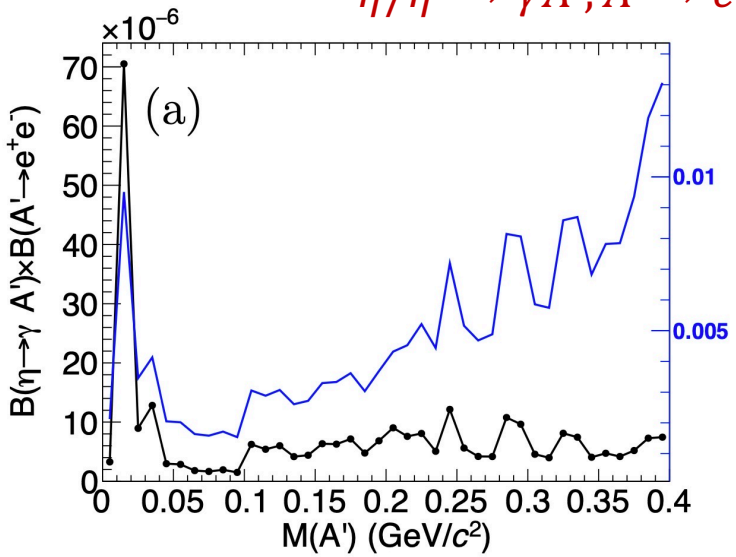


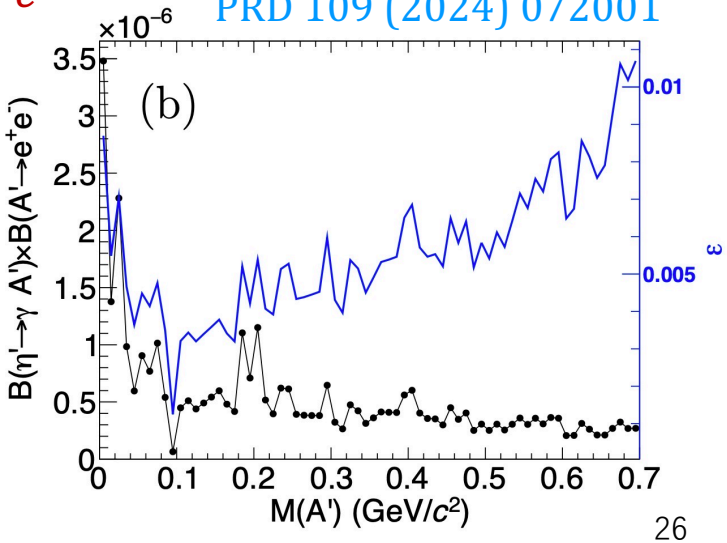
JHEP 07 (2024) 135

$\eta/\eta' \to \gamma A', A' \to e^+e^-$

PRD 109 (2024) 072001







Summary

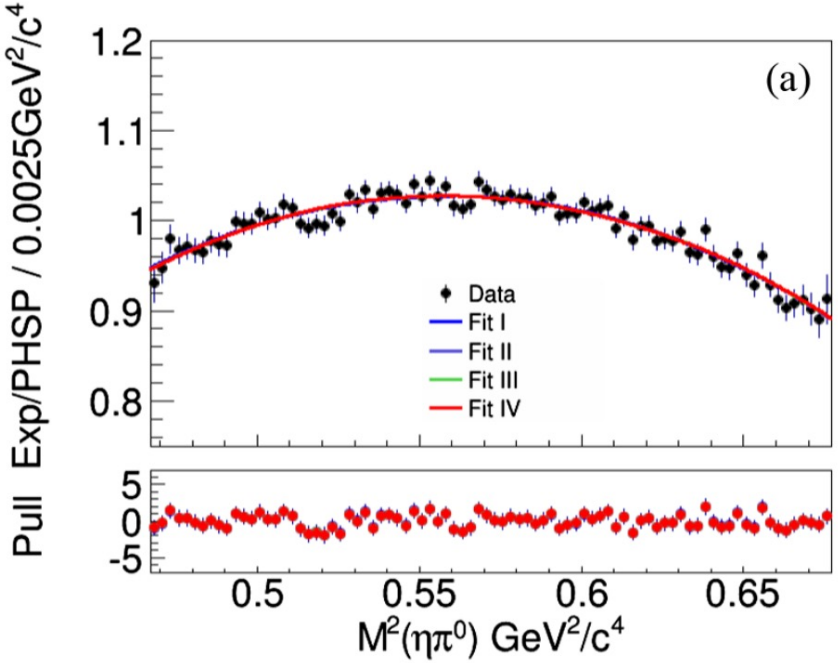
- **Worldwide unique laboratory for η/η' with unprecedented statistics**
  - ✓ **Significant progresses on decay mechanisms, TFFs, ……**
- **More results are expected to come soon**
  - ✓ **Precision measurement of $\eta' \to \eta\pi^+\pi^-$, $\eta' \to \pi^+\pi^-\pi^0$ …**
  - ✓ **Rare or forbidden decays of η**
  - ✓ **……**
- **Together with other Exps, the light meson physics will be into a precision era**

Thanks for your attention!!!

Back up

First evidence of cusp effect in $\eta' \to \pi^0\pi^0\eta$

BESIII: PRL130, 081901(2023)



Cusp effect with ~3.5 σ!

With cusp effect

Parameters	Fit I	Fit II	Fit III	Fit IV
a	$-0.075 \pm 0.003 \pm 0.001$	-0.207 ± 0.013	-0.143 ± 0.010	$-0.077 \pm 0.003 \pm 0.001$
b	$-0.073 \pm 0.005 \pm 0.001$	-0.051 ± 0.014	-0.038 ± 0.006	$-0.066 \pm 0.006 \pm 0.001$
d	$-0.066 \pm 0.003 \pm 0.001$	-0.068 ± 0.004	-0.067 ± 0.003	$-0.068 \pm 0.004 \pm 0.001$
$a_0 - a_2$	-	0.174 ± 0.066	0.225 ± 0.062	$0.226 \pm 0.060 \pm 0.012$
a_0	-	0.497 ± 0.094	-	-
a_2	-	0.322 ± 0.129	-	-
Statistical Significance	-	3.4σ	3.7σ	3.6σ

Decay Amplitude of $\eta' \to \pi^+\pi^- l^+ l^-$

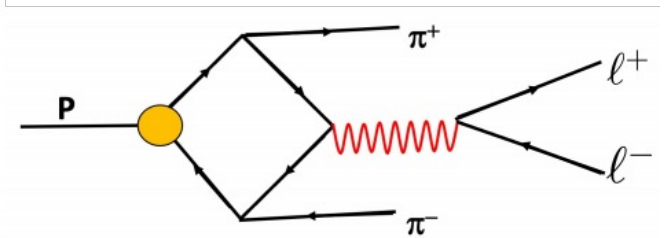
$$\overline{\left|\mathcal{A}_{\eta'\to\pi^+\pi^- l^+ l^-}\right|}^2(s_{\pi\pi}, s_{ll}, \theta_\pi, \theta_1, \phi) = \frac{e^2}{8k^2}|M(s_{\pi\pi}, s_{ll})|^2 \times \lambda\left(m_{\eta'}^2, s_{\pi\pi}, s_{ll}\right) \times \left[1 - \beta_1^2 \sin^2\theta_1 \sin^2\phi\right] s_{\pi\pi}\beta_\pi^2 \sin^2\theta_\pi$$

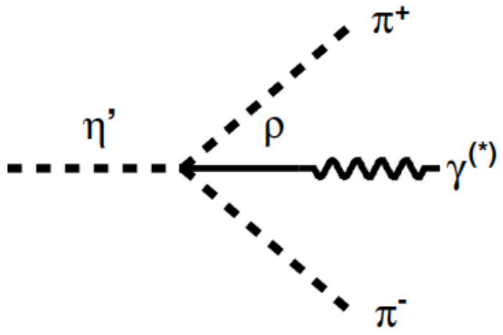
A. Faessler, C. Fuchs, M. I. Krivoruchenko, PRC 61, 035206 (2000)
B. Borasoy, R. Nissler, EPJA 33, 95 (2007)
T. Petri, arXiv:1010.2378

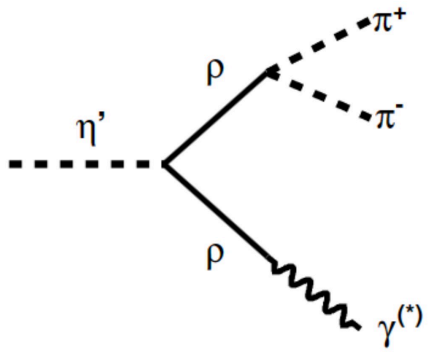
$$M(s_{\pi\pi}, s_{ll}) = M_{mix} \times VMD(s_{\pi\pi}, s_{ll})$$

contains the information of the decaying particle and the form factor

$$VMD(s_{\pi\pi}, s_{ll}) = \underbrace{1 - \frac{3}{4}(c_1 - c_2 + c_3)}_{\text{Box anomaly}} + \underbrace{\frac{3}{4}(c_1 - c_2 - c_3)\frac{m_V^2}{m_V^2 - s_{ll} - i m_V \Gamma(s_{ll})}}_{\text{VMD contribution}} + \underbrace{\frac{3}{2}c_3 \frac{m_V^2}{m_V^2 - s_{ll} - i m_V \Gamma(s_{ll})} \frac{m_{V,\pi}^2}{m_{V,\pi}^2 - s_{\pi\pi} - i m_{V,\pi}\Gamma(s_{\pi\pi})}}_{\text{VMD contribution}}$$







Various VMD models can be switch by adjusting the $c_{1,2,3}$ values

Amplitude analysis result of $\eta' \to \pi^+\pi^- l^+ l^-$

BESIII: JHEP 07, 135 (2024)

	Hidden gauge	Full VMD	Modified VMD
$\eta' \to \pi^+\pi^- e^+ e^-$	Model I	Model II	Model III
	$c_1 - c_2 = c_3 = 1$	$c_1 - c_2 = 1/3, c_3 = 1$	$c_1 - c_2 \neq c_3$
$m_V(\mathrm{MeV}/c^2)$	$954.3 \pm 87.8 \pm 36.4$	857.4 ± 76.5	787.5 ± 173.9
$m_{V,\pi}(\mathrm{MeV}/c^2)$	$765.3 \pm 1.2 \pm 20.2$	765.4 ± 1.2	764.8 ± 1.3
$m_\omega(\mathrm{MeV}/c^2)$	$778.7 \pm 1.3 \pm 17.3$	778.7 ± 1.3	778.7 ± 1.4
$\beta(10^{-3})$	$8.5 \pm 1.4 \pm 0.7$	8.5 ± 1.4	8.1 ± 1.5
θ	$1.4 \pm 0.3 \pm 0.1$	1.4 ± 0.3	1.4 ± 0.3
$c_1 - c_2$	1	$1/3$	-0.03 ± 1.09
c_3	1	1	1.03 ± 0.03
$\chi^2/ndof(e^+e^-, \pi^+\pi^-)$	77.9/82.0, 47.8/65.0	78.7/82.0, 47.6/65.0	79.4/82.0, 45.1/65.0
$b_{\eta'}(\mathrm{GeV}/c^2)^{-2}$	$1.10 \pm 0.20 \pm 0.07$	1.36 ± 0.24	1.61 ± 0.71

	Hidden gauge	Full VMD	Modified VMD
$\eta' \to \pi^+\pi^- \mu^+ \mu^-$	Model I	Model II	Model III
	$c_1 - c_2 = c_3 = 1$	$c_1 - c_2 = 1/3, c_3 = 1$	$c_1 - c_2 \neq c_3$
$m_V(\mathrm{MeV}/c^2)$	$649.4 \pm 55.9 \pm 35.6$	601.6 ± 25.7	589.6 ± 25.9
$m_{V,\pi}(\mathrm{MeV}/c^2)$	$757.3 \pm 24.1 \pm 18.0$	765.4 ± 18.8	774.4 ± 43.5
$c_1 - c_2$	1	$1/3$	0.01 ± 0.45
c_3	1	1	0.98 ± 0.40
$\chi^2/ndof(\mu^+\mu^-, \pi^+\pi^-)$	48.1/34.0, 32.9/46.0	48.3/34.0, 32.9/46.0	49.7/35.0, 32.4/46.0
$b_{\eta'}(\mathrm{GeV}/c^2)^{-2}$	$2.37 \pm 0.41 \pm 0.27$	2.76 ± 0.24	2.88 ± 0.25