Global Data-Driven Determination of Baryon Transition Form Factors

The 9th Workshop on Chiral Effective Field Theory

@ Changsha

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Philosophy

Part I: Introduction

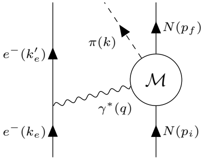
- Structures of matters $\rightarrow$ fundamental task of physics
- Structures of hadrons at middle energies $\rightarrow$ extremely difficult
  - Not direct observables
  - Non-perturbative nature
  - Uncertainties of the hadron spectroscopy
- A bridge between the data and the structures?
  World data $\rightarrow$??? $\rightarrow$ Spectra $\rightarrow$ Indications of the structures
- The bridge here: Comprehensive models
  $\rightarrow$ the Jülich-Bonn/Jülich-Bonn-Washington Model for N^* and Δ
- This work $\rightarrow$ **electromagnetic transition form factors (TFFs) of the nucleon to N^* and Δ's**
  [Y.F. Wang *et. al.* (Jülich-Bonn-Washington Collaboration), Phys. Rev. Lett. 133, 101901 (2024)]

Baryon Transition Form Factors

Part I: Introduction

Electromagnetic Probes

- EM interactions $\to$ clean probes of structures
- $\gamma N \to$ states coupling weakly to πN
  [Ireland, Pasyuk, Strakovsky, Prog. Part. Nucl. Phys. 111, 103752 (2020)]
- Electroproduction $\to$ energy scale $Q^2 \equiv -q^2$
- Transition form factors (TFFs) $\to$ "pictures of hadrons"
  [Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]
  - Lower Q^2: meson clouds etc.
  - Higher Q^2: the quark core
  - Related to quark transverse charge densities
    [Tiator & Vanderhaeghen, PLB 672, 344 (2009)]



Towards the TFFs

- Predictions at quark level
  - Quark models & Dyson-Schwinger equations
    [Segovia *et. al.*, Few Body Syst. 55, 1185 (2014)]
    [Burkert, Roberts RMP 91, 011003 (2019)]
    [Eichmann *et. al.*, Prog. Part. Nucl. Phys. 91, 1 (2016)]
  - Lattice QCD [Agadjanov *et. al.*, NPB 886, 1199 (2014)]
- Extraction from data
  - Experimental facilities & data accumulation
    [Mokeev *et. al.*., PRC 93, 025206 (2016)]
  - Unitary isobar models $\to$ real valued, depending on BW parameters
    [Drechsel, Kamalov, Tiator, EPJA 34, 69 (2007)]
    [Tiator *et. al.*, EPJST 198, 141 (2011)]
  - Dynamical models $\to$ complex TFFs at the poles
    [Kamano, Few Body Syst. 59, 24 (2018)]

  & **this work**

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Part II: Methodology

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The Jülich-Bonn Model

Part II: Methodology

A comprehensive coupled-channel model, fitting to a worldwide collection of data

Hadronic part ($\pi N \to \cdots$)

- Early origins $\to$ studies of $K^- N$ and $\pi\pi$
  [Müller-Groeling *et. al.*, NPA 513, 557 (1990)] [Lohse *et. al.*, NPA 516, 513 (1990)][Pearce *et. al.*, NPA 541, 663 (1992)]
- The πN elastic scatterings [Schütz *et. al.*, PRC 51, 1374 (1995)] [Schütz *et. al.*, PRC 49, 2671 (1994)]
- Extended to $\pi\pi N$ and ηN [Schütz *et. al.*, PRC 57, 1464 (1998)] [Krehl *et. al.*, PRC 62, 025207 (2000)] [Gasparyan *et. al.*, PRC 68, 045207 (2003)]
- Extended to $K\Lambda$ and $K\Sigma$ [Döring *et. al.*, NPA 851, 58 (2011)] [Rönchen *et. al.*, EPJA 49, 44 (2013)]
- Extended to ωN [Wang *et. al.*, PRD 106, 094031 (2022)]
- Analytical continuation for searching poles [Döring *et. al.*, NPA 829, 170 (2009)]

Photo- & Electroproduction

- Photoproduction
  [Rönchen *et. al.*, EPJA 50, 101 (2014)] [Rönchen *et. al.*, EPJA 51, 70 (2015)][Rönchen *et. al.*, EPJA 54, 110 (2018)] [Rönchen *et. al.*, EPJA 558, 229 (2022)]
- Electroproduction (Jülich-Bonn-Washington Model)
  [Mai *et. al.*, PRC 103, 065204 (2021)] [Mai *et. al.*, PRC 106, 015201 (2022)] [Mai *et. al.*, EPJA 59, 286 (2023)]

Formulations

Part II: Methodology

Hadron dynamics

Lippmann-Schwinger-like equation

$$T_{\mu\nu}(p'',p',z) = V_{\mu\nu}(p'',p',z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'',p,z) G_\kappa(p,z) T_{\kappa\nu}(p,p',z)$$

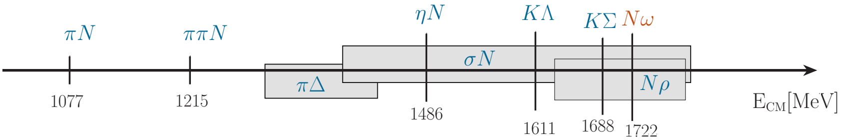
- One-dimensional: time-ordered perturbation theory $+$ JLS basis [Jacob & Wick, Annals Phys. 7, 404 (1959)]
- $T = T^P + T^{NP} \to$ s-channel vertices $+$ t/u-channel exchanges etc.
- $V \to$ SU(3), **ChEFT**, CP...
- Effective three-body channels: $\rho N, \sigma N, \pi\Delta$

Photo- & electroproduction

Construction from Watson's final state theorem

$$M_{\mu\gamma^*}(Q^2) = V_{\mu\gamma^*}(Q^2) + \sum_\kappa \int p^2 dp T_{\mu\kappa} G_\kappa V_{\kappa\gamma^*}(Q^2)$$

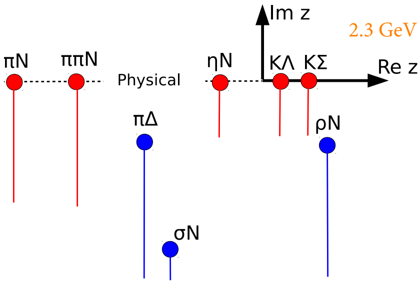
- γ^*: the $\gamma^* N$ channel for electroproduction
- Q^2: photon virtuality
- $V_{\kappa\gamma^*} \to$ phenomenologically parameterized
- Further constraints: Siegert's theorem (gauge invariance), kinematics, etc.
- Photoproduction $\to Q^2 = 0$

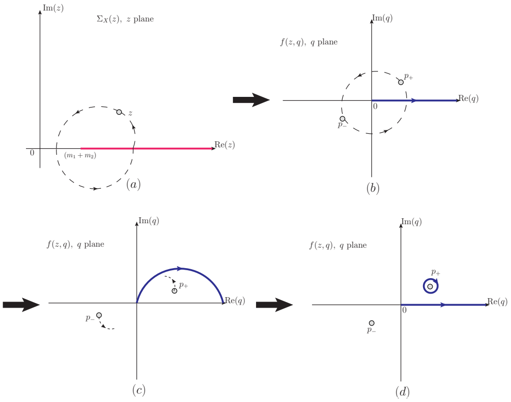


Pole searching

Part II: Methodology

- Resonances $\rightarrow$ poles on the second Riemann sheet
- Analytical continuation $\rightarrow$ contour deformation
- Pole position $z_r = M_r - i\Gamma_r/2$





$p_{\pm}$: singularities in $G = (z - E_1 - E_2)^{-1}$

Transition form factors

Part II: Methodology

Origianl definition

[Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]

$$A_h = \sqrt{\frac{2\pi\alpha}{K}} \left\langle R, h \middle| \epsilon_+ \cdot J \middle| N, h-1 \right\rangle$$

$$S_{\frac{1}{2}} = \frac{|\mathbf{q}|}{Q} \sqrt{\frac{2\pi\alpha}{K}} \left\langle R, \tfrac{1}{2} \middle| \epsilon_0 \cdot J \middle| N, \tfrac{1}{2} \right\rangle$$

- A, S: helicity transition amplitudes
- $h = 1/2, 3/2$: the helicity
- α: fine structure constant
- $\epsilon(J)$: virtual photon polarization vector (current)
- $\mathbf{q}$: 3-momentum of the virtual photon
- $M_R(m_N)$: mass of the excitation state R (nucleon)
- $K = (M_R^2 - m_N^2)/(2M_R)$

At the pole

[Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]

$$H_h = C_I \sqrt{\frac{p_{\pi N}}{\omega_0} \frac{2\pi(2J+1)z_p}{m_N \widetilde{R}}} \widetilde{\mathcal{H}}_h$$

- H is either A or S
- C_I: isospin factor, $C_{1/2} = -\sqrt{3}$ and $C_{3/2} = \sqrt{2/3}$
- $p_{\pi N}$: πN c.m. momentum
- ω_0: photon energy at $Q^2 = 0$
- $z_p = M_R - i\Gamma_R/2$ the pole position
- $\widetilde{R}$, $\widetilde{\mathcal{H}}$: the residues of πN, $\gamma^* N$ channels
- Understanding: the $|R\rangle \to |R)$ Gamow state

  [Gamow, Zeitschrift für Physik 51, 204 (1928)]
- Complex-valued

Numerical details

Part II: Methodology

The latest JBW results

- $\gamma^* p$ initial state
- Coupled-channel study of πN, ηN, and $K\Lambda$
  [Mai *et. al.*, EPJA 59, 286 (2023)]
- Based on the JüBo2017 solution
  [Rönchen *et. al.*, EPJA 54, 110 (2018)]
- C.M. energy range $z \in [1.13, 1.8]$ GeV
- Virtuality $Q^2 \in [0, 8]$ GeV^2
- Orbital angular momentum $L \leq 3$

Database & errors

- Database [Mai *et. al.*, EPJA 59, 286 (2023)]
  - 10^5 data points vs 533 fit parameters
  - 5×10^4 data points from the photoproduction/hadronic part
- Four solutions $\longrightarrow$ fully explored parameter space
  - weighted vs unweighted χ^2
  - different local minima
- To solve $\longrightarrow$ matrix inversion
- Fit $\longrightarrow$ supercomputers

	χ^2_{dof}	$\chi^2_{\text{pp}}(\pi^0 p)$	$\chi^2_{\text{pp}}(\pi^+ n)$	$\chi^2_{\text{pp}}(\eta p)$	$\chi^2_{\text{pp}}(K^+\Lambda)$
FIT$_1$	1.42	1.40	1.47	1.49	0.70
FIT$_2$	1.35	1.38	1.35	1.40	0.58
	$\chi^2_{\text{wt,dof}}$	$\chi^2_{\text{pp}}(\pi^0 p)$	$\chi^2_{\text{pp}}(\pi^+ n)$	$\chi^2_{\text{pp}}(\eta p)$	$\chi^2_{\text{pp}}(K^+\Lambda)$
FIT$_3$	1.12	1.44	1.61	1.08	0.33
FIT$_4$	1.06	1.42	1.44	1.09	0.32

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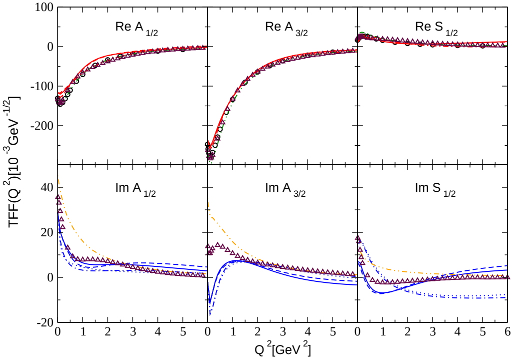
Results of $\Delta(1232)$

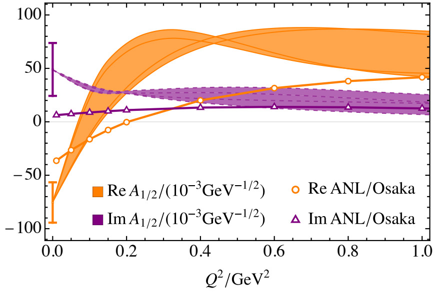
Part III: Results & Discussions

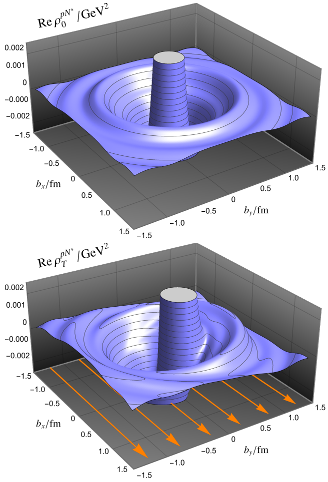
- Solid, dashed, dotted, dash-dotted curves: four fit solutions
- Dash-double-dotted curves: "L+P" extraction from MAID analyses [Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]
- Triangles: ANL-Osaka[Kamano, Few Body Syst. 59, 24 (2018)]



Results of $N^*(1440)$

- A zero crossing!!
- ρ_0, ρ_T [Tiator & Vanderhaeghen, PLB 672, 344 (2009)]





Summary of the results

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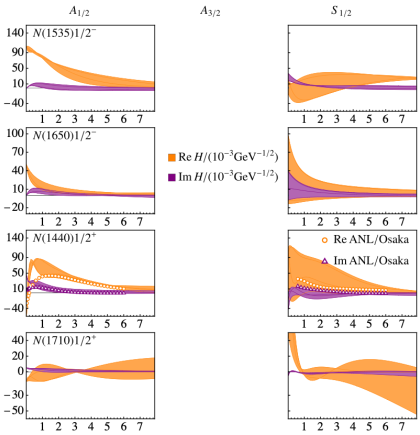
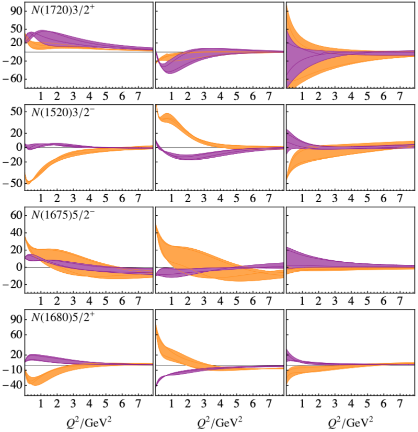
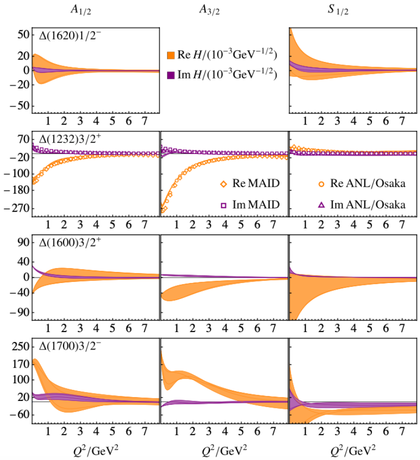




N(1720)3/2+
N(1520)3/2−
N(1675)5/2−
N(1680)5/2+
Q²/GeV²
Q²/GeV²
Q²/GeV²

Summary of the results

Part III: Results & Discussions



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Conclusion & Outlook

Part IV: Conclusion & Outlook

Conclusions

- The Jülich-Bonn(-Washington) Model
  - Comprehensive dynamical coupled-channel approaches
  - Data driven PWA $\longrightarrow$ resonance spectra
  - Connecting experimental observations to hadron structures!
- EM transition from factors of N^* and Δ's
  - First time determined by multi-channel data
  - Defined at the poles
  - Realistic uncertainties
  - Outputs for twelve states

Outlook

- A extension of the model
  - energy range up to 1.95 GeV
  - TFFs of higher states
  - more outputs of the transition charge densities
- ωN photoproduction underway $\longrightarrow$ more modern data!
- Other studies of the structure

  $\longrightarrow$ Weinberg's criterion & extension
  - study of the N^* and Δ states (already done!) [Wang *et. al.*, PRC 109, 015202 (2024)]
  - study of the P_c states $\longrightarrow$ underway [Shen *et. al.*, EPJC 84, 764 (2024)]
  - hyperons...


Thank you

Backups

Details of the scattering equation

The Lippmann-Schwinger-like equation

$T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'', p, z) G_\kappa(p, z) T_{\kappa\nu}(p, p', z)$

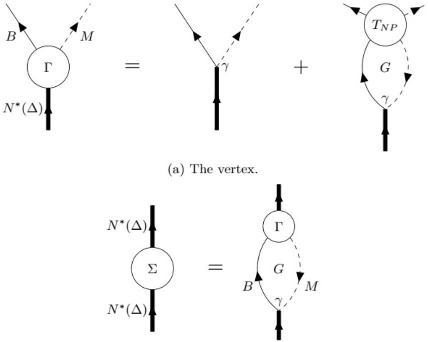
- Reaction channels $\nu \to \mu$ (after PW and isospin projection, JLS basis [Jacob & Wick, Annals Phys. 7, 404 (1959)],$J \leq 9/2$)
- Intermediate channel: κ
- CM initial (final) momentum: $p'(p'')$. CM energy: z
- Potential (kernel): V. Amplitude: T ($\to$ observables)
- Propagator: G ($\pi\pi N$ channel: effective channels $\rho N, \sigma N, \pi\Delta$. E/ω - energy of the baryon/meson.)

$$G_\kappa(z, p) = \begin{cases} (z - E_\kappa - \omega_\kappa + i0^+)^{-1} & \text{(if } \kappa \text{ is a two-body channel)}\,, \\ \left[z - E_\kappa - \omega_\kappa - \Sigma_\kappa(z, p) + i0^+\right]^{-1} & \text{(if } \kappa \text{ is an effective channel)}\,. \end{cases}$$

Details of the scattering equation

Backups

- Separating the amplitude $\rightarrow$ with/without s-channel poles $T = T^P + T^{NP}$
- Reconstruction of the amplitude $\rightarrow T^{NP} = V^{NP} + \sum \int p^2 dp V^{NP} G T^{NP}$,
  $T^P_{\mu\nu}(p'', p', z) = \sum_{i,j} \Gamma^a_{\mu,i}(p'') D_{ij}(z) \Gamma^c_{\nu,j}(p')$, $(D^{-1})_{ij} = \delta_{ij}(z - m_i^b) - \Sigma_{ij}(z)$
  - $\Gamma(\gamma)$: the dressed (bare) vertices (a - annihilation, c - creation)
  - Σ: coupled-channel self-energy functions of the s-channel states



(a) The vertex.

(b) The self energy.

Details of the scattering equation

Backups

Potentials $\longrightarrow$ field-theoretical construction

Parameters $\longrightarrow$ determined by fits

The NP part

- Tree-level potentials
  - t-channel $+$ u-channel $+$ contact
  - Stemming from effective Lagrangians $\longrightarrow$ SU(3) flavour symmetry, CP conservation, chiral symmetry
  - Established by time-ordered perturbation theory (TOPT) $\longrightarrow$ stationary perturbation in Schrödinger picture
  - TOPT+partial wave $\longrightarrow$ one-dimensional integral $\int p^2 dp$
  - Regulators for every vertex $\longrightarrow$ to make the integral converge: $F(q) \sim \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + q^2}\right)^n$

    m: the mass of the exchanged particle. Λ: cut-off (fit parameter)
- Beyond tree-level $\longrightarrow$ correlated two-pion exchanges [Schütz *et. al.*, PRC 49, 2671 (1994)] [Schütz *et. al.*, PRC 51, 1374 (1995)]

The P part

- Stemming from effective Lagrangians with CP conservation (tree-level bare vertices)
- Phenomenological contact terms $\longrightarrow D \sim (1 - \Sigma)^{-1}$ [Rönchen *et. al.*, EPJA 51, 70 (2015)]
- Renormalization of the nucleon mass

Phenomenological parameterizations

Backups

- Details
  - Hadronic part: [Wang *et. al.*, PRD 106, 094031 (2022)]
  - Photoproduction: [Rönchen, Döring, and Meißner, EPJA 54, 110 (2018)]
  - Electroproduction: [Mai *et. al.*, EPJA 59, 286 (2023)]
- Transverse $\gamma^* N$ potentials:

  $V_{\gamma^*\mu} = \alpha_{\gamma^*\mu} + \sum_i \frac{\gamma^a_{\mu i}\gamma^c_{\gamma^* i}}{W - m^b_i}\,, \quad \alpha_{\gamma^*\mu} = \tilde{F}_\mu(Q^2)\alpha_{\gamma\mu}\,, \quad \gamma^c_{\gamma^* i} = \tilde{F}_i(Q^2)\gamma^c_{\gamma i}$
  - $\tilde{F} \to$ Exponential $\times$ polynomial $\times$ Woods-Saxon FF $(1 + Q^2/(0.71\mathrm{GeV}^2))^{-1}$
  - $\alpha_{\gamma\mu}, \gamma_{\gamma i} \to$ Exponential $\times$ polynomial
- Longitudinal $\gamma^* N$ potentials $\to$ constructed from transverse ones with constraints from Siegert's theorem

  [Siegert, PR 52, 787 (1937)]

$$\frac{E_{l+}}{L_{l+}}\bigg|_{\mathrm{PT}-} = 1\,, \quad \frac{E_{l-}}{L_{l-}}\bigg|_{\mathrm{PT}-} = \frac{l}{1-l}\;(l \neq 1)$$

  "PT-": $Q^2 = -(W - m_N)^2$
- Kinematic constraints: Multipoles $M \to RM$ with R the Blatt-Weisskopf barrier-penetration factor

  [J. Blatt, V. Weisskopf, Theoretical Nuclear Physics, John Wiley & Sons, New York, 1952]

Transverse charge distributions: definition
Backups

[Tiator & Vanderhaeghen, PLB 672, 344 (2009)]

- The light front frame:
  - Large momentum along $P = (p_{N^*} + p_N)/2$ (as z-axis)
  - Light front component $v^{\pm} \equiv v^0 \pm v^3$
  - Symmetric frame $q^+_{\gamma^*} = 0$, the transverse component on xOy plane $\mathbf{q}^2_\perp = Q^2$
- The transverse charge density for the transition:

$$\rho(\mathbf{b}) \equiv \int \frac{d^2\mathbf{b}}{(2\pi)^2} \frac{1}{2P^+} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}} \left\langle P^+, \frac{\mathbf{q}_\perp}{2}, \lambda_{N^*} \middle| J^+(0) \middle| P^+, -\frac{\mathbf{q}_\perp}{2}, \lambda_N \right\rangle$$

  - λ: helicity
  - J^+: quark charge current, "$+$" component
  - $\mathbf{b}$: 2D position on xOy plane
- The quark charge distribution that is responsible for the $N \to N^*$ transition
- Two independent densities
  - ρ_0: unpolarized $\to$ only depends on $|\mathbf{b}|$
  - ρ_T: polarized along x-axis, $|\lambda\rangle = \frac{1}{\sqrt{2}} \left(|+\frac{1}{2}\rangle + |-\frac{1}{2}\rangle\right)$

Transverse charge distributions: calculation

Backups

[Tiator & Vanderhaeghen, PLB 672, 344 (2009)]

- Helicity TFFs in terms of Pauli-Dirac TFFs

$$A_{1/2} = \frac{eQ_-}{\sqrt{4Km_N M_R}}(F_1 + F_2)$$

$$S_{1/2} = \frac{eQ_-}{\sqrt{8Km_N M_R}} \frac{Q_+ Q_-}{2M_R} \frac{M_R + m_N}{Q^2} \left[F_1 - \frac{Q^2}{(m_N + M_R)^2} F_2\right]$$

with $Q_\pm = \sqrt{(M_R \pm m_N)^2 + Q^2}$

- Unpolarized (J_n: cylindrical Bessel function)

$$\rho_0(\mathbf{b}) = \int_0^{+\infty} \frac{dQ}{2\pi} Q\, J_0(|\mathbf{b}|Q)\, F_1(Q^2)$$

- Polarized ($\sin\phi = b_y/|\mathbf{b}|$)

$$\rho_T(\mathbf{b}) = \rho_0(\mathbf{b}) + \sin\phi \int_0^{+\infty} \frac{dQ}{2\pi} \frac{Q^2}{m_N + M_R} J_1(|\mathbf{b}|Q)\, F_2(Q^2)$$