Dispersive Determination of Nucleon Gravitational Form Factors

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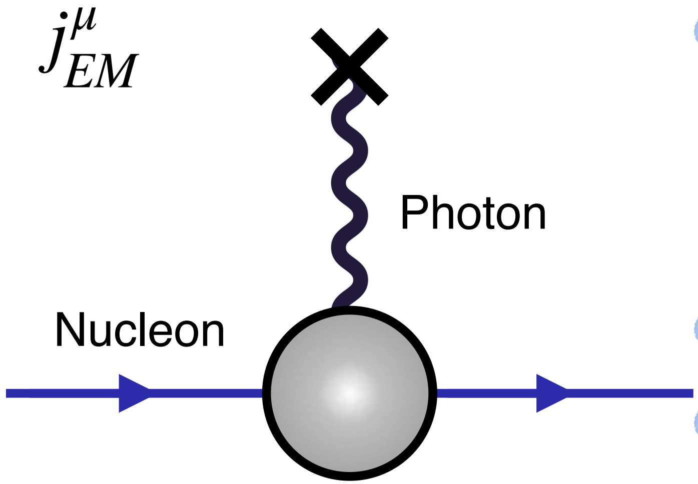
EM structure of nucleons



- Definition: $t \equiv q^2 = (p' - p)^2$

$$\langle N(p') | j^{\mu}_{EM} | N(p) \rangle = \bar{u}(p') \left[F_1(t)\gamma^{\mu} + i\frac{F_2(t)}{2m_N}\sigma^{\mu\nu}q_{\nu} \right] u(p)$$

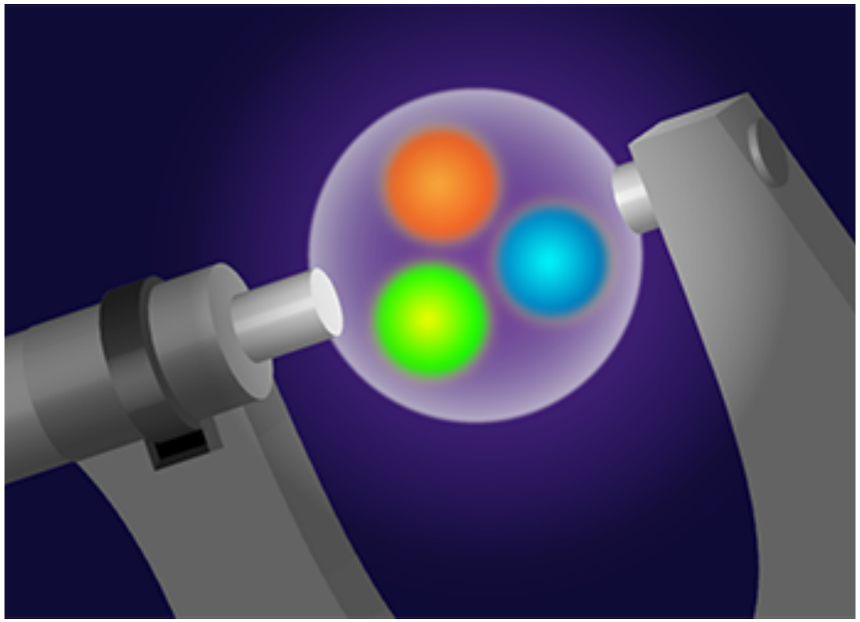
- Normalization: $F_1^p(0) = 1, F_1^n(0) = 0, F_2^p(0) = \kappa_p, F_2^n(0) = \kappa_n$
- Sachs form factors: $G_E = F_1 + \frac{t}{4m^2}F_2, \quad G_M = F_1 + F_2$

- Charge radius (proton):

  $G_E(t) = 1 + t\langle r_C^2 \rangle/6 + \ldots$
- Extracted from the lepton-nucleon elastic scattering or the hydrogen(-like) atom spectroscopy

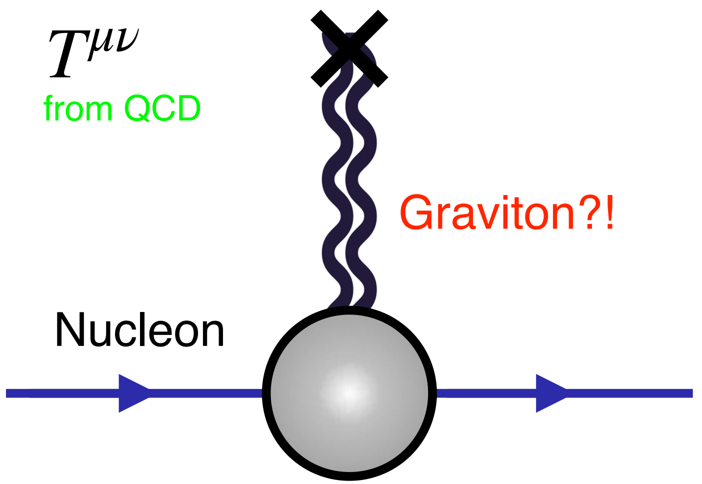
$\Rightarrow$ *"proton radius puzzle"*

0.84 fm v.s. 0.88 fm

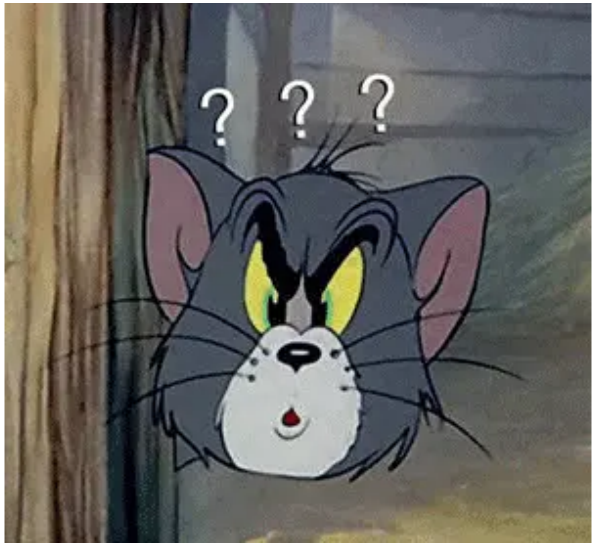


https://physics.aps.org/articles/v12/s28

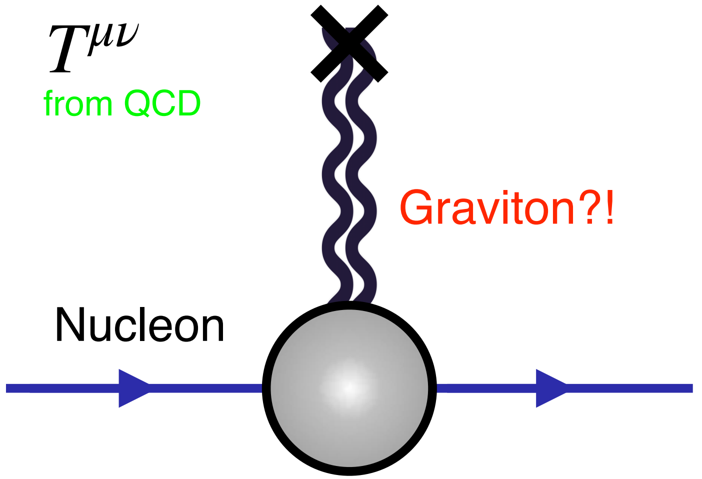
Gravitational structure of nucleons



- Gravity couples to matter through energy-momentum tensor (EMT) $T^{\mu\nu}$
- No direct experiment for detection of the nucleon-graviton interaction (10^{-39} times weaker than electromagnetic interaction)



Gravitational structure of nucleons



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Total (quark+gluon) QCD EMT matrix element is renormalization-scale-independent

- Definition: $a_{\{\mu}b_{\nu\}} = a_\mu b_\nu + a_\nu b_\mu, P = p' + p, \Delta = p' - p$

$$\left\langle N(p') \middle| T^{\mu\nu} \middle| N(p) \right\rangle = \frac{1}{4m_N}\bar{u}(p')\left[A(t)P^\mu P^\nu + J(t)\left(iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho\right) + D(t)\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right)\right]u(p)$$

Kobzarev, Okun (1962); Pagels (1966)
M.V. Polyakov, P. Schweitzer, (2018)

- Mass normalization:

$$m_N = \int d^3r T_{00}(r)$$

$A(0) = 1$

- Spin normalization:

$$J^i = \epsilon^{ijk}\int d^3r r^j T_{0k}(r)$$

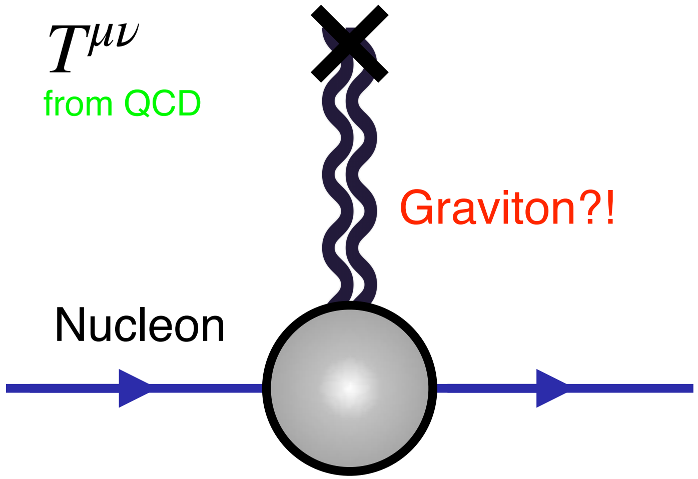
$J(0) = 1/2$

- D-term: $D \equiv D(0)$

$$D = -\frac{m_N}{2}\int d^3r\left(r^i r^j - \frac{1}{3}\delta_{ij}\right)T_{ij}(r)$$

$D = ?$

Gravitational structure of nucleons



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D-term as the "last unknown global property"

em:	$\partial_\mu J^\mu_{\mathrm{em}} = 0$	$\langle N'\vert J^\mu_{\mathbf{em}}\vert N\rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\mathrm{C}$ $\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N'\vert J^\mu_{\mathbf{weak}}\vert N\rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\vert T^{\mu\nu}_{\mathbf{grav}}\vert N\rangle$	$\longrightarrow$	$m = 938.272013(23)\,\mathrm{MeV}/c^2$ $J = \frac{1}{2}$ $\boldsymbol{D} = \boldsymbol{?}$

M.V. Polyakov, P. Schweitzer, (2018)

Mass radius puzzle

- Trace FF:

$$\left\langle N(p') \middle| T^{\mu}_{\mu} \middle| N(p) \right\rangle = m_N \bar{u}(p') \left[A(t) - \frac{t}{4m_N^2}[A(t) - 2J(t) + 3D(t)] \right] u(p) \equiv \bar{u}(p')\Theta(t)u(p)$$

- Trace anomaly in QCD: $T^{\mu}_{\mu} \equiv \frac{\beta(g)}{2g} F^{a,\mu\nu} F^{a,}{}_{\mu\nu} + (1 + \gamma_m) \sum_q m_q \bar{\psi}_q \psi_q$

>90% <10% due to the small σ-term, ~60 MeV≪940 MeV

Hoferichter et.al., (2015)…

- Mass radius: $\Theta(t) = 1 + t\langle r_M^2 \rangle/6 + \ldots$

Mass radius puzzle

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- Mass radius: $\Theta(t) = 1 + t\langle r_M^2 \rangle /6 + \ldots$

Mass (scalar, trace or dilatation) radius v.s. Energy radius

- Energy FF:

$$\left\langle N(p') \left| T^{00} \right| N(p) \right\rangle = m_N \bar{u}(p') \left[A(t) - \frac{t}{4m_N^2}[A(t) - 2J(t) + D(t)] \right] u(p) \equiv \bar{u}(p')E(t)u(p)$$

- Energy radius: $E(t) = 1 + t\langle r_E^2 \rangle /6 + \ldots$
- Reference frame dependent

Mass radius puzzle

- Trace FF:

$$\left\langle N(p') \middle| T^{\mu}_{\ \mu} \middle| N(p) \right\rangle = m_N \bar{u}(p') \left[A(t) - \frac{t}{4m_N^2}[A(t) - 2J(t) + 3D(t)] \right] u(p) \equiv \bar{u}(p')\Theta(t)u(p)$$

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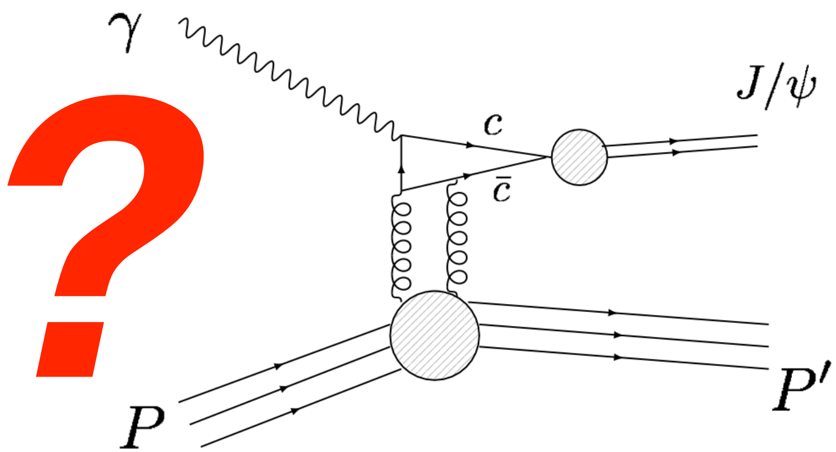
- Mass radius: $\Theta(t) = 1 + t\langle r_M^2 \rangle/6 + \dots$

- Kharzeev proposed it can be extracted from the threshold photoproduction of the vector-meson, e.g., J/ψ, and the fit result is ~0.5 fm
  Kharzeev, (2021)……

- Recently, two LQCD calculations at near physical quark mass result in a large mass radius, ~1 fm
  Hackett et al., (2024); Wang et al., (2024)



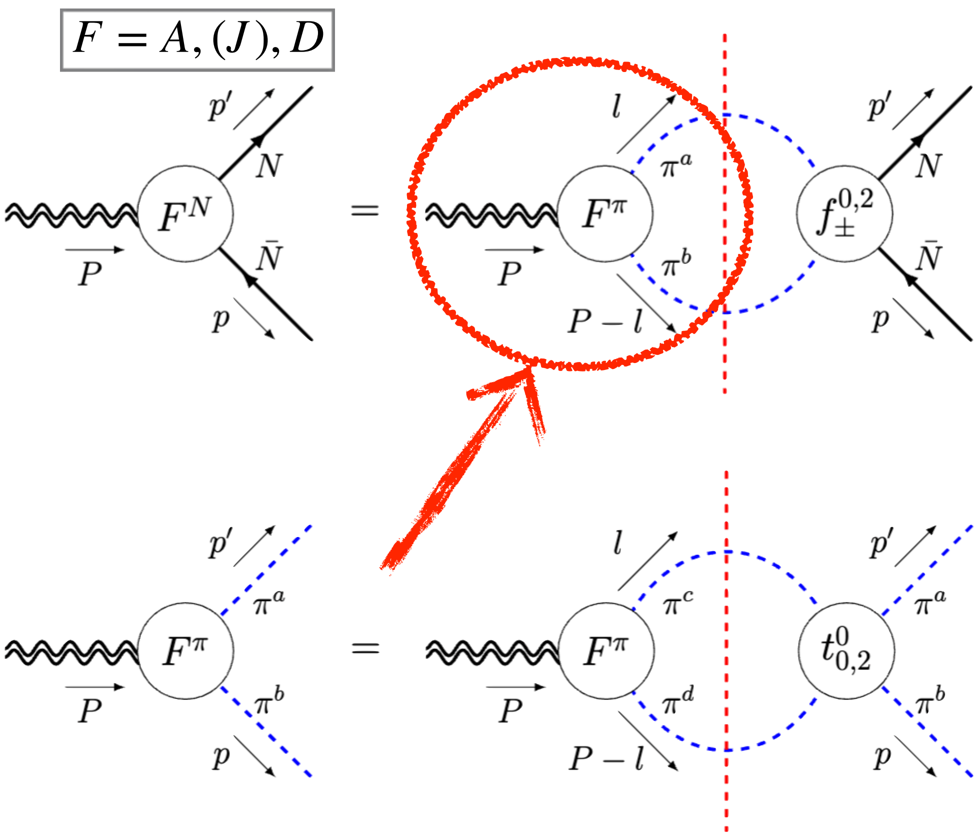
fits to low-t data ⇔ dispersive analyses of "all" data

Why data driven dispersion theory?

- Based on fundamental principles: unitarity, analyticity and crossing symmetry
- Simultaneous analysis of all four FFs (A, J, D and Θ)
- Connects FFs over full range of momentum transfers: time-like and space-like data
- Connects to data from other processes ($\pi\pi$, $K\bar{K}$ and πN, KN scatterings…)
- Constraints from χPT, pQCD
- ***Model-independedt extraction of nucleon radii based on broad theoretical and experimental input***

Dispersive representations

- Crossing: space-like $\left\langle N(p') \middle| T^{\mu\nu} \middle| N(p) \right\rangle \Leftrightarrow$ time-like $\left\langle N(p')\bar{N}(p) \middle| T^{\mu\nu} \middle| 0 \right\rangle$



Take $A(t)$ as an example

- Nucleon sector:

$$\text{Im}\, A(t) = \frac{3p_\pi^5}{\sqrt{6t}} \left[f_-^2(t) + \sqrt{\frac{3}{2}} \frac{m_N}{p_N^2} \Gamma^2(t) \right]^* A^\pi(t) \,,$$

$\ldots$

$$\Gamma^2(t) = m_N \sqrt{\frac{2}{3}} f_-^2(t) - f_+^2(t)$$

- Meson sector:

$$\text{Im}\, A^\pi(t) = \rho_\pi(t) \left(t_2^0(t)\right)^* A^\pi(t) \,,$$

$\ldots$

πN amplitudes $f_\pm^{0,2}$ from modern Roy-Steiner eq. analyses

C. Ditsche, et al., JHEP (2012); M. Hoferichter et.al., JHEP (2012); M. Hoferichter, et al., PRL 115, 092301(2015); PRL 115, 192301 (2015); Phys. Rept. (2016); PLB (2016); EPJA (2016); J. Ruiz de Elvira et.al., JPG (2018); M. Hoferichter, et al., PRL (2018); **XHC, et.al., JHEP (2022)**; M. Hoferichter, et al., PLB (2024)…

Gravitational form factors of pion

- Meson sector: $\mathrm{Im}\, A^{\pi}(t) = \rho_{\pi}(t)\left(t_2^0(t)\right)^* A^{\pi}(t)$

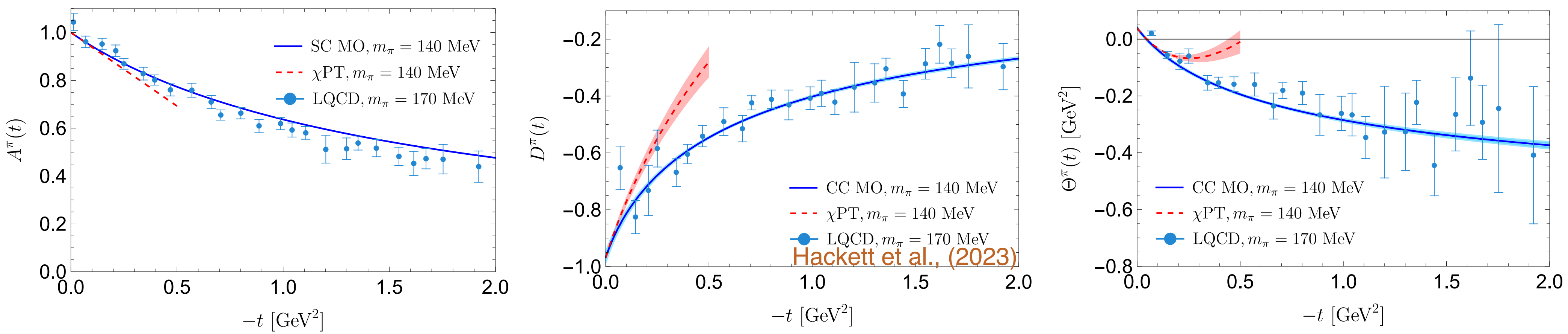
Single (D-wave) and couple (S-wave) channel Muskhelishvili-Omnes problem

- Meson sector: $A^{\pi}(t) = (1+\alpha t)\Omega_2^0(t)\ , \quad \Omega_2^0(t) \equiv \exp\left\{\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}t'}{t'}\frac{\delta_2^0(t')}{t'-t}\right\}$

Match to χPT $\mathcal{O}(p^4)$ result

Donoghue & Leutwyler, (1991)

D-wave phase-shift from experiment



Gravitational form factors of nucleon

- Nucleon sector: $\mathrm{Im}\, A(t) = \frac{3p_\pi^5}{\sqrt{6t}}\left[f_-^2(t) + \sqrt{\frac{3}{2}}\frac{m_N}{p_N^2}\Gamma^2(t)\right]^* A^\pi(t)$

Rigorous πN Roy-Steiner equation analysis

Muskhelishvili-Omnes formalism

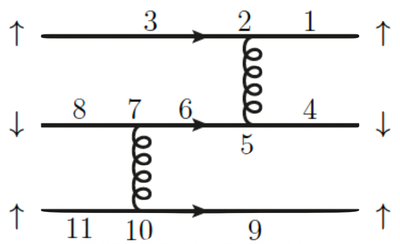
- Unsubtracted dispersion relation: $A(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} dt' \frac{\mathrm{Im}\, A(t')}{t'-t}$

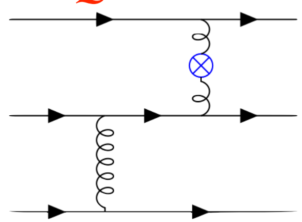
- Constraints:
  - Normalizations: mass m_N, spin $1/2$

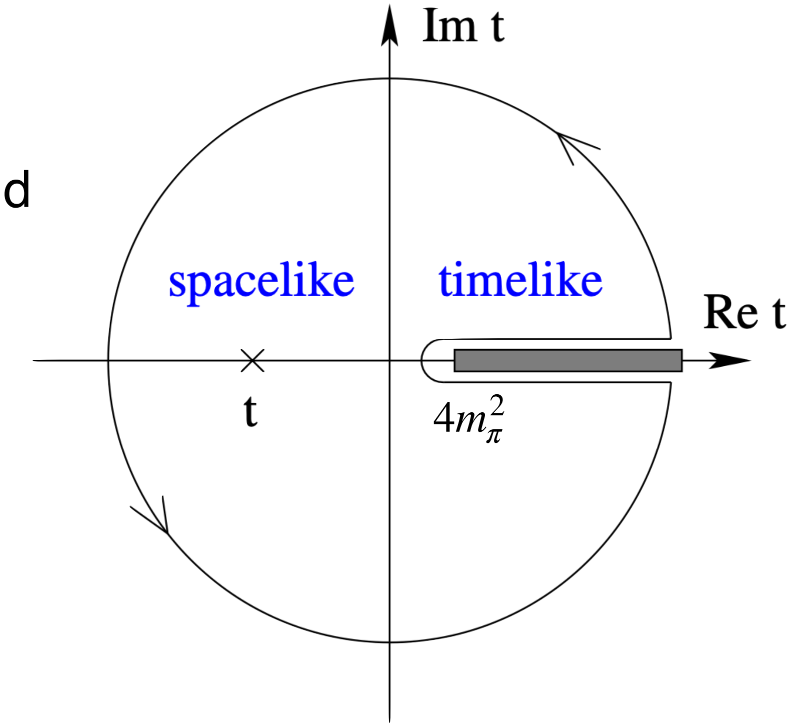
    $\Rightarrow$ sum rules saturated by $\pi\pi, K\bar{K}$ continuum and some higher mass states $(f_0(1500)\ldots; f_2(1565)\ldots)$
  - pQCD behavior for large momentum transfer (modulo some logs):

    Tanaka, (2018); Tong, et al. (2021) (2022)

    $\Rightarrow A(Q^2) \sim J(Q^2) \sim \Theta(Q^2) \sim \frac{1}{Q^2}$ and $D(Q^2) \sim \frac{1}{Q^6}$

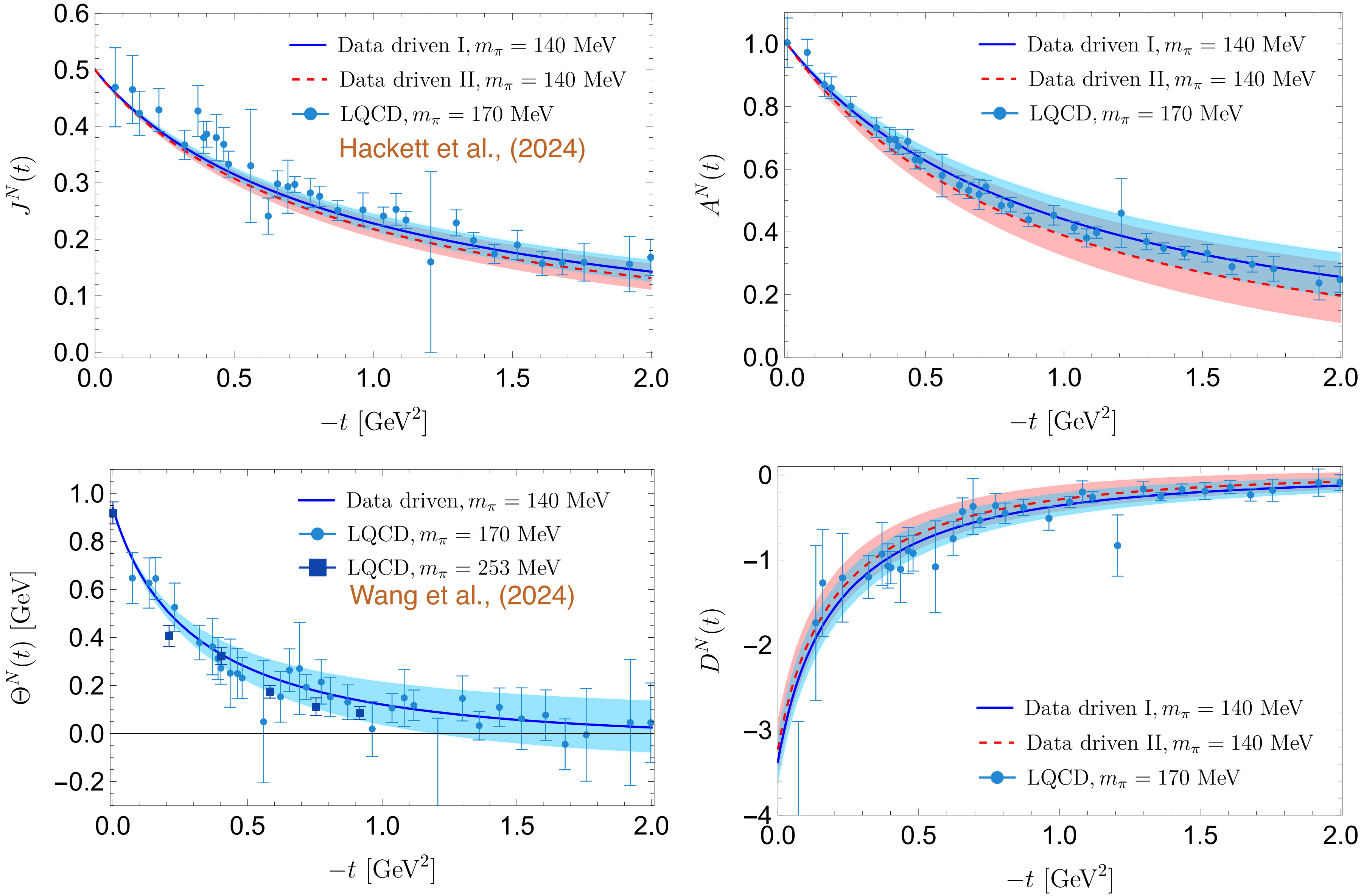






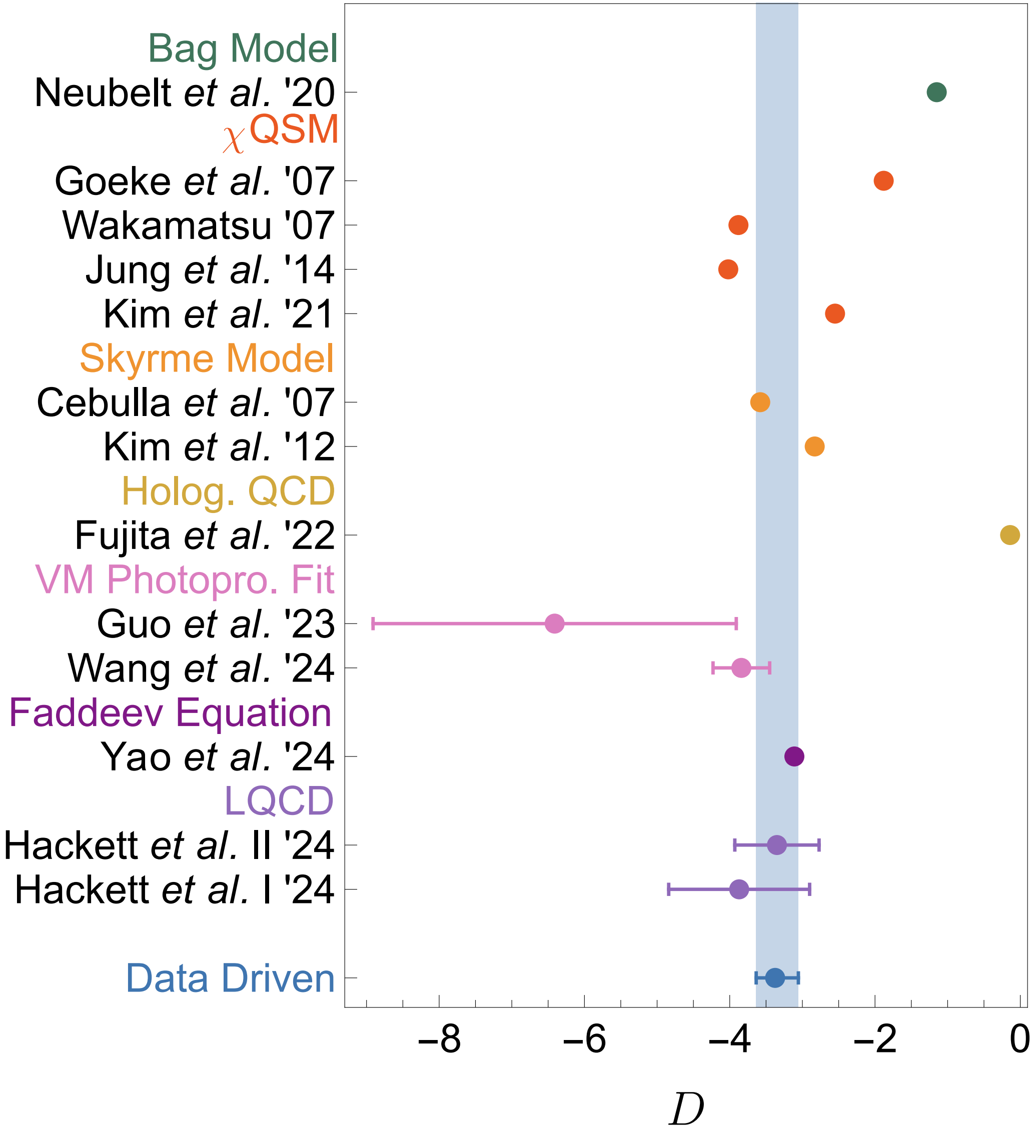
Results: space-like GFFs

- Nucleon gravitational form factors:



Results: D-term

- Nucleon D-term: $D = -\left(3.38^{+0.26}_{-0.32}\right)$



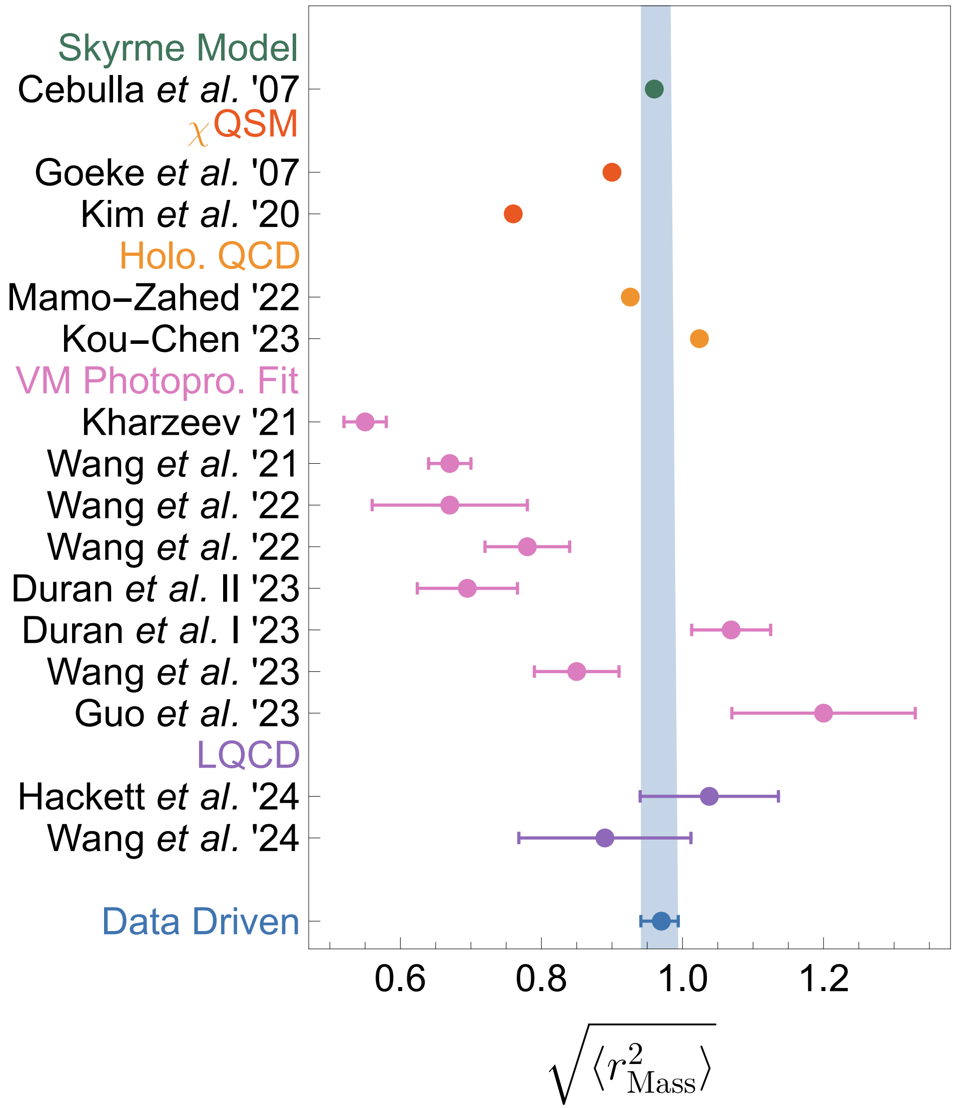
- Positivity bound:

$$D < -0.2$$

Gegelia & Polyakov, (2021)

Results: mass radius

- Nucleon mass radius: $\left\langle r^2_{\text{Mass}} \right\rangle = \frac{6\Theta'(0)}{m_N} = \left(0.97^{+0.02}_{-0.03}\ \text{fm}\right)^2$



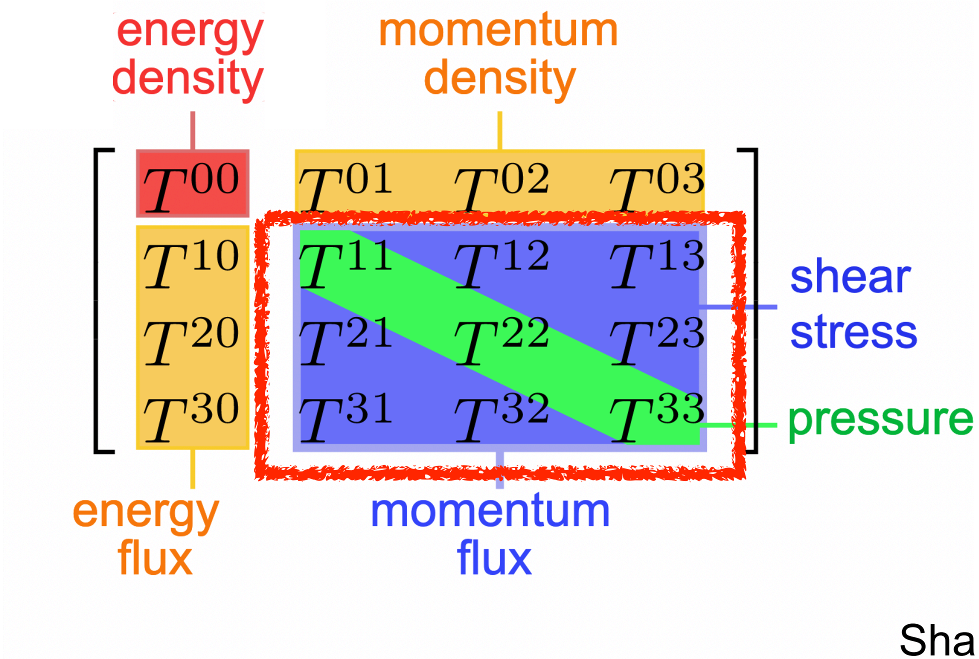
- Dispersive analyses have always found a "large" mass radius ~ 1 fm

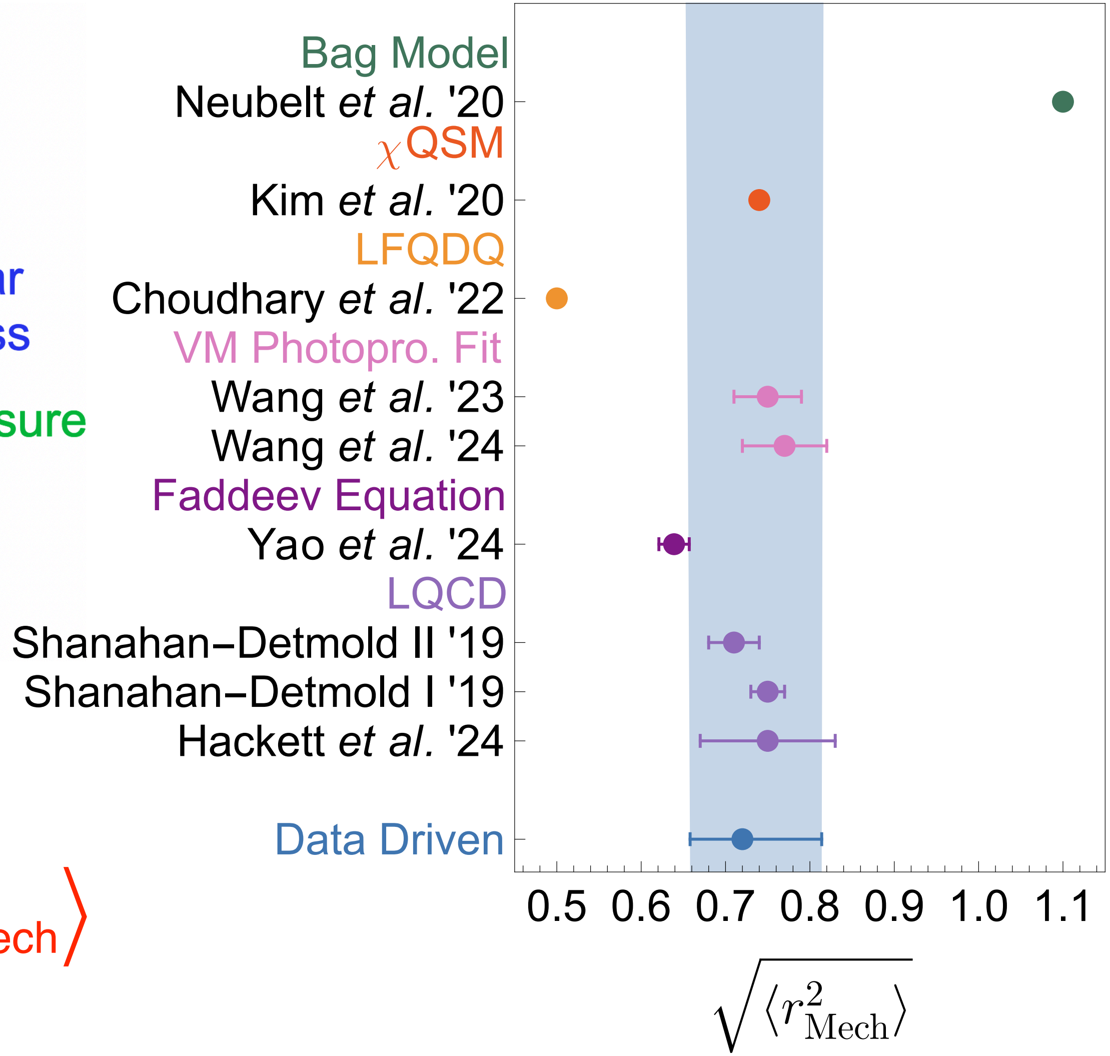
- Notice that the proton charge radius: $\left\langle r^2_{\text{Char}} \right\rangle \sim (0.84\ \text{fm})^2$

$$\Rightarrow \left\langle r^2_{\text{Mass}} \right\rangle > \left\langle r^2_{\text{Char}} \right\rangle$$

Results: mechanical radius

- Nucleon mass radius: $\left\langle r^2_{\text{Mech}} \right\rangle = \dfrac{6D}{\int_{-\infty}^{0} dt\ D(t)} = \left(0.72^{+0.09}_{-0.06}\ \text{fm}\right)^2$





- Ordering of nucleon radii

$$\left\langle r^2_{\text{Mass}} \right\rangle > \left\langle r^2_{\text{Char}} \right\rangle > \left\langle r^2_{\text{Mech}} \right\rangle$$

Summary and outlook

- ☑ The unity of dispersive techniques and experiment data is powerful to investigate nucleon FF
- ☑ Quantification of systematic and theoretical uncertainties
- ☑ Predictions for various proton radii, $\langle r^2_{\text{Mass}} \rangle > \langle r^2_{\text{Char}} \rangle > \langle r^2_{\text{Mech}} \rangle$
- ☐ Pion-mass dependence of pion and nucleon GFFs (in progress)
- ☐ Matching the results to χPT $\Rightarrow$ pure gravitational LECs c_8 and c_9 (in progress)
  Alharazin et al., (2020)
- ☐ 3D (static) and 2D (light-front) distributions (in progress)
  Lorcé et al., (2019)
- ☐ Hyperon gravitational structure……

Thank you for your attention!