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Isospin Violation Effect and Three-Body Decay of T_{cc}

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Z. F. Sun, N. Li, X. Liu, arXiv: 2405.00525

Outline

- **Motivation**
- **The Isospin violation effect**
- **The three-body decay**
- **Summary**

Motivation

- T_{cc} was observed in the final states: $D^0D^0\pi^+$
- Mass and width:

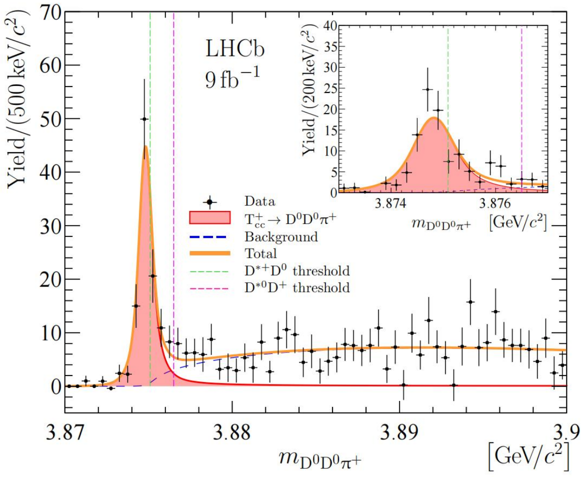
$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{BW} = -273 \pm 61 \pm 5\,^{+11}_{-14}\,\mathrm{keV}/c^2$$
$$\Gamma_{BW} = 410 \pm 165 \pm 43\,^{+18}_{-38}\,\mathrm{keV},$$

→ relativistic P-wave two-body Breit Wigner function with a Blatt-Weisskopf form factor

$$\delta m_U = -360 \pm 40^{+4}_{-0}\ \mathrm{keV}$$
$$\Gamma_U = 48 \pm 2^{+0}_{-14}\ \mathrm{keV}.$$

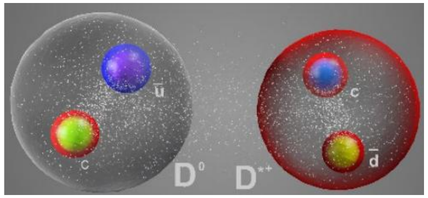
→ unitarised Breit Wigner profile



Nature Phys. 18, 751 (2022)
Nature Commun. 13, 3351 (2022)

- Very close to the D^0D^{*+}/D^+D^{*0} threshold
- Good candidate of D^0D^{*+}/D^+D^{*0} molecular state
- Isospin violation effect can not be neglected

$$m_{D^+D^{*0}} - m_{D^0D^{*+}} \sim 1.4\ \mathrm{MeV} > \delta m$$



- To confirm the molecular explanation
  ⟹ Three-body decay

Motivation

Our prediction in 2013:

PHYSICAL REVIEW D **88**, 114008 (2013)

Coupled-channel analysis of the possible $D^{(*)}D^{(*)}$, $\bar{B}^{(*)}\bar{B}^{(*)}$ and $D^{(*)}\bar{B}^{(*)}$ molecular states

Ning Li,[1,2,*] Zhi-Feng Sun,[3,4,†] Xiang Liu,[3,4,‡] and Shi-Lin Zhu[1,5,6,§]

We perform a coupled-channel study of the possible deuteron-like molecules with two heavy flavor quarks, including the systems of $D^{(*)}D^{(*)}$ with double charm, $\bar{B}^{(*)}\bar{B}^{(*)}$ with double bottom, and $D^{(*)}\bar{B}^{(*)}$ with both charm and bottom, within the one-boson-exchange potential model. In our study, we take into account the S-D mixing which plays an important role in the formation of the loosely bound deuteron, and particularly, the coupled-channel effect in the flavor space. According to our results, the state $D^{(*)}D^{(*)}[I(J^P)=0(1^+)]$ with double charm, the states $\bar{B}^{(*)}\bar{B}^{(*)}[I(J^P)=0(1^+), 1(1^+)]$, $(\bar{B}^{(*)}\bar{B}^{(*)})_s[J^P=1^+, 2^+]$ and $(\bar{B}^{(*)}\bar{B}^{(*)})_{ss}[J^P=1^+, 2^+]$ with double bottom, and the states $D^{(*)}\bar{B}^{(*)}[I(J^P)=0(1^+), 0(2^+)]$ with both charm and bottom might be good molecule candidates. However, the states $D^{(*)}D^{(*)}[I(J^P)=0(2^+), 1(0^+), 1(1^+), 1(2^+)]$, $(D^{(*)}D^{(*)})_s[J^P=0^+, 2^+]$ and $(D^{(*)}D^{(*)})_{ss}[J^P=0^+, 1^+, 2^+]$ with double charm and the state $D^{(*)}\bar{B}^{(*)}[I(J^P)=1(1^+)]$ with both charm and bottom are not supported to be

I	J^P		$D^{(*)}D^{(*)}$ OPE				$D^{(*)}D^{(*)}$ OBE			
	0^+		* * *				* * *			
		Λ (GeV)	1.05	1.10	1.15	1.20	0.95	1.00	1.05	1.10
		B.E. (MeV)	1.24	4.63	11.02	20.98	0.47	5.44	18.72	42.82
		M (MeV)	3874.61	3871.22	3864.83	3854.87	3875.38	3870.41	3857.13	3833.03
		r_{rms} (fm)	3.11	1.68	1.12	0.84	4.46	1.58	0.91	0.64
0	1^+	P_1 (%)	96.39	92.71	88.22	83.34	97.97	92.94	85.64	77.88
		P_2 (%)	0.73	0.72	0.57	0.42	0.58	0.55	0.32	0.15
		P_3 (%)	2.79	6.45	11.07	16.11	1.41	6.42	13.97	21.91
		P_4 (%)	0.08	0.13	0.14	0.13	0.04	0.09	0.08	0.05

Binding energy

LHCb result of T_{cc}

273 (360) keV

Our prediction

470 keV

Perfect DD^* molecular prediction matching the T_{cc} observation at LHCb

Motivation

Studies on double-charm tetraquark before LHCb's observation:

E. Braaten, L. He, PRD 103, 016001 (2021)

J. Cheng, S. Y. Li, Y. R. Liu, Z. G. SI, T. Yao, CPC 45 (2021) 043102

R. Faustov, V. Galkin, E. M. Savchenko, Universe 7 (2021) 4, 94

M. Z. Liu, J. J. Xie, L. S. Geng, PRD 102 (2020) 091502

Q. F Lv, D. Y Chen, Y. B Dong, PRD 102034012 (2020)

C. Deng, H. Chen, J. Ping, EPJA 56 (2020) 9

P. Junnarkar, N. Mathur, and M. Padmanath, PRD 99 (2019) 034057

W. park, S. Noh, S. Lee, NPA 983 (2019) 1

Z. G. Wang, ACTA Physica Polonica B 49 (2018) 1781

E. J. Eichten, and C. Quigg, PRL 119 (2017) 202002

M. Karliner, J. L. Rosner, PRL 119 (2017) 202001

HAL QCD Collaboration, PLB 729 (2014) 85

G. Q. Feng, X. H. Guo, B. S. Zou, arXiv:1309.7813 (2013)

……

Isospin violation

- spontaneously breaking $[U(3)_L \otimes U(3)_R]_{global} \otimes [U(3)_V]_{local}$ symmetry

  π, K, η, η' as Goldstone bosons

  ρ, ω, K^*, ϕ as gauge bosons

- Lagrangians

$$\mathcal{L}_{P^{(*)}P^{(*)}M} = -i\frac{2g}{f_\pi}\epsilon_{\alpha\mu\nu\lambda}v^\alpha P^{*\mu}\partial^\nu M P^{*\lambda\dagger} - \frac{2g}{f_\pi}(P\partial^\lambda M P^{*\dagger}_\lambda + P^*_\lambda \partial^\lambda M P^\dagger),$$

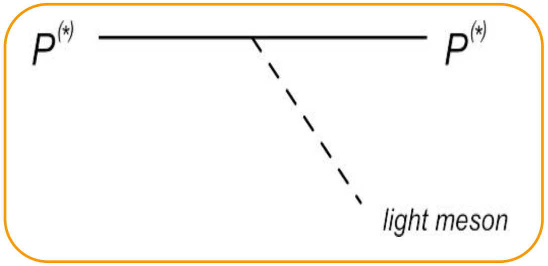
$$\mathcal{L}_{P^{(*)}P^{(*)}V} = -\sqrt{2}\beta g_V P(v\cdot\hat{\rho})P^\dagger - 2\sqrt{2}\lambda g_V \epsilon_{\lambda\mu\alpha\beta}v^\lambda(P\partial^\alpha\hat{\rho}^\beta P^{*\mu\dagger} + P^{*\mu}\partial^\alpha\hat{\rho}^\beta P^\dagger)$$

$$+\sqrt{2}\beta g_V P^{\mu *}(v\cdot\hat{\rho})P^{*\dagger}_\mu - i2\sqrt{2}\lambda g_V P^{*\mu}(\partial_\mu\hat{\rho}_\nu - \partial_\nu\hat{\rho}_\mu)P^{*\nu\dagger}$$

$$\hat{\rho}^\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}^\mu$$

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta + \frac{\eta'}{\sqrt{3}} \end{pmatrix}$$

$$P = (D^0, D^+, D_s^+) \qquad P^* = (D^{*0}, D^{*+}, D_s^{*+})$$



Phys. Rept. 281, 145-238 (1997)

Phys. Lett. B 292, 371-376 (1992)

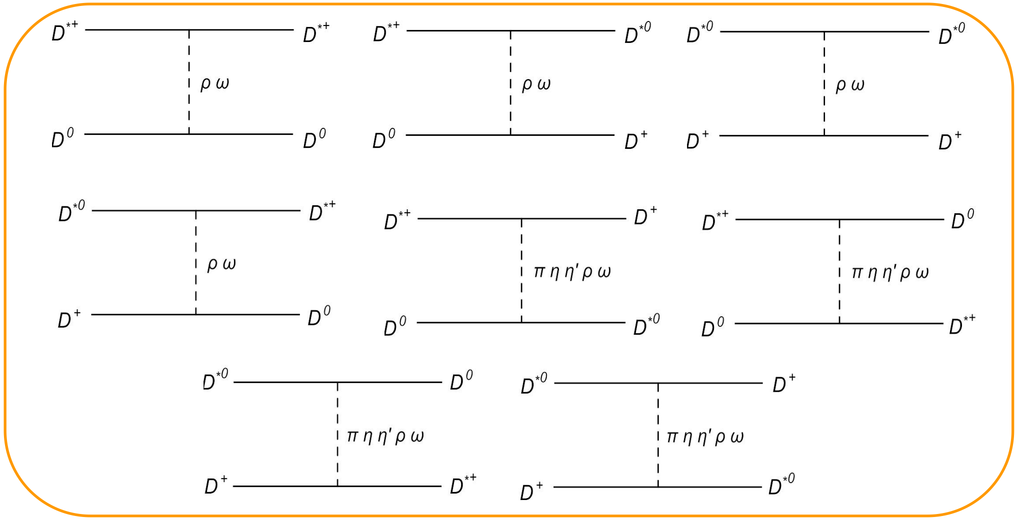
Isospin violation

- Breit approximation:

$$V(\boldsymbol{q}) = -\frac{\mathcal{M}(\boldsymbol{q})}{4\sqrt{m_1 m_2 m_3 m_4}}$$

- Fourier transformation:

$$V(r) = \frac{1}{(2\pi)^3}\int d^3q e^{i\boldsymbol{q}\cdot\boldsymbol{r}} V(\boldsymbol{q}) F^2(\boldsymbol{q})$$



- Monopole form factor:

$$F_M(\boldsymbol{q}) = \frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 - q_0^2 + \boldsymbol{q}^2}$$

suppress the amplitude when m_{ex} is large

- Exponential form factor is used in our work:

$$F(\boldsymbol{q}) = e^{(q_0^2 - \boldsymbol{q}^2)/\Lambda^2}$$

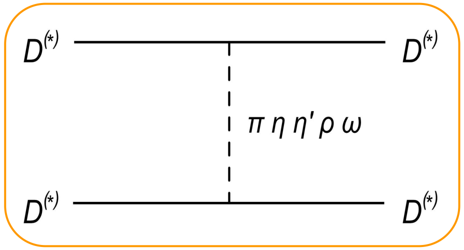
Isospin violation

- Potentials with exponential form factor:

$$
\begin{aligned}
Y(\Lambda,\mu,q_0,r) &= \int \frac{d^3q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \frac{1}{\boldsymbol{q}^2+\mu^2-i\epsilon} e^{2(q_0^2-\boldsymbol{q}^2)/\Lambda^2} \\
&= -\frac{e^{2q_0^2/\Lambda^2}}{(2\pi)^2 r}\frac{\partial}{\partial r}\left\{\frac{\pi}{2\mu}\left[e^{-\mu r}+e^{\mu r}+e^{-\mu r}\,\mathrm{erf}\left(\frac{r\Lambda}{2\sqrt{2}}-\frac{\sqrt{2}\mu}{\Lambda}\right)-e^{\mu r}\mathrm{erf}\left(\frac{r\Lambda}{2\sqrt{2}}+\frac{\sqrt{2}\mu}{\Lambda}\right)\right]e^{2\mu^2/\Lambda^2}\right\}
\end{aligned}
$$

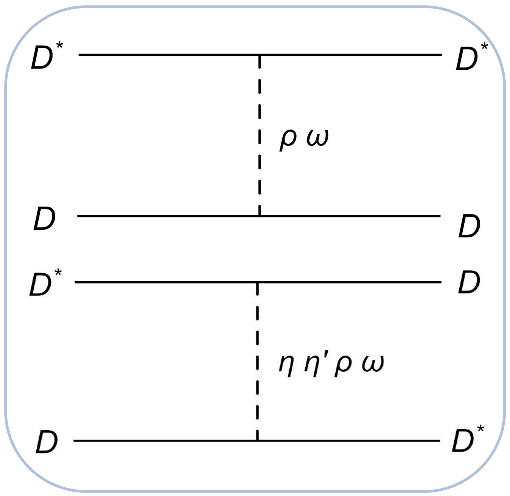
$$
\begin{aligned}
U(\Lambda,\mu,q_0,r) &= \int \frac{d^3q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \frac{1}{\boldsymbol{q}^2-\mu^2-i\epsilon} e^{2(q_0^2-\boldsymbol{q}^2)/\Lambda^2} \\
&= \frac{e^{2q_0^2/\Lambda^2}}{(2\pi)^2 r}\frac{\partial}{\partial r}\left\{\pi\left[-\frac{1}{2i\mu}\left(e^{-i\mu r}\mathrm{erf}\left(\frac{r\Lambda}{2\sqrt{2}}-\frac{\sqrt{2}i\mu}{\Lambda}\right)-e^{i\mu r}\mathrm{erf}\left(\frac{r\Lambda}{2\sqrt{2}}+\frac{\sqrt{2}i\mu}{\Lambda}\right)\right)-\frac{i}{\mu}\cos(\mu r)\right]e^{-2\mu^2/\Lambda^2}\right\}
\end{aligned}
$$

$$
\begin{aligned}
V_V^D &= \frac{1}{4}\beta^2 g_V^2(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)Y(\Lambda,m_V,q_0,r), \\
V_V^C &= 2\lambda^2 g_V^2\left[\frac{2}{3}\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_4^\dagger\,\nabla^2\,Y(\Lambda,\tilde{m}_V,q_0,r)-\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_1,\boldsymbol{\epsilon}_4^\dagger)r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}Y(\Lambda,\tilde{m}_V,q_0,r)\right], \\
V_p^C &= \frac{g^2}{f_\pi^2}\left[\frac{1}{3}\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_4^\dagger\,\nabla^2\,Y(\Lambda,\tilde{m}_p,q_0,r)+\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_1,\boldsymbol{\epsilon}_4^\dagger)r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}Y(\Lambda,\tilde{m}_p,q_0,r)\right], \\
\tilde{V}_p^C &= \frac{g^2}{f_\pi^2}\left[\frac{1}{3}\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_4^\dagger\,\nabla^2\,U(\Lambda,\tilde{m}_p',q_0,r)+\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_1,\boldsymbol{\epsilon}_4^\dagger)r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}U(\Lambda,\tilde{m}_p',q_0,r)\right].
\end{aligned}
$$

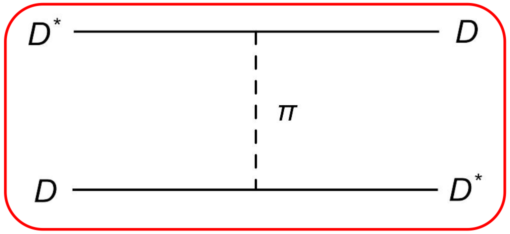


Isospin violation

- Discussion of the potentials



The potentials are real

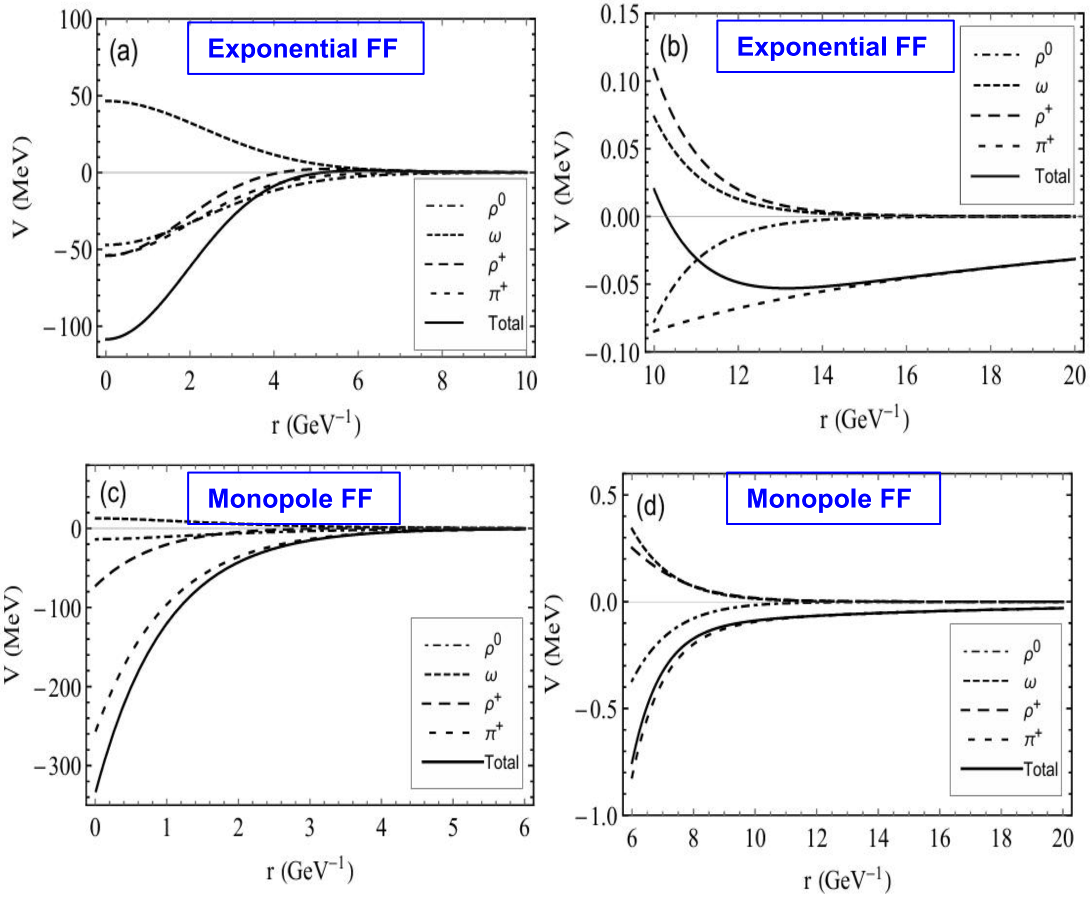


The potentials are complex!!!

We first neglect the imaginary part, and discuss it later.

Isospin violation

- $D^{*+}D^0$ Potentials with exponential and monopole form factors



Exponential FF

- short and medium range:
  the vector meson-exchange contributions are comparable to that of pion-exchange.
- long range:
  pion-exchange contributionis dominant.

Monopole FF

- whole range:
  pion-exchange contributionis dominant;
  other contributions are much small due to the suppression by the numerator of Monopole FF.

Long range: >2 fm; Midium range: 0.3-2 fm; short range: <0.3 fm

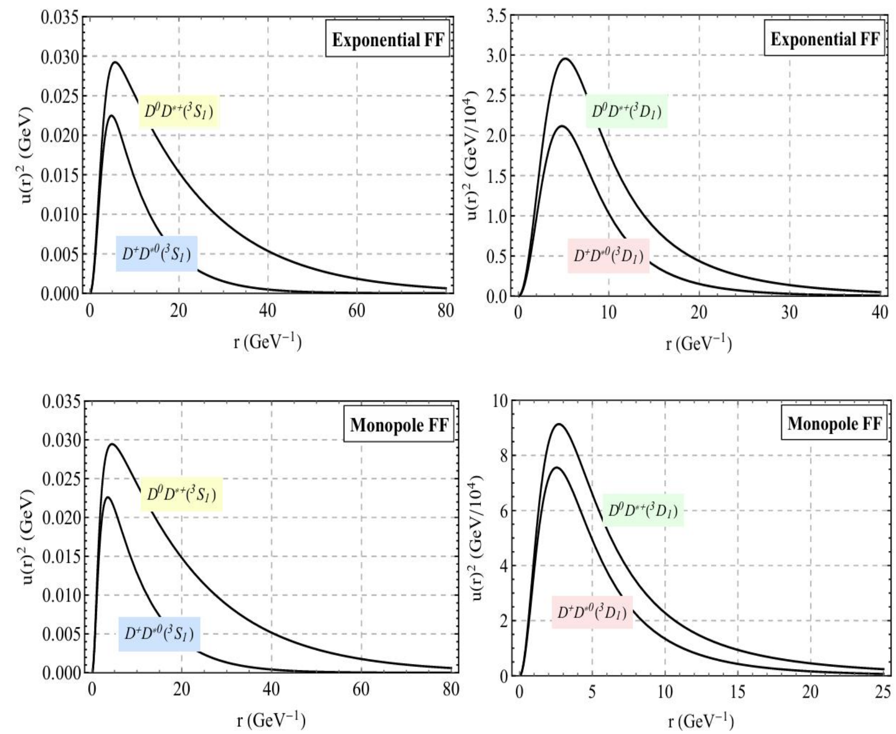
Isospin violation

- Schrodinger equation

$$\left(\hat{K}+\hat{M}+\hat{V}\right)\Psi = E\Psi$$

$$\psi^{T}_{T^{+}_{cc}} = \left(\frac{u_S^{D^{*0}D^{+}}}{r}|^3S_1\rangle, \frac{u_D^{D^{*0}D^{+}}}{r}|^3D_1\rangle, \frac{u_S^{D^{*+}D^{0}}}{r}|^3S_1\rangle, \frac{u_D^{D^{*+}D^{0}}}{r}|^3D_1\rangle\right)$$

- Probabilities with exponential and monopole form factors
  - P_S^{Mo} and P_S^{Ex} are almost the same
  - P_D^{Mo} is larger than P_D^{Ex}



- Numerical result
  - $\Lambda = 782 - 798$ MeV: binding energy 200.3 - 358.6 keV
  - $D^{*+}D^0$ component is dominant
  - S-wave contribution is dominant
  - Different choises of the FFs do not affect the result.

Λ	E	R_{rms}	$P_{D^{*0}D^{+}(^3S_1)}$	$P_{D^{*0}D^{+}(^3D_1)}$	$P_{D^{*+}D^{0}(^3S_1)}$	$P_{D^{*+}D^{0}(^3D_1)}$
782	200.3	6.7	23.9	0.1	75.7	0.2
786	236.2	6.2	25.3	0.1	74.3	0.2
790	274.5	5.7	26.6	0.1	73.0	0.2
794	315.3	5.4	27.8	0.1	71.9	0.2
798	358.6	5.1	28.9	0.1	70.7	0.2
(MeV)	**(keV)**	**(fm)**	**(%)**	**(%)**	**(%)**	**(%)**

Isospin violation

- Representation transformation

$$\psi_{T_{cc}^+} = \begin{pmatrix} \frac{u_S^{D^{*0}D^+}}{r}|^3S_1\rangle \\ \frac{u_D^{D^{*0}D^+}}{r}|^3D_1\rangle \\ \frac{u_S^{D^{*+}D^0}}{r}|^3S_1\rangle \\ \frac{u_D^{D^{*+}D^0}}{r}|^3D_1\rangle \end{pmatrix} \xrightarrow{\psi'_{T_{cc}^+} = K\psi_{T_{cc}^+}} \psi'_{T_{cc}^+} = \begin{pmatrix} -\frac{u_S^{D^{*0}D^+}+u_S^{D^{*+}D^0}}{\sqrt{2}r}|^3S_1\rangle \\ -\frac{u_D^{D^{*0}D^+}+u_D^{D^{*+}D^0}}{\sqrt{2}r}|^3D_1\rangle \\ \frac{u_S^{D^{*0}D^+}-u_S^{D^{*+}D^0}}{\sqrt{2}r}|^3S_1\rangle \\ \frac{u_D^{D^{*0}D^+}-u_D^{D^{*+}D^0}}{\sqrt{2}r}|^3D_1\rangle \end{pmatrix}$$

- The probabilities of isovector and isoscalar

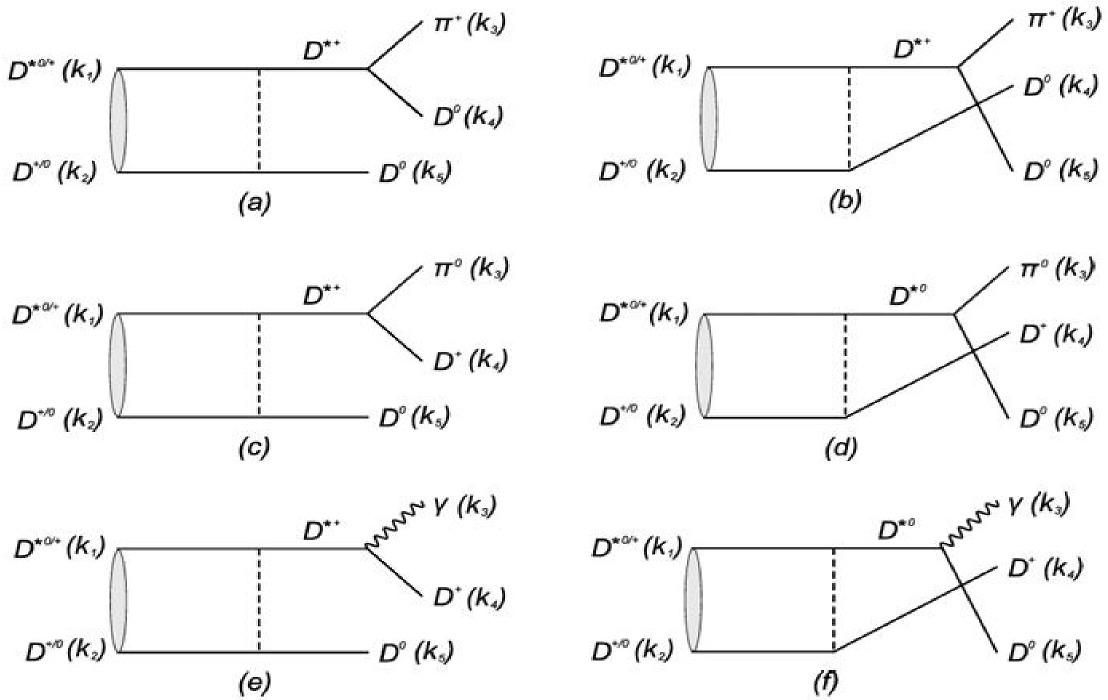
$$\rho_{10} = \int dr \frac{\left[u_S^{D^{*0}D^+} + u_S^{D^{*+}D^0}\right]^2 + \left[u_D^{D^{*0}D^+} + u_D^{D^{*+}D^0}\right]^2}{2} \rightarrow \text{Isovector}$$

$$\rho_{00} = \int dr \frac{\left[u_S^{D^{*0}D^+} - u_S^{D^{*+}D^0}\right]^2 + \left[u_D^{D^{*0}D^+} - u_D^{D^{*+}D^0}\right]^2}{2} \rightarrow \text{Isoscalar}$$

- $\Lambda = 790 - 798$ MeV: $\rho_{00} = 90.2\% - 92.2\%$, $\rho_{10} = 9.8\% - 7.8\%$
- Isoscalar component is dominant

Three-body decay

- Feynman diagrams of the three-body decays



- The decay occur via one-boson exchange
- The molecular state is depicted by the wave function of the Schrodinger equation
- Only consider S-wave contribution

Three-body decay

- The amplitudes of strong decay and radiative decay

$$\mathcal{M}_{T_{cc}^{+}\to\pi^{+}D^{0}D^{0}} = -\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*0}}E_{D^{+}}}}\int_{0}^{\infty} dr\, r\, j_{0}(k_{5}r)u_{S}^{D^{*0}D^{+}}(r)\mathcal{A}_{\rho^{-}}^{(a)} - \sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*0}}E_{D^{+}}}}\int_{0}^{\infty} dr\, r\, j_{0}(k_{4}r)u_{S}^{D^{*0}D^{+}}(r)\mathcal{A}_{\rho^{-}}^{(b)},$$

$$\mathcal{M}_{T_{cc}^{+}\to\pi^{0}D^{+}D^{0}} = -\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*0}}E_{D^{+}}}}\int_{0}^{\infty} dr\, r\, j_{0}(k_{5}r)u_{S}^{D^{*0}D^{+}}(r)\mathcal{A}_{\rho^{-}}^{(c)} - \sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*+}}E_{D^{0}}}}\int_{0}^{\infty} dr\, r\, j_{0}(k_{4}r)u_{S}^{D^{*+}D^{0}}(r)\mathcal{A}_{\rho^{+}}^{(d)},$$

$$\mathcal{M}_{T_{cc}^{+}\to\gamma D^{+}D^{0}} = -\sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*0}}E_{D^{+}}}}\int_{0}^{\infty} dr\, r\, j_{0}(k_{5}r)u_{S}^{D^{*0}D^{+}}(r)\mathcal{A}_{\rho^{-}}^{(e)} - \sqrt{\frac{2\pi m_{T_{cc}^{+}}}{E_{D^{*+}}E_{D^{0}}}}\int_{0}^{\infty} dr\, r\, j_{0}(k_{4}r)u_{S}^{D^{*+}D^{0}}(r)\mathcal{A}_{\rho^{+}}^{(f)}$$

- The widths of strong decay and radiative decay

$$\Gamma_{T_{cc}^{+}\to\pi^{+,0}D^{0,+}D^{0}} = \frac{1}{(2\pi)^{3}}\frac{1}{32m_{T_{cc}^{+}}^{3}}\int dm_{34}^{2}dm_{45}^{2}\,\frac{1}{3}\sum_{S_{z}^{T}}|\mathcal{M}_{T_{cc}^{+}\to\pi^{+,0}D^{0,+}D^{0}}|^{2}\frac{1}{S},$$

$$\Gamma_{T_{cc}^{+}\to\gamma D^{+}D^{0}} = \frac{1}{(2\pi)^{3}}\frac{1}{32m_{T_{cc}^{+}}^{3}}\int dm_{34}^{2}dm_{45}^{2}\,\frac{1}{3}\sum_{S_{z}^{T},S_{z}^{\gamma}}|\mathcal{M}_{T_{cc}^{+}\to\gamma D^{+}D^{0}}|^{2}$$

- Numerical results:

Λ (MeV)	782	786	790	794	798
Γ_R (keV)	0.7	0.8	0.9	0.9	1.0
Γ_S (keV)	17.3	18.6	19.9	21.2	22.4
Γ_{tot} (keV)	18.0	19.4	20.7	22.1	23.4

- ✓ mass difference of D^0 and D^+ leads to $\Gamma_{\pi^+D^0D^0} \neq \Gamma_{\pi^0D^+D^0}$
- ✓ The total decay width is close to the lower limit of the experimental value
- ✓ The result suports the molecular explanation of T_{cc}

$$\delta m_U = -360 \pm 40^{+4}_{-0}\ \text{keV},\quad \Gamma_U = 48 \pm 2^{+0}_{-14}\ \text{keV}.$$

Discussion

➤ Using perturbation theory to deal with the imaginary part of potentials:

$$H_{ab}\psi_b = E\psi_a$$

$$\psi_a = \phi_{a0} + \phi_{a1} + \phi_{a2} + \cdots$$
$$E = E_0 + E_1 + E_2 + \cdots$$

$$H_{ab} = H_0 + H'_{ab}$$

$$H'_{ab} = \mathrm{Im}(V_{ab})$$

consider only the 1st order →

$$E_1 = \int \phi^*_{a0} H'_{ab} \phi_{b0} dV.$$

- numerical result:

$$E_1 = -38.6i \text{ keV, i.e., } \Gamma = 77.2 \text{ keV}$$

- comparable with the experimental value:

$$\Gamma_U = 48 \pm 2^{+0}_{-14} \text{ keV.}$$

Summary

- Our predicted binding energy of T_{cc} in 2013 is consistent with the LHCb's result (PRD 88 (2013) 114008)
- Isospin violation effect within one-boson-exchange model
  - We use the exponential FF, and get the analytic form of potentials
  - The probability of isoscalar component is about 91%, that of isovector component is about 9%
- We develop a method to calculate the three-body decays of T_{cc}
  - mass difference of D^0 and D^+ leads to $\Gamma_{\pi^+ D^0 D^0} \neq \Gamma_{\pi^0 D^+ D^0}$
  - The total decay width is close to the lower limit of the experimental value
  - The result suports the molecular explanation of T_{cc}
- The imaginary part of potentials generate the width, but not affect the mass (up to the first order)

Z. F. Sun, N. Li, X. Liu, arXiv:2405.00525

谢谢！