New insight into the OZI suppression and the $X_0(4140)$, $X_1(4140)$ and $X_1(4685)$ as hadronic molecules

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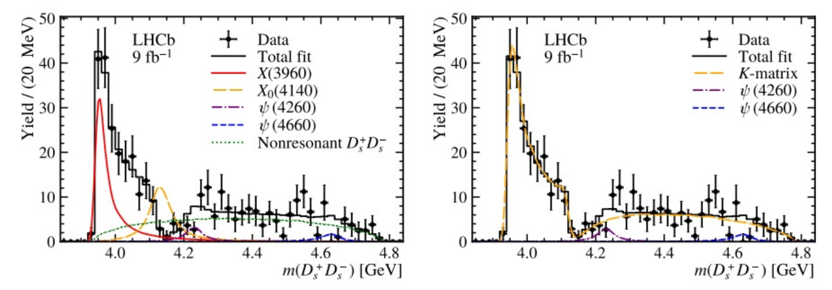
Motivation

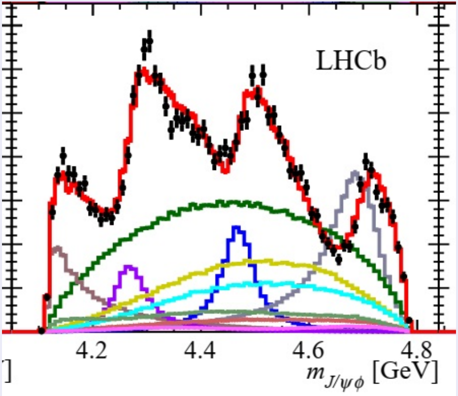
Observation and explanations on $X_0(4140)$ with $J^{PC} = 0^{++}$



- $X_0(4140)$ is a BW (4.1 σ) or dip (3.7 σ)
- BW: $(M, \Gamma) = 4133 \pm 6 \pm 6, 67 \pm 17 \pm 7\,\mathrm{MeV}$
- K-matrix: $(M, \Gamma) = 3960, < 50\,\mathrm{MeV}, g_{X,J/\psi\phi} = 0$, LHCb, PRL.131,2210.15153

Motivation

Observation and explanation on $X_1(4140)$ with $J^{PC} = 1^{++}$



- BW: $X_1(4140)$ (13 σ), $(M, \Gamma) = 4118 \pm 11^{+19}_{-36},\ 162 \pm 27^{+24}_{-49}\,\mathrm{MeV}$,

LHCb, PRL127, 2103.01803

$D_s\bar{D}_s - J/\psi\phi - D_s^*\bar{D}_s^*$ scattering in $J^{PC}=0^{++}$ sector

EFT and RGE

- $\{\Lambda_{QCD}, m_v\}$ and $\frac{d}{d\Lambda}\langle\Psi|C(\Lambda)|\Psi\rangle = 0$, $\Lambda_0 = m_\phi$ is adapted.

$D_s^{(*)}\bar{D}_s^{(*)}$ scattering

- HQSS, $H = \frac{1}{\sqrt{2}}\left[D_s + \vec{\sigma}\cdot\vec{D}_s^*\right]$, $\mathcal{C}|D_s\rangle = |\bar{D}_s\rangle$ and $\mathcal{C}|D_s^*\rangle = |\bar{D}_s^*\rangle$
- $\mathcal{L}_c = C_0 \,\mathrm{Tr}\left[H^\dagger H\right] \mathrm{Tr}\left[\bar{H}^\dagger \bar{H}\right] + C_1 \,\mathrm{Tr}\left[H^\dagger \sigma_k H\right] \mathrm{Tr}\left[\bar{H}^\dagger \sigma_k \bar{H}\right]$
- $|\vec{q}| = \sqrt{m_{D_s^*}\Delta} \simeq 478\,\mathrm{MeV}$ with $\Delta = 2m_{D_s^*} - m_{J/\psi} - m_\phi$, $\frac{\vec{q}^2}{m_\phi^2} \simeq 0.22$, $V_{D_s\bar{D}_s, D_s\bar{D}_s} = V_{D_s^*\bar{D}_s^*, D_s^*\bar{D}_s^*}$

$J/\psi\phi$ scattering

- $\mathcal{L}_d = D_0\left(J/\psi\cdot J/\psi^\dagger\right)\left(\phi\cdot\phi^\dagger\right) + D_1\left(J/\psi\vec{S}_1 J/\psi^\dagger\right)\left(\phi\vec{S}_2\phi^\dagger\right)$
- $V_{C_d}(p) = D_0 + D_1\left(\vec{S}_1\cdot\vec{p}\right)\left(\vec{S}_2\cdot\vec{p}\right)$, the D_1 term is switched off.

$D_s\bar{D}_s - J/\psi\phi - D_s^*\bar{D}_s^*$ scattering in $J^{PC} = 0^{++}$ sector

Inelastic scattering with HQSS

- $\mathcal{L}_e = E_0 \bar{H}\epsilon_k J/\psi\phi\epsilon^k H, + E_1 \eta_c \eta^{(\prime)} \bar{H} H$, with a vanished E_1 corresponding to the excluded $\eta_c\eta^{(\prime)}$ channel in present study
- Spin decomposition:
$$\begin{pmatrix} |D_s\bar{D}_s\rangle \\ |D_s^*\bar{D}_s^*\rangle \end{pmatrix}_{J=0} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} |0_l \otimes 0_Q\rangle \\ |1_l \otimes 1_Q\rangle \end{pmatrix}_{J=0},$$
with subscripts l and Q standing for the light and heavy degrees of freedom in the scattering.
- $V_{D_s\bar{D}_s, J/\psi\phi} = \sqrt{3}E_0, \; V_{D_s^*\bar{D}_s^*, J/\psi\phi} = -E_0.$
- $D_s\bar{D}_s - D^*\bar{D}^*$ is counted into $V_{ij}\,(a_{ij})$ after fitting.

T-matrix

- $T_{ij} = \left[(1 - VG)^{-1}V\right]_{ij},$
$$G_i^\Lambda = \int_0^\Lambda \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{2\mu_i}{\mathbf{q}^2 - k_i^2 - i\epsilon} = i\frac{\mu_i}{2\pi}k_i + \frac{\mu_i}{\pi^2}\Lambda\left[1 + O\left(\frac{k_i^2}{\Lambda^2}\right)\right]$$

$D_s\bar{D}_s - J/\psi\phi - D_s^*\bar{D}_s^*$ scattering in $J^{PC} = 0^{++}$ sector

T-Matrix in ERE in three coupled-channel scattering

- $V_{13} = 0$, $V_{11} = V_{33}$, $V_{im}V_{mj}$ attributed to the subleading term
- $$\frac{T}{8\pi t h_2} = \begin{pmatrix} \frac{1}{a_{11}} - ik_1 & \frac{1}{a_{12}} & \frac{1}{a_{13}} \\ \frac{1}{a_{12}} & \frac{1}{a_{22}} - ik_2 & \frac{1}{a_{23}} \\ \frac{1}{a_{13}} & \frac{1}{a_{23}} & \frac{1}{a_{33}} - ik_3 \end{pmatrix}^{-1},$$
  where $i = 1$, 2 and 3 denote the scattering channels, respectively, μ_i denotes the reduced mass, and the p_i is the NR momentum in c.m. frame. S. Sakai et al, PLB808, 2004.09824

Production amplitude

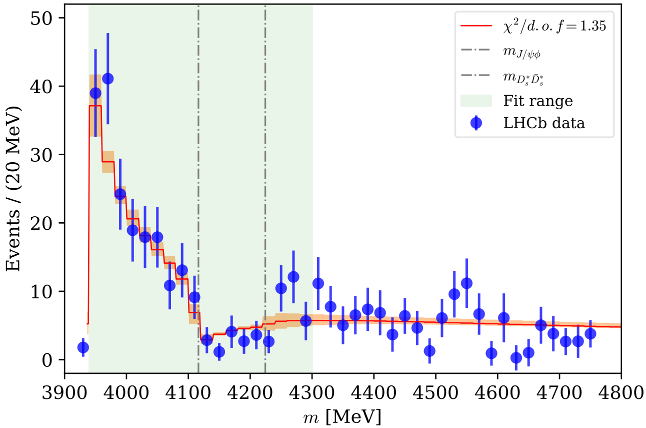
$$\mathcal{M}_a = P_1^\Lambda + P_1^\Lambda G_1^\Lambda T_{11} + P_1^\Lambda G_2^\Lambda T_{21} + P_3^\Lambda G_3^\Lambda T_{31}$$
$$= P_1 T_{11}(w) + P_2 T_{21}(w) + P_3 T_{31}(w).$$

X. K. Dong et al, PRL126, 2011.14517

Lineshape in $D_s\bar{D}_s$ invariant mass distributions

$$\frac{dN}{dm} = |\mathcal{M}_a|^2 \frac{|\vec{k}_1|}{8\pi m}$$

Fitting results



a_{11} [fm]	a_{22} [fm]	a_{23} [fm]	P_1	P_3
1.48 ± 0.17	1.11 ± 0.65	0.87 ± 0.10	-0.29 ± 0.02	0.04 ± 0.03

Fitting results

Scattering length

- a_{11} is consistent with $a_{11}^{Ji} = 1.87^{+0.30}_{-0.25}\,\mathrm{fm}$ extracted with $\Lambda = 1.0\,\mathrm{GeV}$ in the Gaussian regulator. T. Ji et al,Sci.Bull. 68 (2023)
- *Effective* $J/\psi\phi$ scattering length: $0.12^{+0.20}_{-0.10} + i0.78^{+0.20}_{-0.40}\,\mathrm{fm}$
- The $D^{(*)}\bar{D}^{(*)}$ interactions are not included in the analysis dircetly, and partly counted into δa_{22},
  $$\delta a_{22}^{D^*\bar{D}^*} = \frac{|\langle X(3960)|D^*\bar{D}^*\rangle|^2}{|\langle X(3960)|D_s\bar{D}_s\rangle|^2} a_{11} = \frac{|g_{X(3960),D^*\bar{D}^*}|^2}{|g_{X(3960),D_s\bar{D}_s}|^2} a_{11} = 0.30 \pm 0.03\,\mathrm{fm}.$$

Poles

(-++)	(-++)	(--+)
$3904.4^{+5.2}_{-14.7}$	$4106.0^{+10.4}_{-58.3} + i24.6^{-6.7}_{+29.1}$	$4223.5^{+1.9}_{-4.3} + i4.3^{-0.7}_{+14.5}$

The pole on RS $(-++)$ is a bound state w.r.t $J/\psi\phi$, where $J/\psi\phi - D_s^*\bar{D}_s^*$ plays a crucial role.

$J/\psi\phi$ scattering in $J^{PC}=1^{++}$ sector

HQSS

$$X\left(1^{++}\right)=\frac{1}{\sqrt{2}}\left(|D_s^*\bar{D}_s\rangle+|D_s\bar{D}_s^*\rangle\right),$$

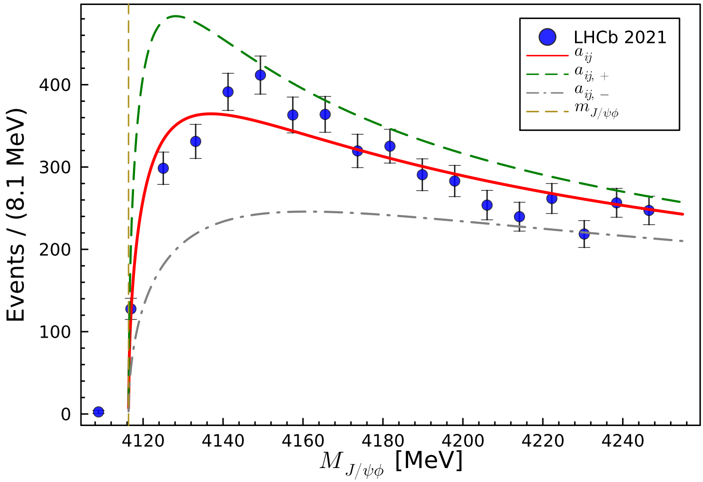
$$\left(\begin{array}{c}|D_s^*\bar{D}_s\rangle\\|D_s\bar{D}_s^*\rangle\\|D_s^*\bar{D}_s^*\rangle\end{array}\right)_{J=1}=\left(\begin{array}{ccc}-\frac{1}{2}&\frac{1}{2}&\frac{1}{\sqrt{2}}\\\frac{1}{2}&-\frac{1}{2}&\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}&0\end{array}\right)\left(\begin{array}{c}|0_l\otimes 1_Q\rangle\\|1_l\otimes 0_Q\rangle\\|1_l\otimes 1_Q\rangle\end{array}\right)_{J=1},$$

$$V_{J/\psi\phi,D_s\bar{D}_s^*}=\sqrt{2}V_{J/\psi\phi,\bar{D}_s^*\bar{D}_s^*}$$

Production amplitude

- $\mathcal{M}_b=P_{1^{++}}\frac{th_2}{8\pi}\frac{1}{1/a^{eff}-ik_2}$, $P_{1^{++}}$ is unknown.
- a virtual pole $4106.7^{-4.0}_{+15.5}+i\,8.3^{+2.6}_{-6.2}\,\mathrm{MeV}$ near $J/\psi\phi$ threshold.
- $\frac{dN}{dM_{J/\psi\phi}}=|\mathcal{M}_b|^2\frac{|\vec{k}_2|}{8\pi M_{J/\psi\phi}}$

$J/\psi\phi$ scattering in $J^{PC} = 1^{++}$ sector



$P_{1^{++}} = 1.16\,\mathrm{MeV}^{-1/2}$.

$X_1(4685)$ in $J/\psi\phi - \psi(2S)\phi$ scattering

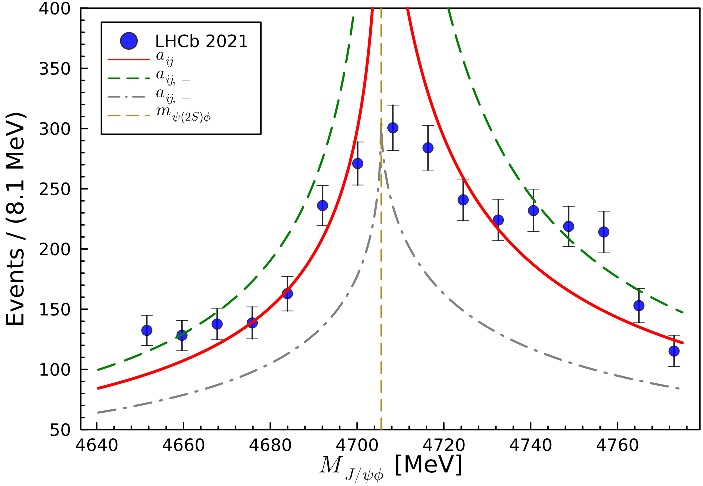
HQSS

- $\mathcal{L}_{J/\psi\phi_i,\psi(2S)\phi_f} = D_0 \frac{E_{\phi_i}}{m_{\phi_f}} \vec{\epsilon}_{J/\psi} \cdot \vec{\epsilon}_{\psi(2S)} \vec{\epsilon}_{\phi_i} \cdot \vec{\epsilon}_{\phi_f},$
- $D_s^{(*)} \bar{D}_s^{(*)}$ are far away from $\psi(2S)\phi$ threshold, where the transtions are kinematically suppressed.
- $\{J/\psi\phi,\ \psi(2S)\phi\}$ is considered around $\psi(2S)\phi$ threshold.

Production amplitude

- $\mathcal{M}_c = P^{J/\psi\phi} t_{11} + P^{\psi(2S)\phi} t_{21}$
- a virtual pole $4690.9^{-5.7}_{+5.1} + i7.0^{+13.5}_{-7.4}\,\mathrm{MeV}$ near $\psi(2S)\phi$ threshold.
- $\frac{dN}{dM_{J/\psi\phi}} = |\mathcal{M}_c|^2 \frac{|\vec{k}_2|}{8\pi M_{J/\psi\phi}}$

$X_1\,(4685)$ in $J/\psi\phi - \psi(2S)\phi$ scattering



$P^{J/\psi\phi}_{1^{++}} = 0$ and $P^{\psi(2S)\phi}_{1^{++}} = 0.48\,\mathrm{MeV}^{-1/2}$.

Summary

1. HQSS inspired EFT is introduced to study $D_s^{(*)}\bar{D}_s^{(*)} - J/\psi\phi$ scattering in the LO approach.
2. The dip around $J/\psi\phi$ threshold in $D_s\bar{D}_s$ invariant mass distribution provides a hint about $J/\psi\phi$ interaction, where a virtual state appears in elastic $J/\psi\phi$ scattering and is a counterpart of X(6200) in di-J/ψ scattering.
3. The $X_1(4140)$ may be a $J/\psi\phi$ virtual state.
4. The X(4685) may be a cusp effect, driven by a virtual pole, at $\psi(2S)\phi$ threshold.
5. The 2^{++} pole in $J/\psi\phi$ scattering might be observed in 2γ final states.
6. Coupled channel effect and OZI interaction work together in low energy scattering.

Summary

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Thanks !

Backup: K-matrix parameteriztion

2 coupled-channel scattering

$\mathcal{K}_{ba}(m) = \sum_R \frac{g_b^R g_a^R}{M_R^2 - m^2} + f_{ba}$, $\mathcal{P}_b(m) = \sum_R \frac{\beta_R g_b^R}{M_R^2 - m^2} + \beta_b$, cplx. valued β_i.
$\mathcal{M}_a = \sum_b (I - i\rho\mathcal{K})^{-1}_{ab} \mathcal{P}_b$,

Table S3: Main results found from the K-matrix fit. Uncertainties are statistical only.

Contribution	J^{PC}	M_R (MeV)	g_1^R (MeV)	Γ_0 (MeV)	$\mathcal{F}$ (%)
$\lvert\mathcal{M}_1\rvert^2$	0^{++}	3957 ± 14	1350 ± 344		94.7 ± 0.4
$\psi(4260)$	1^{--}	4230 [59]		55 [59]	3.2 ± 0.5
$\psi(4660)$	1^{--}	4633 [31]		64 [31]	2.1 ± 0.2
β_R		$(1, 0i)$	β_1	$(-1.2, 2.5i) \pm (4.5, 3.1i)$	
β_2		$(-137.2, -1.5i) \pm (2.7, 218.6i)$	f_{11}	0.8 ± 1.2	
$f_{12} = f_{21}$		0.1 ± 0.1	f_{22}	8.0 ± 5.1	

Backup: ERE and effective potential

IAM

$$T^{-1} = V^{-1} - G = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{12} & V_{22} & V_{23} \\ V_{13} & V_{23} & V_{33} \end{pmatrix}^{-1} - \begin{pmatrix} G_{11} & 0 & 0 \\ 0 & G_{22} & 0 \\ 0 & 0 & G_{33} \end{pmatrix}$$

$V_{13} = 0$, $V_{11} = V_{33}$, $V_{im}V_{mj}$ attributed to the subleading term

- $(T^{-1})_{11} = \frac{1}{V_{11}} - G_{11}$
- $(T^{-1})_{22} = \frac{1}{V_{22}} - G_{22}$
- $(T^{-1})_{33} = \frac{1}{V_{33}} - G_{33}$
- $(T^{-1})_{12} = \frac{V_{12}}{V_{12}^2 + V_{23}^2 - V_{11}V_{22}} \sim -\frac{V_{12}}{V_{11}V_{22}}$
- $(T^{-1})_{23} = \frac{V_{23}}{V_{12}^2 + V_{23}^2 - V_{11}V_{22}} \sim -\frac{V_{23}}{V_{11}V_{22}}$