Study of the structure near proton-antiproton threshold in $J/\psi \to \gamma 3(\pi^+\pi^-)$

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Summary

J/ψ decay

- There is structure near $p\bar{p}$ threshold on the $\eta'\pi^+\pi^-$ invariant spectrum in $J/\psi \to \gamma\eta'\pi^+\pi^-$, the $X(1835)$ was firstly discovered by BES. [Phys.Rev.Lett.95:262001(2005)]

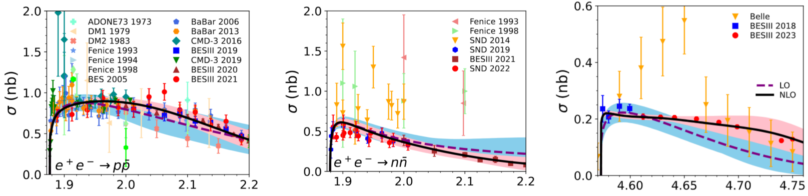
- The anomalous structures near $p\bar{p}$ threshold were found in the processes $J/\psi \to \gamma 3(\pi^+\pi^-)$, $J/\psi \to \gamma K_S^0 K_S^0 \eta$ and $J/\psi \to \gamma\phi$ by BESIII. [Phys.Rev.D 88(9):091502(2013), Phys.Rev.Lett.95:262001(2005), Phys.Rev.D 97(5):051101(2018)]

- The latest measurement for $J/\psi \to \gamma 3(\pi^+\pi^-)$ was performed by BESIII. [Phys.Rev.Lett.132(15):151901(2024)]
  - Higher statistics and more precision.
  - $X(1880)$ and $X(1835)$ are reported.

Threshold effect

The threshold enhancement effect:

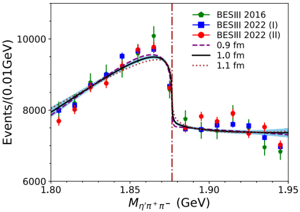
- The enhancement effect of cross section, for example the processes of $e^+e^- \to p\bar{p}/n\bar{n}$, $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$, $J/\psi \to \gamma p\bar{p}$ and etc, can be described by final-state interaction.
- Phys.Lett.B 761:456-461(2016)
  J.Haidenbauer, X.-W. Kang Nucl.Phys.A929:102-118(2014)
  J.Haidenbauer Phys.Rev.D103(1):014028(2021)
  X. Cao Phys.Rev.D105(7):L071503(2022)
  C. Chen, J.J.Xie Chin.Phys.Lett.41(2):021302(2024)
  R.-Q. Qian, X.Liu Phys.Rev.D107(9):L091502(2023)
  L.-Y. Dai Phys.Rev.D96(11):116001(2017)

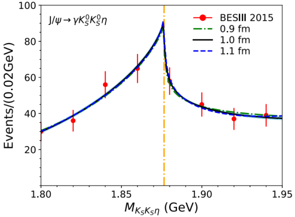


Threshold effect

Anomalous behaviors around the threshold

- For example there are kinks near the threshold for the processes of $J/\psi \to \gamma\eta'\pi^+\pi^-$, $J/\psi \to \gamma K_S^0 K_S^0 \eta$ and $J/\psi \to \gamma\phi$.
- X.-W. Kang Phys.Rev.D91(7):074003(2015)
  J. P. Dedonder Phys.Rev.C97(6):065026(2018)
  A.I. Milstein Nucl.Phys.A966:54-63(2017)
  L.-Y.Dai Phys.Rev.D98(1):014005(2018)

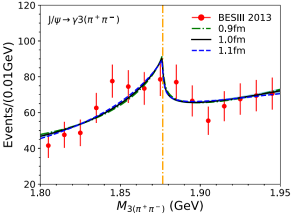


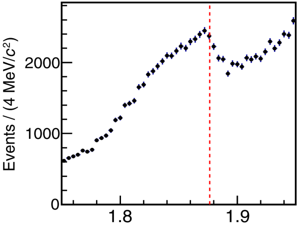


For $J/\psi \to \gamma 3(\pi^+\pi^-)$

- Can anomalous behaviors around the threshold for $J/\psi \to \gamma 3(\pi^+\pi^-)$ be described by threshold effect?

- Can other partial wave described more better than $^1S_0$ partial wave?

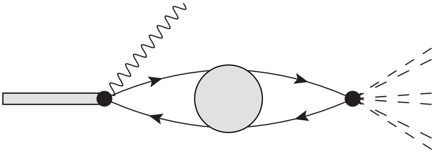




[L.Y.Dai Phys.Rev.D98(1):014005(2018), Q.H.-Yang Phys.Rev.D107(3):034030(2023)]

$N\bar{N}$ scattering

The Feynman diagram for $J/\psi \to \gamma 3(\pi^+\pi^-)$



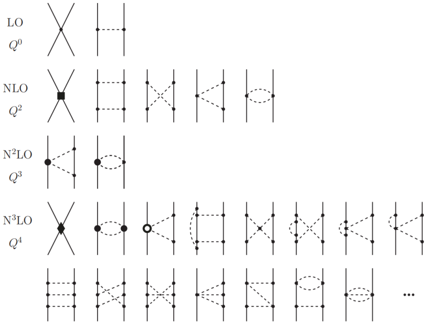
Lippmann-Schwinger equation

$$T_{L''L'}(p'', p'; E_k) = V_{L''L'}(p'', p') + \sum_L \int \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'', p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p, p'; E_k)$$

$V_{L''L'}(p'', p')$ is $SU(2)$ interaction potential.
For single channel $L'' = L' = L$

The scattering potential of $N\bar{N}$

The Feynman diagram of $N\bar{N}$ scattering up to N^{3}LO



- Meson exchange potential: one pion, two pion and three pion exchange;
- Contact potential;
- Annihilation potential.

Regulation

Meson exchange potential

- Transform to position space by Fourier transformation
- Regulating in position space by following function

$$f(r) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6$$

where the cut-off are took R=0.9,1.0,1.1 fm

- Transform to momentum space by inverse Fourier transformation

Contact and annihilation potential

- Regulating in momentum space by following function

$$f(p', p) = \exp\left(-\frac{p'^2 + p^2}{\Lambda^2}\right)$$

where the cut off $\Lambda = 2R^{-1}$

Two step DWBA

The amplitude of $J/\psi \to \gamma 3(\pi^+\pi^-)$ can be obtained through solving the following set of couple equations

$$F_{J/\psi\to\gamma 3(\pi^+\pi^-)}(Q) = A^0_{J/\psi\to\gamma 3(\pi^+\pi^-)}(Q) + \int_0^\infty \frac{\mathrm{d}k k^2}{(2\pi)^3} F_{J/\psi\to\gamma N\bar{N}}(E_k)\frac{1}{Q-2E_k+i\epsilon}V_{N\bar{N}\to 3(\pi^+\pi^-)}(k)\,,$$

$$F_{N\bar{N}\to 3(\pi^+\pi^-)}(Q) = V_{N\bar{N}\to 3(\pi^+\pi^-)}(p) + \int_0^\infty \frac{\mathrm{d}k k^2}{(2\pi)^3} T_{N\bar{N}\to N\bar{N}}(p,k;E_k)\frac{1}{2E_k-Q+i\epsilon}V_{N\bar{N}\to 3(\pi^+\pi^-)}(k)\,.$$

The transition Born amplitude $A^0_{J/\psi\to\gamma 3(\pi^+\pi^-)}$ and annihilation potential $V_{N\bar{N}\to 3(\pi^+\pi^-)}$

$$\begin{aligned} V_{N\bar{N}\to 3(\pi^+\pi^-)}(p) &= \tilde{C}_{N\bar{N}\to 3(\pi^+\pi^-)} + C_{N\bar{N}\to 3(\pi^+\pi^-)} p^2\,, \\ A^0_{J/\psi\to\gamma 3(\pi^+\pi^-)}(Q) &= \tilde{C}_{J/\psi\to\gamma 3(\pi^+\pi^-)} + C_{J/\psi\to\gamma 3(\pi^+\pi^-)} Q\,, \end{aligned}$$

The decay rate and cross section

The Lorentz invariant amplitudes $\mathcal{M}_{J/\psi\to\gamma 3(\pi^+\pi^-)}$ and $\mathcal{M}_{p\bar{p}\to 3(\pi^+\pi^-)}$

$$\begin{aligned}\mathcal{M}_{J/\psi\to\gamma 3(\pi^+\pi^-)} &= -32\pi^{\frac{7}{2}}\sqrt{E_\gamma E_{J/\psi} E_1 E_2 E_3} F_{J/\psi},\\ \mathcal{M}_{N\bar{N}\to 3(\pi^+\pi^-)} &= -32\pi^{\frac{7}{2}} E_N \sqrt{E_1 E_2 E_3} F_{N\bar{N}}.\end{aligned}$$

In order to simplify phase integration, $\pi^+\pi^-$ regard as a whole.
$E_i (i = 1, 2, 3)$ denote the energy of three $(\pi^+\pi^-)$ in the final state.
The decay rate and cross section

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}Q} &= \int_{\beta(Q)} \mathrm{d}t_1 \mathrm{d}t_2 \frac{(m_{J/\psi}^2 - Q^2)|\mathcal{M}_{J/\psi\to\gamma 3(\pi^+\pi^-)}|^2}{6144\tilde{N}\pi^5 m_{J/\psi}^3 Q},\\ \sigma(Q) &= \int_{\beta(Q)} \mathrm{d}t_1 \mathrm{d}t_2 \frac{|\mathcal{M}_{p\bar{p}\to 3(\pi^+\pi^-)}|^2}{1024\tilde{N}\pi^3 Q^3\sqrt{Q^2 - 4m_p^2}}.\end{aligned}$$

where Q is both the invariant mass $M_{3(\pi^+\pi^-)}$ and the center-mass energy of $N\bar{N}$.

The model parameters

- The LECs $\tilde{C}_i$, C_i and D_i in contact potential, and $\tilde{C}_i^a$, C_i^a and D_i^a in annihilation potential

$$V({}^1S_0) = \tilde{C}_{{}^1S_0} + C_{{}^1S_0}(p^2 + p'^2) + D^1_{{}^1S_0}p^2p'^2 + D^2_{{}^1S_0}(p^4 + p'^4)$$
$$V_{\text{ann}}({}^1S_0) = -i(\tilde{C}^a_{{}^1S_0} + C^a_{{}^1S_0}p^2 + D^a_{{}^1S_0}p'^4)(\tilde{C}^a_{{}^1S_0} + C^a_{{}^1S_0}p'^2 + D^a_{{}^1S_0}p'^4)$$

They are took the results from L.-Y.Dai JHEP07(2017)

- $\tilde{C}_i$, C_i in transition Born amplitude $A^0_{J/\psi\to\gamma 3(\pi^+\pi^-)}$ and annihilation potential $V_{N\bar{N}\to 3(\pi^+\pi^-)}$

- Some normalization factor.

The partial wave

The partial wave and quantum number for $p\bar{p}$ system
($P=(-1)^{L+1}$,$C=(-1)^{L+S}$)

$J=0$		$^1S_0(0^{-+})$	$^3P_0(1^{++})$	
$J=1$	$^1P_1(1^{+-})$	$^3P_1(1^{++})$	$^3S_1(1^{--})$	$^3D_1(1^{--})$
$J=2$	$^1D_2(2^{-+})$	$^3D_2(2^{--})$	$^3P_2(1^{++})$	$\cdots$

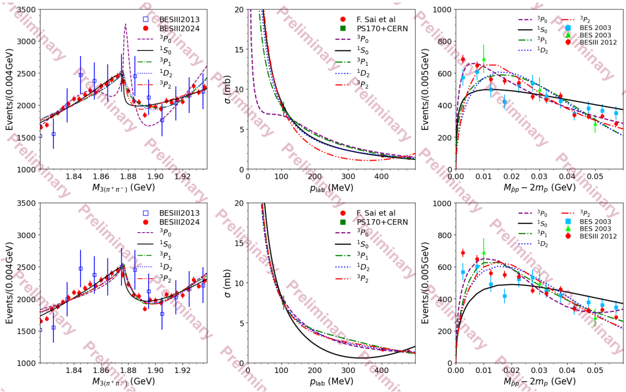
The quantum number of both J/ψ and γ: $J^{PC}=1^{--}$.

The allowed partial wave of $3(\pi^+\pi^-)$ and $p\bar{p}$ system:

- $^1S_0$, $^3P_0$, $^3P_1$, $^1D_2$, $^3P_2$;
- Higher partial wave are ignored.

The fitting for isospin $I=0,1$ are considered.

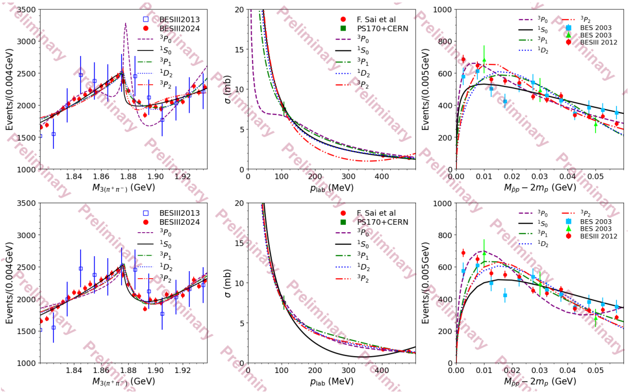
The fitting results for R=0.9 fm



Better fits

- $I = 0$: $^3P_1$;
- $I = 1$: $^3P_0$, $^3P_1$

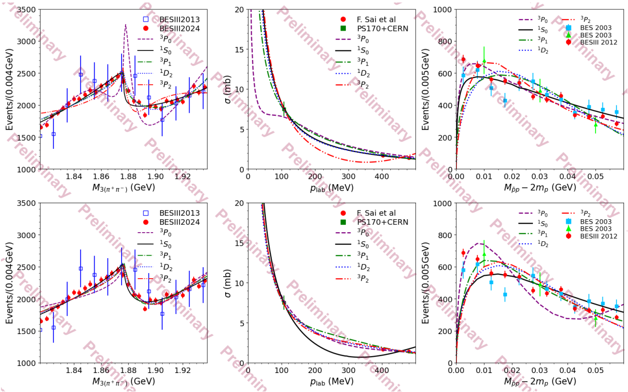
The fitting results for R=1.0 fm



Better fits

- $I = 0$: $^1S_0$, $^3P_1$, $^1D_2$;
- $I = 1$: $^3P_0$, $^3P_1$.

The fitting results for R=1.1 fm



Better fits

- $I = 0$: $^1S_0$, $^3P_1$;
- $I = 1$: $^3P_1$;

χ^2/N

			$^1S_0$	$^3P_0$	$^3P_1$	$^1D_2$	$^3P_2$
R=0.9	χ^2/N	$I=0$	1.86	12.84	2.15	2.06	1.94
		$I=1$	1.35	3.07	2.18	2.03	2.22
		$I=0$	0.93	3.40	0.69	0.02	11.27
		$I=1$	24.08	1.04	2.04	0.04	0.79
		$I=0$	10.33	2.05	8.81	12.18	9.60
		$I=1$	17.38	5.67	8.31	10.99	11.61
R=1.0	χ^2/N	$I=0$	1.86	13.06	2.00	2.02	3.21
		$I=1$	1.49	3.63	2.23	1.96	2.23
		$I=0$	0.66	3.41	0.66	0.04	13.19
		$I=1$	19.44	0.89	1.98	0.07	0.81
		$I=0$	6.70	2.06	8.87	11.78	9.49
		$I=1$	12.51	6.88	7.97	10.63	11.45
R=1.1	χ^2/N	$I=0$	1.85	12.73	1.93	2.00	5.05
		$I=1$	1.64	4.25	2.28	1.94	2.25
		$I=0$	0.36	4.10	0.76	0.05	16.17
		$I=1$	19.00	0.87	2.10	0.10	0.81
		$I=0$	3.58	2.02	8.70	11.33	9.58
		$I=1$	8.56	10.62	7.69	10.17	11.38

The partial wave for best fit: $^3P_1(I=1,0)$.

Summary

- The amplitudes of the processes $J/\psi \to \gamma 3(\pi^+\pi^-)$, $J/\psi \to \gamma p\bar{p}$ and $p\bar{p} \to 3(\pi^+\pi^-)$ are obtained through solving LS equation and a set of couple equation of DWBA;

- Fitting the $J/\psi \to \gamma 3(\pi^+\pi^-)$ latest measurement results, analysing the results of different partial wave.

- The latest data can be described very well by threshold effect, the partial wave ${}^3P_1(I=1,0)$ give the best fitting.

Thank you for your patience!