Transition formfactors of pseudoscalars for HLBL

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- Definition of g-factor: the strength of coupling to magnetic field

$$\vec{\mu} = -\mathrm{g_l}\frac{\mathrm{e}}{2\mathrm{m_l}}\vec{\mathrm{S}} \tag{1}$$

in 1928, Dirac got g=2 by using his equation.

- Anomalous magnetic moment: the deviation of g-factor from 2

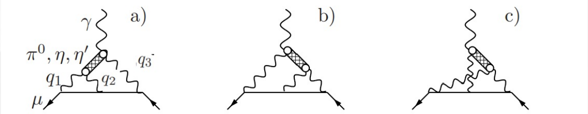
$$\mathrm{a_l} = \frac{\mathrm{g_l} - 2}{2} \tag{2}$$

in 1948, Schwinger got $\mathrm{a_l} = \frac{\alpha}{\pi}$ by using QED.

- Experimental value[1]:

$$(\mathrm{g} - 2)_\mu = 0.00233184110 \pm 0.00000000043 \pm 0.00000000019 \tag{3}$$

[1] arXiv:2402.15410 [hep-ex]



- Projection formula of a_l:[2]

$$a_l = -\frac{1}{48 m_l} \mathrm{Tr}\left\{ (\not{p} + m)[\gamma^\mu, \gamma^\nu](\not{p} + m) \frac{\partial \Gamma_\nu(k^2)}{\partial k^\mu} \right\} \tag{4}$$

- Pole contribution $a_\mu^{\mathrm{HLbL};P^0}$:

$$\begin{aligned} & -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 \left[(p+q_1)^2 - m^2\right] \left[(p-q_2)^2 - m^2\right]} \\ & \times \left[\frac{\mathcal{F}_{P^0\gamma^*\gamma^*}\left(q_1^2, (q_1+q_2)^2\right) \mathcal{F}_{P^0\gamma^*\gamma^*}\left(q_2^2, 0\right)}{q_2^2 - M_{P^0}^2} T_1(q_1, q_2; p) \right. \\ & \left. + \frac{\mathcal{F}_{P^0\gamma^*\gamma^*}\left(q_1^2, q_2^2\right) \mathcal{F}_{P^0\gamma^*\gamma^*}\left((q_1+q_2)^2, 0\right)}{(q_1+q_2)^2 - M_{P^0}^2} T_2(q_1, q_2; p) \right] \end{aligned} \tag{5}$$

[2] arXiv:0902.3360 [hep-ph]

- Resonance Chiral Lagrangian[3] [4]:

$$(\mathcal{L}^{\mathrm{GB}}_{(2)} + \mathcal{L}_{\mathrm{WZW}}) + \mathcal{L}^{\mathrm{R}}_{\mathrm{kin}} + \mathcal{L}^{\mathrm{R}}_{(2)} + \mathcal{L}^{\mathrm{R}}_{(4)} + \mathcal{L}^{\mathrm{RR}}_{(2)} \tag{6}$$

Where:

$$\mathcal{L}^{\mathrm{GB}}_{(2)} = \frac{F^2}{4}\langle \tilde{u}_\mu \tilde{u}^\mu \rangle + \frac{F^2}{4}\langle \tilde{\chi}_+ \rangle + \frac{F^2}{3} M_0^2 \ln^2 \det\tilde{u}, \tag{7}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{WZW}} = &-\frac{iN_C}{240\pi^2}\int d\sigma^{ijklm}\left\langle \Sigma^L_i \Sigma^L_j \Sigma^L_k \Sigma^L_l \Sigma^L_m \right\rangle \\ &-\frac{iN_C}{48\pi^2}\int d^4x \varepsilon_{\mu\nu\alpha\beta}\left(W(\tilde{U},\ell,r)^{\mu\nu\alpha\beta} - W(\mathbf{1},\ell,r)^{\mu\nu\alpha\beta}\right),\end{aligned} \tag{8}$$

$$\begin{aligned}\mathcal{L}^{\mathrm{R}}_{\mathrm{kin}} = &-\frac{1}{2}\left\langle \nabla^\lambda V_{\lambda\mu}\nabla_\nu V^{\nu\mu} - \frac{M_V^2}{2} V_{\mu\nu}V^{\mu\nu}\right\rangle \\ &- e^V_m \langle V^{\mu\nu}V_{\mu\nu}\tilde{\chi}_+\rangle + k^V_m V_0^{\mu\nu}\left\langle \hat{V}_{\mu\nu}\tilde{\chi}_+\right\rangle - \frac{\gamma_V M_V^2}{2} V_0^{\mu\nu}V_{0\mu\nu}\end{aligned}, \tag{9}$$

$$\mathcal{L}^{\mathrm{R}}_{(2)} = \frac{F_V}{2\sqrt{2}}\left\langle V_{\mu\nu}\tilde{f}_+^{\mu\nu}\right\rangle + \frac{\lambda_V}{\sqrt{2}}\left\langle V_{\mu\nu}\left\{\tilde{f}_+^{\mu\nu},\tilde{\chi}_+\right\}\right\rangle, \mathcal{L}^{\mathrm{R}}_{(4)} = \sum_{j=1}^{8}\frac{\tilde{c}_j}{M_V}\tilde{\mathcal{O}}^j_{\mathrm{VJP}},\ \mathcal{L}^{\mathrm{RR}}_{(2)} = \sum_{i=1}^{5}\tilde{d}_i\tilde{\mathcal{O}}^i_{\mathrm{VVP}}. \tag{10}$$

- In order to make the analysis more complete, this work follow the literature[5] by including the mixing of goldstone meson and pseudoscalar resonance, and their contributions could be treated as corrections of VJP and VVP LECS:

$$\tilde{c}_3 \longrightarrow \tilde{c}_3^* = \tilde{c}_3 + \frac{d_m M_V \kappa_3^{PV}}{M^2_{P'}},\ \tilde{d}_2 \longrightarrow \tilde{d}_2^* = \tilde{d}_2 + \frac{d_m M_V \kappa^{PVV}}{2M^2_{P'}}. \tag{11}$$

[3] arXiv:1201.2135[hep-ph]
[4] JHEP 0306 (2003) 012
[5] arXiv:1803.08099 [hep-ph]

- The gold-stone fields in the two-angle mixing scheme is

$$\tilde{\Phi} = \begin{pmatrix} \frac{\eta' C_q{}' + \eta C_q + \pi_0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta' C_q{}' + \eta C_q - \pi_0}{\sqrt{2}} & K_0 \\ K^- & \bar{K}_0 & \eta' C_S{}' - \eta C_S \end{pmatrix} . \tag{12}$$

where

$$\begin{aligned} C_q &= \frac{F}{\sqrt{3}\cos(\theta_8 - \theta_0)} \left(\frac{\cos\theta_0}{F_8} - \frac{\sqrt{2}\sin\theta_8}{F_0} \right), \\ C_q' &= \frac{F}{\sqrt{3}\cos(\theta_8 - \theta_0)} \left(\frac{\sqrt{2}\cos\theta_8}{F_0} + \frac{\sin\theta_0}{F_8} \right), \\ C_s &= \frac{F}{\sqrt{3}\cos(\theta_8 - \theta_0)} \left(\frac{\sqrt{2}\cos\theta_0}{F_8} + \frac{\sin\theta_8}{F_0} \right), \\ C_s' &= \frac{F}{\sqrt{3}\cos(\theta_8 - \theta_0)} \left(\frac{\cos\theta_8}{F_0} - \frac{\sqrt{2}\sin\theta_0}{F_8} \right). \end{aligned} \tag{13}$$

- $\rho^0 - \omega$ mixing[6] [7]:

$$\begin{pmatrix} |\bar{\rho}^0\rangle \\ |\bar{\omega}\rangle \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta\frac{M_\rho\Gamma_\rho}{M_\rho^2-q^2+iM_\rho\Gamma_\rho} \\ \sin\delta\frac{M_\rho\Gamma_\rho}{M_\rho^2-q^2-iM_\rho\Gamma_\rho} & \cos\delta \end{pmatrix} \begin{pmatrix} |\rho^0\rangle \\ |\omega\rangle \end{pmatrix}$$
$$\equiv \begin{pmatrix} \cos\delta & -\sin\delta\omega(q^2) \\ \sin\delta\rho(q^2) & \cos\delta \end{pmatrix} \begin{pmatrix} |\rho^0\rangle \\ |\omega\rangle \end{pmatrix}. \tag{14}$$

where δ is $\rho - \omega$ mixing angle, and

$$\sin\delta\omega(q^2) = -\sin(\delta)\frac{M_\rho\Gamma_\rho}{M_\rho^2 - q^2 + iM_\rho\Gamma_\rho} \tag{15}$$

$$\sin\delta\rho(q^2) = \sin(\delta)\frac{M_\rho\Gamma_\rho}{M_\rho^2 - q^2 - iM_\rho\Gamma_\rho} \tag{16}$$

- $\omega - \phi$ mixing:

$$V_{11} = \frac{1}{\sqrt{2}}(\rho^0 + \frac{\sqrt{2}}{\sqrt{3}}(\frac{\cos(\theta v)}{\sqrt{2}} - \sin(\theta v))\phi + \frac{\sqrt{2}}{\sqrt{3}}(\frac{\sin(\theta v)}{\sqrt{2}} + \cos(\theta v)\omega) \tag{17}$$

$$V_{22} = -\frac{1}{\sqrt{2}}(\rho^0 - \frac{\sqrt{2}}{\sqrt{3}}(\frac{\cos(\theta v)}{\sqrt{2}} - \sin(\theta v))\phi - \frac{\sqrt{2}}{\sqrt{3}}(\frac{\sin(\theta v)}{\sqrt{2}} + \cos(\theta v))\omega) \tag{18}$$

$$V_{33} = \frac{\left(\cos(\theta v) - \sqrt{2}\sin(\theta v)\right)\omega}{\sqrt{3}} - \frac{\left(\sin(\theta v) + \sqrt{2}\cos(\theta v)\right)\phi}{\sqrt{3}} \tag{19}$$

[6] DOI:10.1016/0370-1573(82)90035-7
[7] arXiv:2302.08859 [hep-ph]

- High-energy constraints:

$$\tilde{c}_{125} = \tilde{c}_{1235} = 0,\ \tilde{c}_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V},\ \tilde{c}^*_{1235} = 8\frac{d_m M_V \kappa_3^{PV}}{M^2_{P'}}, \tag{20}$$

where

$$\tilde{c}_{125} = \tilde{c}_1 - \tilde{c}_2 + \tilde{c}_5\ ,\ \tilde{c}^*_{1235} = \tilde{c}_1 + \tilde{c}_2 + 8\tilde{c}^*_3 - \tilde{c}_5\ ,\ \tilde{c}_{1256} = \tilde{c}_1 - \tilde{c}_2 - \tilde{c}_5 + 2\tilde{c}_6. \tag{21}$$

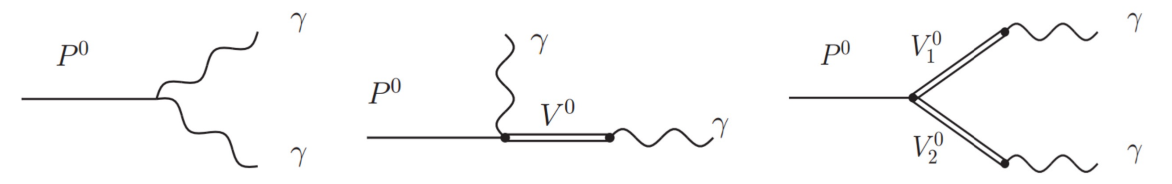
$$\tilde{d}^*_{123} = \frac{F^2}{8F_V^2} + 4\frac{d_m M_V \kappa^{PVV}}{M^2_{P'}}\ ,\ \tilde{d}_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2} \tag{22}$$

where

$$\tilde{d}^*_{123} = \tilde{d}_1 + 8\tilde{d}^*_2 - \tilde{d}_3, \tag{23}$$

and

$$\gamma_V = 0\ ,\ \lambda_V = 0. \tag{24}$$



- $P^0 \to \gamma^*\gamma^*$ amplitude:
  the amplitude is given by doubly-virtual Transition Form Factor(TFF)

$$|\mathcal{M}_{P^0\gamma^*\gamma^*}| = e^2|\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon_{1\rho}\epsilon_{2\sigma}| \cdot |\mathcal{F}_{P^0\gamma^*\gamma^*}(q_1^2, q_2^2)| \quad (25)$$

- TFF:

$$\mathcal{F}_{P^0\gamma^*\gamma^*} = \mathcal{F}^{\text{local}}_{P^0\gamma^*\gamma^*} + \mathcal{F}^{1R}_{P^0\gamma^*\gamma^*} + \mathcal{F}^{2R}_{P^0\gamma^*\gamma^*} \quad (26)$$

- Local term part:

$$\mathcal{M}^{\pi^0}_{\text{local}} = ie^2\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\left(-\frac{N_C}{12\pi^2F_\pi}\right) \quad (27)$$

$$\mathcal{M}^{\eta}_{\text{local}} = ie^2\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\left(-(5C_q - \sqrt{2}C_s)\frac{N_C}{36\pi^2F_\pi}\right) \quad (28)$$

$$\mathcal{M}^{\eta'}_{\text{local}} = ie^2\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\left(-(5C'_q + \sqrt{2}C'_s)\frac{N_C}{36\pi^2F_\pi}\right) \quad (29)$$

$$\mathcal{M}^{1R}_{\pi^0} = \mathcal{M}^{\rho}_{\pi^0} + \mathcal{M}^{\omega}_{\pi^0} + \mathcal{M}^{\phi}_{\pi^0} \tag{30}$$

$$\mathcal{M}^{\rho}_{\pi^0} = -i\frac{2e^2}{3FM_V}\varepsilon^{\mu\nu\rho\sigma} q_{1\mu} q_{2\nu} \epsilon^*_{1\rho}\epsilon^*_{2\sigma}[(\tilde{c}^*_{1235} m^2_\pi + \tilde{c}_{125} q_1^2 - \tilde{c}_{1256} q_2^2)\left(F\rho(q_2^2)BW(\rho 0, q_2^2)\left(\sqrt{2}\cos(\delta) - \sqrt{3}Sin\delta\rho(q_2^2)\left(\sqrt{2}\sin(\theta v) + 2\cos(\theta v)\right)\right)\right)$$
$$+ (q_1 \to q_2, q_2 \to q_1)] \tag{31}$$

$$\mathcal{M}^{\omega}_{\pi^0} = -i\frac{2e^2}{3FM_V}\varepsilon^{\mu\nu\rho\sigma} q_{1\mu} q_{2\nu} \epsilon^*_{1\rho}\epsilon^*_{2\sigma}[(\tilde{c}^*_{1235} m^2_\pi + \tilde{c}_{125} q_1^2 - \tilde{c}_{1256} q_2^2)\left(F\omega(q_2^2)BW(\omega, q_2^2)\left(2\sqrt{3}\cos(\delta)\cos(\theta v) + \sqrt{6}\cos(\delta)\sin(\theta v) + \sqrt{2}Sin\delta\omega(q_2^2)\right)\right)$$
$$+ (q_1 \to q_2, q_2 \to q_1)] \tag{32}$$

$$\mathcal{M}^{\phi}_{\pi^0} = -i\frac{2e^2}{3FM_V}\varepsilon^{\mu\nu\rho\sigma} q_{1\mu} q_{2\nu} \epsilon^*_{1\rho}\epsilon^*_{2\sigma}[(\tilde{c}^*_{1235} m^2_\pi + \tilde{c}_{125} q_1^2 - \tilde{c}_{1256} q_2^2)\left(\sqrt{3}F\phi(q_2^2)\left(\sqrt{2}\cos(\theta v) - 2\sin(\theta v)\right)BW(\phi, q_2^2)\right)$$
$$+ (q_1 \to q_2, q_2 \to q_1)] \tag{33}$$

where

$$F\rho(q^2) = \frac{1}{9}\left(9\cos(\delta)\left(F_V + 8\lambda_V m_\pi^2\right) - \sqrt{3}Sin\delta\rho(q^2)\left(3F_V\sin(\theta v) - 16\lambda_V m_K^2\left(\sqrt{2}\cos(\theta v) - 2\sin(\theta v)\right) + 8\lambda_V m_\pi^2\left(2\sqrt{2}\cos(\theta v) - \sin(\theta v)\right)\right)\right) \tag{34}$$

$$F\omega(q^2) = \frac{1}{9}\left(\sqrt{3}\cos(\delta)\left(3F_V\sin(\theta v) - 16\lambda_V m_K^2\left(\sqrt{2}\cos(\theta v) - 2\sin(\theta v)\right) + 8\lambda_V m_\pi^2\left(2\sqrt{2}\cos(\theta v) - \sin(\theta v)\right)\right) + 9Sin\delta\omega(q^2)\left(F_V + 8\lambda_V m_\pi^2\right)\right) \tag{35}$$

$$F\phi(q^2) = \frac{\left(3F_V\cos(\theta v) + 16\lambda_V m_K^2\left(\sqrt{2}\sin(\theta v) + 2\cos(\theta v)\right) - 8\lambda_V m_\pi^2\left(2\sqrt{2}\sin(\theta v) + \cos(\theta v)\right)\right)}{3\sqrt{3}} \tag{36}$$

$$\mathcal{M}^{2R}_{\pi^0} = \mathcal{M}^{\rho\rho}_{\pi^0} + \mathcal{M}^{\rho\omega}_{\pi^0} + \mathcal{M}^{\rho\phi}_{\pi^0} + \mathcal{M}^{\omega\omega}_{\pi^0} + \mathcal{M}^{\omega\phi}_{\pi^0} \tag{37}$$

and

$$\begin{aligned}\mathcal{M}^{\rho\rho}_{\pi^0} = & -i\frac{4e^2}{\sqrt{3}F}\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\left(\tilde{d}^*_{123}m^2_\pi + \tilde{d}_3\left(q_1^2+q_2^2\right)\right)\\ & \left[\cos(\delta)F\rho(q_1^2)F\rho(q_2^2)\left(\sin(\theta v)+\sqrt{2}\cos(\theta v)\right)(Sin\delta\rho(q_1^2)+Sin\delta\rho(q_2^2))BW(\rho0,q_1^2)BW(\rho0,q_2^2)\right]\end{aligned} \tag{38}$$

$$\begin{aligned}\mathcal{M}^{\rho\omega}_{\pi^0} = & \, i\frac{2e^2}{\sqrt{3}F}\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\left(\tilde{d}^*_{123}m^2_\pi + \tilde{d}_3\left(q_1^2+q_2^2\right)\right)\\ & [\left(\sin(\theta v)+\sqrt{2}\cos(\theta v)\right)(F\rho(q_2^2)F\omega(q_1^2)BW(\omega,q_1^2)BW(\rho0,q_2^2)(\cos(2\delta)-2Sin\delta\omega(q_1^2)Sin\delta\rho(q_2^2)+1)\\ & +F\rho(q_1^2)F\omega(q_2^2)BW(\rho0,q_1^2)BW(\omega,q_2^2)(\cos(2\delta)-2Sin\delta\rho(q_1^2)Sin\delta\omega(q_2^2)+1))]\end{aligned} \tag{39}$$

$$\begin{aligned}\mathcal{M}^{\rho\phi}_{\pi^0} = & -i\frac{4e^2}{\sqrt{3}F}\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\left(\tilde{d}^*_{123}m^2_\pi + \tilde{d}_3\left(q_1^2+q_2^2\right)\right)\\ & \left[\cos(\delta)\left(\sqrt{2}\sin(\theta v)-\cos(\theta v)\right)\left(F\rho(q_2^2)F\phi(q_1^2)BW(\phi,q_1^2)BW(\rho0,q_2^2)+F\rho(q_1^2)F\phi(q_2^2)BW(\rho0,q_1^2)BW(\phi,q_2^2)\right)\right]\end{aligned} \tag{40}$$

$$\begin{aligned}\mathcal{M}^{\omega\omega}_{\pi^0} = & \, i\frac{4e^2}{\sqrt{3}F}\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\left(\tilde{d}^*_{123}m^2_\pi + \tilde{d}_3\left(q_1^2+q_2^2\right)\right)\\ & \left[\cos(\delta)F\omega(q_1^2)F\omega(q_2^2)\left(\sin(\theta v)+\sqrt{2}\cos(\theta v)\right)\left(Sin\delta\omega(q_1^2)+Sin\delta\omega(q_2^2)\right)\right]\end{aligned} \tag{41}$$

$$\begin{aligned}\mathcal{M}^{\omega\phi}_{\pi^0} = & -i\frac{4e^2}{\sqrt{3}F}\varepsilon^{\mu\nu\rho\sigma}q_{1\mu}q_{2\nu}\epsilon^*_{1\rho}\epsilon^*_{2\sigma}\left(\tilde{d}^*_{123}m^2_\pi + \tilde{d}_3\left(q_1^2+q_2^2\right)\right)\\ & \left[\left(\sqrt{2}\sin(\theta v)-\cos(\theta v)\right)\left(F\omega(q_1^2)F\phi(q_2^2)Sin\delta\omega(q_1^2)BW(\omega,q_1^2)BW(\phi,q_2^2)+F\omega(q_2^2)F\phi(q_1^2)Sin\delta\omega(q_2^2)BW(\phi,q_1^2)BW(\omega,q_2^2)\right)\right]\end{aligned} \tag{42}$$

- Including higher resonances:
  In order to extend the above analysis to higher energy region, one could add heavier vector resonance multiplets, V′ and V″. This work follows pervious method[8], by applying the extension to the Breit-Wigner(BW) propagators

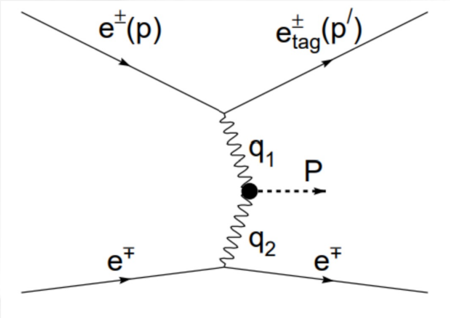
$$\mathrm{BW}(\mathrm{V},\mathrm{x}) = \frac{1}{\Delta_{\mathrm{V}}(\mathrm{x})} \longrightarrow \frac{1}{\Delta_{\mathrm{V}}(\mathrm{x})} + \frac{\beta'_{\mathrm{P}^0\gamma}}{\Delta_{\mathrm{V}'}(\mathrm{x})} + \frac{\beta''_{\mathrm{P}^0\gamma}}{\Delta_{\mathrm{V}''}(\mathrm{x})} \tag{43}$$

- Off-shell widths of V, V′ and V″:

$$\begin{aligned}
\Gamma_\rho\left(\mathrm{q}^2\right) &= \frac{\mathrm{M}_\rho \mathrm{q}^2}{96\pi \mathrm{F}^2}\left[\sigma_\pi^3\left(\mathrm{q}^2\right)\theta\left(\mathrm{q}^2-4\mathrm{m}_\pi^2\right)+\frac{1}{2}\sigma_{\mathrm{K}}^3\left(\mathrm{q}^2\right)\theta\left(\mathrm{q}^2-4\mathrm{m}_{\mathrm{K}}^2\right)\right] \\
\Gamma_{\rho'}\left(\mathrm{q}^2\right) &= \Gamma_{\rho'_0}\left(\mathrm{M}_{\rho'}^2\right)\frac{\sqrt{\mathrm{q}^2}}{\mathrm{M}_{\rho'}}\left(\frac{\sigma_\pi\left(\mathrm{q}^2\right)}{\sigma_\pi\left(\mathrm{M}_{\rho'}^2\right)}\right)^3\theta\left(\mathrm{q}^2-4\mathrm{m}_\pi^2\right) \\
\Gamma_{\rho''}\left(\mathrm{q}^2\right) &= \Gamma_{\rho''_0}\left(\mathrm{M}_{\rho''}^2\right)\frac{\sqrt{\mathrm{q}^2}}{\mathrm{M}_{\rho''}}\left(\frac{\sigma_\pi\left(\mathrm{q}^2\right)}{\sigma_\pi\left(\mathrm{M}_{\rho''}^2\right)}\right)^3\theta\left(\mathrm{q}^2-4\mathrm{m}_\pi^2\right)
\end{aligned} \tag{44}$$

[8] arXiv:2302.08859 [hep-ph]

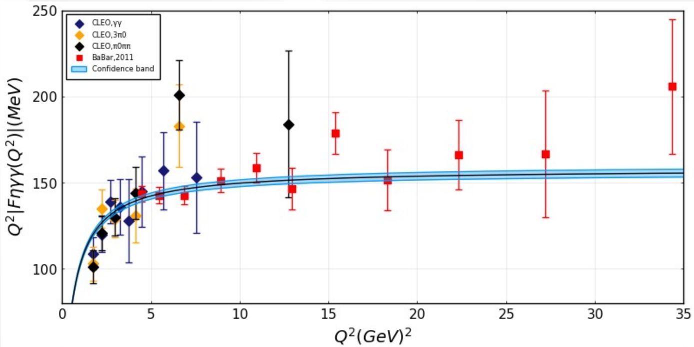
- The space-like data are mostly measured by $e^+e^- \to e^+\gamma^*\gamma e^- \to e^+P^0e^-$

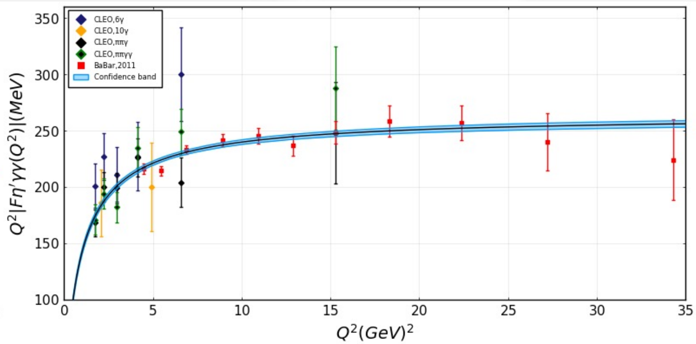


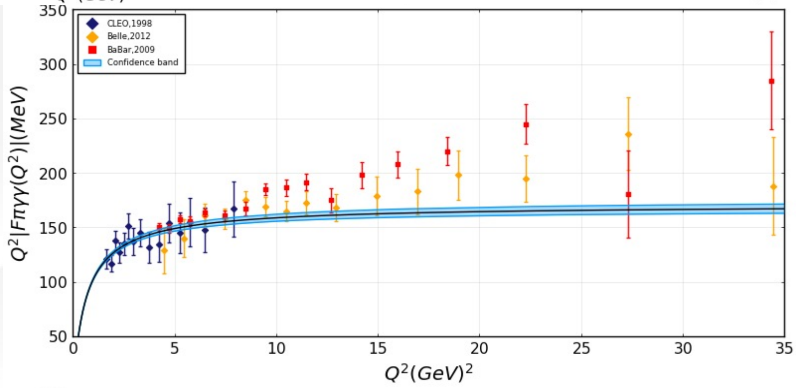
(45)

one of the photons is nearly on-shell.

Space-like region(2):TFFs







- Normalized invariant mass spectrum:

$$\frac{d\Gamma_{P^0\to l^+l^-\gamma}}{dq^2\Gamma_{P^0\to\gamma\gamma}} = \frac{2\alpha}{3\pi}\frac{1}{q^2}\sqrt{1-\frac{4m_l^2}{q^2}}(1+\frac{2m_l^2}{q^2})(1-\frac{q^2}{m_{P^0}^2})^3|F_{P^0}(q^2)|^2 \tag{46}$$

where $|F_{P^0}(q^2)|$ is the Normalized Form Factor

$$|F_{P^0}(q^2)| = |\mathcal{F}_{P^0\gamma^*\gamma^*}(q^2,0)/\mathcal{F}_{P^0\gamma^*\gamma^*}(0,0)|. \tag{47}$$

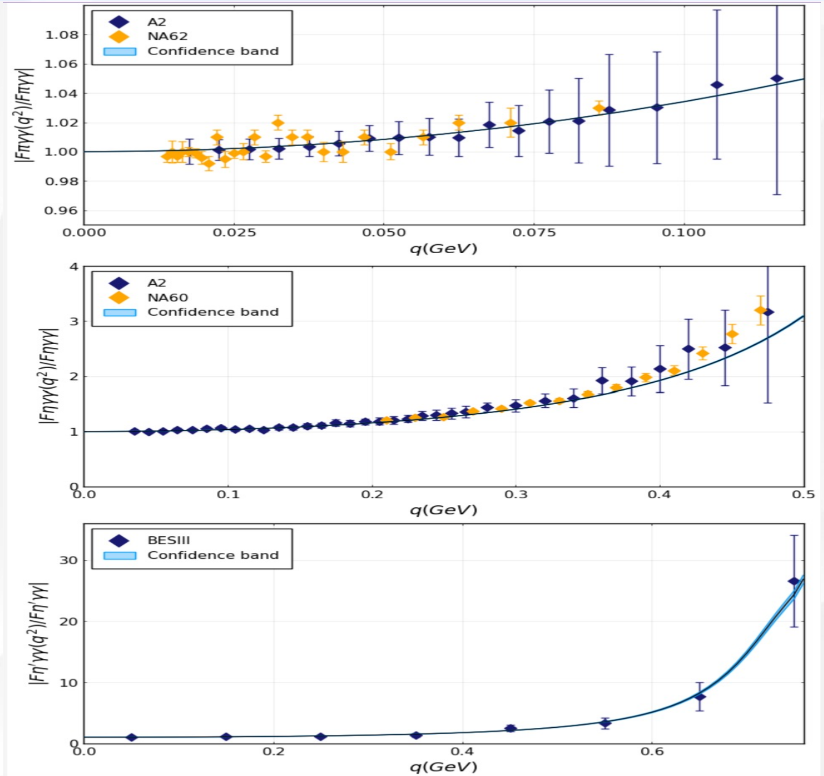
- Single-pole form factor:

$$F_{P^0}^{single-pole}(q^2) = \frac{1}{1-q^2/\Lambda_{P^0}^2} \tag{48}$$

the pole parameter is

$$b_{P^0} = \Lambda_{P^0}^{-2} = \frac{dF_{P^0}(q^2)}{dq^2}|_{q^2=0}. \tag{49}$$

Single-Dalitz decay(2): Normalized Form Factors

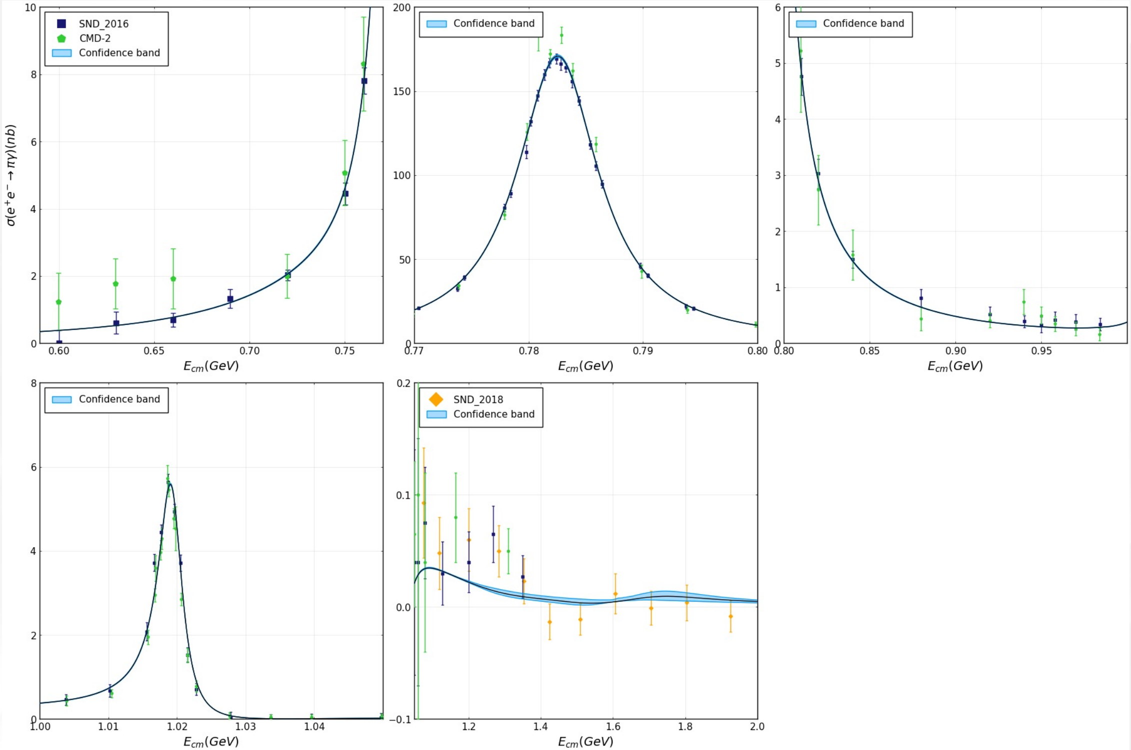


- The total cross-section is

$$\sigma_{l^+l^-\to P^0\gamma} = \frac{2\pi^2\alpha^3|\mathcal{F}_{P^0\gamma^*\gamma^*}(0,s)|^2\left(2m_l^2+s\right)\left(\frac{\left(s-m_{P^0}^2\right)^2}{s}\right)^{3/2}}{3s^2\sqrt{s-4m_l^2}}. \tag{50}$$

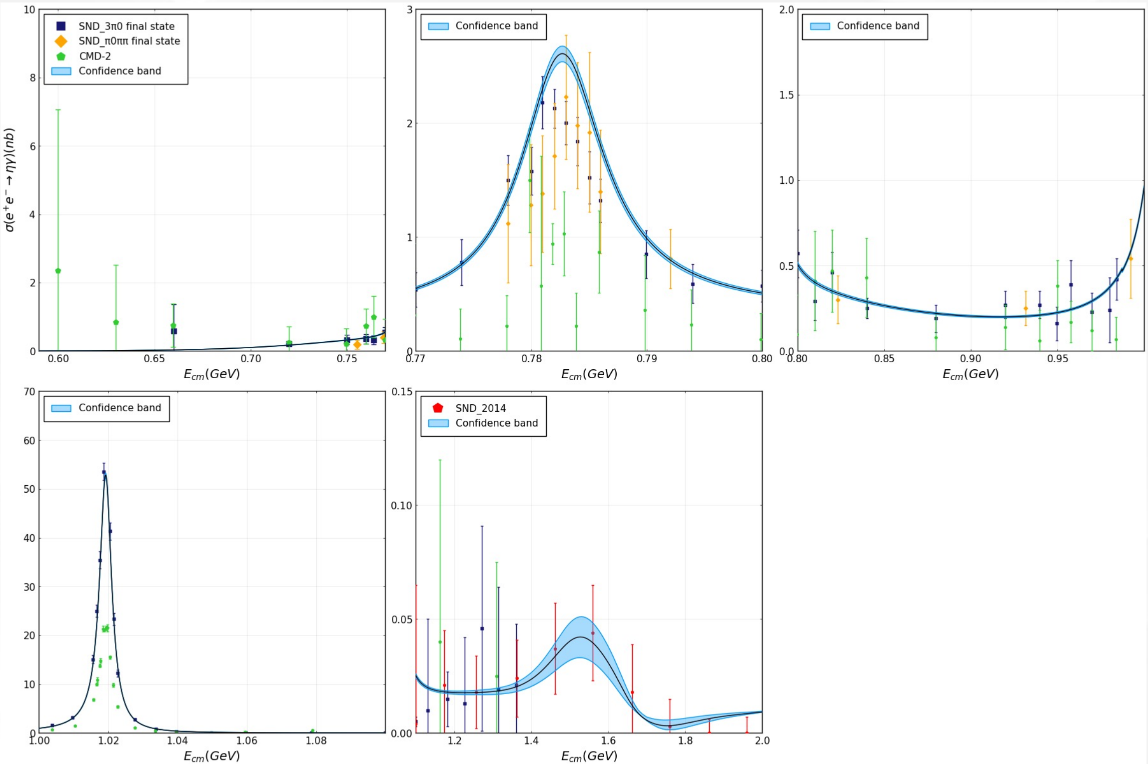
Since $s > m_{P^0}^2 >> m_e^2$, one has

$$\sigma_{e^+e^-\to P^0\gamma} = \frac{2}{3}\pi^2\alpha^3|\mathcal{F}_{P^0\gamma^*\gamma^*}(0,s)|^2\left(1-\frac{m_{P^0}^2}{s}\right)^3 \tag{51}$$



SND_2016
CMD-2
Confidence band
σ(e+e− → πγ)(nb)
Ecm(GeV)
Confidence band
Ecm(GeV)
Confidence band
Ecm(GeV)
Confidence band
Ecm(GeV)
SND_2018
Confidence band
Ecm(GeV)

$e^+e^- \to \eta\gamma$ cross section



- Pole contributions

$$a_{\mu}^{\pi^0} = 5.61 \times 10^{-10},\ a_{\mu}^{\eta} = 1.61 \times 10^{-10},\ a_{\mu}^{\eta'} = 1.49 \times 10^{-10} \tag{52}$$

- Futher improvements:
(1)Allow the break of the Brodsky Lepage (BL) limit of the singly-virtual form factor;
(2)Add data of doubly-virtual form factor;
(3)Replace $\mathcal{F}_{\mathrm{P}^0\gamma^*\gamma^*}$ by $\mathcal{F}_{\mathrm{P}^{0*}\gamma^*\gamma^*}$;
(4)More systematic treatment of vector meson excited states

Thank you for your patience!