Two-pseudoscalar-meson decay of resonances dynamically generated by two vector mesons

arXiv:2409.05302v1 [hep-ph]

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Thanks： Xiang Liu, Eulogio Oset

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- **Background**
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Part 1

Background

- Vector meson-vector meson interaction
- Dynamically generated states
- Decay

Background

Vector meson-vector meson interaction

L. S. Geng and E. Oset. Phys. Rev. D 79 (2009) 7, 074009

Hidden-gauge Lagrangian

$$\mathcal{L} = -\frac{1}{4}\left\langle \bar{V}_{uv}\bar{V}^{uv}\right\rangle + \frac{1}{2}M_V^2\left\langle\left[V_u - \frac{i}{g}\Gamma_u\right]^2\right\rangle$$

where

$$\bar{V}_{uv} = \partial_u V_v - \partial_v V_u - ig[V_u, V_v]$$

$$\Gamma_u = \frac{1}{2}\{u^+[\partial_u - i(v_u + a_u)]u + u[\partial_u - i(v_u - a_u)]u^+\}$$

$$u^2 = U = e^{\frac{i\sqrt{2}P}{f}} \qquad g = \frac{M_V}{2f}$$

$$V_u = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\pi^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \Phi \end{pmatrix}_u$$

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

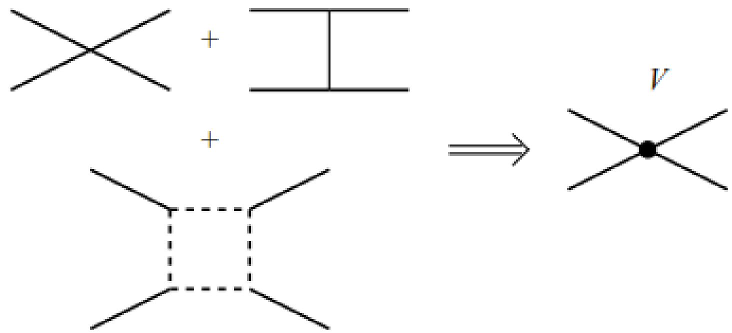
$$\mathcal{L}_{VVVV} = \frac{1}{2}g^2\langle [V_u, V_v]V^u V^v\rangle$$

$$\mathcal{L}_{VVV} = ig\langle (V^u\partial_v V_u - \partial_v V_u V^u)V^v\rangle$$

$$\mathcal{L}_{VPP} = -ig\langle V_u[P, \partial^u P]\rangle$$

Background

Dynamically generated states



$I^G(J^{PC})$	Theory			PDG data		
	Pole position	Real axis		Name	Mass	Width
		$\Lambda_b = 1.4$ GeV	$\Lambda_b = 1.5$ GeV			
$0^+(0^{++})$	(1512,51)	(1523,257)	(1517,396)	$f_0(1370)$	1200∼1500	200∼500
$0^+(0^{++})$	(1726,28)	(1721,133)	(1717,151)	$f_0(1710)$	1724 ± 7	137 ± 8
$0^-(1^{+-})$	(1802,78)	(1802,49)		h_1		
$0^+(2^{++})$	(1275,2)	(1276,97)	(1275,111)	$f_2(1270)$	1275.1 ± 1.2	$185.0^{+2.9}_{-2.4}$
$0^+(2^{++})$	(1525,6)	(1525,45)	(1525,51)	$f_2'(1525)$	1525 ± 5	73^{+6}_{-5}
$1^-(0^{++})$	(1780,133)	(1777,148)	(1777,172)	a_0		
$1^+(1^{+-})$	(1679,235)	(1703,188)		b_1		
$1^-(2^{++})$	(1569,32)	(1567,47)	(1566,51)	$a_2(1700)$??		
$1/2(0^+)$	(1643,47)	(1639,139)	(1637,162)	K_0^*		
$1/2(1^+)$	(1737,165)	(1743,126)		$K_1(1650)$?		
$1/2(2^+)$	(1431,1)	(1431,56)	(1431,63)	$K_2^*(1430)$	1429 ± 1.4	104 ± 4

Background

decay

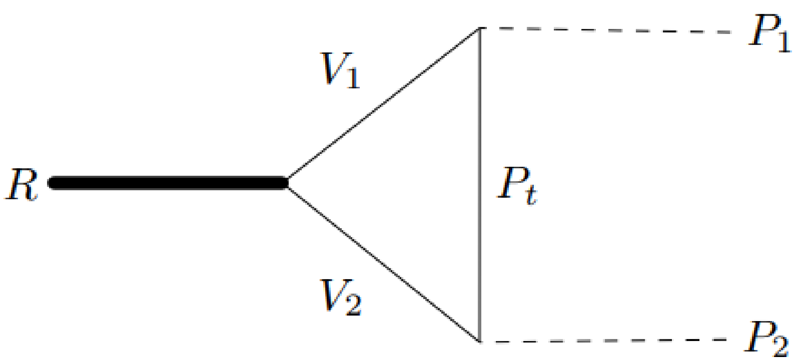
Process	Reference	Process	Reference
$R \to \gamma\gamma$	Phys. Rev. D 81, 054037 (2010)	$J/\psi \to VR$	Phys. Lett. *B* 680 (2009) 310-315
$R \to \gamma P$	Phys. Rev. D 81, 054037 (2010)		Eur. Phys. J. A 56 (2020) 6, 173
	Phys. Rev. *D* 83 (2011) 094030		Phys. Lett. *B* 843 (2023) 137999
$R \to V\gamma P$	Phys. Lett. *B* 690 (2010) 376-381	$J/\psi \to \gamma R$	Eur. Phys. J. A 44 (2010) 305-311
$\overline{B}_0/\overline{B}_s^0 \to J/\Psi\ R$	Phys. Rev. D 90 (2014) 9, 094006	$\chi_{c1} \to PR$	Phys. Rev. D 100 (2019) 11, 114011
$\Psi(nS)/\Upsilon(nS) \to VR$	Eur. Phys. J. A 49 (2013) 130	$\gamma p \to RB$	Phys. Rev. C 93 (2016) 2, 025202
	Phys. Rev. D 91 (2015) 9, 094013	$D_s^+ \to \pi^+ K_s^0 K_s^0$	Eur. Phys. J. C 82 (2022) 3, 225
$\Psi(nS)/\Upsilon(nS) \to \gamma R$	Phys. Rev. D 91 (2015) 9, 094013		Phys. Rev. D 105 (2022) 11, 116010
$\eta_c \to \overline{K}^0 K^+ \pi^-$	Phys. Rev. D 108 (2023) 11, 114004	$D_s^+ \to K_s^0 K^+ \pi^-$	Phys. Rev. D 107 (2023) 11, 116018

Part 2

Formalism

Formalism

$$R \to PP$$



RVV vertex:

$$\mathcal{L}_{RVV} = g_{RVV}\mathcal{P}^{(s)}$$

VPP vertex:

$$\mathcal{L}_{VPP} = -ig\langle V_u[P, \partial^u P]\rangle$$

form factor:

$$F(t^2) = \frac{\Lambda^2 - m_t^2}{\Lambda^2 - t^2}$$

projection operator:

$$\mathcal{P}^{(1)} = \frac{1}{\sqrt{3}}\epsilon_i\epsilon_i \qquad \mathcal{P}^{(2)} = \frac{1}{2}[\epsilon_i\epsilon_j - \epsilon_j\epsilon_i] - \frac{1}{3}\epsilon_m\epsilon_m\delta_{ij}$$

Phys.Rev.D 78 (2008) 114018

Phys.Rev.D 79 (2009) 074009

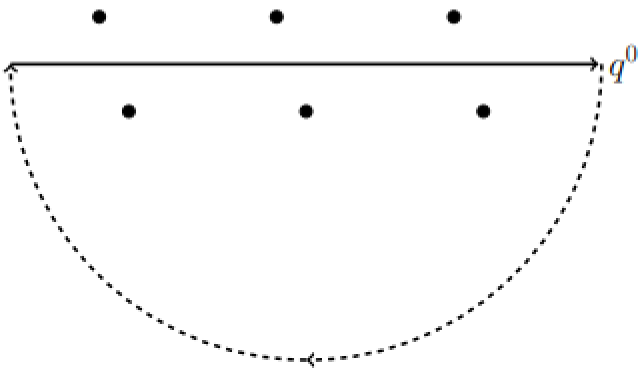
Eur.Phys.J.C 83 (2023) 12, 1106

Phys.Rev.D 80 (2009) 014025

Phys.Rev.D 80 (2009) 114013

Formalism

PhD thesis of Feng-Kun Guo



$$t_{V_1P_1P_t} = -igC_2\epsilon(V_1)\cdot(2k_1 - q)$$

$$t_{V_2P_2P_t} = -igC_3\epsilon(V_2)\cdot(k_2 - k_1 + q)$$

$$t_a = \mathcal{P}_J^*(V_1V_2)\epsilon(V_1)\cdot(2k_1 - q)\epsilon(V_2)\cdot(k_2 - k_1 + q)$$

$$t = Cg_{RV_1V_2}g^2\int\frac{d^4q}{(2\pi)^4}t_aF^2\frac{1}{q^2 - M_{V_1}^2 + i\varepsilon}\frac{1}{(P-q)^2 - M_{V_2}^2 + i\varepsilon}\frac{1}{(q-k_1)^2 - m_{P_t}^2 + i\varepsilon}$$

$$\Gamma_{R\to P_1P_2} = \frac{1}{2J+1}\frac{|\vec{k}_1|}{8\pi M^2}\sum_{pol}|t|^2$$

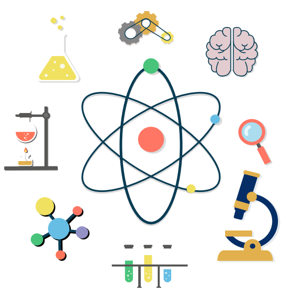
$$\Lambda = 1350(\pm 100)\ \text{MeV}$$

Width of vector meson:

$$\omega_{V_1} \to \omega_{V_1} + \frac{\Gamma_{V_1}}{4} \qquad \omega_{V_2} \to \omega_{V_2} - \frac{\Gamma_{V_2}}{4}$$

Phys.Rev.D 78 (2008) 114018

Phys.Rev.D 79 (2009) 074009



Part 3

Results

- Discussions
- Partial decay width
- Branching ratios

Results
Discussions

$f_2(1270)$

This work	
$f_2(1270) \to \pi\pi$	$141.1^{+68.8}_{-48.0}$
$f_2(1270) \to K\bar{K}$	$15.63^{+10.29}_{-6.68}$
$f_2(1270) \to \eta\eta$	$0.87^{+0.61}_{-0.39}$
$R^{K\bar{K}/\pi\pi}_{f_2(1270)}$	0.11 ± 0.02
$R^{\eta\eta/\pi\pi}_{f_2(1270)}$	0.006 ± 0.001

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	$f_2(1270) \to \pi\pi$	$f_2(1270) \to K\bar{K}$	$f_2(1270) \to \eta\eta$	$R^{K\bar{K}/\pi\pi}_{f_2(1270)}$	$R^{\eta\eta/\pi\pi}_{f_2(1270)}$
Value (average/fit)	$157.2^{+4.0}_{-1.1}$	8.6 ± 0.8	0.75 ± 0.14	$0.041^{+0.004}_{-0.005}$	
Exp.	$157.0^{+6.0}_{-1.0}$	$9.0^{+0.7}_{-0.3}$	1.0 ± 0.1	$0.02 - 0.06$	0.003 ± 0.001
	152 ± 8	7.5 ± 2.0	1.8 ± 0.4		

Results

Discussions

$f_2'(1525)$

This work	
$f_2'(1525) \to \pi\pi$	$5.5^{+1.0}_{-1.0}$
$f_2'(1525) \to K\bar{K}$	$17.6^{+10.1}_{-6.9}$
$f_2'(1525) \to \eta\eta$	$6.6^{+4.3}_{-2.9}$
$R^{\pi\pi/K\bar{K}}_{f_2'(1525)}$	$0.31^{+0.11}_{-0.08}$
$R^{\eta\eta/K\bar{K}}_{f_2'(1525)}$	$0.37^{+0.03}_{-0.02}$

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	$f_2'(1525) \to \pi\pi$	$f_2'(1525) \to K\bar{K}$	$f_2'(1525) \to \eta\eta$	$R^{\pi\pi/K\bar{K}}_{f_2'(1525)}$	$R^{\eta\eta/K\bar{K}}_{f_2'(1525)}$
Value (average/fit)	0.71 ± 0.14	75 ± 4	9.9 ± 1.9	0.0092 ± 0.0018	0.115 ± 0.028
Exp.	$1.4^{+1.0}_{-0.7}$	63^{+6}_{-5}	24^{+3}_{-1}	0.075 ± 0.035	$0.119 \pm 0.015 \pm 0.036$
	$0.2^{+1.0}_{-0.2}$		5.0 ± 8		0.11 ± 0.04

$K_2^*(1430)$

This work	
$K_2^*(1430)\to K\pi$	$34.6^{+16.5}_{-11.7}$
$K_2^*(1430)\to K\eta$	$2.62^{+1.65}_{-1.09}$
$R^{K\eta/K\pi}_{K_2^*(1430)}$	0.075 ± 0.009

The Review of Particle Physics			
	$K_2^*(1430)\to K\pi$	$K_2^*(1430)\to K\eta$	$R^{K\eta/K\pi}_{K_2^*(1430)}$
Value (average/fit)	53.2 ± 2.9	$0.16^{+0.36}_{-0.11}$	$0.0030^{+0.0070}_{-0.0020}$
Exp.			0 ± 0.0056

Results

Discussions

$f_0(1710)$

This work	
$f_0(1710)\to\pi\pi$	$5.76^{+2.04}_{-1.31}$
$f_0(1710)\to K\bar{K}$	$57.9^{+28.8}_{-20.5}$
$f_0(1710)\to\eta\eta$	$32.7^{+18.8}_{-12.9}$
$R^{\pi\pi/K\bar{K}}_{f_0(1710)}$	0.10 ± 0.02
$R^{\eta\eta/K\bar{K}}_{f_0(1710)}$	0.56 ± 0.04

The Review of Particle Physics					
	$f_0(1710)\to\pi\pi$	$f_0(1710)\to K\bar{K}$	$f_0(1710)\to\eta\eta$	$R^{\pi\pi/K\bar{K}}_{f_0(1710)}$	$R^{\eta\eta/K\bar{K}}_{f_0(1710)}$
Value (average /fit)	$5.85^{+0.30}_{-4}$	54^{+18}_{-19}	33 ± 18	0.23 ± 0.05	
Exp.				0.64 ± 0.27 ±0.18	0.48 ± 0.15
				$0.41^{+0.11}_{-0.17}$	$0.46^{+0.70}_{-0.38}$
				0.2 ± 0.024 ±0.036	
				0.39 ± 0.14	
				0.32 ± 0.14	
				$5.8^{+9.1}_{-5.5}$	

Results

Discussions

$a_0(1710)$

This work	
$a_0(1710) \to K\bar{K}$	$40.2^{+26.8}_{-17.6}$
$a_0(1710) \to \pi\eta$	$38.4^{+19.9}_{-14.2}$
$R^{K\eta/K\pi}_{a_0(1710)}$	$0.96^{+0.14}_{-0.09}$

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	$a_0(1710) \to K\bar{K}$	$a_0(1710) \to \pi\eta$	$R^{K\eta/K\pi}_{a_0(1710)}$
Value (average/fit)			
Exp.			

$f_0(1500)$ ？

This work	
$f_0(1500) \to \pi\pi$	632^{+269}_{-198}
$f_0(1500) \to K\bar{K}$	$51.0^{+30.6}_{-20.5}$
$f_0(1500) \to \eta\eta$	$0.54^{+0.37}_{-0.24}$
$R^{K\bar{K}/\pi\pi}_{f_0(1500)}$	0.08 ± 0.01
$R^{\eta\eta/\pi\pi}_{f_0(1500)}$	$(8.6 \pm 1.5) \times 10^{-4}$

The Review of Particle Physics

Mass: 1522 ± 25 MeV Width: 108 ± 33 MeV

	$f_0(1500) \to \pi\pi$	$f_0(1500) \to K\bar{K}$	$f_0(1500) \to \eta\eta$	$R^{K\bar{K}/\pi\pi}_{f_0(1500)}$	$R^{\eta\eta/\pi\pi}_{f_0(1500)}$
Value (average/fit)	37 ± 12	9.2 ± 3.0	$0.54^{+0.37}_{-0.24}$	0.236 ± 0.026	0.175 ± 0.027
Exp.				0.25 ± 0.03	0.18 ± 0.03
				$0.33 \pm 0.03 \pm 0.07$	0.11 ± 0.03
				0.16 ± 0.05	0.157 ± 0.060
				0.19 ± 0.07	0.080 ± 0.033
				0.20 ± 0.08	0.078 ± 0.013
					0.230 ± 0.097

Results
Discussions

$f_0(1370)$?

This work	
$f_0(1370) \to \pi\pi$	632^{+269}_{-198}
$f_0(1370) \to K\bar{K}$	$51.0^{+30.6}_{-20.5}$
$f_0(1370) \to \eta\eta$	$0.54^{+0.37}_{-0.24}$
$R^{K\bar{K}/\pi\pi}_{f_0(1370)}$	0.08 ± 0.01
$R^{\eta\eta/\pi\pi}_{f_0(1370)}$	$(8.6 \pm 1.5) \times 10^{-4}$

The Review of Particle Physics					
Mass: $1200-1500$ MeV Width: $200-500$ MeV					
	$R^{\pi\pi/Total}_{f_0(1370)}$	$R^{K\bar{K}/Total}_{f_0(1370)}$	$R^{\eta\eta/Total}_{f_0(1370)}$	$R^{K\bar{K}/\pi\pi}_{f_0(1370)}$	$R^{\eta\eta/\pi\pi}_{f_0(1370)}$
Value (average/fit)					
Exp.	0.26 ± 0.09	0.35 ± 0.13		0.91 ± 0.20	
				$0.46 \pm 0.15 \pm 0.11$	
				0.12 ± 0.06	
				0.08 ± 0.08	

Results
Discussions

$\Gamma(f_0(1370) \to \pi\pi)/\Gamma_{\text{total}}$ — Γ_1/Γ

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •					
< 0.10	95	OCHS	2013	RVUE	
0.26 ± 0.09		BUGG	1996	RVUE	
< 0.15		[1] AMSLER	1994	CBAR	$\bar{p}\,p \to \pi^+\pi^- 3\,\pi^0$
< 0.06		GASPERO	1993	DBC	0.0 $\bar{p}\,n \to$ hadrons

Results

Partial decay width

Tensor

Decay mode	This work	RPP	Exp.	
$f_2(1270) \to \pi\pi$	$141.1^{+68.8}_{-48.0}$	$157.2^{+4.0}_{-1.1}$	$157.0^{+6.0}_{-1.0}$	Phys. Lett. B 177,223 (1986)
			152 ± 8	Eur. Phys. J. A 27,207 (2006)
$f_2(1270) \to K\bar{K}$	$15.63^{+10.29}_{-6.68}$	8.6 ± 0.8	$9.0^{+0.7}_{-0.3}$	Phys. Lett. B 177,223(1986)
			7.5 ± 2.0	Eur. Phys. J. A 27,207 (2006)
$f_2(1270) \to \eta\eta$	$0.87^{+0.61}_{-0.39}$	0.75 ± 0.14	1.0 ± 0.1	Phys. Lett. B 177,223(1986)
			1.8 ± 0.4	Eur. Phys. J. A 27,207 (2006)
$f_2'(1525) \to \pi\pi$	$5.5^{+1.0}_{-1.0}$	0.71 ± 0.14	$1.4^{+1.0}_{-0.7}$	Phys. Lett. B 177,223(1986)
			$0.2^{+1.0}_{-0.2}$	Eur. Phys. J. A 27,207 (2006)
$f_2'(1525) \to K\bar{K}$	$17.6^{+10.1}_{-6.9}$	75 ± 4	63^{+6}_{-5}	Phys. Lett. B 177,223(1986)
$f_2'(1525) \to \eta\eta$	$6.6^{+4.3}_{-2.9}$	9.9 ± 1.9	24^{+3}_{-1}	Phys. Lett. B 177,223(1986)
			5.0 ± 8	Eur. Phys. J. A 27,207 (2006)
$K_2^*(1430) \to K\pi$	$34.6^{+16.5}_{-11.7}$	53.2 ± 2.9		
$K_2^*(1430) \to K\eta$	$2.62^{+1.65}_{-1.09}$	$0.16^{+0.36}_{-0.11}$		

Results

Partial decay width

Scalar

Decay mode	This work	RPP	Exp.
$f_0(1710) \to \pi\pi$	$5.76^{+2.04}_{-1.31}$	$5.85^{+0.30}_{-4}$	
$f_0(1710) \to K\bar{K}$	$57.9^{+28.8}_{-20.5}$	54^{+18}_{-19}	
$f_0(1710) \to \eta\eta$	$32.7^{+18.8}_{-12.9}$	33 ± 18	
$a_0(1710) \to K\bar{K}$	$40.2^{+26.8}_{-17.6}$		
$a_0(1710) \to \pi\eta$	$38.4^{+19.9}_{-14.2}$		
$f_0(1500) \to \pi\pi$	632^{+269}_{-198}	37 ± 12	
$f_0(1500) \to K\bar{K}$	$51.0^{+30.6}_{-20.5}$	9.2 ± 3.0	
$f_0(1500) \to \eta\eta$	$0.54^{+0.37}_{-0.24}$	6.5 ± 2.2	

Results

Branching ratios

Tensor

Ratio		This work	RPP	EXP.	
$f_2(1270)$	$R^{K\bar{K}/\pi\pi}_{f_2(1270)}$	0.11 ± 0.02	$0.041^{+0.004}_{-0.005}$	$0.02-0.06$	
	$R^{\eta\eta/\pi\pi}_{f_2(1270)}$	0.006 ± 0.001		0.003 ± 0.001	Phys. Lett. B 479, 59(2000)
$f_2{}'(1525)$	$R^{\pi\pi/K\bar{K}}_{f_2{}'(1525)}$	$0.31^{+0.11}_{-0.08}$	0.0092 ± 0.0018	0.075 ± 0.035	Z. Phys. C 36,369(1987)
	$R^{\eta\eta/K\bar{K}}_{f_2{}'(1525)}$	$0.37^{+0.03}_{-0.02}$	0.115 ± 0.028	$0.119\pm0.015\pm0.036$	Phys.Atom.Nucl.70,1713(2007)
				0.11 ± 0.04	Sov.Phys.Dokl.36 (1991) 155-158
$K_2^*(1430)$	$R^{K\eta/K\pi}_{K_2^*(1430)}$	0.075 ± 0.009	$0.0030^{+0.0070}_{-0.0020}$	0 ± 0.00561988	Phys. Lett. B 201,169(1988)

Results

Branching ratios

Scalar

Ratio		This work	RPP	Exp.	
$f_0(1710)$	$R^{\pi\pi/K\bar{K}}_{f_0(1710)}$	0.10 ± 0.02	0.23 ± 0.05	$0.64 \pm 0.27 \pm 0.18$	Phys. Rev. D 97, 112006(2018)
				$0.41^{+0.11}_{-0.17}$	Phys. Lett. B 642,441(2006)
				$0.2 \pm 0.024 \pm 0.036$	Phys. Lett. B 642, 462(1999)
				0.39 ± 0.14	Z. Phys. C 51, 351(1991)
				0.32 ± 0.14	Phys. Rev. Lett.101,252002
				$5.8^{+9.1}_{-5.5}$	Phys. Atom. Nucl.65, 1545(2002)
	$R^{\eta\eta/K\bar{K}}_{f_0(1710)}$	0.56 ± 0.04		0.48 ± 0.15	Phys. Lett. B 479, 59(2000)
				$0.46^{+0.70}_{-0.38}$	Phys. Atom. Nucl.65, 1545(2002)
$a_0(1710)$	$R^{K\eta/K\pi}_{a_0(1710)}$	$0.96^{+0.14}_{-0.09}$			

Results

Branching ratios

Scalar

Ratio		This work	RPP	Exp.	
$f_0(1500)$	$R^{K\bar{K}/\pi\pi}_{f_0(1500)}$	0.08 ± 0.01	0.236 ± 0.026	0.25 ± 0.03	Eur. Phys. J. C 26,371(2003)
				$0.33 \pm 0.03 \pm 0.07$	Phys. Lett. B 462,462 (1999)
				0.16 ± 0.05	Phys. Atom. Nucl. 65,1545(2002)
				0.19 ± 0.07	Phys. Rev. D 57,3860(1998)
				0.20 ± 0.08	Phys. Lett . B 385,425(1996)
	$R^{\eta\eta/\pi\pi}_{f_0(1500)}$	$(8.6 \pm 1.5) \times 10^{-4}$	0.175 ± 0.027	0.18 ± 0.03	Phys. Lett. B 479, 59(2000)
				0.11 ± 0.03	Phys. Atom. Nucl. 65,1545(2002)
				0.157 ± 0.060	Phys. Lett. B 355,425(1995)
				0.080 ± 0.033	Eur. Phys. J. C 23, 29(2002)
				0.078 ± 0.013	Nucl. Phys. A 609, 562 (1996)
				0.230 ± 0.097	Phys. Lett. B 353,571(1995)

Part 4

Summary

Summary

- The partial widths and branching ratios for different two-pseudoscalar-meson decay channels of states dynamically generated by two vector meson are calculated by triangular diagrams, and interference between different channels and neglected real part of previous box diagrams are considered.

- Our results are similar with previous ones, which are obtain by box diagram and real axis method.

- Data from experiment is limited and old, and some of them are not consistent with each other. We expect new measurements from BES III, Belle II and LHCb collaborations can check old and limited data in RPP for obtaining more reliable data to test our results.

THANKS

Backup

Backup

Coupling

	pole position	$K^*\bar{K}^*$	$\rho\rho$	$\omega\omega$	$\omega\phi$	$\phi\phi$
strangeness=0 and isospin=0						
$f_2(1270)$	$(1275, -i)$	$(4733, -53i)$	$(10889, -99i)$	$(-440, 7i)$	$(777, -13i)$	$(-675, 11i)$
$f_2'(1525)$	$(1525, -3i)$	$(10121, 101i)$	$(-2443, 649i)$	$(-2709, 8i)$	$(5016, -17i)$	$(-4615, 17i)$
$f_0(1710)$	$(1726, -14i)$	$(7124, 96i)$	$(-1030, 1086i)$	$(-1763, 108i)$	$(3010, -210i)$	$(-2493, -204i)$
$f_0(1500)$	$(1512, -26i)$	$(1208, -419i)$	$(7906, -1084i)$	$(-40, i30)$	$(34, -42i)$	$(12, 24i)$

strangeness=0 and isospin=1

	pole position	$K^*\bar{K}^*$	$\rho\omega$	$\rho\phi$
$a_0(1710)$	$(1780, -66i)$	$(7526, -1525i)$	$(-4042, 1389i)$	$(4998, -1869i)$

strangeness=1 and isospin=1/2

	pole position	ρK^*	$K^*\omega$	$K^*\phi$
$K_2^*(1430)$	$(1431, -i)$	$(10901, -71i)$	$(2267, -13i)$	$(-2898, 17i)$

Backup
Data from RPP

$f_2(1270)$

$\Gamma(f_2(1270) \to \pi\pi)$ Γ_1

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{157.2^{+5.0}_{-1.0}}$	OUR FIT				
$157.0^{+6.0}_{-1.0}$		[1] LONGACRE	1986	MPS	$22\ \pi^- p \to n2K^0_S$
• • We do not use the following data for averages, fits, limits, etc. • •					
152 ± 8	870	[2] SCHEGELSKY	2006A	RVUE	$\gamma\gamma \to K^0_S K^0_S$

$\Gamma(f_2(1270) \to K\overline{K})$ Γ_3

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{8.5 \pm 0.8}$	OUR FIT Error includes scale factor of 2.8.				
$9.0^{+0.7}_{-0.3}$		[1] LONGACRE	1986	MPS	$22\ \pi^- p \to n2K^0_S$
• • We do not use the following data for averages, fits, limits, etc. • •					
7.5 ± 2.0	870	[2] SCHEGELSKY	2006A	RVUE	$\gamma\gamma \to K^0_S K^0_S$

$\Gamma(f_2(1270) \to \eta\eta)$ Γ_5

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{0.75 \pm 0.14}$	OUR FIT Error includes scale factor of 2.1.				
1.0 ± 0.1		[1] LONGACRE	1986	MPS	$22\ \pi^- p \to n2K^0_S$
• • We do not use the following data for averages, fits, limits, etc. • •					
1.8 ± 0.4	870	[2] SCHEGELSKY	2006A	RVUE	$\gamma\gamma \to K^0_S K^0_S$

Backup
Data from RPP

$\Gamma(f_2(1270) \to \pi\pi)/\Gamma_{\text{total}}$ Γ_1/Γ

VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{0.843^{+0.028}_{-0.010}}$	OUR FIT Error includes scale factor of 1.2.				
$\mathbf{0.837 \pm 0.020}$	OUR AVERAGE				
0.849 ± 0.025		CHABAUD	1983	ASPK	17 $\pi^- p$ polarized
0.85 ± 0.05	250	BEAUPRE	1971	HBC	8 $\pi^+ p \to \Delta^{++} f_2$
0.8 ± 0.04	600	OH	1970	HBC	1.26 $\pi^- p \to \pi^+\pi^- n$
• • We do not use the following data for averages, fits, limits, etc. • •					
$0.856 \pm 0.001 \pm 0.05$		[1] ALBRECHT	2020	RVUE	0.9 $\bar{p} p \to \pi^0\pi^0\eta$, $\pi^0\eta\eta$, $\pi^0 K^+ K^-$

Backup

Data from RPP

$\Gamma(f_2(1270) \to K\overline{K})/\Gamma_{total}$ — Γ_3/Γ

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
• • We do not use the following data for averages, fits, limits, etc. • •				
0.033 ±0.001 ±0.005	[1] ALBRECHT	2020	RVUE	0.9 $\overline{p}\,p \to \pi^0\pi^0\eta$, $\pi^0\eta\eta$, $\pi^0 K^+ K^-$

$\Gamma(f_2(1270) \to \eta\eta)/\Gamma_{total}$ — Γ_5/Γ

VALUE (10^{-3})	*DOCUMENT ID*		*TECN*	*COMMENT*
4.0 ± 0.8	**OUR FIT** Error includes scale factor of 2.1.			
2.9 ± 0.5	**OUR AVERAGE**			
2.7 ±0.7	BINON	2005	GAMS	33 $\pi^- p \to \eta\eta n$
2.8 ±0.7	ALDE	1986D	GAM4	100 $\pi^- p \to 2\,\eta n$
5.2 ±1.7	BINON	1983	GAM2	38 $\pi^- p \to 2\,\eta n$
• • We do not use the following data for averages, fits, limits, etc. • •				
4.0 ±1.0 ±2.0	[1] ALBRECHT	2020	RVUE	0.9 $\overline{p}\,p \to \pi^0\pi^0\eta$, $\pi^0\eta\eta$, $\pi^0 K^+ K^-$

Backup
Data from RPP

$\Gamma(f_2(1270) \to K\overline{K})/\Gamma(f_2(1270) \to \pi\pi)$ Γ_3/Γ_1

We average only experiments which either take into account $f_2(1270) - a_2(1320)$ interference explicitly or demonstrate that $a_2(1320)$ production is negligible.

VALUE	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
$\mathbf{0.054^{+0.005}_{-0.006}}$	OUR FIT Error includes scale factor of 2.7.				
$\mathbf{0.041^{+0.004}_{-0.005}}$	OUR AVERAGE				
0.045 ± 0.01		[1] BARGIOTTI	2003	OBLX	$\overline{p}p$
$0.037^{+0.008}_{-0.021}$		ETKIN	1982B	MPS	23 $\pi^- p \to n2 K_S^0$
0.045 ± 0.009		CHABAUD	1981	ASPK	17 $\pi^- p$ polarized
0.039 ± 0.008		LOVERRE	1980	HBC	4 $\pi^- p \to K\overline{K}N$
• • We do not use the following data for averages, fits, limits, etc. • •					
0.052 ± 0.025		ABLIKIM	2004E	BES2	$J/\psi \to \omega K^+ K^-$
0.036 ± 0.005		[2] COSTA	1980	OMEG	1–2.2 $\pi^- p \to K^+ K^- n$
0.030 ± 0.005		[3] MARTIN	1979	RVUE	
0.027 ± 0.009		[4] POLYCHRONAKOS	1979	STRC	7 $\pi^- p \to n2 K_S^0$
0.025 ± 0.015		EMMS	1975D	DBC	4 $\pi^+ n \to pf_2$
0.031 ± 0.012	20	ADERHOLZ	1969	HBC	8 $\pi^+ p \to K^+ K^- \pi^+ p$

[1] Coupled channel analysis of $\pi^+\pi^-\pi^0$, $K^+K^-\pi^0$, and $K^\pm K^0 \pi^\mp$

Backup
Data from RPP

$\Gamma(f_2(1270) \to \eta\eta)/\Gamma(f_2(1270) \to \pi\pi)$ Γ_5/Γ_1

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
0.003 ± 0.001		BARBERIS	2000E		450 $p\,p \to p_f \eta\eta p_s$
• • We do not use the following data for averages, fits, limits, etc. • •					
< 0.05	95	EDWARDS	1982F	CBAL	$e^+ e^- \to e^+ e^- 2\,\eta$
< 0.016	95	EMMS	1975D	DBC	4 $\pi^+ n \to p f_2$
< 0.09	95	EISENBERG	1974	HBC	4.9 $\pi^+ p \to \Delta^{++} f_2$

Backup
Data from RPP

$f_2'(1525)$

$\Gamma(f_2'(1525) \to K\overline{K})$ Γ_1

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
$\mathbf{64^{+6}_{-5}}$ **OUR FIT**				
$63\ ^{+6}_{-5}$	[1] LONGACRE	1986	MPS	$22\ \pi^- p \to K^0_S K^0_S n$

$\Gamma(f_2'(1525) \to \pi\pi)$ Γ_3

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{0.59 \pm 0.12}$ **OUR FIT**					
$1.4\ ^{+1.0}_{-0.5}$		[1] LONGACRE	1986	MPS	$22\ \pi^- p \to K^0_S K^0_S n$
• • We do not use the following data for averages, fits, limits, etc. • •					
$0.2\ ^{+1.0}_{-0.2}$	870	[2] SCHEGELSKY	2006A	RVUE	$\gamma\gamma \to K^0_S K^0_S$

$\Gamma(f_2'(1525) \to \eta\eta)$ Γ_2

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{7.4 \pm 1.9}$ **OUR FIT**					
• • We do not use the following data for averages, fits, limits, etc. • •					
5.0 ± 0.8	870	[1] SCHEGELSKY	2006A	RVUE	$\gamma\gamma \to K^0_S K^0_S$
$24\ ^{+3}_{-1}$		[2] LONGACRE	1986	MPS	$22\ \pi^- p \to K^0_S K^0_S n$

Backup
Data from RPP

$\Gamma(f_2'(1525) \to K\overline{K})/\Gamma_{total}$ — Γ_1/Γ

VALUE	DOCUMENT ID		TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •				
$0.746 \pm 0.002 ^{+0.166}_{-0.162}$	[1] ALBRECHT	2020	RVUE	$0.9\ \overline{p}\,p \to \pi^0\pi^0\eta\,,\ \pi^0\eta\eta\,,\ \pi^0 K^+ K^-$

$\Gamma(f_2'(1525) \to \eta\eta)/\Gamma_{total}$ — Γ_2/Γ

VALUE	DOCUMENT ID		TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •				
$0.059 \pm 0.003 \pm 0.026$	[1] ALBRECHT	2020	RVUE	$0.9\ \overline{p}\,p \to \pi^0\pi^0\eta\,,\ \pi^0\eta\eta\,,\ \pi^0 K^+ K^-$
seen	UEHARA	2010A	BELL	$10.6\ e^+ e^- \to e^+ e^- \eta\eta$
0.10 ± 0.03	[2] PROKOSHKIN	1991	GAM4	$300\ \pi^- p \to \pi^- p \eta\eta$

Backup
Data from RPP

$\Gamma(f_2'(1525) \to \pi\pi)/\Gamma_{total}$ Γ_3/Γ

VALUE (10^{-2})	CL%	DOCUMENT ID		TECN	COMMENT
$\mathbf{0.82 \pm 0.16}$	OUR FIT				
$\mathbf{0.75 \pm 0.16}$	OUR AVERAGE				
0.7 ± 0.2		COSTA	1980	OMEG	$10\ \pi^- p \to K^+ K^- n$
$2.7^{+7.1}_{-1.3}$		[1] GORLICH	1980	ASPK	$17{,}18\ \pi^- p$
0.75 ± 0.25		[1,2] MARTIN	1979	RVUE	
• • We do not use the following data for averages, fits, limits, etc. • •					
$3.4 \pm 1.5 \pm 1.0$		[3] ALBRECHT	2020	RVUE	$0.9\ \bar{p} p \to \pi^0\pi^0\eta$, $\pi^0\eta\eta$, $\pi^0 K^+ K^-$
< 6	95	AGUILAR-BENIT..	1981B	HBC	$4.2\ K^- p \to \Lambda K^+ K^-$
19 ± 3		CORDEN	1979	OMEG	$12\text{–}15\ \pi^- p \to \pi^+\pi^- n$
< 4.5	95	BARREIRO	1977	HBC	$4.15\ K^- p \to \Lambda K^0_S K^0_S$
1.2 ± 0.4		[1] PAWLICKI	1977	SPEC	$6\ \pi N \to K^+ K^- N$
< 6.3	90	BRANDENBURG	1976C	ASPK	$13\ K^- p \to K^+ K^- (\Lambda, \Sigma)$
< 0.86		[1] BEUSCH	1975B	OSPK	$8.9\ \pi^- p \to K^0 \overline{K}^0 n$

Backup
Data from RPP

$\Gamma(f_2'(1525) \to \pi\pi)/\Gamma(f_2'(1525) \to K\overline{K})$ Γ_3/Γ_1

VALUE		DOCUMENT ID		TECN	COMMENT
0.0092 ± 0.0018	OUR FIT				
0.075 ± 0.035		AUGUSTIN	1987	DM2	$J/\psi \to \gamma\pi^+\pi^-$

$\Gamma(f_2'(1525) \to \eta\eta)/\Gamma(f_2'(1525) \to K\overline{K})$ Γ_2/Γ_1

VALUE		CL%	EVTS	DOCUMENT ID		TECN	COMMENT
0.116 ± 0.028	OUR FIT						
0.115 ± 0.028	OUR AVERAGE						
$0.119 \pm 0.015 \pm 0.036$			61	[1] BINON	2007	GAMS	32.5 $K^- p \to \eta\eta(\Lambda / \Sigma^0)$
0.11 ± 0.04				[2] PROKOSHKIN	1991	GAM4	300 $\pi^- p \to \pi^- p\eta\eta$
• • We do not use the following data for averages, fits, limits, etc. • •							
< 0.14		90		BARBERIS	2000E		450 $p p \to p_f\eta\eta p_s$
< 0.50				BARNES	1967	HBC	4.6,5.0 $K^- p$

Backup
Data from RPP

$K_2^*(1430)$

$\Gamma(K_2^*(1430) \to K\pi)/\Gamma_{total}$ Γ_1/Γ

VALUE	DOCUMENT ID		TECN	CHG	COMMENT
$\mathbf{0.499 \pm 0.012}$	OUR FIT				
$\mathbf{0.488 \pm 0.014}$	OUR AVERAGE				
$0.485 \pm 0.006 \pm 0.020$	[1] ASTON	1988	LASS	0	11 $K^- p \to K^- \pi^+ n$
0.49 ± 0.02	[1] ESTABROOKS	1978	ASPK	$\pm$	13 $K^\pm p \to pK\pi$

$\Gamma(K_2^*(1430) \to K\eta)/\Gamma(K_2^*(1430) \to K^*(892)\pi)$ Γ_7/Γ_2

VALUE	DOCUMENT ID		TECN	CHG	COMMENT
$\mathbf{0.006^{+0.014}_{-0.004}}$	OUR FIT Error includes scale factor of 1.2.				
0.07 ± 0.04	FIELD	1967	HBC	-	3.8 $K^- p$

Data from RPP

$\Gamma(K_2^*(1430) \to K\eta)/\Gamma(K_2^*(1430) \to K\pi)$ Γ_7/Γ_1

	CL%	DOCUMENT ID		TECN	CHG	COMMENT
$\mathbf{0.0030^{+0.0070}_{-0.0020}}$		**OUR FIT** Error includes scale factor of 1.3.				
0 ± 0.0056		[1] ASTON	1988B	LASS	-	11 $K^- p \to K^- \eta p$
• • We do not use the following data for averages, fits, limits, etc. • •						
< 0.04	95	AGUILAR-BENIT..	1971B	HBC		3.9,4.6 $K^- p$
< 0.065		[2] BASSOMPIERRE	1969	HBC		5.0 $K^+ p$
< 0.02		BISHOP	1969	HBC		3.5 $K^+ p$

Backup
Data from RPP

$f_0(1710)$

$\Gamma(f_0(1710) \to K\overline{K})/\Gamma_{total}$ — Γ_1/Γ

VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •					
seen	1004	[1] DOBBS	2015		$J/\psi \to \gamma K^+ K^-$
seen	349	[1] DOBBS	2015		$\psi(2S) \to \gamma K^+ K^-$
0.36 ± 0.12		ALBALADEJO	2008	RVUE	
$0.38^{+0.09}_{-0.19}$		[2] LONGACRE	1986	MPS	$22\ \pi^- p \to n2K^0_S$

$\Gamma(f_0(1710) \to \eta\eta)/\Gamma_{total}$ — Γ_2/Γ

VALUE	DOCUMENT ID		TECN
• • We do not use the following data for averages, fits, limits, etc. • •			
0.22 ± 0.12	ALBALADEJO	2008	RVUE
$0.18^{+0.03}_{-0.13}$	[1] LONGACRE	1986	RVUE

$\Gamma(f_0(1710) \to \pi\pi)/\Gamma_{total}$ — Γ_4/Γ

VALUE	EVTS	DOCUMENT ID		TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •					
seen	381	[1] DOBBS	2015		$J/\psi \to \gamma\pi^+\pi^-$
seen	237	[1] DOBBS	2015		$\psi(2S) \to \gamma\pi^+\pi^-$
not seen		AMSLER	2002	CBAR	$0.9\ \overline{p} p \to \pi^0\eta\eta,\ \pi^0\pi^0\pi^0$
$0.039^{+0.002}_{-0.024}$		[2] LONGACRE	1986	RVUE	

Backup
Data from RPP

$\Gamma(f_0(1710) \to \pi\pi)/\Gamma(f_0(1710) \to K\overline{K})$ Γ_4/Γ_1

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
$\mathbf{0.23 \pm 0.05}$	OUR AVERAGE	Error includes scale factor of 1.2.			
$0.64 \pm 0.27 \pm 0.18$		LEES	2018A	BABR	$\Upsilon(1S) \to \gamma\pi^+\pi^-, \gamma K^+K^-$
$0.41^{+0.11}_{-0.17}$		ABLIKIM	2006V	BES2	$e^+e^- \to J/\psi \to \gamma\pi^+\pi^-$
$0.2 \pm 0.024 \pm 0.036$		BARBERIS	1999D	OMEG	450 $pp \to K^+K^-, \pi^+\pi^-$
0.39 ± 0.14		ARMSTRONG	1991	OMEG	300 $pp \to pp\pi\pi, ppK\overline{K}$
• • We do not use the following data for averages, fits, limits, etc. • •					
0.32 ± 0.14		ALBALADEJO	2008	RVUE	
< 0.11	95	[1] ABLIKIM	2004E	BES2	$J/\psi \to \omega K^+K^-$
$5.8^{+9.1}_{-5.5}$		[2] ANISOVICH	2002D	SPEC	Combined fit

[1] Using data from ABLIKIM 2004A.

Backup
Data from RPP

$\Gamma(f_0(1710) \to \eta\eta)/\Gamma(f_0(1710) \to K\overline{K})$ Γ_2/Γ_1

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
0.48 ± 0.15		BARBERIS	2000E		450 $p\,p \to p_f \eta\eta p_s$
• • We do not use the following data for averages, fits, limits, etc. • •					
$0.46\ ^{+0.70}_{-0.38}$		[1] ANISOVICH	2002D	SPEC	Combined fit
< 0.02	90	[2] PROKOSHKIN	1991	GA24	300 $\pi^-\ p \to \pi^- p\eta\eta$

Backup
Data from RPP

$f_0(1500)$

	Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_1	$f_0(1500) \to \pi\pi$	$(34.5 \pm 2.2) \times 10^{-2}$	S=1.2	749

$\Gamma(f_0(1500) \to \pi\pi)/\Gamma_{total}$ — Γ_1/Γ

VALUE	DOCUMENT ID		TECN
• • We do not use the following data for averages, fits, limits, etc. • •			
0.454 ± 0.104	BUGG	1996	RVUE

References

Backup
Data from RPP

	Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_{13}	$f_0(1500) \to K\overline{K}$	$(8.5 \pm 1.0) \times 10^{-2}$	S=1.1	579

$\Gamma(f_0(1500) \to K\overline{K})/\Gamma_{total}$ Γ_{13}/Γ

VALUE	DOCUMENT ID		TECN
• • We do not use the following data for averages, fits, limits, etc. • •			
0.044 ± 0.021	BUGG	1996	RVUE

Backup
Data from RPP

$f_0(1500)$

	Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_{11}	$f_0(1500) \to \eta\eta$	$(6.0 \pm 0.9) \times 10^{-2}$	S=1.1	528

$\Gamma(f_0(1500) \to \eta\eta)/\Gamma_{total}$ Γ_{11}/Γ

VALUE	DOCUMENT ID		TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •				
large	ALDE	1988	GAM4	300 $\pi^- N \to \eta\eta\pi^- N$
large	BINON	1983	GAM2	38 $\pi^- p \to 2\eta n$

Backup
Data from RPP

$\Gamma(f_0(1500) \to K\overline{K})/\Gamma(f_0(1500) \to \pi\pi)$ — Γ_{13}/Γ_1

VALUE		DOCUMENT ID		TECN	COMMENT
$\mathbf{0.246 \pm 0.025}$	OUR FIT				
$\mathbf{0.236 \pm 0.026}$	OUR AVERAGE				
0.25 ± 0.03		[1] BARGIOTTI	2003	OBLX	$\overline{p}p$
0.19 ± 0.07		[2] ABELE	1998	CBAR	0.0 $\overline{p}\,p \to K_L^0\,K^\pm\pi^\mp$
0.20 ± 0.08		[3] ABELE	1996B	CBAR	0.0 $\overline{p}\,p \to \pi^0 K_L^0\,K_L^0$
• • We do not use the following data for averages, fits, limits, etc. • •					
0.16 ± 0.05		[4] ANISOVICH	2002D	SPEC	Combined fit
$0.33 \pm 0.03 \pm 0.07$		BARBERIS	1999D	OMEG	450 $p\,p \to K^+K^-, \pi^+\pi^-$

Backup
Data from RPP

$\Gamma(f_0(1500) \to \eta\eta)/\Gamma(f_0(1500) \to \pi\pi)$ — Γ_{11}/Γ_1

VALUE		DOCUMENT ID		TECN	COMMENT
$\mathbf{0.173 \pm 0.024}$	OUR FIT Error includes scale factor of 1.1.				
$\mathbf{0.175 \pm 0.027}$	OUR AVERAGE				
0.18 ± 0.03		BARBERIS	2000E		450 $p\,p \to p_f \eta\eta p_s$
0.157 ± 0.060	[1]	AMSLER	1995D	CBAR	0.0 $\bar{p}\,p \to \pi^0\pi^0\pi^0,\ \pi^0\eta\eta,\ \pi^0\pi^0\eta$
• • We do not use the following data for averages, fits, limits, etc. • •					
0.080 ± 0.033		AMSLER	2002	CBAR	0.9 $\bar{p}\,p \to \pi^0\eta\eta,\ \pi^0\pi^0\pi^0$
0.11 ± 0.03	[2]	ANISOVICH	2002D	SPEC	Combined fit
0.078 ± 0.013	[3]	ABELE	1996C	RVUE	Compilation
0.230 ± 0.097	[4]	AMSLER	1995C	CBAR	0.0 $\bar{p}\,p \to \eta\eta\pi^0$

Backup
Data from RPP

$\Gamma(f_0(1500) \to K\overline{K})/\Gamma(f_0(1500) \to \eta\eta)$ Γ_{13}/Γ_{11}

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
1.43 ± 0.24	OUR FIT	Error includes scale factor of 1.1.			
1.85 ± 0.41		BARBERIS	2000E		450 $p\,p \to p_f \eta\eta p_s$
• • We do not use the following data for averages, fits, limits, etc. • •					
1.5 ± 0.6		[1] ANISOVICH	2002D	SPEC	Combined fit
< 0.4	90	[2] PROKOSHKIN	1991	GAM4	300 $\pi^-\,p \to \pi^- p\eta\eta$
< 0.6		[3] BINON	1983	GAM2	38 $\pi^-\,p \to 2\,\eta n$

Backup
Data from RPP

$f_0(1370)$

$\Gamma(f_0(1370) \to \pi\pi)/\Gamma_{total}$ — Γ_1/Γ

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •					
< 0.10	95	OCHS	2013	RVUE	
0.26 ± 0.09		BUGG	1996	RVUE	
< 0.15		[1] AMSLER	1994	CBAR	$\bar{p}p \to \pi^+\pi^- 3\pi^0$
< 0.06		GASPERO	1993	DBC	$0.0\ \bar{p}n \to$ hadrons

$\Gamma(f_0(1370) \to K\bar{K})/\Gamma_{total}$ — Γ_{11}/Γ

VALUE	DOCUMENT ID		TECN
• • We do not use the following data for averages, fits, limits, etc. • •			
0.35 ± 0.13	BUGG	1996	RVUE

Backup
Data from RPP

$\Gamma(f_0(1370) \to K\overline{K})/\Gamma(f_0(1370) \to \pi\pi)$ Γ_{11}/Γ_1

VALUE	DOCUMENT ID		TECN	COMMENT
• • We do not use the following data for averages, fits, limits, etc. • •				
0.08 ±0.08	ABLIKIM	2005	BES2	$J/\psi \to \phi\pi^+\pi^-$, ϕK^+K^-
0.91 ±0.20	[1] BARGIOTTI	2003	OBLX	$\overline{p}p$
0.12 ±0.06	[2] ANISOVICH	2002D	SPEC	Combined fit
0.46 ±0.15 ±0.11	BARBERIS	1999D	OMEG	450 $p\,p \to K^+K^-$, $\pi^+\pi^-$

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Our treatment of the box amplitudes enables us to obtain the decay branching ratios of the generated states into two pseudoscalar mesons using the real-axis method in the following way:

- First, we calculate the width of the selected state with and without the contributions of the box diagrams, $\Gamma(\text{total}) = \Gamma(\text{VV}) + \Gamma(\text{PP})$ and $\Gamma(\text{VV})$.
- Second, we estimate the partial decay width into a particular two-pseudoscalar channel of the state by including only the contribution of the particular channel. Taking the $\pi\pi$ channel as an example, this way we obtain $\Gamma(\text{w}\pi\pi)$. The contribution of the $\pi\pi$ channel is then determined as $\Gamma(\pi\pi) = \Gamma(\text{w}\pi\pi) - \Gamma(\text{VV})$ and its branching ratio is calculated as $\Gamma(\pi\pi)/\Gamma(\text{total})$. The partial decay branching ratios into other two-pseudoscalar channels are calculated similarly. It should be noted that we have assumed that interference between contributions of different channels is small, which seems to be justified since the sum of the calculated partial decay widths agrees well with the total decay width.

Backup

Previous results of box diagram

$f_2(1270)$

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$\Gamma(\pi\pi)/\Gamma(total)$	$\sim 88\%$
$\Gamma(K\bar{K})/\Gamma(total)$	$\sim 10\%$
$\Gamma(\eta\eta)/\Gamma(total)$	$< 10\%$
$\Gamma(VV)/\Gamma(total)$	$< 1\%$

$f_2'(1525)$

PoS EFT09 (2009) 040	
$\Gamma(\pi\pi)/\Gamma(total)$	$< 1\%$
$\Gamma(K\bar{K})/\Gamma(total)$	$\sim 66\%$
$\Gamma(\eta\eta)/\Gamma(total)$	$\sim 21\%$
$\Gamma(VV)/\Gamma(total)$	$\sim 13\%$

$K_2^*(1430)$

PoS EFT09 (2009) 040	
$\Gamma(K\pi)/\Gamma(total)$	$\sim 93\%$
$\Gamma(K\eta)/\Gamma(total)$	$\sim 5\%$
$\Gamma(VV)/\Gamma(total)$	$\sim 2\%$

Backup

Previous results of box diagram

$f_0(1710)$

PoS EFT09 (2009) 040	
$\Gamma(\pi\pi)/\Gamma(total)$	$<1\%$
$\Gamma(K\bar{K})/\Gamma(total)$	$\sim 55\%$
$\Gamma(\eta\eta)/\Gamma(total)$	$\sim 27\%$
$\Gamma(VV)/\Gamma(total)$	$\sim 18\%$

$a_0(1710)$

PoS EFT09 (2009) 040	
$\Gamma(K\bar{K})/\Gamma(total)$	$\sim 27\%$
$\Gamma(\pi\eta)/\Gamma(total)$	$\sim 23\%$
$\Gamma(VV)/\Gamma(total)$	$\sim 50\%$

f_0

PoS EFT09 (2009) 040	
$\Gamma(\pi\pi)/\Gamma(total)$	$\sim 72\%$
$\Gamma(K\bar{K})/\Gamma(total)$	$\sim 10\%$
$\Gamma(\eta\eta)/\Gamma(total)$	$<1\%$
$\Gamma(VV)/\Gamma(total)$	$\sim 18\%$

A chiral covariant approach to $\rho\rho$ scattering

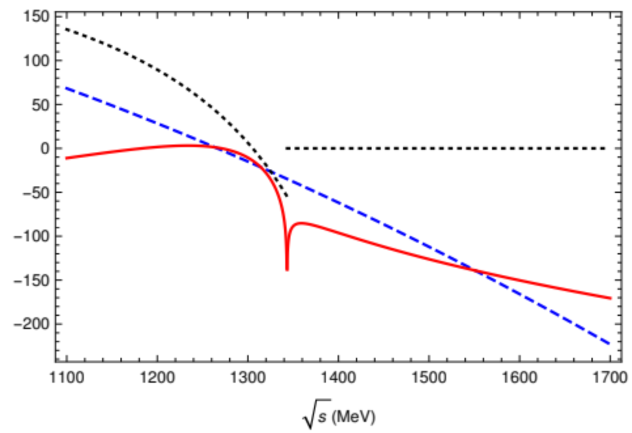
Eur. Phys. J. C 77 (2017) 7, 460

Left hand cut no $f_2(1270)$

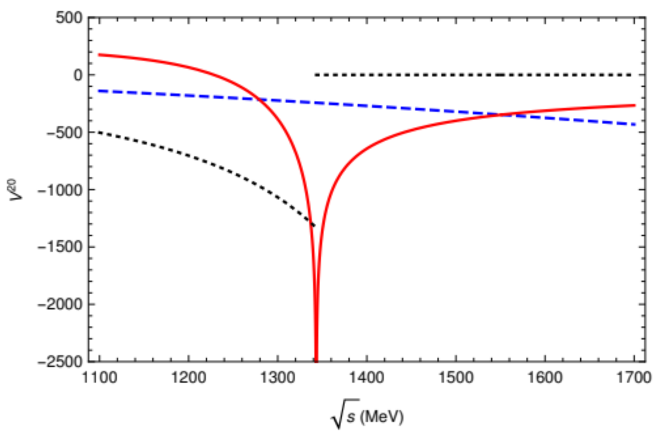
$$T^{(JI)}_{\ell S;\bar{\ell}\bar{S}}(s) = \frac{Y^0_{\bar{\ell}}(\hat{\mathbf{z}})}{2(2J+1)} \sum_{\substack{\sigma_1,\sigma_2,\bar{\sigma}_1\\ \bar{\sigma}_2, m}} \int d\hat{\mathbf{p}}''\, Y^m_{\ell}(\mathbf{p}'')^* \,(\sigma_1\sigma_2 M|s_1 s_2 S)(m M \bar{M}|\ell S J)(\bar{\sigma}_1\bar{\sigma}_2\bar{M}|\bar{s}_1\bar{s}_2\bar{S})(0\bar{M}\bar{M}|\bar{\ell}\bar{S}J)$$
$$\times T^{(I)}(p_1,p_2,p_3,p_4;\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4)\ , \qquad (17)$$

with s the usual Mandelstam variable, $\mathbf{p}_1 = |\mathbf{p}|\hat{\mathbf{z}}$, $\mathbf{p}_2 = -|\mathbf{p}|\hat{\mathbf{z}}$, $\mathbf{p}_3 = \mathbf{p}''$ and $\mathbf{p}_4 = -\mathbf{p}''$, $M = \sigma_1 + \sigma_2$ and $\bar{M} =$

$$T^{(JI)}(s) = \left[I - V^{(JI)}(s)\cdot G(s)\right]^{-1} \cdot V^{(JI)}(s)\ .$$



(a) $V^{(00)}(s)$



(b) $V^{(20)}(s)$

Backup
Argument on V-V interaction

According to the N/D method [54] a partial-wave amplitude can be written as

$$T = \frac{N(s)}{D(s)} \,, \tag{34}$$

where the function $D(s)$ has only the unitarity or right-hand cut (RHC) while $N(s)$ only has the left-hand cut (LHC). The secular equation for obtaining resonances and bound states corresponds to look for the zeros of $D(s)$,

$$D(s_i) = 0 \,. \tag{35}$$

Below threshold along the real s axis this equation is purely real because $D(s)$ has a non-vanishing imaginary part only for $s > s_{\rm th}$.

However, with our unitarization procedure from leading-order unitary chiral perturbation theory (UChPT) we have obtained the approximation

$$T^{(JI)}(s) = \frac{V^{(JI)}(s)}{1 - V^{(JI)}(s) g_c(s)} \,, \tag{36}$$

and the resulting equation to look for the bound states is

$$D_U(s) = 1 - V(s) g_c(s) = 0 \,. \tag{37}$$

Notice that Eq. (36), contrary to the general Eq. (35), has an imaginary part below the branch-point singularity at $s = 3m^2$.

Backup
Argument on V-V interaction

$$N(s) = V^{(JI)}(s) \ ,$$

$$D(s) = \gamma_0 + \gamma_1(s - s_{th}) + \frac{1}{2}\gamma_2(s - s_{s_{\rm th}})^2 + \frac{(s - s_{\rm th})s^2}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{\rho(s')V^{(JI)}(s')}{(s' - s_{\rm th})(s' - s)(s')^2} \ ,$$

$$T_{ND}(s) = \frac{N(s)}{D(s)} \ ,$$

$$\gamma_0 + \gamma_1(s - s_{th}) + \frac{1}{2}\gamma_2(s - s_{s_{\rm th}})^2 = 1 - V(s)g_c(s) - \frac{(s - s_{\rm th})s^2}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{\rho(s')V(s')}{(s' - s_{\rm th})(s' - s)(s')^2}$$
$$\equiv \omega(s) \ .$$

In this way,

$$\gamma_0 = 1 - V(s_{\rm th})g_c(s_{\rm th}) \ ,$$
$$\gamma_1 = \omega'(s_{\rm th}) \ ,$$
$$\gamma_2 = \omega''(s_{\rm th}) \ .$$

Backup
Argument on V-V interaction

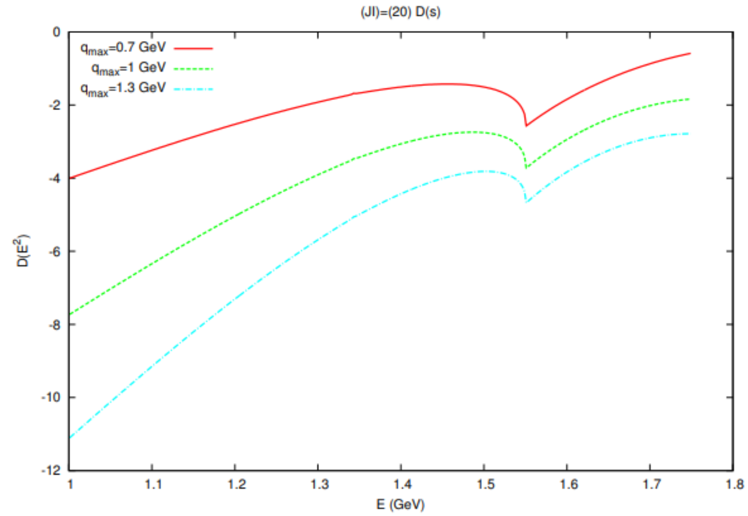


Figure 10: $D(s)$ function, Eq. (38), for $(J, I) = (2, 0)$. Above threshold only the real part is shown.

Interactions between vector mesons and dynamically generated resonances

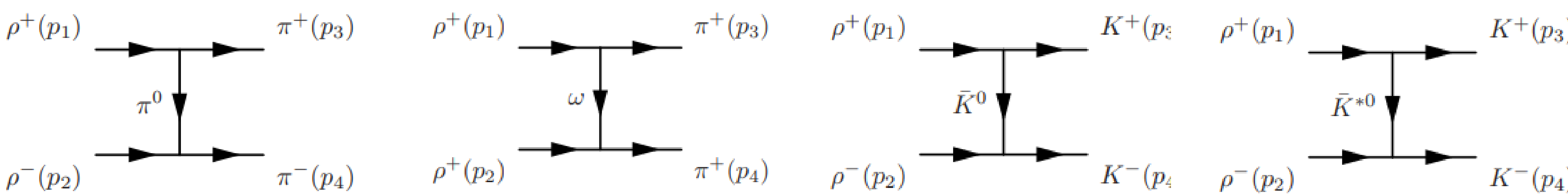
Eur. Phys. J. C 78 (2018) 12, 988

S	$I^G(J^{PC})$	Pole [GeV]	PDG	Mass [GeV] [29]	Pole [GeV] [45]
	$0^+(0^{++})$	$[1.41-1.50]$	$f_0(1370)$	$[1.2-1.5]$	1.512
			$f_0(1500)$	1.504 ± 0.006	
0	$0^+(0^{++})$	$[1.58-1.76]$	$f_0(1710)$	$1.723^{+0.006}_{-0.005}$	1.726
	$0^-(1^{+-})$	$[1.77-1.78]$	–	–	1.802
	$1^-(0^{++})$	$[1.75-1.79]$	$a_0(1950)$?	1.931 ± 0.026	1.780
	$1^+(1^{+-})$	$[1.44-1.50]$	–	–	1.679
1	$1/2(0^+)$	$[1.58-1.66]$	–	–	1.643
2	$0(1^+)$	$[1.68-1.74]$	–	–	–

$\rho\rho$ scattering revisited with coupled-channels of pseudoscalar mesons

Phys. Rev. D 99 (2019) 9, 096014

$f_0(1500)$ rather than $f_0(1370)$



$q_{max}(GeV)$	0.875	1.0	1.2	1.4
$\rho\rho$ only	1494.8	1467.2	1427.3	1395.0
π	$1530.0-4.9i$	$1519.5-8.4i$	$1501.5-12.3i$	$1488.6-14.6i$
ω	$1492.2-0.7i$	$1466.5-1.0i$	$1428.1-1.1i$	$1400.0-1.1i$
K	$1497.8-3.3i$	$1473.9-4.1i$	$1437.2-4.4i$	$1410.0-4.2i$
K^*	$1489.6-0.5i$	$1463.3-0.5i$	$1424.5-0.4i$	$1396.1-0.3i$
3-channels	$1529.8-4.9i$	$1519.0-8.6i$	$1500.9-13.5i$	$1488.4-16.7i$

Table 1: Pole position for coupled-channels

Backup
Argument on V-V interaction

Further study of $f_0(1710)$ with coupled-channel approach and hadron molecular picture

Phys. Rev. D 104 (2021) 11, 114001

$q_{max}(GeV)$	0.7	0.8	0.9	1.0	1.1
$Pole(GeV)$	$1.77-0.015i$	$1.75-0.028i$	$1.73-0.035i$	$1.72-0.045i$	$1.70-0.053i$

Table 3: The resonance pole for different cutoffs

$$\frac{\Gamma(f_0(1710)\to\pi\pi)}{\Gamma(f_0(1710\to K\bar{K}))}=0.289\pm0.092\ (0.23\pm0.05),$$
$$\frac{\Gamma(f_0(1710)\to\eta\eta)}{\Gamma(f_0(1710\to K\bar{K}))}=0.294\pm0.048\ (0.48\pm0.15).$$

Backup
Argument on V-V interaction

Two dynamical generated a_0 resonances by interactions between vector mesons

Eur. Phys. J. C 82 (2022) 6, 509

$q_{max}(GeV)$	0.9	1.0	1.1
$Pole(GeV)$	$1.76 - 0.09i$	$1.72 - 0.10i$	$1.69 - 0.11i$

Table 3: The resonance pole for different cutoffs

$\Gamma(K^*\bar{K}^* \to \rho\omega)$	$\Gamma(K^*\bar{K}^* \to K\bar{K})$	$\Gamma(K^*\bar{K}^* \to \pi\eta)$
$61.0MeV$	$74.4MeV$	$66.9MeV$
$\Gamma(\rho\phi \to \rho\omega)$	$\Gamma(\rho\phi \to K\bar{K})$	$\Gamma(\rho\phi \to \pi\eta)$
$60.8MeV$	$74.2MeV$	$66.6MeV$

Table 4: The partial decay widths

lower to be nearly degenerate with its isoscalar partner $f_0(1710)$. The mass is consistent with the $a_0(1710)$ newly reported by Babar Collaboration [17] and BESII Collaboration [18]. The ratio of its decays to $\rho\omega, K\bar{K}, \pi\eta$ is predicted to be about 1 : 1 : 1. With pseudo-scalar

Backup

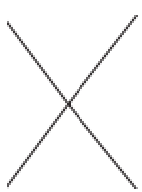
Argument on V-V interaction

On the chiral covariant approach to $\rho\rho$ scattering

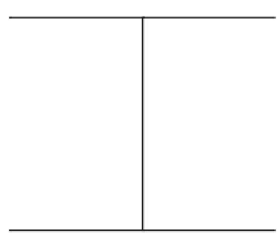
Chin. Phys. C 41 (2017) 12, 124101

$$\frac{1}{2}\int_{-1}^{+1} d\cos\theta \frac{1}{-2\mathbf{p}^2(1-\cos\theta)-m_\rho^2+i\varepsilon} = -\frac{1}{4\mathbf{p}^2}\log\left(\frac{4\mathbf{p}^2+m_\rho^2}{m_\rho^2}+\frac{4\mathbf{p}^2}{m_\rho^4}i\varepsilon\right)$$

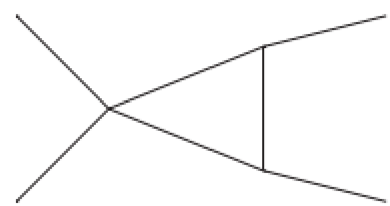
resonance $f_2(1270)$ does not appear with the potential of Eq. (8) is correct. The problem is that this is a clear situation where the on shell factorization cannot be done since the "on- shell" potential seats on top of a singularity of the extrapolated amplitude below threshold.



V_c



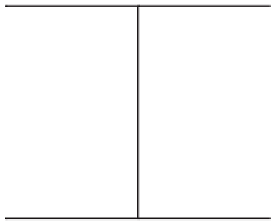
V_t



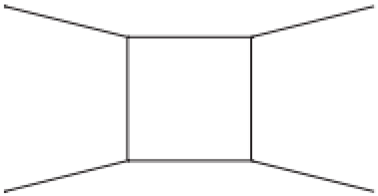
$V_c G V_t \to$ no singularity

Backup

Argument on V-V interaction



V_t



$V_t G V_t \rightarrow$ no singularity

A method is developed, evaluating the loops with full ρ propagators, and we show that they do not develop singularities and do not have an imaginary part below threshold.

Chin. Phys. C 41 (2017) 12, 124101

PTEP 2019 (2019) 10, 103B05   Look back this paper.

Comments on the dispersion relation method to vector-vector interaction

PTEP 2019 (2019) 10, 103B05

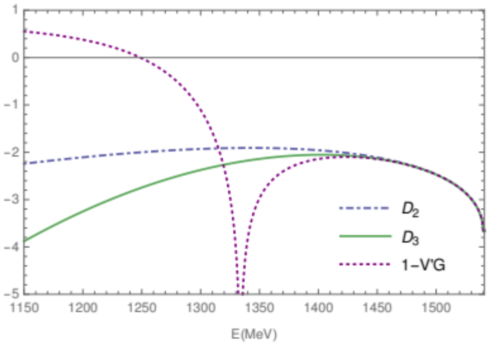


FIG. 7: The real part of the functions D_2 and D_3 from Eqs. (18) and (23) in comparison with $1 - V'G$.

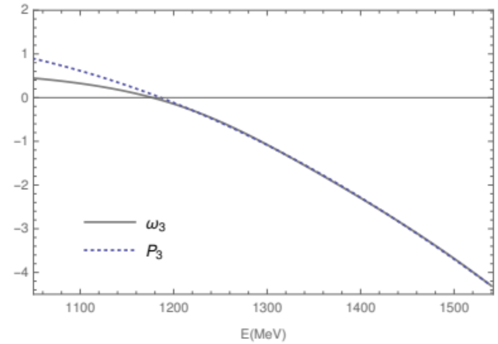


FIG. 9: The real part of the function ω_3 in comparison with P_3 from Eqs. (24), (25), and (26), with the convoluted potential V.

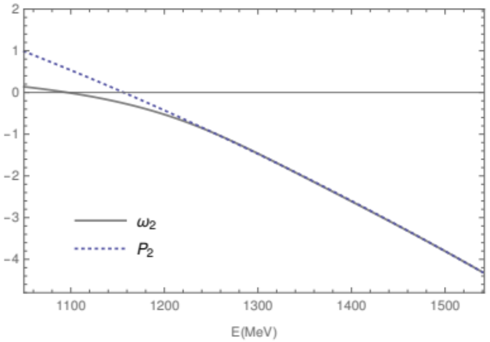


FIG. 8: The real part of the function ω_2 in comparison with P_2, which appear in Eqs. (20), (21), and (22), with the convoluted potential V.

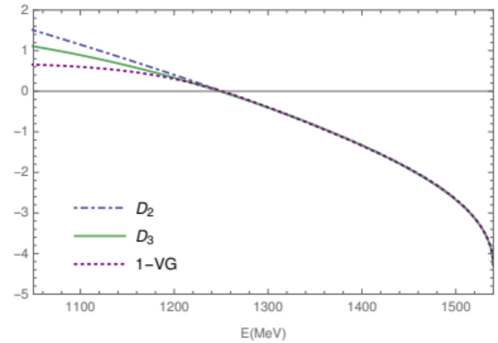


FIG. 10: The real part of the functions D_2 and D_3 from Eqs. (18) and (23) in comparison with the real part of $1 - VG$, with V the convoluted potential.

larity. Certainly, there is no convergence of the different orders in the region below the singularity and hence, neither D_2 nor D_3, nor any higher order expansion, can be taken as a representation of a realistic D function below the singular peak. Thus, the claim that the $f_2(1270)$ does not appear from the $\rho\rho$ interaction based on the approach of Ref. [8] is not justified.

Summary

Some similar conclusion can be obtained by different methods in light meson sector:

$$f_0(1710), a_0(1710) \text{ and a lower } f_0 \text{ around } 1500MeV$$

Backup

On shell factorization

Nucl. Phys. A 620 (1997) 438-456

PhD thesis of Feng-Kun Guo

[34, 35] in the meson-baryon sector. Our channels are labelled 1 for the $K\bar{K}$ and 2 for the $\pi\pi$ states in $T=0$ and 1 for $K\bar{K}$, 2 for $\pi\eta$ in $T=1$. The coupled channel equations then become

$$
\begin{aligned}
t_{11} &= V_{11} + V_{11}G_{11}t_{11} + V_{12}G_{22}t_{21} \\
t_{21} &= V_{21} + V_{21}G_{11}t_{11} + V_{22}G_{22}t_{21} \\
t_{22} &= V_{22} + V_{21}G_{11}t_{12} + V_{22}G_{22}t_{22}
\end{aligned} \tag{20}
$$

$$G_{ii} = i\frac{1}{q^2 - m_{1i}^2 + i\epsilon}\ \frac{1}{(P-q)^2 - m_{2i}^2 + i\epsilon} \tag{21}$$

where P is the total fourmomentum of the meson-meson systems and q the fourmomentum of one intermediate meson. The terms VGT in eq. (20) actually mean

$$VGT = \int \frac{d^4q}{(2\pi)^4} V(k,p;q)G(P,q)t(q;k',p') \tag{22}$$

Note that as G_{ii} has no angular dependence and V_{ij} is purely S-wave then t_{ij} is S-wave.

Backup
On shell factorization

In principle one would have to solve the integrals in eqs. (20) by taking V and t off shell. However, this is not the case, as we show below, at least when dealing with S-wave, and we only need the on shell information. The argument goes as follows: As we can see from eqs. (18),(19), the on shell amplitudes are obtained by taking $p_i^2 = m_i^2$ and then we can write the off shell amplitudes as

$$V = V_{on} + \beta \sum_i (p_i^2 - m_i^2)$$

In order to illustrate the procedure let us simplify to one loop and one channel (the procedure is easily generalized to two channels). Hence we have

$$V^2 = V_{on}^2 + 2\beta V_{on} \sum_i (p_i^2 - m_i^2) + \beta^2 \sum_{ij} (p_i^2 - m_i^2)(p_j^2 - m_j^2) \qquad (23)$$

Backup

On shell factorization

When performing the q^0 integration in the loop we have two poles, one for $q^0 = w_1(q)$ and the other one for $q^0 = P^0 + w_2(q)$, where the indices 1 and 2 indicate the two mesons inside the loop. Let us take the contribution from the first pole (the procedure follows analogously for the second pole). From the second term in eq. (23) we get the contribution

$$\frac{2\beta V_{on}}{(2\pi)^3}\int \frac{d^3q}{2w_1(q)}\frac{(P^0-w_1)^2-w_2^2}{(P^0-w_1)^2-w_2^2} = \frac{\beta V_{on}}{(2\pi)^3}\int dw_1 q \tag{24}$$

which for large Λ compared to the masses goes as $V_{on}\Lambda^2$ and has the same structure in the dynamical variables as the tree diagram. The third term in eq. (23) gives rise to the integral

$$\frac{\beta^2}{(2\pi)^3}\int \frac{d^3q}{2w_1(q)}[(P^0-w_1)^2-w_2^2] = \frac{\beta^2}{(2\pi)^3}\int \frac{d^3q}{2w_1(q)}[P^{02}-2w_1P^0+(m_1^2-m_2^2)] \tag{25}$$

ɔming from the first pole ($q^0 = w_1(q)$) and

$$\frac{\beta^2}{(2\pi)^3}\int \frac{d^3q}{2w_2(q)}[P^{02}+2w_2P^0+(m_2^2-m_1^2)]$$

coming from the second pole ($q^0 = P^0 + w_2(q)$). As we can see, the terms linear in P^0 cancel exactly, respecting the chiral structure which does not allow linear terms in P^0.

Backup

On shell factorization

The term proportional to $P^{02} + m_1^2 - m_2^2$ in (25) leads again to a structure of the type $[P^{02} + (m_1^2 - m_2^2)]\Lambda^2$ and similarly happens with the quadratic contribution form the second pole which leads to $[P^{02} + (m_2^2 - m_1^2)]\Lambda^2$. These terms, together the $V_{on}\Lambda^2$ which we obtained before, combine with the tree level contribution, giving rise to an amplitude with the same structure as the tree level one but with renormalized parameters f and masses. However, since we are taking physical values for $f (f = f_\pi = 93\,MeV)$ and the masses in the potential, these terms should be omitted. One can proceed like that to higher orders with the same conclusions.

Since we are taking V and t on shells they factorize outside the q integral. Thus the term VGT of eq. (22), after the q^0 integration is performed by choosing the contour in the lower half of the complex plane, is given by

$$V_{ij}G_{jj}t_{jk} = V_{ij}(s)t_{jk}(s)G_{jj}(s)$$
$$G_{jj}(s) = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1+\omega_2}{\omega_1\omega_2[P^{02}-(\omega_1+\omega_2)^2+i\epsilon]} \tag{26}$$

where $\omega_i = (\vec{q}^{\,2} + m_i^2)^{1/2}$ and $P^{02} = s$ and the subindex i stands for the two intermediate mesons of the j channel.

Thus the coupled channel Lippmann Schwinger equations get reduced to a set of algebraic equations:

$$At = V \tag{27}$$

Backup

On shell factorization

where

$$
t = \begin{pmatrix} t_{11} \\ t_{21} \\ t_{22} \end{pmatrix} \qquad V = \begin{pmatrix} V_{11} \\ V_{21} \\ V_{22} \end{pmatrix}
$$

$$
A = \begin{pmatrix} 1 - V_{11}t_{11} & -V_{12}G_{22} & 0 \\ -V_{21}G_{11} & 1 - V_{22}G_{22} & 0 \\ 0 & -V_{21}G_{11} & 1 - V_{22}G_{22} \end{pmatrix} \tag{28}
$$

Backup

Two-point loop function

被积分部分可以费曼参数化

$$\frac{1}{q^2-m_1^2+i\epsilon}\frac{1}{(P-q)^2-m_2^2+i\epsilon}=\int_0^1 dx\frac{1}{[xA+(1-x)B]^2}=\int_0^1 dx\frac{1}{[l^2-\Delta]^2} \tag{3.51}$$

其中

$$A=(P-q)^2-m_2^2,\quad B=q^2-m_1^2,\quad l=q-xP$$
$$\Delta=x(x-1)p^2+x(m_2^2-m_1^2)+m_1^2 \tag{3.52}$$

然后，利用发散函数将无穷大直接分离出来

$$\begin{aligned}
i\int_0^1 dx\int\frac{dq^4}{(2\pi)^2}\frac{1}{(l^2-\Delta)^2} &= i\int_0^1\frac{(-1)^2 i}{(2\pi)^{\frac{4-2\epsilon}{2}}}\frac{\Gamma(2-\frac{4-2\epsilon}{2})}{\Gamma(2)}(\frac{1}{\Delta})^{2-\frac{4-2\epsilon}{2}}\\
&=\frac{-1}{16\pi^2}\int_0^1 dx(\frac{1}{\epsilon}-\ln\frac{\Delta}{u^2})\\
&=\frac{-1}{16\pi^2}\int_0^1 dx(\frac{1}{\epsilon}+\ln u^2-\ln[x(x-1)P^2+x(m_2^2-m_1^2)+m_1^2])\\
&=\frac{-1}{16\pi^2}\int_0^1 dx(\frac{1}{\epsilon}+\ln\frac{u^2}{P^2}-\ln[x(x-1)+x\frac{m_2^2-m_1^2}{P^2}+\frac{m_1^2}{P^2}])\\
&=\frac{-1}{16\pi^2}(\frac{1}{\epsilon}+\ln u^2-C(p))
\end{aligned} \tag{3.53}$$

这里

$$C(p)=\int_0^1 \ln[x(x-1)+x\frac{m_2^2-m_1^2}{P^2}+\frac{m_1^2}{P^2}] \tag{3.54}$$

被积函数对数中的二次多项式可以整理成标准形式

$$x(x-1)+x\frac{m_2^2-m_1^2}{P^2}+\frac{m_1^2}{P^2}=x^2+x\frac{m_2^2-m_1^2-P^2}{P^2}+\frac{m_1^2}{P^2} \tag{3.55}$$

为了将对数积分处理掉对其进行因式分解，令

$$x^2+x\frac{m_2^2-m_1^2-P^2}{P^2}+\frac{m_1^2}{P^2}=0 \tag{3.56}$$

即

$$x^2P^2+x(m_2^2-m_1^2-P^2)+m_1^2=0 \tag{3.57}$$

考虑两个根存在的情况，可以得到其两个根为

$$\begin{aligned} x_{1,2} &= \frac{-(m_2^2-m_1^2-P^2)\pm\sqrt{(m_2^2-m_1^2-P^2)^2-4P^2m_1^2}}{2P^2} \\ &= \frac{(P^2+m_1^2-m_2^2)\pm\sqrt{(m_2^2-m_1^2-P^2)^2-4P^2m_1^2}}{2P^2} \end{aligned} \tag{3.58}$$

Backup

Two-point loop function

注意到根号里面的项

$$(P^2+m_1^2-m_2^2)^2-4P^2m_1^2 = P^4+m_1^4+m_2^4+2P^2m_1^2-2P^2m_2^2-2m_1^2m_2^2-4P^2m_1^2$$
$$= P^4+m_1^4+m_2^4-2P^2m_1^2-2P^2m_2^2-2m_1^2m_2^2$$
$$= \lambda(P^2,m_1^2,m_2^2) \tag{3.59}$$

这一项对于两个质量对称，故

$$(P^2+m_2^2-m_1^2)^2-4P^2m_2^2 = \lambda(P^2,m_1^2,m_2^2) \tag{3.60}$$

对于两体末态粒子

$$p = \frac{\sqrt{\lambda(P^2,m_1^2,m_2^2)}}{2|P|} \tag{3.61}$$

令 $P^2 = s$, 则

$$p = \frac{\sqrt{\lambda(s,m_1^2,m_2^2)}}{2|P|} \tag{3.62}$$

也可以将其写成全对称形式

$$p = \frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{2\sqrt{s}} \tag{3.63}$$

Backup

Two-point loop function

因为

$$(s-(m_1+m_2)^2)(s-(m_1-m_2)^2) = s^2 - [(m_1+m_2)^2+(m_1-m_2)^2]s + (m_1^2-m_2^2)^2$$
$$= s^2 + m_1^4 + m_2^4 - 2sm_1^2 - 2sm_2^2 - 2m_1^2m_2^2$$
$$= \lambda(s, m_1^2, m_2^2) \tag{3.64}$$

由 (3.63) 可以得到

$$\sqrt{\lambda(s, m_1^2, m_2^2)} = 2p\sqrt{s} \tag{3.65}$$

因此，(3.58) 简化为

$$x_{1,2} = \frac{(s+m_1^2-m_2^2) \pm 2p\sqrt{s}}{2s} \tag{3.66}$$

对数函数进行因式分解

$$\ln(x-x_1)(x-x_2) = \ln(x-x_1) + \ln(x-x_2) \tag{3.67}$$

并借助对数积分公式

$$\int \ln x dx = x\ln x - x + C \tag{3.68}$$

Backup

Two-point loop function

最后，原式变成了

$$G_{ii} = -\frac{1}{16\pi^2}[\frac{1}{\epsilon} + \ln u^2 + 2 - \ln\frac{m_2^2}{u^2} - \frac{p}{\sqrt{s}}(\ln\frac{s - m_1^2 + m_2^2 + 2p\sqrt{s}}{-s + m_1^2 - m_2^2 + 2p\sqrt{s}} + \ln\frac{s + m_1^2 - m_2^2 + 2p\sqrt{s}}{-s - m_1^2 + m_2^2 + 2p\sqrt{s}}) - \frac{s + m_1^2 - m_2^2}{2s}\ln\frac{m_1^2}{m_2^2}] \tag{3.73}$$

在某一能标 $u = u_0$ 下引入抵消项 $G_{ii}^{\Lambda} = -\frac{1}{16\pi^2}(\frac{1}{\epsilon} + \ln u_0^2 + 2)$ 抵消掉发散并引入截断常数，并省略 u_0 的下标都写作 u 则

$$G_{ii} = \frac{1}{16\pi^2}[\alpha_i + \ln\frac{m_2^2}{u^2} + \frac{p}{\sqrt{s}}(\ln\frac{s - m_1^2 + m_2^2 + 2p\sqrt{s}}{-s + m_1^2 - m_2^2 + 2p\sqrt{s}} + \ln\frac{s + m_1^2 - m_2^2 + 2p\sqrt{s}}{-s - m_1^2 + m_2^2 + 2p\sqrt{s}}) + \frac{s + m_1^2 - m_2^2}{2s}\ln\frac{m_1^2}{m_2^2}] \tag{3.74}$$

此即两点圈函数的精确解析表达式。这里 u 是重整化标度，a 是减除常数, 是 u 的相关函数。注意该式结果只依赖与 u, 因为 u 的改变可以通过 $a(u') = a(u) + 2In\frac{u'}{u}$ 吸收进 $a(u)$ 里。