Pion condensation in dense QCD

余青

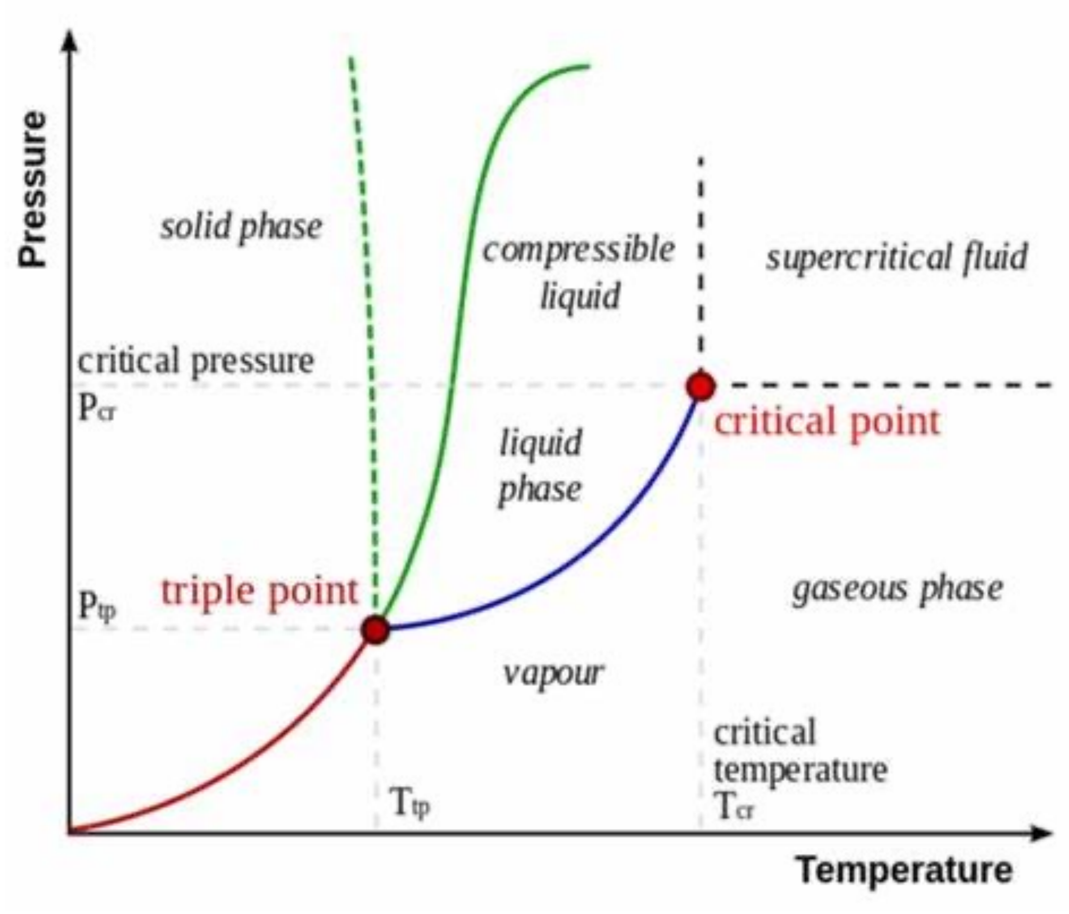
湖南 • 长沙

2024.10.18-2024.10.22

Phys. Rev. D 107, 014010 (2023) ; Phys. Rev. D 109, 034022 (2024) ; arXiv: 2312.13092 [hep-ph]

Contents

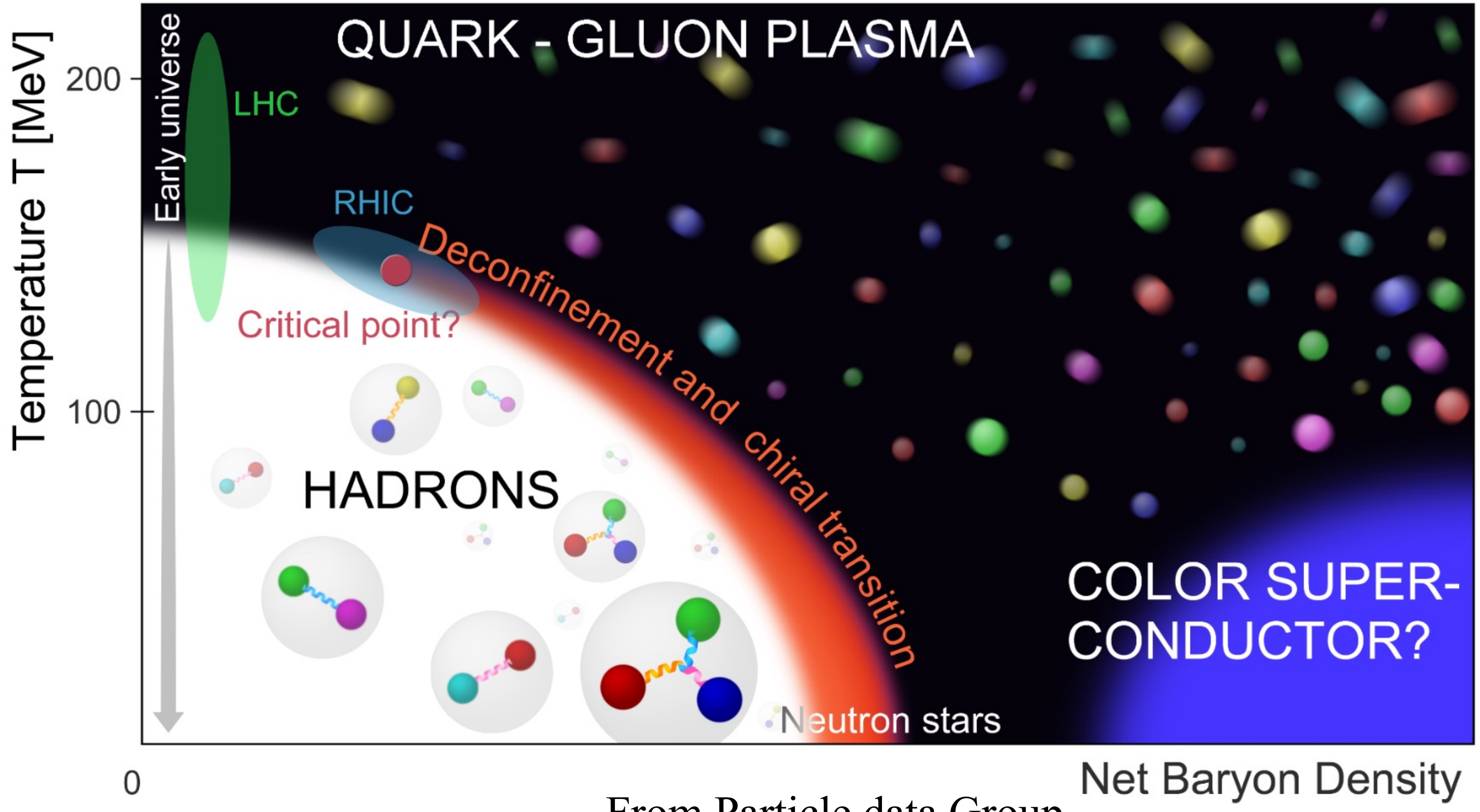
1. QCD phase diagram
2. Chiral perturbation theory
3. Ground state and fluctuations at finite density
4. Thermodynamics quantities in dense QCD
5. Summery


Pressure
solid phase
compressible
liquid
supercritical fluid
critical pressure
P_{cr}
critical point
liquid
phase
triple point
P_{tp}
gaseous phase
vapour
critical
temperature
T_{cr}
T_{tp}
Temperature

Two important features of **QCD** in the low energy region:

Quark confinement

Dynamical chiral symmetry breaking



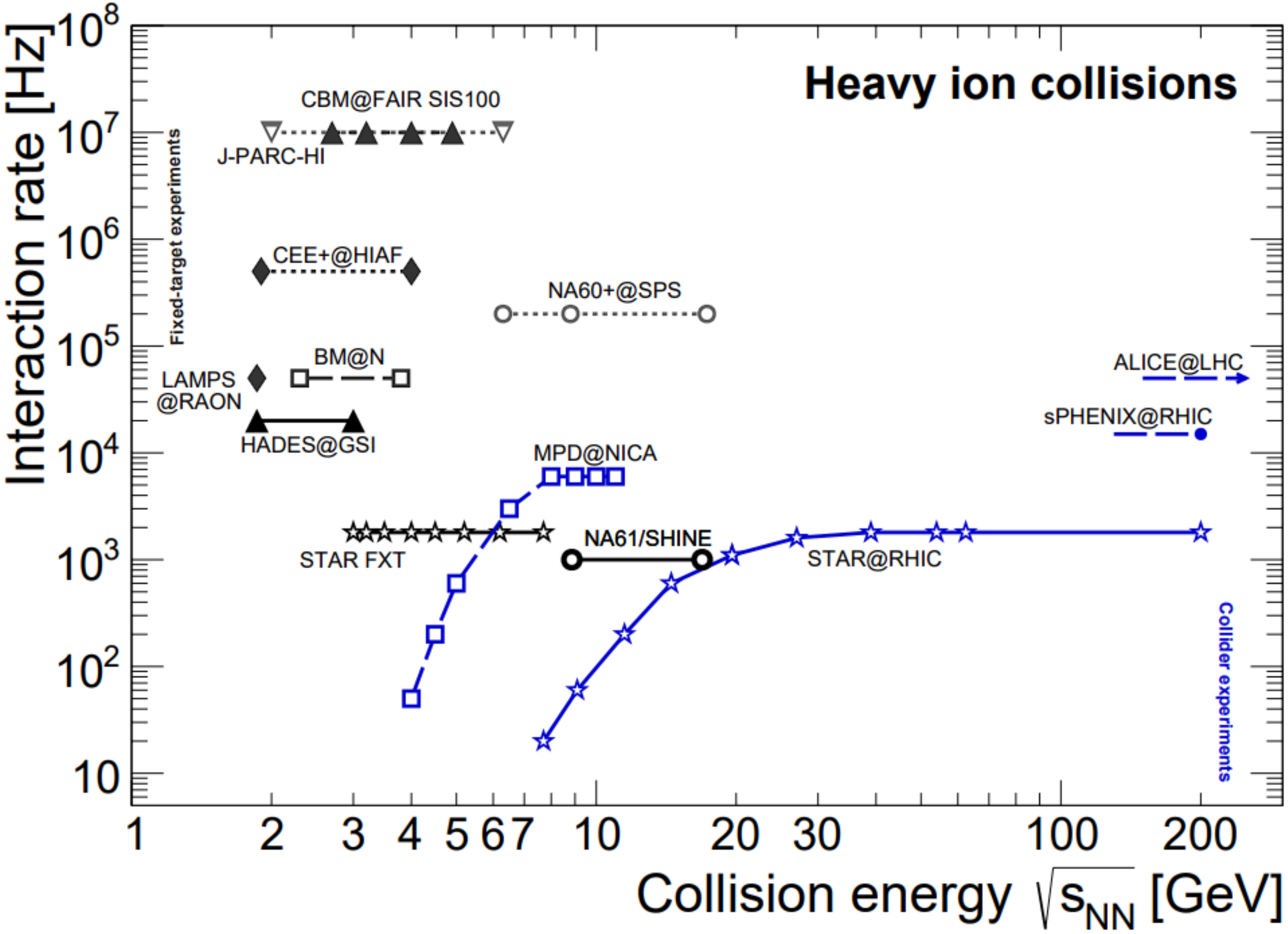
From Particle data Group

Phase Transitions in Particle Physics

Prog.Part.Nucl.Phys. 133 (2023) 104070

Nucl. Phys. A 982 (2019) 163–169.

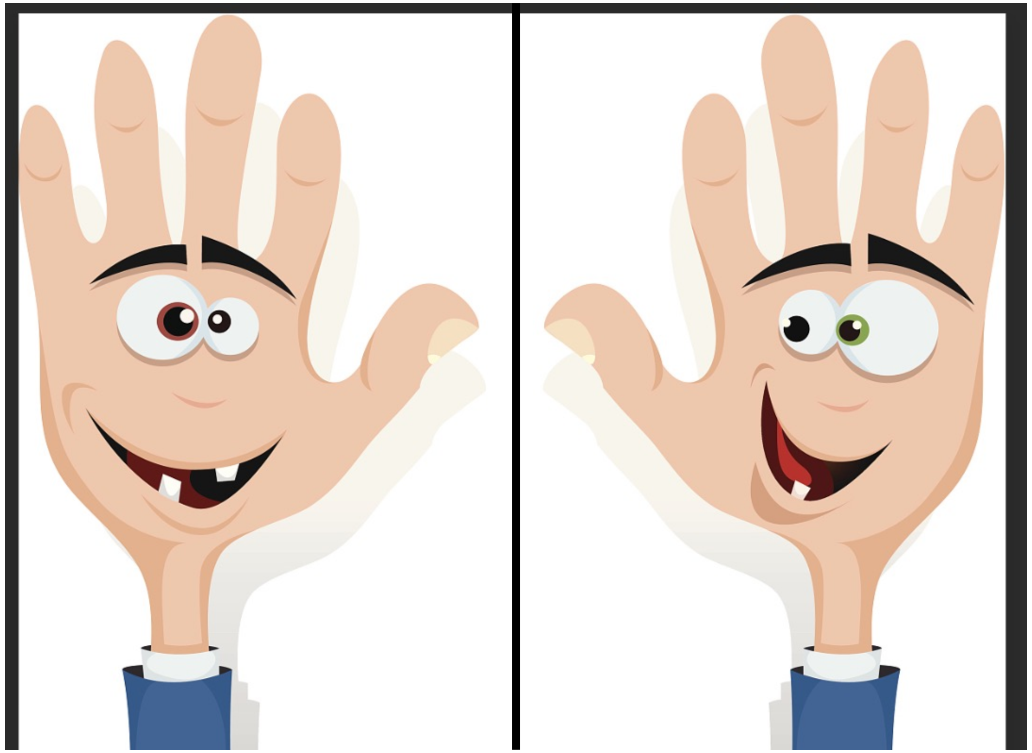
Eur. Phys. J. A 53 (3) (2017) 60.



Overview over future and past experiments taking data from heavy ion collisions

Chiral symmetry

$\vec{s}$ and $\vec{p}$
anti-parallel



$\vec{s}$ and $\vec{p}$
parallel

Spin ($\vec{s}$) and momentum ($\vec{p}$) of quarks

$$\begin{matrix} \overline{m}_u(2\ \mathrm{GeV}) \approx 2\ \mathrm{MeV} \\ \overline{m}_d(2\ \mathrm{GeV}) \approx 5\ \mathrm{MeV} \\ \overline{m}_s(2\ \mathrm{GeV}) \approx 93\ \mathrm{MeV} \end{matrix} \ll 1\ \mathrm{GeV} \ll \begin{matrix} \overline{m}_c \approx 1.27\ \mathrm{GeV} \\ \overline{m}_b \approx 4.18\ \mathrm{GeV} \\ \overline{m}_t \approx 172.69\ \mathrm{GeV} \end{matrix}$$

Dynamical chiral symmetry breaking

If we use the approximation $m_u = m_d = m_s \neq 0$, then we obtain a manifestly U(3)-flavor symmetric theory.

chiral limit

$$m_u = m_d = m_s = 0$$

Under chiral limit

$$\mathcal{L}_{QCD^0} = \sum \bar{q}_f \left(i\not{D}\right) q_f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}.$$

$$\boxed{SU_L(N)\times SU_R(N)}$$

The relation between the Dirac field q and its right and left-handed field components q_R and q_L can now be written as

$$q_R = P_R q, \quad q_L = P_L q, \quad \bar{q}_R = \bar{q} P_L, \quad \bar{q}_L = \bar{q} P_R.$$

$$\mathcal{L}_{QCD^0} = \sum \bar{q}_{R,f} \left(i\not{D}\right) q_{R,f} + \bar{q}_{L,f} \left(i\not{D}\right) q_{L,f} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}.$$

$$\boxed{SU_L(N)\times SU_R(N) + U_L(1)\times U_R(1)}$$

$$\Downarrow$$

$$\boxed{U_V(1) + SU_L(N)\times SU_R(N)} \xrightarrow{\text{vacuum}} SU_V(N)$$

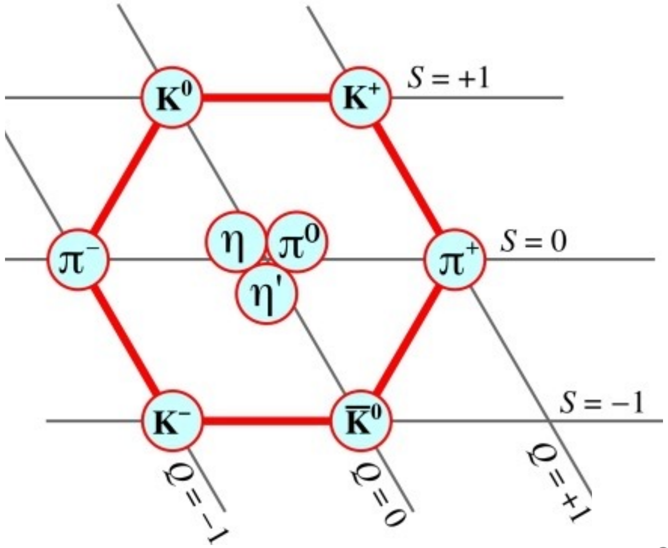
Quark confinement $\xrightarrow{\approx 1\ \text{GeV}}$ **The perturbation theory**
is not available below 1 GeV

8 pseudo-Goldstone particles

$$\phi = \phi^\dagger = \sum_{a=1}^{8} \phi_a \lambda_a \equiv \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

The vacuum manifold of ChiPT can be parametrized as:

$$\Sigma = \Sigma_0 e^{i\phi_a X_a / f}$$



S. Weinberg, Physica (Amsterdam) A 96, 327 (1979).
J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158, (142) (1984).
J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985).

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2g^2}\langle G_{\mu\nu}G^{\mu\nu}\rangle + i\bar{q}\gamma^{\mu}D_{\mu}q + \bar{q}\gamma^{\mu}\left(v_{\mu}+\gamma^5 a_{\mu}\right)q - \bar{q}(s - i\gamma^5 p)q$$

The chiral effective lagrangian must satisfies Chiral symmetry, P, C and Lorentz invariance.

$$\nabla_\mu \Sigma \equiv \partial_\mu \Sigma - i\underbrace{(v_\mu + a_\mu)\Sigma + i\Sigma(v_\mu - a_\mu)}_{\text{others}}$$

Mass term: $\frac{1}{4}f^2\langle\Sigma^\dagger\chi + \chi^\dagger\Sigma\rangle \quad \chi = 2B_0(s+ip)$

Power Counting rules

∂_μ	v_μ	a_μ	s	p	Σ
P^1	P^1	P^1	P^2	P^2	P^0

Effective Lagrangian

$$\boxed{\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots\cdots}$$

S. Weinberg, Physica (Amsterdam) A 96, 327 (1979).
J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158, (142) (1984).
J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985).

The 3-flavor chiral Lagrangian

$$\mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}f^2\langle\nabla_\mu\Sigma^\dagger\nabla^\mu\Sigma\rangle + \frac{1}{4}f^2\langle\Sigma^\dagger\chi + \chi^\dagger\Sigma\rangle + C\langle Q\Sigma Q\Sigma^\dagger\rangle + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm ghost} - eA_\mu J^\mu_{\rm back} ,$$

$$\mathcal{L}_2 = \mathcal{L}_2^{\rm static} + \mathcal{L}_2^{\rm linear} + \mathcal{L}_2^{\rm quadratic} + \mathcal{L}_2^{\rm cubic} + \mathcal{L}_2^{\rm quartic} + \cdots ,$$

$$\mathcal{L}_2^{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - \frac{1}{2}m_a^2\phi_a^2 + \frac{1}{\sqrt{3}}\Delta m^2\phi_3\phi_8 + \partial_\mu\bar{c}\partial^\mu c - \frac{1}{2\xi}(\partial_\mu A^\mu)^2$$

$$m^2_{\pi^0/\eta^0} = \frac{1}{3}\left(m^2_{K^\pm,0} + m^2_{K^0,0} + m^2_{\pi,0} \mp\sqrt{\left(m^2_{K^\pm,0} + m^2_{K^0,0} - 2m^2_{\pi,0}\right)^2 + 3(\Delta m^2)^2}\right)$$

$$m^2_{\pi^\pm} = m^2_{\pi,0} + \Delta m^2_{\rm EM} ,$$
$$m^2_{K^0} = m^2_{K^\pm,0} + \Delta m^2 ,$$
$$m^2_{K^\pm} = m^2_{K^\pm,0} + \Delta m^2_{\rm EM} .$$

$$m_{\pi^0} = 134.98\,\text{MeV} , \quad m_{\pi^\pm} = 139.57\,\text{MeV} ,$$
$$m_{K^\pm} = 493.68\,\text{MeV} , \quad m_{K^0} = 497.61\,\text{MeV} .$$

$$f_\pi = 92.07\text{MeV} , \quad e = 0.3028,$$

$$\boxed{\begin{aligned} m_{\pi,0} &= 135.09\ \text{MeV}, & m_{K^\pm,0} &= 492.43\ \text{MeV} ,\\ \Delta m^2 &= (71.60\ \text{MeV})^2 , & \Delta m^2_{\rm EM} &= (35.09\ \text{MeV})^2 \end{aligned}}$$

arXiv: 2312.13092

For 2-flavor

$$\mathcal{L}_4 = \frac{1}{4} l_1 \langle \nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma \rangle^2 + \frac{1}{4} l_2 \langle \nabla_\mu \Sigma^\dagger \nabla_\nu \Sigma \rangle \langle \nabla^\mu \Sigma^\dagger \nabla^\nu \Sigma \rangle$$
$$+ \frac{1}{16}(l_3 + l_4) \langle \chi^\dagger \Sigma + \Sigma^\dagger \chi \rangle^2$$
$$+ \frac{1}{8} l_4 \langle \nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma \rangle \langle \chi^\dagger \Sigma + \Sigma^\dagger \chi \rangle$$
$$- \frac{1}{16} l_7 \langle \chi^\dagger \Sigma - \Sigma^\dagger \chi \rangle^2 + \frac{1}{2} h_1 \langle \chi^\dagger \chi \rangle \,,$$

For 3-flavor

$$\mathcal{L}_4 = L_1 \langle \nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma \rangle^2 + L_2 \langle \nabla_\mu \Sigma^\dagger \nabla_\nu \Sigma \rangle \langle \nabla^\mu \Sigma^\dagger \nabla^\nu \Sigma \rangle$$
$$+ L_3 \langle \nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma \nabla_\nu \Sigma^\dagger \nabla^\nu \Sigma \rangle$$
$$+ L_4 \langle \nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma \rangle \langle \chi^\dagger \Sigma + \chi \Sigma^\dagger \rangle$$
$$+ L_5 \langle \nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma (\chi^\dagger \Sigma + \chi \Sigma^\dagger) \rangle$$
$$+ L_6 \langle \chi^\dagger \Sigma + \chi \Sigma^\dagger \rangle^2 + L_7 \langle \chi \Sigma - \chi \Sigma^\dagger \rangle^2$$
$$+ L_8 \langle \chi^\dagger \Sigma \chi^\dagger \Sigma + \chi \Sigma^\dagger \chi \Sigma^\dagger \rangle + H_2 \langle \chi \chi^\dagger \rangle \,.$$

At $O(p^6)$, the Lagrangian contains a larger number of terms, 57 for SU(2) and 94 for SU(3)

$$\mathcal{L}_6 = C_{24} \langle (\nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma)^3 \rangle$$
$$+ C_{25} \langle \nabla_\rho \Sigma^\dagger \nabla^\rho \Sigma \nabla_\mu \Sigma^\dagger \nabla_\nu \Sigma \nabla^\mu \Sigma^\dagger \nabla^\nu \Sigma \rangle$$
$$+ C_{26} \langle \nabla_\mu \Sigma^\dagger \nabla_\nu \Sigma \nabla_\rho \Sigma^\dagger \nabla^\mu \Sigma \nabla^\nu \Sigma^\dagger \nabla^\rho \Sigma \rangle \,,$$

J. Bijnens et al, Ann. Phys. 280, 100 (2000).
J. Bijnens et al, JHEP 02 020 (1999).

Renormalization?

Two flavor:

$$l_i = l_i^r - \frac{\gamma_i \Lambda^{-2\epsilon}}{2(4\pi)^2}\left[\frac{1}{\epsilon}+1\right]$$

$$h_i = h_i^r - \frac{\delta_i \Lambda^{-2\epsilon}}{2(4\pi)^2}\left[\frac{1}{\epsilon}+1\right]$$

$$l_i^r(\Lambda) = \frac{\gamma_i}{2(4\pi)^2}\left[\bar{l}_i + \log\frac{m_{\pi,0}^2}{\Lambda^2}\right]$$

Three flavor:

$$L_i = L_i^r - \frac{\Gamma_i \Lambda^{-2\epsilon}}{2(4\pi)^2}\left[\frac{1}{\epsilon}+1\right] ,$$

$$H_i = H_i^r - \frac{\Delta_i \Lambda^{-2\epsilon}}{2(4\pi)^2}\left[\frac{1}{\epsilon}+1\right] .$$

$$\Lambda\frac{dL_i^r}{d\Lambda} = -\frac{\Gamma_i}{(4\pi)^2} , \qquad \Lambda\frac{dH_i^r}{d\Lambda} = -\frac{\Delta_i}{(4\pi)^2} .$$

J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985).

	Fundamental theory	Effective field theory
Theoretical framework	QCD	ChPT
Degrees of freedom	Quarks and gluons	Goldstone bosons (+ other hadrons)
Parameters	g_3 + quark masses	Low-energy coupling constants + quark masses

- All the interaction terms are coming from $P^4, P^6 \cdots$, and the $O(P^n)$ terms are instructed by symmetries and renormalizations. Thus the divergence part all canceled.
- Limited theoretical accuracy requires only a finite number of calculations, and an infinite number of higher-order loop diagrams do not contribute to finite power results.

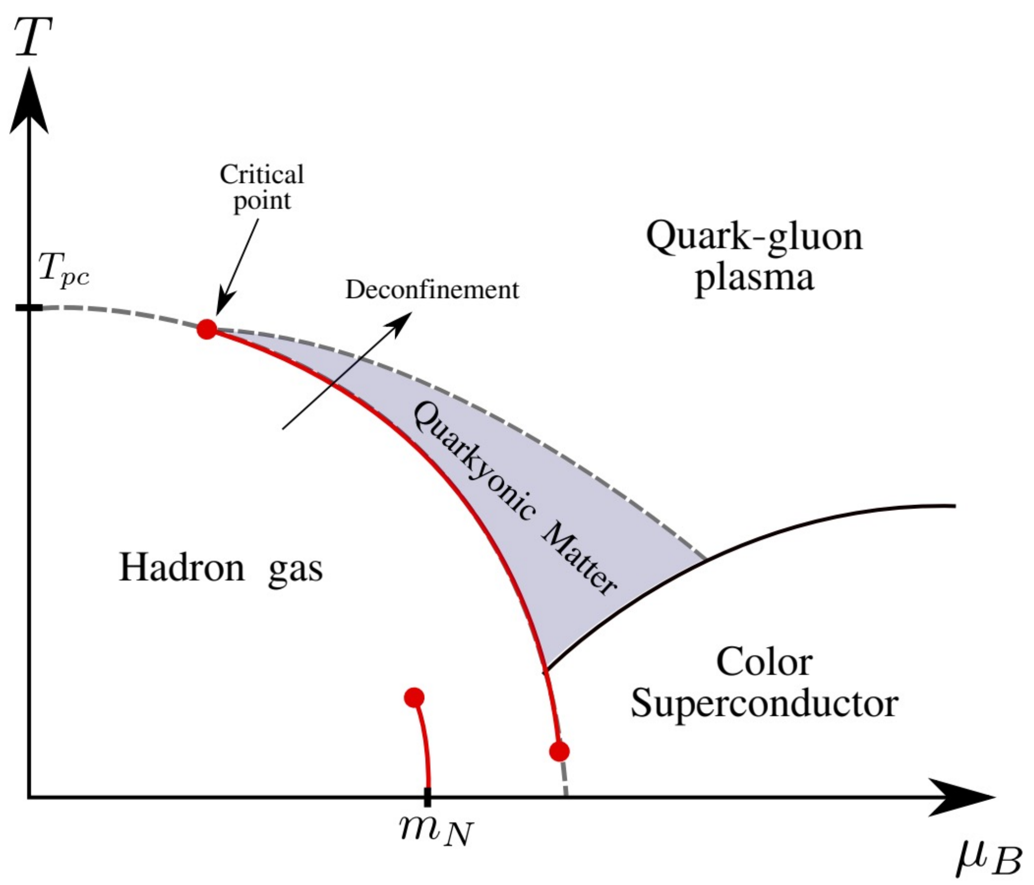


FIG.1 QCD phase diagram in μ_B and T plane.

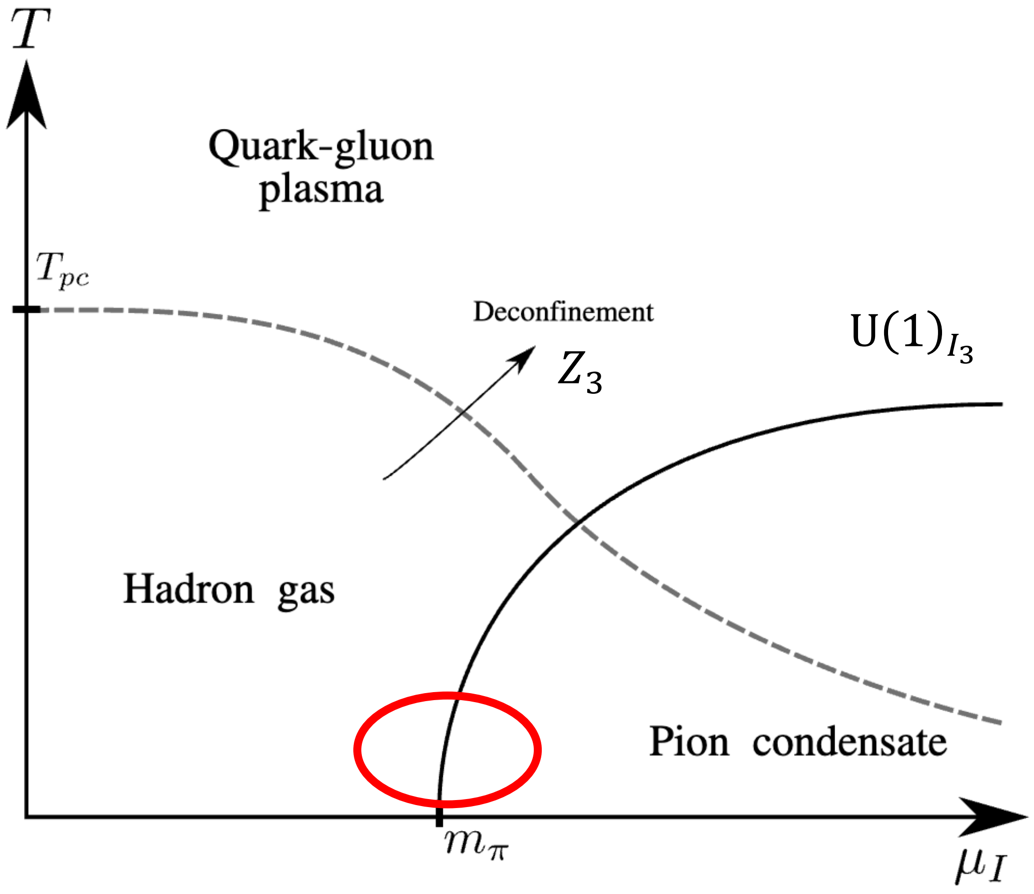


FIG.2 QCD phase diagram in μ_I and T plane.

A. W. Steiner et al. Phys. Rev. D 66, 094007 (2002).
M. Alford and K. Rajagopal, JHEP 06, 031 (2002).
S. B. R¨uster et al, Phys. Rev. D 72, 034004 (2005).
H. Abuki and T. Kunihiro, Nucl. Phys. A 768, 118 (2006).

$$\boxed{\begin{aligned}\partial_\mu\Sigma &\to \nabla_\mu\Sigma = \partial_\mu\Sigma - \underline{ir_\mu\Sigma + i\Sigma l_\mu} \ ,\\ \partial_\mu\Sigma^\dagger &\to \nabla_\mu\Sigma^\dagger = \partial_\mu\Sigma^\dagger + \underline{i\Sigma^\dagger r_\mu - il_\mu\Sigma^\dagger}\end{aligned}}$$

$$\begin{aligned} l_\mu &= v_\mu - a_\mu = \delta_{0\mu}\mathrm{diag}(\mu_u,\mu_d,\mu_s) + Q_L A_\mu \\ r_\mu &= v_\mu + a_\mu = \delta_{0\mu}\mathrm{diag}(\mu_u,\mu_d,\mu_s) + Q_R A_\mu \end{aligned}$$

$$\mathrm{diag}(\mu_u,\mu_d,\mu_s) = \frac{1}{3}(\mu_B-\mu_S)\mathbb{1} + \frac{1}{2}\mu_I\lambda_3 + \frac{1}{\sqrt{3}}\mu_S\lambda_8$$

$$\boxed{\begin{aligned}\mu_B &= \frac{3}{2}(\mu_u+\mu_d) \ ,\\ \mu_I &= \mu_u - \mu_d \ ,\\ \mu_S &= \frac{1}{2}(\mu_u+\mu_d-2\mu_s) \ .\end{aligned}}$$

The parametrization of the fluctuations around the ground-state configuration:

Two-flavor: $\Sigma_\alpha = e^{i\hat{\phi}_a\tau_a\alpha}$

$$\Sigma_\alpha = e^{i\tau_2\alpha} = \mathbb{1}\cos\alpha + i\tau_2\sin\alpha$$

Three-flavor: $\Sigma_\alpha = e^{i\hat{\phi}_a\lambda_a\alpha}$

superfluid $\Sigma_\alpha^{\pi^\pm} = e^{i\lambda_2\alpha} = \frac{1+2\cos\alpha}{3}\mathbb{1} + \frac{\cos\alpha-1}{\sqrt{3}}\lambda_8 + i\lambda_2\sin\alpha \ ,$

superfluid

$$\Sigma_\alpha^{K^\pm} = e^{i\lambda_5\alpha} = \frac{1+2\cos\alpha}{3}\mathbb{1} + \frac{\cos\alpha-1}{2\sqrt{3}}\left(\sqrt{3}\lambda_3-\lambda_8\right) + i\lambda_5\sin\alpha \ ,$$

superfluid and superconductor

$$\Sigma_\alpha^{K^0/\bar{K}^0} = e^{i\lambda_7\alpha} = \frac{1+2\cos\alpha}{3}\mathbb{1} + \frac{1-\cos\alpha}{2\sqrt{3}}\left(\sqrt{3}\lambda_3+\lambda_8\right) + i\lambda_7\sin\alpha \ . \quad (89)$$

$$\mathrm{U}(1)_{I_3}\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_B \to \mathrm{U}(1)_Q\times\mathrm{U}(1)_B$$

Dense QCD:

2-flavor:Dover Books on Physics (2003), J. Phys. (USSR) 11, 23 (1947)
3-flavor:Eur. Phys. J. B 11, 143 (1999), Phys. Rev. 105, 1119 (1957)

In order to calculate the quasiparticles masses, we expand the LO chiral Lagrangian to the second order in the fields+**in the pion-condensed phase**.

$$
\begin{aligned}
\mathcal{L}_2^{(2)} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2\eta_{\mu\nu}A^\mu A^\nu + \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a \\
&+\frac{1}{2}m_{ab}\phi_a\partial_0\phi_b - m_{\phi A}^2\phi_2 A^0 - \frac{1}{2}m_a^2\phi_a^2 \\
&-ef\sin\alpha\partial_\mu A^\mu\phi_1 + \frac{1}{\sqrt{3}}\Delta m^2\phi_3\phi_8 + \partial_\mu\bar{c}\partial^\mu c \\
&-m_c^2\bar{c}c - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \,,
\end{aligned}
$$

$$
m_{\pi^0/\eta}^2 = \frac{1}{3}\Bigg(m_{K^\pm,0}^2 + m_{K^0,0}^2 + m_{\pi,0}^2 \mp \sqrt{(m_{K^\pm,0}^2 + m_{K^0,0}^2 - 2m_{\pi,0}^2)^2 + 3(\Delta m^2)^2}\Bigg)\,,
$$

$$
m_{\tilde{\pi}^\pm}^2 = \left(\sqrt{m_{\pi,0}^2 + \Delta m_{\mathrm{EM}}^2} \mp \mu_I\right)^2 \,,
$$

$$
m_{\tilde{K}^0/\tilde{\bar{K}}^0}^2 = \left(m_{K^0,0} \mp \frac{1}{2}\mu_I\right)^2
$$

$$
m_{\tilde{K}^\pm}^2 = \left(\sqrt{m_{K^\pm,0}^2 + \Delta m_{\mathrm{EM}}^2} \mp \frac{1}{2}\mu_I\right)^2 \,,
$$

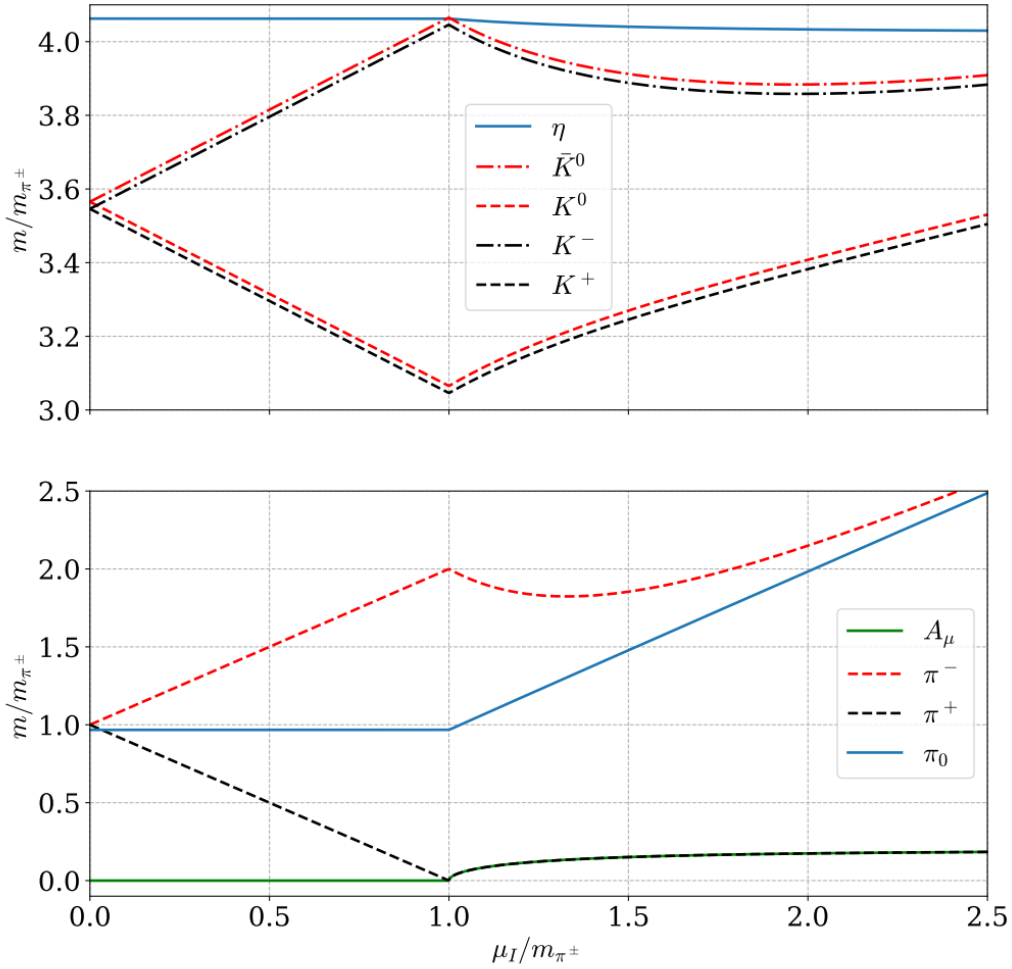


FIG.3 Quasi-particle masses at LO versus the isospin chemical potential.

arXiv: 2312.13092

The thermodynamic potential:

$$\Omega(\mu,\alpha)=\Omega_0(\mu,\alpha)+\Omega_1(\mu,\alpha)+\cdots$$

$$\Omega_0(\mu_I,\alpha)=-f^2\left[m_{\pi,0}^2\cos\alpha+B_0m_s+\frac{1}{3}\Delta m_{\rm EM}^2+\frac{1}{2}(\mu_I^2-\Delta m_{\rm EM}^2)\sin^2\alpha\right]$$

The pressure:

$$\mathcal{P}(\mu)=-\Omega(\mu,\alpha^*)$$

Pion condensed phase $\boxed{\cos\alpha_0=\frac{m_{\pi,0}^2}{\mu_I^2-\Delta m_{\rm EM}^2}=\frac{m_{\pi,0}^2}{\mu_{I,\rm eff}^2}}$

$$\boxed{\mathcal{P}=\frac{1}{2}f^2\mu_{I,\rm eff}^2\left[1-\frac{m_{\pi,0}^2}{\mu_{I,\rm eff}^2}\right]^2}$$

The charge density:

$$n(\mu)=\frac{d\mathcal{P}}{d\mu}$$

What about using the corresponding parametrization of the ground states $\Sigma_\alpha^{K^\pm}$ and $\Sigma_\alpha^{\overline{K^0}/K^0}$?

The energy density:

$$\mathcal{E}(n)=-\mathcal{P}(\mu)+n(\mu)\mu$$

$$\boxed{\mathcal{P}=\frac{1}{2}f^2\mu_{K^\pm,\rm eff}^2\left[1-\frac{m_{K^\pm,0}^2}{\mu_{K^\pm,\rm eff}^2}\right]^2}\qquad\boxed{\mathcal{P}=\frac{1}{2}f^2\mu_{K^0}^2\left[1-\frac{m_{K^0,0}^2}{\mu_{K^0}^2}\right]^2}$$

Pion condensation phase at T=0

$$\mathcal{P}_0 = \frac{1}{2} f^2 \mu_I^2 \left[1 - \frac{m_{\pi,0}^2}{\mu_I^2} \right]^2$$

$$\mathcal{P}_1^{\text{loop}}(\mu_I) = \frac{1}{2} I_0'(m_2^2) + \frac{1}{2} I_0'(m_3^2) + \frac{\Gamma(2-\epsilon)}{2\Gamma(\frac{1}{2})} \times \sum_{n=1}^{\infty} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+2-\epsilon)} \frac{(-1)^n m_{12}^{2n}}{n} I_n(m_2^2)$$

$$\Omega_1^{\text{static}}(\mu_I, \alpha) = -(l_1 + l_2)\mu_I^4 \sin^4\alpha - l_4 m_{\pi,0}^2 \mu_I^2 \cos\alpha \sin^2\alpha - (l_3 + l_4) m_{\pi,0}^4 \cos^2\alpha \,,$$

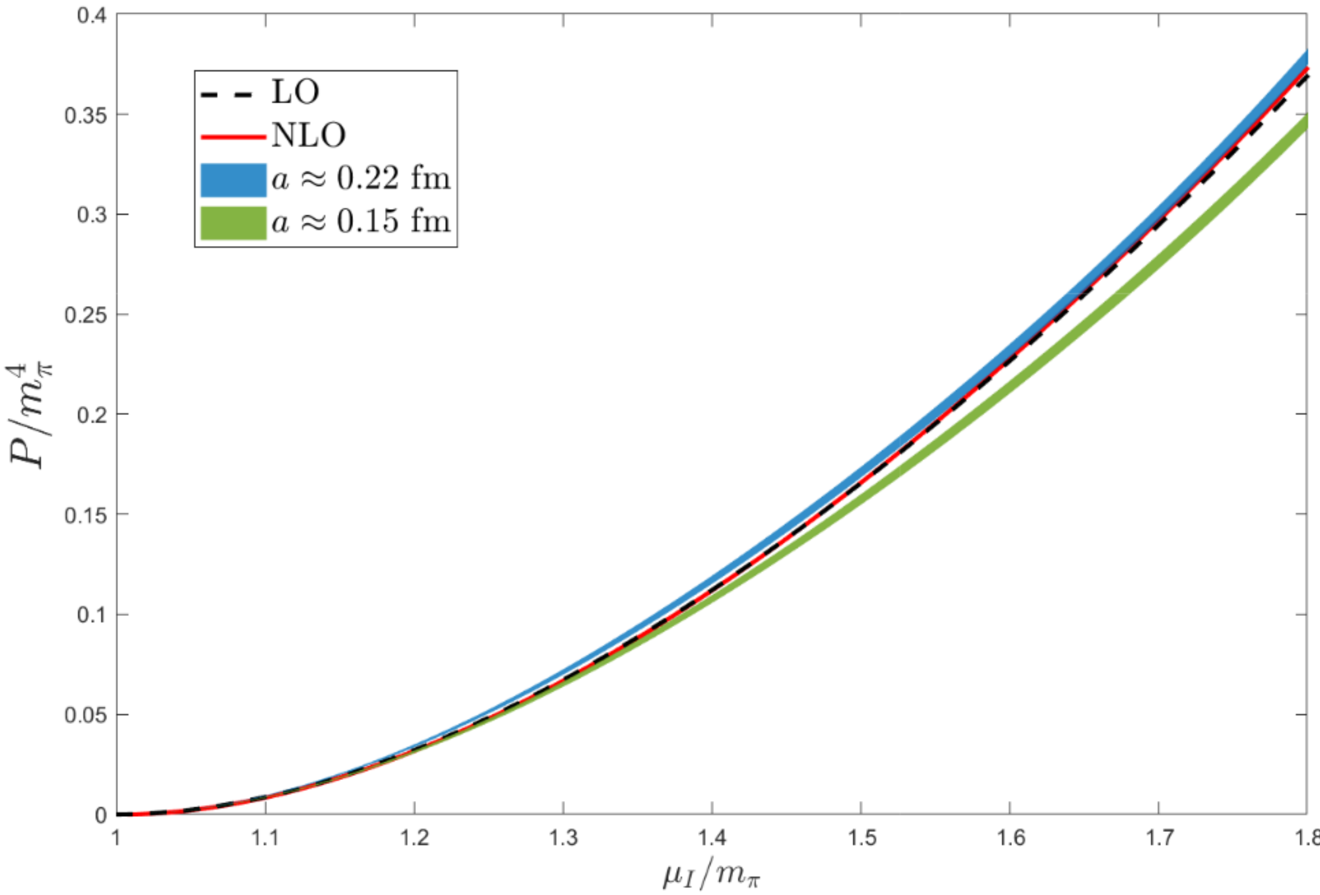
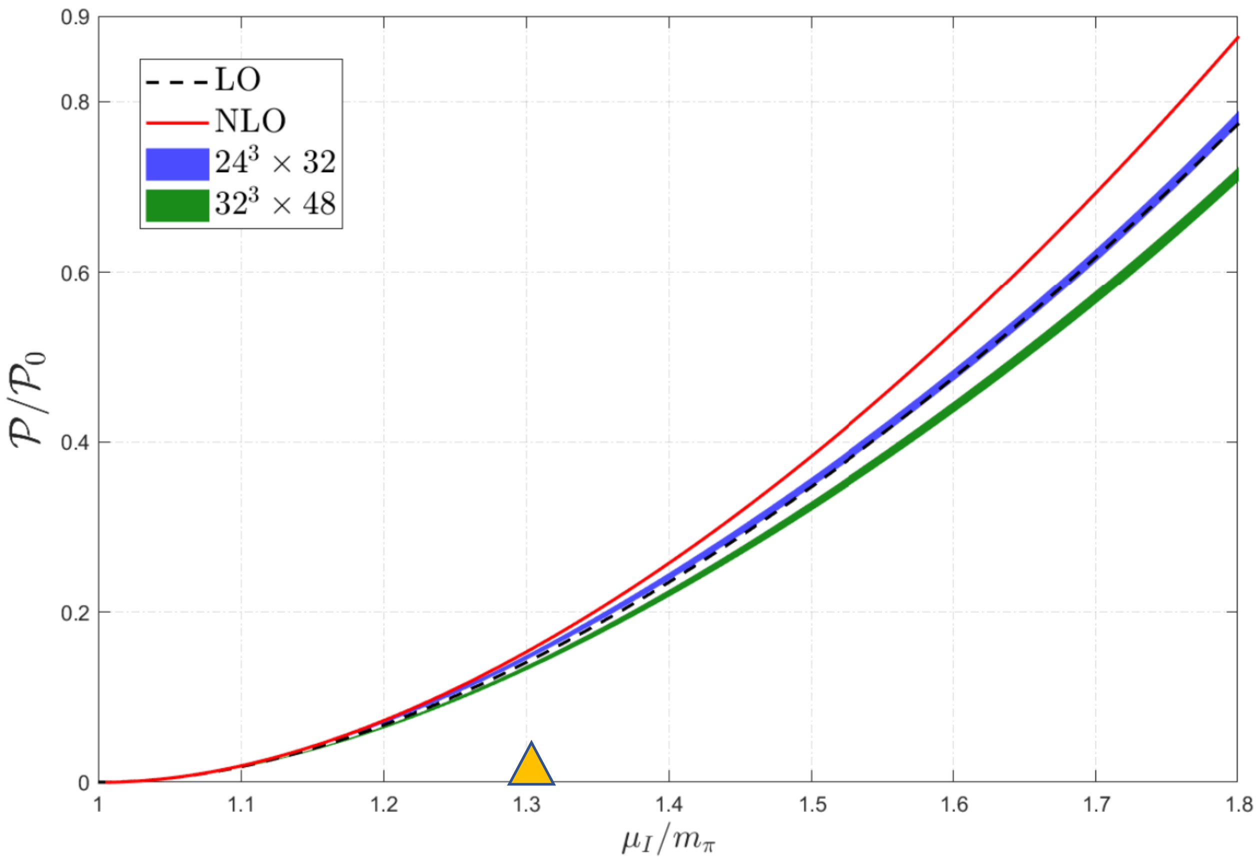


FIG.4 The normalized pressure as a function of the normalized isospin chemical potential **in 2-flavor ChiPT**.

Phys. Rev. D 109, 034022 (2024)

Lattice: B. B. Brandt et al, JHEP 07 , 055 (2023).



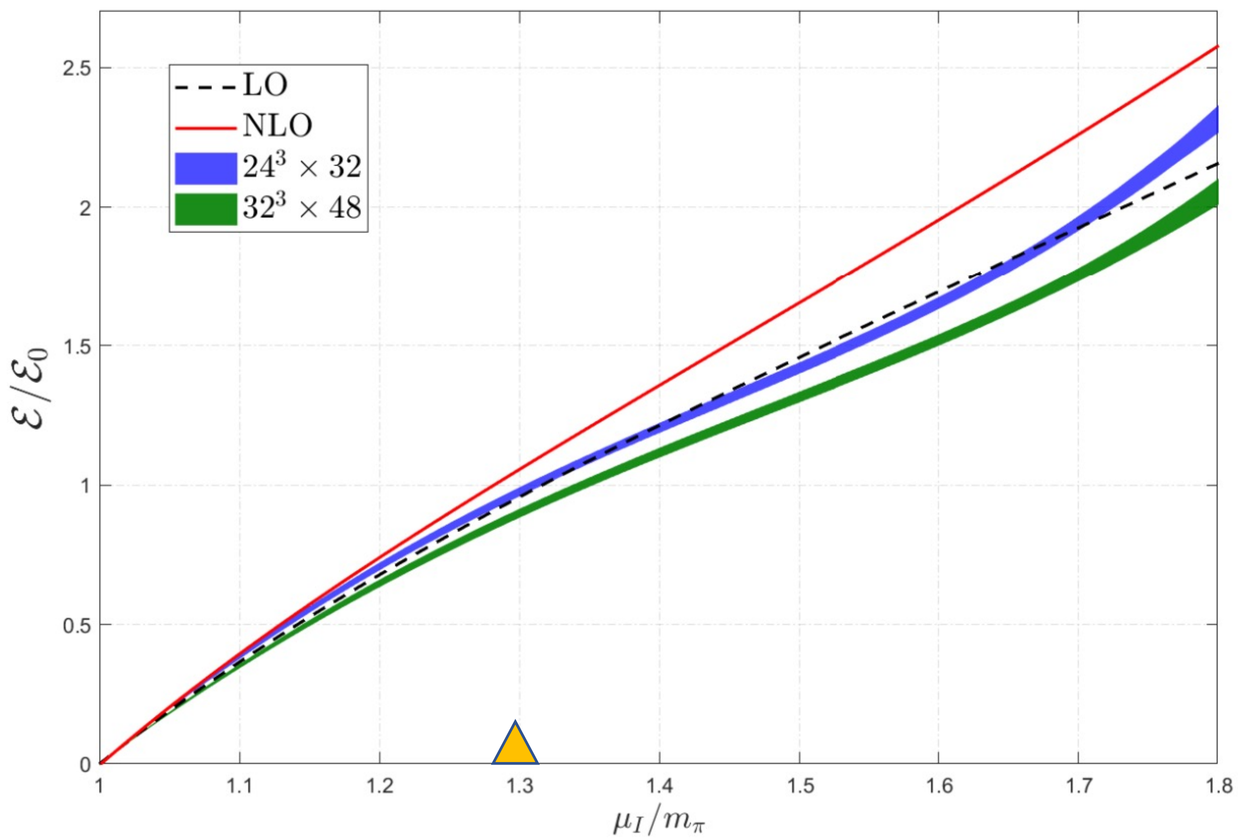


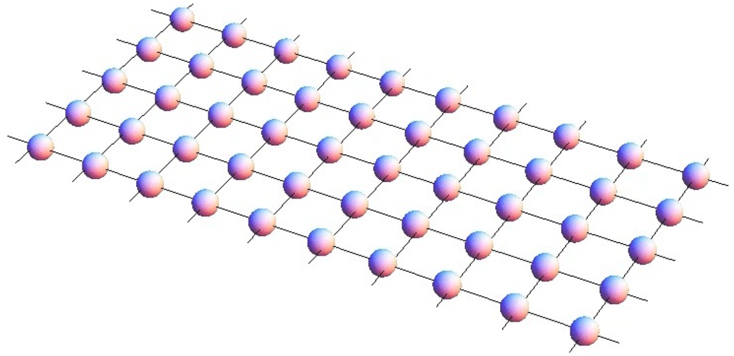
FIG.5 LO and NLO results for the normalized pressure **in 3-flavor ChiPT**.

FIG.6 LO and NLO results for the normalized energy density **in 3-flavor ChiPT**.

Lattice: B. B. Brandt et al, JHEP 07, 055 (2023).

arXiv: 2312.13092 [hep-ph]

Son's low-energy effective theory for the superfluid phonons



$$\mathcal{L}_{\text{phonon}} = \mathcal{P}(\sqrt{\nabla_\mu \phi \nabla^\mu \phi})$$

$$\nabla_\mu \phi = \partial_\mu \phi - \delta_{0\mu} \mu_I$$

D. T. Son, e-Print:hep-ph/0204199 [hep-ph]

An effective low-energy theory for the massless mode in dense QCD at finite isospin.

$$\mathcal{L} = \frac{1}{2}\partial_0 \phi^2 - \frac{1}{2} c_s^2 (\nabla \phi)^2 - c_1 (\partial_0 \phi)^3 + c_1 \partial_0 \phi (\nabla \phi)^2$$

the speed of sound or phonon speed

$$c_s = \sqrt{\frac{\mu_I^4 - m_{\pi,0}^4}{3m_{\pi,0}^4 + \mu_I^4}},$$

$$c_1 = \frac{2m_{\pi,0}^4 \mu_I}{f} \frac{1}{(3m_{\pi,0}^4 + \mu_I^4)^{\frac{3}{2}}}$$

ultrarelativistic limit

$$c_s = 1$$

nonrelativistic limit

$$c_s = \sqrt{\mu_{NR}/m_{\pi,0}}$$

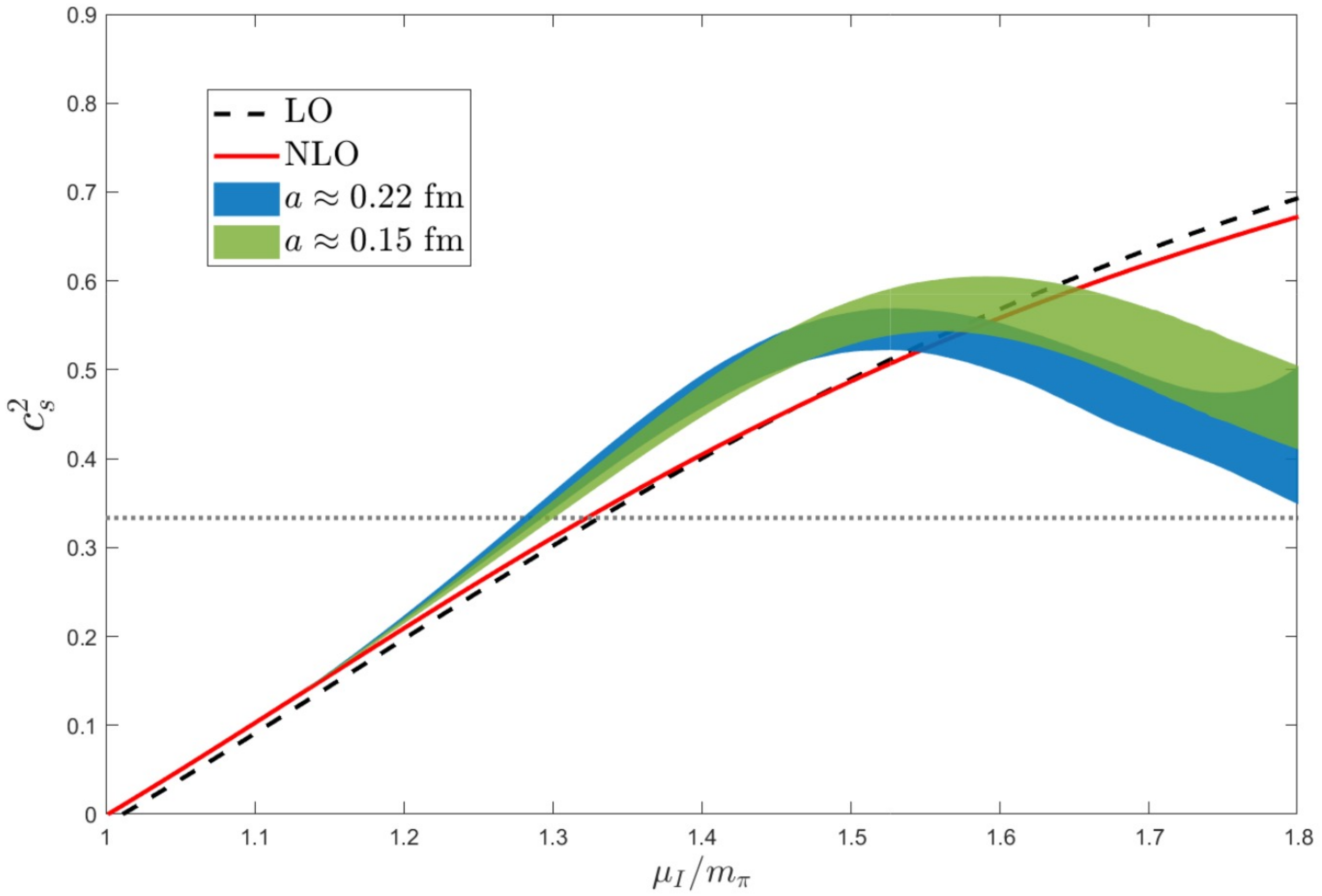


FIG.12 Speed of sound squared c_s^2 as a function of the normalized isospin chemical potential. See. main text for details.

Lattice: JHEP 07, 055 (2023).
Phys. Rev. D 109, 034022 (2024);
arXiv: 2312.13092 [hep-ph]

Finite temperature T and zero chemical potential

HADRON RESONANCE GAS MODEL

$$\mathcal{P} = \sum_h \mathcal{P}_h$$
$$= \mp \frac{8T}{(4\pi)^2} \sum_h d_h (2s+1) \int_0^\infty dp\, p^2 \log\left[1 \mp e^{-\beta\sqrt{p^2+m_h^2}}\right]$$

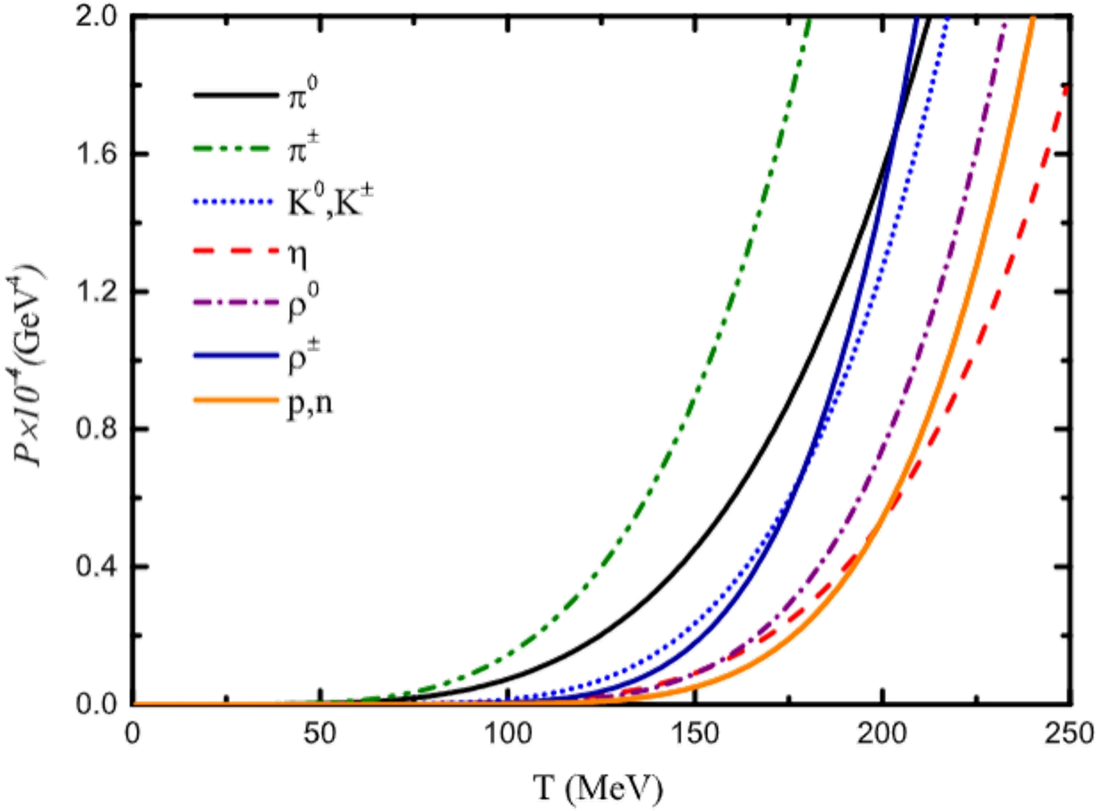


FIG.7 Individual contributions to the pressure in the HRG model.

Phys. Rev. D 107, 014010 (2023)

Finite temperature T and zero chemical potential

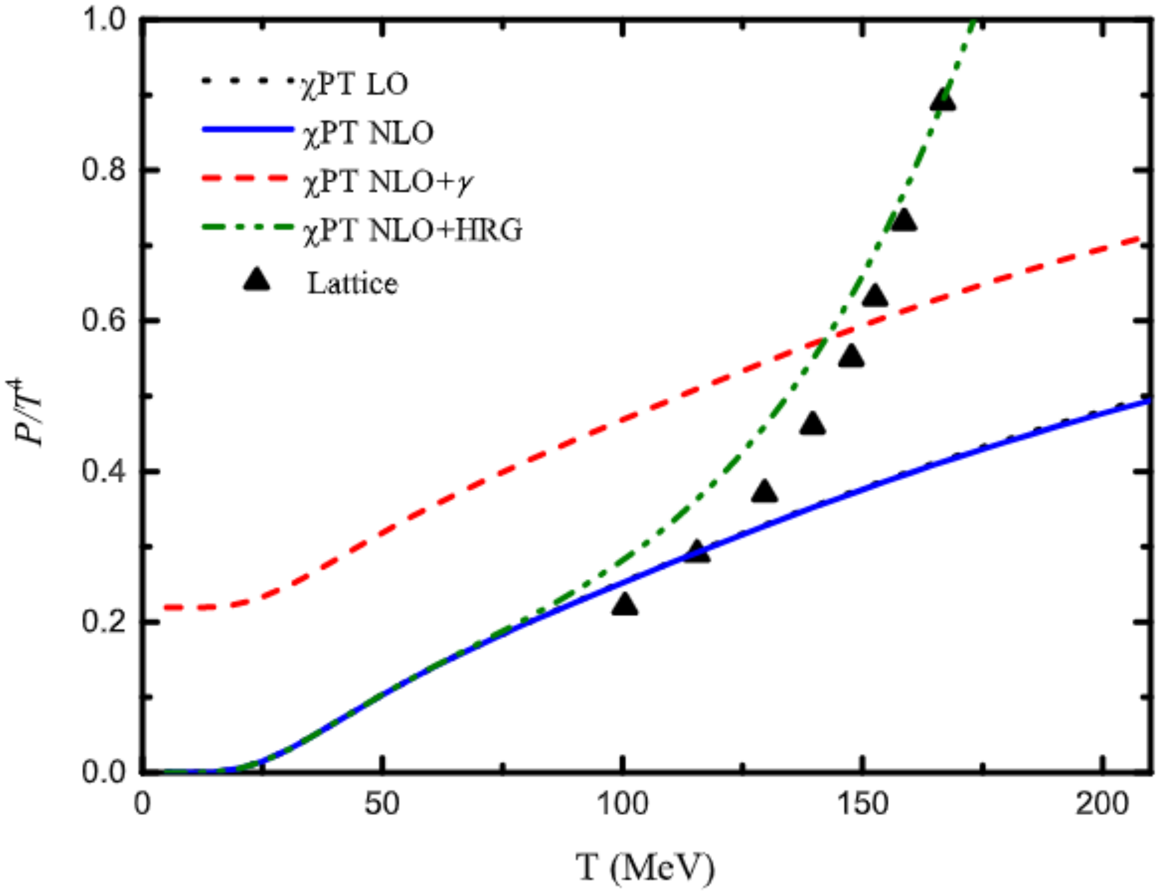


FIG.8 Pressure normalized by T^4 as a function of the temperature in MeV. See main text for details.

Phys. Rev. D 107, 014010 (2023)
Lattice:S. Borsanyi et al. (Wuppertal-Budapest Collaboration) JHEP 09 (2010) 073.

QUARK CONDENSATES

$$\langle \bar{u}u \rangle_0 = \frac{\partial V}{\partial m_u},$$

$$\langle \bar{d}d \rangle_0 = \frac{\partial V}{\partial m_d},$$

$$\langle \bar{s}s \rangle_0 = \frac{\partial V}{\partial m_s},$$

$$\langle \bar{u}u \rangle_0 + \langle \bar{d}d \rangle_0 = \langle \bar{q}q \rangle_0 = \frac{\partial V}{\partial m},$$

$$\langle \bar{u}u \rangle_0 - \langle \bar{d}d \rangle_0 = \frac{\partial V}{\partial \Delta m}.$$

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 \left[1 + \sum_a \frac{c_a}{f^2} \frac{\partial \mathcal{P}}{\partial m_a^2} \right] ,$$

$$\langle \bar{s}s \rangle = \langle \bar{s}s \rangle_0 \left[1 + \sum_a \frac{c_{sa}}{f^2} \frac{\partial \mathcal{P}}{\partial m_a^2} \right] ,$$

Finite temperature T and zero chemical potential

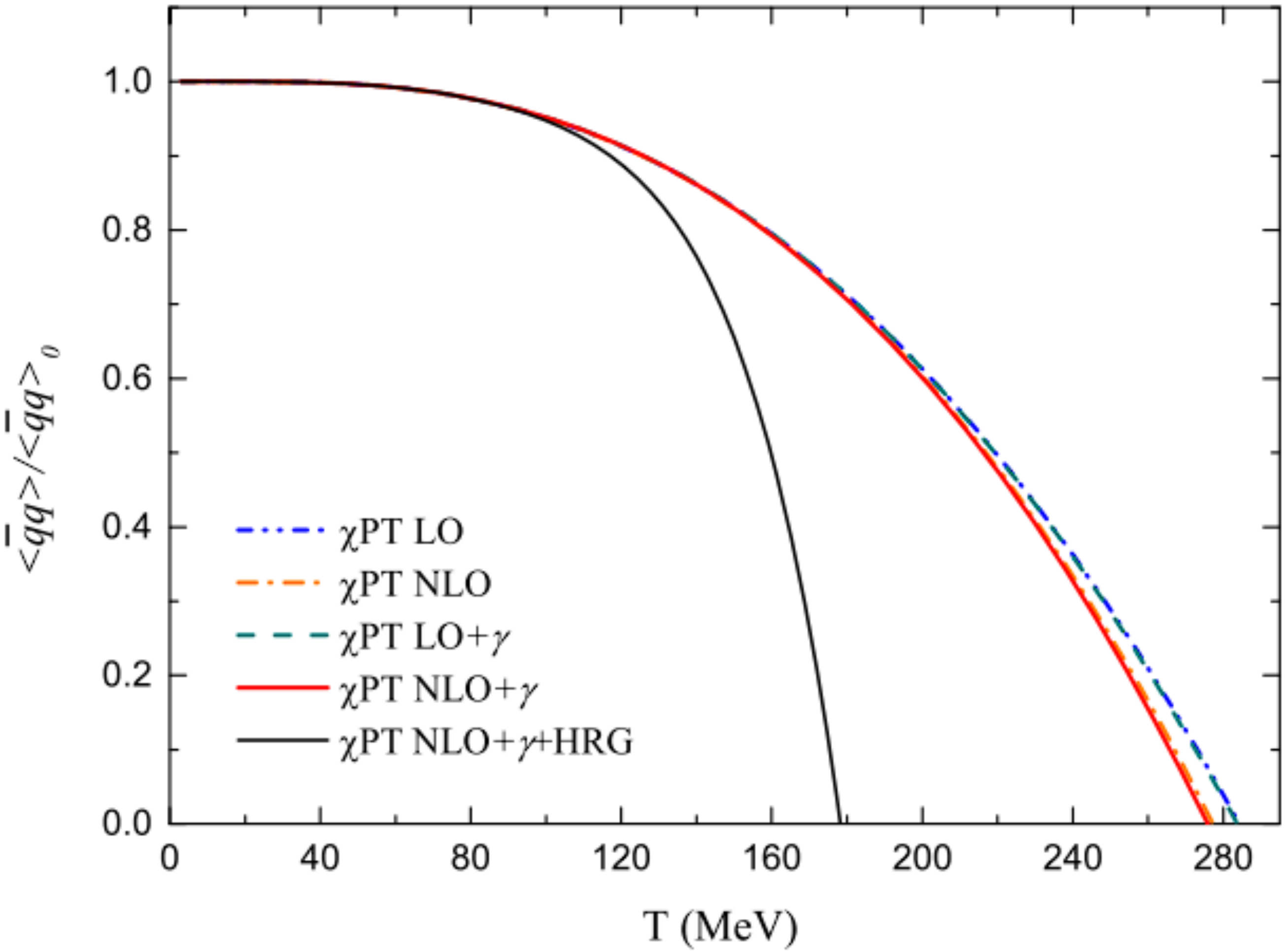


FIG.9 Normalized light quark condensate as a function of the temperature in MeV.

Phys. Rev. D 107, 014010 (2023)

Finite temperature T and zero chemical potential

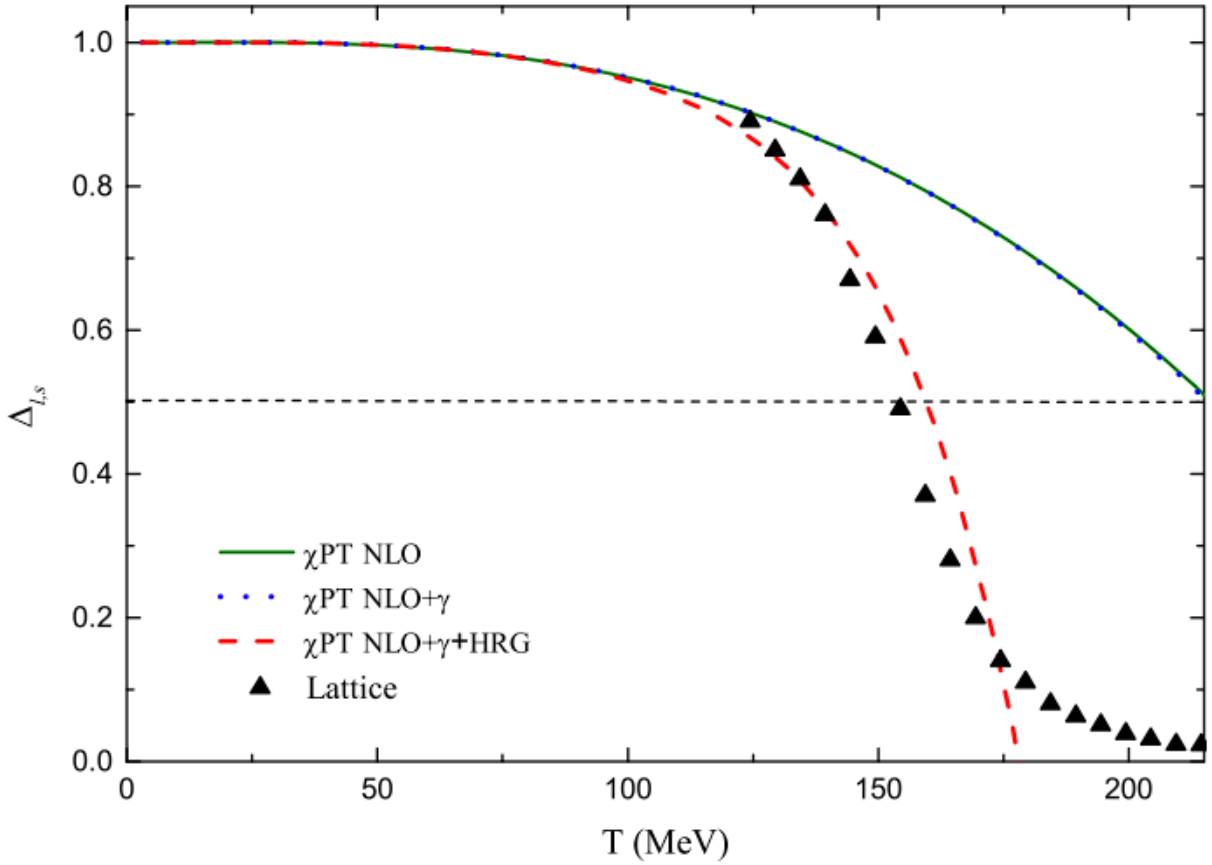


FIG.10 $\Delta_{l,s}$ as a function of the temperature in MeV.

$$\Delta_{l,s} = \frac{\langle \bar{q}q \rangle_T - \frac{m}{m_s}\langle \bar{s}s \rangle_T}{\langle \bar{q}q \rangle_0 - \frac{m}{m_s}\langle \bar{s}s \rangle_0}.$$

$$\boxed{T_{pc} = 160.1 \text{ MeV}}$$

Lattice:

$$T_{pc} = 157.3 \text{ MeV}$$

Phys. Rev. D 107, 014010 (2023)

Lattice:S. Borsanyi et al. (Wuppertal-Budapest Collaboration), JHEP 09 (2010) 073.

- Our results are compared with lattice simulations and the agreement is very good for temperatures below 170 MeV, in contrast to the results from χPT which agree with the lattice only up to $T \approx 120$ MeV.
- Our value for the chiral crossover temperature is 160.1 MeV, which compares favorably to the lattice result of 157.3 MeV.
- The spontaneous breakdown of the global internal symmetry $U(1)_{I_3}$ gives rise to a massless Goldstone boson or phonon.
- Comparing our results for the pressure and the speed of sound with recent lattice simulations with 2+1 flavors, the agreement is very good for isospin chemical potentials up to 180-200 MeV.

Qing Yu
Lecturer
Southwest U. Sci. Technol.
yuq@swust.edu.cn

Jens Oluf Andersen
Professor
Institutt for fysikk

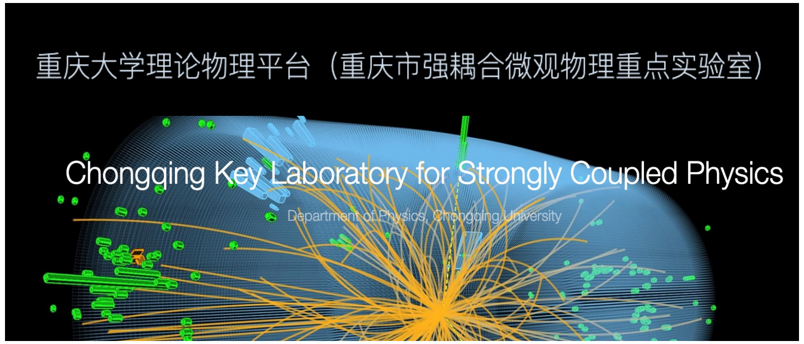
jens.andersen@ntnu.no
46478747
Realfagbygget, D5-119, Gløshaugen, Høgskoleringen 5

www.nt.ntnu.no/users/jensoa/

Hua Zhou
Lecturer
Southwest U. Sci. Technol.
zhouhua@cqu.edu.cn

西南科技大学
Southwest University of Science and Technology

NTNU
Norwegian University of Science and Technology



Thank You!