国科大杭州高等研究院
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Insights into Neutron Star Equation of State by Machine Learning

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Guo L J, Xiong J Y, Ma Y, et al. Insights into neutron star equation of state by machine learning[J]. The Astrophysical Journal, 2024, 965(1): 47.

In preparation for 23-par AI model

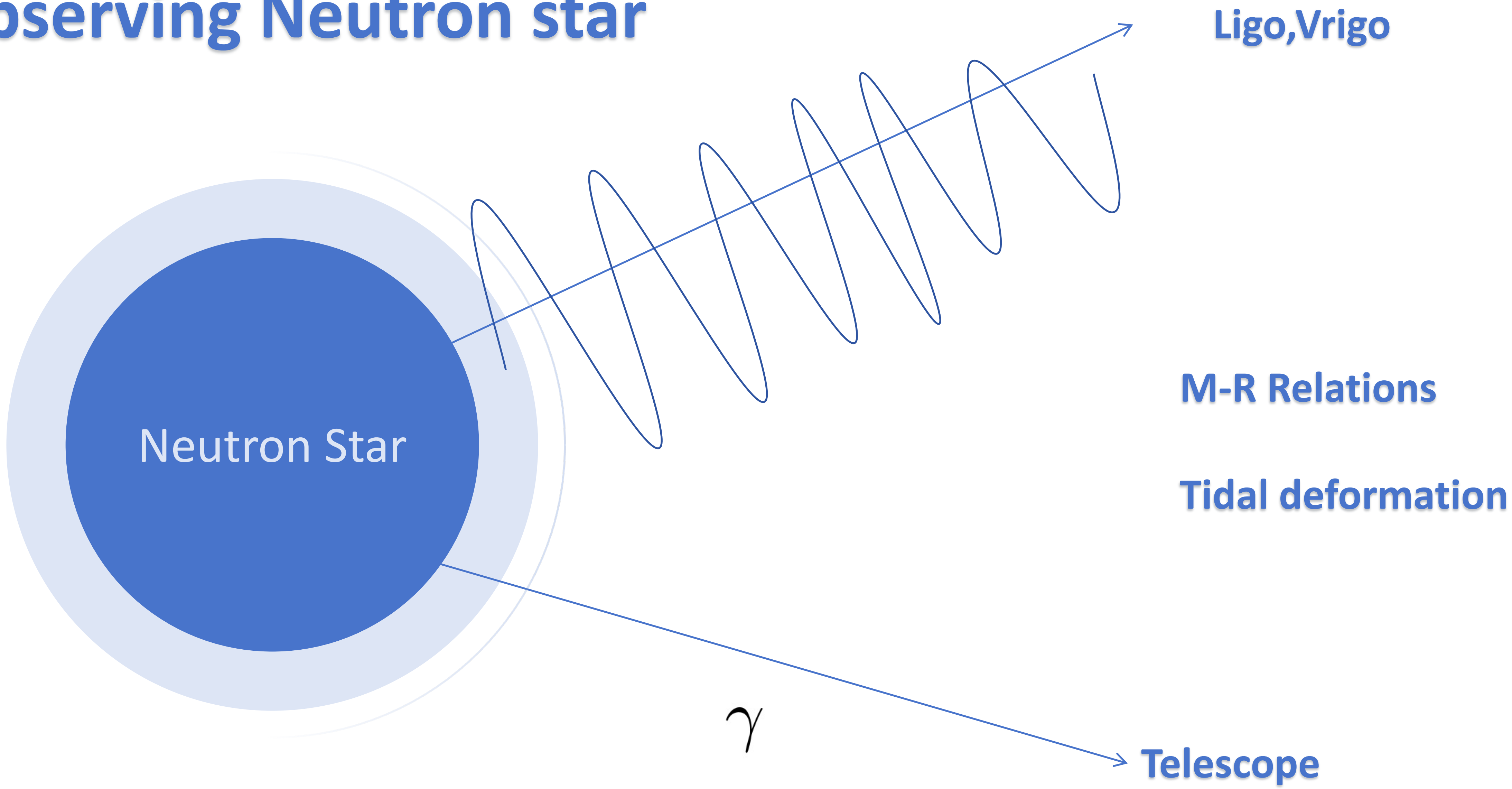
Artificial Intelligence

AI for Everything

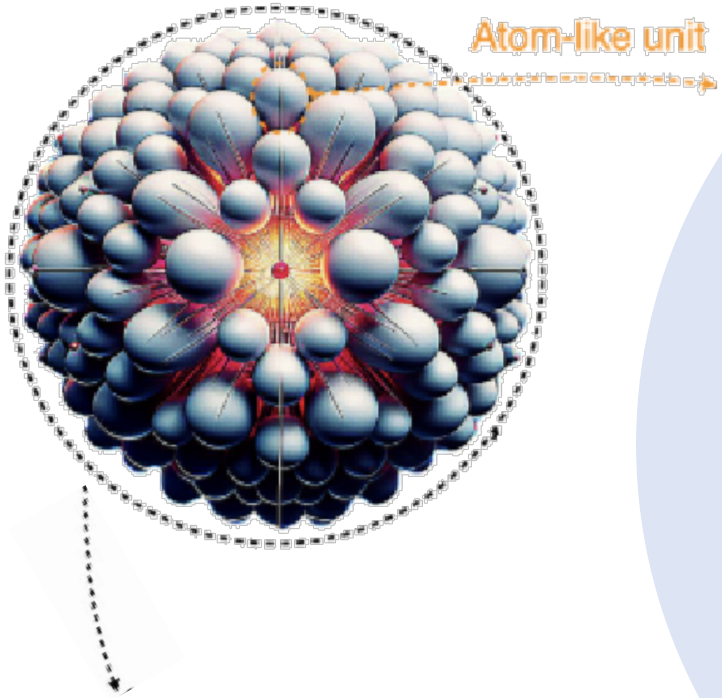
Physics	2024 Nobel Prize
Chemistry	2024 Nobel Prize
AutoDriving	End-to-End training
ChatGPT	World model

……

Observing Neutron star



From nuclei to white dwarf



White Dwarf Star

Nuclei

$$n_0 \to 0.155 \pm 0.050 \left(\mathrm{fm}^{-3}\right)$$
$$E_0(n_0) \to -15.0 \pm 1.0(\mathrm{MeV})$$
$$E_{sym}(n_0) \to 30.9 \pm 1.9(\mathrm{MeV})$$
$$K_0 \to 230 \pm 30(\mathrm{MeV})$$
$$L_0 \to 52.5 \pm 17.5(\mathrm{MeV})$$
$$J_0 \to -700 \pm 500(\mathrm{MeV})$$

White Dwarf Structure and Binary Inspiral Gravitational Waves from Quantum Hadrodynamics

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ABSTRACT

White dwarf, one of the compact objects in the universe, is not only pivotal for astrophysical inquiries but also furnishes a dynamic arena for nuclear physics exploration. In this work, we extend the relativistic mean field approach using a Walecka-type quantum hadrodynamics model to encapsulate the intricate structure of white dwarfs. This methodological advancement facilitates a cohesive analysis of white dwarfs, neutron stars, and the nuclear pasta within a unified theoretical framework. By meticulously calibrating the model parameters to nuclear matter properties, we successfully replicate the structures of various nuclei, such as ^{4}He, isotopes of C, and ^{16}O. Subsequently, we predict the properties of white dwarfs composed of atom-like units and the gravitational waves stemming from binary white dwarf inspirals incorporating tidal deformability contributions up to the 2.5 post-Newtonian order. These results shed light on the structure of white dwarfs and offer more clues for future gravitational wave detection.

arxiv.org/pdf/2410.06088

The Journey of a Model

Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{RMF}} = & \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu} - e\gamma_\mu\frac{1-\tau_3}{2}A^\mu\right]\psi \\
& + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
& - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\
& - \frac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},
\end{aligned}
$$

where

$$
\begin{aligned}
W^{\mu\nu\nu} &= \partial^\mu\omega^\nu - \partial^\nu\omega^\mu , \\
R^{a,\mu\nu} &= \partial^\mu\rho^{a\nu} - \partial^\nu\rho^{a\mu} + g_\rho\epsilon^{abc}\rho^{b\mu}\rho^{c\nu} , \\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu .
\end{aligned}
$$

RMF

EC

$$
\begin{aligned}
m_\sigma^2\sigma + g_2\sigma^2 + g_3\sigma^3 &= -g_\sigma(\rho_{n,s} + \rho_{p,s}), \\
m_\omega^2\omega + c_3\omega^3 &= g_\omega(\rho_p + \rho_n), \\
m_\rho^2\rho &= g_\rho(\rho_p - \rho_n),
\end{aligned}
$$

$$
\begin{aligned}
\rho_{n(p),s} &= \frac{m_N^{*3}}{\pi^2}\int_0^{t_{n(p)}} \mathrm{d}x \frac{x^2}{\sqrt{1+x^2}} \\
&= \frac{m_N^{*3}}{\pi^2}\left[\frac{1}{2}\left(t_{n(p)}\sqrt{1+t_{n(p)}^2} - \mathrm{arcsinh}t_{n(p)}\right)\right]
\end{aligned}
$$

EOS:

$$
\begin{aligned}
\mathcal{E} = & \frac{m_N^{*4}}{8\pi^2}\sum_{i=n,p}\left[t_i\sqrt{t_i^2+1}\left(1+2t_i^2\right) - \mathrm{arcsinh}t_i\right] + \\
& g_\omega\omega(\rho_n+\rho_p) + g_\rho\rho(\rho_p-\rho_n) + \\
& \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \\
& \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{4}c_3\omega^4 - \frac{1}{2}m_\rho^2\rho^2 .
\end{aligned}
$$

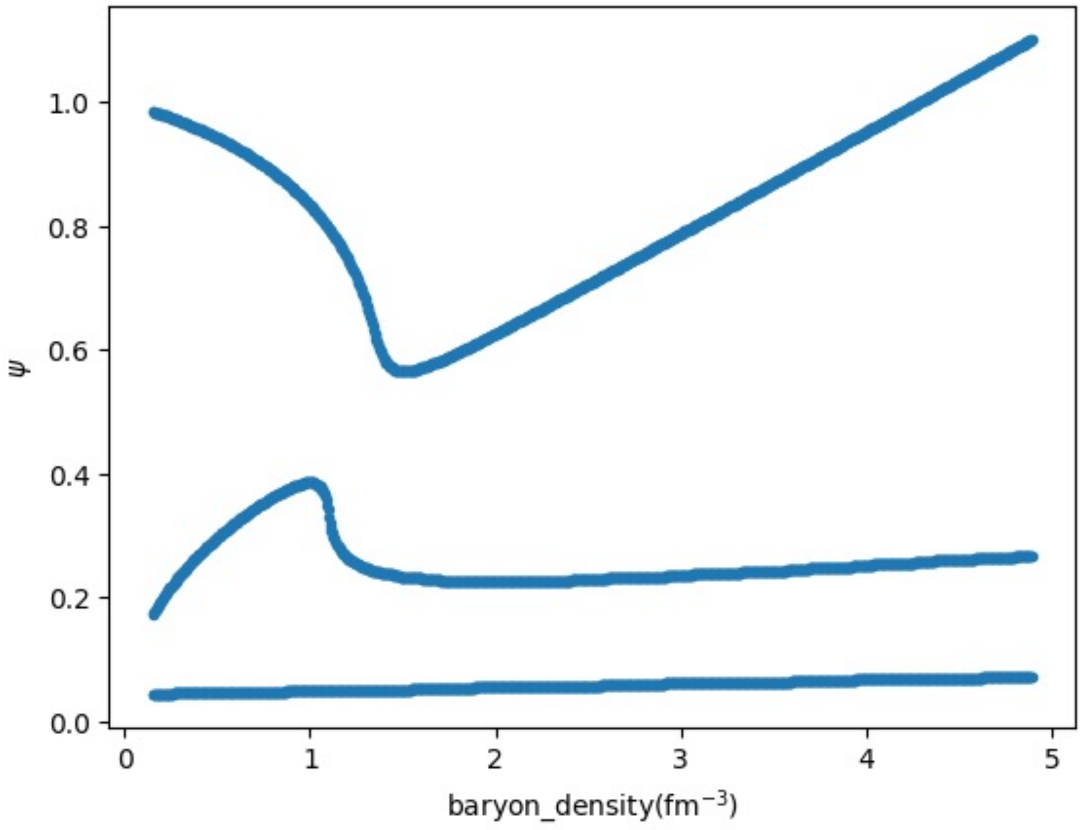
$$
P = -\mathcal{E} + \rho_N\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}\rho_N}
$$

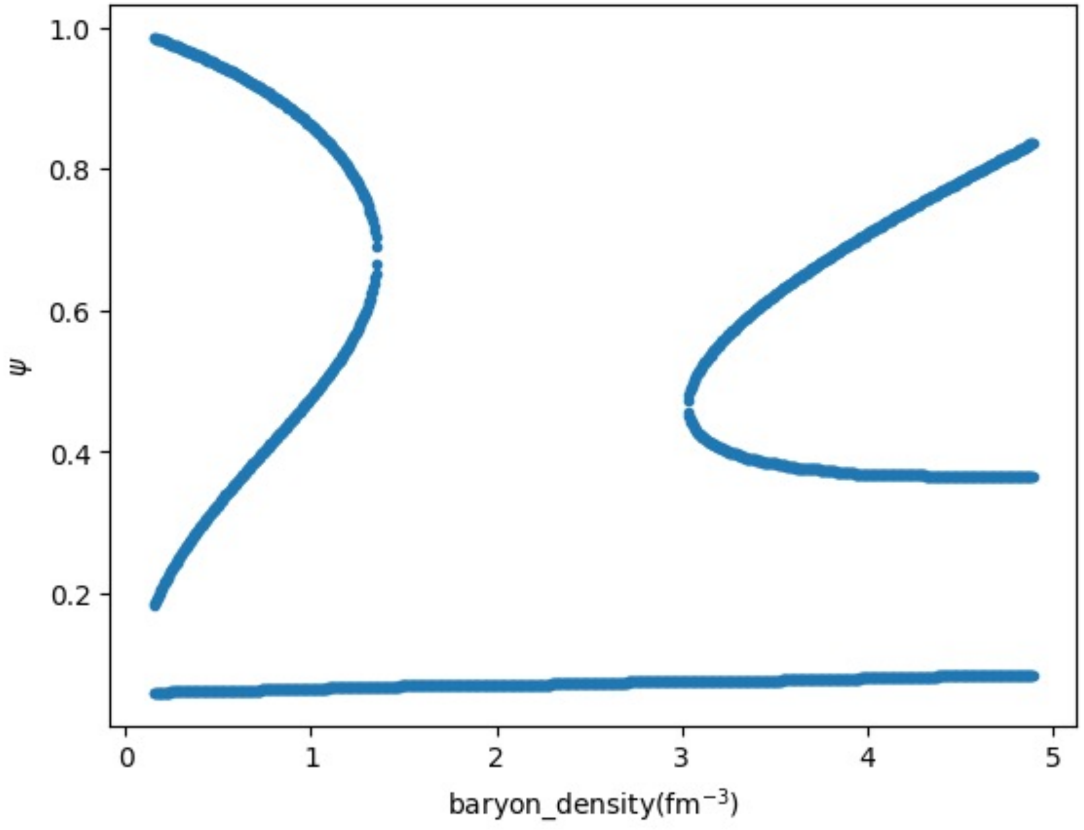
TOV:

$$
\begin{aligned}
\frac{dP}{dr} &= \left(\rho\left(1+\epsilon\right)+p\right)\frac{m+4\pi r^3 p}{r\left(r-2m\right)}, \\
\frac{dm(r)}{dr} &= 4\pi\rho\left(1+\epsilon\right)r^2.
\end{aligned}
$$

The Journey of a Model

Multi solution problem





The Journey of a Model

Coupling coefficients

Lagrangian → EOS → TOV

Coupling coefficients:

$g_\sigma \quad g_\omega \quad g_\rho$

$g_3 \quad c_3$

...

Nuclear properties:

incompressibility coefficient

symmetry energy

Binding energy

...

M—R relations:

GW107817
GRB 200415A
PSR-J0740+6620
PSR-J0030+045

...

Bayesian inference

$$g_\sigma \quad g_\omega \quad g_\rho$$
$$g_3 \quad c_3$$

$$P(H \mid E) = \frac{P(E \mid H) \cdot P(H)}{P(E)}$$

$$p(\theta \mid \mathbf{X}, \alpha) = \frac{p(\mathbf{X} \mid \theta) p(\theta \mid \alpha)}{p(\mathbf{X} \mid \alpha)} \propto p(\mathbf{X} \mid \theta) p(\theta \mid \alpha)$$

Drischler, C., Furnstahl, R. J., Melendez, J. A., & Phillips, D. R. (2020). How well do we know the neutron-matter equation of state at the densities inside neutron stars? A Bayesian approach with correlated uncertainties. Physical Review Letters, 125(20), 202702.

Markov Chain Monte Carlo, MCMC
Variational Inference

emcee、PyMC3、ArviZ

Jiang, J.-L., Ecker, C., & Rezzolla, L. 2023, Astrophys. J., 949, 11, doi: 10.3847/1538-4357/acc4be

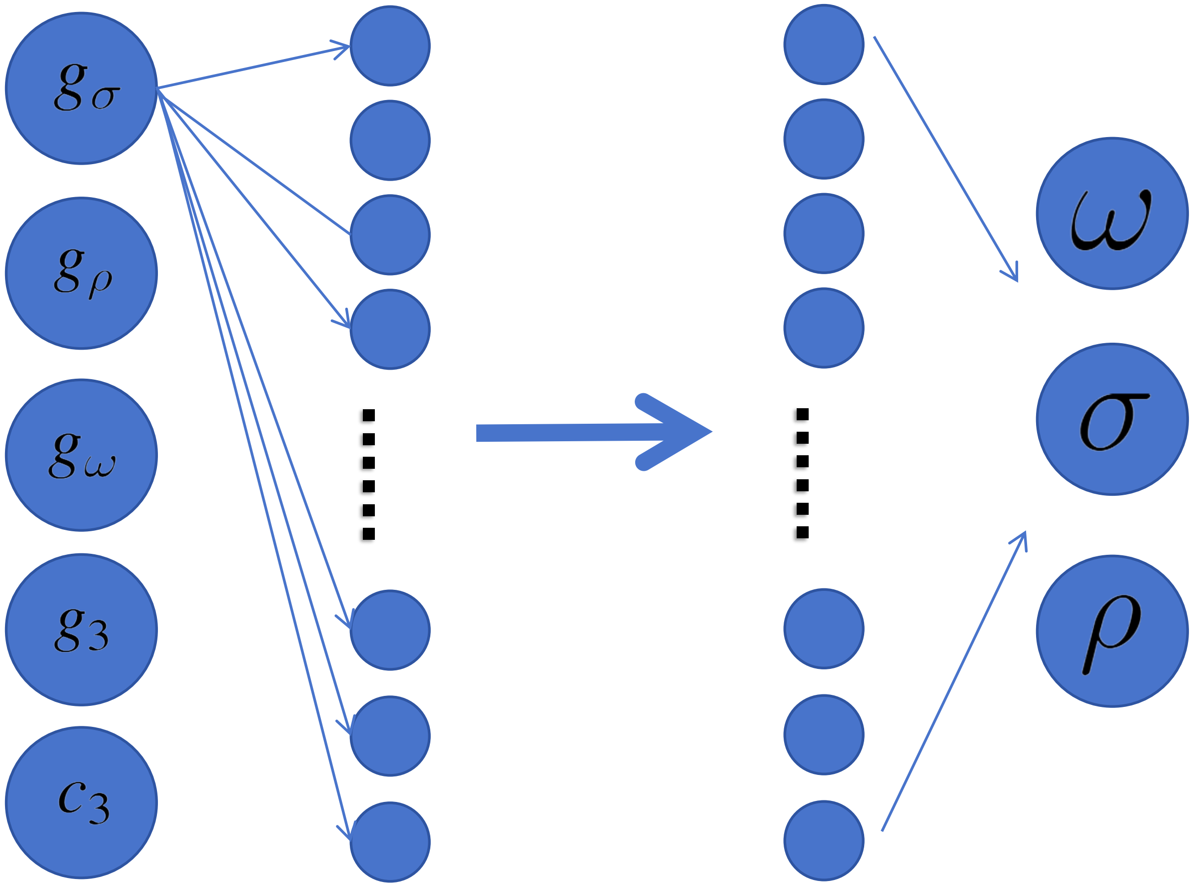
Han, M.-Z., Jiang, J.-L., Tang, S.-P., & Fan, Y.-Z. 2021, Astrophys. J., 919, 11, doi: 10.3847/1538-4357/ac11f8

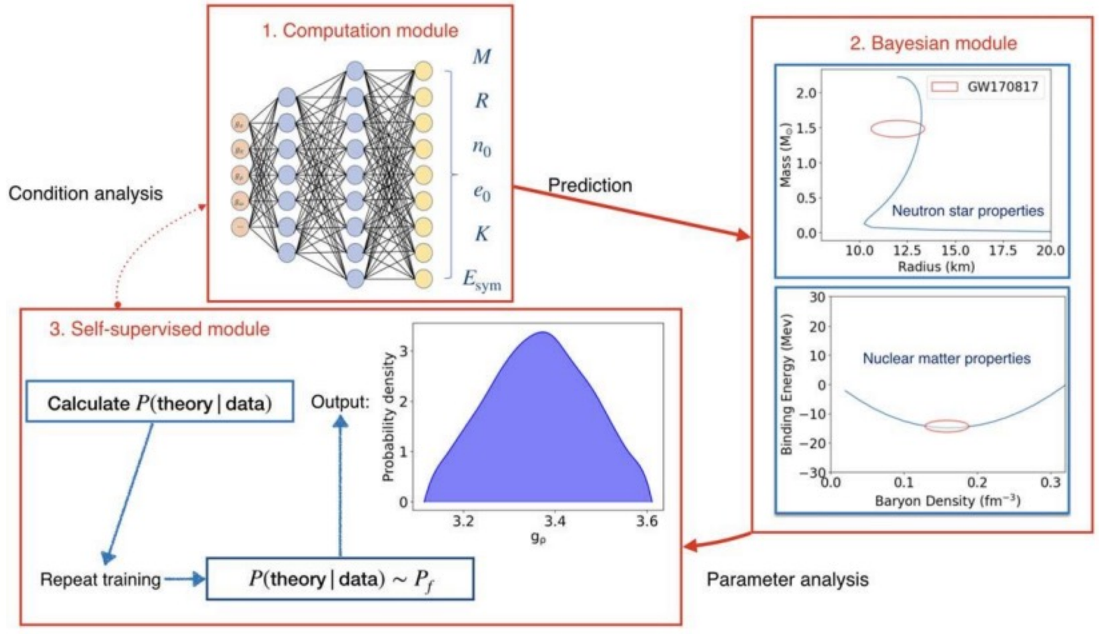
The Journey of a Model

Machine Learning Approach

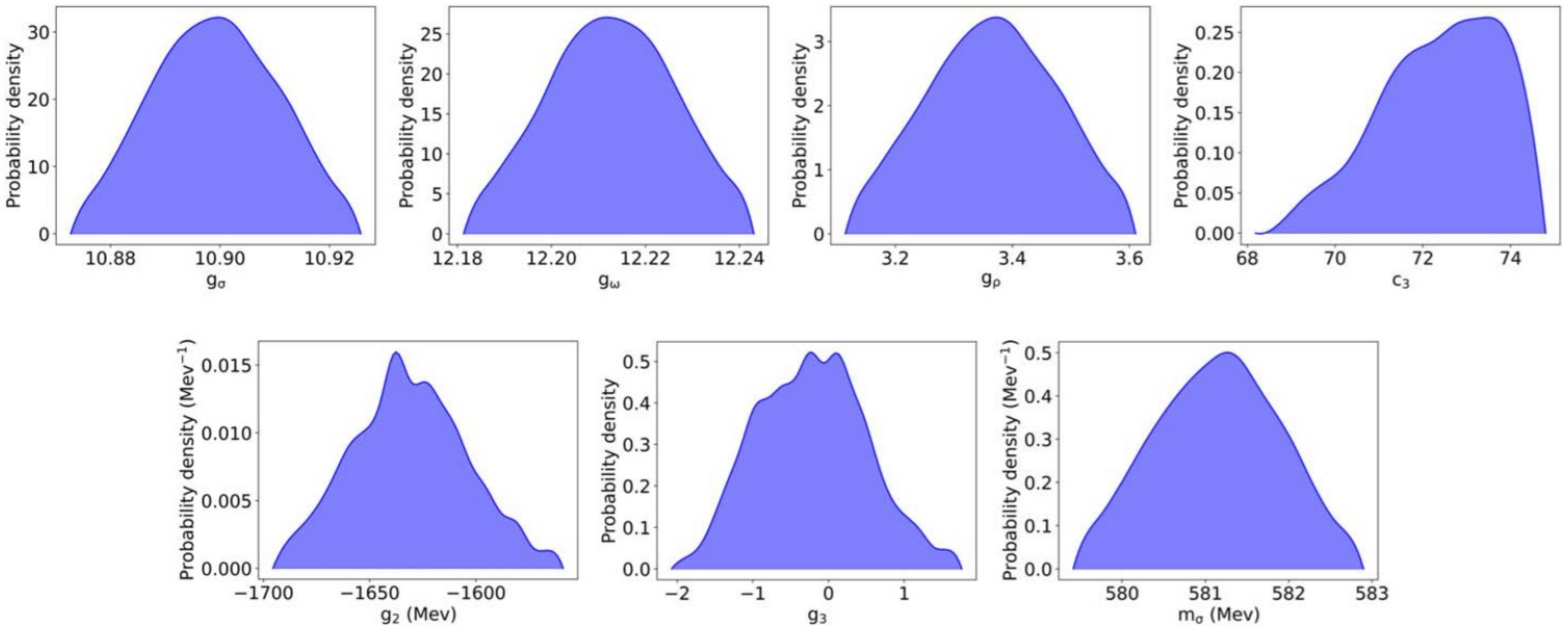
$$f_{\mathrm{loss},\sigma} = \left[m_\sigma^2\sigma + g_2\sigma^2 + g_3\sigma^3 + g_\sigma(\rho_{n,s} + \rho_{p,s})\right]^2$$

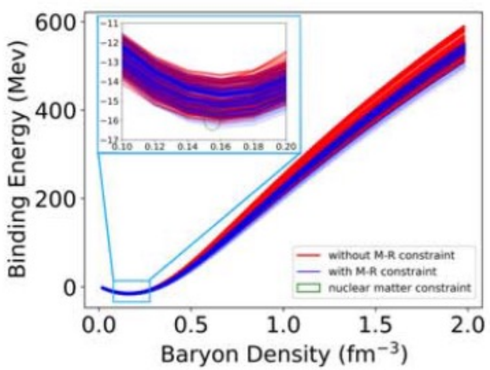
$$f_{\mathrm{loss},\omega} = \left[m_\omega^2\omega + c_3\omega^3 - g_\omega(\rho_p + \rho_n)\right]^2$$



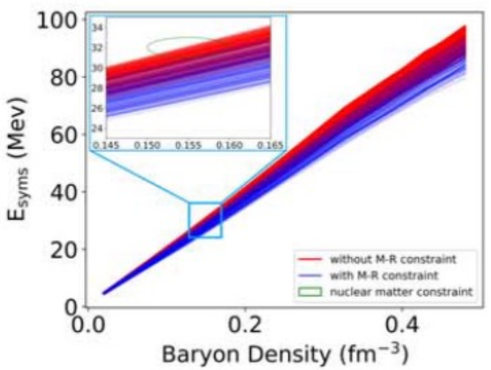


Bayesian Parameter Estimation

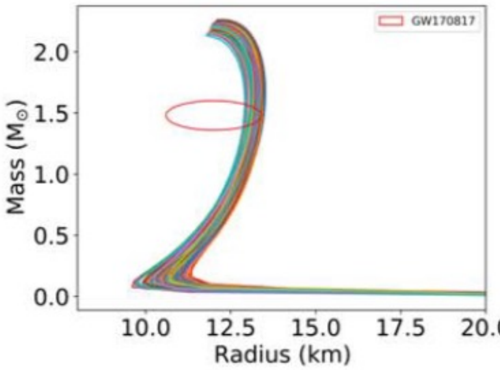




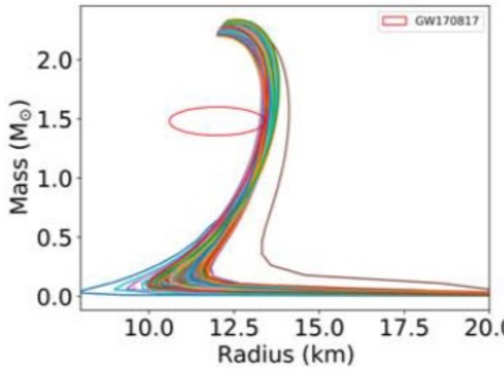
(a) e_0 with NM property constraints.



(b) E_{sym} with NM property constraints.



(c) MR relations with MR constraints.



(d) MR relations without MR constraints.

21-parameters-model for AI

$$\mathcal{L}_N = \bar{\Psi}(i\gamma^\mu\partial_\mu - m_N)\Psi$$

$$\mathcal{L}_\sigma = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - \frac{1}{2}g_4\sigma^2\omega^\mu\omega_\mu - \frac{1}{2}g_5\sigma^2\mathrm{Tr}(\vec{\rho}^\mu\cdot\vec{\rho}_\mu)$$

$$\mathcal{L}_\omega = -\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu + \frac{1}{4}c_3(\omega^\mu\omega_\mu)^2 + \frac{1}{2}c_4\omega^\mu\omega_\mu\mathrm{Tr}(\vec{\rho}^\mu\cdot\vec{\rho}_\mu)$$

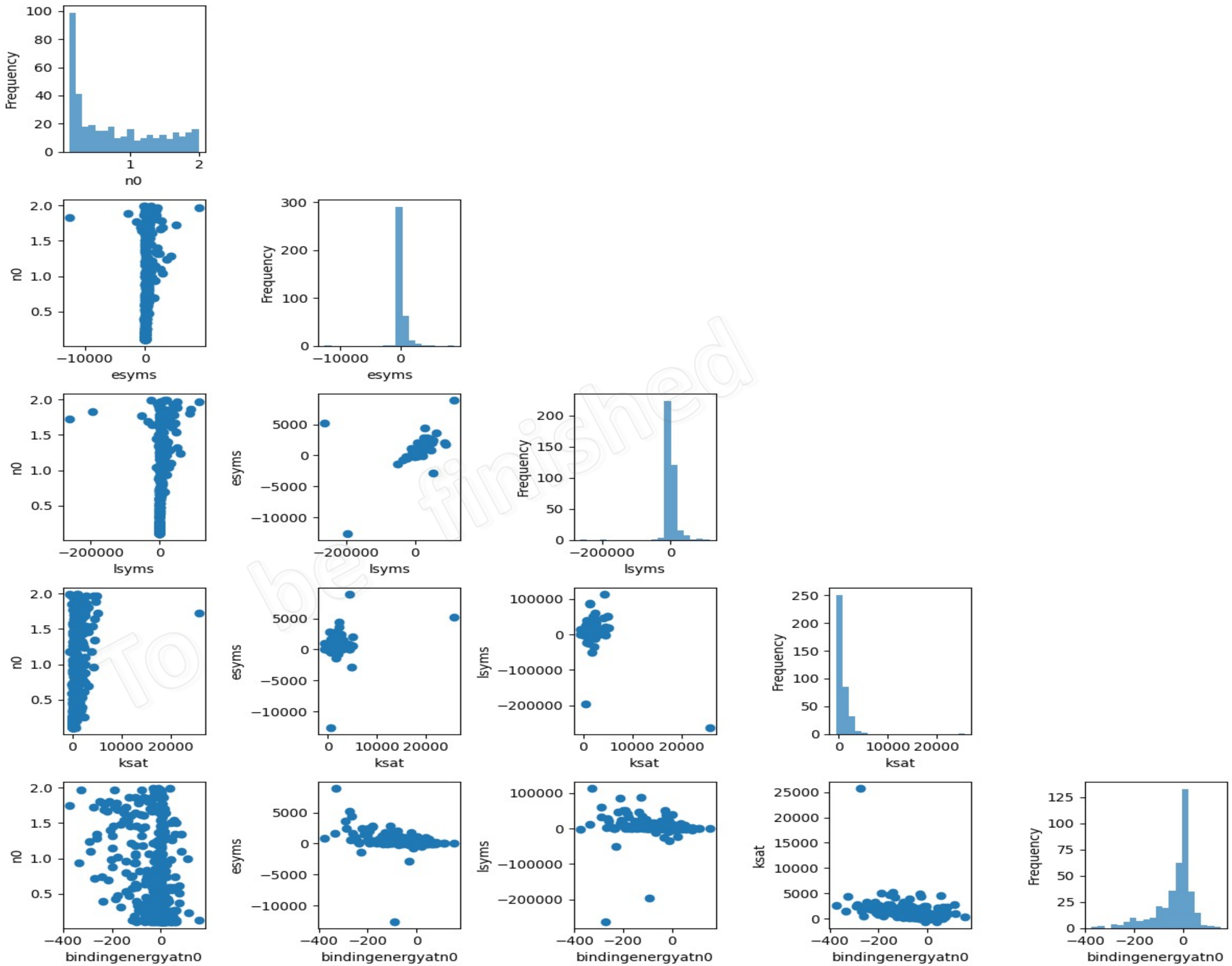
$$\mathcal{L}_\rho = -\frac{1}{4}\mathrm{Tr}(P^{\mu\nu}\cdot P_{\mu\nu}) + \frac{1}{2}m_\rho^2\mathrm{Tr}(\vec{\rho}^\mu\cdot\vec{\rho}_\mu) + \frac{1}{4}d_3\mathrm{Tr}[(\vec{\rho}^\mu\cdot\vec{\rho}_\mu)^2]$$

$$\mathcal{L}_{a_0} = \frac{1}{2}\mathrm{Tr}(\partial_\mu\vec{a_0}\partial^\mu\vec{a_0} - m_{a_0}^2\vec{a_0}\vec{a_0}) + \frac{1}{4}b_3\mathrm{Tr}[(\vec{a_0}\vec{a_0})^2] + \frac{1}{2}b_4\mathrm{Tr}[(\vec{a_0}\vec{a_0})(\vec{\rho^\mu}\vec{\rho_\mu})]$$
$$+\frac{1}{2}b_6\mathrm{Tr}[\vec{a_0}\vec{a_0}]\sigma^2 + \frac{1}{2}b_7\mathrm{Tr}[\vec{a_0}\vec{a_0}]\omega^\mu\omega_\mu + b_8\mathrm{Tr}[\vec{a_0}\vec{\rho^\mu}]\omega_\mu\sigma$$

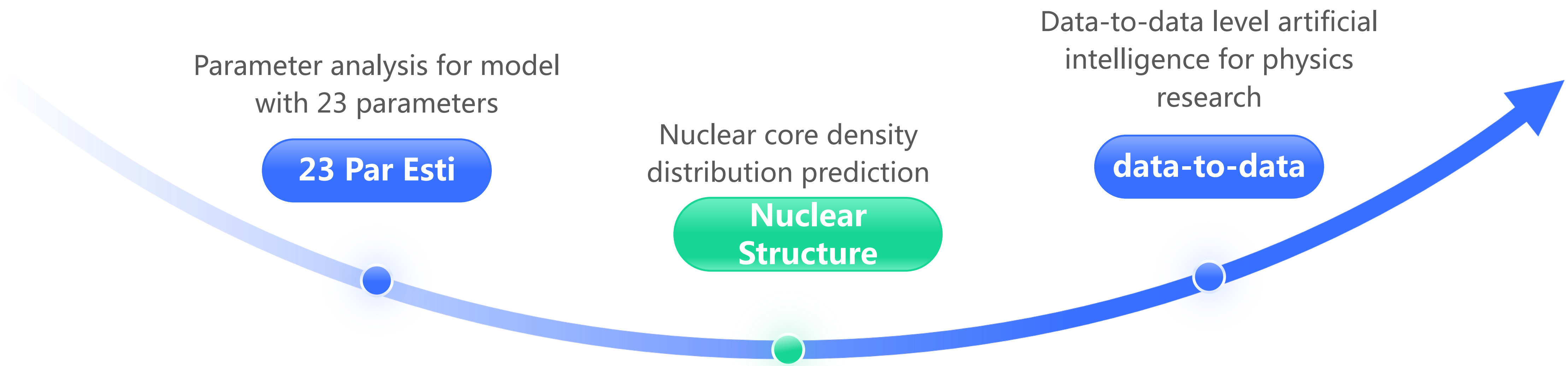
$$\mathcal{L}_I = \bar{\Psi}(-g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma^\mu\vec{\rho}_\mu - g_{a_0}\vec{a_0})\Psi$$

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix}, \quad \vec{\rho}^\mu = \rho_i^\mu\tau_i, \quad \vec{a}^\mu = a_i^\mu\tau_i, \quad \Omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu, \quad P^{\mu\nu} = \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu$$

Preliminary results



AI For Physics



Outlook

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Thank you