第九届手征有效场论研讨会

Baryons as Vortexes on the η' Domain Wall

arXiv:2310.16438 [hep-th] or JHEP 05 (2024) 270

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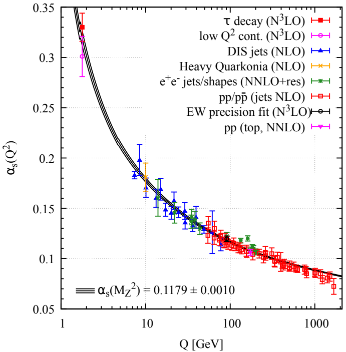
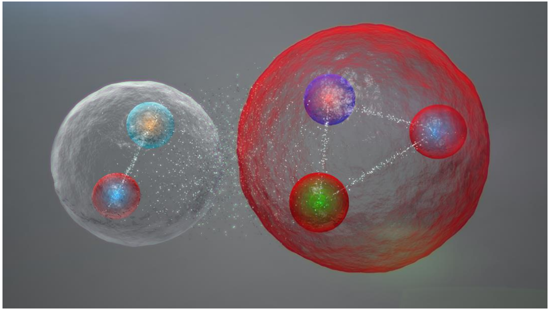
2024 年 10 月 21 日

Quantum Chromodynamics (QCD)

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f \right) \psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}.$$





* Runing coupling of QCD.

* At low-energy region, Mesons and Baryons are the basic degrees of freedom of QCD.

[P.D.G. and CERN]

Baryon as Topological Soliton

* Skyrme's Model with $U(x) = \exp\left(2i\pi^a T^a / f_\pi\right) \in SU(N_f)$: [T.H.R.Skyrme]

$$\mathcal{L}_{\rm Skyr} = \frac{f_\pi^2}{4}\,{\rm tr}(\partial^\mu U^\dagger \partial_\mu U) + \frac{1}{32e^2}\,{\rm tr}([U^\dagger\partial_\mu U, U^\dagger\partial_\nu U][U^\dagger\partial^\mu U, U^\dagger\partial^\nu U]).$$

* There exists static chiral field $U(\mathbf{x}) : \mathbb{R}^3 \to SU(N_f)$, $N_f \geq 2$ possessing non-trivial topological soliton structure:

$$\pi_3(SU(N_f)) = \mathbb{Z}, \quad \Longrightarrow \quad B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\,{\rm tr}(U^\dagger\partial_\nu U U^\dagger\partial_\alpha U U^\dagger\partial_\beta U).$$

The skyrmion is regarded as baryon by the same large N_c behavior.

* For $N_f = 1$ case, baryon has spin $\frac{N_c}{2}$, No Nambu-Goldstone boson exists.

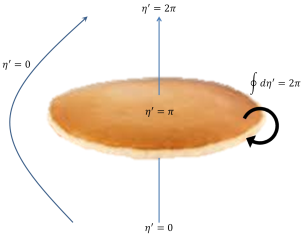
* Even in the large N_c limit, η' becomes massless but $\pi_3(U(N_f = 1)) = 0$, still No non-trivial topological soliton structure exists.

Baryon as Hall Droplet on the η' Domain Wall

* Level/Rank duality: For $N_f = 1$, the domain wall supports a Level -1 and Rank N_c Chern-Simons (CS) theory about gluon $\bar{a} \in \mathfrak{su}(N_c)$, this can be dual to an abelian CS theory $a \in \mathfrak{u}(1)$:

$$SU(N_c)_{-1}, \quad \longleftrightarrow \quad U(1)_{N_c}.$$

$$\frac{-1}{4\pi}\int_{\mathcal{M}_3} \mathrm{Tr}\left(\bar{a}\wedge d\bar{a} - \mathrm{i}\frac{2}{3}\bar{a}^3\right), \quad \longleftrightarrow \quad \int_{\mathcal{M}_3}\frac{N_c}{4\pi} a\wedge \mathrm{d}a.$$



spin $\sim \frac{N_c}{2}$. $B = \frac{1}{2\pi}\int \partial_\theta \varpi$

* If a boundary is imposed, the sheet is in fact a droplet realizing the fractional quantum hall effect with filling number $1/N$, A Quantum Hall Droplet!

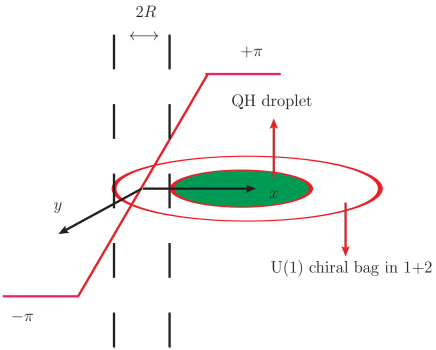
* The droplet with one baryon number carried by chiral mode has spin $N_c/2$.

[Z. Komargodski, N. Seiberg, D. Gaiotto, A. Kapustin and et al.]

Baryon as Chiral Bag by CCP

$*$ One-flavor baryon also can be understood as a annuls chiral bag using the Cheshire cat principle (physics is independent on the bag size).

$*$ This $(2+1)$-dimensional chiral bag spins $N_c/2$ and carries one baryon number. In the limit of the zero bag radius, the bag becomes a Hall Droplet suggested by Komargodski.



$*$ The chiral bag carries quark field with quark number 1, and is clouded by η' field, so the charge leaks through the anomaly inflow/outflow. When the radius decreases to zero, a Chern-Simons thoery emerges

$$\frac{N_c}{4\pi}\int_{2+1} a\mathrm{d}a,$$

which gives quark fractional spin $1/2N_c$.

[Yong-Liang Ma, Maciej A. Nowak, Mannque Rho, and Ismail Zahed.]

Hall Liquid Theory on η' Domain Wall

* Complete level/rank duality for $N_f = 1$ (fermion bosonization $\psi^\dagger\psi \sim \bar{\phi}\phi$):

$$\text{non-abelian } SU(N_c)_{-1} + \text{ one fermions } \psi \longleftrightarrow \text{abelian } U(1)_{N_c} + \text{ one scalars } \phi.$$

* For one-flavor case, we conjecture the effective theory on η' domain wall should be:

$$S_A[a,\phi] = \int \mathrm{d}^3x \, |\partial_\mu\phi - \mathrm{i}a_\mu\phi|^2 - V(\phi^*\phi) + \frac{N_c}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho,$$

where Higgs-type potential — Chern-Simons-Higgs theory

$$V(\phi^*\phi) = N_c \sum_{I\in\mathbb{N}} c_I\Big(\frac{\phi^*\phi}{N_c} - v^2\Big)^I, v > 0,$$

and the coefficients c_I are subject to strong constraints to ensure a non-zero vacuum expectation value of $\phi_c^*\phi_c$ equal to $N_c v^2$ with a positive v.

Vortexes are Anyons

* The Chern-Simons term brings a topological current:

$$j^{\mu} = \frac{N_c}{2\pi}\epsilon^{\mu\nu\rho}\partial_{\nu}a_{\rho}.$$

* This theory has non-trivial topological soliton solutions: $\phi(r \to \infty) = v\mathrm{e}^{\mathrm{i}n\theta}$.

* The vortex solutions of this theory carry both flux quanta and topological charges:

$$\Phi = \int \epsilon^{0\nu\rho}\partial_{\nu}a_{\rho}dxdy = 2\pi n, \quad \Longrightarrow \quad Q = \int j^0 dxdy = nN_c.$$

* The object with both charge and flux is Anyon.

If the wave-function of two anyons with charge q and flux ϕ exchanges, a phase angle is generated.

$$\psi(\boldsymbol{r}_2, \boldsymbol{r}_2) = e^{2\pi s}\psi(\boldsymbol{r}_2, \boldsymbol{r}_2), \quad \Longrightarrow \quad \text{induced spin } s = \frac{q\phi}{4\pi}$$

Our vortexes have spin $s = \frac{Q\Phi}{4\pi} = \frac{N_c}{2}n^2, n \in \mathbb{Z}; s = \frac{N_c}{2}$ for $n = \pm 1$.

Baryons as Vortexes

$*$ The ϕ depicts the quark density (or quasi-particles density in quantum Hall effct), and the CS field a_μ propagates the quark number. We can consider vortexes with different winding number n as baryons or multi-baryon structures located on the domain wall.

$$\text{baryon number :} \quad B = \frac{Q}{N_c} = \frac{nN_c}{N_c} = n.$$

$*$ In the large N_c limit, we have demanded that quark density of every color $\phi_c^*\phi_c = \phi^*\phi/N_c$ keeps finite. In the leading N_c order, the effective action can be expressed as

$$S_A = \int \mathrm{d}^3x\, N_c \left[|\partial_\mu\phi_c - \mathrm{i}a_\mu\phi_c|^2 - V_c(\phi_c^*\phi_c) + \frac{1}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho \right],$$

where

$$V_c(\phi_c^*\phi_c) = V(\phi^*\phi)/N_c = \sum_{I\in\mathbb{N}^+} c_I(\frac{\phi^*\phi}{N_c} - v^2)^I = \sum_{I\in\mathbb{N}^+} c_I(\phi_c^*\phi_c - v^2)^I, v > 0.$$

The Large N_c Behavior of Baryons as Vortexes

* The sizes of the vortexes are determined by the masses of ϕ, a_μ, which only depend on v^2:

$$\text{Size} \sim \max\{m_\phi^{-1}, m_{a_\mu}^{-1}\} \sim N_c^0.$$

* Furthermore, since effective action $S_A \sim N_c$, this implies that the energy/mass of the vortexes behaves

$$\text{Energy/Mass} \sim H_A \sim S_A \sim N_c.$$

* If the interaction between two vortexes is taken into account, the interaction energy is also proportional to N_c, meaning that the vortex-vortex scattering amplitudes

$$\mathcal{M} \sim e^{S_A} \sim S_A \sim N_c.$$

All large N_c behavior of the vortexes are the same as those of baryons!

Mesons on the Domain Wall

$*$ In large N_c QCD, the meson operators are $\mathcal{J}(x) = \sqrt{N_c}\bar{\psi}F^m\psi$. On the domain wall, the meson operators should be $\hat{\mathcal{J}}(x) = \sqrt{N_c}\phi^* f^m\phi$.

$*$ Estimating the the correlation function of mesons with Higgs-type potential and quark density relation: $\phi^*\phi \sim \psi^\dagger\psi$

$$V(\phi^*\phi) = N_c \sum_{I\in\mathbb{N}^+} c_I(\frac{\phi^*\phi}{N_c} - v^2)^I, v > 0,$$

The relation between two-point function and the first term of the potential

$$\langle\mathcal{J}\mathcal{J}\rangle \sim N_c^0 \quad \longleftrightarrow \quad c_2 N_c(\frac{\phi^*\phi}{N_c} - v^2)^2 \sim c_2(\sqrt{N_c}\phi_c^*\phi_c)^2 \sim c_2\hat{\mathcal{J}}\hat{\mathcal{J}},$$

Similarly, the four-point function of the meson fields is

$$\langle\mathcal{J}\mathcal{J}\mathcal{J}\mathcal{J}\rangle \sim \frac{1}{N_c} \quad \longleftrightarrow \quad c_4 N_c(\frac{\phi^*\phi}{N_c} - v^2)^4 \sim \frac{c_4}{N_c}(\sqrt{N_c}\phi_c^*\phi_c)^4 \sim \frac{c_4}{N_c}\hat{\mathcal{J}}\hat{\mathcal{J}}\hat{\mathcal{J}}\hat{\mathcal{J}}.$$

Particle-Vertex Duality

∗ The Chern-Simons-Higgs theory S_A on the η' domain wall:

$$S_A[a,\phi] = \int \mathrm{d}^3x\,|\partial_\mu\phi - \mathrm{i}a_\mu\phi|^2 - V(\phi^*\phi) + \frac{N_c}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho.$$

∗ By the Particle-Vortex duality, a dual description is presented:

$$S_B[\tilde{a},\tilde{\phi}] = \int \mathrm{d}^3x\,|\partial_\mu\tilde{\phi} - \mathrm{i}\tilde{a}_\mu\tilde{\phi}|^2 - \tilde{V}(\tilde{\phi}^*\tilde{\phi}) + \frac{1}{4\pi N_c}\epsilon^{\mu\nu\rho}\tilde{a}_\mu\partial_\nu\tilde{a}_\rho.$$

∗ We still demand the potential is Higgs-type. Now the $\tilde{\phi}$ describes the baryon number density, and the CS field $\tilde{a}_\mu$ propagates the baryon number.

∗ $S_B[\tilde{a},\tilde{\phi}]$ exactly is Zhang-Hansson-Kivelson theory (ZHK theory), initially proposed to describe fractional quantum hall effect with filling $1/N$.

[S. C. Zhang, T. Hansson and S. Kivelson.]

Fractional Spin

* Dual ZHK theory on the η' domain wall:

$$S_B[\tilde{a}, \tilde{\phi}] = \int \mathrm{d}^3 x \, |\partial_\mu \tilde{\phi} - \mathrm{i} \tilde{a}_\mu \tilde{\phi}|^2 - \tilde{V}(\tilde{\phi}^* \tilde{\phi}) + \frac{1}{4\pi N_c} \epsilon^{\mu\nu\rho} \tilde{a}_\mu \partial_\nu \tilde{a}_\rho,$$

* ZHK theory also exists similar vortex solutions, but with different topological currents and charges

$$\tilde{j}^\mu = \frac{1}{2\pi N_c} \epsilon^{\mu\nu\rho} \partial_\nu \tilde{a}_\rho, \qquad \tilde{Q} = \int \tilde{j}^0 \, dxdy = \frac{\tilde{n}}{N_c} = B \, (\text{baryon number}),$$

which implies the vortexes carry fractional spin:

$$\tilde{s} = \frac{\tilde{Q}\Phi}{4\pi} = \frac{\tilde{n}^2}{2N_c}, \tilde{n} \in \mathbb{N} \quad \Longrightarrow \quad \begin{array}{ll} \tilde{s} = \dfrac{1}{2N_c} & \text{when } \tilde{n} = \pm 1 \quad \text{single quark.} \\ \tilde{s} = \dfrac{N_c}{2} & \text{when } \tilde{n} = \pm N_c \text{ single baryon.} \end{array}$$

[S. C. Zhang, T. Hansson and S. Kivelson.]

More on the Level/Rank Duality

$\ast$ Hidden local symmetry introduces the vector mesons in chiral effective theory, the most general hidden Wess-Zumino terms (contracted with ϵ tensor) can be added:

$$\mathcal{L}_{hWZ} = \frac{N_c}{16\pi^2}\sum_i c_i\mathcal{L}_i \quad \supset \quad \frac{\partial_\nu \eta'}{2\pi} \times \frac{N_c}{2\pi}\epsilon^{\mu\nu\rho\sigma}\omega_\mu\partial_\rho\omega_\sigma \sim \omega_\mu B^\mu,$$

the flux of vector meson field ω_μ actually measures the baryon density after integrating the η' derivative.

[T. Kugo, K. Yamawaki, T. Yanagida et al.]

$\ast$ On the η' domain wall, the Level/Rank duality actually is between non-abelian gluon field G and abelian vector meson field ω (for $N_f = 1$).

$$SU(N_c)_{-1} \quad \longleftrightarrow \quad U(1)_{N_c},$$
$$\frac{-1}{4\pi}\left(G\mathrm{d}G - \mathrm{i}\frac{2}{3}G^3\right) \quad \longleftrightarrow \quad \frac{N_c}{4\pi}\omega\mathrm{d}\omega.$$

Vector mesons field inherits some topological properties of the gluon field.

[Avner Karasik.]

Summary: Baryons as Vortexes on the η' Domain Wall

- A Chern-Simons-Higgs theory lives on the η' domain wall.
- The vortex solutions with one winding number carry an unit topological charge and induce a spin of $N_c/2$.
- All vortex solutions share the same large N_c behavior as the baryons' mass, radius, and interaction. Estimation of mesons also coincides.
- Through the particle-vortex duality, the Chern-Simons-Higgs theory is dual to the Zhang-Hansson-Kivelson theory, implies that quarks on the domain wall obey fractional statistics.
- Skyrmion picture is proper for baryon $N_f \geq 2$ case, and the quantum Hall droplet/vortex is more proper for $N_f = 1$ case.

Acknowledgment

Thanks!