Charmonium-like states with the exotic quantum number $J^{PC}=3^{-+}$

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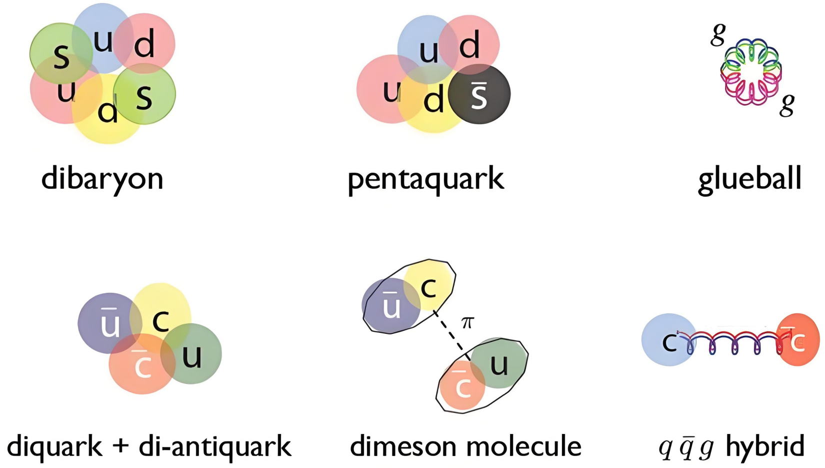
第九届手征有效场论研讨会

Outline

Background

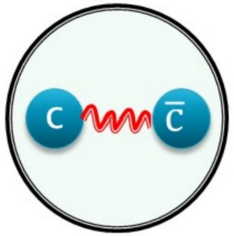
- **Traditional quark model: mesons($\bar{q}q$), baryons(qqq).**
- **QCD also allows for the existence of the exotic hadron states: tetraquark states, pentaquark states, glueballs, hybrid states, etc.**
- **There exist some exotic hadron states can not be explained as traditional hadrons with such quantum numbers $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}$, etc.**



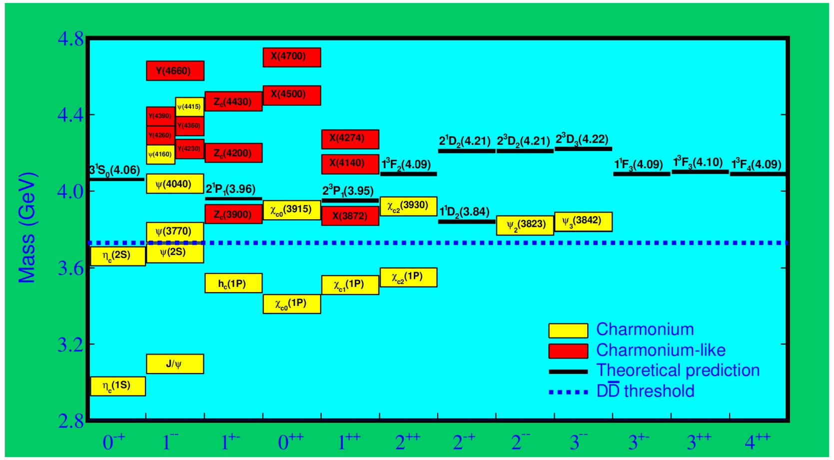
L	S = 0	S = 1
0	0^{-+}	1^{--}
1	1^{+-}	$0^{++}, 1^{++}, 2^{++}$
2	2^{-+}	$1^{--}, 2^{--}, 3^{--}$
3	3^{+-}	$2^{++}, 3^{++}, 4^{++}$

Background

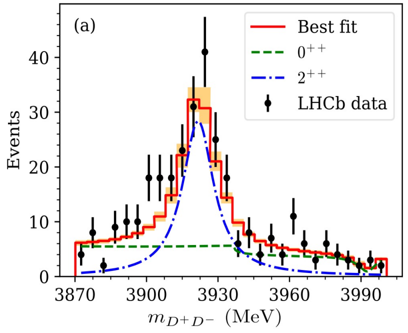
Charmonium (-like) states



- Below the $D\bar{D}$ threshold, charmonium states are well understood, while above this threshold many charmonium-like states (XYZ states) have been observed.



arXiv:2212.00631



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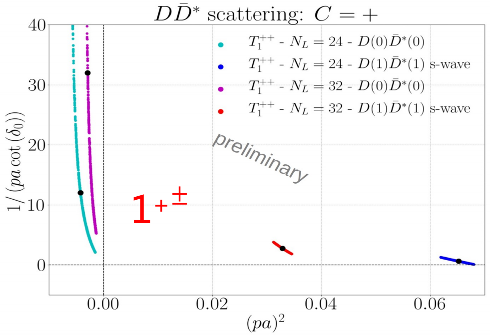
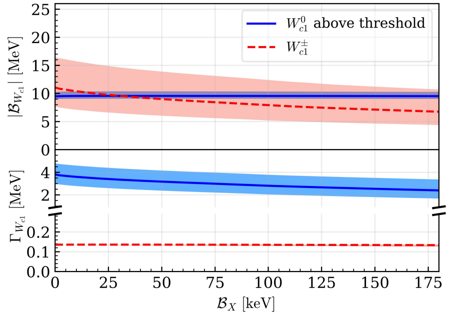
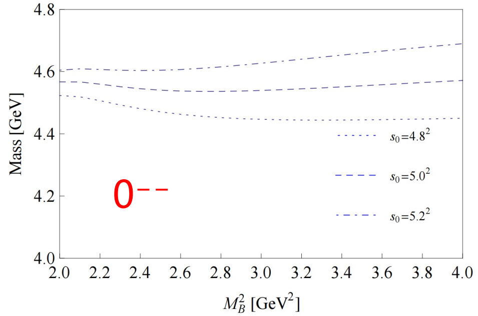


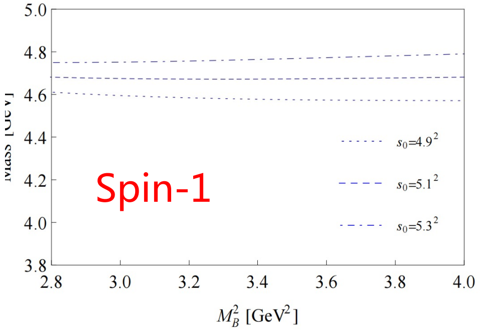
JHEP 08 (2024) 130



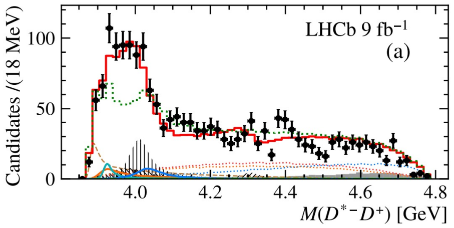
Phys.Rev.D81:105018



Phys.Rev.D83:034010



Phys. Rev. Lett. 133, 131902

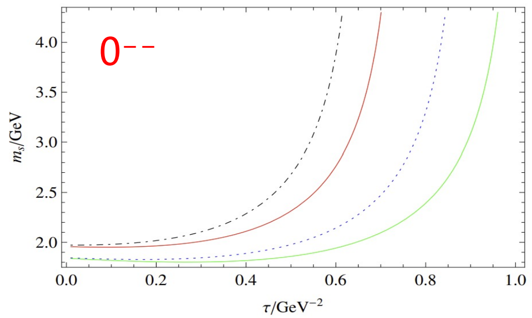


- "X" states: Neutral, $J^{PC} \neq 1^{--}$
- "Y" states: Neutral, $J^{PC} = 1^{--}$
- "Z" states: Charged, isospin triplet

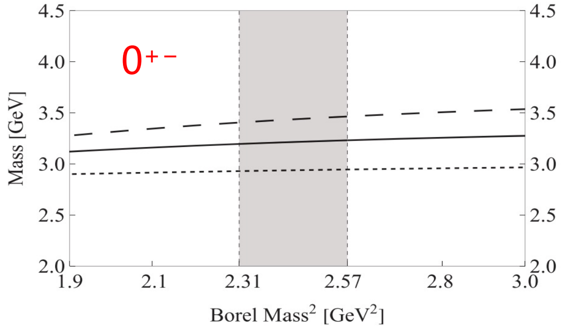
Background

➢ **Some tetraquark states with exotic quantum number have been investigated using the QCD sum rules.**

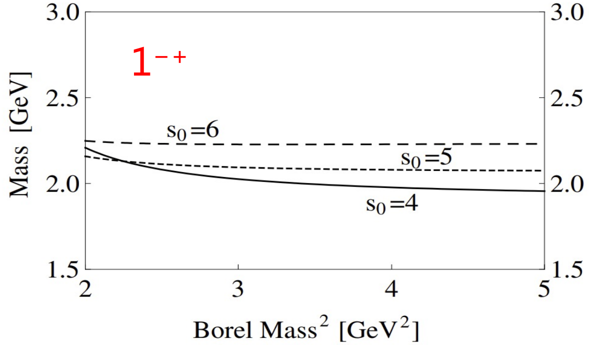
Phys. Rev. D 95：076017



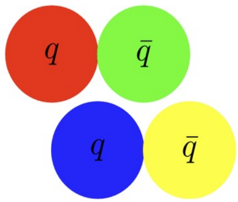
Phys.Rev. D 108：094019



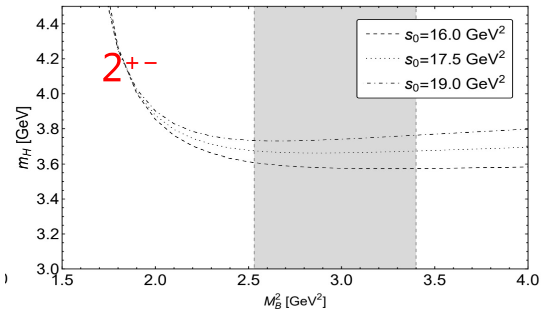
Phys. Rev. D 78：117502



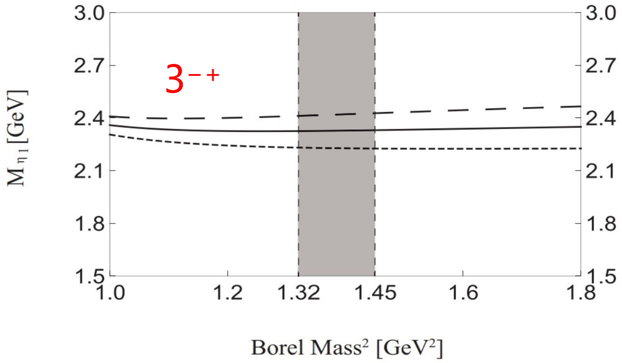
$qq\bar{q}\bar{q}$



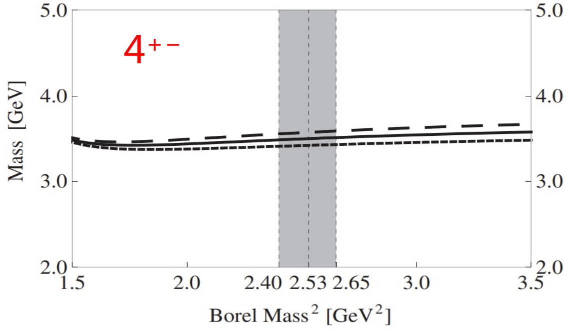
Phys. Rev. D 110：034022



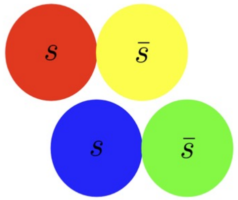
Phys. Rev. D 103：054006



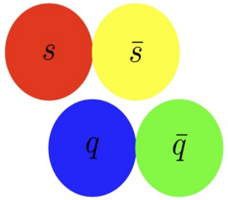
Eur. Phys. J. C82 (2022) 983



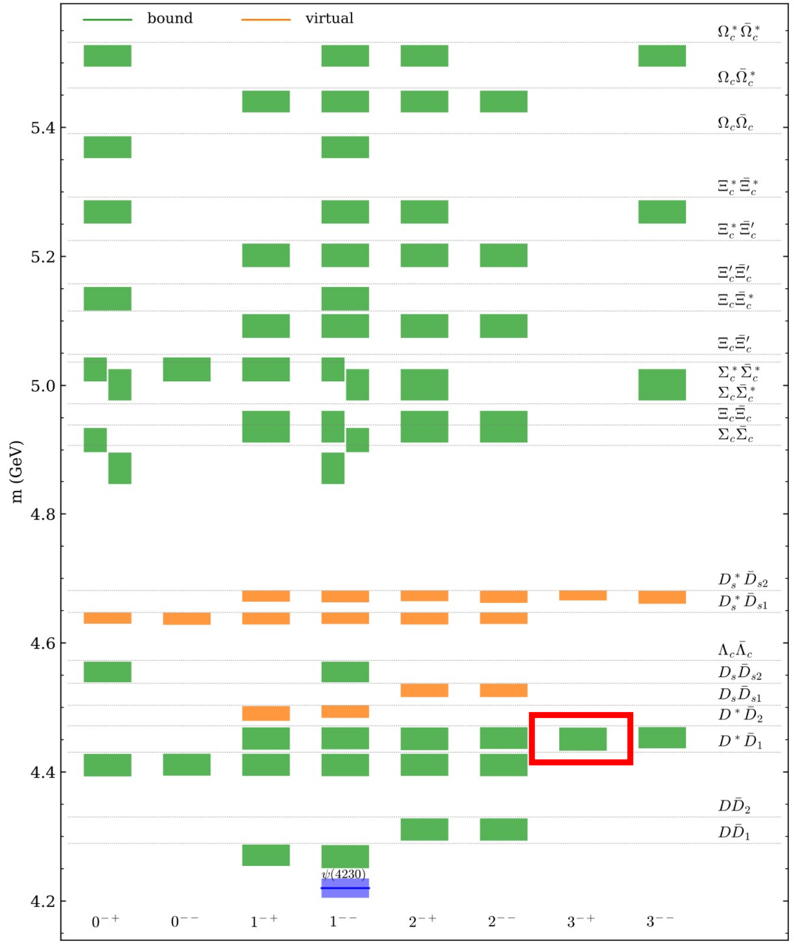
$ss\bar{s}\bar{s}$



$sq\bar{s}\bar{q}$



Background



X. K. Dong, F. K. Guo and B. S. Zou, Progr. Phys. 41, 65-93

- **229 molecular states predicted by solving the Bethe-Salpeter equation.**
- **A study on the $D^*\bar{D}_2^*$ molecular state with $J^{PC} = 3^{-+}$ by OBE model before.** Chin. Phys. C39 (2) 023101
- **A study on the glueball with $J^{PC} = 3^{-+}$ by Lattice QCD forty years ago.** HEPNP 8 :573–578
- **No theoretical study on the $cq\bar{c}\bar{q}$ tetraquark state with $J^{PC} = 3^{-+}$ by QCD sum rules yet.**

Method: QCD sum rules

- We consider the **two-point correlation function** (η is an interpolating current) :

$$\begin{aligned}&\Pi^{ii}_{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}(q^2)\\ \equiv\ & i\int d^4x e^{iqx}\langle 0|\mathbf{T}[\eta^i_{\alpha_1\alpha_2\alpha_3}(x)\eta^{i,\dagger}_{\beta_1\beta_2\beta_3}(0)]|0\rangle\\ =\ &(-1)^J\ \mathcal{S}'[\tilde{g}_{\alpha_1\beta_1}\tilde{g}_{\alpha_2\beta_2}\tilde{g}_{\alpha_3\beta_3}]\ \Pi_{ii}(q^2)\end{aligned}$$

- **At the hadron level**, the correlation function can be expressed by **the dispersion relation**:

$$\Pi(q^2)=\int_{4m_c^2}^{\infty}\frac{\rho^{\text{phen}}(s)}{s-q^2-i\varepsilon}ds$$

When $s\to\infty$, it could not be neglected!

- **At the quark-gluon level**, we perform the **Operator Product Expansion (OPE)** .

Method: QCD sum rules

- We perform the **Borel transformation** at both the hadron and quark-gluon levels to obtain：

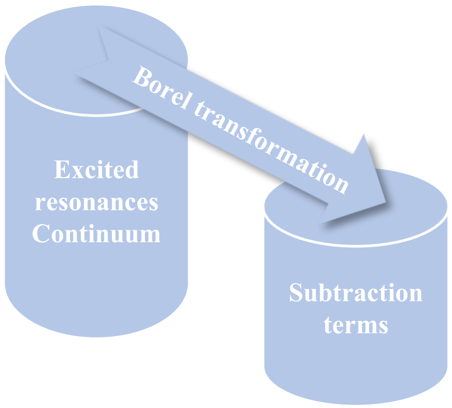
$$\Pi(s_0, M_B^2) \equiv f^2 e^{-M^2/M_B^2} = \int_{4m_c^2}^{s_0} e^{-s/M_B^2} \rho(s) ds$$

$$M^2(s_0, M_B) = \frac{\frac{\partial}{\partial(-1/M_B^2)} \Pi(s_0, M_B^2)}{\Pi(s_0, M_B^2)} = \frac{\int_{4m_c^2}^{s_0} e^{-s/M_B^2} s\rho(s) ds}{\int_{4m_c^2}^{s_0} e^{-s/M_B^2} \rho(s) ds}$$

- Providing $\frac{1}{(k-1)!}$ suppression factor to improve the convergence of OPE and multiple differentiation to eliminate subtraction terms with q^2 polynomials.

- Two phenomenological parameters:

Threshold s_0
Borel mass M_B



Method: QCD sum rules

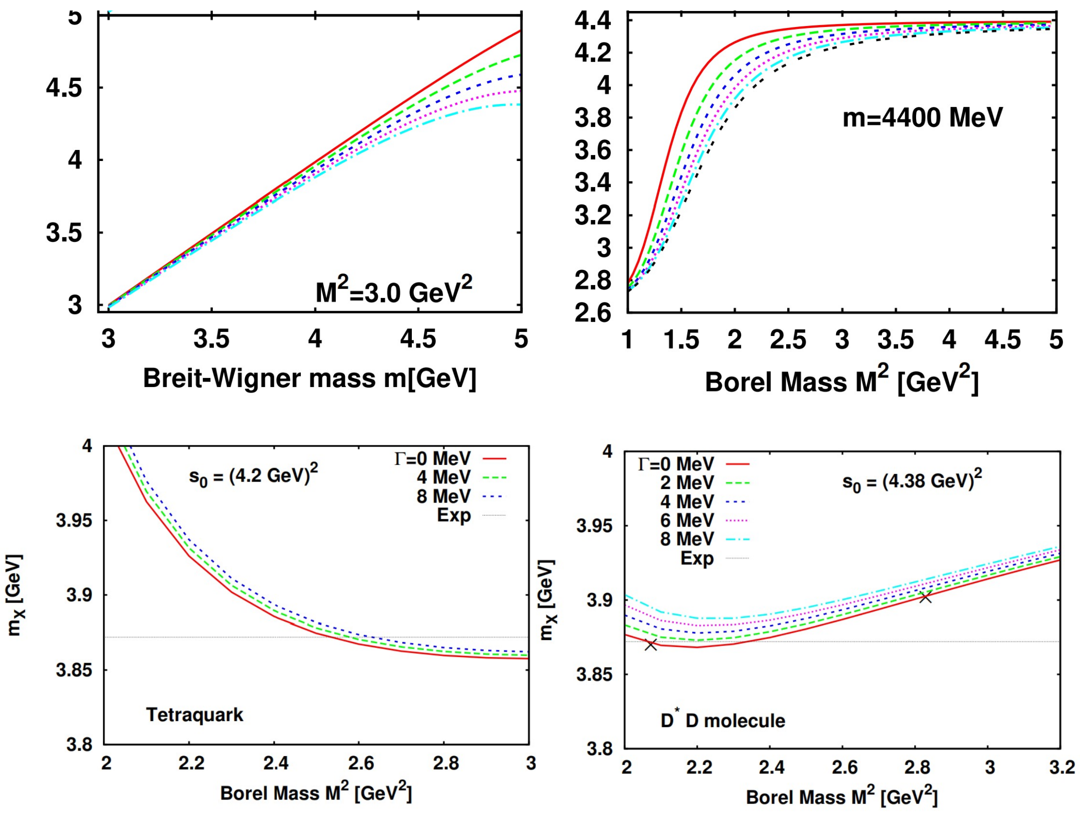
- **Hadronic spectral density with one pole dominance for the state and a continuum contribution:**

$$\rho^{\text{phen}}(s) = \rho^{\text{pole}}(s) + \rho^{\text{cont}}(s)$$

$$\rho^{\text{pole}}(s) = \lambda^2 \delta(s - m^2)$$

- **Some researchers also have examined finite-width spectral densities (such as the Breit-Wigner Distribution Function) .**
  - Increasing the mass at a small Borel window region.
  - Strongly favoring the molecular description of these states instead of the tetraquark description.

Phys.Rev.D78:076001



For the charmonium-like states X(3872), Z(4430), Z_2(4250).

Method: QCD sum rules

➤ **Interpolating currents (for diquark-antidiquark currents)**

- $\mathbf{D_\alpha = \partial_\alpha + ig_s A_\alpha}$
- $\mathcal{S}$: **symmetrization and subtracting trace terms in $\{\alpha_1\alpha_2\alpha_3\}$**

$$\eta = [c_a^T C\Gamma_1 \overleftrightarrow{D}_\alpha q_b](\bar{c}_c\Gamma_2 C\bar{q}_d^T)$$
$$\eta' = (c_a^T C\Gamma_1 q_b)[\bar{c}_c\Gamma_2 C\overleftrightarrow{D}_\alpha \bar{q}_d^T]$$

$$\eta^1_{\alpha_1\alpha_2\alpha_3} = \epsilon^{abe}\epsilon^{cde} \times \mathcal{S}\Big\{[c_a^T C\gamma_{\alpha_1}\overleftrightarrow{D}_{\alpha_3} q_b](\bar{c}_c\gamma_{\alpha_2} C\bar{q}_d^T) + (c_a^T C\gamma_{\alpha_1} q_b)[\bar{c}_c\gamma_{\alpha_2} C\overleftrightarrow{D}_{\alpha_3}\bar{q}_d^T]\Big\},$$
$$\eta^2_{\alpha_1\alpha_2\alpha_3} = (\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}) \times \mathcal{S}\Big\{[c_a^T C\gamma_{\alpha_1}\overleftrightarrow{D}_{\alpha_3} q_b](\bar{c}_c\gamma_{\alpha_2} C\bar{q}_d^T) + (c_a^T C\gamma_{\alpha_1} q_b)[\bar{c}_c\gamma_{\alpha_2} C\overleftrightarrow{D}_{\alpha_3}\bar{q}_d^T]\Big\},$$
$$\eta^3_{\alpha_1\alpha_2\alpha_3} = \epsilon^{abe}\epsilon^{cde} \times \mathcal{S}\Big\{[c_a^T C\gamma_{\alpha_1}\gamma_5\overleftrightarrow{D}_{\alpha_3} q_b](\bar{c}_c\gamma_{\alpha_2}\gamma_5 C\bar{q}_d^T) + (c_a^T C\gamma_{\alpha_1}\gamma_5 q_b)[\bar{c}_c\gamma_{\alpha_2}\gamma_5 C\overleftrightarrow{D}_{\alpha_3}\bar{q}_d^T]\Big\},$$
$$\eta^4_{\alpha_1\alpha_2\alpha_3} = (\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}) \times \mathcal{S}\Big\{[c_a^T C\gamma_{\alpha_1}\gamma_5\overleftrightarrow{D}_{\alpha_3} q_b](\bar{c}_c\gamma_{\alpha_2}\gamma_5 C\bar{q}_d^T) + (c_a^T C\gamma_{\alpha_1}\gamma_5 q_b)[\bar{c}_c\gamma_{\alpha_2}\gamma_5 C\overleftrightarrow{D}_{\alpha_3}\bar{q}_d^T]\Big\}$$
$$\eta^5_{\alpha_1\alpha_2\alpha_3} = \epsilon^{abe}\epsilon^{cde} \times g^{\mu\nu}\mathcal{S}\Big\{[c_a^T C\sigma_{\alpha_1\mu}\overleftrightarrow{D}_{\alpha_3} q_b](\bar{c}_c\sigma_{\alpha_2\nu} C\bar{q}_d^T) + (c_a^T C\sigma_{\alpha_1\mu} q_b)[\bar{c}_c\sigma_{\alpha_2\nu} C\overleftrightarrow{D}_{\alpha_3}\bar{q}_d^T]\Big\},$$
$$\eta^6_{\alpha_1\alpha_2\alpha_3} = (\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}) \times g^{\mu\nu}\mathcal{S}\Big\{[c_a^T C\sigma_{\alpha_1\mu}\overleftrightarrow{D}_{\alpha_3} q_b](\bar{c}_c\sigma_{\alpha_2\nu} C\bar{q}_d^T) + (c_a^T C\sigma_{\alpha_1\mu} q_b)[\bar{c}_c\sigma_{\alpha_2\nu} C\overleftrightarrow{D}_{\alpha_3}\bar{q}_d^T]\Big\}$$

Method: QCD sum rules

➢ **Interpolating currents (for meson-antimeson currents)**

$$
\begin{aligned}
\xi^1_{\alpha_1\alpha_2\alpha_3} &= \mathcal{S}\Big\{(\bar{c}_a\gamma_{\alpha_1}c_a)\overleftrightarrow{D}_{\alpha_3}(\bar{q}_b\gamma_{\alpha_2}q_b)\Big\}\\
\xi^2_{\alpha_1\alpha_2\alpha_3} &= \mathcal{S}\Big\{(\bar{c}_a\gamma_{\alpha_1}\gamma_5c_a)\overleftrightarrow{D}_{\alpha_3}(\bar{q}_b\gamma_{\alpha_2}\gamma_5q_b)\Big\}\\
\xi^3_{\alpha_1\alpha_2\alpha_3} &= g^{\mu\nu}\mathcal{S}\Big\{(\bar{c}_a\sigma_{\alpha_1\mu}c_a)\overleftrightarrow{D}_{\alpha_3}(\bar{q}_b\sigma_{\alpha_2\nu}q_b)\Big\}\\
\xi^4_{\alpha_1\alpha_2\alpha_3} &= \mathcal{S}\Big\{\big[\bar{c}_a\gamma_{\alpha_1}\overleftrightarrow{D}_{\alpha_3}q_a\big](\bar{q}_b\gamma_{\alpha_2}c_b)-(\bar{c}_a\gamma_{\alpha_1}q_a)\big[\bar{q}_b\gamma_{\alpha_2}\overleftrightarrow{D}_{\alpha_3}c_b\big]\Big\}\\
\xi^5_{\alpha_1\alpha_2\alpha_3} &= \mathcal{S}\Big\{\big[\bar{c}_a\gamma_{\alpha_1}\gamma_5\overleftrightarrow{D}_{\alpha_3}q_a\big](\bar{q}_b\gamma_{\alpha_2}\gamma_5c_b)-(\bar{c}_a\gamma_{\alpha_1}\gamma_5q_a)\big[\bar{q}_b\gamma_{\alpha_2}\gamma_5\overleftrightarrow{D}_{\alpha_3}c_b\big]\Big\}\\
\xi^6_{\alpha_1\alpha_2\alpha_3} &= g^{\mu\nu}\mathcal{S}\Big\{\big[\bar{c}_a\sigma_{\alpha_1\mu}\overleftrightarrow{D}_{\alpha_3}q_a\big](\bar{q}_b\sigma_{\alpha_2\nu}c_b)-(\bar{c}_a\sigma_{\alpha_1\mu}q_a)\big[\bar{q}_b\sigma_{\alpha_2\nu}\overleftrightarrow{D}_{\alpha_3}c_b\big]\Big\}
\end{aligned}
$$

$$\xi = [\bar{c}c]\overleftrightarrow{D}[\bar{q}q]$$

$$\xi' = [\bar{c}\overleftrightarrow{D}q]\,[\bar{q}c] - [\bar{c}q]\,[\bar{q}\overleftrightarrow{D}c]$$

Method: QCD sum rules

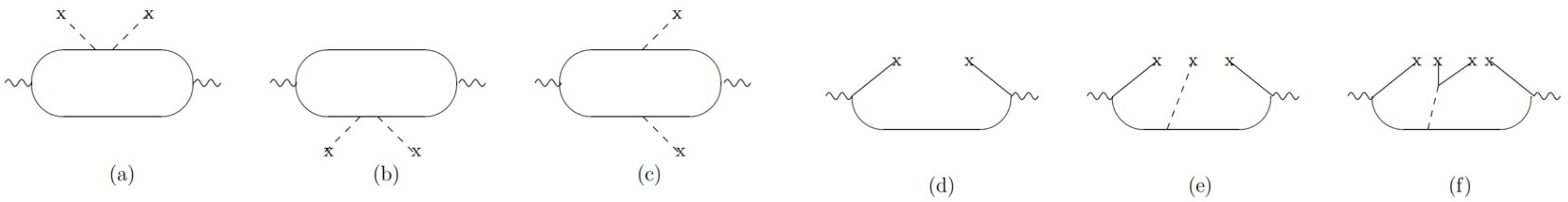
Fierz rearrangement

$$\begin{pmatrix} \eta^1_{\alpha_1\alpha_2\alpha_3} \\ \eta^2_{\alpha_1\alpha_2\alpha_3} \\ \eta^3_{\alpha_1\alpha_2\alpha_3} \\ \eta^4_{\alpha_1\alpha_2\alpha_3} \\ \eta^5_{\alpha_1\alpha_2\alpha_3} \\ \eta^6_{\alpha_1\alpha_2\alpha_3} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} \xi^1_{\alpha_1\alpha_2\alpha_3} \\ \xi^2_{\alpha_1\alpha_2\alpha_3} \\ \xi^3_{\alpha_1\alpha_2\alpha_3} \\ \xi^4_{\alpha_1\alpha_2\alpha_3} \\ \xi^5_{\alpha_1\alpha_2\alpha_3} \\ \xi^6_{\alpha_1\alpha_2\alpha_3} \end{pmatrix}$$

- **Since any given four-fermion couplings, the fermion fields entering them can always be rearranged by the Fierz transformations.**
- **This transformation indicates that these two configurations are equivalent, which will be applied to study the decay properties later.**

Method: QCD sum rules

- **The QCD spectral density ρ(s) at the leading order of α_s and up to the dimension ten (D = 10)：**



$$\rho(s) = \rho^{pert}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle GG\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle^2}(s) + \rho^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle^2}(s)$$

- **Some vacuum condensate values in QCD sum rules：**

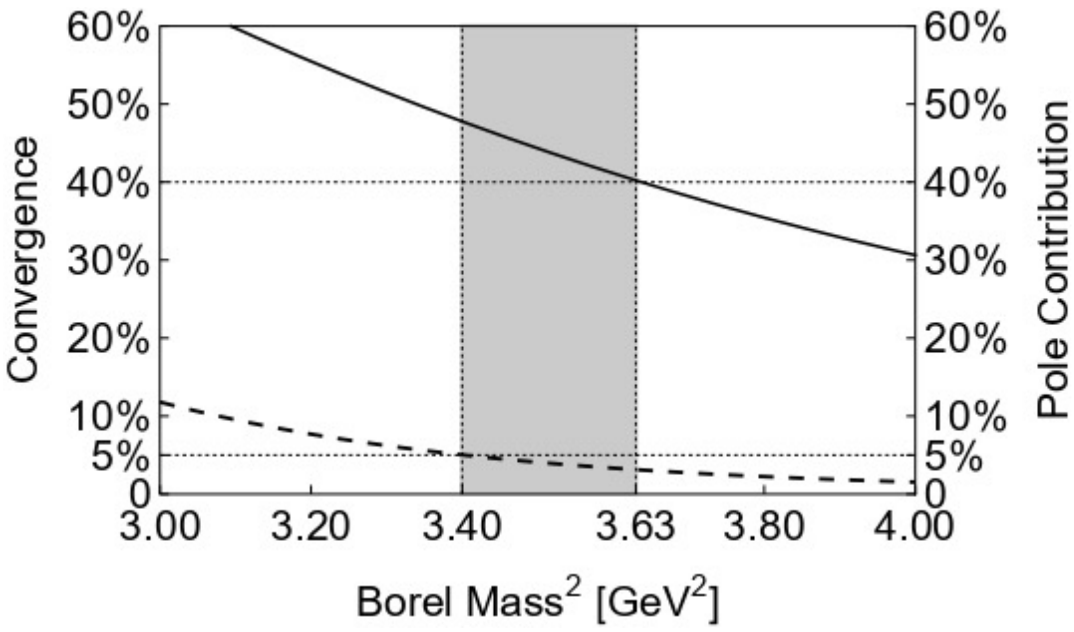
$$\begin{aligned}
\langle\alpha_s GG\rangle &= (6.35 \pm 0.35) \times 10^{-2}\ \text{GeV}^4 \\
\langle\bar{q}q\rangle &= -(0.240 \pm 0.010)^3\ \text{GeV}^3 \\
\langle g_s\bar{q}\sigma Gq\rangle &= -M_0^2 \times \langle\bar{q}q\rangle \\
M_0^2 &= (0.8 \pm 0.2)\ \text{GeV}^2 \\
m_c(m_c) &= 1.275^{+0.025}_{-0.035}\ \text{GeV}
\end{aligned}$$

➢ Pole Dominance

$$\mathrm{PC} \equiv \left| \frac{\Pi_{11}(s_0, M_B^2)}{\Pi_{11}(\infty, M_B^2)} \right| \geq 40\%$$

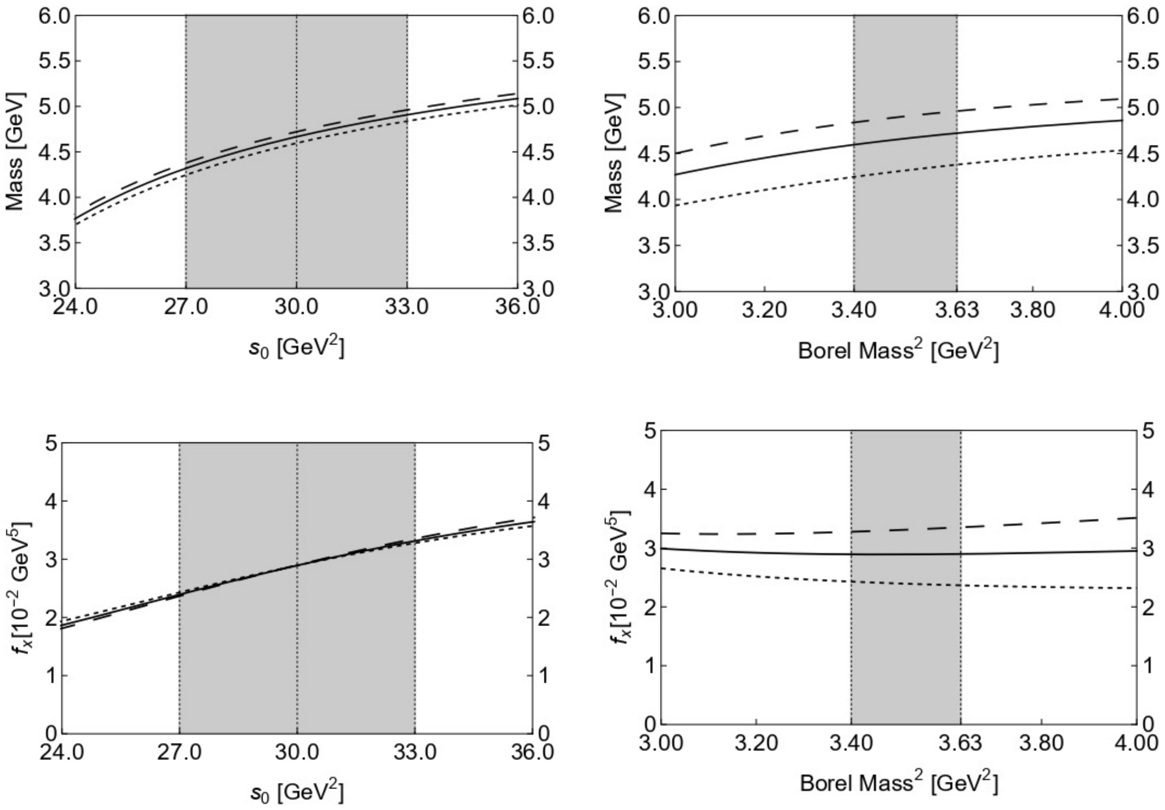
➢ OPE Convergence

$$\mathrm{CVG} \equiv \left| \frac{\Pi_{11}^{D=10}(\infty, M_B^2)}{\Pi_{11}(\infty, M_B^2)} \right| \leq 5\%$$



These requirements together determine that the Borel window to be 3.40 GeV² ≤ M_B^2 ≤ 3.63 GeV² when setting s_0 = 30.0 GeV².

➢ Stability



- **There are non-vanishing Borel windows as long as $s_0 \geq 28.4\ \mathrm{GeV}^2$.**
- **Around s_0 ~30.0 GeV^2,we choose the threshold value to be $27.0\ \mathrm{GeV}^2 \leq s_0 \leq 33.0\ \mathrm{GeV}^2$.**
- **Within this working regions, we obtain:**

$$M_1 = 4.66^{+0.49}_{-0.46}\ \mathrm{GeV}$$

$$f_1 = \left(2.89^{+2.26}_{-1.46}\right) \times 10^{-2}\,\mathrm{GeV}^5$$

Results and Discussion

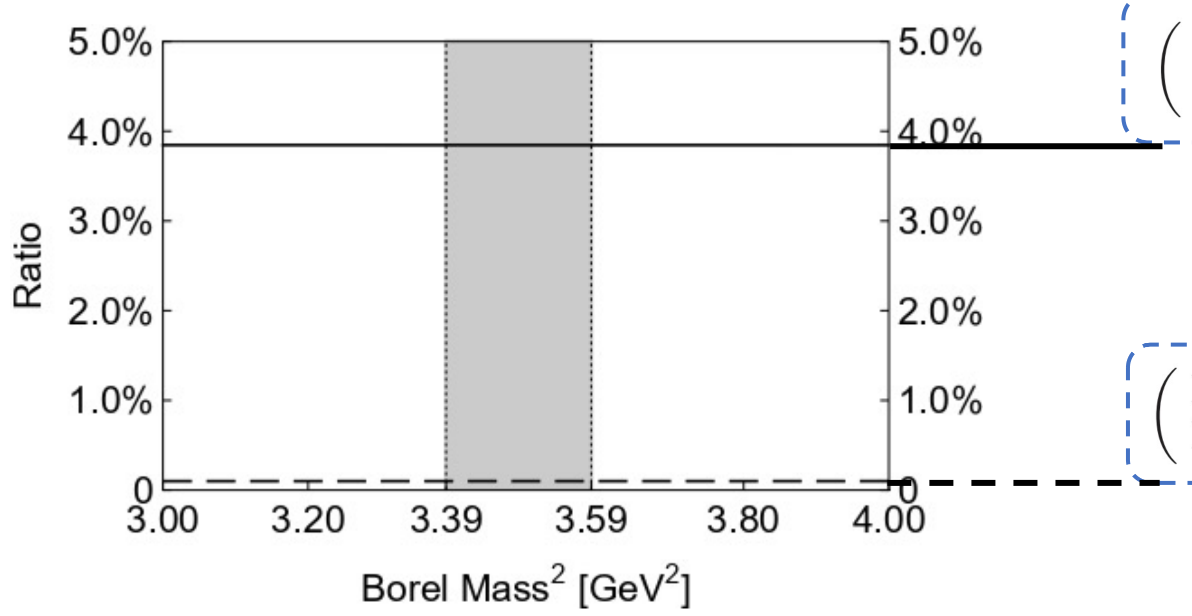
Summary table

TABLE I: QCD sum rule results extracted from the diquark-antidiquark currents $\eta^{1\cdots6}_{\alpha_1\alpha_2\alpha_3}$ and the mixing currents $J^{1\cdots2}_{\alpha_1\alpha_2\alpha_3}$ with the exotic quantum number $J^{PC}=3^{-+}$.

Currents	M_B^2 [GeV^2]	s_0 [GeV^2]	Pole [%]	Mass [GeV]	f_X [GeV^5]
$\eta^1_{\alpha_1\alpha_2\alpha_3}$	3.40-3.63	30.0 ± 3.0	40-48	$4.66^{+0.49}_{-0.46}$	$(2.89^{+2.26}_{-1.46})\times10^{-2}$
$\eta^2_{\alpha_1\alpha_2\alpha_3}$	3.40-3.60	29.0 ± 3.0	40-47	$4.50^{+0.45}_{-0.41}$	$(3.37^{+2.47}_{-1.69})\times10^{-2}$
$\eta^3_{\alpha_1\alpha_2\alpha_3}$	3.63-4.00	35.0 ± 3.0	40-46	$5.75^{+0.21}_{-0.14}$	$(11.71^{+4.53}_{-3.07})\times10^{-2}$
$\eta^4_{\alpha_1\alpha_2\alpha_3}$	3.65-4.05	35.0 ± 3.0	40-47	$5.71^{+0.21}_{-0.14}$	$(16.07^{+6.09}_{-4.35})\times10^{-2}$
$\eta^5_{\alpha_1\alpha_2\alpha_3}$	3.57-3.78	32.0 ± 3.0	40-45	$5.12^{+0.28}_{-0.28}$	$(7.15^{+4.16}_{-3.20})\times10^{-2}$
$\eta^6_{\alpha_1\alpha_2\alpha_3}$	3.35-3.80	32.0 ± 3.0	40-53	$5.08^{+0.27}_{-0.28}$	$(10.10^{+5.68}_{-4.46})\times10^{-2}$
$J^1_{\alpha_1\alpha_2\alpha_3}$	3.40-3.61	30.0 ± 3.0	40-47	$4.67^{+0.51}_{-0.42}$	$(2.87^{+2.31}_{-1.49})\times10^{-2}$
$J^2_{\alpha_1\alpha_2\alpha_3}$	3.39-3.59	29.0 ± 3.0	40-47	$4.49^{+0.45}_{-0.41}$	$(3.35^{+2.43}_{-1.67})\times10^{-2}$

Results and Discussion

- We also investigated the mixing of $\eta^1_{\alpha_1\alpha_2\alpha_3}$ and $\eta^2_{\alpha_1\alpha_2\alpha_3}$ by calculating their off-diagonal correlation function $\Pi^{12}_{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}(q^2)$ by setting **$s_0 = 29.0\ GeV^2$ and $M_B^2 = 3.40\ GeV^2$**.



$$\begin{pmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{pmatrix} = \begin{pmatrix} 70.27 & 3.88 \\ 3.88 & 145.14 \end{pmatrix} \times 10^{-5}\ GeV^{10}$$

Diagonalization

$$\begin{pmatrix} \Pi'_{11} & \Pi'_{12} \\ \Pi'_{21} & \Pi'_{22} \end{pmatrix} = \begin{pmatrix} 145.34 & 0 \\ 0 & 70.07 \end{pmatrix} \times 10^{-5}\ GeV^{10}$$

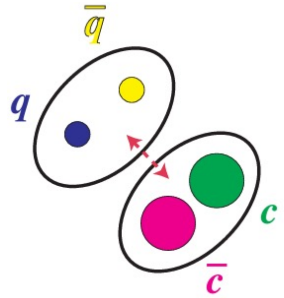
$$\begin{pmatrix} J^1_{\alpha_1\alpha_2\alpha_3} \\ J^2_{\alpha_1\alpha_2\alpha_3} \end{pmatrix} = \begin{pmatrix} 1.00 & -0.05 \\ 0.05 & 1.00 \end{pmatrix} \begin{pmatrix} \eta^1_{\alpha_1\alpha_2\alpha_3} \\ \eta^2_{\alpha_1\alpha_2\alpha_3} \end{pmatrix}$$

$$M'_2 = 4.49^{+0.45}_{-0.41}\ GeV$$

$$f'_2 = \left(3.35^{+2.43}_{-1.67}\right) \times 10^{-2}\ GeV^5$$

Decay behavior

1



$$\xi = [\bar{c}c]\overleftrightarrow{D}[\bar{q}q]$$

↓

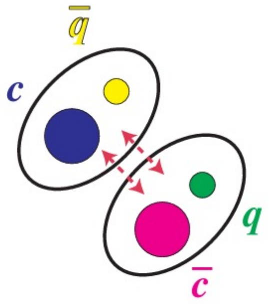
P-wave $(\bar{c}c)_{S-\text{wave}}(\bar{q}q)_{S-\text{wave}}$

↓

P-wave $[\rho J/\psi]/[\omega J/\psi]$

S-wave $[\rho\chi_{c2}]/[\omega\chi_{c2}]/[J/\psi f_2(1270)]$ ✗

2



$$\xi' = [\bar{c}\overleftrightarrow{D}q][\bar{q}c] - [\bar{c}q][\bar{q}\overleftrightarrow{D}c]$$

↓

S-wave $(\bar{c}q)_{S-\text{wave}}(\bar{q}c)_{P-\text{wave}}$

↓

S-wave $[D^*\bar{D}_2^*]$ ≈ 4472 MeV

P-wave $[D^*\bar{D}^*]$ ✗

Summary

- We investigated the charmonium-like states with $J^{PC}=3^{-+}$ by QCD sum rules and find the corresponding interpolating currents are composed of two quark fields and two antiquark fields as well as one covariant derivative operator.

- Through the identification of an effective Borel window, we find the mass of the lowest-lying state is around 4.49 GeV.

- Above the open-charm threshold and combined with the Fierz rearrangement, we analyzed potential decay channels around the $D^*\overline{D}_2^*$ threshold: $X\to[\omega J/\psi]/[\rho J/\psi]$.

- We hope that such states can be found in future experiments at BESIII, Belle-II, LHCb, etc., to further enrich the study of charmonium-like hadronic states.

Thanks for your attention！