基于手征有效场论的双粲重子研究

-ChPT studies of doubly charmed baryons-

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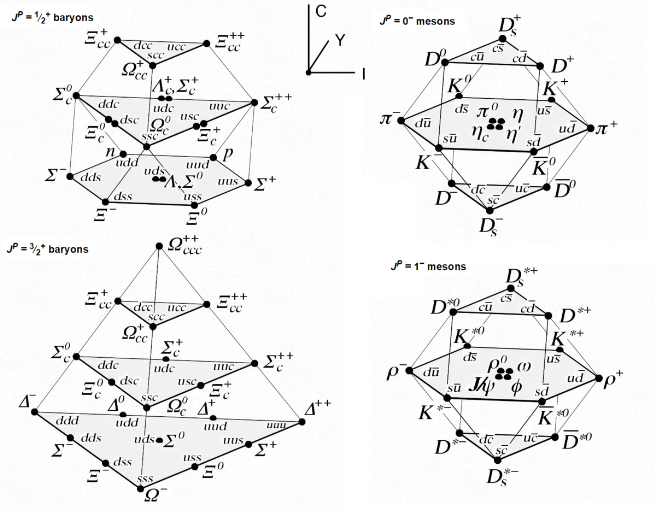
I. Introduction

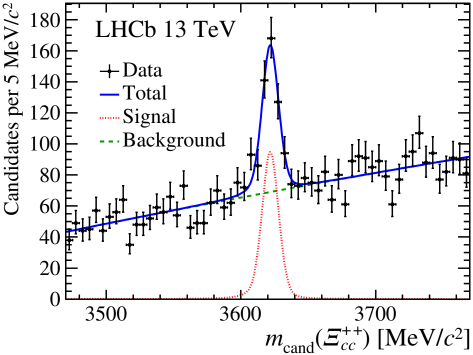
Doubly charmed baryons in quark model

- **Quark model with u, d, s, c**
  - Mesons: $\mathbf{4} \otimes \bar{\mathbf{4}} = \mathbf{15} \oplus \mathbf{1}$
  - Baryons: $\mathbf{4} \otimes \mathbf{4} \otimes \mathbf{4} = \mathbf{20}_S \oplus \mathbf{20}_M \oplus \mathbf{20}_M \oplus \mathbf{4}_A$



Experimental efforts

- In 2002, the SELEX Collaboration firstly reported that the Ξ_{cc}^{+} state was observed with measured mass 3519 ± 2 MeV [M. Mattson, PRL89(2002)112001].
- Other experimental groups: FOCUS [S. Ratti, Nucl.Phys.B, Proc.Suppl.115(2003)33], BABAR [B. Aubert, PRD74(2006)011103], Belle [R. Chistov, PRL97(2006)162001], LHCb [R. Aaij, JHEP12(2013)090].
- In 2017, the LHCb Collaboration announced the observation of the Ξ_{cc}^{++}, via the decay mode $\Lambda_c^+ K^- \pi^+ \pi^+$ [R. Aaij, PRL119(2017)112001].



- Reported value of mass (Ξ_{cc}^{++}): $3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV

Inspired by a theoretical work

[F.-S. Yu, H.-Y. Jiang, R.-H. Li, C.-D. Lü, W. Wang, and Z.-X. Zhao, CPC42(2018)]

Theoretical studies

- Various theoretical studies on the properties of doubly charmed (DC) baryons:
  - Heavy quark effective theory, e.g. [J. Korner, M. Kramer, and D. Pirjol, Prog.Part.Nucl.Phys.33(1994)787]
  - Quark model, e.g. [D. Ebert, R. Faustov, V. Galkin, and A. Martynenko, PRD66(2002)014008] [L.-Y. Xiao, K.-L. Wang, Q.-F. Lu, X.-H. Zhong, and S.-L. Zhu, PRD96(2017)094005]
  - Effective potential method, e.g. [M. Karliner and J. L. Rosner, PRD90(2014)094007]
  - Lattice QCD, e.g. [L. Liu, et al, PRD81(2010)094505] [Z. S. Brown, et al, PRD90(2014)094507]
  - Light-front approach, e.g. [W. Wang, Z.-P. Xing, and J. Xu, EPJC77(2017)800] [W. Wang, F.-S. Yu, and Z.-X. Zhao, EPJC77(2017)781]
  - Chiral effective field theory
  - etc.
- Recent Reviews, e.g.
  [D.-L. Yao, L.-Y. Dai, H.-Q. Zheng and Z.-Y. Zhou, Rept. Prog. Phys. **84**, 076201 (2021)]
  [L. Meng, B. Wang, G.-J. Wang and S.-L. Zhu, Phys. Rept. 1019 (2023) 1-149]

Status in chiral perturbation theory

- Chiral effective field theory (or Chiral perturbation theory)
  - Heavy baryon approach
    - Magnetic moments [H.-S. Li, L. Meng, Z.-W. Liu, and S.-L. Zhu, PLB 777(2018)]
    - Strong and radiative decays [L.-Y. Xiao, K.-L. Wang, Q.-F. Lu, X.-H. Zhong, and S.-L. Zhu, PRD 96(2017)]]
  - Covariant formalism with EOMS scheme
    - Masses [Z.-F. Sun and M.J. Vicente Vacas, PRD93(2016)] [D.-L. Yao, PRD97(2018)]
    - Electromagnetic form factors, etc.
      [Hiller Blin, Z.-F. Sun, and Vicente Vacas, PRD98(2018)] [R.-X. Shi, Y. Xiao and L.-S. Geng, PRD100(2019)]
  - The spectroscopy of the DC baryons [tree level and unitarization]
    e.g. [Z.-H. Guo, PRD96(2017)074004] [M.-J. Yan, et al, PRD98 (2018)]
- Our works:
  - Scattering lengths at one-loop level $\longrightarrow$ DC baryon spectrum
    [Z.-R. Liang, P.-C. Qiu and D.-L. Yao, JHEP07(2023)124]
  - A lattice QCD study of the S-wave scattering lengths and phase shifts
    [J.-Y. Yi, Z.-R. Liang, Q.-Z. Li, L. Liu, P. Sun, X. Xiong, Y.-B. Yang and D.-L. Yao, work in progress]

II. One-loop analysis of the interactions between DCBs and GBs

Chiral effective Lagrangian

- For $\psi\phi$ scattering, only a subset of operators is needed

$$\mathcal{L}_{\rm eff}=\sum_{i=1}^{2}\mathcal{L}_{\phi\phi}^{(2i)}+\sum_{j=1}^{3}\mathcal{L}_{\psi\phi}^{(j)},$$

  - Purely mesonic sector → required by renormalization
  - Baryonic sector (unknown LECs: $1\ g + 7\ b_j + 10\ c_k$)

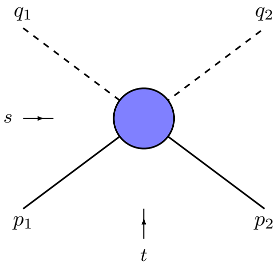
$$\mathcal{L}_{\psi\phi}^{(1)}=\bar\psi\left(i\not{D}-m\right)\psi+\frac{g}{2}\bar\psi\not{u}\gamma_5\psi\ ,$$

$$\mathcal{L}_{\psi\phi}^{(2)}=b_1\bar\psi\langle\chi_+\rangle\psi+b_2\bar\psi\widetilde{\chi}_+\psi+b_3\bar\psi u^2\psi+b_4\bar\psi\left\langle u^2\right\rangle\psi+\frac{b_5}{m^2}\bar\psi\left(\{u^\mu,u^\nu\}D_{\mu\nu}+H.c.\right)\psi$$
$$+\frac{b_6}{m^2}\bar\psi\left(\langle u^\mu u^\nu\rangle D_{\mu\nu}+H.c.\right)\psi+ib_7\bar\psi\left[u^\mu,u^\nu\right]\sigma_{\mu\nu}\psi\ ,$$

$$\mathcal{L}_{\psi\phi}^{(3)}=ic_{11}\bar\psi\left[u_\mu,h^{\mu\nu}\right]\gamma_\nu\psi+\frac{c_{12}}{m^2}\bar\psi\left(i\left[u^\mu,h^{\nu\rho}\right]\gamma_\mu D_{\nu\rho}+H.c.\right)\psi+\frac{c_{13}}{m}\bar\psi\left(i\left\{u^\mu,h^{\nu\rho}\right\}\right.$$
$$\left.\times\sigma_{\mu\nu}D_\rho+H.c.\right)\psi+\frac{c_{14}}{m}\bar\psi\left(i\sigma_{\mu\nu}\langle u^\mu h^{\nu\rho}\rangle D_\rho+H.c.\right)\psi+c_{15}\bar\psi\left\{u^\mu,\widetilde{\chi}_+\right\}\gamma_5\gamma_\mu\psi$$
$$+c_{16}\bar\psi u^\mu\gamma_5\gamma_\mu\langle\chi_+\rangle\psi+c_{17}\bar\psi\gamma_5\gamma_\mu\left\langle u^\mu\widetilde{\chi}_+\right\rangle\psi+ic_{18}\bar\psi\gamma_5\gamma_\mu\left[D^\mu,\widetilde{\chi}_-\right]\psi$$
$$+ic_{19}\bar\psi\gamma_5\gamma_\mu\langle[D^\mu,\chi_-]\rangle\psi+c_{20}\bar\psi\left[\widetilde{\chi}_-,u^\mu\right]\gamma_\mu\psi\ .$$

Generic structure of scattering amplitude

Kinematics



$= i\mathcal{T}_{\psi_1\phi_1\to\psi_2\phi_2}(s,t)$

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}.$$

$$\psi = \begin{pmatrix} \Xi_{cc}^{++} \\ \Xi_{cc}^{+} \\ \Omega_{cc}^{+} \end{pmatrix}.$$

$3 \otimes 8 = 15 \oplus \bar{6} \oplus 3$

$SU(3)$ symmetry constraint: $SU(3) \to SU(2) \otimes U(1)$

- Classify the physics amplitudes into 16 different one with given (S, I), e.g.

$$\mathcal{T}^{(-2,\frac{1}{2})}_{\Omega_{cc}\bar{K}\to\Omega_{cc}\bar{K}}(s,t,u) = \mathcal{T}_{\Omega_{cc}^+K^-\to\Omega_{cc}^+K^-}(s,t,u), \cdots$$

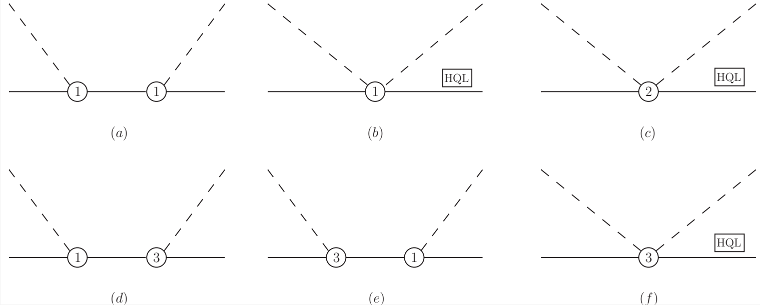
Lorentz decomposition:

$$\mathcal{T}^{(S,I)}_{\psi_1\phi_1\to\psi_2\phi_2}(s,t) = \bar{u}(p',\sigma')\left\{A^{(S,I)}(s,t) + \frac{1}{2}(\not{q} + \not{q}')B^{(S,I)}(s,t)\right\}u(p,\sigma),$$

- A and B are two independent scalar functions.

Tree amplitudes

$$\bar{\psi}\left\{\frac{g}{2}\not{u}\gamma_5\right\}\psi \overset{\text{HQL}}{\Longrightarrow} \bar{\mathcal{N}}_v\left\{gS_\nu\cdot u\right\}\mathcal{N}_v + \alpha_1 m^{-1} + \cdots$$



- LO contribution

$$A^{(1)}_{\text{tree}} = \frac{g^2}{8F^2}\left[\mathcal{C}^{(1)}_S\mathcal{F}(s) + \mathcal{C}^{(1)}_U\mathcal{F}(u)\right], \quad B^{(1)}_{\text{tree}} = \frac{\mathcal{C}^{(1)}_{\text{WT}}}{4F^2} - \frac{g^2}{4F^2}\left[\mathcal{C}^{(1)}_S\mathcal{G}(s) - \mathcal{C}^{(1)}_U\mathcal{G}(u)\right].$$

- NLO contribution

$$A^{(2)}_{\text{tree}} = \frac{\mathcal{C}^{(2)}_1}{6F^2} + \mathcal{C}^{(2)}_2\frac{m^2_{\phi_1}+m^2_{\phi_2}-t}{2F^2} + \frac{\mathcal{C}^{(2)}_3\mathcal{H}(s,t)}{2m^2F^2} + \mathcal{C}^{(2)}_4\frac{u-s}{F^2},$$

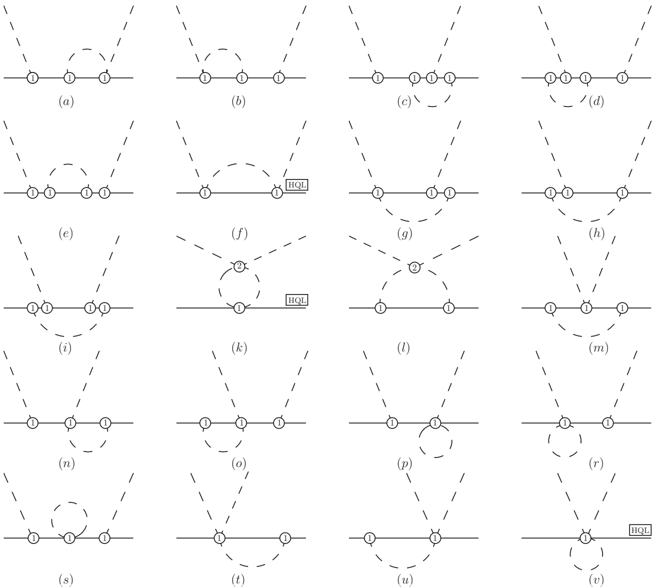
$$B^{(2)}_{\text{tree}} = \mathcal{C}^{(2)}_4\frac{2(m_{\psi_1}+m_{\psi_2})}{F^2}.$$

- The NNLO tree amplitude can be derived straightforwardly, but in more complicated form.

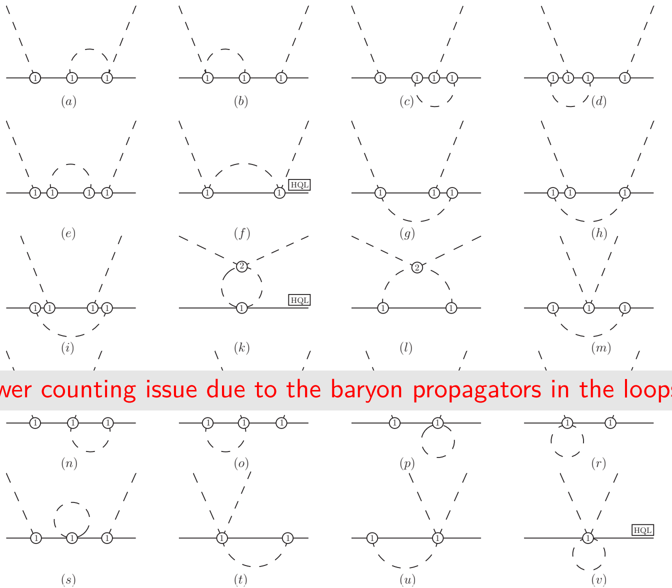
Coefficients in tree amplitudes

(S, I)	Processes	$O(p^1)$			$O(p^2)$
		$\mathcal{C}^{(1)}_{\rm WT}$	$\mathcal{C}^{(1)}_{S}$	$\mathcal{C}^{(1)}_{U}$	$\mathcal{C}^{(2)}_{1}$ $[\mathcal{C}^{(2)}_{2}, \cdots]$
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	-2	0	$2\ [\Xi_{cc}]$	$-4(6b_1 + b_2)m_K^2$
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	-2	0	$2\ [\Omega_{cc}]$	$-4(6b_1 + b_2)m_K^2$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	2	0	$-2\ [\Omega_{cc}]$	$-4(6b_1 + b_2)m_K^2$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	-2	0	$2\ [\Xi_{cc}]$	$-4(6b_1 - 5b_2)m_K^2$
$(-1, 0)$	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	4	$4\ [\Omega_{cc}]$	0	$-8(3b_1 + 2b_2)m_K^2$
	$\Omega_{cc}\eta \to \Omega_{cc}\eta$	0	$\frac{4}{3}\ [\Omega_{cc}]$	$\frac{4}{3}\ [\Omega_{cc}]$	$-\frac{32}{3}(3b_1 + 2b_2)m_K^2 + (8b_1 + \frac{40}{3}b_2)m_\pi^2$
	$\Xi_{cc}\bar{K} \to \Omega_{cc}\eta$	$-2\sqrt{3}$	$-\frac{4}{\sqrt{3}}\ [\Omega_{cc}]$	$\frac{2}{\sqrt{3}}\ [\Xi_{cc}]$	$2\sqrt{3}b_2(5m_K^2 - 3m_\pi^2)$
$(-1, 1)$	$\Omega_{cc}\pi \to \Omega_{cc}\pi$	0	0	0	$2\sqrt{3}b_2(5m_K^2 - 3m_\pi^2)$
	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	0	0	0	$-8(3b_1 - b_2)m_K^2$
	$\Omega_{cc}\pi \to \Xi_{cc}\bar{K}$	-2	0	$2\ [\Xi_{cc}]$	$-6b_2(m_K^2 + m_\pi^2)$
$(0, \frac{1}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	4	$3\ [\Xi_{cc}]$	$-1\ [\Xi_{cc}]$	$-4(6b_1 + b_2)m_\pi^2$
	$\Xi_{cc}\eta \to \Xi_{cc}\eta$	0	$\frac{1}{3}\ [\Xi_{cc}]$	$\frac{1}{3}\ [\Xi_{cc}]$	$-\frac{32}{3}(3b_1 - b_2)m_K^2 + (8b_1 - \frac{20}{3}b_2)m_\pi^2$
	$\Omega_{cc}K \to \Omega_{cc}K$	2	$2\ [\Xi_{cc}]$	0	$-4(6b_1 + b_2)m_K^2$
	$\Xi_{cc}\pi \to \Xi_{cc}\eta$	0	$1\ [\Xi_{cc}]$	$1\ [\Xi_{cc}]$	$-12b_2m_\pi^2$
	$\Xi_{cc}\pi \to \Omega_{cc}K$	$\sqrt{6}$	$\sqrt{6}\ [\Xi_{cc}]$	0	$-3\sqrt{6}b_2(m_K^2 + m_\pi^2)$
	$\Xi_{cc}\eta \to \Omega_{cc}K$	$\sqrt{6}$	$\frac{\sqrt{6}}{3}\ [\Xi_{cc}]$	$-\frac{2\sqrt{6}}{3}\ [\Omega_{cc}]$	$\sqrt{6}b_2(5m_K^2 - 3m_\pi^2)$

Loop amplitudes



Loop amplitudes



Power counting issue due to the baryon propagators in the loops !

Power counting breaking problem & Solutions

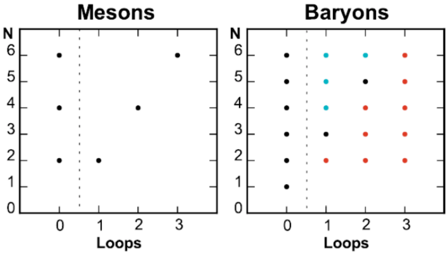
- **PCB problem in Baryon Chiral Perturbation Theory (BChPT)**

Feynman Diagram

⇩

$\mathcal{O}(p^N)$

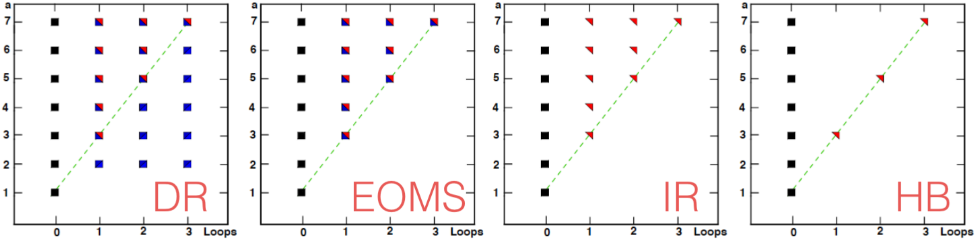
How important?



red dots denote possible PCB terms

Chiral order: $N = 4L - 2N_M - N_B + \Sigma_k k V_k$

- **Solutions (we prefer EOMS)**



Extended-On-Mass-Shell (EOMS) scheme

- **Essence:** two-step renormalization ($\widetilde{\mathrm{MS}}$+finite)

1. UV subtraction:

$$m = m^r(\mu) + \beta_m \frac{R}{16\pi^2 F^2} ,$$
$$g = g^r(\mu) + \beta_g \frac{R}{16\pi^2 F^2} ,$$
$$b_i = b_i^r(\mu) + \beta_{b_i} \frac{R}{16\pi^2 F^2} ,$$
$$c_j = c_j^r(\mu) + \beta_{c_j} \frac{R}{16\pi^2 F^2} .$$

2. Finite subtraction:

$$m^r(\mu) = \widetilde{m} + \frac{\widetilde{\beta}_m}{16\pi^2 F^2} ,$$
$$g^r(\mu) = \widetilde{g} + \frac{\widetilde{\beta}_g}{16\pi^2 F^2} ,$$
$$b_i^r(\mu) = \widetilde{b}_i + \frac{\widetilde{\beta}_{b_i}}{16\pi^2 F^2} .$$

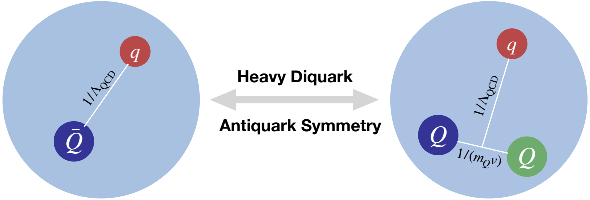
- **Advantages:**
  - Power counting is restored $\longrightarrow$ predictive power
  - Respect original analytic properties $\longrightarrow$ spectroscopy (poles and cuts), chiral extrapolation, finite volume corrections
  - Fast convergency behaviour in many cases, w.r.t. IR, HB, etc

Low energy constants

❑ Priority principle for the LEC determination

- ☞ Experimental or lattice QCD data $\Longrightarrow$ $[b_1, b_2]$
- ☞ Symmetries & resonance saturation $\xrightarrow{\text{HDAS}}$ $[g, b_3, b_4, b_5, b_6, c_{11}, c_{12}, c_{20}]$
- ☞ Naturalness $\Longrightarrow$ $[b_7, c_{13-19}]$

❑ Heavy Diquark Antiquark Symmetry (HDAS)



- ☞ In HQL, the heavy diquark belongs to the color $\overline{3}_c$ representation and serves as a static color source for the light quarks.
- ☞ The same color dynamics arises in the mesons containing a single heavy antiquark.

Heavy diquark antiquark symmetry

- **Unknown LECs connected with the ones in the charmed sector.**
  - Lagrangian with super field $\mathcal{S} \equiv \mathrm{diag}\{\tilde{\bar{H}}, T\}$, where $\psi^{(\prime)} \in T$ and $D^{(*)} \in H$.
  - The $D\phi$ interaction is well determined!

Relations by HDAS

$$g = -\frac{1}{3\overline{m}_D} g_0,$$

$$b_1 = -\frac{1}{2\overline{m}_D}(h_0 + \frac{1}{3}h_1), \quad b_2 = -\frac{1}{2\overline{m}_D}h_1,$$

$$b_3 = -\frac{1}{2\overline{m}_D}h_3, \quad b_4 = \frac{1}{2\overline{m}_D}h_2,$$

$$b_5 = \frac{\overline{m}_D}{8}h_5, \quad b_6 = -\frac{\overline{m}_D}{8}h_4,$$

$$c_{11} = \frac{g_2}{2}, \quad c_{12} = \frac{\overline{m}_D^2}{4}g_3, \quad c_{20} = -\frac{g_1}{2}.$$

Being able to make predictions now!

$\psi\phi$ scattering		$D\phi$ scattering	
LECs	Value	LECs	Value
$\tilde{g}$	-0.19	$\tilde{g}_0$	1.095
$\tilde{b}_1$	-0.04	$\tilde{h}_0$	0.0172
$\tilde{b}_2$	-0.11	$\tilde{h}_1$	0.4266
$\tilde{b}_3$	$-1.46^{+0.43}_{-0.46}$	$\tilde{h}_3$	$5.59^{-2.07}_{-1.96}$
$\tilde{b}_4$	0.66 ± 0.19	$\tilde{h}_2$	$2.52^{+0.73}_{-0.74}$
$\tilde{b}_5$	$-0.17^{+0.05}_{-0.06}$	$\tilde{h}_5$	$-0.71^{+0.23}_{-0.24}$
$\tilde{b}_6$	0.11 ± 0.04	$\tilde{h}_4$	$-0.47^{+0.17}_{-0.17}$
$\tilde{c}_{11}$	$-0.08^{+0.21}_{-0.14}$	$\tilde{g}_2$	$-0.16^{+0.52}_{-0.39}$
$\tilde{c}_{12}$	$0.08^{+0.03}_{-0.02}$	$\tilde{g}_3$	$0.08^{+0.03}_{-0.03}$
$\tilde{c}_{20}$	$0.49^{+0.09}_{-0.15}$	$\tilde{g}_1$	$-0.99^{+0.30}_{-0.18}$

Basics of scattering lengths

- Partial-wave amplitude:

$$f_{\ell\pm}^{(S,I)}(s) = \frac{1}{16\pi\sqrt{s}} \left\{ (E+m_\psi)\left[A_\ell^{(S,I)}(s) + \left(\sqrt{s}-m_\psi\right) B_\ell^{(S,I)}(s)\right] + (E-m_\psi)\left[-A_{\ell\pm1}^{(S,I)}(s) + \left(\sqrt{s}+m_\psi\right) B_{\ell\pm1}^{(S,I)}(s)\right] \right\}.$$

- Generic definition of scattering lengths:

$$a_{\ell\pm} = \lim_{|\mathbf{q}|\to 0} \frac{f_{\ell\pm}(s)}{\mathbf{q}^{2\ell}} \qquad \text{(numerical } 0/0 \text{ problem)}$$

- Formulae for S- and P-wave scattering lengths

$$a_{0+}^{(S,I)} = \frac{m_\psi}{4\pi(m_\psi+m_\phi)} \left\{ \left[A^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} + m_\phi \left[B^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} \right\},$$

$$a_{1+}^{(S,I)} = \frac{m_\psi}{6\pi(m_\psi+m_\phi)} \left\{ \left[\partial_t A^{(S,I)}(s,t)\right]_{t=0,\mathbf{q}^2=0} + m_\phi \left[\partial_t B^{(S,I)}(s,t)\right]_{t=0,\mathbf{q}^2=0} \right\},$$

$$a_{1-}^{(S,I)} = a_{1+}^{(S,I)} - \frac{1}{16\pi m_\psi (m_\psi+m_\phi)} \left\{ \left[A^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} - (2m_\psi+m_\phi) \left[B^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} \right\}.$$

Therefore, the S- and P-wave scattering lengths can be calculated analytically!

S-wave scattering lengths: $J^P = (1/2)^-$

(S,I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$ Tree	$\mathcal{O}(p^3)$ Loop	Total-EOMS	HB
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K}\to\Omega_{cc}\bar{K}$	-0.27	0.29	-0.11	-0.001	$-0.09^{+0.12}_{-0.13}$	$-0.20(1)$
$(1,1)$	$\Xi_{cc}K\to\Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	$-0.25(1)$
$(1,0)$	$\Xi_{cc}K\to\Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	$0.92(2)$
$(0,\frac{3}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	$-0.10(2)$
$(-1,0)$	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	0.54	0.24	0.25	0.16	$1.19^{+0.22}_{-0.21}$	$2.15(11)$
	$\Omega_{cc}\eta\to\Omega_{cc}\eta$	-0.001	0.37	0.0	$0.05+0.55i$	$0.42^{+0.18}_{-0.19}+0.55i$	$0.57(3)+0.21i$
$(-1,1)$	$\Omega_{cc}\pi\to\Omega_{cc}\pi$	0.0	0.04	0.0	-0.04	-0.01 ± 0.02	$-0.002(1)$
	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	0.0	0.31	0.0	$-0.04+0.10i$	$0.27^{+0.13}_{-0.13}+0.10i$	$0.26(1)+0.19i$
$(0,\frac{1}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	0.25	0.04	0.01	0.04	0.34 ± 0.02	$0.36(1)$
	$\Xi_{cc}\eta\to\Xi_{cc}\eta$	-0.001	0.32	0.0	-0.26	$0.06^{+0.14}_{-0.15}$	$0.34(1)+0.10i$
	$\Omega_{cc}K\to\Omega_{cc}K$	0.27	0.29	0.11	$-0.01+0.55i$	$0.66^{+0.13}_{-0.13}+0.55i$	$1.18(6)+0.29i$

- **Differences between EOMS and HB results:**
  - ☞ Rel. corrections;
  - ☞ HQL-vanishing diagrams;
  - ☞ Res.-exchange contributions

P-wave scattering lengths: $J^P = (3/2)^+$

(S,I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$ Tree	$\mathcal{O}(p^3)$ Loop	Total
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K}\to\Omega_{cc}\bar{K}$	0.16	0.60	-0.22	-3.00	$-2.47^{+3.04}_{-2.64}$
$(1,1)$	$\Xi_{cc}K\to\Xi_{cc}K$	0.10	0.59	-0.22	-1.19	$-0.73^{+3.02}_{-2.64}$
$(1,0)$	$\Xi_{cc}K\to\Xi_{cc}K$	-0.10	-8.77	0.22	1.71	$-6.93^{+2.83}_{-3.21}$
$(0,\frac{3}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	0.62	0.75	-0.18	-41.8	$-40.6^{+3.20}_{-2.97}$
$(-1,0)$	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	0.0	5.27	0.45	0.48	$6.19^{+4.78}_{-5.40}$
	$\Omega_{cc}\eta\to\Omega_{cc}\eta$	0.07	2.0	0.0	$-1.13+0.01i$	$0.93^{+2.04}_{-1.96}+0.01i$
$(-1,1)$	$\Omega_{cc}\pi\to\Omega_{cc}\pi$	0.0	-6.23	0.001	-0.10	$-6.32^{+1.85}_{-1.82}$
	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	0.0	-4.09	0.0	$-0.11+0.01i$	$-4.2^{+1.23}_{-1.21}+0.01i$
$(0,\frac{1}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	-0.31	0.75	0.35	21.2	$21.9^{+3.39}_{-3.70}$
	$\Xi_{cc}\eta\to\Xi_{cc}\eta$	0.02	-2.31	0.0	$-0.01+0.01i$	$-2.30^{+1.13}_{-1.13}+0.01i$
	$\Omega_{cc}K\to\Omega_{cc}K$	0.0	0.6	0.22	$0.19+0.01i$	$1.0^{+2.67}_{-3.01}+0.01i$

- The NNLO loop contribution turns out to be large for some channels
  - The size of NNLO trees might be underestimated, due to the poor information on the NNLO LECs.

P-wave scattering lengths: $J^P = (1/2)^+$

(S, I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$ Tree	$\mathcal{O}(p^3)$ Loop	Total
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	-0.38	0.58	-0.34	0.02	$-0.13^{+3.03}_{-2.64}$
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	-0.36	0.57	-0.37	-1.74	$-1.90^{+3.01}_{-2.61}$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	0.36	-8.80	0.37	0.46	$-7.59^{+2.82}_{-3.20}$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	-0.80	0.75	-0.2	19.5	$19.3^{+3.19}_{-2.97}$
$(-1, 0)$	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	0.16	5.25	0.74	-9.77	$-3.61^{+4.77}_{-5.37}$
	$\Omega_{cc}\eta \to \Omega_{cc}\eta$	-0.13	1.97	0.0	$-2.16 + 0.01i$	$-0.32^{+2.03}_{-1.95} + 0.01i$
$(-1, 1)$	$\Omega_{cc}\pi \to \Omega_{cc}\pi$	0.0	-6.23	0.0	-0.53	$-6.75^{+1.85}_{-1.82}$
	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	0.0	-4.11	0.0	$-0.60 + 0.01i$	$-4.72^{+1.24}_{-1.22} + 0.01i$
$(0, \frac{1}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	-0.27	0.75	0.39	-104.9	$-104.1^{+3.38}_{-3.70}$
	$\Xi_{cc}\eta \to \Xi_{cc}\eta$	-0.03	-2.33	0.0	$-1.43 + 0.01i$	$-3.79^{+1.13}_{-1.14} + 0.01i$
	$\Omega_{cc}K \to \Omega_{cc}K$	0.16	0.58	0.34	$-3.77 + 0.01i$	$-2.69^{+2.67}_{-3.00} + 0.01i$

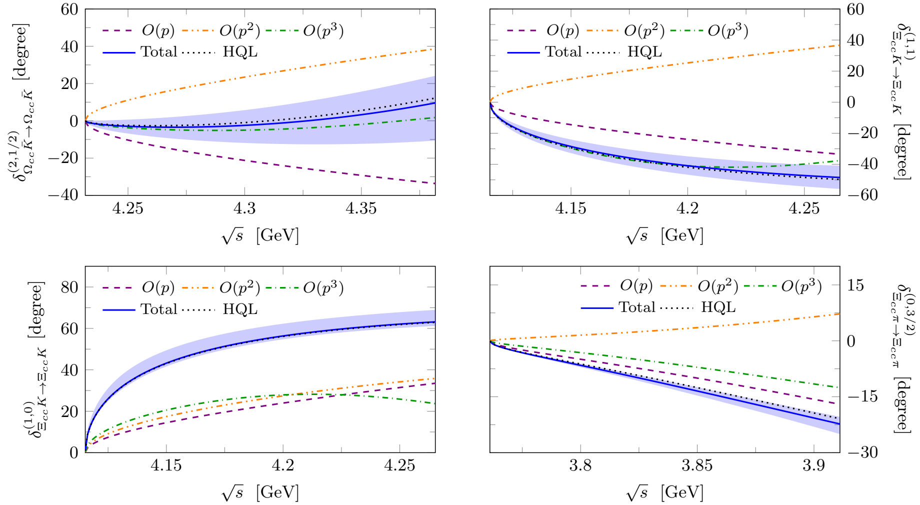
Results for P-wave scattering lengths for DCBs for the first time.

Full results vs HQL results

(S,I)	Processes	a_{0+} $[J^P=\frac{1}{2}^-]$ Full results	a_{0+} $[J^P=\frac{1}{2}^-]$ HQL results	a_{1+} $[J^P=\frac{3}{2}^+]$ Full results	a_{1+} $[J^P=\frac{3}{2}^+]$ HQL results
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K}$	$-0.09^{+0.12}_{-0.13}$	$-0.08^{+0.12}_{-0.13}$	$-2.47^{+3.04}_{-2.64}$	$0.34^{+3.04}_{-2.64}$
$(1,1)$	$\Xi_{cc}K$	-0.60 ± 0.13	-0.62 ± 0.13	$-0.73^{+3.02}_{-2.64}$	$0.39^{+3.02}_{-2.62}$
$(1,0)$	$\Xi_{cc}K$	1.03 ± 0.19	1.03 ± 0.19	$-6.93^{+2.83}_{-3.21}$	$-8.19^{+2.83}_{-3.21}$
$(0,\frac{3}{2})$	$\Xi_{cc}\pi$	-0.16 ± 0.02	-0.15 ± 0.02	$-40.6^{+3.20}_{-2.97}$	$0.63^{+3.20}_{-2.97}$
$(-1,0)$	$\Xi_{cc}\bar{K}$	$1.19^{+0.22}_{-0.21}$	$1.19^{+0.22}_{-0.21}$	$6.19^{+4.78}_{-5.40}$	$6.36^{+4.78}_{-5.40}$
	$\Omega_{cc}\eta$	$0.42^{+0.18}_{-0.19}+0.55i$	$0.42^{+0.18}_{-0.19}+0.56i$	$0.93^{+2.04}_{-1.96}+0.01i$	$2.03^{+2.04}_{-1.96}$
$(-1,1)$	$\Omega_{cc}\pi$	-0.01 ± 0.02	0.0 ± 0.02	$-6.32^{+1.85}_{-1.82}$	$-6.23^{+1.85}_{-1.82}$
	$\Xi_{cc}\bar{K}$	$0.27^{+0.13}_{-0.13}+0.10i$	$0.28^{+0.13}_{-0.13}+0.10i$	$-4.2^{+1.23}_{-1.21}+0.01i$	$-4.14^{+1.23}_{-1.21}$
$(0,\frac{1}{2})$	$\Xi_{cc}\pi$	0.34 ± 0.02	0.33 ± 0.02	$21.9^{+3.39}_{-3.70}$	$1.41^{+3.39}_{-3.70}$
	$\Xi_{cc}\eta$	$0.06^{+0.14}_{-0.15}$	$0.05^{+0.14}_{-0.15}$	$-2.30^{+1.13}_{-1.13}+0.01i$	-2.16 ± 1.13
	$\Omega_{cc}K$	$0.66^{+0.13}_{-0.13}+0.55i$	$0.64^{+0.13}_{-0.13}+0.55i$	$-2.30^{+1.13}_{-1.13}+0.01i$	$1.14^{+2.67}_{-3.01}$

Those HQL-vanishing diagrams are significant for P-wave scattering lengths.

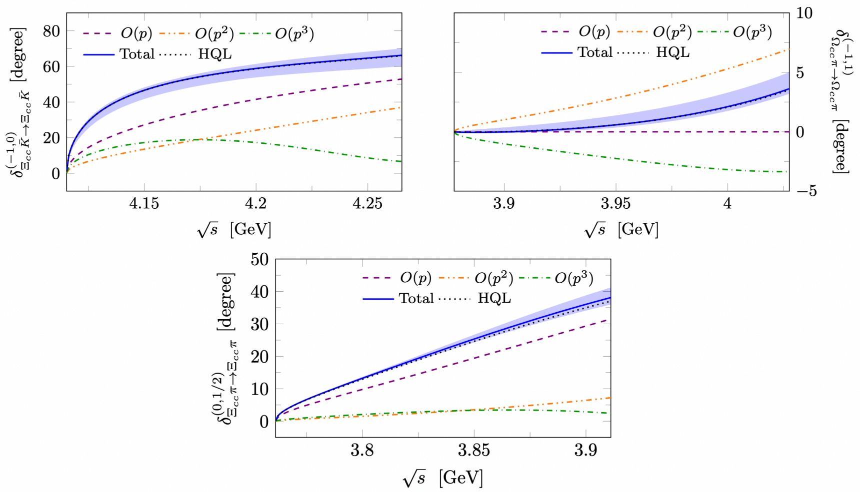
S-wave phase shifts: single channels



- Cancellation exists between the LO and NLO contributions.
- Suffer from poor convergence property as usual in SU(3) ChPT!
- The HQL-vanishing diagrams contribute negligible!

S-wave phase shifts: coupled channels

- Only for elastic processes in elastic scattering regions



☞ Same conclusion can be drawn as the single-channel cases, except for the $\Xi_{cc}\pi(I=1/2)$ channel.

III. Summary and Outlook

Summary and outlook

- **One-loop analysis of the $\psi\phi$ interactions within relativistic BChPT using EOMS**
  - Analytical chiral amplitudes are obtained, HDAS is used to estimate the low-energy constants.
  - S- and P-wave scattering lengths are predicted.
  - S-wave phase shifts for elastic scattering processes are presented in the energy region near threshold.
- **To draw more solid conclusions, one needs more data from Lattice QCD.**
  - Lattice computation of all the single channels is ongoing.
  - Coupled channels with dis-connected diagrams, more types of interpolating operators.

ChPT & Lattice QCD $\rightarrow$ the spectroscopy of doubly charmed baryons

Thank you very much!

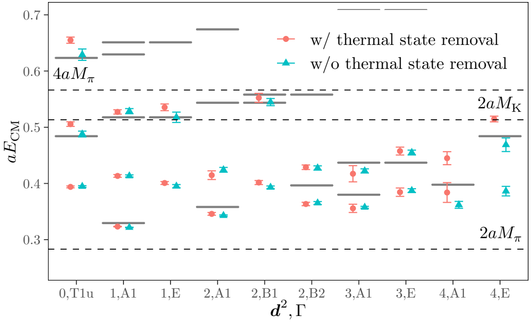
Backup

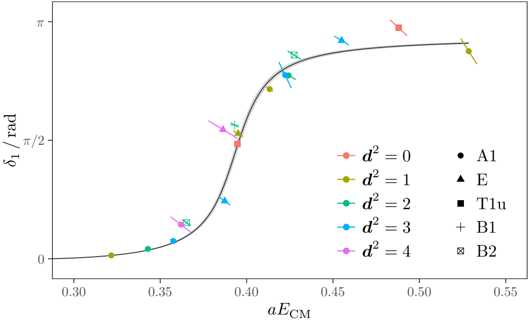
Why phase shifts?

- **Disclaimer:**
  - Challenging to measure phase shifts experimentally!
  - Yet, Lüscher formula links phase shifts to energy levels from lattice simulations!
- **Lüscher formula**

$$\det[\mathbf{1} + \mathrm{i}\rho \cdot \mathbf{t} \cdot (1 + \mathrm{i}\mathbf{M})] = 0$$

  - $\mathbf{t}$ is infinite-volume scattering matrix (ERE, S-matrix parametrization)
  - $\mathbf{M}(E_{cm}, L)$ encodes finite-volume information





ρ meson from $\pi\pi$ scatteing [M. Werner, et al., Eur.Phys.J.A 56 (2020) 2, 61]