



Correlation function and the inverse problem in the BD interaction

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2024 年 10 月 21 日

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Based on: Chin.Phys.C 48 (2024) 5, 053107

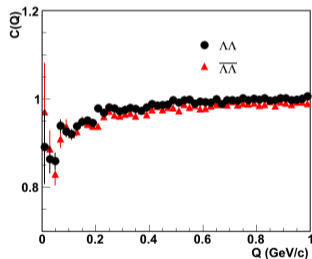


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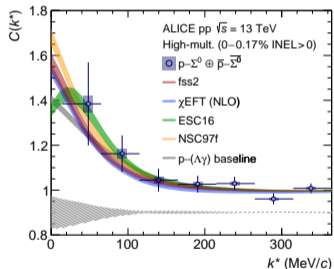
1. Background and motivation



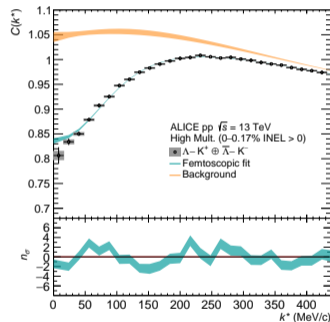
Increasing experimental data on correlation functions.



STAR /PRL 114 (2015)



ALICE /PLB 805 (2020)



ALICE /PLB 845 (2023)

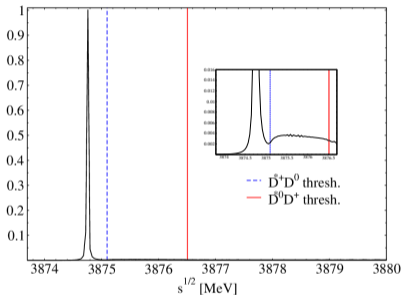
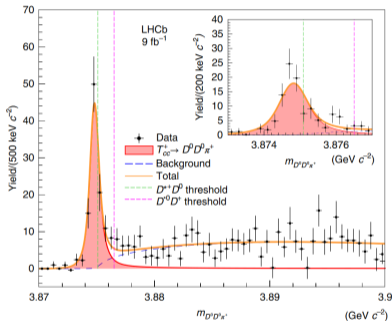


1. Background and motivation

Studied D^*D and $B(B^*)D(D^*)$ interaction within the chiral unitary approach framework.

LHCb / NP 18 (2022) 7
 $T_{cc}^+ \rightarrow D^0 D^0 \pi^+$

A. Feijoo, W.H. Liang, Eulogio Oset / PRD 104 (2021)
 $\Gamma(T_{cc} \rightarrow D^0 D^0 \pi^+)$



S. Sakai, L. Roca, E. Oset / Phys.Rev.D 96 (2017) 5, 054023

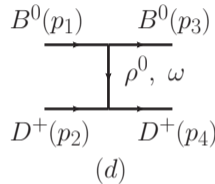
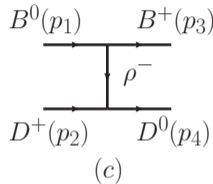
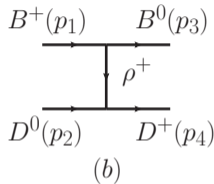
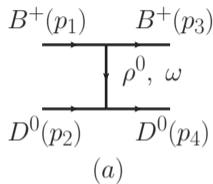
	$I(J^P)$	$\sqrt{s_p}$	B	g	a [fm]
BD	$0(0^+)$	7133 7111	15 38	33484 49867	-1.78 -1.45
B^*D	$0(1^+)$	7179 7156	15 38	33742 50243	-1.78 -1.45



2. Formalism

The chiral unitary approach

Two coupled channels: $D^0 B^+$, $D^+ B^0$ (without Coulomb interaction).



$$V_{ij} = -\frac{1}{4f^2} C_{ij} (p_1 + p_3) \cdot (p_2 + p_4); \quad f = 93 \text{ MeV}, \quad C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

This formalism is similar to that used for $D^* D$ interaction.



2. Formalism

The chiral unitary approach

Amplitude

$$T = [1 - VG]^{-1}V,$$

$$G_i(\sqrt{s}) = \frac{\int_{|\vec{q}| < q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1(q) + \omega_2(q)}{2\omega_1(q)\omega_2(q)}}{1 - \frac{1}{s - [\omega_1(q) + \omega_2(q)]^2 + i\varepsilon}},$$

$$\omega_1(q) = \sqrt{\vec{q}^2 + m_i^2}, \quad \omega_2(q) = \sqrt{\vec{q}^2 + M_i^2}.$$

Only one free parameter q_{\max} ,
used $q_{\max} = 420$ MeV

Couplings

$$T_{ij} = \frac{g_i g_j}{s - s_p},$$

$$g_1^2 = \lim_{s \rightarrow s_p} (s - s_p) T_{11},$$

$$g_1 g_j = \lim_{s \rightarrow s_p} (s - s_p) T_{1j},$$



2. Formalism

The chiral unitary approach

Probabilities and the wave function at the origin

$$\mathcal{P}_i = -g_i^2 \left. \frac{\partial G_i}{\partial s} \right|_{s=s_p}.$$

$$\psi_i(r=0) = g_i G_i|_{s=s_p}.$$

If $P_1 + P_2 = 1$, completely molecular

Observables

$$T = -8\pi\sqrt{s} f^{QM} \\ \simeq -8\pi\sqrt{s} \frac{1}{-\frac{1}{a} + \frac{1}{2}r_0k^2 - ik},$$

$$-\frac{1}{a_i} = (-8\pi\sqrt{s} T_{ii}^{-1})|_{s_{th,i}},$$

$$r_{0,i} = \left[\frac{2\sqrt{s}}{\mu_i} \frac{\partial}{\partial s} (-8\pi\sqrt{s} T_{ii}^{-1} + ik_i) \right]_{s_{th,i}},$$



2. Formalism

Correlation functions and inverse problem

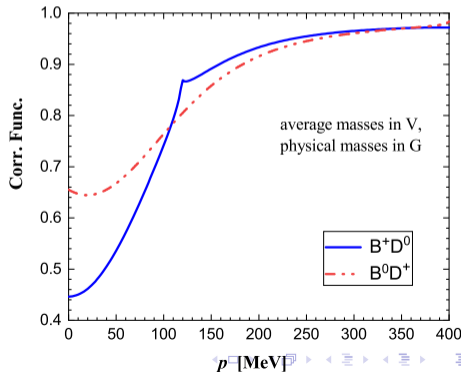
$$C_i(p) = 1 + 4\pi\theta(q_{\max} - p) \int dr^2 S_{12}(r) \left\{ \left| j_0(pr) + T_{ii}(E) \tilde{G}^i(r, E) \right|^2 + \sum_{j \neq i} \left| T_{ij}(E) \tilde{G}^j(r, E) \right|^2 - j_0^2(pr) \right\}$$

$$\tilde{G}^{(i)}(r; E) = \int \frac{d^3q}{(2\pi)^3} \frac{\omega_1(q) + \omega_2(q)}{2\omega_1(q)\omega_2(q)} \frac{j_0(qr)}{s - [\omega_1(q) + \omega_2(q)]^2 + i\varepsilon}, \quad E = \sqrt{s}$$

Source function

$$S_{12}(r) = \frac{1}{(\sqrt{4\pi} R)^3} e^{-(r^2/4R^2)},$$

$R = 1 \text{ fm}$

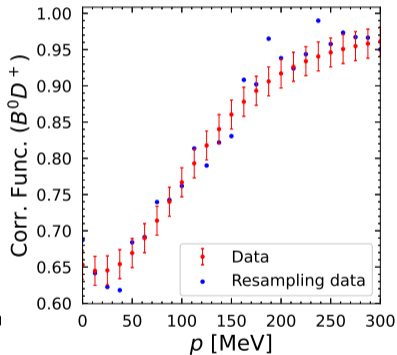
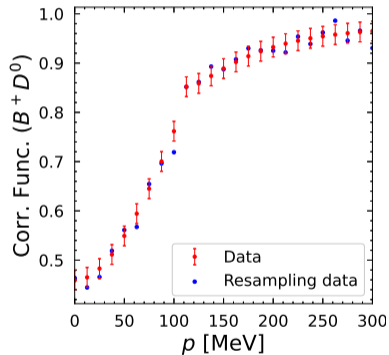


2. Formalism

Correlation function and inverse problem



Resampling



Resample 25 points for each channel for fitting, and run the process **50 times**. Assuming errors at the order of ± 0.02 .



2. Formalism

Correlation functions and inverse problem

General framework

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{pmatrix},$$

considering isospin symmetric

$$\langle BD, I = 0 | V | BD, I = 1 \rangle = \frac{1}{2}(V_{11} - V_{22}) = 0, \quad \Rightarrow \quad V_{22} = V_{11}.$$

$$V_{11} = V'_{11} + \frac{\alpha}{m_V^2}(s - s_{\text{th},1}), \quad V_{12} = V'_{12} + \frac{\beta}{m_V^2}(s - s_{\text{th},1}), \quad M_V = 800 \text{ MeV}$$

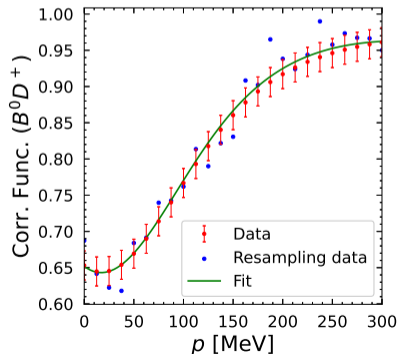
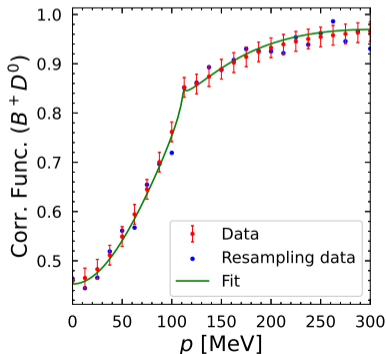
6 free parameters: V'_{11} , V'_{23} , α , β , q_{max} and R .

3. Results and dicussions



Average values and dispersion for the parameters.

V'_{11}	V'_{12}	α
-1537.54 ± 918.93	-1512.32 ± 913.55	-31.89 ± 10.00
β	q_{\max} (MeV)	R (fm)
-165.16 ± 133.31	407.9 ± 60.8	1.01 ± 0.04





Fitting results: Average values and dispersion of the pole position and couplings. [in units of MeV]

$\sqrt{s_p}$	$g_1(D^0 B^+)$	$g_2(D^+ B^0)$
7107.84 ± 17.79	34623.08 ± 14300.15	34506.64 ± 14304.57

Theoretical results: Pole position and couplings with $q_{\max} = 420$ MeV. [in units of MeV]

$\sqrt{s_p}$	$g_1(D^0 B^+)$	$g_2(D^+ B^0)$
$(7110.41 + 0i)$	31636.8	31631.0

3. Results and discussions



Fitting results: Average value and dispersion for the probability \mathcal{P}_i and wave function at the origin $\psi_i(r=0)$ for channel i .

$\mathcal{P}_1(D^0 B^+)$	$\mathcal{P}_2(D^+ B^0)$	$\psi_1(r=0)$	$\psi_2(r=0)$
0.49 ± 0.03	0.42 ± 0.03	-13.61 ± 2.65	-12.61 ± 2.423

$$\mathcal{P}_1 + \mathcal{P}_2 = 0.91$$

Theoretical results: Probability \mathcal{P}_i and wave function at the origin $\psi_i(r=0)$ for channel i .

$\mathcal{P}_1(D^0 B^+)$	$\mathcal{P}_2(D^+ B^0)$	$\psi_1(r=0)$	$\psi_2(r=0)$
0.52	0.44	-14.75	-13.61

$$\mathcal{P}_1 + \mathcal{P}_2 = 0.96$$

3. Results and discussions



Fitting results: Average value and dispersion for scattering length a_i and effective range r_i for channel i . [in units of fm]

$a_1(D^0 B^+)$	$a_2(D^+ B^0)$
0.72 ± 0.03	$(0.51 \pm 0.02) - (0.17 \pm 0.01)i$
r_1	r_2
-0.61 ± 0.19	$(1.41 \pm 0.28) - (1.65 \pm 0.07)i$

Theoretical results: Scattering length a_i and effective range r_i for channel i . [in units of fm]

a_1	a_2	r_1	r_2
0.71	$0.50 - 0.16i$	-0.61	$1.22 - 1.77i$



- Studied BD interaction within the chiral unitary approach framework and obtained a bound state at 7110 MeV, indicating a fully molecular state.
- Studied the inverse problem of obtaining observables from correlation functions. The correlation functions contain information below the threshold and can serve as a useful tool for studying hadron interactions.

Thank You!