

# Isospin-breaking contribution to the model-independent axion-photon-photon coupling in $U(3)$ chiral theory

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## Abstract

We study the mixing of the QCD/QCD-like axion and light-flavor mesons  $\pi^0, \eta, \eta'$  within the framework of  $U(3)$  chiral perturbation theory up to next-to-leading order and the first-order term of isospin breaking in this work. The axion-meson mixing formulas are calculated order by order in the  $U(3)$   $\delta$ -expansion scheme, namely the joint expansions of the momentum, light-quark masses and  $1/N_c$ . The two-photon couplings of the light-flavor mesons, together with the axion, are also investigated in the  $U(3)$  chiral theory up to next-to-leading order and the first-order term of isospin breaking in the  $\delta$ -counting scheme.

$$\mathcal{L}^{\text{LO}} = \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{12} M_0^2 X^2$$

$$\mathcal{L}^{\text{NLO}} = L_5 \langle u^\mu u_\mu \chi_+ \rangle + \frac{L_8}{2} \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle - \frac{F^2 \Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2 \Lambda_2}{12} X \langle \chi_- \rangle$$

This  $U(3)$  chiral perturbation theory Lagrangian is used to calculate the axion-meson mixing.

$$\mathcal{L}_{WZW}^{\text{LO}} = -\frac{3\sqrt{2}e^2}{8\pi^2 F} \varepsilon_{\mu\nu\rho\sigma} \partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2 \Phi \rangle$$

$$\mathcal{L}_{WZW}^{\text{NLO}} = it_1 \varepsilon_{\mu\nu\rho\sigma} \langle f_+^{\mu\nu} f_+^{\rho\sigma} \chi_- \rangle - ik_3 \varepsilon_{\mu\nu\rho\sigma} \langle f_+^{\mu\nu} f_+^{\rho\sigma} \rangle X$$

This  $U(3)$  chiral perturbation theory Lagrangian is used to calculate the  $\phi$  ( $\phi = \pi^0, \eta, \eta', a$ ) to  $\gamma\gamma$ .

## Results and Discussions

Up to NLO the relations between the physical states (denoted by the hatted fields on the left) and the bare ones (on the right) reduce to

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = \begin{pmatrix} 1+z_{11} & c_\theta(-v_{12}+z_{12})+s_\theta(-v_{13}+z_{13}) & -s_\theta(-v_{12}+z_{12})+c_\theta(-v_{13}+z_{13}) & -v_{41}+z_{14} \\ v_{12}+z_{21} & c_\theta(1+z_{22})+s_\theta(z_{23}-v_{23}) & -s_\theta(1+z_{22})+c_\theta(z_{23}-v_{23}) & -v_{42}+z_{24} \\ v_{13}+z_{31} & c_\theta(z_{32}+v_{23})+s_\theta(1+z_{33}) & -s_\theta(z_{32}+v_{23})+c_\theta(1+z_{33}) & -v_{43}+z_{34} \\ v_{14}+z_{41} & c_\theta(v_{24}+z_{42})+s_\theta(v_{34}+z_{43}) & -s_\theta(v_{24}+z_{42})+c_\theta(v_{34}+z_{43}) & 1+v_{44}+z_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

where the NLO corrections are collected in the  $z_{ij}$

In our previous work, we fix the unknown low-energy constants (LECs) of  $\chi_{PT}$  by performing fits to relevant lattice data

Parameters	NLO Fit
$F(\text{MeV})$	$91.05^{+0.39}_{-0.43}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.05}$
$10^3 \times L_8$	$0.89^{+0.03}_{-0.04}$
$\Lambda_1$	$-0.17^{+0.04}_{-0.05}$
$\Lambda_2$	$0.06^{+0.07}_{-0.08}$
$\chi^2/(d.o.f)$	221.0/(111-5)

Table.1: The values of the LECs from the NLO fit

Two-photon couplings:

The two-photon decay amplitude of  $\phi \rightarrow \gamma(k_1)\gamma(k_2)$  with  $\phi = \pi^0, \eta, \eta'$  and  $a$  can be written as

$$T_{\phi \rightarrow \gamma\gamma} = e^2 \varepsilon_{\mu\nu\rho\sigma} k_1^\mu \epsilon_1^\nu k_2^\rho \epsilon_2^\sigma F_{\phi\gamma\gamma}$$

We use the decay widths of  $\pi^0 \rightarrow \gamma\gamma$ ,  $\eta \rightarrow \gamma\gamma$  and  $\eta' \rightarrow \gamma\gamma$  from the most recent PDG average

The explicit values of the NLO LECs from the WZW Lagrangian turn out to be

$$t_1 = -(3.4 \pm 2.4) \times 10^{-4} \text{GeV}^{-2}, \quad k_3 = (1.19 \pm 0.23) \times 10^{-4} \text{ (preliminary)}$$

leading to

$$F_{\pi^0\gamma\gamma} = 0.280 \pm 0.0004 \text{GeV}^{-1},$$

$$F_{\eta\gamma\gamma} = 0.276 \pm 0.009 \text{GeV}^{-1}, \quad \text{(preliminary)}$$

$$F_{\eta'\gamma\gamma} = 0.342 \pm 0.012 \text{GeV}^{-1},$$

We can now give our prediction to the two-photon coupling of the axion  $F_{a\gamma\gamma}$  up to NLO in the  $\delta$ .

$$F_{a\gamma\gamma} = -\frac{[20.1 + 3.4 + 0.5 \pm 0.1] \times 10^{-3}}{f_a} \quad \text{(preliminary)}$$

where the first entry in the numerator on the right hand side corresponds to the LO contribution and the second one denotes the LO first order IB contribution and the third denotes the NLO contribution.  $F_{a\gamma\gamma}$  is related with the  $g_{a\gamma\gamma}$  used via:

$$g_{a\gamma\gamma} = 4\pi\alpha_{em}F_{a\gamma\gamma} = -\frac{\alpha_{em}}{2\pi f_a} (1.89 \pm 0.01) \quad \text{(preliminary)}$$

This can be compared with the the numbers ( $1.63 \pm 0.01$ ) inside the bracket in our previous work that did not consider isospin breaking from the  $U(3)$   $\chi_{PT}$  and the numbers is  $1.92 \pm 0.04$  and  $2.05 \pm 0.03$  from the  $SU(2)$  and  $SU(3)$   $\chi_{PT}$  analyses up to NLO, respectively.

## Conclusion

1. Compared to our previous work, in this study, we calculated the axion-meson mixing matrix to the next-to-leading order and the first order of IB within the framework of  $U(3)$  chiral perturbation theory. Then, we predicted the process of axion to  $\gamma\gamma$ .

2. When we additionally retain the first-order IB terms, the value of  $g_{a\gamma\gamma}$  obtained is consistent with the results in the  $SU(2)$  and  $SU(3)$  cases; however, the value from our previous work is somewhat smaller.

## References

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