Revisiting O(N) σ model at unphysical pion masses and high temperatures

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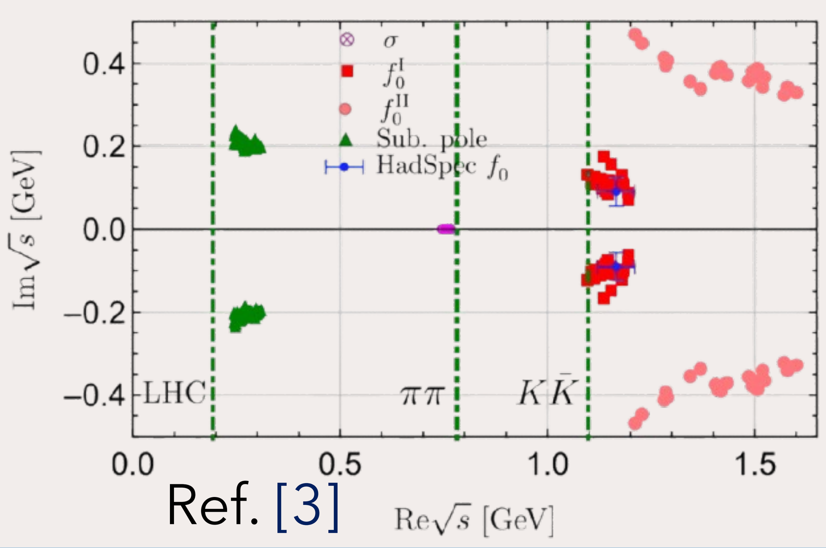
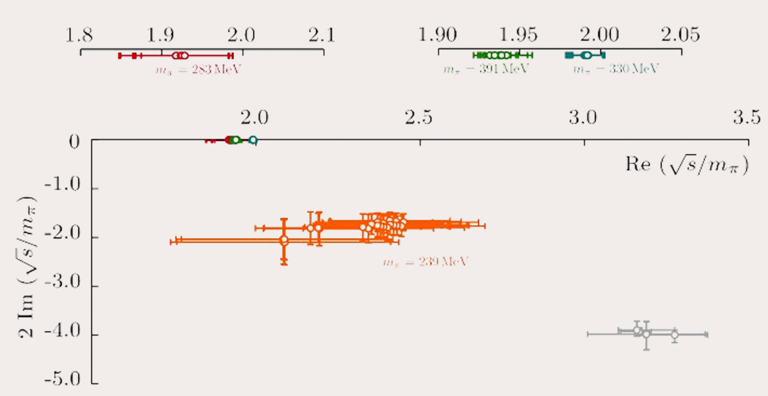
Introduction

- Lattice-QCD studies found that $\sigma/f_0(500)$ transforms into a **bound state** at large unphysical m_π (~ 391, 330 MeV) [1, 2].
- Roy equation study of $\pi\pi$ scattering lattice data reaveals the pole structure with large pion masses [3, 4]. In addition, there is a pair of ***subthreshold*** poles generated by crossing symmetry [3].
- This study in Refs. [5, 6] uses the N/D method to partially recover crossing symmetry of the O(N) linear σ model amplitude at leading order of 1/N expansion, and qualitatively reproduce the pole structure and pole trajectories with varying pion masses as revealed by Roy-equation analyses. The σ pole trajectory with varying temperature is also discussed and found to be similar to its properties when varying m_π.



Ref. [3]



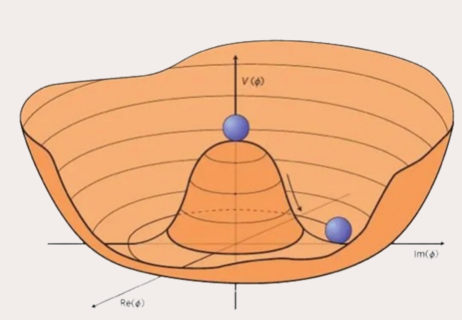
Ref. [4]

O(N) model at large N limit

- O(N) model (with explicit symmetry breaking)

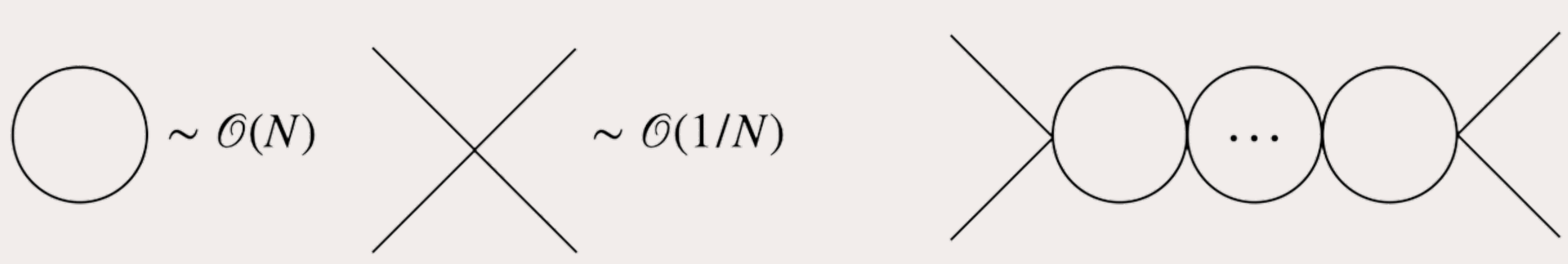
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - \frac{1}{2}\mu_0^2\phi_a\phi_a - \frac{\lambda_0}{8N}(\phi_a\phi_a)^2 + \alpha\phi_N$$

In the broken phase $<\phi_N> = v = f_\pi$. Redefinition of fields is adopted as $\pi_a \equiv \phi_a\ (a < N)$ and $\sigma = \phi_N - v$.



$O(4) \simeq SU(2)_L \times SU(2)_R \to O(3)$

- Large N expansion

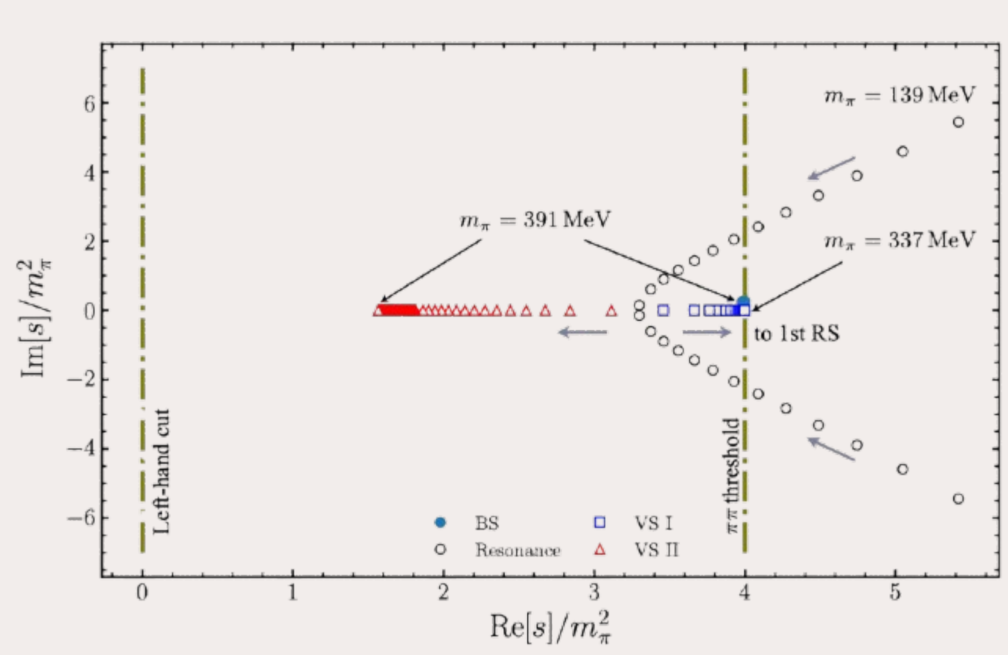


Large N expansion breaks ***crossing symmetry*!**

$$\mathcal{T}_{I=0}(s,t,u) = NA(s) - A(s) + A(t) + A(u), \qquad A(s), A(t), A(u) \sim \mathcal{O}(1/N)$$

leading 1/N order contribution

- σ pole trajectory with varying m_π (at large N limit)



The left-hand cut branch point is always at $s = 0$ and there are **no *subthreshold*** resonance poles at large m_π values.

N/D modified O(N) model

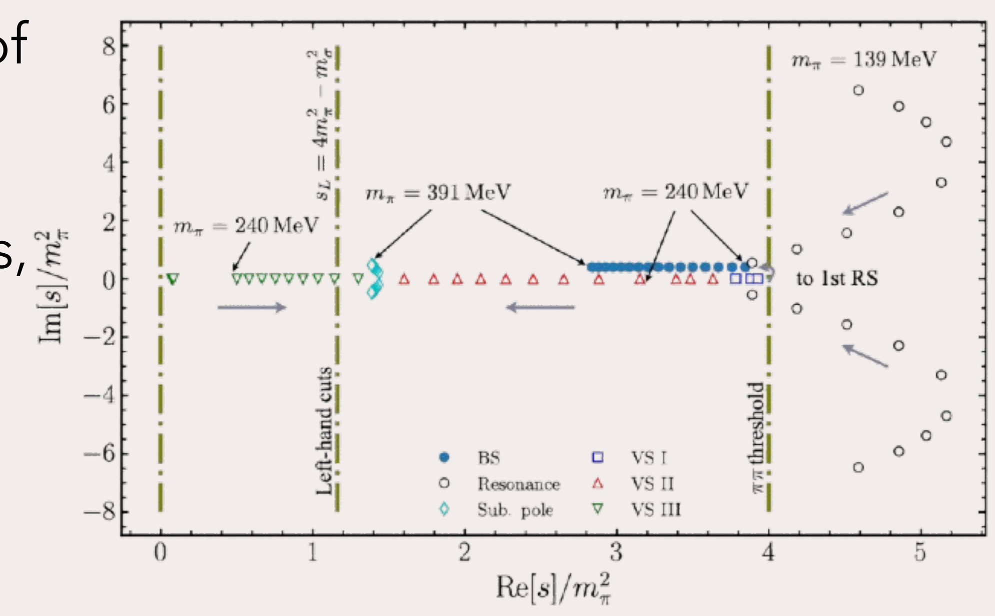
- Partially incorporating cross-channel contributions
- The unitarized scattering amplitude can be obtained through a version of the N/D method. The twice-subtracted dispersion relations are numerically solved with O(N) inputs:

$$\mathcal{T}(s) = \frac{N(s)}{D(s)}$$

$$N(s) = b_0\frac{s - s_A}{s_0 - s_A} + g_N\frac{s - s_0}{s_A - s_0} + \frac{(s - s_0)(s - s_A)}{\pi}\int_L \frac{D(s')\mathrm{Im}_L\mathcal{T}(s')}{(s' - s)(s' - s_0)(s' - s_A)}\mathrm{d}s'$$

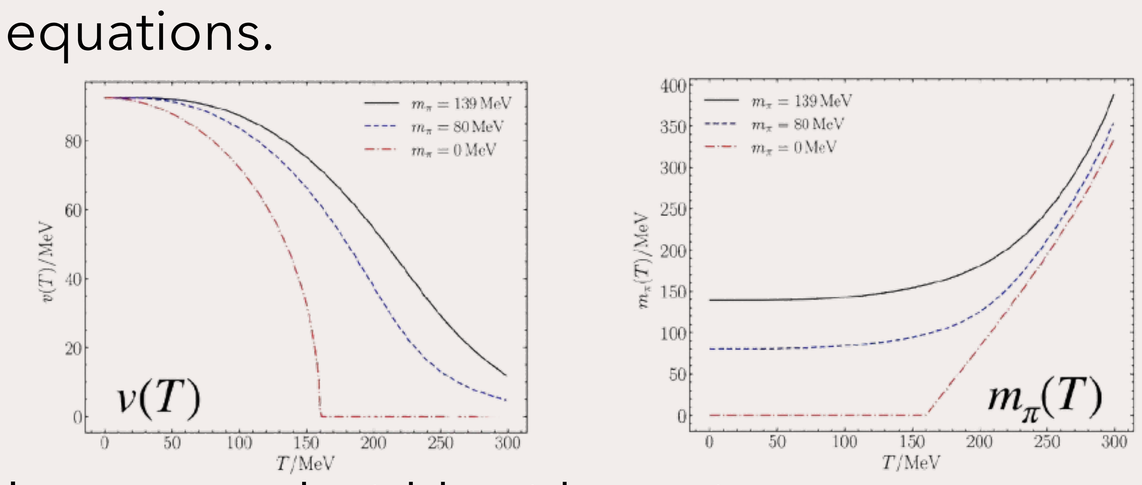
$$D(s) = \frac{s - s_A}{s_0 - s_A} + g_D\frac{s - s_0}{s_A - s_0} - \frac{(s - s_0)(s - s_A)}{\pi}\int_R \frac{\rho(s')N(s')}{(s' - s)(s' - s_0)(s' - s_A)}\mathrm{d}s'$$

- There is a pair of ***subthreshold*** poles found at large m_π values, similar to the case in Ref. [3].

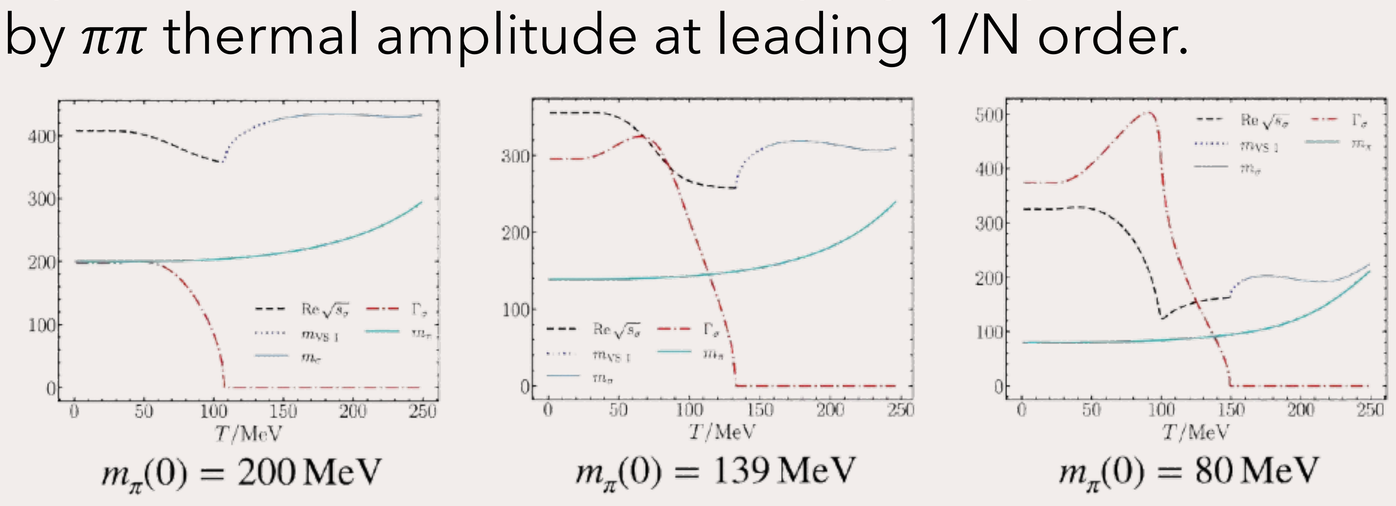


σ pole thermal trajectory

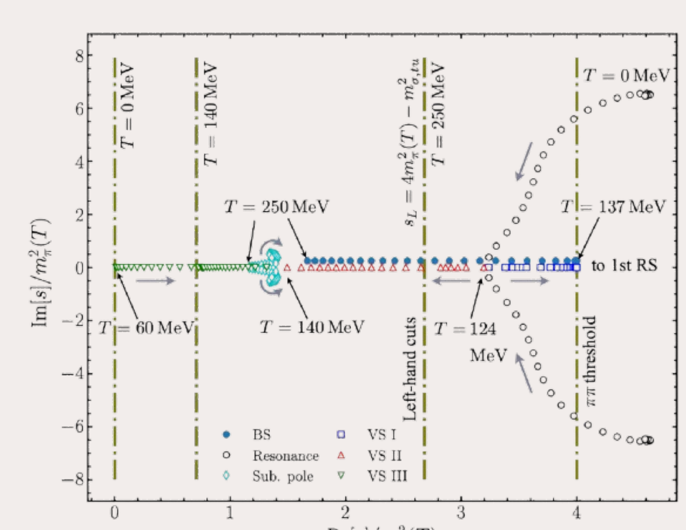
- The pion mass $m_\pi(T)$ and condensate $v(T)$ solved from gap equations.



- σ pole mass and width with varying temperature obtained by $\pi\pi$ thermal amplitude at leading 1/N order.



- σ thermal trajectory with **cross-channel improvements** [N/D modified O(N) model]



Conclusion

- Our results provide further evidences that the lowest f_0 state extracted from experiments and lattice data, *at the qualitative level*, plays the role of σ meson in O(N) model in the spontaneous breaking of chiral symmetry.

References

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