2024复旦大学“QCD与重离子碰撞物理”暑期学校

QCD自旋物理基础
Basics for QCD Spin Physics

第二部分：部分子分布函数和碎裂函数基础
Basics for Parton Distribution Functions (PDFs) and Fragmentation Functions (FFs)

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Based on a short review by K.B. Chen, S.Y. Wei and ZTL, Front. Phys. 10, 101204 (2015)

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强相互作用物理

电 弱 → 弱电统一理论

强 ⟹ 量子色动力学 (QCD)

- 微扰部分 → 已被实验充分检验（1979、2004诺贝尔奖）
- 非微扰部分 ⟺ 尚未解决的重大难题

强相互作用物理
- pQCD高精度计算与应用
- 强子结构与强子产生
- 强相互作用物质形态
- ………

是当代 粒子物理 原子核物理 共同的前沿之一

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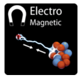
强相互作用物理：强相互作用物质形态

电磁



气体
液体
固体
晶体
超导体
超流体
等离子体
电磁波
………

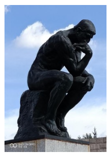
强

核子/强子 (nucleon/hadron)
原子核 (nuclei)
} 束缚态

色超导体？(color super conductor)
色玻璃体？(color glass condensate)
夸克胶子等离子体？(quark gluon plasmas)
………
} 特殊条件下

原子分子物理
凝聚态物理
光学
等离子体物理
声学
无线电物理

材料科学
电子科学
化学



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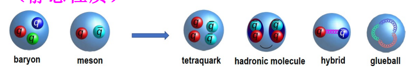
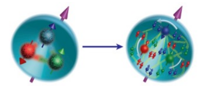
强相互作用物理：强子结构

夸克模型 → 强子多重态、重子反常磁矩（静态性质）→ 强子质量谱、奇特强子态



量子场论的基本性质：真空涨落与激发 (vacuum excitation)
(Lamb位移 —— QED)



高速运动的核子的内部结构——夸克部分子模型

✷强相互作用性质研究的重要场所 ✷高能反应的初始条件

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Introduction: QCD and hadron physics

Two important quantities: parton distribution function (PDF) ⟷ hadron structure
fragmentation function (FF) ⟷ hadronization

hadron structure (PDFs)
hadronization (FFs)

theory

the measurable quantities (cross section, correlations, azimuthal asymmetries, polarizations, ...)

the Standard Model

to provide an accurate description of high energy reaction
to study the properties of QCD
to test the standard model precisely
to provide "initial conditions" to search for new physics

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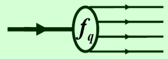
Introduction: Intuitive definition of TMD PDFs

One-dimensional PDFs:

Parton model: A fast moving proton == A beam of partons



$f_q(x)$: number density of parton in proton, parton distribution function (PDF)
$x = k/p$: fractional momentum carried by the parton

Including spin ⟹ spin dependent PDFs:

$$f_1(x,s_q;S) = f_1(x) + \lambda_q \lambda\, g_{1L}(x) + \vec{s}_{Tq}\cdot\vec{S}_T\; h_{1T}(x)$$

helicity distribution transversity

Including transverse momentum ⟹ three-dimensional (or TMD) PDFs:

$$f(x,k_\perp,s_q;p,S) = f_1(x,k_\perp) + \frac{1}{M}\vec{S}_T\cdot(\hat{p}\times\vec{k}_\perp) f_{1T}^\perp(x,k_\perp) + \lambda_q\lambda g_{1L}(x,k_\perp) + \lambda_q \frac{1}{M}(\vec{S}_T\cdot\vec{k}_\perp) g_{1T}^\perp(x,k_\perp)$$
$$+\frac{1}{M}\vec{s}_{\perp q}\cdot(\hat{p}\times\vec{k}_\perp)h_1^\perp(x,k_\perp) + \vec{s}_{\perp q}\cdot\vec{S}_T h_{1T}(x,k_\perp) + \frac{1}{M^2}(\vec{s}_{\perp q}\cdot\vec{k}_\perp)(\vec{S}_T\cdot\vec{k}_\perp)h_{1T}^\perp(x,k_\perp) + \frac{1}{M}(\vec{s}_{\perp q}\cdot\vec{k}_\perp)\lambda h_{1L}^\perp(x,k_\perp)$$

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部分子分布函数与碎裂函数：prelude --- before we really start

Parton distribution functions (PDFs)

$$f_1(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p|\bar{\psi}(0)\mathcal{L}(0,z^-)\frac{\gamma^+}{2}\psi(0,z^-,\vec{0}_\perp)|p\rangle$$

$$\mathcal{L}(0,z) = \mathcal{L}^\dagger(-\infty,0)\mathcal{L}(-\infty,z),$$

gauge link

$$\mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp)}$$
$$= 1 + ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp) + \frac{1}{2}(ig)^2 \int_{-\infty}^{z^-} dy^- \int_{-\infty}^{y^-} dy'^- A^+(0,y^-,\vec{0}_\perp)A^+(0,y'^-,\vec{0}_\perp) + \ldots\ldots$$

Why? Where does it come from?

How does it look like in the three dimensional case ?

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Contents

Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

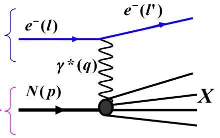
Our knowledge of parton model started from inclusive DIS

当代卢瑟福散射实验

The differential cross section

$$d\sigma = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l,\lambda_l,l',\lambda_l') W_{\mu\nu}(q,p,S) \frac{d^3 l'}{2E'}$$

leptonic tensor   hadronic tensor

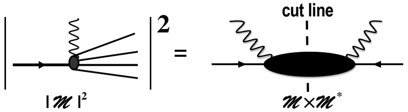


$Q^2 \equiv -q^2$ $x_B \equiv \frac{Q^2}{2q\cdot p}$ $y \equiv \frac{q\cdot p}{l\cdot p}$

强子张量，包含核子结构的信息

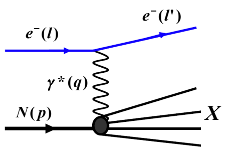
The hadronic tensor: $W_{\mu\nu}(q,p,S) = \sum_X \langle p,S|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p,S\rangle (2\pi)^4 \delta^4(p+q-p_X)$

$W_{\mu\nu}(q,p,S) =$



Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

The derivation of the differential cross section



$$d\sigma = \frac{1}{4s}\frac{|\mathcal{M}|^2}{TV}\frac{d^3 l'}{(2\pi)^3(2E')} \qquad \mathcal{M} = \langle f|\hat{S}|i\rangle = \langle e_f^- X|\hat{S}|e_i^- N\rangle$$

$$\hat{S} = Te^{i\int d^4x \mathcal{H}_I(x)} = 1 + i\int d^4x \mathcal{H}_I(x) + \frac{i^2}{2}T\int d^4x d^4y \mathcal{H}_I(x)\mathcal{H}_I(y) + \ldots\ldots\ldots$$

$$\mathcal{H}_I(x) = eJ_\mu(x)A^\mu(x) \qquad J_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$$

$$\mathcal{M} \approx \frac{i^2}{2}\langle e_f^- X|T\int d^4x d^4y \mathcal{H}_I(x)\mathcal{H}_I(y)|e_i^- N\rangle = \frac{i^2}{2}\langle e_f^- X|T\int d^4x d^4y J_\mu(x)A^\mu(x)J_\nu(y)A^\nu(y)|e_i^- N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\langle e_f^- X|\int d^4x d^4y e^{iq(x-y)}J^\mu(x)J_\mu(y)|e_i^- N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\int d^4x d^4y e^{iq(x-y)}\langle e_f^-|J^\mu(x)|e_i^-\rangle\langle X|J_\mu(y)|N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\int d^4x d^4y e^{-i(l-l'-q)x}e^{-i(p+q-p_X)y}\langle e_f^-|J^\mu(0)|e_i^-\rangle\langle X|J_\mu(0)|N\rangle$$

$$= \frac{i}{q^2}\langle e_f^-|J^\mu(0)|e_i^-\rangle\langle X|J_\mu(0)|N\rangle(2\pi)^4\delta^4(l+p-l'-p_X)$$

$$\langle 0|TA^\mu(x)A^\nu(y)|0\rangle = \int\frac{d^4q}{(2\pi)^4}D_F^{\mu\nu}(q)e^{iq(x-y)}$$

$$D_F^{\mu\nu}(q) = \frac{-ig^{\mu\nu}}{q^2}$$

$$\langle 0|\psi(0)|l\rangle = u(l)$$

Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

Kinematic analysis: find the complete set of the "basic Lorentz tensors" and the general form of the hadronic tensor

The constraints: Gauge invariance $q^\mu W_{\mu\nu}(q,p,S) = 0$ Hermiticity $W_{\mu\nu}^*(q,p,S) = W_{\nu\mu}(q,p,S)$

Parity invariance $W_{\mu\nu}(\tilde{q},\tilde{p},-\tilde{S}) = W^{\mu\nu}(q,p,S)$

The unpolarized set: $\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$, $(q+2xp)_\mu(q+2xp)_\nu$

The polarized (spin dependent) set: $\varepsilon_{\mu\nu\rho\sigma}q^\sigma S^\sigma$, $\varepsilon_{\mu\nu\rho\sigma}q^\sigma\left(S^\sigma - \frac{S\cdot q}{p\cdot q}p^\sigma\right)$

$$\Longrightarrow W_{\mu\nu}(q,p,S) = W_{\mu\nu}^{(S)}(q,p) + iW_{\mu\nu}^{(A)}(q,p,S)$$

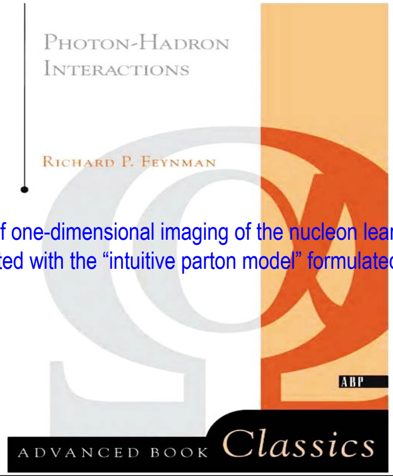
$$W_{\mu\nu}^{(S)}(q,p) = 2\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \frac{1}{xQ^2}(q+2xp)_\mu(q+2xp)_\nu F_2(x,Q^2)$$

$$W_{\mu\nu}^{(A)}(q,p,S) = \frac{2M}{p\cdot q}\varepsilon_{\mu\nu\rho\sigma}q^\sigma S^\sigma g_1(x,Q^2) + \frac{2M}{p\cdot q}\varepsilon_{\mu\nu\rho\sigma}q^\sigma\left(S^\sigma - \frac{S\cdot q}{p\cdot q}p^\sigma\right)g_2(x,Q^2)$$

4 independent "basic Lorentz tensors" and correspondingly 4 structure functions

包含核子结构的信息

"Original / Intuitive" Parton Model



Our knowledge of one-dimensional imaging of the nucleon learned from DIS experiments started with the "intuitive parton model" formulated e.g. in this book.

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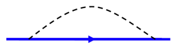
"Original / Intuitive" Parton Model

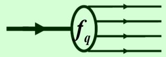
The model:

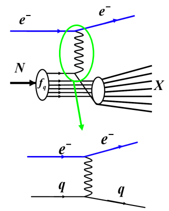
Feynman (1969); Bjorken & Paschos (1969)

Virtual processes such as 

Because of time dilatation, in the infinite momentum frame, they exist forever.

A fast moving proton = A beam of free partons





$$|\mathcal{M}(eN \to eX)|^2 = \sum_q \int dx f_q(x) |\hat{\mathcal{M}}(eq \to eq)|^2$$

scattering amplitude squared

$x = k/p$: momentum fraction carried by the parton

$f_q(x)$: parton number density, known as Parton Distribution Function (PDF)

E.g.: $F_2(x) = 2xF_1(x) = \sum_q e_q^2 f_q(x) \qquad g_1(x) = \sum_q e_q^2 \Delta f_q(x)$

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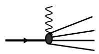
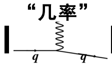
"Original / Intuitive" Parton Model

It is just the impulse approximation!

$W_{\mu\nu}(q,p,S) =$ |  $|^2 = f(x) \otimes$ |  $|^2$

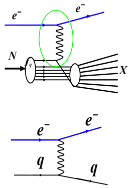
数密度 "几率"

Impulse Approximation (冲量/脉冲近似):

(1) during the interaction of lepton with parton, interaction between partons is neglected;

(2) lepton interacts only with one single parton;

(3) interaction with different partons adds incoherently.



Approximation: What is neglected? Controllable?

Parton distribution function (PDF): A proper (quantum field theoretical) definition?

⇨ A quantum field theoretical formulation ?

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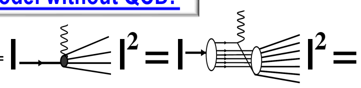
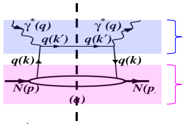
Quantum field theoretical formulation of parton model

Parton model without QCD:

$W_{\mu\nu}(q,p,S) = \left| \; \right|^2 = \left| \; \right|^2 =$





$$W_{\mu\nu}(q,p,S)=\sum_X \langle p,S|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p,S\rangle(2\pi)^4\delta^4(p+q-p_X)$$

$$=\sum_X\int d^4z\langle p,S|J_\mu(0)|X\rangle\langle X|J_\nu(z)|p,S\rangle e^{-iqz}$$

$|X\rangle = |X'\rangle|k'\rangle$
$J_\mu(x) = \psi(x)\gamma_\mu\psi(x)$
$\psi(x)|X'\rangle|k'\rangle = u(k')e^{ik'\cdot x}|X'\rangle$

$$=\int\frac{d^4k'}{(2\pi)^4}(2\pi)\delta_+(k'^2)\sum_{X'}\int d^4z e^{-iqz}\langle p,S|\bar\psi(0)|X'\rangle\gamma_\mu u(k')\bar u(k')\gamma_\nu e^{ik'z}\langle X'|\psi(z)|p,S\rangle$$

$$=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}(k,q)\hat\phi(k,p,S)\right]$$

the calculable hard part $\hat H_{\mu\nu}(k,q)=\gamma_\mu(\not k+\not q)\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

the quark-quark correlator $\hat\phi(k,p,S)=\int d^4z e^{ikz}\langle p,S|\bar\psi(0)\psi(z)|p,S\rangle$

4x4 matrix: $\phi_{\alpha\beta}(k,p,S) = \int d^4z\, e^{ikz}\langle p,S|\psi_\beta(0)\psi_\alpha(z)|p,S\rangle$

根本不象部分子模型？

no local (color) gauge invariance!

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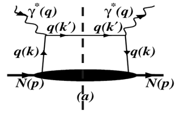
Quantum field theoretical formulation of parton model

$|X\rangle=|X',k'\rangle,\qquad \sum_X=\sum_{X'}\int\frac{d^3k'}{(2\pi)^3 2E_k'}\qquad \int\frac{d^3k'}{(2\pi)^3 2E_k'}=\int\frac{d^4k'}{(2\pi)^4}\delta_+(k'^2)$



$$W_{\mu\nu}(q,p)=\sum_X\langle p|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p\rangle(2\pi)^4\delta^4(p+q-p_X)$$

$$=\sum_{X'}\int\frac{d^3k'}{(2\pi)^3 2E_k'}\langle p|\bar\psi(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|\bar\psi(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$=\int d^4z\sum_{X'}\int\frac{d^3k'}{(2\pi)^3 2E_k'}e^{i(p+q-p_{X'}-k')\cdot z}\langle p|\bar\psi(0)\gamma_\mu|X'\rangle\not k'\langle X'|\gamma_\nu\psi(0)|p\rangle$$

$J_\mu(x)=\bar\psi(x)\gamma_\mu\psi(x),$
$\psi(x)|X',k'\rangle=u(k')e^{-ik'\cdot x}|X'\rangle$
$u(k')\bar u(k')=\not k'$

$$=\int d^4z\frac{d^4k}{(2\pi)^4}e^{ikz}\langle p|\psi(0)\gamma_\mu(\not q+\not k)\gamma_\nu\psi(z)|p\rangle(2\pi)\delta_+\left((k+q)^2\right)$$

$$=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}(k,q)\hat\phi(k,p,S)\right]$$

$\hat H_{\mu\nu}(k,q)=\gamma_\mu(\not k+\not q)\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)\qquad \hat\phi_{\alpha\beta}(k,p,S)=\int d^4z\,e^{ikz}\langle p,S|\psi_\beta(0)\psi_\alpha(z)|p,S\rangle$

具体推导

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Quantum field theoretical formulation of parton model

Parton model without QCD (continued):

$x=k^+/p^+$

$k^\pm=\frac{1}{\sqrt2}(k_0\pm k_3)$

$n=(0,1,\vec 0_\perp)$

$\bar n=(1,0,\vec 0_\perp)$

Collinear approximation(共线近似): $p\approx p^+\bar n,\quad k\approx xp$

$$\hat H_{\mu\nu}(k,q)\approx\hat H_{\mu\nu}(x)\equiv\hat H_{\mu\nu}(k=xp,q)=\gamma_\mu\not n\gamma_\nu\delta(x-x_B)$$

$$W_{\mu\nu}(q,p)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}(k,q)\hat\phi(k,p)\right]\approx\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}(x)\hat\phi(k,p)\right]=\int dx\mathrm{Tr}\left[\hat H_{\mu\nu}(x)\hat\phi(x,p)\right]$$

$$\hat\phi(x;p)\equiv\int\frac{d^4k}{(2\pi)^4}\delta(x-k^+/p^+)\,\hat\phi(k,p)=\frac12 p^+\not{\bar n}f_1(x)+\dots$$

$$\Longrightarrow\quad W_{\mu\nu}(q,p)\approx\left[(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2})+\frac{1}{2xq\cdot p}(q+2xp)_\mu(q+2xp)_\nu\right]f_1(x)$$

operator expression of the number density : $f_1(x)=\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle p|\bar\psi(0)\frac{\gamma^+}{2}\psi(z)|p\rangle$

正是部分子模型的结果！

no local (color) gauge invariance!

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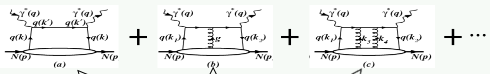
Inclusive DIS with "multiple gluon scattering"

To get the gauge invariance, we need to take the "multiple gluon scattering" into account

$W_{\mu\nu}(q,p,S)=$



$$W_{\mu\nu}(q,p,S)=W^{(0)}_{\mu\nu}(q,p,S)+W^{(1)}_{\mu\nu}(q,p,S)+W^{(2)}_{\mu\nu}(q,p,S)+\dots$$

$$W^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H^{(0)}_{\mu\nu}(k,q)\hat\phi^{(0)}(k,p,S)\right]$$

$$W^{(1)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat H^{(1)\rho}_{\mu\nu}(k_1,k_2,q)\hat\phi^{(1)}_\rho(k_1,k_2,p,S)\right]\qquad \hat H^{(1)\rho}_{\mu\nu}=\hat H^{(1,L)\rho}_{\mu\nu}+\hat H^{(1,R)\rho}_{\mu\nu}$$

the calculable hard part: $\hat H^{(0)}_{\mu\nu}(k,q)=\gamma_\mu(\not k+\not q)\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

$$\hat H^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q)=\gamma_\mu\frac{(\not k_2+\not q)\gamma^\rho(\not k_1+\not q)}{(k_2+q)^2-i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_1+q)^2\right)$$

the quark-quark correlator: $\hat\phi^{(0)}(k;p,S)=\int d^4z\,e^{ikz}\langle p,S|\psi(0)\psi(z)|p,S\rangle$

the quark-gluon-quark correlator: $\hat\phi^{(1)}(k_1,k_2;p,S)=\int d^4y\,d^4z\,e^{ik_1z+ik_2(y-z)}\langle p,S|\psi(0)A_\rho(y)\psi(z)|p,S\rangle$

no (local) gauge invariance!

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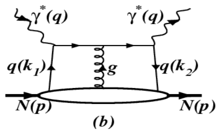
Inclusive DIS with QCD interaction

Consider QCD interaction: first order

$$\mathcal{H}_I(y) = \mathcal{H}_I^{QED}(y) + \mathcal{H}_I^{QCD}(y)$$

$$\mathcal{H}_I^{QED}(y) = e\bar{\psi}(y)\gamma_\mu\psi(y)A^\mu_{em}(y)$$

$$\mathcal{H}_I^{QCD}(y) = g\bar{\psi}(y)\gamma^\rho\psi(y)A_\rho(y) + \ldots\ldots$$



$$J_\mu(x) \to T\int d^4y\, \mathcal{H}_I^{QCD}(y)\bar{\psi}(x)\gamma_\mu\psi(x),$$

$$W^{(1,R)}_{\mu\nu}(q,p) = T\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E_k'}\langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|\mathcal{H}_I^{QCD}(y)\bar{\psi}(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E_k'}\langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|T\bar{\psi}(y)\gamma^\rho\psi(y)A_\rho(y)\bar{\psi}(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E_k'}\langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|\bar{\psi}(y)\gamma^\rho\psi(y)A_\rho(y)\bar{\psi}(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y\, d^4z \frac{d^4k'}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} e^{-i(q-k')\cdot z}e^{-i(k-k')\cdot y}\langle p|\bar{\psi}(0)\gamma_\mu \not{k}'\gamma^\rho S_F(k)A_\rho(y+z)\gamma_\nu\psi(z)|p\rangle$$

具体推导

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Inclusive DIS with QCD interaction

Consider QCD interaction: first order

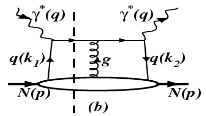
$$\Longrightarrow\quad W^{(1,R)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}[\hat{\phi}^{(1)}_\rho(k_1,k_2;p,S)\hat{H}^{(1,R)\rho}_{\mu\nu}(k_1,k_2,q)]$$

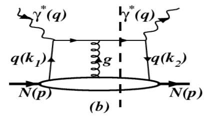
$$\hat{H}^{(1,R)\rho}_{\mu\nu}(k,q) = \gamma_\mu\frac{(\not{k}_2+\not{q})\gamma^\rho(\not{k}_1+\not{q})}{(k_1+q)^2+i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_2+q)^2\right)$$

$$\hat{\phi}^{(1)}_\rho(k_1,k_2;p,S) = \int d^4z d^4y\, e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)gA_\rho(y)\psi(z)|p,S\rangle$$

Similarly: $$W^{(1,L)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}[\hat{\phi}^{(1)}_\rho(k_1,k_2;p,S)\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q)]$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(k,q) = \gamma_\mu\frac{(\not{k}_2+\not{q})\gamma^\rho(\not{k}_1+\not{q})}{(k_2+q)^2-i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_1+q)^2\right)$$





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Inclusive DIS: LO pQCD, leading twist

Collinear approximation:

$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$

$\hat{H}^{(1)}_{\mu\nu}(x_1,x_2) \equiv \hat{H}^{(1)}_{\mu\nu}(k_1 = x_1p, k_2 = x_2p, q$

- Approximating the **hard part** as that at $k = xp$:

$$\hat{H}^{(0)}_{\mu\nu}(k,q) \approx \hat{H}^{(0)}_{\mu\nu}(x)$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) \approx \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)$$

$$x = k^+/p^+$$

$$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$

$$n = (0,1,\vec{0}_\perp)$$

$$\bar{n} = (1,0,\vec{0}_\perp)$$

- Keep only the longitudinal component of the gluon field:

$$A_\rho(y) \approx n\cdot A(y)\frac{p_\rho}{n\cdot p} = A^+(y)\frac{p_\rho}{p^+}$$

- Using the Ward identities such as,

$$p_\rho\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2-x_1-i\varepsilon}$$

to replace hard parts for diagrams with multiple gluon scatterings by $\hat{H}^{(0)}_{\mu\nu}(x)$.

- Adding all terms together ⟹

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Inclusive DIS: LO pQCD, leading twist

$$\Longrightarrow\quad W_{\mu\nu}(q,p,S) \approx \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(0)}(k;p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

LO & leading twist

$$\hat{\Phi}^{(0)}(k;p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

The gauge invariant un-integrated quark-quark correlator: contain QCD interaction!

gauge link

$$\mathcal{L}(0,z) = \mathcal{L}^\dagger(-\infty,0)\mathcal{L}(-\infty,z),$$

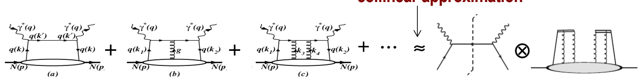
$$\mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-}dy^-A^+(0,y^-,\vec{0}_\perp)}$$

$$= 1 + ig\int_{-\infty}^{z^-}dy^-A^+(0,y^-,\vec{0}_\perp) + \frac{1}{2}(ig)^2\int_{-\infty}^{z^-}dy^-\int_{-\infty}^{y^-}dy'^-A^+(0,y^-,\vec{0}_\perp)A^+(0,y'^-,\vec{0}_\perp) + \ldots\ldots$$

Gauge link comes from the multiple gluon scattering.

Graphically:

collinear approximation



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Inclusive DIS: LO pQCD, leading & higher twists

Collinear expansion:

Ellis, Furmanski, Petronzio (1982,1983); Qiu, Sterman (1990,1991)

- Expanding the hard part at $k = xp$:

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho}\omega_\rho{}^{\rho'}k_{\rho'} + ...$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) = \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_1^\sigma}\omega_\sigma{}^{\sigma'}k_{1\sigma'} + ...$$

$$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$$

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} \equiv \left.\frac{\partial \hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^\rho}\right|_{k=xp}$$

$$x = k^+/p^+$$
$$\omega_\rho{}^{\rho'} \equiv g_\rho{}^{\rho'} - \bar{n}_\rho n^{\rho'}$$
$$\omega_\rho{}^{\rho'}k_{\rho'} = (k - xp)_\rho$$
$$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$
$$n = (0,1,\vec{0}_\perp)$$
$$\bar{n} = (1,0,\vec{0}_\perp)$$

- Decomposition of the gluon field:

$$A_\rho(y) = n\cdot A(y)\frac{p_\rho}{n\cdot p} + \omega_\rho{}^{\rho'}A_{\rho'}(y)$$

- Using the Ward identities such as,

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} = -\hat{H}^{(1)\rho}_{\mu\nu}(x,x), \quad p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace the derivatives etc.

- Adding all terms with the same hard part together ⇨

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Inclusive DIS: LO pQCD, leading & higher twists

$$W_{\mu\nu}(q,p,S) = \tilde{W}^{(0)}_{\mu\nu}(q,p,S) + \tilde{W}^{(1)}_{\mu\nu}(q,p,S) + \tilde{W}^{(2)}_{\mu\nu}(q,p,S) + ...$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

$$\hat{\Phi}^{(0)}(k;p,S) = \int d^4z\, e^{ikz}\langle p,S|\psi(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho{}^{\rho'}\right]$$

$$\hat{\Phi}^{(1)}(k_1,k_2;p,S) = \int d^4y\, d^4z\, e^{ik_1z+ik_2(y-z)}\langle p,S|\psi(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

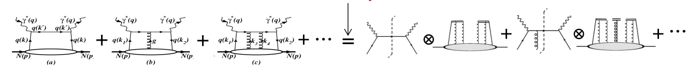
$$D_\rho(y) = -i\partial_\rho + gA_\rho(y)$$

gauge invariant quark-gluon-quark correlator

⇨ A consistent framework for inclusive DIS $e^-N \to e^-X$ including leading & higher twists

Graphically

collinear expansion



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Inclusive DIS: LO pQCD, leading & higher twists

Simplified expressions for hadronic tensors

The "collinearly expanded hard parts" take the simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu}\delta(x - x_B), \quad \hat{h}^{(0)}_{\mu\nu} = \gamma_\mu \not{n}\gamma_\nu$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho{}^{\rho'} = \frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\delta(x_1 - x_B), \quad \hat{h}^{(1)\rho}_{\mu\nu} = \gamma_\mu \bar{\not{n}}\gamma^\rho \not{n}\gamma_\nu$$

depends only on ONE variable!

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int dx \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x;p,S)\, h^{(0)}_{\mu\nu}\right]\delta(x - x_B)$$

$$\hat{\Phi}^{(0)}(x;p,S) \equiv \int \frac{d^4k}{(2\pi)^4}\delta(x - \frac{k^+}{p^+})\hat{\Phi}^{(0)}(k;p,S) = \int \frac{p^+dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

one-dimensional gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S) = \frac{\pi}{2q\cdot p}\mathrm{Re}\int dx \mathrm{Tr}\left[\hat{\varphi}^{(1)}_{\rho'}(x;p,S)\, h^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\right]\delta(x - x_B)$$

$$\hat{\varphi}^{(1)}_\rho(x;p,S) \equiv \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(x - \frac{k_1^+}{p^+})\hat{\Phi}^{(1)}_\rho(k_1,k_2;p,S) = \int \frac{p^+dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

the involved one-dimensional gauge invariant quark-gluon-quark correlator

⇨ Only ONE dimensional imaging of the nucleon is involved in inclusive DIS.

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PDFs defined via quark-quark correlator

- Expand the quark-quark correlator in terms of the Γ-matrices:

$$\hat{\Phi}^{(0)}(x;p,S) = \frac{1}{2}\left[\Phi^{(0)}(x;p,S) + i\gamma_5\tilde{\Phi}^{(0)}(x;p,S) + \gamma^\alpha\Phi^{(0)}_\alpha(x;p,S) + \gamma_5\gamma^\alpha\tilde{\Phi}^{(0)}_\alpha(x;p,S) + i\gamma_5{}^\alpha\sigma^{\alpha\beta}\Phi^{(0)}_{\alpha\beta}(x;p,S)\right]$$

(scalar) (pseudo-scalar) (vector) (axial vector) (tensor)

- Make Lorentz decompositions

$$p = p^+\bar{n} + \frac{M^2}{2p^+}n, \quad S = \lambda\frac{p^+}{M}\bar{n} + S_T - \lambda\frac{M^2}{2p^+}n$$

$$\Phi^{(0)}(x;p,S) = Me(x)$$

$$\tilde{\Phi}^{(0)}(x;p,S) = \lambda Me_L(x)$$

$$\Phi^{(0)}_\alpha(x;p,S) = p^+\bar{n}_\alpha f_1(x) + M\varepsilon_{\perp\alpha\rho}S_T^\rho f_T(x) + \frac{M^2}{p^+}n_\alpha f_3(x)$$

$$\tilde{\Phi}^{(0)}_\alpha(x;p,S) = \lambda p^+\bar{n}_\alpha g_{1L}(x) + MS_{T\alpha}g_T(x) + \lambda\frac{M^2}{p^+}n_\alpha g_{3L}(x)$$

$$\Phi^{(0)}_{\rho\alpha}(x;p,S) = p^+\bar{n}_{[\rho}S_{T\alpha]}h_{1T}(x) - M\varepsilon_{T\rho\alpha}h_T(x) + \lambda M\bar{n}_{[\rho}n_{\alpha]}h_L(x) + \frac{M^2}{p^+}n_{[\rho}S_{T\alpha]}h_{3T}(x)$$

3+6+3

blue: twist-2

black: twist-3, M/Q suppressed

brown: twist-4, $(M/Q)^2$ suppressed

$$A_{[\alpha}B_{\beta]} \equiv A_\alpha B_\beta - A_\beta B_\alpha$$
$$\varepsilon_{\perp\alpha\beta} \equiv \varepsilon_{\rho\sigma\alpha\beta}\bar{n}^\rho n^\sigma$$

the scalar functions are the one-dimensional PDFs, e.g.,

$$f_1(x) = \frac{1}{p^+}n^\alpha\Phi^{(0)}_\alpha(x;p,S) = \int \frac{dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\frac{\gamma^+}{2}\psi(z^-)|p,S\rangle$$

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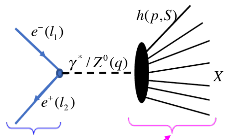
Inclusive hadron production in e^+e^--annihilation $e^- + e^+ \to h + X$

The differential cross section ⟹ definition of the fragmentation function

$$d\sigma = \chi \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l_1,\lambda_1,l_2,\lambda_2) W_{\mu\nu}(q,p,S) \frac{d^3p}{2E}$$

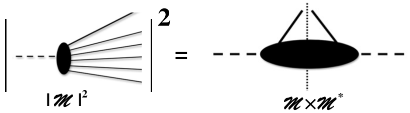
leptonic tensor   hadronic tensor



$-q^2 = Q^2$ $z_B = \frac{2q\cdot p}{Q^2}$

The hadronic tensor: $W_{\mu\nu}(q,p,S) = \sum_X \langle p,S;X|J_\mu(0)|0\rangle\langle 0|J_\nu(0)|p,S;X\rangle(2\pi)^4\delta^4(q-p-p_X)$

$W_{\mu\nu}(q,p,S) =$



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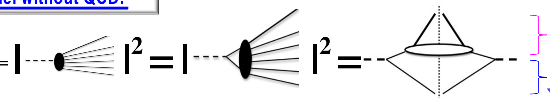
Quantum field theoretical formulation $e^- + e^+ \to h + X$

Parton model without QCD:

$W_{\mu\nu}(q,p,S) =$



$$W_{\mu\nu}(q,p,S) = \sum_X \langle p,S;X|J_\mu(0)|0\rangle\langle 0|J_\nu(0)|p,S;X\rangle(2\pi)^4\delta^4(q-p-p_X)$$

$$= \sum_X \int d^4z \langle p,S;X|J_\mu(0)|X\rangle\langle 0|J_\nu(z)|p,S;X\rangle e^{-iqz}$$

$J_\mu(x) = \bar\psi(x)\Gamma_\mu\psi(x)$,
$|X\rangle = |X'\rangle|k'\rangle$,
$\bar\psi(x)|X'\rangle|k'\rangle = \bar v(k')e^{ik'\cdot x}|X'\rangle$

$$= \int \frac{d^4k'}{(2\pi)^4}(2\pi)\delta_+(k'^2)\sum_{X'}\int d^4z e^{-iqz}\langle p,S;X'|\bar\psi(0)|0\rangle\Gamma_\mu v(k')\bar v(k')\Gamma_\nu\langle 0|\psi(z)|p,S;X'\rangle e^{-ik'z}$$

$$= \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}(k,q)\hat\Pi(k,p,S)\right]$$

$$\Gamma_\mu = \begin{cases}\gamma_\mu \\ \gamma_\mu(c_V - c_A\gamma_5)\end{cases}$$

the calculable hard part $\hat H_{\mu\nu}(k,q) = \Gamma_\mu(\not q - \not k)\Gamma_\nu(2\pi)\delta_+((q-k)^2)$

the quark-quark correlator $\hat\Pi(k;p,S) = \sum_X \int d^4z e^{-ikz}\langle 0|\psi(z)|p,S;X\rangle\langle p,S;X|\bar\psi(0)|0\rangle$

no local (color) gauge invariance!

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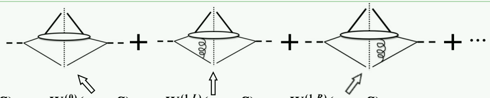
Inclusive e^+e^--annihilation with "multiple gluon scattering"

To get the gauge invariance, we need to take the "multiple gluon scattering" into account

$W_{\mu\nu}(q,p,S) =$



$$W_{\mu\nu}(q,p,S) = W_{\mu\nu}^{(0)}(q,p,S) + W_{\mu\nu}^{(1,L)}(q,p,S) + W_{\mu\nu}^{(1,R)}(q,p,S) + \ldots$$

$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}^{(0)}(k,q)\Pi^{(0)}(k,p,S)\right]$$

$$W_{\mu\nu}^{(1,L)}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}^{(1,L)\rho}(k_1,k_2,q)\Pi_\rho^{(1,L)}(k_1,k_2,p,S)\right]$$

the quark-quark correlator: $\hat\Pi^{(0)}(k;p,S) = \sum_X \int d^4z e^{-ikz}\langle 0|\psi(z)|hX\rangle\langle hX|\bar\psi(0)|0\rangle$

the quark-gluon-quark correlator:

$$\hat\Pi_\rho^{(1,L)}(k_1,k_2;p,S) = \sum_X g\int d^4\xi d^4\eta e^{-ik_1\xi}e^{-i(k_2-k_1)\eta}\langle 0|A_\rho(\eta)\psi(0)|hX\rangle\langle hX|\bar\psi(0)|0\rangle$$

no (local) gauge invariance!

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Inclusive e^+e^-: LO pQCD, leading & higher twists

Collinear expansion: S.Y. Wei, Y.K. Song and ZTL, PRD89, 014024 (2014).

- Expanding the hard part at $k = p/z$:

$$\hat H_{\mu\nu}^{(0)}(z) \equiv \hat H_{\mu\nu}^{(0)}(k = p/z, q)$$

$$\hat H_{\mu\nu}^{(0)}(k,q) = \hat H_{\mu\nu}^{(0)}(z) + \frac{\partial \hat H_{\mu\nu}^{(0)}(z)}{\partial k^\rho}\omega_\rho^{\ \rho'}k_{\rho'} + \ldots\ldots$$

$$\frac{\partial \hat H_{\mu\nu}^{(0)}(z)}{\partial k^\rho} \equiv \left.\frac{\partial \hat H_{\mu\nu}^{(0)}(k,q)}{\partial k^\rho}\right|_{k=p/z}$$

$$\hat H_{\mu\nu}^{(1,L)\rho}(k_1,k_2,q) = \hat H_{\mu\nu}^{(1,L)\rho}(z_1,z_2) + \frac{\partial \hat H_{\mu\nu}^{(1)\rho}(z_1,z_2)}{\partial k_1^\sigma}\omega_\sigma^{\ \sigma'}k_{1\sigma'} + \ldots\ldots$$

$$z = p^+/k^+$$

- Decomposition of the gluon field:

$$A_\rho(y) = n\cdot A(y)\frac{p_\rho}{n\cdot p} + \omega_\rho^{\ \rho'}A_{\rho'}(y)$$

- Using the Ward identities such as,

$$p_\rho \hat H_{\mu\nu}^{(1,L)\rho}(z_1,z_2) = -\frac{z_1z_2}{z_2 - z_1 - i\varepsilon}\hat H_{\mu\nu}^{(0)}(z_1) \qquad p_\rho \hat H_{\mu\nu}^{(1,R)\rho}(z_1,z_2) = -\frac{z_1z_2}{z_2 - z_1 + i\varepsilon}\hat H_{\mu\nu}^{(0)}(z_2)$$

to replace the derivatives etc.

- Adding all terms with the same hard part together ⟹

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Inclusive e^+e^-: LO pQCD, leading & higher twists

$$W_{\mu\nu}(q,p,S)=\tilde{W}^{(0)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1,L)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1,R)}_{\mu\nu}(q,p,S)+\ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Xi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(z)\right]$$

$$\hat{\Xi}^{(0)}(k;p,S)=\sum_X\int d^4\xi e^{ik\xi}\langle hX|\bar{\psi}(0)\mathcal{L}(0,\infty)|0\rangle\langle 0|\mathcal{L}^\dagger(\xi,\infty)\psi(\xi)|hX\rangle$$

gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1,L)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Xi}^{(1,L)}_\rho(k_1k_2;p,S)\omega_{\rho'}{}^{\rho}\hat{H}^{(1,L)\rho'}_{\mu\nu}(z_1,z_2)\right]$$

$$\hat{\Xi}^{(1,L)}_\rho(k_1,k_2;p,S)=\sum_X\int d^4\xi d^4\eta e^{-ik_1\xi}e^{-i(k_2-k_1)\eta}\langle 0|\mathcal{L}^\dagger(\eta,\infty)D_\rho(\eta)\mathcal{L}^\dagger(0,\eta)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$D_\rho(\eta)=-i\partial_\rho+gA_\rho(\eta)$

gauge invariant quark-gluon-quark correlator

⟹ A consistent framework for $e^+e^-\to hX$ including leading & higher twists

Inclusive e^+e^-: LO pQCD, leading & higher twists

Simplified expressions for hadronic tensors

The "collinearly expanded hard parts" take the simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(z)=z_B^2\hat{h}^{(0)}_{\mu\nu}\delta(z-z_B),\quad \hat{h}^{(0)}_{\mu\nu}=\Gamma_\mu \not{n}\Gamma_\nu/p^+$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(z_1,z_2)\omega_\rho{}^{\rho'}=-\frac{\pi}{2q\cdot p}z_B^2\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\delta(z_1-z_B),\quad \hat{h}^{(1)\rho}_{\mu\nu}=\Gamma_\mu\not{n}\gamma^\rho\not{\bar{n}}\Gamma_\nu$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\frac{1}{2}\int dz\mathrm{Tr}\left[\hat{\Xi}^{(0)}(z;p,S)\,\hat{h}^{(0)}_{\mu\nu}\right]\delta(z-z_B)$$

$$\hat{\Xi}^{(0)}(z;p,S)\equiv\int\frac{d^4k}{(2\pi)^4}\delta(z-\frac{p^+}{k^+})\hat{\Xi}^{(0)}(k;p,S)=\sum_X\int\frac{p^+d\xi^-}{2\pi}e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

one-dimensional gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1,L)}_{\mu\nu}(q,p,S)=-\frac{\pi}{4q\cdot p}\mathrm{Re}\int dz\mathrm{Tr}\left[\hat{\Xi}^{(1)}_{\rho'}(z;p,S)\,h^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\right]\delta(z-z_B)$$

$$\hat{\Xi}^{(1)}_\rho(z;p,S)\equiv\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(z-\frac{p^+}{k_1^+})\hat{\Xi}^{(1)}_\rho(k_1,k_2;p,S)$$
$$=\int\frac{p^+d\xi^-}{2\pi}e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)\psi(\xi^-)|0\rangle$$

the involved one-dimensional gauge invariant quark-gluon-quark correlator

⟹ Only one-dimensional fragmentation functions are involved in inclusive e^+e^- annihilations

Description of polarization of particles with different spins

Spin 1/2 hadrons:

The spin density matrix is 2x2: $\rho=\begin{pmatrix}\rho_{++} & \rho_{+-}\\ \rho_{-+} & \rho_{--}\end{pmatrix}=\frac{1}{2}(1+\vec{S}\cdot\vec{\sigma})$

Vector polarization: $S^\mu=(0,\vec{S}_T,\lambda)$

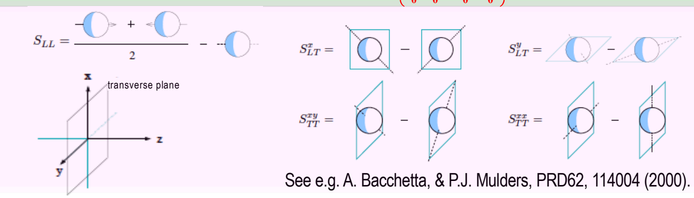
Spin 1 hadrons:

The spin density matrix is 3x3: $\rho=\begin{pmatrix}\rho_{11} & \rho_{10} & \rho_{1-1}\\ \rho_{01} & \rho_{00} & \rho_{0-1}\\ \rho_{-11} & \rho_{-10} & \rho_{-1-1}\end{pmatrix}=\frac{1}{3}(1+\frac{3}{2}\vec{S}\cdot\vec{\Sigma}+3T^{ij}\Sigma^{ij})$

Vector polarization: $S^\mu=(0,\vec{S}_T,\lambda)$ — 3

Tensor polarization: S_{LL}, $S^\mu_{LT}=(0,S^x_{LT},S^y_{LT},0)$, $S^{\mu\nu}_{TT}=\begin{pmatrix}0&0&0&0\\0&S^{xx}_{TT}&S^{xy}_{TT}&0\\0&S^{xy}_{TT}&-S^{xx}_{TT}&0\\0&0&0&0\end{pmatrix}$ — 5

3 + 5 = 8 independent components.



See e.g. A. Bacchetta, & P.J. Mulders, PRD62, 114004 (2000).

One dimensional FFs defined via quark-quark correlator

- **Expand the quark-quark correlator in terms of the Γ-matrices:**

$$\hat{\Xi}^{(0)}(z;p,S)=\frac{1}{2}\left[\Xi^{(0)}(z;p,S)+i\gamma_5\tilde{\Xi}^{(0)}(z;p,S)+\gamma^\alpha\Xi^{(0)}_\alpha(z;p,S)+\gamma_5\gamma^\alpha\tilde{\Xi}^{(0)}_\alpha(z;p,S)+i\gamma_5{}^\alpha\sigma^{\alpha\beta}\Xi^{(0)}_{\alpha\beta}(z;p,S)\right]$$

(scalar) (pseudo-scalar) (vector) (tensor)

- **Make Lorentz decompositions**

5+10+5

blue: twist-2
black: twist-3, M/Q suppressed
brown: twist-4, $(M/Q)^2$ suppressed

$$z\Xi^{(0)}(z;p,S)=ME(z)+MS_{LL}E_{LL}(z)$$

$$z\tilde{\Xi}^{(0)}(z;p,S)=\lambda ME_L(z)$$

$$z\Xi^{(0)}_\alpha(z;p,S)=p^+\bar{n}_\alpha D_1(z)+p^+\bar{n}_\alpha S_{LL}D_{1LL}(z)-M\tilde{S}_{T\alpha}D_T(z)+MS_{LT\alpha}D_{LT}(z)+\frac{M^2}{p^+}n_\alpha D_3(z)+\frac{M^2}{p^+}n_\alpha S_{LL}D_{3LL}(z)$$

$$z\tilde{\Xi}^{(0)}_\alpha(z;p,S)=\lambda p^+\bar{n}_\alpha G_{1L}(z)-MS_{T\alpha}G_T(z)-M\tilde{S}_{LT\alpha}G_{LT}(z)+\lambda\frac{M^2}{p^+}n_\alpha G_{3L}(z)$$

$$z\Xi^{(0)}_{\rho\alpha}(z;p,S)=p^+\bar{n}_{[\rho}S_{T\alpha]}H_{1T}(z)-p^+\bar{n}_{[\rho}\tilde{S}_{LT\alpha]}H_{1LT}(z)-M\varepsilon_{T\rho\alpha}H_T(z)+\lambda M\bar{n}_{[\rho}n_{\alpha]}H_L(z)+MS_{LL}\varepsilon_{T\rho\alpha}H_{LL}(z)$$
$$+\frac{M^2}{p^+}n_{[\rho}S_{T\alpha]}H_{3T}(z)-\frac{M^2}{p^+}n_{[\rho}\tilde{S}_{LT\alpha]}H_{3LT}(z)$$

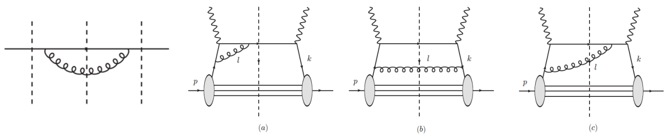
$A_{[\alpha}B_{\beta]}\equiv A_\alpha B_\beta-A_\beta B_\alpha$

$\varepsilon_{\perp\alpha\beta}\equiv\varepsilon_{\rho\sigma\alpha\beta}\bar{n}^\rho n^\sigma$ $\quad\tilde{A}_{T\alpha}\equiv\varepsilon_{\perp\alpha\beta}A^\beta_T$

Inclusive DIS: Higher order pQCD

Factorization theorem and QCD evolution of PDFs

"Loop diagram contributions"



factorization & resummation ⇒

- **Higher order pQCD contributions;**
- **Evolution of PDFs (DGLAP equation)**

Not covered in these lectures.

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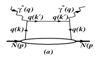
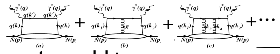
Inclusive DIS and parton model: brief summary

List of to do's --- the recipe

kinematics (symmetries, … …) → general form of the cross section



parton model without QCD interaction → (collinear approximation) → leading order pQCD, leading twist, no evolution, no gauge invariance



parton model + "multiple gluon scattering" → (collinear expansion) → leading order pQCD, leading & higher twist, no evolution, but gauge invariance

parton model + "multiple gluon scattering" + "loop diagram contributions" → (collinear expansion + factorization & resummation) → leading & higher order pQCD, leading & higher twist, evolution & gauge invariance

experiments; calculations: model, LQCD, etc → (global QCD fit 整体QCD拟合) → parameterizations (PDFLib)

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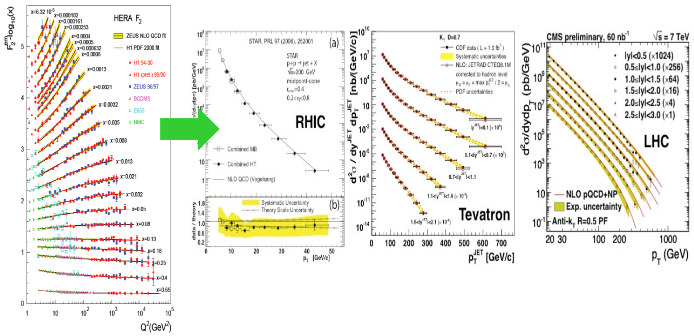
Global QCD analysis and PDFLIB

Very successful!



parameterize at 0.3 TeV e-p (HERA), predict p-p and p-p-bar at 0.2, 1.96, and 7 TeV.

J.W. Qiu, lectures at Weihai High Energy Physics Summer School(WHEPS2015), 2015, Weihai, China.

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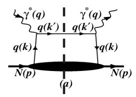
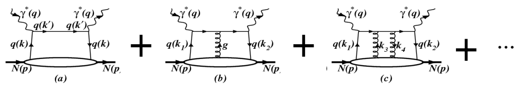
Inclusive DIS and parton model: brief summary

- (Gauge invariant) PDF is not merely 

but 

i.e., it always contains "intrinsic motion" and "multiple gluon scattering".

- "Multiple gluon scattering" gives rise to the gauge link.
- Collinear expansion is the necessary procedure to obtain the correct formulism in terms of gauge invariant parton distribution functions (PDFs).
- Collinear expansion ⟺ power $\left(\frac{M}{Q}\right)^n$ expansion

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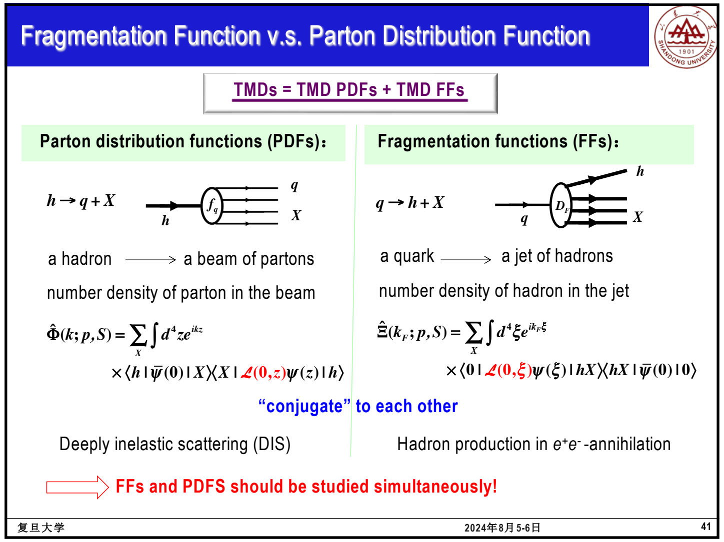
Fragmentation Function v.s. Parton Distribution Function
TMDs = TMD PDFs + TMD FFs
Parton distribution functions (PDFs):
$h \to q + X$
a hadron → a beam of partons
number density of parton in the beam
$\hat{\Phi}(k;p,S) = \sum_X \int d^4z e^{ikz} \times \langle h|\bar{\psi}(0)|X\rangle\langle X|\mathcal{L}(0,z)\psi(z)|h\rangle$
Fragmentation functions (FFs):
$q \to h + X$
a quark → a jet of hadrons
number density of hadron in the jet
$\hat{\Xi}(k_F;p,S) = \sum_X \int d^4\xi e^{ik_F\xi} \times \langle 0|\mathcal{L}(0,\xi)\psi(\xi)|hX\rangle\langle hX|\bar{\psi}(0)|0\rangle$
"conjugate" to each other
Deeply inelastic scattering (DIS)
Hadron production in e^+e^--annihilation
FFs and PDFS should be studied simultaneously!
复旦大学
2024年8月5-6日
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