QCD自旋物理基础
Basics for QCD Spin Physics

第二部分：部分子分布函数和碎裂函数基础
Basics for Parton Distribution Functions (PDFs) and Fragmentation Functions (FFs)

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Based on a short review by K.B. Chen, S.Y. Wei and ZTL, Front. Phys. 10, 101204 (2015)

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TMD PDFs defined via quark-quark correlator

The quark-quark correlator $\hat{\Phi}^{(0)}(k;p,S)=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

integrate over k^-: $\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\int dz^- d^2z_\perp e^{i(xp^+z^- -\vec{k}_\perp\cdot\vec{z}_\perp)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

Expansion in terms of the Γ-matrices

$$\begin{aligned}\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\frac{1}{2}\Big[\ &\Phi^{(0)}(x,k_\perp;p,S) && \text{— scalar}\\ &+i\gamma_5\,\tilde{\Phi}^{(0)}(x,k_\perp;p,S) && \text{— pseudo-scalar}\\ &+\lambda^\alpha\,\Phi^{(0)}_\alpha(x,k_\perp;p,S) && \text{— vector}\\ &+\gamma_5\lambda^\alpha\,\tilde{\Phi}^{(0)}_\alpha(x,k_\perp;p,S) && \text{— axial vector}\\ &+i\gamma_5\sigma^{\alpha\beta}\,\Phi^{(0)}_{\alpha\beta}(x,k_\perp;p,S)\Big] && \text{— tensor}\end{aligned}$$

e.g.: $\Phi^{(0)}_\alpha(x,k_\perp;p,S)=\frac{1}{2}\mathrm{Tr}\left[\gamma_\alpha\hat{\Phi}^{(0)}(x,k_\perp;p,S)\right]$

$$=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\frac{\gamma_\alpha}{2}\psi(z)|p,S\rangle$$

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TMD PDFs defined via quark-quark correlator

The Lorentz decomposition

totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\Phi^{(0)}_S(x,k_\perp;p,S)=M\left[e(x,k_\perp)+\frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M}e_T^\perp(x,k_\perp)\right] \quad \leftarrow \text{twist-3}$$

$$\begin{aligned}\Phi^{(0)}_\alpha(x,k_\perp;p,S)=\ &p^+\bar{n}_\alpha\left[f_1(x,k_\perp)+\frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M}f_{1T}^\perp(x,k_\perp)\right] \quad \leftarrow \text{twist-2}\\ &+k_{\perp\alpha}f^\perp(x,k_\perp)+M\varepsilon_{\perp\alpha\sigma}S_T^\sigma f_T(x,k_\perp)+\varepsilon_{\perp\alpha\rho}k_\perp^\rho\left[\lambda f_L^\perp(x,k_\perp)+\frac{k_\perp\cdot S_T}{M}f_T^\perp(x,k_\perp)\right]\\ &+\frac{M^2}{p^+}n_\alpha\left[f_3(x,k_\perp)+\frac{\varepsilon_{\perp\rho\sigma}k_\perp^\rho S_T^\sigma}{M}f_{3T}^\perp(x,k_\perp)\right] \quad \leftarrow \text{twist-4}\end{aligned}$$

$$p=p^+\bar{n}+\frac{M^2}{2p^+}n,\qquad S=\lambda\frac{p^+}{M}\bar{n}+S_T-\lambda\frac{M^2}{2p^+}n$$

See e.g., K. Goeke, A. Metz, M. Schlegel, PLB 618, 90 (2005);
P. J. Mulders, lectures in 17th Taiwan nuclear physics summer school, August, 2014.

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TMD PDFs defined via quark-quark correlator

The Lorentz decomposition

totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\tilde{\Phi}^{(0)}(x,k_\perp;p,S)=M\left[\lambda e_L(x,k_\perp)+\frac{k_\perp\cdot S_T}{M}e_T(x,k_\perp)\right]$$

$$\tilde{\Phi}^{(0)}_\alpha(x,k_\perp;p,S)=p^+\bar{n}_\alpha\left[\lambda g_{1L}(x,k_\perp)+\frac{k_\perp\cdot S_T}{M}g_{1T}^\perp(x,k_\perp)\right]$$
$$-MS_{T\alpha}g_T(x,k_\perp)-k_{\perp\alpha}\left[\lambda g_L^\perp(x,k_\perp)+\frac{k_\perp\cdot S_T}{M}g_T^\perp(x,k_\perp)\right]+\varepsilon_{\perp\alpha\beta}k_\perp^\beta g^\perp(x,k_\perp)$$
$$+\frac{M^2}{p^+}n_\alpha\left[\lambda g_{3L}(x,k_\perp)+\frac{k_\perp\cdot S_T}{M}g_{3T}^\perp(x,k_\perp)\right]$$

$$\Phi^{(0)}_{\rho\alpha}(x,k_\perp;p,S)=p^+\bar{n}_{[\rho}S_{T\alpha]}h_{1T}(x,k_\perp)+\frac{p^+\bar{n}_{[\rho}k_{\perp\alpha]}}{M}\left[\lambda h_{1L}^\perp(x,k_\perp)+\frac{k_\perp\cdot S_T}{M}h_{1T}^\perp(x,k_\perp)\right]+\frac{p^+\bar{n}_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta}{M}h_1^\perp(x,k_\perp)$$
$$+S_{T[\rho}k_{\perp\alpha]}h_T^{\perp'}(x,k_\perp)+M\varepsilon_{\perp\rho\alpha}h(x,k_\perp)-\bar{n}_{[\rho}n_{\alpha]}\left[M\lambda h_L(x,k_\perp)-(k_\perp\cdot S_T)h_T^\perp(x,k_\perp)\right]$$
$$+\frac{M^2}{p^+}\left\{n_{[\rho}S_{T\alpha]}h_{3T}(x,k_\perp)+\frac{n_{[\rho}k_{\perp\alpha]}}{M}\left[\lambda h_{3L}^\perp(x,k_\perp)+\frac{k_\perp\cdot S_T}{M}h_{3T}^\perp(x,k_\perp)\right]+\frac{n_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta}{M}h_3^\perp(x,k_\perp)\right\}$$

twist-3

twist-2

twist-4

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Twist-2 TMD PDFs defined via quark-quark correlator

Leading twist (twist 2) f, g, h: quark un-, longitudinally, transversely polarized

polarization: quark	nucleon	pictorially	TMD PDFs (8)	if no gauge link	integrated over $k_\perp$	name
U	U		$f_1(x,k_\perp)$		$q(x)$	number density
	T		$f_{1T}^\perp(x,k_\perp)$	0	×	Sivers function
L	L		$g_{1L}(x,k_\perp)$		$\Delta q(x)$	helicity distribution
	T		$g_{1T}^\perp(x,k_\perp)$		×	worm gear/trans-helicity
T	U		$h_1^\perp(x,k_\perp)$	0	×	Boer-Mulders function
	$T(//)$		$h_{1T}(x,k_\perp)$		$\delta q(x)$	transversity distribution
	$T(\perp)$		$h_{1T}^\perp(x,k_\perp)$			pretzelocity
	L		$h_{1L}^\perp(x,k_\perp)$		×	worm gear/ longi-transversity

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Twist-3 TMD PDFs defined via quark-quark correlator

Next to the leading twist (twist-3)

they are NOT probability distributions but contribute in different polarization.

polarization: quark	nucleon	pictorially	TMD PDFs (16)	if no gauge link	integrated over $k_\perp$	name
U	U		$e(x,k_\perp)$, $f^\perp(x,k_\perp)$	0	$e(x)$, ×	number density
	L		$f_L^\perp(x,k_\perp)$	0	×	Sivers function
	T		$e_T^\perp(x,k_\perp)$,	0	×	
			$f_T(x,k_\perp)$, $f_T^\perp(x,k_\perp)$	0 0	$f_T(x)$	
L	U		$g^\perp(x,k_\perp)$	0	×	helicity distribution
	L		$e_L(x,k_\perp)$, $g_L^\perp(x,k_\perp)$	0 $\frac{g_{1L}(x,k_\perp)}{x}$	$e_L(x)$, ×	
	T		$e'^\perp_T(x,k_\perp)$,	0	×	worm gear/trans-helicity
			$g_T(x,k_\perp)$, $g_T^\perp(x,k_\perp)$	0 $\frac{g_{1T}(x,k_\perp)}{x}$	$g'_T(x)$	
T	U		$h(x,k_\perp)$	0	$h(x)$	Boer-Mulders function
	$T(//)$		$h_T^\perp(x,k_\perp)$	$\frac{h_{1T}^\perp(x,k_\perp)}{x}$	×	transversity distribution
	$T(\perp)$		$h_T^{\perp'}(x,k_\perp)$	$\frac{k_\perp^2 h_{1T}^\perp(x,k_\perp)}{M^2x}$	×	pretzelocity
	L		$h_L(x,k_\perp)$	$\frac{k_\perp^2 h_{1L}^\perp(x,k_\perp)}{M^2x}$	$h_L(x)$	worm gear/ longi-transversity

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TMD PDFs defined via quark-quark correlator

Twist-2 TMD PDFs

nucleon polarization ↓ / quark polarization →	U	L	T
U	$f_1(x,k_\perp)$ number density		$h_1^\perp(x,k_\perp)$ Boer-Mulders function
L		$g_{1L}(x,k_\perp)$ helicity distribution	$h_{1L}^\perp(x,k_\perp)$ Worm-gear/longi-transversity
T	$f_{1T}^\perp(x,k_\perp)$ Sivers function	$g_{1T}^\perp(x,k_\perp)$ Worm-gear/trans-helicity	$h_{1T}(x,k_\perp)$ transversity distribution; $h_{1T}^\perp(x,k_\perp)$ pretzelosity

Twist-3 TMD PDFs

nucleon polarization ↓ / quark polarization →	U	L	T
U	$e(x,k_\perp),f^\perp(x,k_\perp)$ number density	$g^\perp(x,k_\perp)$	$h(x,k_\perp)$ Boer-Mulders function
L	$f_L^\perp(x,k_\perp)$	$e_L(x,k_\perp),g_L^\perp(x,k_\perp)$ helicity distribution	$h_L(x,k_\perp)$ Worm gear/ longi-transversity
T	$e_T^\perp(x,k_\perp)$, $f_T^{\perp 1}(x,k_\perp),f_T^{\perp 2}(x,k_\perp)$ Sivers function	$e_T(x,k_\perp)$, $g_T(x,k_\perp),g_T^\perp(x,k_\perp)$ Worm gear/ trans-helicity	$h_T^\perp(x,k_\perp)$ transversity distribution; $h_T(x,k_\perp)$ pretzelosity

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TMD PDFs defined intuitively (equivalent to twist-2)

In the 1-dimensional case:

$$f_q(x, S_q; p, S) = f_q(x) + \lambda_q \lambda\, \Delta f_q(x) + (\vec{S}_{\perp q} \cdot \vec{S}_T)\, \delta f_q(x)$$

In the 3-dimensional case:

$$f_q(x, k_\perp, S_q; p, S) = f_q(x, k_\perp) + \lambda_q \lambda\, \Delta f_q(x, k_\perp) + (\vec{S}_{\perp q} \cdot \vec{S}_T)\, \delta f_q(x, k_\perp)$$
$$+ \vec{S}_T \cdot (\hat{p} \times \hat{k}_\perp) \Delta^N f(x, k_\perp) + \frac{1}{M} \vec{S}_{\perp q} \cdot (\hat{p} \times \vec{k}_\perp) h_1^\perp(x, k_\perp)$$
$$+ \frac{1}{M^2} (\vec{S}_{\perp q} \cdot \vec{k}_\perp)(\vec{S}_T \cdot \vec{k}_\perp) h_{1T}^\perp(x, k_\perp) + \frac{1}{M} (\vec{S}_{\perp q} \cdot \vec{k}_\perp)\, \lambda\, h_{1L}^\perp(x, k_\perp)$$
$$+ \lambda_q \frac{1}{M} (\vec{S}_T \cdot \vec{k}_\perp)\, g_{1T}^\perp(x, k_\perp)$$

$$\delta f_q(x, k_\perp) = h_{1T}(x, k_\perp), \qquad \Delta^N f(x, k_\perp) = -\frac{|\vec{k}_\perp|}{M} f_{1T}^\perp(x, k_\perp)$$

Twist-2 TMD FFs defined via quark-quark correlator

Leading twist (twist 2) D, G, H: quark un-, longitudinally, transversely polarized

quark	hadron (polarization)	pictorially	TMD FFs (8)	integrated over $k_{F\perp}$	name
U	U		$D_1(z, k_{F\perp})$	$D_1(z)$	number density
	T		$D_{1T}^\perp(z, k_{F\perp})$	×	Sivers-type function
L	L		$G_{1L}(z, k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T		$G_{1T}^\perp(x, k_\perp)$	×	
T	U		$H_1^\perp(z, k_{F\perp})$	×	Collins function
	T(//)		$H_{1T}(z, k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	T(⊥)		$H_{1T}^\perp(z, k_{F\perp})$		
	L		$H_{1L}^\perp(z, k_{F\perp})$	×	

Twist-2 TMD FFs defined via quark-quark correlator (spin-1)

Quark pol	Hadron pol		TMD FFs (2+6+10=18)	integrated over $k_{F\perp}$	name
U	U		$D_1(z, k_{F\perp})$	$D_1(z)$	number density
	T		$D_{1T}^\perp(z, k_{F\perp})$	×	Sivers-type function
	LL		$D_{1LL}(z, k_{F\perp})$	$D_{1LL}(z)$	spin alignment
	LT		$D_{1LT}^\perp(z, k_{F\perp})$	×	
	TT		$D_{1TT}^\perp(z, k_{F\perp})$	×	
L	L		$G_{1L}(z, k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T		$G_{1T}^\perp(z, k_{F\perp})$	×	
	LT		$G_{1LT}^\perp(z, k_{F\perp})$	×	
	TT		$G_{1TT}^\perp(z, k_{F\perp})$	×	
T	U		$H_1^\perp(z, k_{F\perp})$	×	Collins function
	T(//)		$H_{1T}(z, k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	T(⊥)		$H_{1T}^\perp(z, k_{F\perp})$		
	L		$H_{1L}^\perp(z, k_{F\perp})$	×	
	LL		$H_{1LL}^\perp(z, k_{F\perp})$	×	
	LT		$H_{1LT}(z, k_{F\perp})$, $H_{1LT}^\perp(z, k_{F\perp})$	$H_{1LT}(z)$	
	TT		$H_{1TT}^\perp(z, k_{F\perp})$, $H'^{\perp}_{1TT}(z, k_{F\perp})$	×, ×	

See e.g., K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD94, 034003 (2016).

Twist-2 TMD FFs defined via quark-quark correlator (spin-1)

Classified according to the polarization of the quark:

Unpolaried quark	$D_1, D_{1T}^\perp, D_{1LL}, D_{1LT}^\perp, D_{1TT}^\perp$
Longitudinally polarized quark	$G_{1L}, G_{1T}^\perp, G_{1LT}^\perp, G_{1TT}^\perp$
Transversely polarized quark	$H_1^\perp, H_{1T}, H_{1T}^\perp, H_{1L}^\perp, H_{1LL}^\perp, H_{1LT}, H_{1LT}^\perp, H_{1TT}^\perp, H'^{\perp}_{1TT}$

Classified according to the polarization of the hadron:

$$\hat{\Xi}(z, k_{F\perp}; p, S) = \hat{\Xi}(z, k_{F\perp}; p \mid Upol) + \hat{\Xi}(z, k_{F\perp}; p \mid Vpol) + \hat{\Xi}(z, k_{F\perp}; p \mid Tpol)$$

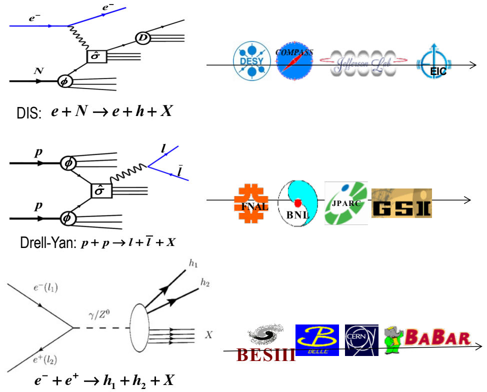
	Upol	Vpol	Tpol
number density:	$D_1, H_1^\perp$		
induced polarization:		$D_{1T}^\perp$	$D_{1LL}, D_{1LT}^\perp, D_{1TT}^\perp$
spin transfer – "direct":		$G_{1L}; H_{1T}, H_{1T}^\perp$	
spin transfer – "worm gear":		$G_{1T}^\perp; H_{1L}^\perp$	$G_{1LT}^\perp, G_{1TT}^\perp; H_{1LL}^\perp, H_{1LT}, H_{1LT}^\perp, H_{1TT}^\perp, H'^{\perp}_{1TT}$

Their contributions to the cross section are additive.

Contents

Access TMDs via semi-inclusive high energy reactions

Semi-inclusive reactions



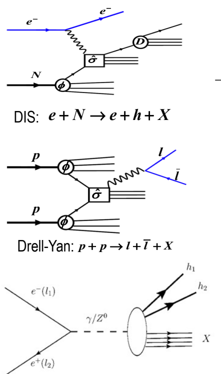
DIS: $e+N\to e+h+X$ — TMD PDFs: $f_1, f_{1T}^\perp, g_{1L}, g_{1T}, h_1, h_1^\perp, h_{1L}^\perp, h_{1T}^\perp$... TMD FFs: $D_1, H_1^\perp$,...

Drell-Yan: $p+p\to l+\bar{l}+X$ — TMD PDFs: $f_1, f_{1T}^\perp, g_{1L}, g_{1T}, h_1, h_1^\perp, h_{1L}^\perp, h_{1T}^\perp$...

$e^-+e^+\to h_1+h_2+X$ — TMD FFs: $D_1, H_1^\perp$,...

Semi-inclusive high energy reactions: Kinematics

Semi-inclusive reactions: general form of the hadronic tensors and cross sections



DIS: $e+N\to e+h+X$
Gourdin, NPB 49, 501 (1972); Kotzinian, NPB 441, 234 (1995); Diehl, Sapeta, EPJ C41, 515 (2005); Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02, 093 (2007); ...... → 18 independent "structure functions" for spinless hadron h

Drell-Yan: $p+p\to l+\bar{l}+X$
Arnold, Metz, Schlegel, Phys. Rev. D79 ,034005 (2009). → 48 independent "structure functions"

$e^-+e^+\to h_1+h_2+X$
Pitonyak, Schlegel, Metz, PRD89, 054032 (2014). → 36 independent "structure functions" for spin-1/2 hadrons
K.B. Chen, W.H. Yang, S.Y. Wei, & ZTL, PRD94, 034003 (2016). → 81 independent "structure functions" for spin-1 hadrons

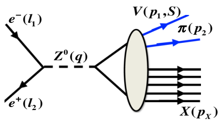
Kinematic analysis for $e^+e^-\to Z\to V\pi X$

$e^-e^+\to Z\to V(p_1,S)\pi(p_2)X$: the best place to study tensor polarization dependent FFs

The differential cross section:

$$\frac{2E_1E_2}{d^3p_1d^3p_2}=\frac{\alpha^2}{sQ^4}\chi L_{\mu\nu}(l_1,l_2)W^{\mu\nu}(q,p_1,S,p_2)$$

$$L_{\mu\nu}(l_1,l_2)=c_1^e\left[l_{1\mu}l_{2\nu}+l_{1\nu}l_{2\mu}-(l_1\cdot l_2)g_{\mu\nu}\right]+ic_3^e\varepsilon_{\mu\nu\rho\sigma}l_1^\rho l_2^\sigma$$



The hadronic tensor:

$$W_{\mu\nu}(q,p_1,S,p_2)=W^{S\mu\nu}\ \text{(the Symmetric part)}\ +iW^{A\mu\nu}\ \text{(the Anti-symmetric part)}$$

$$=\sum_{\sigma,i}W_{\sigma i}^S h_{\sigma i}^{S\mu\nu}+\sum_{\sigma,j}\tilde{W}_{\sigma j}^S\tilde{h}_{\sigma j}^{S\mu\nu}+i\sum_{\sigma,i}W_{\sigma i}^A h_{\sigma i}^{A\mu\nu}+i\sum_{\sigma,j}\tilde{W}_{\sigma j}^A\tilde{h}_{\sigma j}^{A\mu\nu}$$

$\sigma=U,V,S_{LL},S_{LT},S_{TT}$ polarization

the basic Lorentz tensors: $h_{\sigma i}^{S\mu\nu}=h_{\sigma i}^{S\nu\mu}$, $h_{\sigma i}^{A\mu\nu}=-h_{\sigma i}^{A\nu\mu}$ space reflection P-even: $\hat{\mathcal{P}}h^{\mu\nu}=h_{\mu\nu}$

$\tilde{h}_{\sigma i}^{S\mu\nu}=\tilde{h}_{\sigma i}^{S\nu\mu}$, $\tilde{h}_{\sigma i}^{A\mu\nu}=-\tilde{h}_{\sigma i}^{A\nu\mu}$ space reflection P-odd: $\hat{\mathcal{P}}\tilde{h}^{\mu\nu}=-\tilde{h}_{\mu\nu}$

Constraints: $W^{\mu\nu*}=W^{\nu\mu}$ (hermiticity), $q_\mu W^{\mu\nu}=q_\nu W^{\mu\nu}=0$ (current conservation)

K.B. Chen, W.H. Yang, S.Y. Wei, & ZTL, PRD94, 034003 (2016) (spin-1).

Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The basic Lorentz tensor sets for the hadronic tensor

unpolarized: 5+4=9

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2},\ p_{1q}^\mu p_{1q}^\nu,\ p_{2q}^\mu p_{2q}^\nu,\ p_{1q}^{\{\mu} p_{2q}^{\nu\}} \right\}$$ symmetric (S), P-even

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{ \varepsilon^{\{\mu q p_1 p_2}(p_{1q}^{\nu\}},\ p_{2q}^{\nu\}}) \right\}$$ symmetric (S), P-odd

$$h_U^{A\mu\nu} = p_{1q}^{[\mu} p_{2q}^{\nu]}$$ anti-symmetric (A), P-even

$$\tilde{h}_{Ui}^{A\mu\nu} = \left\{ \varepsilon^{\mu\nu q p_1},\ \varepsilon^{\mu\nu q p_2} \right\}$$ anti-symmetric (A), P-odd

$a^{[\alpha}b^{\beta]} \equiv a^\alpha b^\beta - a^\beta b^\alpha$

$a^{\{\alpha}b^{\beta\}} \equiv a^\alpha b^\beta + a^\beta b^\alpha$

$\varepsilon^{\mu\nu\alpha p} \equiv \varepsilon^{\mu\nu\alpha\beta} p_\beta$, $\varepsilon_\perp^{\mu\nu} \equiv \varepsilon^{\mu\nu\alpha\beta}\bar{n}_\alpha n_\beta$

$p_q \equiv p - \frac{p\cdot q}{q^2} q \quad (p_q \cdot q = 0)$

A regularity: (spin dependent Lorentz tensor set) = (spin dependent Lorentz (pseudo)scalar) × (the unpolarized set)

Unpolarized 5+4=9

$$\begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix}$$

Vector polarization S-dependent: 13+14=27

longitudinal polarization $\lambda \sim \vec{p}_1 \cdot \vec{S}$

$$\begin{pmatrix} h_{Li}^{S\mu\nu} \\ \tilde{h}_{Li}^{S\mu\nu} \\ h_{Li}^{A\mu\nu} \\ \tilde{h}_{Li}^{A\mu\nu} \end{pmatrix} = \lambda \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix}$$

transverse polarization

$$\begin{pmatrix} h_{Ti}^{S\mu\nu} \\ \tilde{h}_{Ti}^{S\mu\nu} \\ h_{Ti}^{A\mu\nu} \\ \tilde{h}_{Ti}^{A\mu\nu} \end{pmatrix} = \left\{ (p_2\cdot S) \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix},\quad \varepsilon^{Sqp_1p_2} \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix} \right\}$$

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Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The basic Lorentz tensor sets for the hadronic tensor (continued)

$S_{TT}^{p\beta} \equiv p_\alpha S_{TT}^{\alpha\beta}$

$\varepsilon^{abcd} \equiv \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$

S_{LL}-dependent part: 5+4=9

$S_{LL}^{\mathcal{P}} = S_{LL}$

$$\begin{pmatrix} h_{LLi}^{S\mu\nu} \\ \tilde{h}_{LLi}^{S\mu\nu} \\ h_{LLi}^{A\mu\nu} \\ \tilde{h}_{LLi}^{A\mu\nu} \end{pmatrix} = S_{LL} \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix}$$

S_{LT}-dependent part: 9+9=18

$S_{LT} = (0, S_{LT}^x, S_{LT}^y, 0)$

$p_1 \cdot S_{LT} = 0,\quad q\cdot S_{LT} = 0 \qquad S_{LT\mu}^{\mathcal{P}} = S_{LT}^\mu$

$$\begin{pmatrix} h_{LTi}^{S\mu\nu} \\ \tilde{h}_{LTi}^{S\mu\nu} \\ h_{LTi}^{A\mu\nu} \\ \tilde{h}_{LTi}^{A\mu\nu} \end{pmatrix} = \left\{ (p_2\cdot S_{LT}) \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix},\quad \varepsilon^{S_{LT}qp_1p_2} \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix} \right\}$$

S_{TT}-dependent part: 9+9=18

$$S_{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{TT}^{xx} & S_{TT}^{xy} & 0 \\ 0 & S_{TT}^{xy} & -S_{TT}^{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$S_{TT\mu\nu}^{\mathcal{P}} = S_{TT}^{\mu\nu}$

$S_{TT}^{p_1\beta} = S_{TT}^{\alpha p_1} = 0$

$S_{TT}^{q\beta} = S_{TT}^{\alpha q} = 0$

$$\begin{pmatrix} h_{TTi}^{S\mu\nu} \\ \tilde{h}_{TTi}^{S\mu\nu} \\ h_{TTi}^{A\mu\nu} \\ \tilde{h}_{TTi}^{A\mu\nu} \end{pmatrix} = \left\{ S_{TT}^{p_2p_2} \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix},\quad \varepsilon^{S_{TT}^{p_2}qp_1p_2} \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix} \right\}$$

K.B. Chen, W.H. Yang, S.Y. Wei, & ZTL, PRD94, 034003 (2016).

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Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The cross section in Helicity-GJ-frame: unpolarized and longitudinally polarized parts

$$\frac{2E_1E_2 d\sigma^U}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi(\mathcal{F}_U + \tilde{\mathcal{F}}_U)$$

The structure functions: $F_{jxx}^{yy} = F_{jxx}^{yy}(s,\xi_1,\xi_2,p_{2T})$, $\tilde{F}_{jxx}^{yy} = \tilde{F}_{jxx}^{yy}(s,\xi_1,\xi_2,p_{2T})$

$$\mathcal{F}_U = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U} + \cos\varphi[\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi}] + \cos 2\varphi \sin^2\theta F_U^{\cos 2\varphi}$$

1, $\cos\varphi$, $\cos 2\varphi$

$$\tilde{\mathcal{F}}_U = \sin\varphi[\sin\theta \tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2U}^{\sin\varphi}] + \sin 2\varphi \sin^2\theta \tilde{F}_U^{\sin 2\varphi}$$

$\sin\varphi$, $\sin 2\varphi$

$$\frac{2E_1E_2 d\sigma^L}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi\lambda(\mathcal{F}_L + \tilde{\mathcal{F}}_L)$$

$\mathcal{F}_L \leftrightarrow \tilde{\mathcal{F}}_U$, $\tilde{\mathcal{F}}_L \leftrightarrow \mathcal{F}_U$

$F_{jL}^{xxx} \leftrightarrow \tilde{F}_{jU}^{xxx}$, $\tilde{F}_{jL}^{xxx} \leftrightarrow F_{jU}^{xxx}$

$$\mathcal{F}_L = \sin\varphi[\sin\theta F_{1L}^{\sin\varphi} + \sin 2\theta F_{2L}^{\sin\varphi}] + \sin 2\varphi \sin^2\theta F_L^{\sin 2\varphi}$$

$$\tilde{\mathcal{F}}_L = (1+\cos^2\theta)\tilde{F}_{1L} + \sin^2\theta \tilde{F}_{2L} + \cos\theta \tilde{F}_{3L} + \cos\varphi[\sin\theta \tilde{F}_{1L}^{\cos\varphi} + \sin 2\theta \tilde{F}_{2L}^{\cos\varphi}] + \cos 2\varphi \sin^2\theta \tilde{F}_L^{\cos 2\varphi}$$

$$\frac{2E_1E_2 d\sigma^{LL}}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi S_{LL}(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL})$$

$\mathcal{F}_{LL} \leftrightarrow \mathcal{F}_U$, $\tilde{\mathcal{F}}_{LL} \leftrightarrow \tilde{\mathcal{F}}_U$

$F_{jLL}^{xxx} \leftrightarrow F_{jU}^{xxx}$, $\tilde{F}_{jLL}^{xxx} \leftrightarrow \tilde{F}_{jU}^{xxx}$

$$\mathcal{F}_{LL} = (1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL} + \cos\varphi[\sin\theta F_{1LL}^{\cos\varphi} + \sin 2\theta F_{2LL}^{\cos\varphi}] + \cos 2\varphi \sin^2\theta F_{LL}^{\cos 2\varphi}$$

$$\tilde{\mathcal{F}}_{LL} = \sin\varphi[\sin\theta \tilde{F}_{1LL}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2LL}^{\sin\varphi}] + \sin 2\varphi \sin^2\theta \tilde{F}_{LL}^{\sin 2\varphi}$$

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General kinematic analysis for $e^+e^- \to V\pi X$

The cross section in Helicity-GJ-frame: transverse polarization dependent parts

$$\frac{2E_1E_2 d\sigma^T}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi|\vec{S}_T|(\mathcal{F}_T + \tilde{\mathcal{F}}_T)$$

$|\vec{S}_T|^2 = (S_T^x)^2 + (S_T^y)^2$

$\tan\varphi_S = S_T^x / S_T^y$

$$\begin{aligned} \mathcal{F}_T = {} & \sin\varphi_S[\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}] && \sin\varphi_S \\ & + \sin(\varphi_S+\varphi)\sin^2\theta F_T^{\sin(\varphi_S+\varphi)} && \sin(\varphi_S+\varphi) \\ & + \sin(\varphi_S-\varphi)[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)} + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}] && \sin(\varphi_S-\varphi) \\ & + \sin(\varphi_S-2\varphi)[\sin\theta F_{1T}^{\sin(\varphi_S-2\varphi)} + \sin 2\theta F_{2T}^{\sin(\varphi_S-2\varphi)}] && \sin(\varphi_S-2\varphi) \\ & + \sin(\varphi_S-3\varphi)\sin^2\theta F_T^{\sin(\varphi_S-3\varphi)} && \sin(\varphi_S-3\varphi) \end{aligned}$$

$$\begin{aligned} \tilde{\mathcal{F}}_T = {} & \cos\varphi_S[\sin\theta \tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde{F}_{2T}^{\cos\varphi_S}] && \cos\varphi_S \\ & + \cos(\varphi_S+\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S+\varphi)} && \cos(\varphi_S+\varphi) \\ & + \cos(\varphi_S-\varphi)[(1+\cos^2\theta)\tilde{F}_{1T}^{\cos(\varphi_S-\varphi)} + \sin^2\theta \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}] && \cos(\varphi_S-\varphi) \\ & + \cos(\varphi_S-2\varphi)[\sin\theta \tilde{F}_{1T}^{\cos(\varphi_S-2\varphi)} + \sin 2\theta \tilde{F}_{2T}^{\cos(\varphi_S-2\varphi)}] && \cos(\varphi_S-2\varphi) \\ & + \cos(\varphi_S-3\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S-3\varphi)} && \cos(\varphi_S-3\varphi) \end{aligned}$$

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General kinematic analysis for $e^+e^- \to V\pi X$

The cross section in Helicity-GJ-frame: transverse polarization dependent parts

$$\frac{2E_1E_2 d\sigma^T}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi |\vec{S}_T| (\mathcal{F}_T + \tilde{\mathcal{F}}_T)$$

$$|\vec{S}_T|^2 = (S_T^x)^2 + (S_T^y)^2, \quad \tan\varphi_S = S_T^x / S_T^y$$

$$\begin{aligned}\mathcal{F}_T =\ & \sin\varphi_S[\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}] \\ & + \sin(\varphi_S+\varphi)\sin^2\theta F_T^{\sin(\varphi_S+\varphi)} \\ & + \sin(\varphi_S-\varphi)[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)} + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}] \\ & + \sin(\varphi_S-2\varphi)[\sin\theta F_{1T}^{\sin(\varphi_S-2\varphi)} + \sin 2\theta F_{2T}^{\sin(\varphi_S-2\varphi)}] \\ & + \sin(\varphi_S-3\varphi)\sin^2\theta F_T^{\sin(\varphi_S-3\varphi)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{F}}_T =\ & \cos\varphi_S[\sin\theta \tilde F_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde F_{2T}^{\cos\varphi_S}] \\ & + \cos(\varphi_S+\varphi)\sin^2\theta \tilde F_T^{\cos(\varphi_S+\varphi)} \\ & + \cos(\varphi_S-\varphi)[(1+\cos^2\theta)\tilde F_{1T}^{\cos(\varphi_S-\varphi)} + \sin^2\theta \tilde F_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde F_{3T}^{\cos(\varphi_S-\varphi)}] \\ & + \cos(\varphi_S-2\varphi)[\sin\theta \tilde F_{1T}^{\cos(\varphi_S-2\varphi)} + \sin 2\theta \tilde F_{2T}^{\cos(\varphi_S-2\varphi)}] \\ & + \cos(\varphi_S-3\varphi)\sin^2\theta \tilde F_T^{\cos(\varphi_S-3\varphi)}\end{aligned}$$

$$\frac{2E_1E_2 d\sigma^{LT}}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi |\vec{S}_{LT}| (\mathcal{F}_{LT} + \tilde{\mathcal{F}}_{LT})$$

$$|\vec{S}_{LT}|^2 = (S_{LT}^x)^2 + (S_{LT}^y)^2, \quad \tan\varphi_{LT} = S_{LT}^x / S_{LT}^y$$

$$\varphi_S \leftrightarrow \varphi_{LT}; \quad \mathcal{F}_T \leftrightarrow \tilde{\mathcal{F}}_{LT},\ \tilde{\mathcal{F}}_T \leftrightarrow \mathcal{F}_{LT}; \quad F_{jT}^{xxx} \leftrightarrow \tilde F_{jLT}^{xxx},\ \tilde F_{jT}^{xxx} \leftrightarrow F_{jLT}^{xxx}$$

$$\begin{aligned}\mathcal{F}_{LT} =\ & \cos\varphi_{LT}[\sin\theta F_{1LT}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos\varphi_{LT}}] \\ & + \cos(\varphi_{LT}+\varphi)\sin^2\theta F_{LT}^{\cos(\varphi_{LT}+\varphi)} \\ & + \cos(\varphi_{LT}-\varphi)[(1+\cos^2\theta)F_{1LT}^{\cos(\varphi_{LT}-\varphi)} + \sin^2\theta F_{2LT}^{\cos(\varphi_{LT}-\varphi)} + \cos\theta F_{3LT}^{\cos(\varphi_{LT}-\varphi)}] \\ & + \cos(\varphi_{LT}-2\varphi)[\sin\theta F_{1LT}^{\cos(\varphi_{LT}-2\varphi)} + \sin 2\theta F_{2LT}^{\cos(\varphi_{LT}-2\varphi)}] \\ & + \cos(\varphi_{LT}-3\varphi)\sin^2\theta F_{LT}^{\cos(\varphi_{LT}-3\varphi)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{F}}_{LT} =\ & \sin\varphi_{LT}[\sin\theta \tilde F_{1LT}^{\sin\varphi_{LT}} + \sin 2\theta \tilde F_{2LT}^{\sin\varphi_{LT}}] \\ & + \sin(\varphi_{LT}+\varphi)\sin^2\theta F_{LT}^{\sin(\varphi_{LT}+\varphi)} \\ & + \sin(\varphi_{LT}-\varphi)[(1+\cos^2\theta)\tilde F_{1LT}^{\sin(\varphi_{LT}-\varphi)} + \sin^2\theta \tilde F_{2LT}^{\sin(\varphi_{LT}-\varphi)} + \cos\theta \tilde F_{3LT}^{\sin(\varphi_{LT}-\varphi)}] \\ & + \sin(\varphi_{LT}-2\varphi)[\sin\theta \tilde F_{1LT}^{\sin(\varphi_{LT}-2\varphi)} + \sin 2\theta \tilde F_{2LT}^{\sin(\varphi_{LT}-2\varphi)}] \\ & + \sin(\varphi_{LT}-3\varphi)\sin^2\theta \tilde F_{LT}^{\sin(\varphi_{LT}-3\varphi)}\end{aligned}$$

$$\frac{2E_1E_2 d\sigma^{TT}}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi |\vec{S}_{TT}| (\mathcal{F}_{TT} + \tilde{\mathcal{F}}_{TT})$$

$$|\vec{S}_{TT}|^2 = (S_{TT}^{xx})^2 + (S_{TT}^{xy})^2, \quad \tan 2\varphi_{TT} = S_{TT}^{xx} / S_{TT}^{xy}$$

$$(2\varphi_{TT}-\varphi) \leftrightarrow \varphi_{LT}; \quad \mathcal{F}_{TT} \leftrightarrow \mathcal{F}_{LT},\ \tilde{\mathcal{F}}_{TT} \leftrightarrow \tilde{\mathcal{F}}_{LT}; \quad F_{jTT}^{xxx} \leftrightarrow F_{jLT}^{xxx},\ F_{jTT}^{xxx} \leftrightarrow F_{jLT}^{xxx}$$

$$\begin{aligned}\mathcal{F}_{TT} =\ & \cos 2\varphi_{TT}\sin^2\theta F_{TT}^{\cos 2\varphi_{TT}} \\ & + \cos(2\varphi_{TT}-\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-\varphi)} + \sin 2\theta F_{2TT}^{\cos(2\varphi_{TT}-\varphi)}] \\ & + \cos(2\varphi_{TT}-2\varphi)[(1+\cos^2\theta)F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} + \sin^2\theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}] \\ & + \cos(2\varphi_{TT}-3\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-3\varphi)} + \sin 2\theta F_{2TT}^{\cos(2\varphi_{TT}-3\varphi)}] \\ & + \cos(2\varphi_{TT}-4\varphi)\sin^2\theta F_{TT}^{\cos(2\varphi_{TT}-4\varphi)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{F}}_{TT} =\ & \sin 2\varphi_{TT}\sin^2\theta \tilde F_{TT}^{\sin 2\varphi_{TT}} \\ & + \sin(2\varphi_{TT}-\varphi)[\sin\theta \tilde F_{1TT}^{\sin(2\varphi_{TT}-\varphi)} + \sin 2\theta \tilde F_{2TT}^{\sin(2\varphi_{TT}-\varphi)}] \\ & + \sin(2\varphi_{TT}-2\varphi)[(1+\cos^2\theta)\tilde F_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} + \sin^2\theta \tilde F_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta \tilde F_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}] \\ & + \sin(2\varphi_{TT}-3\varphi)[\sin\theta \tilde F_{1TT}^{\sin(2\varphi_{TT}-3\varphi)} + \sin 2\theta \tilde F_{2TT}^{\sin(2\varphi_{TT}-3\varphi)}] \\ & + \sin(2\varphi_{TT}-4\varphi)\sin^2\theta \tilde F_{TT}^{\sin(2\varphi_{TT}-4\varphi)}\end{aligned}$$

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General kinematic analysis for $e^+e^- \to V\pi X$

θ-dep.	$1+\cos^2\theta$	$\sin^2\theta$	$\cos\theta$	$\sin\theta$	$\sin 2\theta$	$\sin^2\theta$	$\sin\theta$	$\sin 2\theta$	$\sin^2\theta$
φ-dep.	1			$\cos\varphi$		$\cos 2\varphi$	$\sin\varphi$		$\sin 2\varphi$
U	F_{1U}	F_{2U}	F_{3U}	$F_{1U}^{\cos\varphi}$	$F_{2U}^{\cos\varphi}$	$F_U^{\cos 2\varphi}$	$\tilde F_{1U}^{\sin\varphi}$	$\tilde F_{2U}^{\sin\varphi}$	$\tilde F_U^{\sin 2\varphi}$
L	$\tilde F_{1L}$	$\tilde F_{2L}$	$\tilde F_{3L}$	$\tilde F_{1L}^{\cos\varphi}$	$\tilde F_{2L}^{\cos\varphi}$	$\tilde F_L^{\cos 2\varphi}$	$F_{1L}^{\sin\varphi}$	$F_{2L}^{\sin\varphi}$	$F_L^{\sin 2\varphi}$
LL	F_{1LL}	F_{2LL}	F_{3LL}	$F_{1LL}^{\cos\varphi}$	$F_{2LL}^{\cos\varphi}$	$F_{LL}^{\cos 2\varphi}$	$\tilde F_{1LL}^{\sin\varphi}$	$\tilde F_{2LL}^{\sin\varphi}$	$\tilde F_{LL}^{\sin 2\varphi}$
T-PC	$F_{1T}^{\sin(\varphi_S-\varphi)}$	$F_{2T}^{\sin(\varphi_S-\varphi)}$	$F_{3T}^{\sin(\varphi_S-\varphi)}$	$F_{1T}^{\sin(\varphi_S-2\varphi)}$	$F_{2T}^{\sin(\varphi_S-2\varphi)}$	$F_T^{\sin(\varphi_S-3\varphi)}$	$F_{1T}^{\sin\varphi_S}$	$F_{2T}^{\sin\varphi_S}$	$F_T^{\sin(\varphi_S+\varphi)}$
φ-dep.	$\sin(\varphi_S-\varphi)$			$\sin(\varphi_S-2\varphi)$		$\sin(\varphi_S-3\varphi)$	$\sin\varphi_S$		$\sin(\varphi_S+\varphi)$
T-PV	$\tilde F_{1T}^{\cos(\varphi_S-\varphi)}$	$\tilde F_{2T}^{\cos(\varphi_S-\varphi)}$	$\tilde F_{3T}^{\cos(\varphi_S-\varphi)}$	$\tilde F_{1T}^{\cos(\varphi_S-2\varphi)}$	$\tilde F_{2T}^{\cos(\varphi_S-2\varphi)}$	$\tilde F_T^{\cos(\varphi_S-3\varphi)}$	$\tilde F_{1T}^{\cos\varphi_S}$	$\tilde F_{2T}^{\cos\varphi_S}$	$\tilde F_T^{\cos(\varphi_S+\varphi)}$
φ-dep.	$\cos(\varphi_S-\varphi)$			$\cos(\varphi_S-2\varphi)$		$\cos(\varphi_S-3\varphi)$	$\cos\varphi_S$		$\cos(\varphi_S+\varphi)$
LT-PC	$F_{1LT}^{\cos(\varphi_{LT}-\varphi)}$	$F_{2LT}^{\cos(\varphi_{LT}-\varphi)}$	$F_{3LT}^{\cos(\varphi_{LT}-\varphi)}$	$F_{1LT}^{\cos(\varphi_{LT}-2\varphi)}$	$F_{2LT}^{\cos(\varphi_{LT}-2\varphi)}$	$F_{LT}^{\cos(\varphi_{LT}-3\varphi)}$	$F_{1LT}^{\cos\varphi_{LT}}$	$F_{2LT}^{\cos\varphi_{LT}}$	$F_{LT}^{\cos(\varphi_{LT}+\varphi)}$
φ-dep.	$\cos(\varphi_{LT}-\varphi)$			$\cos(\varphi_{LT}-2\varphi)$		$\cos(\varphi_{LT}-3\varphi)$	$\cos\varphi_{LT}$		$\cos(\varphi_{LT}+\varphi)$
LT-PV	$\tilde F_{1LT}^{\sin(\varphi_{LT}-\varphi)}$	$\tilde F_{2LT}^{\sin(\varphi_{LT}-\varphi)}$	$\tilde F_{3LT}^{\sin(\varphi_{LT}-\varphi)}$	$\tilde F_{1LT}^{\sin(\varphi_{LT}-2\varphi)}$	$\tilde F_{2LT}^{\sin(\varphi_{LT}-2\varphi)}$	$\tilde F_{LT}^{\sin(\varphi_{LT}-3\varphi)}$	$\tilde F_{1LT}^{\sin\varphi_{LT}}$	$\tilde F_{2LT}^{\sin\varphi_{LT}}$	$\tilde F_{LT}^{\sin(\varphi_{LT}+\varphi)}$
φ-dep.	$\sin(\varphi_{LT}-\varphi)$			$\sin(\varphi_{LT}-2\varphi)$		$\sin(\varphi_{LT}-3\varphi)$	$\sin\varphi_{LT}$		$\sin(\varphi_{LT}+\varphi)$
TT-PC	$F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)}$	$F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)}$	$F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}$	$F_{1TT}^{\cos(2\varphi_{TT}-3\varphi)}$	$F_{2TT}^{\cos(2\varphi_{TT}-3\varphi)}$	$F_{TT}^{\cos(2\varphi_{TT}-4\varphi)}$	$F_{1TT}^{\cos(2\varphi_{TT}-\varphi)}$	$F_{2TT}^{\cos(2\varphi_{TT}-\varphi)}$	$F_{TT}^{\cos 2\varphi_{TT}}$
φ-dep.	$\cos(2\varphi_{TT}-2\varphi)$			$\cos(2\varphi_{TT}-3\varphi)$		$\cos(2\varphi_{TT}-4\varphi)$	$\cos(2\varphi_{TT}-\varphi)$		$\cos 2\varphi_{TT}$
TT-PV	$\tilde F_{1TT}^{\sin(2\varphi_{TT}-2\varphi)}$	$\tilde F_{2TT}^{\sin(2\varphi_{TT}-2\varphi)}$	$\tilde F_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}$	$\tilde F_{TT}^{\sin(2\varphi_{TT}-3\varphi)}$	$\tilde F_{2TT}^{\sin(2\varphi_{TT}-3\varphi)}$	$\tilde F_{TT}^{\sin(2\varphi_{TT}-4\varphi)}$	$\tilde F_{1TT}^{\sin(2\varphi_{TT}-\varphi)}$	$\tilde F_{2TT}^{\sin(2\varphi_{TT}-\varphi)}$	$\tilde F_{TT}^{\sin 2\varphi_{TT}}$
φ-dep.	$\sin(2\varphi_{TT}-2\varphi)$			$\sin(2\varphi_{TT}-3\varphi)$		$\sin(2\varphi_{TT}-4\varphi)$	$\sin(2\varphi_{TT}-\varphi)$		$\sin 2\varphi_{TT}$

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General kinematic analysis for $e^+e^- \to V\pi X$

Azimuthal asymmetries in the unpolarized case

parity conserved: $\langle\cos\varphi\rangle_U = [\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi}]/2F_{Ut}$

$\langle\cos 2\varphi\rangle_U = \sin^2\theta F_U^{\cos 2\varphi}/2F_{Ut}$

parity violated: $\langle\sin\varphi\rangle_U = [\sin\theta \tilde F_{1U}^{\sin\varphi} + \sin 2\theta \tilde F_{2U}^{\sin\varphi}]/2F_{Ut}$

$\langle\sin 2\varphi\rangle_U = \sin^2\theta \tilde F_U^{\sin 2\varphi}/2F_{Ut}$

$$F_{Ut} = \int\frac{d\varphi}{2\pi}(\mathcal{F}_U + \tilde{\mathcal{F}}_U) = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U}$$

Hadron polarizations

E.g.: $\lambda_{ave} = \frac{\mathcal{F}_L + \tilde{\mathcal{F}}_L}{\mathcal{F}_U + \tilde{\mathcal{F}}_U}$ $\quad S_{LL,ave} = \frac{1}{2}\frac{\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL}}{\mathcal{F}_U + \tilde{\mathcal{F}}_U}$ $\quad S_{LT,ave}^i = \frac{2}{3}\frac{\mathcal{F}_{LT}^i + \tilde{\mathcal{F}}_{LT}^i}{\mathcal{F}_U + \tilde{\mathcal{F}}_U}$

In practice, often integrated over the azimuthal angle φ ⟹

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General kinematic analysis for $e^+e^- \to V\pi X$

Integrated over the azimuthal angle φ — inclusive $e^+e^- \to VX$

$$\int\frac{d\varphi}{2\pi}\frac{2E_1E_2 d^6\sigma}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi\Big\{\big(\langle\mathcal{F}_U\rangle + \langle\tilde{\mathcal{F}}_U\rangle\big) + \lambda\big(\langle\mathcal{F}_L\rangle + \langle\tilde{\mathcal{F}}_L\rangle\big) + S_{LL}\big(\langle\mathcal{F}_{LL}\rangle + \langle\tilde{\mathcal{F}}_{LL}\rangle\big) + |\vec{S}_T|\big(\langle\mathcal{F}_T\rangle + \langle\tilde{\mathcal{F}}_T\rangle\big) + |\vec{S}_{LT}|\big(\langle\mathcal{F}_{LT}\rangle + \langle\tilde{\mathcal{F}}_{LT}\rangle\big) + |\vec{S}_{TT}|\big(\langle\mathcal{F}_{TT}\rangle + \langle\tilde{\mathcal{F}}_{TT}\rangle\big)\Big\}$$

$$\frac{2E_1 d^3\sigma_{in}}{d^3p_1} = \frac{\alpha^2}{s^2}\chi\Big\{\big(\mathcal{F}_{U,in} + \tilde{\mathcal{F}}_{U,in}\big) + \lambda\big(\mathcal{F}_{L,in} + \tilde{\mathcal{F}}_{L,in}\big) + S_{LL}\big(\mathcal{F}_{LL,in} + \tilde{\mathcal{F}}_{LL,in}\big) + |\vec{S}_T|\big(\mathcal{F}_{T,in} + \tilde{\mathcal{F}}_{T,in}\big) + |\vec{S}_{LT}|\big(\mathcal{F}_{LT,in} + \tilde{\mathcal{F}}_{LT,in}\big) + |\vec{S}_{TT}|\big(\mathcal{F}_{TT,in} + \tilde{\mathcal{F}}_{TT,in}\big)\Big\}$$

$\langle\mathcal{F}_U\rangle = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U}$	P-even, T-even	$\mathcal{F}_{U,in} = (1+\cos^2\theta)F_{1U,in} + \sin^2\theta F_{2U,in} + \cos\theta F_{3U,in}$
$\langle\tilde{\mathcal{F}}_U\rangle = 0$	P-odd, T-odd	$\tilde{\mathcal{F}}_{U,in} = 0$
$\langle\mathcal{F}_L\rangle = 0$	P-even, T-odd	$\mathcal{F}_{L,in} = 0$
$\langle\tilde{\mathcal{F}}_L\rangle = (1+\cos^2\theta)\tilde F_{1L} + \sin^2\theta \tilde F_{2L} + \cos\theta \tilde F_{3L}$	P-odd, T-even	$\tilde{\mathcal{F}}_{L,in} = (1+\cos^2\theta)\tilde F_{1L,in} + \sin^2\theta \tilde F_{2L,in} + \cos\theta \tilde F_{3L,in}$
$\langle\mathcal{F}_{LL}\rangle = (1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL}$	P-even, T-even	$\mathcal{F}_{LL,in} = (1+\cos^2\theta)F_{1LL,in} + \sin^2\theta F_{2LL,in} + \cos\theta F_{3LL,in}$
$\langle\tilde{\mathcal{F}}_{LL}\rangle = 0$	P-odd, T-odd	$\tilde{\mathcal{F}}_{LL,in} = 0$
$\langle\mathcal{F}_T\rangle = \sin\varphi_S(\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S})$	P-even, T-odd	$\mathcal{F}_{T,in} = \sin\varphi_S(\sin\theta F_{1T,in}^{\sin\varphi_S} + \sin 2\theta F_{2T,in}^{\sin\varphi_S})$
$\langle\tilde{\mathcal{F}}_T\rangle = \cos\varphi_S(\sin\theta \tilde F_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde F_{2T}^{\cos\varphi_S})$	P-odd, T-even	$\tilde{\mathcal{F}}_{T,in} = \cos\varphi_S(\sin\theta \tilde F_{1T,in}^{\cos\varphi_S} + \sin 2\theta \tilde F_{2T,in}^{\cos\varphi_S})$
$\langle\mathcal{F}_{LT}\rangle = \cos\varphi_{LT}(\sin\theta F_{1LT}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos\varphi_{LT}})$	P-even, T-even	$\mathcal{F}_{LT,in} = \cos\varphi_{LT}(\sin\theta F_{1LT,in}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT,in}^{\cos\varphi_{LT}})$
$\langle\tilde{\mathcal{F}}_{LT}\rangle = \sin\varphi_{LT}(\sin\theta \tilde F_{1LT}^{\sin\varphi_{LT}} + \sin 2\theta \tilde F_{2LT}^{\sin\varphi_{LT}})$	P-odd, T-odd	$\tilde{\mathcal{F}}_{LT,in} = \sin\varphi_{LT}(\sin\theta \tilde F_{1LT,in}^{\sin\varphi_{LT}} + \sin 2\theta \tilde F_{2LT,in}^{\sin\varphi_{LT}})$
$\langle\mathcal{F}_{TT}\rangle = \cos 2\varphi_{TT}\sin^2\theta F_{TT}^{\cos 2\varphi_{TT}}$	P-even, T-even	$\mathcal{F}_{TT,in} = \cos 2\varphi_{TT}\sin^2\theta F_{TT,in}^{\cos 2\varphi_{TT}}$
$\langle\tilde{\mathcal{F}}_{TT}\rangle = \sin 2\varphi_{TT}\sin^2\theta \tilde F_{TT}^{\sin 2\varphi_{TT}}$	P-odd, T-odd	$\tilde{\mathcal{F}}_{TT,in} = \sin 2\varphi_{TT}\sin^2\theta \tilde F_{TT,in}^{\sin 2\varphi_{TT}}$

19 "structure functions" left, 11 parity conserved, 8 parity violated.

$$F_{xxx,in}^{yy} = \int\frac{dp_{2T}^2 dp_{2z}}{2E_2}F_{xxx}^{yy}$$

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General kinematic analysis for $e^+e^- \to V\pi X$

Hadron polarizations averaged over the azimuthal angle φ

Longitudinal components

$$\langle\lambda\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde F_{1L} + \sin^2\theta \tilde F_{2L} + \cos\theta \tilde F_{3L}]$$ parity violated

$$\langle S_{LL}\rangle = \frac{1}{2F_{Ut}}[(1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL}]$$ parity conserved

Transversal components

w.r.t. the hadron-hadron plane

$$\langle S_T^n\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)} + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}]$$

$$\langle S_T^t\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde F_{1T}^{\cos(\varphi_S-\varphi)} + \sin^2\theta \tilde F_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde F_{3T}^{\cos(\varphi_S-\varphi)}]$$

$$\langle S_{LT}^n\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde F_{1LT}^{\sin(\varphi_{LT}-\varphi)} + \sin^2\theta \tilde F_{2LT}^{\sin(\varphi_{LT}-\varphi)} + \cos\theta \tilde F_{3LT}^{\sin(\varphi_{LT}-\varphi)}]$$

$$\langle S_{LT}^t\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)F_{1LT}^{\cos(\varphi_{LT}-\varphi)} + \sin^2\theta F_{2LT}^{\cos(\varphi_{LT}-\varphi)} + \cos\theta F_{3LT}^{\cos(\varphi_{LT}-\varphi)}]$$

$$\langle S_{TT}^{nn}\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} + \sin^2\theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}]$$

$$\langle S_{TT}^{nt}\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde F_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} + \sin^2\theta \tilde F_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta \tilde F_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}]$$

w.r.t. the lepton-hadron plane

$$\langle S_T^x\rangle = \frac{2}{3F_{Ut}}[\sin\theta \tilde F_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde F_{2T}^{\cos\varphi_S}]$$

$$\langle S_T^y\rangle = \frac{2}{3F_{Ut}}[\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}]$$

$$\langle S_{LT}^x\rangle = \frac{2}{3F_{Ut}}[\sin\theta F_{1LT}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos\varphi_{LT}}]$$

$$\langle S_{LT}^y\rangle = \frac{2}{3F_{Ut}}[\sin\theta \tilde F_{1LT}^{\sin\varphi_{LT}} + \sin 2\theta \tilde F_{2LT}^{\sin\varphi_{LT}}]$$

$$\langle S_{TT}^{xx}\rangle = \frac{2}{3F_{Ut}}\sin^2\theta F_{1TT}^{\cos 2\varphi_{TT}}$$

$$\langle S_{TT}^{xy}\rangle = \frac{2}{3F_{Ut}}\sin^2\theta \tilde F_{1TT}^{\sin 2\varphi_{TT}}$$

General kinematic analysis for $e^+e^- \to V\pi X$

Hadron polarizations averaged over the azimuthal angle φ

For the semi-inclusive process $e^+e^- \to V\pi X$

Longitudinal components

$$\langle\lambda\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde F_{1L} + \sin^2\theta \tilde F_{2L} + \cos\theta \tilde F_{3L}]$$

$$\langle S_{LL}\rangle = \frac{1}{2F_{Ut}}[(1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL}]$$

Transversal components

w.r.t. the lepton-hadron plane

$$\langle S_T^x\rangle = \frac{2}{3F_{Ut}}[\sin\theta \tilde F_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde F_{2T}^{\cos\varphi_S}]$$

$$\langle S_T^y\rangle = \frac{2}{3F_{Ut}}[\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}]$$

$$\langle S_{LT}^x\rangle = \frac{2}{3F_{Ut}}[\sin\theta F_{1LT}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos\varphi_{LT}}]$$

$$\langle S_{LT}^y\rangle = \frac{2}{3F_{Ut}}[\sin\theta \tilde F_{1LT}^{\sin\varphi_{LT}} + \sin 2\theta \tilde F_{2LT}^{\sin\varphi_{LT}}]$$

$$\langle S_{TT}^{xx}\rangle = \frac{2}{3F_{Ut}}\sin^2\theta F_{1TT}^{\cos 2\varphi_{TT}}$$

$$\langle S_{TT}^{xy}\rangle = \frac{2}{3F_{Ut}}\sin^2\theta \tilde F_{1TT}^{\sin 2\varphi_{TT}}$$

For the inclusive process $e^+e^- \to VX$

Longitudinal components

$$\langle\lambda\rangle_{in} = \frac{2}{3F_{Ut,in}}[(1+\cos^2\theta)\tilde F_{1L,in} + \sin^2\theta \tilde F_{2L,in} + \cos\theta \tilde F_{3L,in}]$$

$$\langle S_{LL}\rangle_{in} = \frac{1}{2F_{Ut,in}}[(1+\cos^2\theta)F_{1LL,in} + \sin^2\theta F_{2LL,in} + \cos\theta F_{3LL,in}]$$

Transversal components

w.r.t. the lepton-hadron plane

$$\langle S_T^x\rangle_{in} = \frac{2}{3F_{Ut,in}}[\sin\theta \tilde F_{1T,in}^{\cos\varphi_S} + \sin 2\theta \tilde F_{2T,in}^{\cos\varphi_S}]$$

$$\langle S_T^y\rangle_{in} = \frac{2}{3F_{Ut,in}}[\sin\theta F_{1T,in}^{\sin\varphi_S} + \sin 2\theta F_{2T,in}^{\sin\varphi_S}]$$

$$\langle S_{LT}^x\rangle_{in} = \frac{2}{3F_{Ut,in}}[\sin\theta F_{1LT,in}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT,in}^{\cos\varphi_{LT}}]$$

$$\langle S_{LT}^y\rangle_{in} = \frac{2}{3F_{Ut,in}}[\sin\theta \tilde F_{1LT,in}^{\sin\varphi_{LT}} + \sin 2\theta \tilde F_{2LT,in}^{\sin\varphi_{LT}}]$$

$$\langle S_{TT}^{xx}\rangle_{in} = \frac{2}{3F_{Ut,in}}\sin^2\theta F_{1TT,in}^{\cos 2\varphi_{TT}}$$

$$\langle S_{TT}^{xy}\rangle_{in} = \frac{2}{3F_{Ut,in}}\sin^2\theta \tilde F_{1TT,in}^{\sin 2\varphi_{TT}}$$

General kinematic analysis for $e^+e^- \to V\pi X$

Hadron polarizations averaged over the azimuthal angle φ and the polar angle θ

$e^+e^- \to V\pi X$

Longitudinal components

$$\bar\lambda = 4(2\tilde F_{1L} + \tilde F_{2L})/9\bar{\mathcal F}_U$$

$$\bar S_{LL} = (2F_{1LL} + F_{2LL})/3\bar{\mathcal F}_U$$

Transversal components

w.r.t. the hadron-hadron plane

$$\bar S_T^n = 4(2F_{1T}^{\sin(\varphi_S-\varphi)} + F_{2T}^{\sin(\varphi_S-\varphi)})/9\bar{\mathcal F}_U$$

$$\bar S_T^t = 4(2\tilde F_{1T}^{\cos(\varphi_S-\varphi)} + \tilde F_{2T}^{\cos(\varphi_S-\varphi)})/9\bar{\mathcal F}_U$$

$$\bar S_{LT}^n = 4(2\tilde F_{1LT}^{\sin(\varphi_{LT}-\varphi)} + \tilde F_{2LT}^{\sin(\varphi_{LT}-\varphi)})/9\bar{\mathcal F}_U$$

$$\bar S_{LT}^t = 4(2F_{1LT}^{\cos(\varphi_{LT}-\varphi)} + F_{2LT}^{\cos(\varphi_{LT}-\varphi)})/9\bar{\mathcal F}_U$$

$$\bar S_{TT}^{nn} = 4(2F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} + F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)})/9\bar{\mathcal F}_U$$

$$\bar S_{TT}^{nt} = 4(2\tilde F_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} + \tilde F_{2TT}^{\sin(2\varphi_{TT}-2\varphi)})/9\bar{\mathcal F}_U$$

w.r.t. the lepton-hadron plane

$$\bar S_T^x = \pi\tilde F_{1T}^{\cos\varphi_S}/6\bar{\mathcal F}_U$$

$$\bar S_T^y = \pi F_{1T}^{\sin\varphi_S}/6\bar{\mathcal F}_U$$

$$\bar S_{LT}^x = \pi F_{1LT}^{\cos\varphi_{LT}}/6\bar{\mathcal F}_U$$

$$\bar S_{LT}^y = \pi\tilde F_{1LT}^{\sin\varphi_{LT}}/6\bar{\mathcal F}_U$$

$$\bar S_{TT}^{xx} = 4F_{1TT}^{\cos 2\varphi_{TT}}/9\bar{\mathcal F}_U$$

$$\bar S_{TT}^{xy} = 4\tilde F_{1TT}^{\sin 2\varphi_{TT}}/9\bar{\mathcal F}_U$$

$e^+e^- \to VX$

$$\bar\lambda_{in} = 4(2\tilde F_{1L,in} + \tilde F_{2L,in})/9\bar{\mathcal F}_{U,in}$$

$$\bar S_{LL,in} = (2F_{1LL,in} + F_{2LL,in})/3\bar{\mathcal F}_{U,in}$$

w.r.t. the lepton-hadron plane

$$\bar S_{T,in}^x = \pi\tilde F_{1T,in}^{\cos\varphi_S}/6\bar{\mathcal F}_{U,in}$$

$$\bar S_{T,in}^y = \pi F_{1T,in}^{\sin\varphi_S}/6\bar{\mathcal F}_{U,in}$$

$$\bar S_{LT,in}^x = \pi F_{1LT,in}^{\cos\varphi_{LT}}/6\bar{\mathcal F}_{U,in}$$

$$\bar S_{LT,in}^y = \pi\tilde F_{1LT,in}^{\sin\varphi_{LT}}/6\bar{\mathcal F}_{U,in}$$

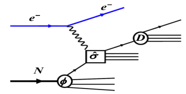
$$\bar S_{TT,in}^{xx} = 4F_{1TT,in}^{\cos 2\varphi_{TT}}/9\bar{\mathcal F}_{U,in}$$

$$\bar S_{TT,in}^{xy} = 4\tilde F_{1TT,in}^{\sin 2\varphi_{TT}}/9\bar{\mathcal F}_{U,in}$$

Semi-inclusive DIS $e^-(\lambda_l) + N(\lambda, S_T) \to e^- + h + X$: Kinematics

The differential cross section:

$$d\sigma = \frac{\alpha^2}{sQ^4}L_{\mu\nu}(l,\lambda_e,l')W^{\mu\nu}(q,p,S,p')\frac{d^3l'}{2E_l'(2\pi)3}\frac{d^3p'}{2E_h'(2\pi)3}$$



$$W_{\mu\nu}(q,p,S,p') = \sum_{\sigma,i} W_{\sigma i}^S h_{\sigma i}^{S\mu\nu} + \sum_{\sigma,j}\tilde W_{\sigma j}^S \tilde h_{\sigma j}^{S\mu\nu} + i\sum_{\sigma,i} W_{\sigma i}^A h_{\sigma i}^{A\mu\nu} + i\sum_{\sigma,j}\tilde W_{\sigma j}^A \tilde h_{\sigma j}^{A\mu\nu}$$

$\sigma = U, V$: polarization

basic Lorentz tensors

The basic Lorentz sets

unpolarized part: 5+4=9

$$h_{Ui}^{S\mu\nu} = \left\{g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde p^\mu \tilde p^\nu, \tilde p^{\{\mu}\tilde p'^{\nu\}}, \tilde p'^\mu \tilde p'^\nu\right\}$$

$$\tilde h_{Ui}^{S\mu\nu} = \left\{\varepsilon^{\{\mu qpp'}(\tilde p^{\nu\}}, \tilde p'^{\nu\}})\right\}$$

$$h_U^{A\mu\nu} = \tilde p^{[\mu}\tilde p'^{\nu]}$$

$$\tilde p = p - \frac{p\cdot q}{q^2}q$$

$$\tilde h_{Ui}^{A\mu\nu} = \left\{\varepsilon^{\mu\nu qp}, \varepsilon^{\mu\nu qp'}\right\}$$

spin dependent part: 13+5=18

$$h_{Vi}^{S\mu\nu} = \left\{\left[(q\cdot S),(p'\cdot S)\right]\tilde h_{Ui}^{S\mu\nu}, \varepsilon^{Sqpp'}h_{Uj}^{S\mu\nu}\right\}$$

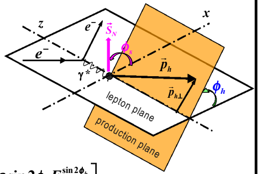
$$\tilde h_{Vi}^{S\mu\nu} = \left\{\left[(q\cdot S),(p'\cdot S)\right]h_{Ui}^{S\mu\nu}, \varepsilon^{Sqpp'}\tilde h_{Uj}^{S\mu\nu}\right\}$$

$$h_{Vi}^{A\mu\nu} = \left\{\left[(q\cdot S),(p'\cdot S)\right]\tilde h_{Ui}^{A\mu\nu}, \varepsilon^{Sqpp'}h_U^{A\mu\nu}\right\}$$

$$\tilde h_{Vi}^{A\mu\nu} = \left\{\left[(q\cdot S),(p'\cdot S)\right]h_U^{A\mu\nu}, \varepsilon^{Sqpp'}h_U^{A\mu\nu}\right\}$$

Semi-inclusive DIS $e^-(\lambda_l) + N(\lambda, S_T) \to e^- + h + X$: Kinematics

General form of the differential cross section



$$\frac{d\sigma}{dxdydzd\phi_S d^2 p_{h\perp}} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}(1+\frac{\gamma^2}{2x})\times$$

$$\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon\cos 2\phi_h F_{UU}^{\cos 2\phi_h}$$

$$+\lambda_l\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h} \quad +\lambda\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon\sin 2\phi_h F_{UL}^{\sin 2\phi_h}\right]$$

$$+\lambda_l\lambda\left[\sqrt{1-\varepsilon^2}F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$

$$+|\vec{S}_T|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\sin(\phi_h+\phi_S)F_{UT,T}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)F_{UT,T}^{\sin(3\phi_h-\phi_S)}$$
$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+\lambda_l|\vec{S}_T|\left[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\left(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\right)\right]\}$$

nucleon
electron

2x3=6 combinations

$$\varepsilon = (1-y-\frac{1}{4}\gamma^2y^2)/(1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2), \quad \gamma = \frac{2Mx}{Q}$$

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Semi-inclusive DIS: LO & Leading twist parton model results

for the structure functions (8 non-zero *F* 's)

$e(\lambda_l) + N(\lambda, S_T) \to e + h + X$

unpolarized electron, unpolarized nucleon	$F_{UU,T} = \mathcal{C}[f_1 D_1]$ $\quad F_{UU,L} = 0$	$F_{UU}^{\cos\phi_h} = 0$	$F_{UU}^{\cos 2\phi_h} = \mathcal{C}[w_1 h_1^\perp H_1^\perp]$
longitudinally polarized electron / longitudinally polarized nucleon	$F_{LU}^{\sin\phi_h} = 0$	$F_{UL}^{\sin\phi_h} = 0$	$F_{UL}^{\sin 2\phi_h} = \mathcal{C}[w_1 h_{1L}^\perp H_1^\perp]$
both longitudinally polarized	$F_{LL} = \mathcal{C}[g_{1L} D_1]$	$F_{LL}^{\cos\phi_h} = 0$	
transversely polarized nucleon	$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -2\mathcal{C}[w_2 f_{1T}^\perp D_1]$ $F_{UT}^{\sin\phi_S} = 0$	$F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0$ $F_{UT}^{\sin(2\phi_h-\phi_S)} = 0$	$F_{UT}^{\sin(\phi_h+\phi_S)} = -2\mathcal{C}[w_3 h_{1T} H_1^\perp]$ $F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}[w_4 h_{1T}^\perp H_1^\perp]$
longitudinally polarized electron, transversely polarized nucleon	$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}[w_2 g_{1T} D_1]$	$F_{LT}^{\cos\phi_S} = 0$	$F_{LT}^{\cos(2\phi_h-\phi_S)} = 0$

nucleon
electron

$$\mathcal{C}[w_i f\, D] \equiv x\sum_q e_q^2 \int d^2k_\perp d^2k_{F\perp}\delta^{(2)}(\vec{k}_\perp - \vec{k}_{F\perp} - \vec{p}_{h\perp}/z)\, w_i\, f_q(x,k_\perp)D_q(z,k_{F\perp})$$

$$w_1 = -\left[2(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp})(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp) - \vec{k}_\perp\cdot\vec{k}_{F\perp}\right]/MM_h, \quad w_2 = \hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp/M, \quad w_3 = \hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp}/M_h$$

See e.g., Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 0702, 093 (2007); ....

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Semi-inclusive DIS: LO & Leading twist parton model results

for the cross section

$e(\lambda_l) + N(\lambda, S_T) \to e + h + X$

$$\frac{d\sigma}{dxdydzd^2 p_{h\perp}} = \frac{\alpha^2}{xyQ^2}\times$$

$$\{(1-y+\frac{1}{2}y^2)\mathcal{C}[f_1 D_1] + (1-y)\cos 2\phi_h\mathcal{C}[w_1 h_1^\perp H_1^\perp]$$

Boer-Mulders ⊗ Collins

$$+\lambda_l\lambda y(1-\frac{1}{2}y)\mathcal{C}[g_{1L} D_1]$$

longi-transversity ⊗Collins

$$+\lambda(1-y)\sin 2\phi_h\mathcal{C}[w_1 h_{1L}^\perp H_1^\perp]$$

Sivers ⊗ unpolarized FF

$$+|\vec{S}_T|\Big((1-y+\frac{1}{2}y^2)\sin(\phi_h-\phi_S)\mathcal{C}[w_2 f_{1T}^\perp D_1]$$
$$+2(1-y)\sin(\phi_h+\phi_S)\mathcal{C}[w_3 h_{1T} H_1^\perp] + 2(1-y)\sin(3\phi_h-\phi_S)\mathcal{C}[w_4 h_{1T}^\perp H_1^\perp]\Big)$$

$$+\lambda_l|\vec{S}_T|y(1-\frac{1}{2}y)\cos(\phi_h-\phi_S)\mathcal{C}[w_2 g_{1T} D_1]\}$$

transversity ⊗ Collins

pretzelosity ⊗ Collins

trans-helicity ⊗ unpolarized FF

nucleon
electron

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Semi-inclusive DIS: LO & Leading twist parton model results

for the azimuthal asymmetries (6 leading twist asymmetries)

$$A_{UU}^{\cos 2\phi_h} = \langle\cos 2\phi_h\rangle_{UU} = \frac{(1-y)}{A(y)}\frac{\mathcal{C}[w_1 h_1^\perp H_1^\perp]}{\mathcal{C}[f_1 D_1]}$$ Boer-Mulders ⊗ Collins

$$A_{UL}^{\sin 2\phi_h} = \langle\sin 2\phi_h\rangle_{UL} = \frac{(1-y)}{A(y)}\frac{\mathcal{C}[w_1 h_{1L}^\perp H_1^\perp]}{\mathcal{C}[f_1 D_1]}$$ longi-transversity ⊗ Collins

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \langle\sin(\phi_h-\phi_S)\rangle_{UT} = \frac{1}{2}\frac{\mathcal{C}[w_2 f_{1T}^\perp D_1]}{\mathcal{C}[f_1 D_1]} \equiv A_{Sivers}$$ Sivers ⊗ unpolarized FF

$$A_{UT}^{\sin(\phi_h+\phi_S)} = \langle\sin(\phi_h+\phi_S)\rangle_{UT} = \frac{(1-y)}{A(y)}\frac{\mathcal{C}[w_3 h_{1T} H_1^\perp]}{\mathcal{C}[f_1 D_1]} \equiv A_{Collins}$$ transversity ⊗ Collins

$$A_{UT}^{\sin(3\phi_h-\phi_S)} = \langle\sin(3\phi_h-\phi_S)\rangle_{UT} = \frac{(1-y)}{A(y)}\frac{\mathcal{C}[w_4 h_{1T}^\perp H_1^\perp]}{\mathcal{C}[f_1 D_1]}$$ pretzelosity ⊗ Collins

$$A_{LT}^{\cos(\phi_h-\phi_S)} = \langle\cos(\phi_h-\phi_S)\rangle_{LT} = \frac{y(2-y)}{2A(y)}\frac{\mathcal{C}[-w_2 g_{1T} D_1]}{\mathcal{C}[f_1 D_1]}$$ trans-helicity ⊗ unpolarized FF

nucleon
electron

$$A(y) \equiv 1 + (1-y)^2$$

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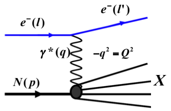
Collinear expansion in high energy reactions

Inclusive DIS $e^- N \to e^- X$

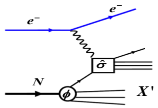


Yes!

where collinear expansion was first formulated.

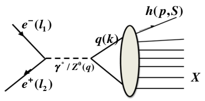
R. K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B207,1 (1982); B212, 29 (1983).

Semi-Inclusive DIS $e + N \to e + q(jet) + X$



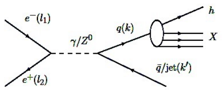
Yes!
ZTL & X.N. Wang, PRD (2007);

Inclusive $e^- + e^+ \to h + X$



Yes!
S.Y. Wei, Y.K. Song, ZTL, PRD (2014);

Semi-Inclusive $e^- + e^+ \to h + \bar{q}(jet) + X$



Yes!
S.Y. Wei, K.B. Chen, Y.K. Song, ZTL, PRD (2015).

Successfully to all processes where only ONE hadron is explicitly involved.

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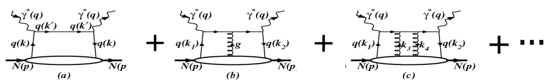
Collinear expansion in semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$ with QCD interaction:

$$W^{(si)}_{\mu\nu}(q,p,S,k') = \sum_X \langle p,S| J_\mu(0)|k',X\rangle\langle k',X|J_\nu(0)|p,S\rangle (2\pi)^4\delta^4(p+q-k'-p_X)$$
$$= W^{(0,si)}_{\mu\nu}(q,p,S,k') + W^{(1,si)}_{\mu\nu}(q,p,S,k') + W^{(2,si)}_{\mu\nu}(q,p,S,k') + \dots$$



$$W^{(0,si)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(0,si)}_{\mu\nu}(k,k',q)\ \hat{\phi}^{(0)}(k,p,S)\right]$$
$$\hat{H}^{(0,si)}_{\mu\nu}(k,k',q) = \gamma_\mu(\not k + \not q)\gamma_\nu (2\pi)^4\delta^4(k'-k-q)$$

c.f.: $$W^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\ \hat{\phi}^{(0)}(k,p,S)\right]$$
$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \gamma_\mu(\not k + \not q)\gamma_\nu (2\pi)\delta_+\left((k+q)^2\right)$$

$$W^{(1,si,L)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(1,si,L)\rho}_{\mu\nu}(k_1,k_2,k',q)\hat{\phi}^1_\rho(k_1,k_2,p,S)\right]$$
$$\hat{H}^{(1,si,L)\rho}_{\mu\nu}(k_1,k_2,k',q) = \gamma_\mu \frac{(\not k_2 + \not q)\gamma^\rho(\not k_1 + \not q)}{(k_2+q)^2 - i\epsilon}\gamma_\nu (2\pi)^4\delta^4(k'-k_1-q)$$

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Collinear expansion in semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

An identity: $$(2\pi)^4\delta^4(k'-k-q) = (2\pi)\delta_+\left((k-q)^2\right)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$

We obtain: $$\hat{H}^{(0,si)}_{\mu\nu}(k,k',q) = \hat{H}^{(0)}_{\mu\nu}(k,q)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$
$$\hat{H}^{(1,c,si)\rho}_{\mu\nu}(k_1,k_2,k',q) = \hat{H}^{(1,c)\rho}_{\mu\nu}(k_1,k_2,q)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}_c-\vec{q}\right)$$

Hence:

$$W^{(0,si)}_{\mu\nu}(q,p,S,k') = \underbrace{\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\hat{\phi}^{(0)}(k,p,S)\right]}_{W^{(0)}_{\mu\nu}(q,p,S)}(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$

a common factor!

$$W^{(1,si)}_{\mu\nu}(q,p,S,k') = \underbrace{\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}\mathrm{Tr}\left[\hat{H}^{(1,c)\rho}_{\mu\nu}(k_1,k_2,q)\hat{\phi}^{(1)}_\rho(k_1,k_2,p,S)\right]}_{W^{(1)}_{\mu\nu}(q,p,S)}(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}_c-\vec{q}\right)$$

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Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

$$W^{(si)}_{\mu\nu}(q,p,S,k') = \tilde{W}^{(0,si)}_{\mu\nu}(q,p,S,k') + \tilde{W}^{(1,si)}_{\mu\nu}(q,p,S,k') + \tilde{W}^{(2,si)}_{\mu\nu}(q,p,S,k') + \dots$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0,si)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)]\ (2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$
$$\hat{\Phi}^{(0)}(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

depends on x only!

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1,si)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}\mathrm{Tr}[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\ \hat{H}^{(1,c)\rho}_{\mu\nu}(x_1,x_2)\ \omega_\rho^{\ \rho'}]\ (2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$
$$\hat{\Phi}^{(1)}_\rho(k_1,k_2,p,S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

⇒ A consistent framework for $e^- N \to e^- + q(jet) + X$ at LO pQCD including higher twists

ZTL & X.N. Wang, PRD (2007); Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2011) & PRD (2014).

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Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

Simplified expressions for hadronic tensors

The "collinearly expanded hard parts" take the simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu}\delta(x - x_B), \qquad \hat{h}^{(0)}_{\mu\nu} = \gamma_\mu \not{n} \gamma_\nu$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1, x_2)\omega_\rho^{\ \rho'} = \frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\delta(x_1 - x_B),$$ where $\hat{h}^{(1)\rho}_{\mu\nu} = \gamma_\mu \bar{\not{n}} \gamma^\rho \not{n} \gamma_\nu$, depends only on x_1!

twist-2, 3 and 4

$$\tilde{W}^{(0,si)}_{\mu\nu}(q,p,S;k_\perp) = \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x_B,k_\perp)\, h^{(0)}_{\mu\nu}\right]$$

$$\hat{\Phi}^{(0)}(x,k_\perp) = \int \frac{p^+ dz^-}{2\pi} d^2 z_\perp e^{ixp^+z^- - ik_\perp\cdot z_\perp}\langle N|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|N\rangle$$

three-dimensional gauge invariant quark-quark correlator

twist-3, 4 and 5

$$\tilde{W}^{(1,si)}_{\mu\nu}(q,p,S;k_\perp) = \frac{\pi}{2q\cdot p}\mathrm{Tr}\left[\hat{\varphi}^{(1)}_{\rho'}(x_B,k_\perp)\, h^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\right]$$

$$\hat{\varphi}^{(1)}_\rho(x,k_\perp) \equiv \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta\left(x - \frac{k_1^+}{p^+}\right)\delta^2(k_{1\perp} - k_\perp)\hat{\Phi}^{(1)}_\rho(k_1,k_2)$$

$$= \int \frac{p^+ dz^-}{2\pi} d^2 z_\perp e^{ixp^+z^- - i\vec{k}_\perp\cdot \vec{z}_\perp}\langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0,z)\psi(z)|N\rangle$$

the involved three-dimensional gauge invariant quark-gluon-quark correlator

THREE dimensional, depend only on ONE parton momentum!

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Semi-Inclusive e⁺e⁻ annihilation: $e^+ + e^- \to h + \bar{q}(jet) + X$

$$W^{(si)}_{\mu\nu}(q,p,S,k') = \tilde{W}^{(0,si)}_{\mu\nu}(q,p,S,k') + \tilde{W}^{(1,si)}_{\mu\nu}(q,p,S,k') + \tilde{W}^{(2,si)}_{\mu\nu}(q,p,S,k') + \ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0,si)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Xi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(z)\right](2E_{k'})(2\pi)^3\delta^3(\vec{k}' - \vec{k} - \vec{q})$$

$$\hat{\Xi}^{(0)}(k,p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi e^{-ik\xi}\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,0)|0\rangle$$

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1,L,si)}_{\mu\nu}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Xi}^{(1,L)}(k_1,k_2;p,S)\hat{H}^{(1,L)\rho}_{\mu\nu}(z_1,z_2)\omega_\rho^{\ \rho'}\right](2E_{k'})(2\pi)^3\delta^3(\vec{k}' - \vec{k}_c - \vec{q})$$

$$\hat{\Xi}^{(1,L)}_\rho(k_1,k_2,p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi d^4\eta e^{-ik_1\xi - i(k_2-k_1)\eta}\langle 0|\mathcal{L}(0,y)D_\rho(\eta)\mathcal{L}^\dagger(y,z)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$D_\rho(y) = -i\partial_\rho + gA_\rho(y)$$

⇨ A consistent framework for $e^-e^+ \to h + \bar{q}(jet) + X$ at LO pQCD including higher twists.

S.Y. Wei, K.B. Chen, Y.K. Song,& ZTL, PRD (2015).

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Semi-Inclusive DIS: $e^- + N \to e^- + q(jet) + X$

Complete results for structure functions up to twist-4

$$\kappa_M \equiv \frac{M}{Q}, \quad \bar{k}_\perp \equiv \frac{|\vec{k}_\perp|}{M}$$

$$W_{UU,T} = xf_1 + 4x^2\kappa_M^2 f_{+3dd}, \quad W_{UU,L} = 8x^3\kappa_M^2 f_3$$

$$W^{\cos2\phi}_{UU} = -2x^2\kappa_M^2\bar{k}_\perp^2 f^\perp_{-3d}$$

$$W^{\sin2\phi}_{UL} = 2x^2\kappa_M^2\bar{k}_\perp^2 f^\perp_{+3dL}$$

$$W_{LL} = xg_{1L} + 4x^2\kappa_M^2 f_{+3ddL}$$

$$W^{\sin(\phi-\phi_S)}_{UT,T} = \bar{k}_\perp(xf^\perp_{1T} + 4x^2\kappa_M^2 f_{+3ddT}), \quad W^{\sin(\phi-\phi_S)}_{UT,L} = 8x^3\kappa_M^2\bar{k}_\perp f^\perp_{3T}$$

$$W^{\sin(\phi+\phi_S)}_{UT} = -x^2\kappa_M^2\bar{k}_\perp^3(f^{\perp4}_{+3dT} + f^{\perp2}_{-3dT})$$

$$W^{\sin(3\phi-\phi_S)}_{UT} = -x^2\kappa_M^2\bar{k}_\perp^3(f^{\perp4}_{+3dT} - f^{\perp2}_{-3dT})$$

$$W^{\cos(\phi-\phi_S)}_{LT} = \bar{k}_\perp(xg^\perp_{1T} + 4x^2\kappa_M^2 f^{\perp3}_{+3ddT})$$

$$W^{\cos\phi}_{UU} = -2x^2\kappa_M\bar{k}_\perp f^\perp$$

$$W^{\sin\phi}_{UL} = -2x^2\kappa_M\bar{k}_\perp f^\perp_L$$

$$W^{\sin\phi}_{LU} = 2x^2\kappa_M\bar{k}_\perp g^\perp$$

$$W^{\cos\phi}_{LL} = -2x^2\kappa_M\bar{k}_\perp g^\perp_L$$

$$W^{\sin\phi_S}_{UT} = -2x^2\kappa_M f_T$$

$$W^{\sin(2\phi-\phi_S)}_{UT} = -x^2\kappa_M\bar{k}_\perp^2 f^\perp_T$$

$$W^{\cos\phi_S}_{LT} = -2x^2\kappa_M g_T$$

$$W^{\cos(2\phi-\phi_S)}_{LT} = -x^2\kappa_M\bar{k}_\perp^2 g^\perp_T$$

(1) twist 2 and 4 ⟺ even number of ϕ and ϕS; twist-3 ⟺ odd number of ϕ and ϕS

(2) Wherever there is twist-2 contribution, there is a twist-4 addendum to it.

S.Y. Wei, Y.K. Song, K.B. Chen, & ZTL, PRD95, 074017 (2017).

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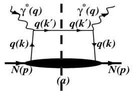
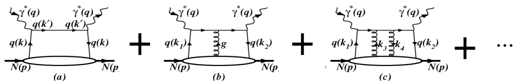
Summary

- (Gauge invariant) PDF is not merely 

but 

i.e., it always contains "intrinsic motion" and "multiple gluon scattering".

- "Multiple gluon scattering" gives rise to the gauge link.
- Collinear expansion is the necessary procedure to obtain the correct formulism in terms of gauge invariant parton distribution functions (PDFs).
- Collinear expansion has been proven to be applicable to all processes where one hadron is explicitly involved.

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