EIC物理唯象研究简介

周剑
山东大学

Outline:

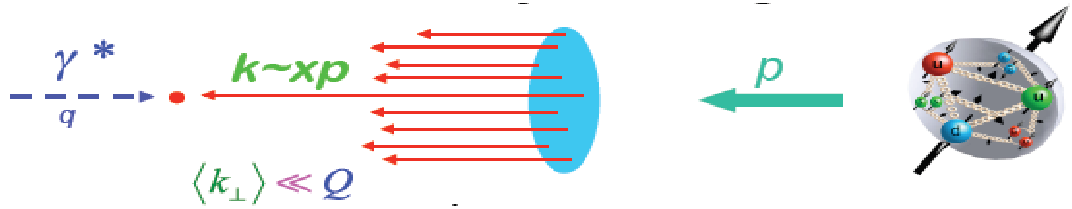
- Introduction
- 3D imaging of proton
- Mass and spin decompositions of proton
- Small x physics

Many interesting topics not covered: proton radius puzzle, Quasi PDFs…

Introduction

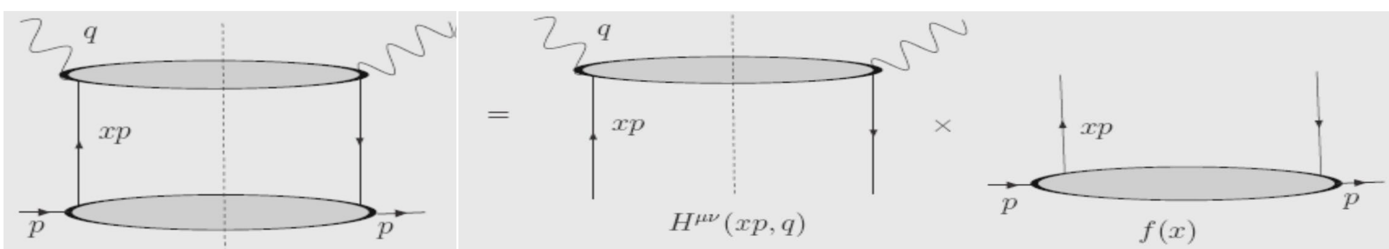
Parton model and QCD factorization

DIS:



Asymptotically Free Partons,

$$Q^2,\ P_T^2 \gg \Lambda^2_{\mathrm{QCD}} \sim [1/\mathrm{fm}]^2 \qquad \sigma_{\mathrm{phy}}(Q) \approx \sum_f \hat{\sigma}_f(Q) \otimes [\varphi_{f/h}(x)] + O\left(\frac{1}{Q}\right)$$

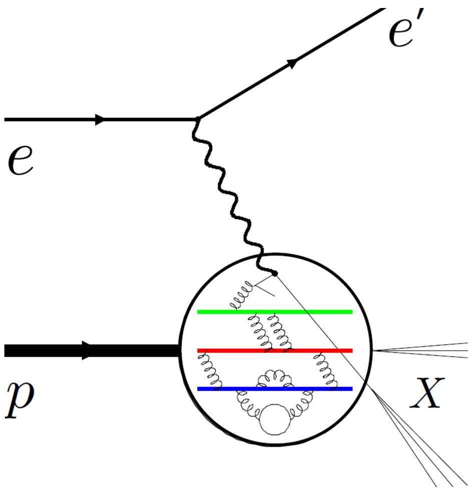


- Predicative power:

  Universal & Scale dependence perturbativly computable

Deep inelastic scattering(DIS)

$$\frac{d^2\sigma^{ep\to eX}}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[\left(1-y+\frac{y^2}{2}\right)F_2(x,Q^2)-\frac{y^2}{2}F_L(x,Q^2)\right]$$



$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_eE'_e\sin^2\left(\frac{\theta'_e}{2}\right)$$

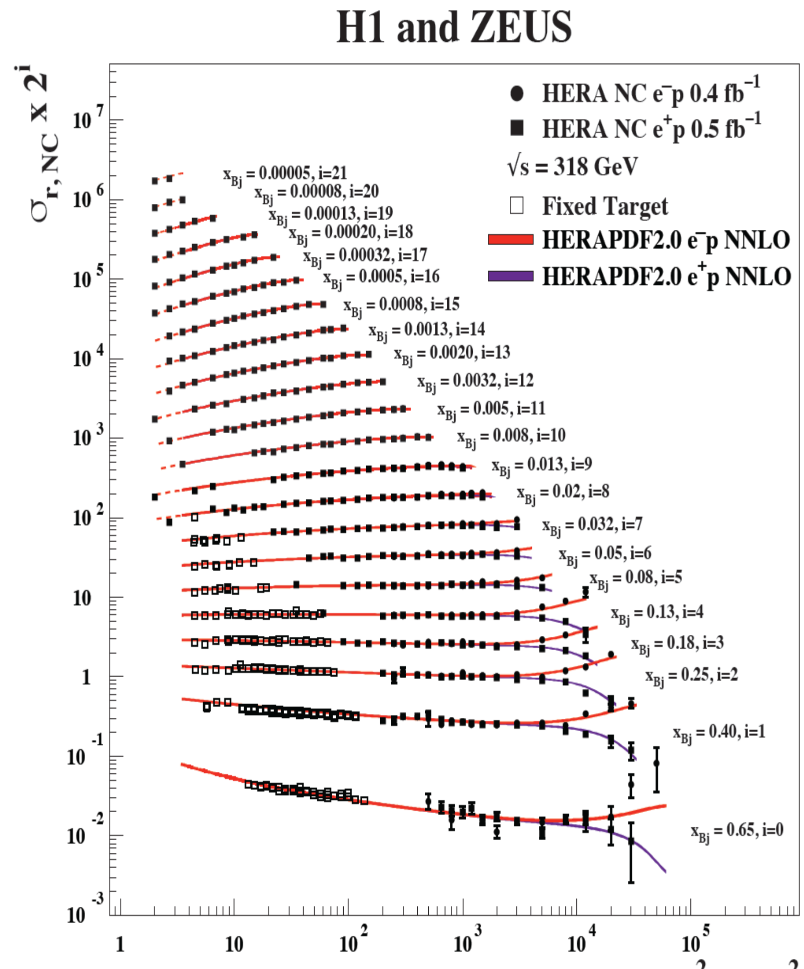
$$y = \frac{pq}{pk} = 1-\frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

- Bjorken scaling

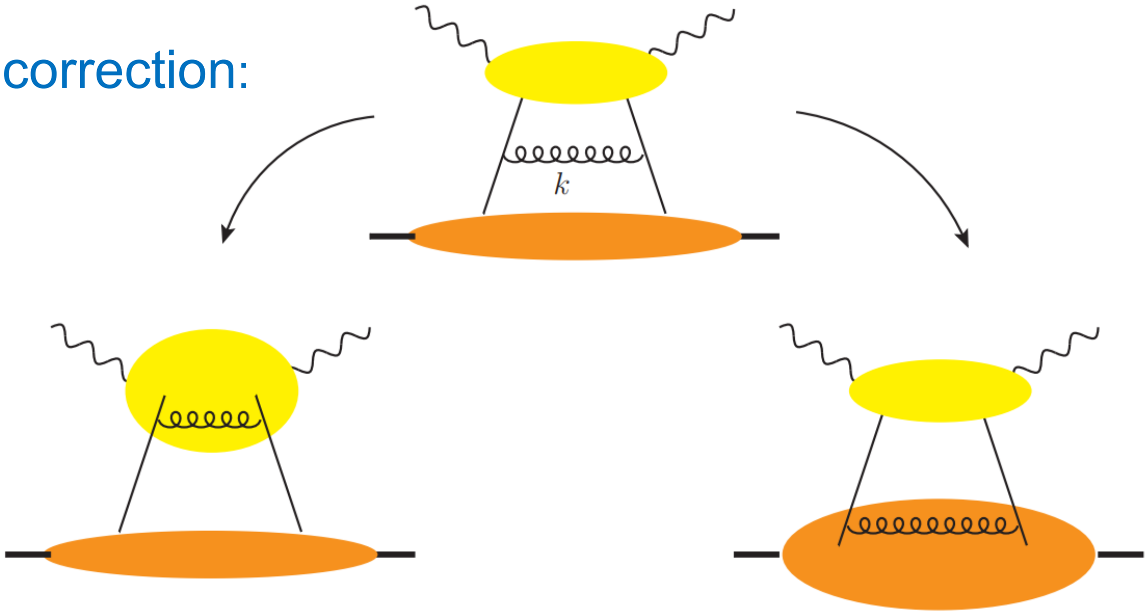
$$F(q^2) \equiv \int e^{\frac{-i\mathbf{q}.\mathbf{R}}{\hbar}}\rho(\mathbf{R})d\tau$$

$$F_2(x,Q^2) = x\sum_q Q_q^2(q+\overline{q})$$



DGLAP evolution

- Radiative correction:



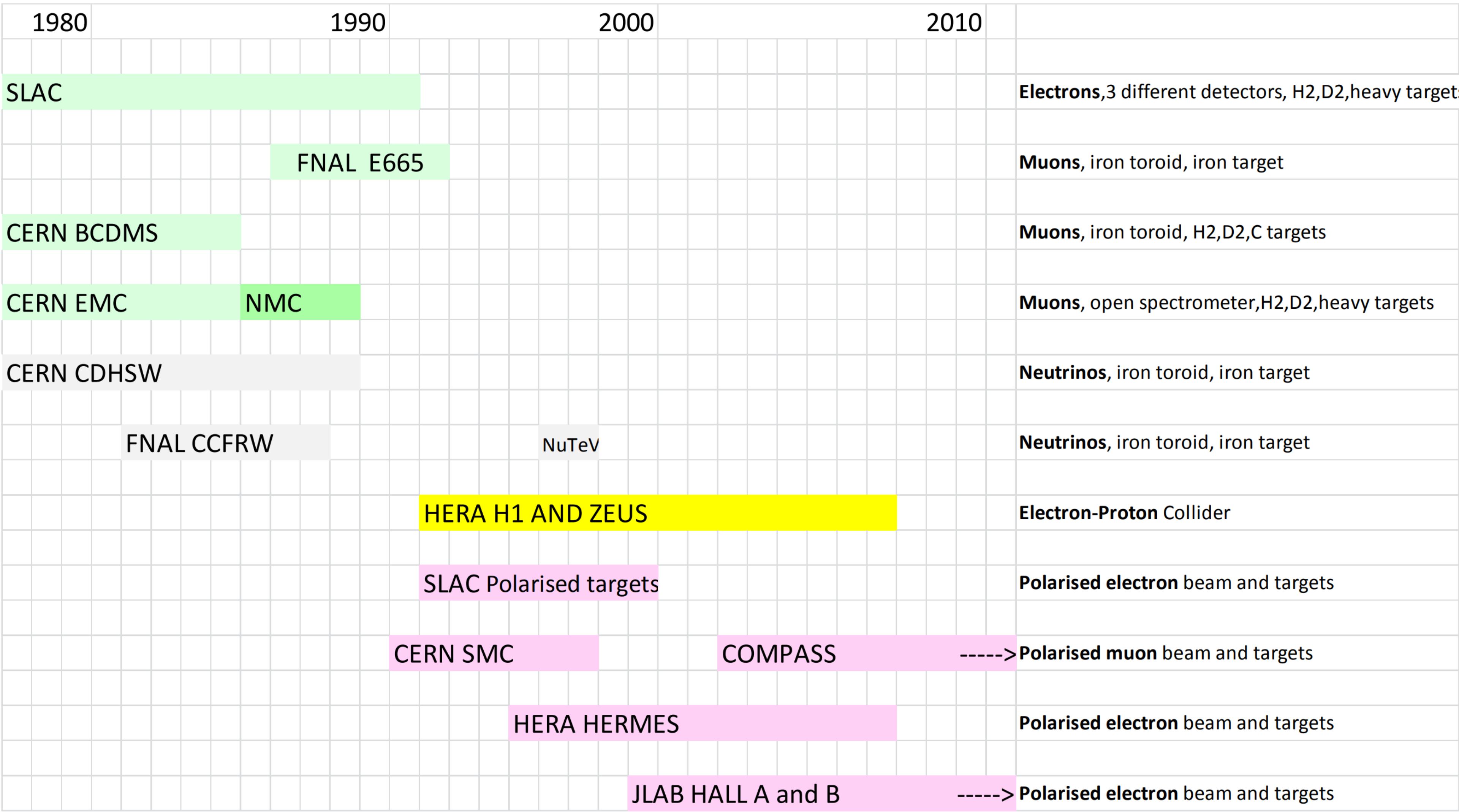
$k_T > \mu$ $\qquad$ $k_T < \mu$

- Contribute to hard part
- Drive the scale evolution of PDF

➢ DGLAP evolution equation:

$$\frac{d}{d\log\mu^2} f(x,\mu) = \int_x^1 \frac{dx'}{x'} P\left(\frac{x}{x'}\right) f(x',\mu)$$

DIS experiments

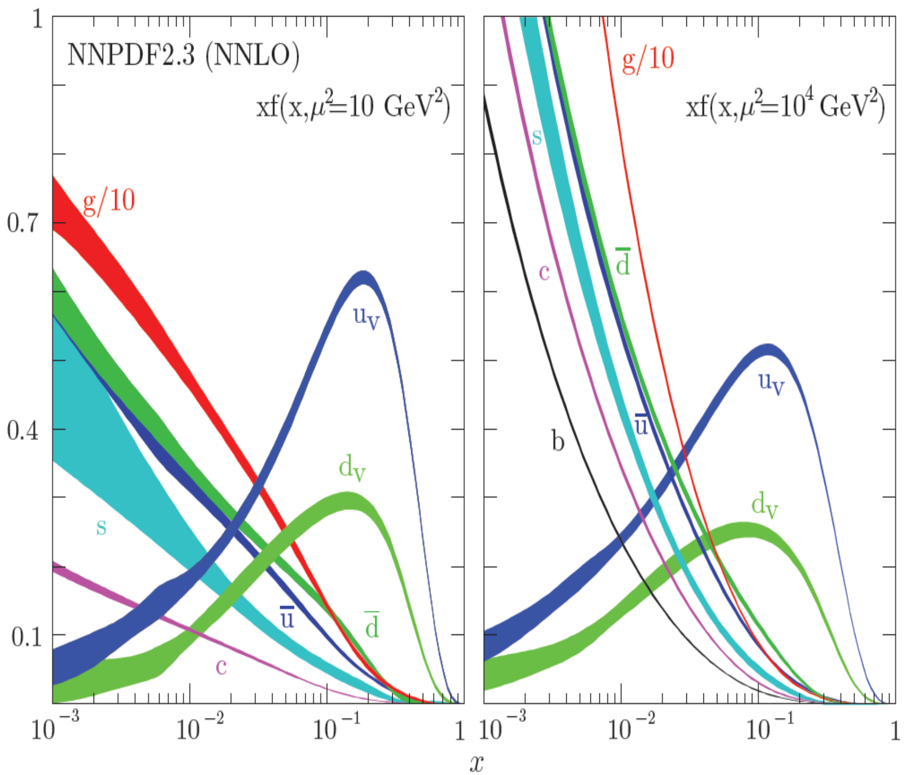


+ input from hadron-hadron collisions: TeVatron RHIC LHC

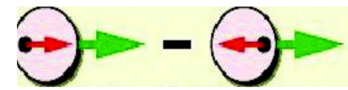
1 dimensional imaging of proton

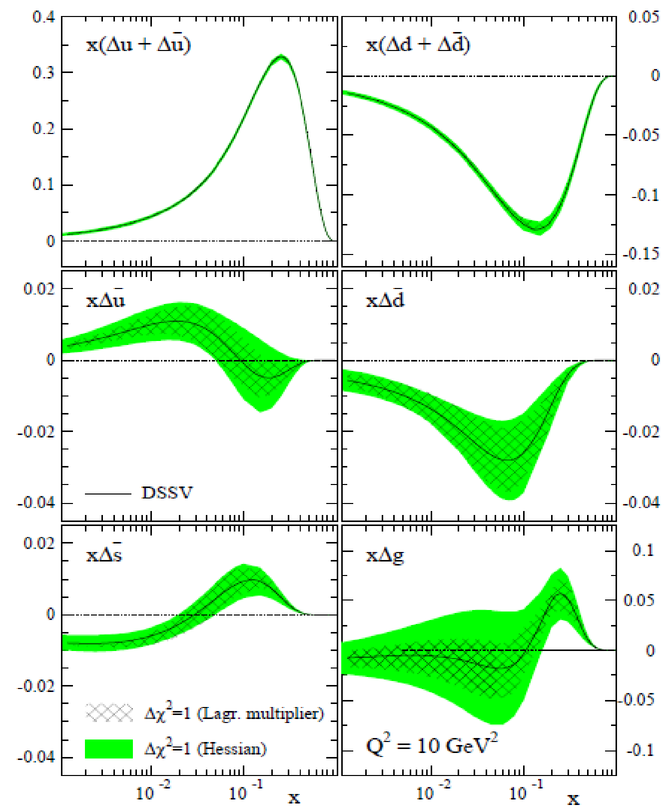
- Extracted PDF set, based on QCD collinear factorization

Unpolarized PDF



Helicity distribution

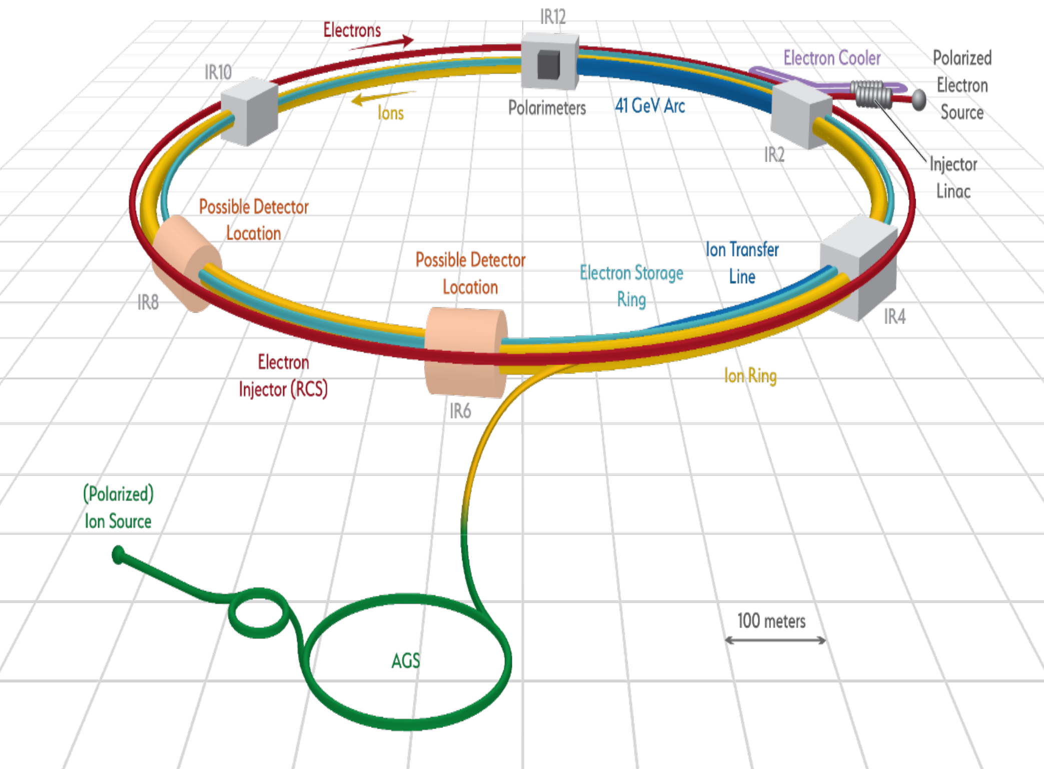




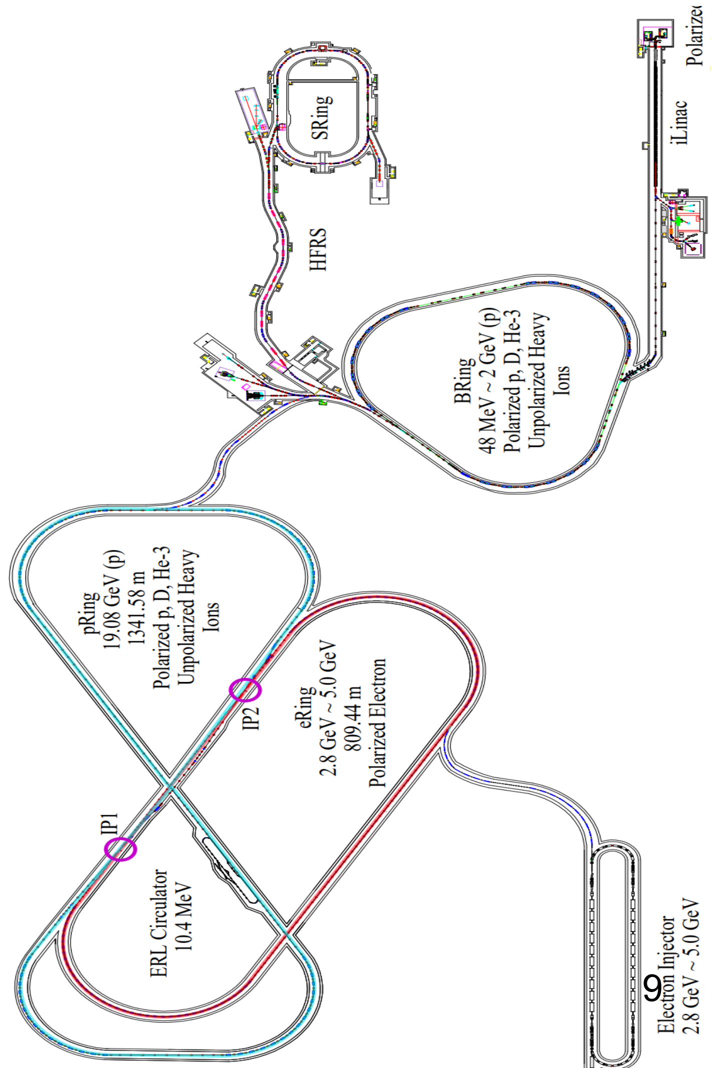
- Quasi-PDF, Ji, 2013
- Quantum computation, see 张旦波,郭星雨's talks

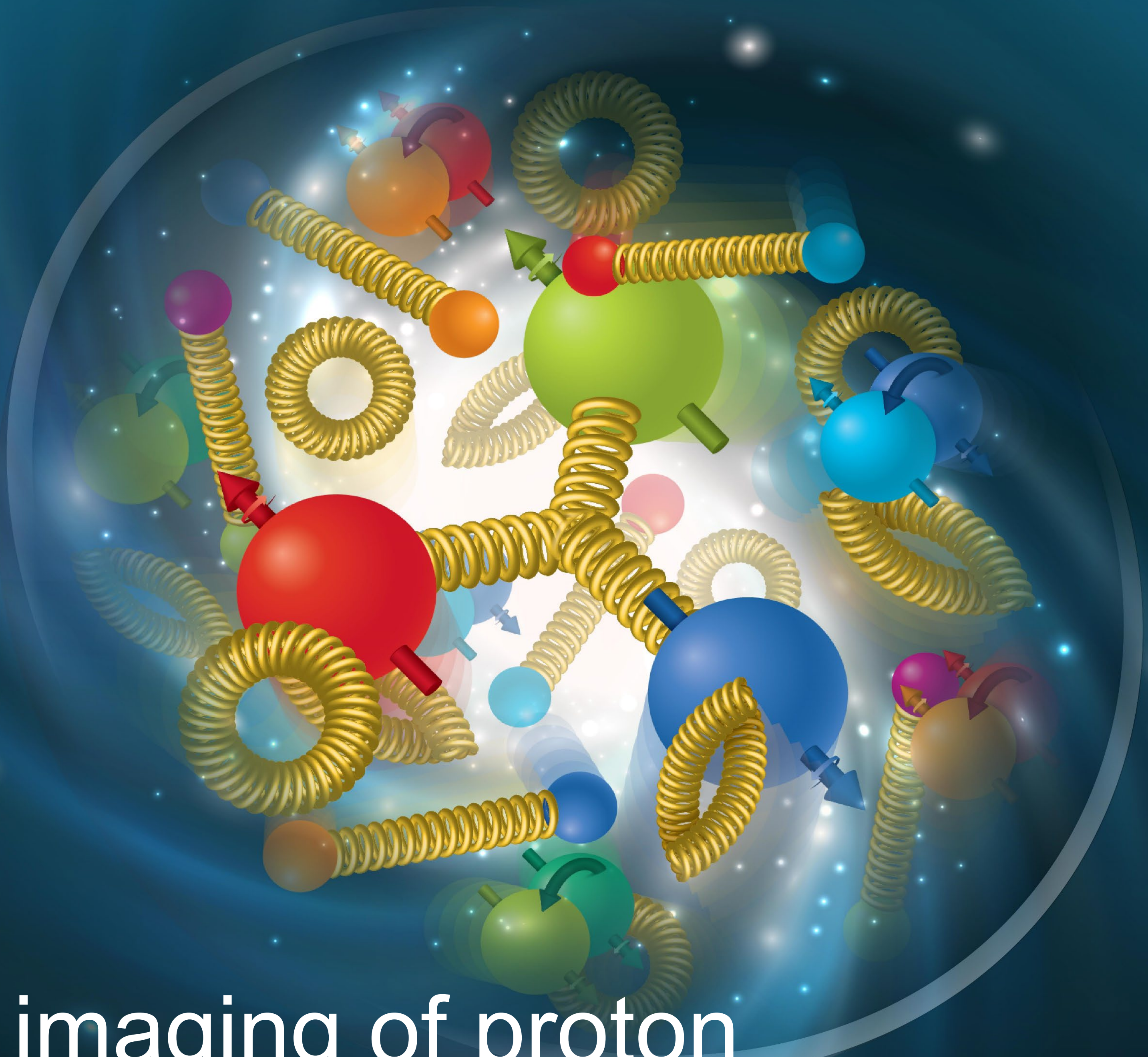
Future experiments

EIC, gluonic matter



EicC, sea quark region

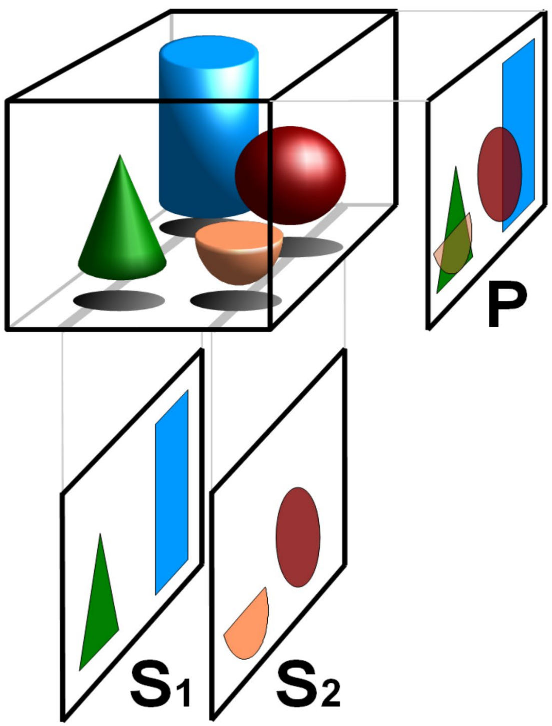




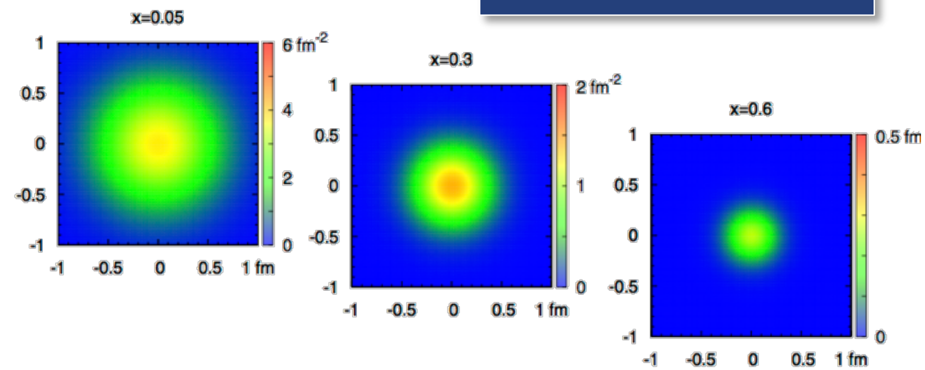
3D imaging of proton

3D tomography of proton

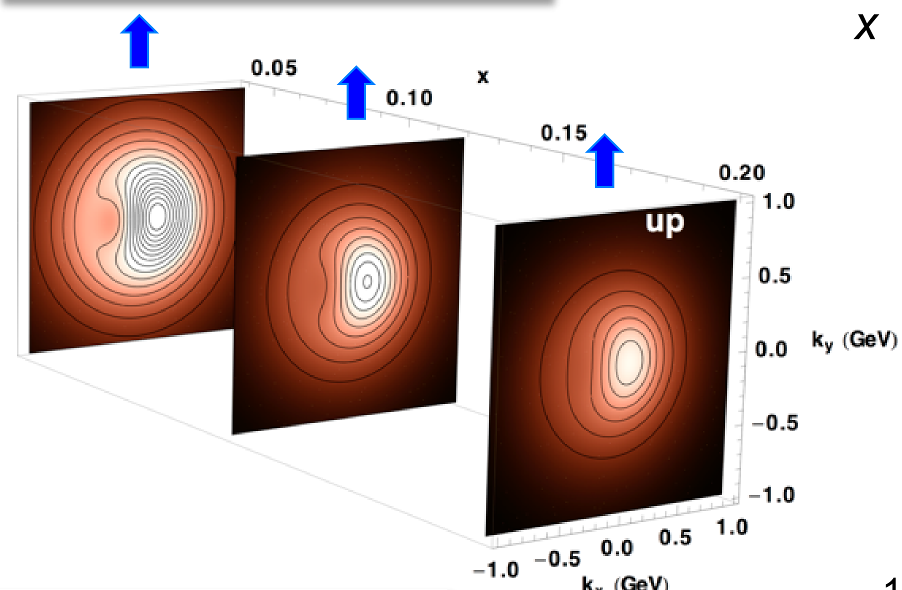
X-ray computed tomography
X射线计算机断层成像



Spatial images



Momentum space images

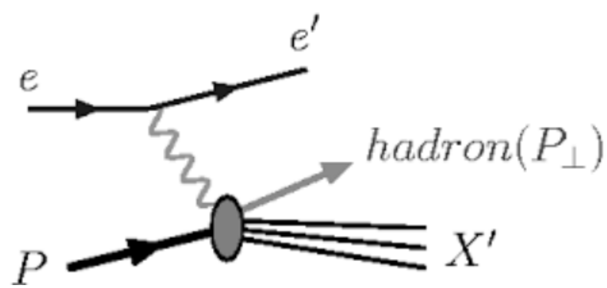


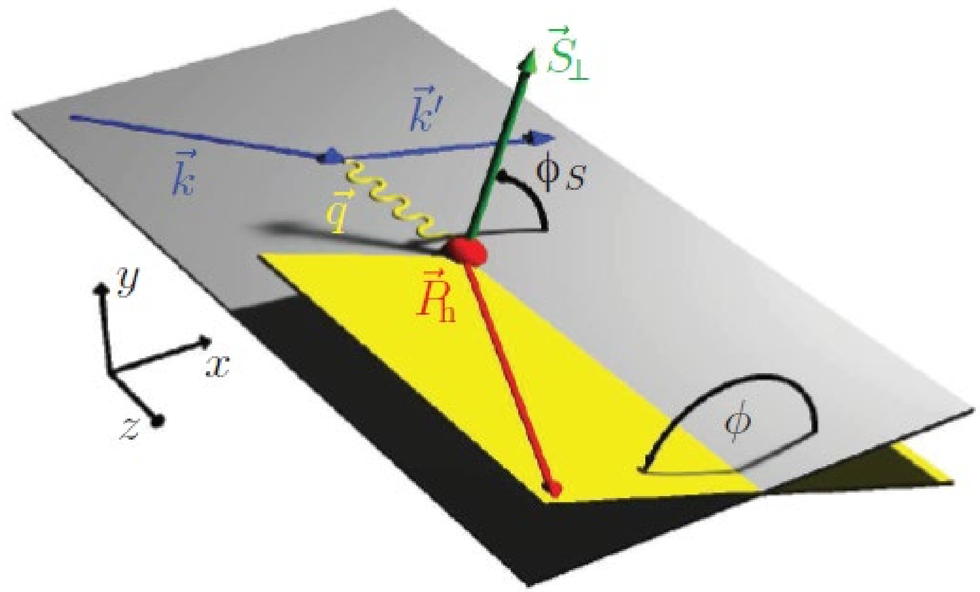
Tomographic Imaging

3D momentum space distributions: TMDs

- The “simplest” TMD is the unpolarized function $f_1(x; k_T)$

SIDIS:





$$\int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{-ixP^+\cdot y^- + i\vec{k}_\perp\cdot\vec{y}_\perp} \langle PS|\overline{\psi}_\beta(y^-, y_\perp)\mathcal{L}_v^\dagger(y^-, y_\perp)\mathcal{L}_v(0)\psi_\alpha(0)|PS\rangle$$

TMD factorization: Collins-Soper 1981, Collins-Soper-Sterman 1985

$kt<<Q$

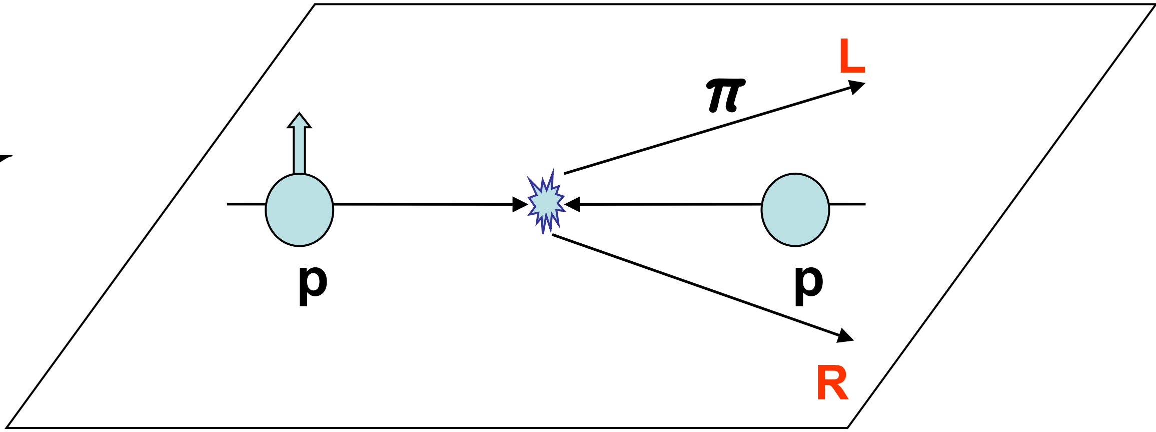
Why TMDs?

- Phenomenogical needs
- Confined motion of partons inside proton
- Access to orbital angular momentum
- Universality issue, QCD factorization

Single spin asymmetry

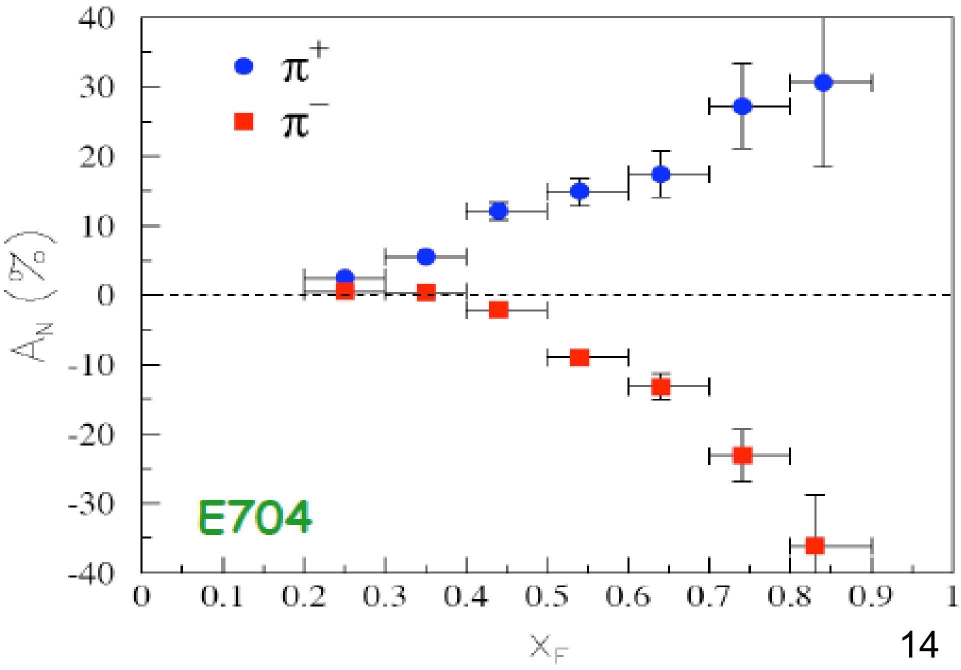
$$p(\uparrow) + p \rightarrow \pi + X$$

$$A_N \equiv (\sigma(S_\perp) - \sigma(-S_\perp))/(\sigma(S_\perp) + \sigma(-S_\perp))$$



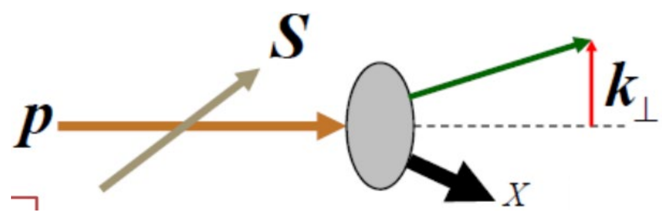
A_N--proportional to quark mass

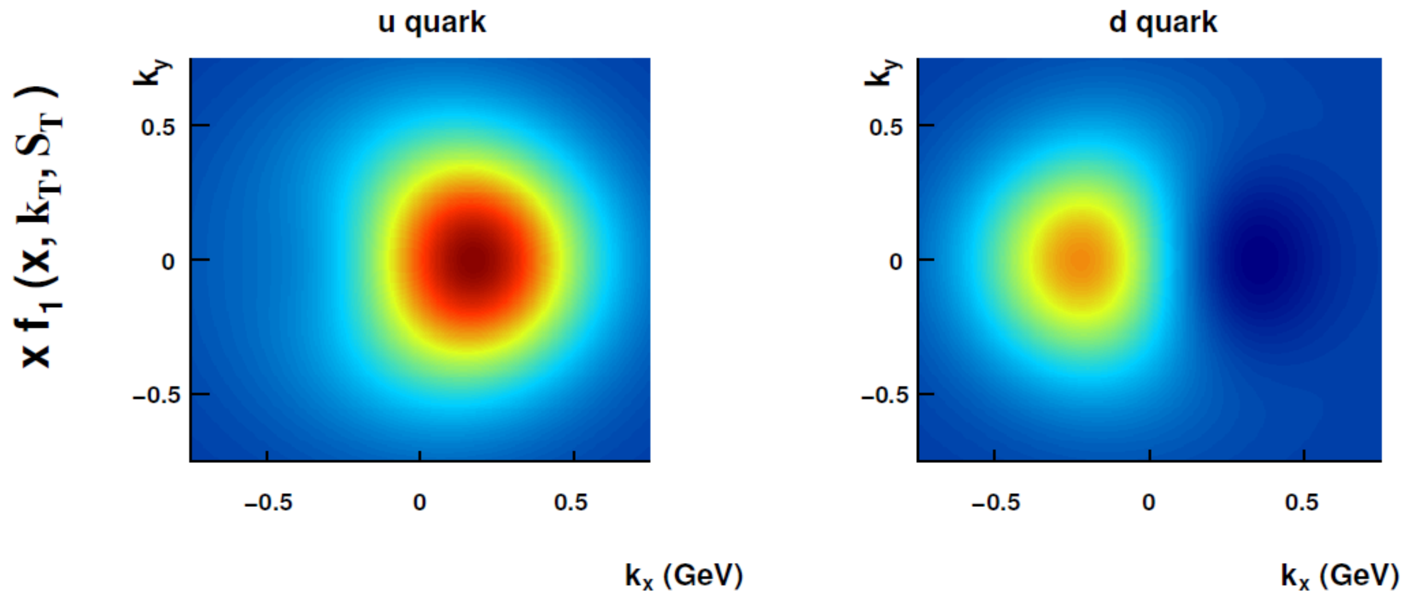
- TMD, Sivers function
- Collinear twist-3, QS function



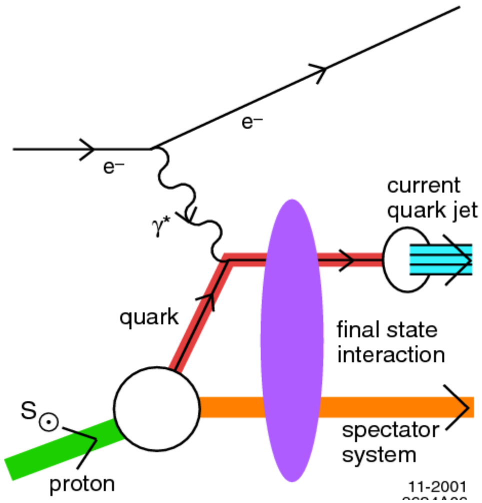
Sivers function

- The most interesting TMD: Sivers function:





The tale of the Sivers function



- The introduction of the Sivers function

  Sivers 1990

- Proof it is zero using time&parity invariance of QCD,

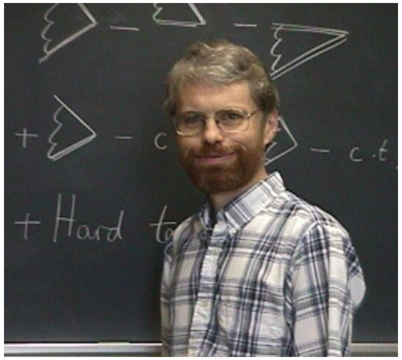
  Collins 1993

- Non-vanishing Sivers function in a model calculation,

  Brodsky-Hwang-Schmidt 2002

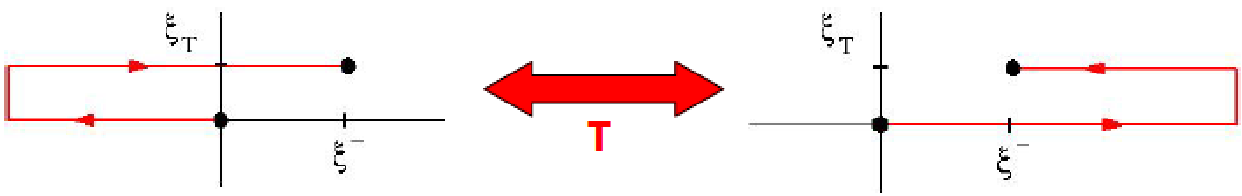
- Including gauge link contribution, prove $f_{1T}^{\perp}|_{DY} = - f_{1T}^{\perp}|_{DIS}$

  Collins 2002



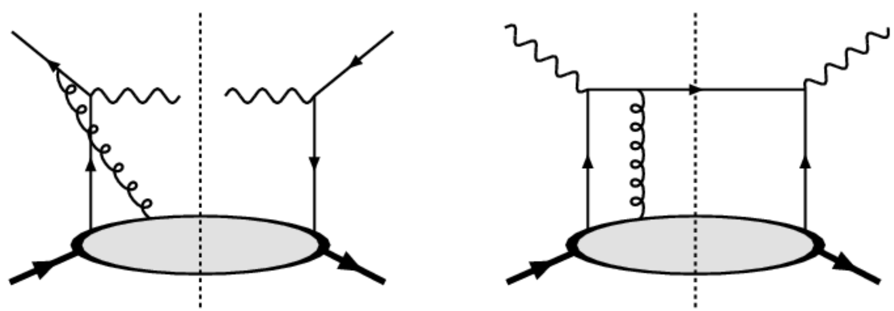
Gauge link

$$\Phi_{ij}(x,\mathbf{k}_\perp) = \int \frac{\mathbf{d}\xi^-}{(\mathbf{2}\pi)}\frac{\mathbf{d}^2\xi_\perp}{(\mathbf{2}\pi)^2}\mathbf{e}^{\mathbf{ixP}^+\xi^-}\,\mathbf{e}^{-\mathbf{ik}_\perp\xi_\perp}\langle \mathbf{P},\mathbf{S_P}|\bar{\psi}_\mathbf{j}(\mathbf{0})\,\mathcal{U}(\mathbf{0},\xi)\,\psi_\mathbf{i}(\xi)|\mathbf{P},\mathbf{S_P}\rangle\Big|_{\xi^+=\mathbf{0}}$$



$$\mathcal{P}\exp\left(-ig\int_{y^-}^{\infty} d\lambda\, n\cdot A(\lambda n)\right)$$

Drell-Yan



DIS

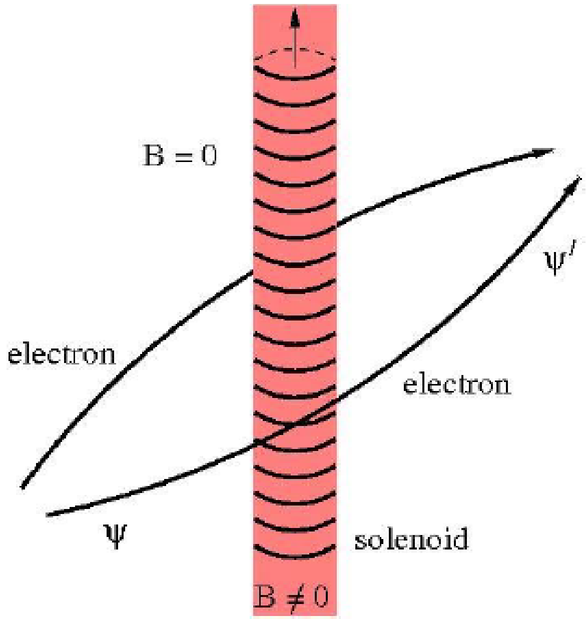
ISI FSI

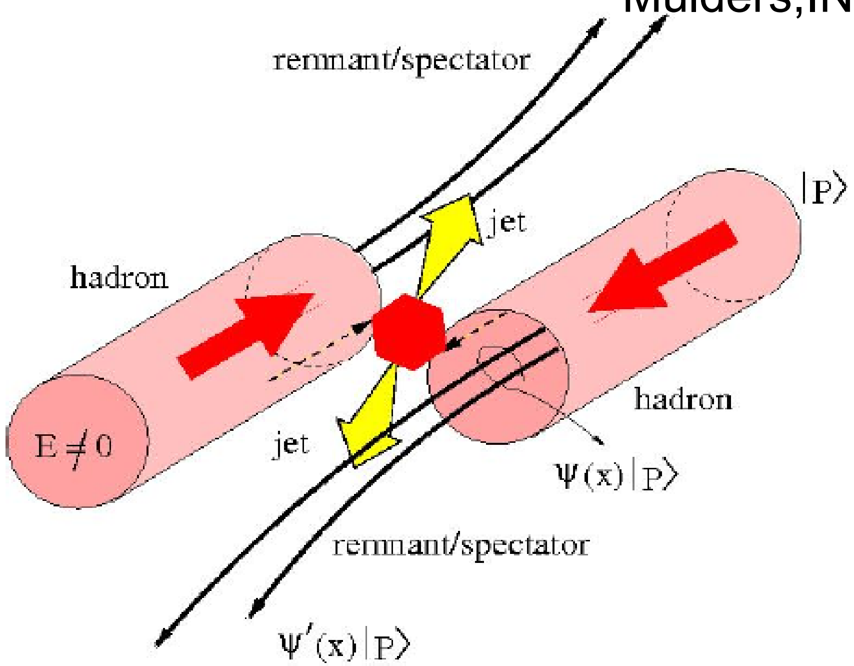
$$\mathrm{Sivers}|_{\mathrm{DY}} = -\mathrm{Sivers}|_{\mathrm{DIS}}$$

$$\int_{-\infty}^{+\infty} \mathrm{d}k_g^+ \frac{i}{-k_g^+ - i\epsilon} A^+(k_g) = \int_0^{-\infty} \mathrm{d}\zeta^- A^+(\zeta^-) \qquad \int_{-\infty}^{+\infty} \mathrm{d}k_g^+ \frac{i}{-k_g^+ + i\epsilon} A^+(k_g) = \int_0^{+\infty} \mathrm{d}\zeta^- A^+(\zeta^-)$$

➢ Gauge link physics

Mulders,INT





Gauge link (Wilson line), pure gauge gluon

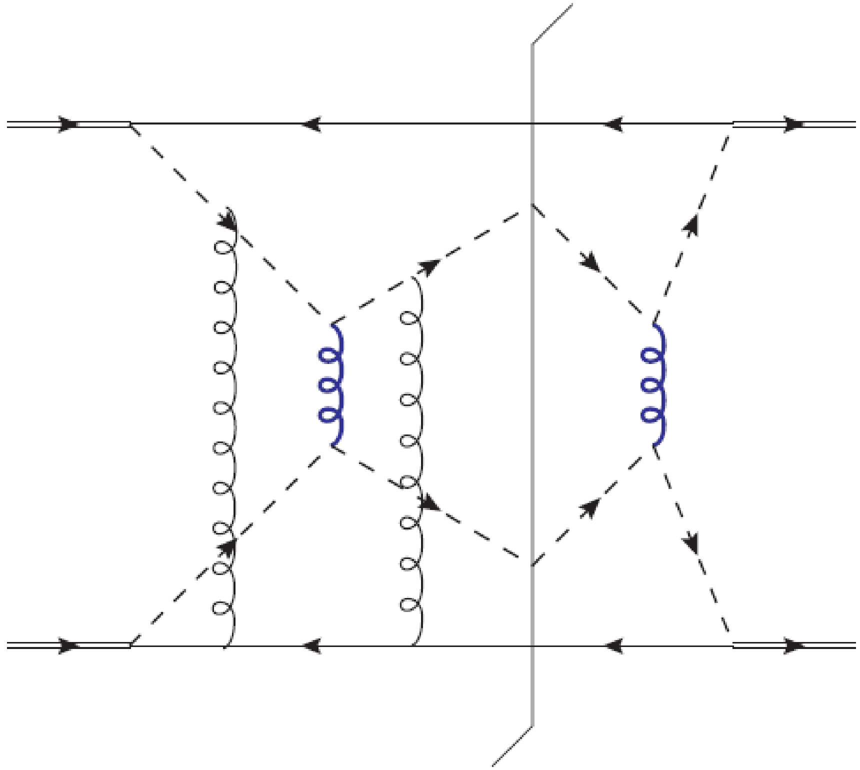
$$\psi' = e^{ie\int ds.A}\psi \qquad \psi_i(x)\left|P\right\rangle = e^{-ig\int_x^{x'} ds_\mu A^\mu}\psi_i(x')\left|P\right\rangle$$

◆ S and P wave interference

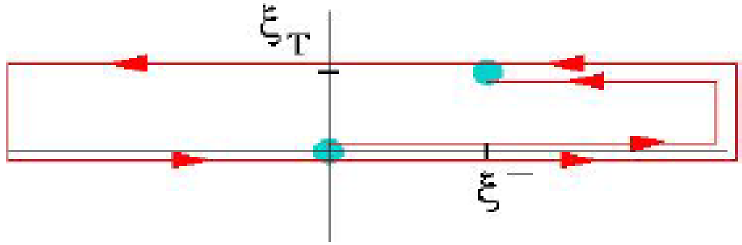
Boris, Liang 1993
Belitsky, Ji, Yuan, 2004

Are parton distributions universal?

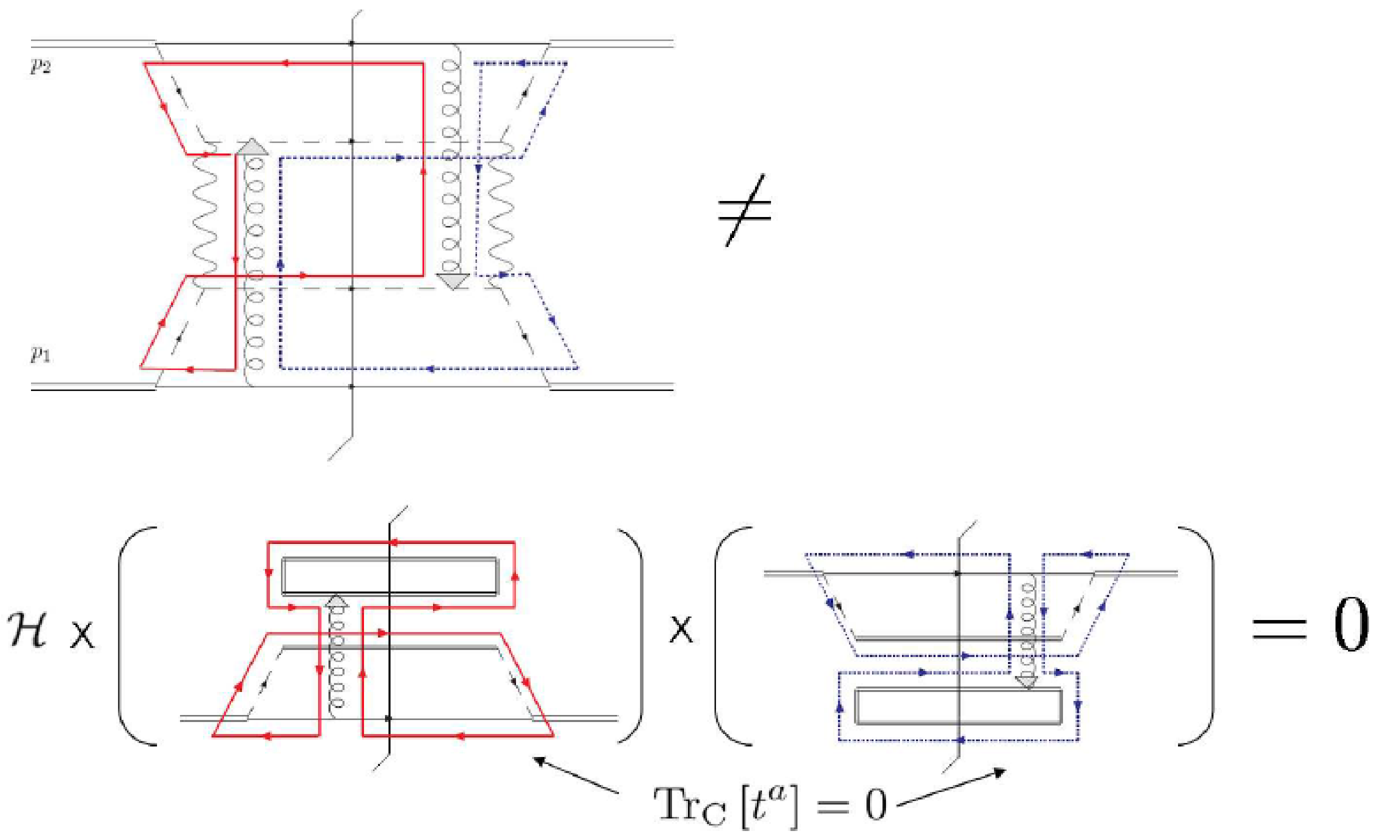
TMD factorization breaks down



Collins, Qiu, 2007

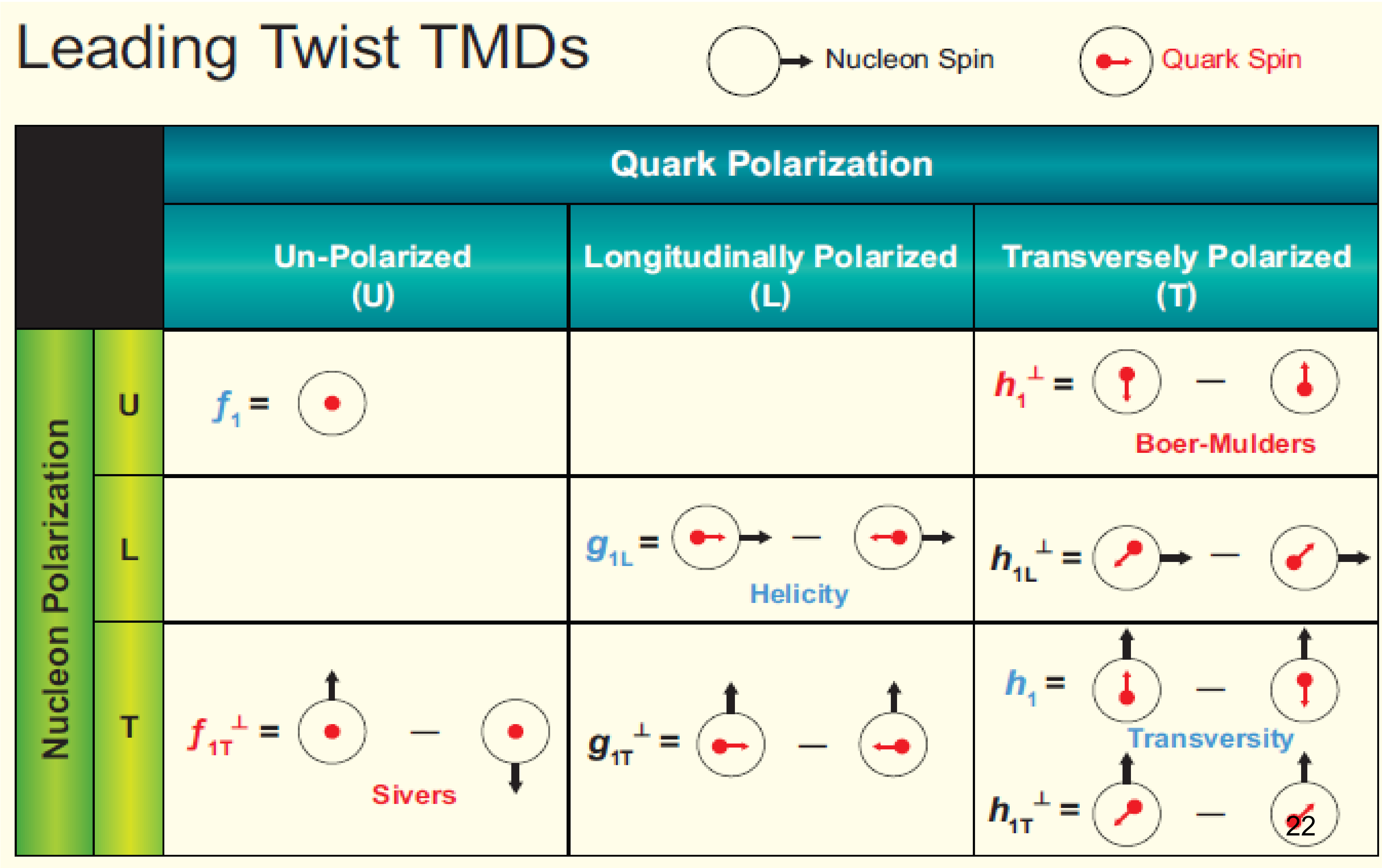


Generalized TMD factorization breaks down



Rogers, Mulders, 2010

Zoo of TMDs

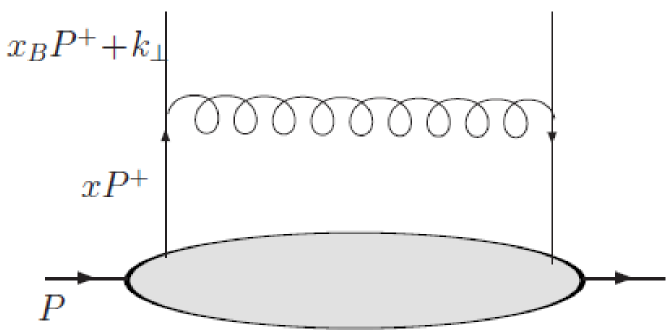

Leading Twist TMDs
Nucleon Spin
Quark Spin
Quark Polarization
Un-Polarized (U)
Longitudinally Polarized (L)
Transversely Polarized (T)
Nucleon Polarization
U
L
T
f_1 =
$h_1^\perp$ = −
Boer-Mulders
g_{1L} = −
Helicity
$h_{1L}^\perp$ = −
$f_{1T}^\perp$ = −
Sivers
$g_{1T}^\perp$ = −
h_1 = −
Transversity
$h_{1T}^\perp$ = −

Large k_T TMD distributions (K_T-even type)

When intrinsic transverse momentum $k_T >> \Lambda_{QCD}$

TMD distributions can be calculated within perturbative QCD,

k_T-even TMD distribution , in the light cone gauge $A^+ = 0$



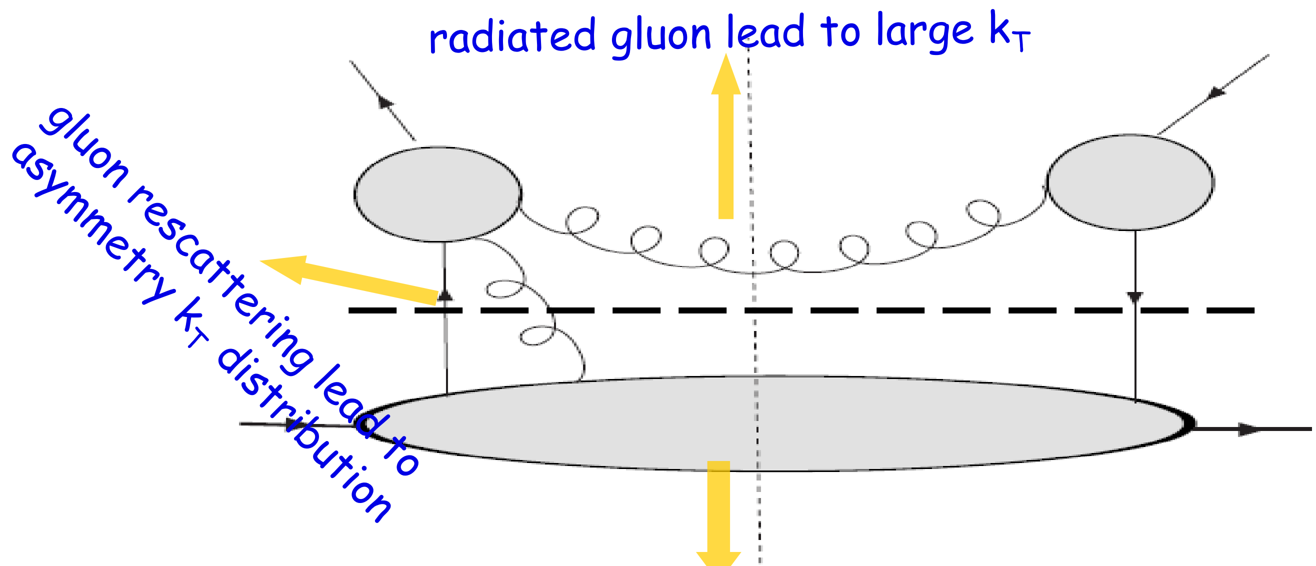
radiated gluon generate
large transverse momentum,

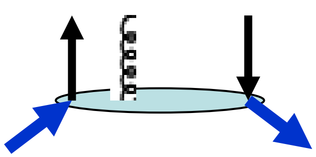
$$f_1(x_B, k_\perp) = \frac{\alpha_s}{2\pi^2}\frac{1}{\vec{k}_\perp^2}C_F\int\frac{dx}{x}f_1(x)\left[\frac{1+\xi^2}{(1-\xi)_+}+\delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2}-1\right)\right]$$

TMD evolution: resum to all orders
using the Collins-Soper equation

k_T-odd TMD distributions at large K_T

In the same spirit,
K_T-odd TMD distributions can be calculated by using collinear approach





factorized into twist-3 collinear functions accordingly, $T_F(x,x_1)$, $T_F^{(\sigma)}(x,x_1)$,etc.

$$M^\mu_{F\alpha\beta}(x,x_1) \equiv \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi} e^{ixP^+y^-} e^{i(x_1-x)P^+y_1^-} \langle P,S|\bar\psi_\beta(0) g F^\mu_{+\perp}(y_1^-)\psi_\alpha(y^-)|P,S\rangle .$$

$$= \frac{M}{2}\left[T_F(x,x_1)\epsilon_\perp^{\nu\mu} S_{\perp\nu} \not{p} + \tilde{T}_F(x,x_1) i S_\perp^\mu \gamma_5 \not{p} + \tilde{T}_F^{(\sigma)}(x,x_1) i\lambda\gamma_5\gamma_\perp^\mu \not{p} + T_F^{(\sigma)}(x,x_1) i\gamma_\perp^\mu \not{p}\right]$$

and two more twist-3 functions: $\tilde{g}(x)$ $\tilde{h}(x)$.

- Sivers and Boer-Mulders

$$f_{1T}^{\perp}|_{\rm DY}(x_B,k_\perp) = \frac{\alpha_s}{\pi}\frac{M^2}{(\vec{k}_\perp^2)^2}\int\frac{dx}{x}\left[A_{f_{1T}^{\perp}} + C_F T_F(x,x)\delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right]$$

$$h_1^{\perp}|_{\rm DY}(x_B,k_\perp) = \frac{\alpha_s}{\pi}\frac{M^2}{(\vec{k}_\perp^2)^2}\int\frac{dx}{x}\left[A_{h_1^{\perp}} + C_F T_F^{(\sigma)}(x,x)\delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2} - 1\right)\right]$$

Ji, Qiu, Vogelsang, Yuan

ZJ, Yuan, Liang,2009

- g_{1T} and h_{1L}

$$g_{1T}(x_B,k_\perp) = \frac{\alpha_s}{\pi^2}\frac{M^2}{(k_\perp^2)^2}\int\frac{dx}{x}\left\{A_{g_{1T}} + C_F\tilde{g}(x)\delta(\xi-1)\left(\ln\frac{x_B^2\zeta^2}{k_\perp^2} - 1\right)\right\}$$

$$h_{1L}(x_B,k_\perp) = \frac{\alpha_s}{\pi^2}\frac{M^2}{(k_\perp^2)^2}\int\frac{dx}{x}\left\{A_{h_{1L}} + C_F\tilde{h}(x)\delta(\xi-1)\left(\ln\frac{x_B^2\zeta^2}{k_\perp^2} - 1\right)\right\}$$

ZJ, Yuan, Liang, 2009

where ,

$$A_{f_{1T}^\perp} = -\frac{1}{2N_c} T_F(x,x) \frac{1+\xi^2}{(1-\xi)_+} + \frac{C_A}{2} T_F(x,x_B) \frac{1+\xi}{(1-\xi)_+} + \frac{C_A}{2} \tilde{T}_F(x_B,x)$$

$$A_{h_1^\perp} = -\frac{1}{2N_c} T_F^{(\sigma)}(x,x) \frac{2\xi}{(1-\xi)_+} + \frac{C_A}{2} T_F^{(\sigma)}(x,x_B) \frac{2}{(1-\xi)_+} .$$

$$\begin{aligned} A_{g_{1T}} = & \int dx_1 \left\{ \frac{1}{2N_C} \tilde{g}(x) \frac{1+\xi^2}{(1-\xi)_+} \delta(x_1 - x) \right. \\ & + \left[C_F \left(\frac{x_B^2}{x^2} + \frac{x_B}{x_1} - \frac{2x_B^2}{x_1 x} - \frac{x_B}{x} - 1 \right) + \frac{C_A}{2} \frac{(x_B^2 + x x_1)(2x_B - x - x_1)}{(x_B - x_1)(x - x_1)x_1} \right] \tilde{G}_D(x,x_1 \\ & \left. + \left[C_F \left(\frac{x_B^2}{x^2} + \frac{x_B}{x_1} - \frac{x_B}{x} - 1 \right) + \frac{C_A}{2} \frac{x_B^2 - x x_1}{(x_1 - x_B)x_1} \right] G_D(x,x_1) \right\} \end{aligned}$$

(

$$\begin{aligned} A_{h_{1L}} = & \int dx_1 \left\{ \frac{1}{2N_C} \tilde{h}(x) \frac{2\xi}{(1-\xi)_+} \delta(x_1 - x) \right. \\ & \left. + \left[C_F \frac{2(x - x_1 - x_B)}{x_1} + \frac{C_A}{2} \frac{2x_B(x_B x + x_B x_1 - x^2 - x_1^2)}{(x_B - x_1)(x - x_1)x_1} \right] H_D(x,x_1) \right\} . \end{aligned}$$

TMD evolution

Two scales problem(formulated in bt space):

$$\frac{d\ln\tilde{f}(x,b;\zeta,\mu)}{d\ln\sqrt{\zeta}} = \tilde{K}(b;\mu),$$

Collins Soper(CS) equation

$$\frac{d\ln\tilde{f}(x,b;\zeta,\mu)}{d\ln\mu} = \gamma_F(g(\mu);\zeta/\mu^2)$$

Renormalization group equation

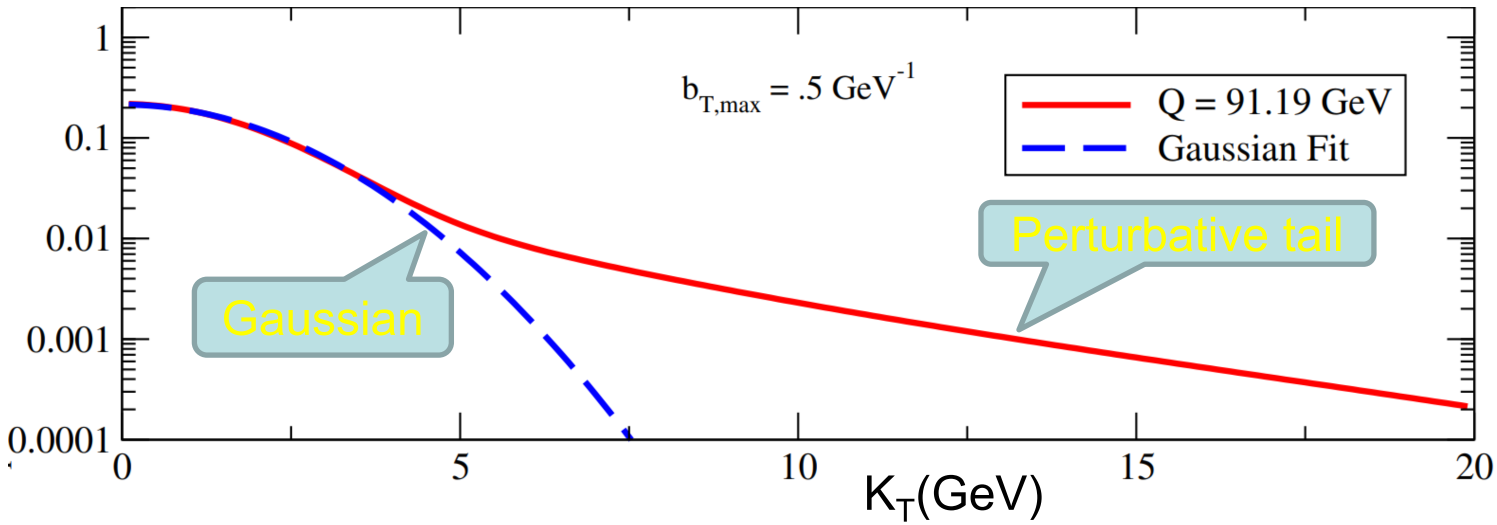
Evolved TMD: $$\tilde{f}_1^a(x,b^2;\zeta_F,\mu) = e^{-S(b,Q)}\,\tilde{f}_1^a(x,b^2;\mu_b^2,\mu_b)$$

With $$S(b,Q) = -\ln\left(\frac{Q^2}{\mu_b^2}\right)\tilde{K}(b,\mu_b) - \int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[\gamma_F(g(\mu);1) - \frac{1}{2}\ln\left(\frac{Q^2}{\mu^2}\right)\gamma_K(g(\mu))\right]$$

- The last step: making Fourier transform back to kt space.

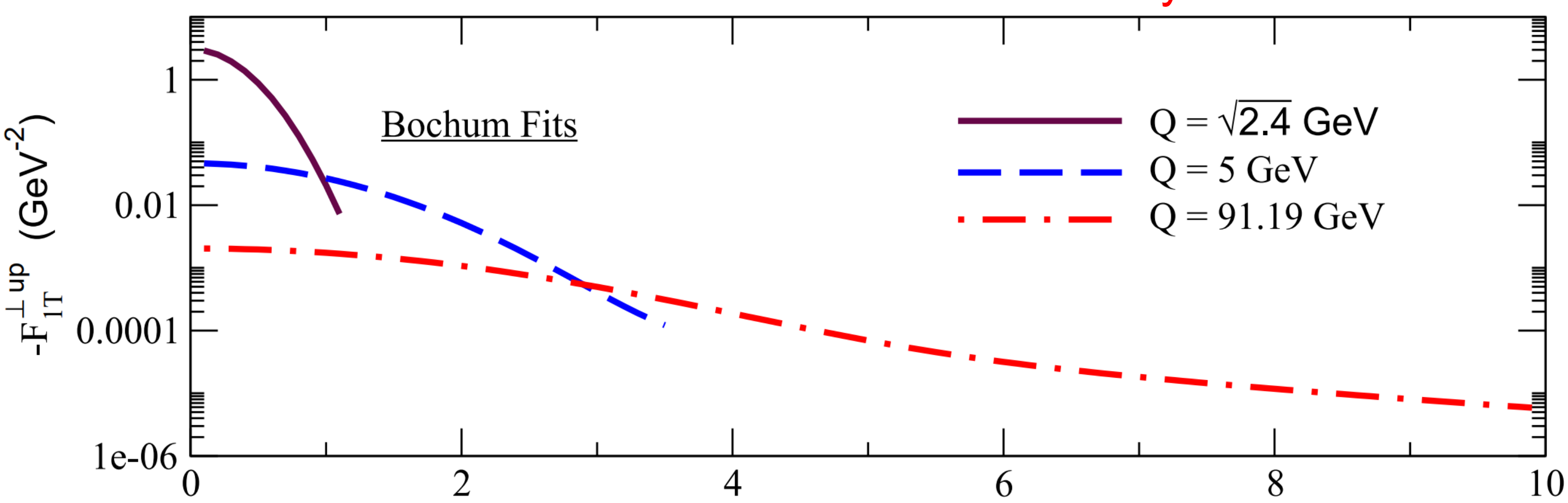
Evolved TMDs

- Up quark TMD, x=0.09:



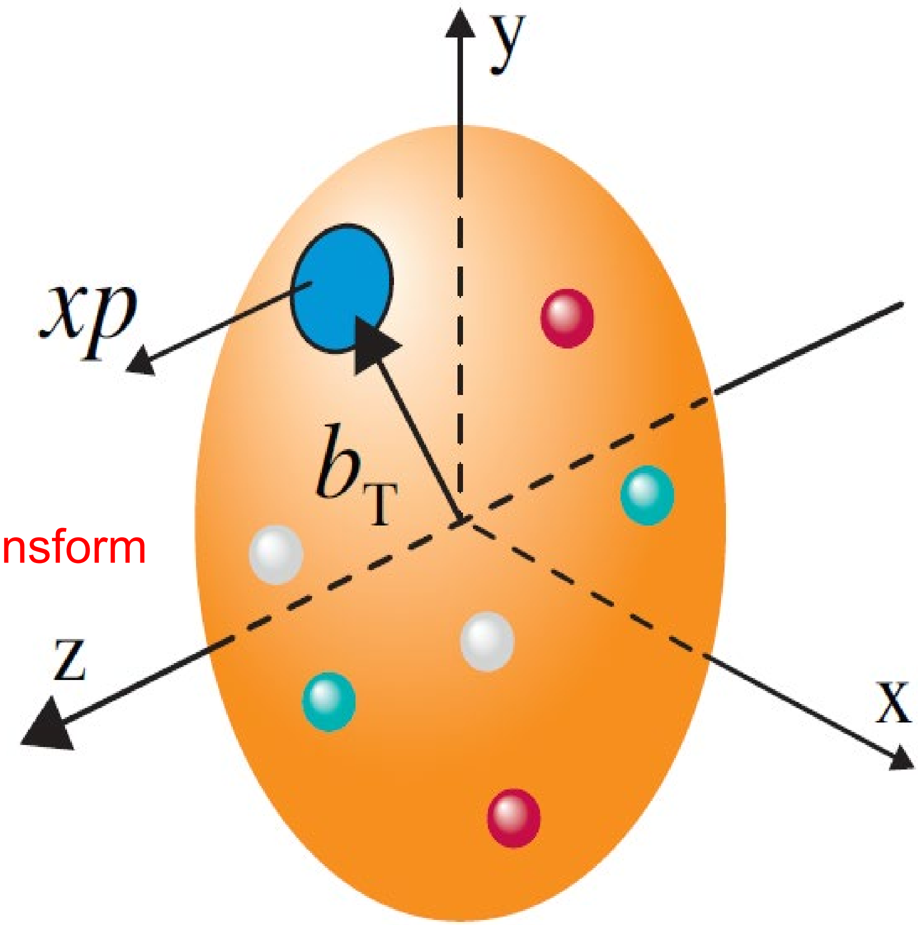
- Up quark Sivers function, x=0.1:

Aybat-Collins-Qiu-Rogers, 2012



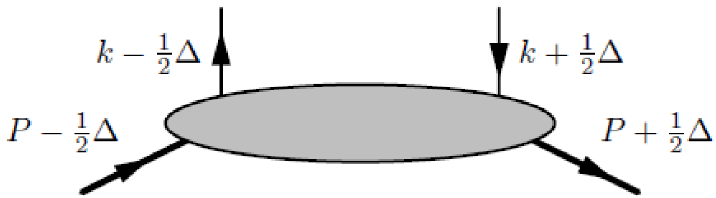
Spatial imaging of Quarks and Gluons

- Longitudinal momentum distribution + transverse spatial distribution: $f(x,b_T)$



- Remark: $f(x,b_T)$ and $f(x,k_T)$ are not related to each other by a Fourier transform

Generalized Parton Distributions(GPDs)



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

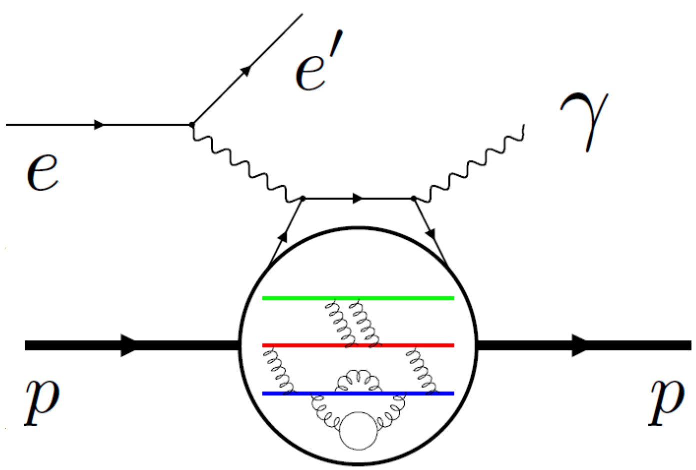
$$\int \frac{d\lambda}{2\pi} e^{ix(Pz)}\, n_{-\alpha} n_{-\beta} \left\langle p' \middle| G^{\alpha\mu}\left(-\frac{z}{2}\right) G^{\beta}_{\mu}\left(\frac{z}{2}\right) \middle| p \right\rangle \Big|_{z=\lambda n_-}$$

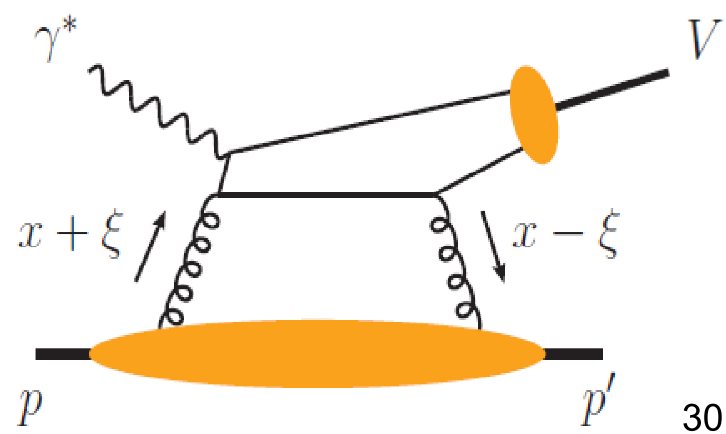
$$= \frac{1}{2}\left[H^g\, \bar{u}(p')\, \not{n}_- u(p) + E^g\, \bar{u}(p') \frac{i\sigma^{\alpha\beta} n_{-\alpha} \Delta_\beta}{2\, m_N} u(p) \right]$$

D. Muller, 94
X. D. Ji, 97
A. V. Radushkin, 97

x, ζ, t

DVCS





Some properties of GPDs

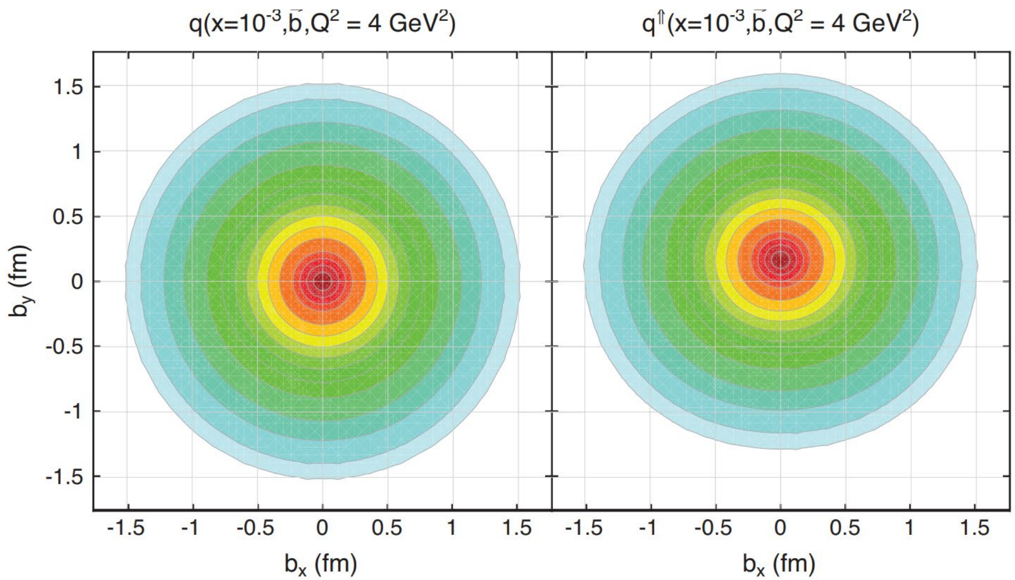
- Form factors

$$\sum_q e_q \int dx\, H^q(x,\xi,t) = F_1^p(t)\,, \qquad \sum_q e_q \int dx\, E^q(x,\xi,t) = F_2^p(t)$$

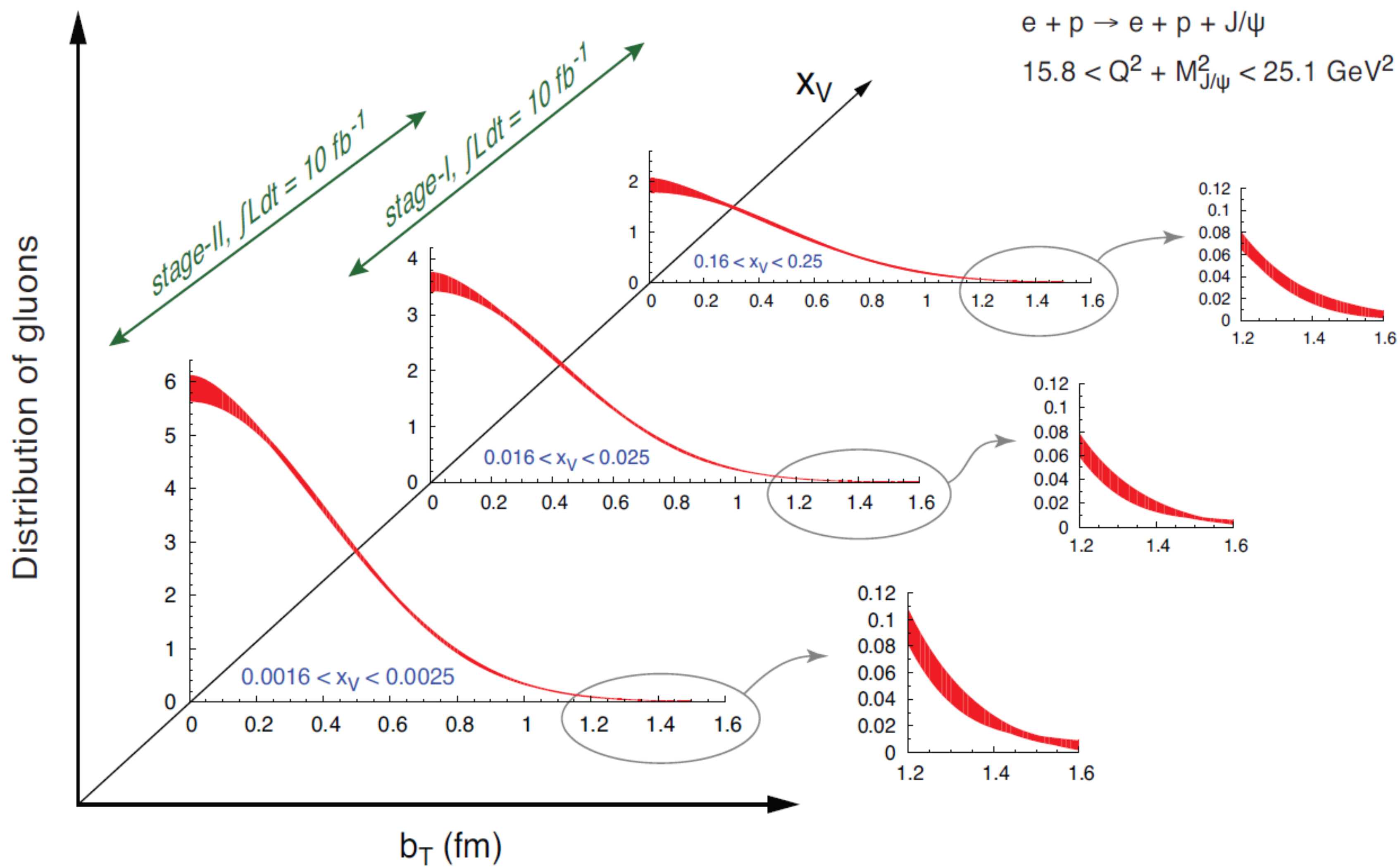
- Transverse spatial distribution

Soper 77 & Burkardt 2000

$$\mathcal{H}^q(x,\vec{b}_T^{\,2}) = \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2}\, e^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, H^q(x,0,-\vec{\Delta}_T^2) \qquad f_q(z,b_{\perp,q}) = \mathcal{H}_q(z,b_{\perp,q}^2) + \frac{1}{M}\epsilon_\perp^{ij} b_{\perp,q}^i S_\perp^j \frac{\partial \mathcal{E}_q(z,b_{\perp,q}^2)}{\partial b_{\perp,q}^2}$$



Transverse spatial distribution of gluons



Information encoded in parton distributions is incomplete

Wave function $c_1 \mid p_T = 0.2\rangle + c_2 e^{i\phi} \mid p_T = 0.4\rangle$

Corresponding Parton distribution function:

$$f(p_T = 0.2) \propto |c_1|^2 \qquad f(p_T = 0.4) \propto |c_2|^2$$

Nontrival correlation could exist:

$$\mathbf{b}_T \times \mathbf{k}_T, \mathbf{b}_T \cdot \mathbf{k}_T$$

Parton Wigner distributions

In quantum mechanics:

$$\widehat{W}^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x) \equiv \frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2 z_\perp}{(2\pi)^3}\, e^{i(xp^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}\,\overline{\psi}(\vec{b}_\perp - \tfrac{z}{2})\Gamma\mathcal{W}\,\psi(\vec{b}_\perp + \tfrac{z}{2})\big|_{z^+=0}$$

Operator defination:

$$\rho^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x,\vec{S}) \equiv \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,\langle p^+,\tfrac{\vec{\Delta}_\perp}{2},\vec{S}|\widehat{W}^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x)|p^+,-\tfrac{\vec{\Delta}_\perp}{2},\vec{S}\rangle.$$

A. Belitisky, X. D. Ji and F. Yuan, 2003

Motivations of studying parton Wigner distributions:

- tomography picture of nucleon
- encode information on parton OAM

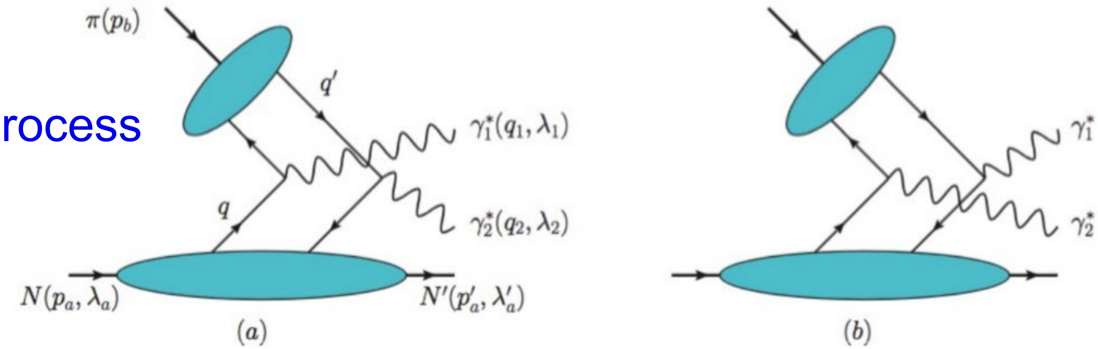
Relation to TMDs and GPDs

$$\int \mathrm{d}^2 b_\perp\,\rho^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x,\vec{S}) = W^{[\Gamma]}(\vec{0}_\perp,\vec{k}_\perp,x,\vec{S})$$ TMD correlator

$$\int \mathrm{d}^2 k_\perp\,\rho^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x,\vec{S}) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,F^{[\Gamma]}(\vec{\Delta}_\perp,x,\vec{S})$$ GPD correlator

How to measure parton Wigner distribution

Quark case: exclusive double DY process



Gluon case: exclusive double quarknioum production.

Bhattacharya, Metz, ZJ 2017

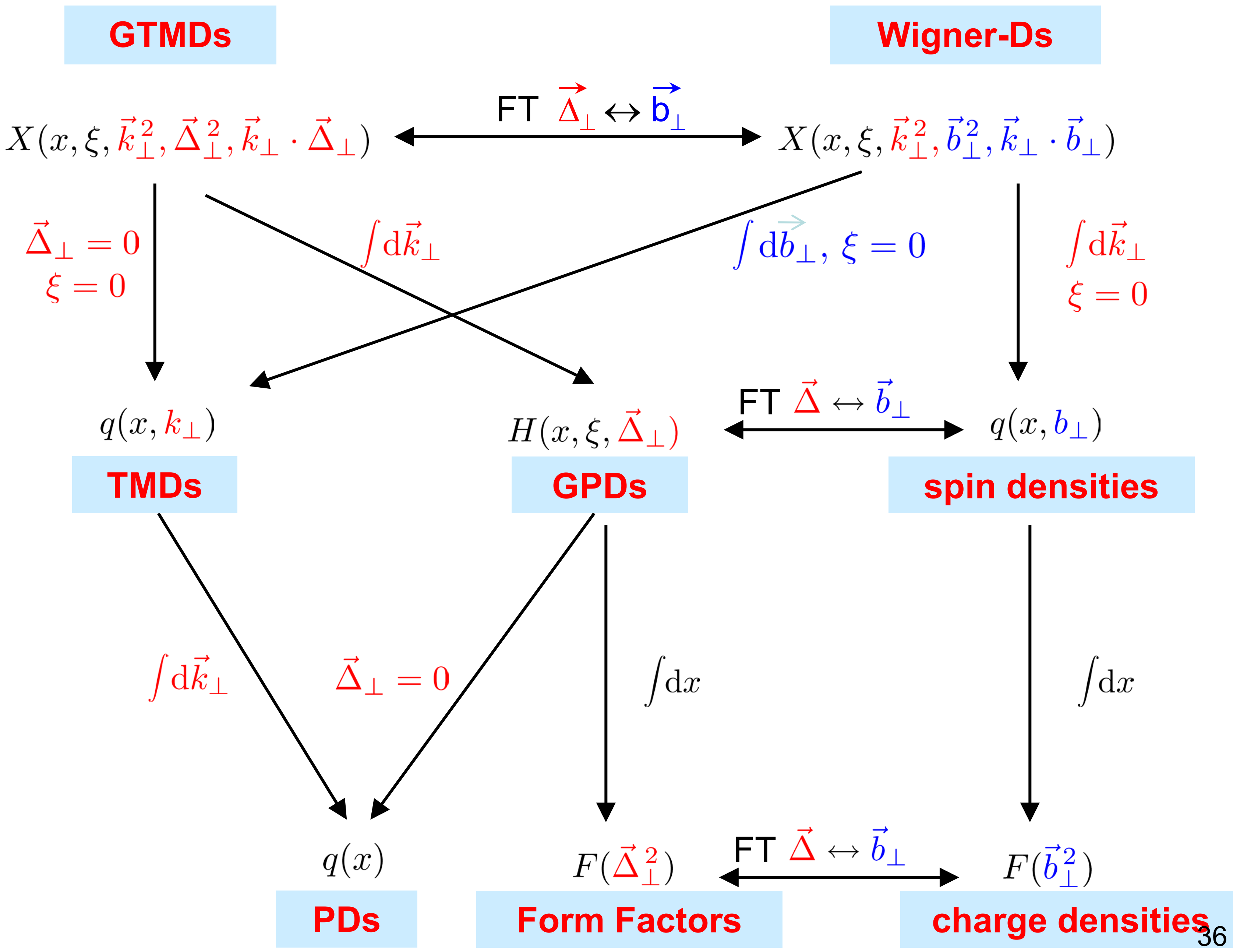
$$\frac{1}{2}(\tau_{UL}+\tau_{LU})$$

$$\approx\ 2\,\mathrm{Im}\Big\{-\frac{1}{M^2}\,(\varepsilon_\perp^{ij}\Delta q_\perp^i\,\Delta_{a\perp}^j)\,C\Big[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}\,F_{1,4}(x_a,\vec{k}_{a\perp})\,F_{1,1}(x_b,\vec{k}_{b\perp})\Big]\,C\Big[F_{1,1}^*(x_a,\vec{p}_{a\perp})\,F_{1,1}^*(x_b,\vec{p}_{b\perp})\Big]$$

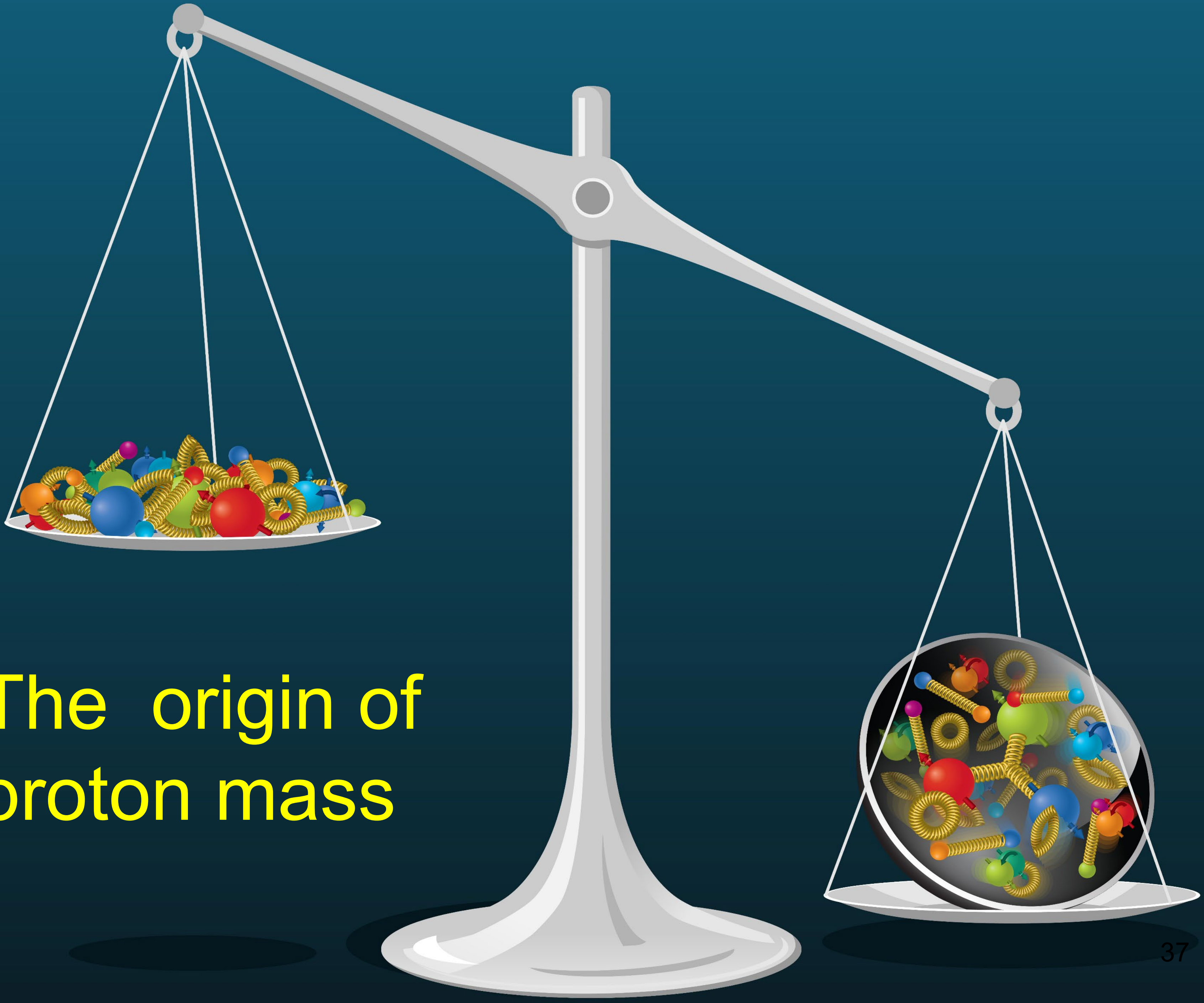
$$+\ \frac{1}{M^2}\,(\varepsilon_\perp^{ij}\Delta q_\perp^i\,\Delta_{a\perp}^j)\,C\Big[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}\,G_{1,1}(x_a,\vec{k}_{a\perp})\,G_{1,4}(x_b,\vec{k}_{b\perp})\Big]\,C\Big[G_{1,4}^*(x_a,\vec{p}_{a\perp})\,G_{1,4}^*(x_b,\vec{p}_{b\perp})\Big]\Big\}$$

$$\approx\ 2\,\mathrm{Im}\Big\{-\frac{1}{M^2}\,(\varepsilon_\perp^{ij}\Delta q_\perp^i\,\Delta_{a\perp}^j)\,C\Big[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}\,F_{1,4}(x_a,\vec{k}_{a\perp})\,F_{1,1}(x_b,\vec{k}_{b\perp})\Big]\,C\Big[F_{1,1}^*(x_a,\vec{p}_{a\perp})\,F_{1,1}^*(x_b,\vec{p}_{b\perp})\Big]\Big\}$$

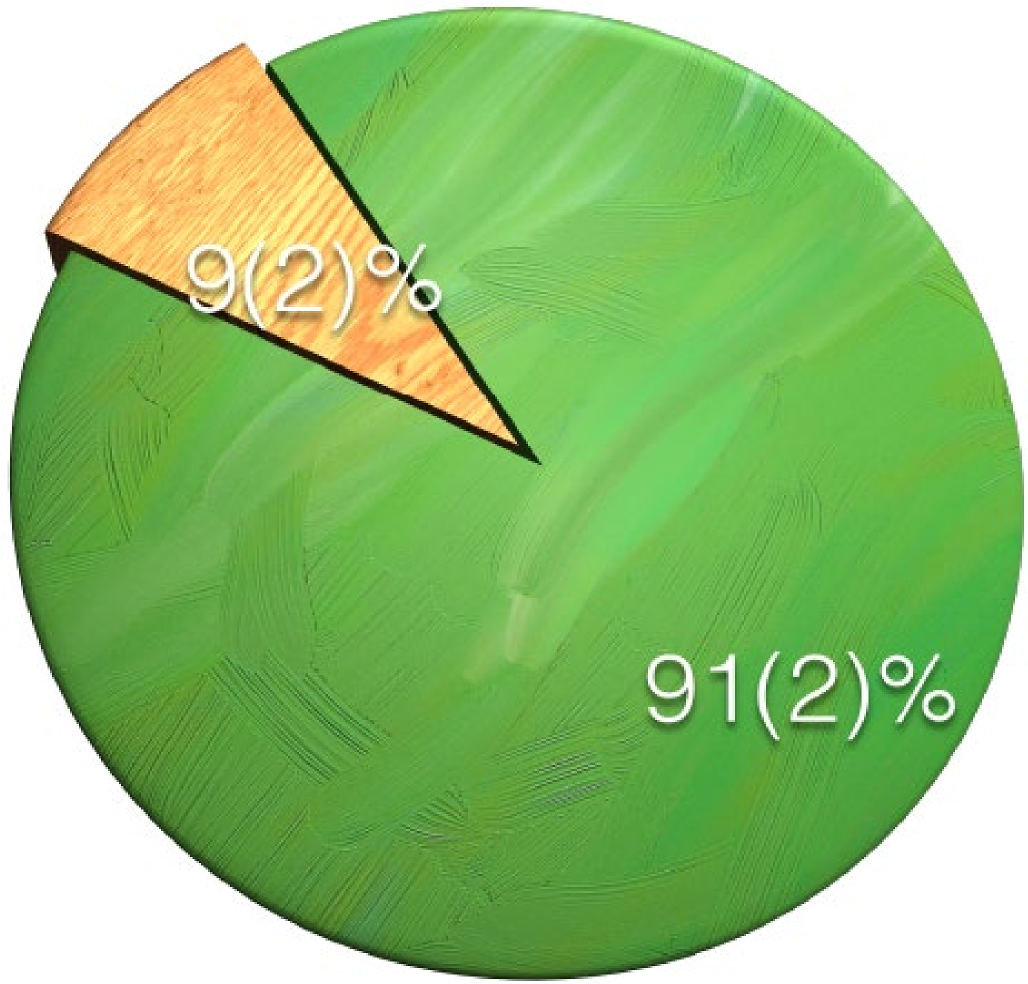
Bhattacharya, Vikash, Metz, Jeng, ZJ 2018


GTMDs
Wigner-Ds
FT $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$
$X(x,\xi,\vec{k}_\perp^{\,2},\vec{\Delta}_\perp^{\,2},\vec{k}_\perp\cdot\vec{\Delta}_\perp)$
$X(x,\xi,\vec{k}_\perp^{\,2},\vec{b}_\perp^{\,2},\vec{k}_\perp\cdot\vec{b}_\perp)$
$\vec{\Delta}_\perp = 0$
$\xi = 0$
$\int d\vec{k}_\perp$
$\int d\vec{b}_\perp,\ \xi = 0$
$\int d\vec{k}_\perp$
$\xi = 0$
$q(x, k_\perp)$
$H(x,\xi,\vec{\Delta}_\perp)$
FT $\vec{\Delta} \leftrightarrow \vec{b}_\perp$
$q(x, b_\perp)$
TMDs
GPDs
spin densities
$\int d\vec{k}_\perp$
$\vec{\Delta}_\perp = 0$
$\int dx$
$\int dx$
$q(x)$
$F(\vec{\Delta}_\perp^{\,2})$
FT $\vec{\Delta} \leftrightarrow \vec{b}_\perp$
$F(\vec{b}_\perp^{\,2})$
PDs
Form Factors
charge densities

The origin of proton mass



Proton mass budget



From Yibo's talk

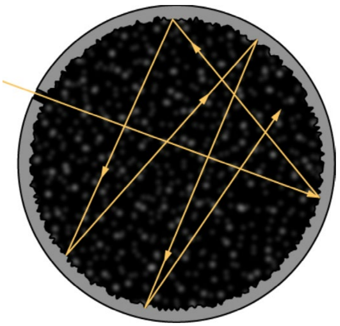
Massless limit

- The mass of blackbody radiation

von Laue's theorem，1911

$$dV\varepsilon + dVp \qquad p = \varepsilon/3$$

$$\frac{4}{3}dV\varepsilon$$



- Mass from Quark and gluon kinetic energy accessible via PDF

$$\int_0^1 dx\, x d(x) \qquad \int_0^1 dx\, x g(x)$$

◆ But only makes up ¾ proton mass!

Trace anomaly

- Energy momentum tensor:

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}$$

- Proton EMT

$$\langle P|T^{\mu\nu}|P\rangle = 2P^\mu P^\nu \qquad \langle P|T^{\alpha}_{\alpha}|P\rangle = 2M^2$$

- The trace of EME in 4-2ε: $T^\mu_\mu = -2\epsilon\frac{F^2}{4} + m\bar{q}q$

Collins, Duncan, Joglekar, 1977
Nielsen, 1977

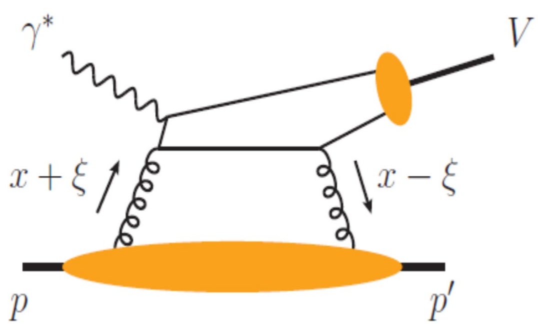
$$(T_g)^\mu_\mu = \frac{\beta}{2g}(F^2)_R + \gamma_m(m\bar{q}q)_m$$

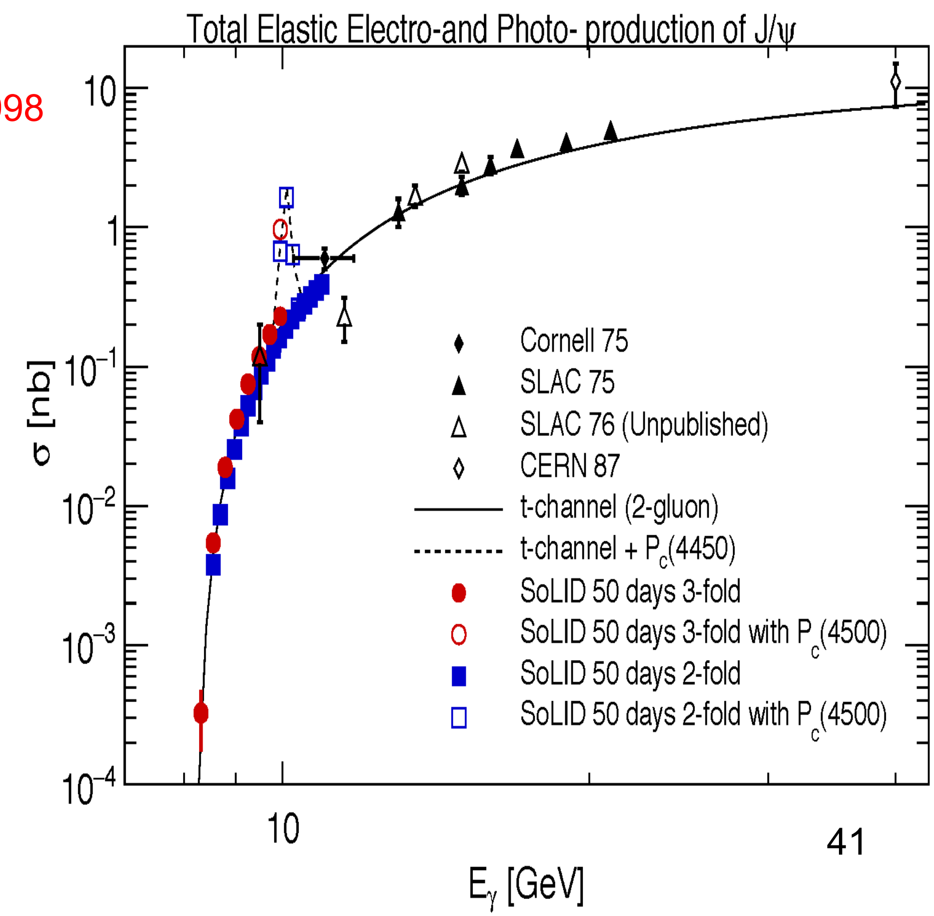
How to measure trace anomaly

- Twist-4 operator: $\langle P' | F^{\mu\nu} F_{\mu\nu} | P \rangle$

- Threshold J/psi production

Kharzeev,Satz,Syamtomov, 1998





- Extractions: Xu-Xie-Wang-Chen, 2020
Wang-Bu-Zeng, 2022

- Intense debates:

Hatta, Ji, Ma, Sun, Tong, Yuan…..

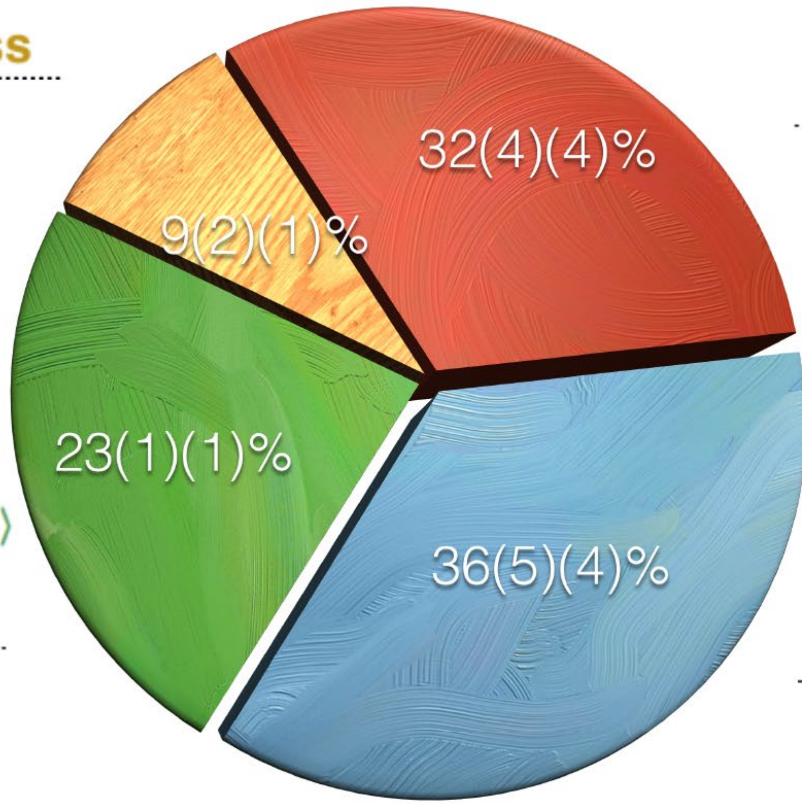
Ji's decomposition

$\langle H_m \rangle / M_N = 9(2)\%$

quark mass

$\langle H_E \rangle = \frac{3}{4} \langle x \rangle_q M - \frac{3}{4} \langle H_m \rangle$

quark energy



$\langle x \rangle_q = 52(7)(5)\%$

$\langle x \rangle_g = 48(7)(5)\%$

$\frac{1}{4} M = -\langle \hat{T}_{44} \rangle = \frac{1}{4} \langle H_m \rangle + \langle H_a \rangle$

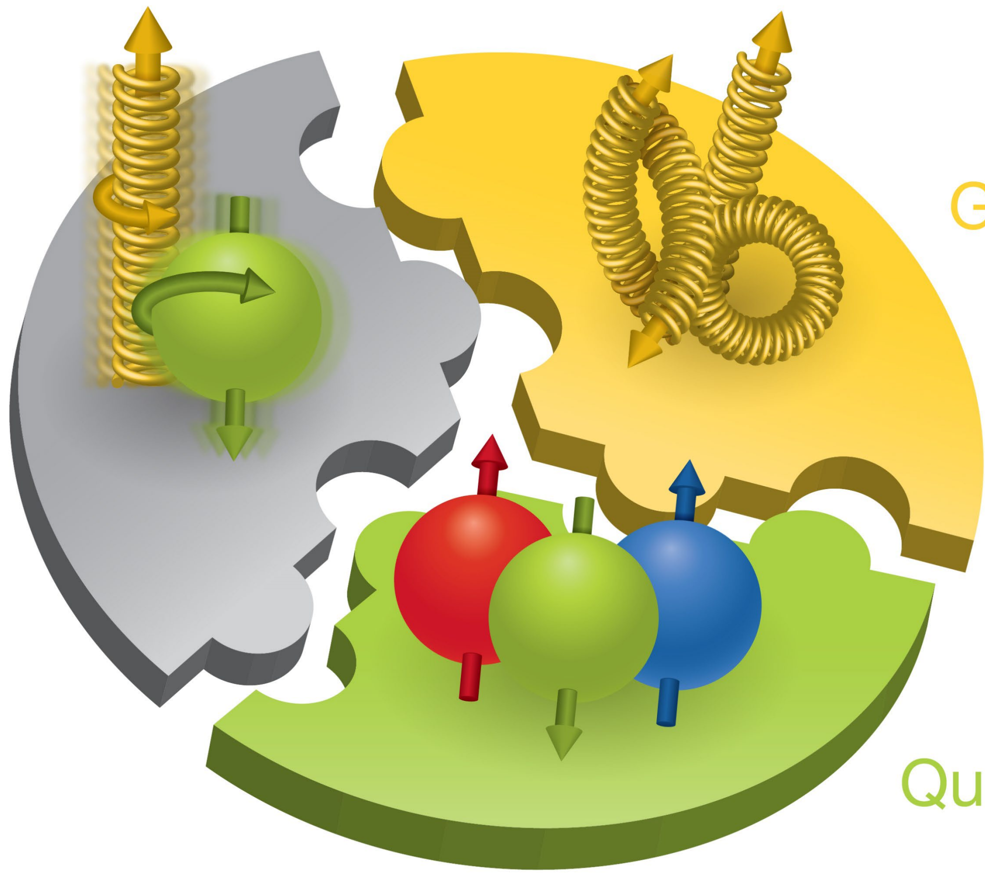
QCD anomaly

$\langle H_g \rangle = \frac{3}{4} \langle x \rangle_g M$

glue energy

- Other decompositions exist: Hatta-Rajan-Tanaka, 2018
  Metz-Pasquini-Rodini, 2020

Proton spin decomposition

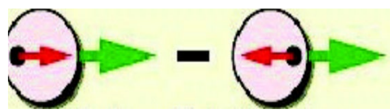


Proton spin crises

- Naïve quark model: proton spin from valance quark

■ Vanishing quark spin contribution-----EMC measurement@CERN, 1988

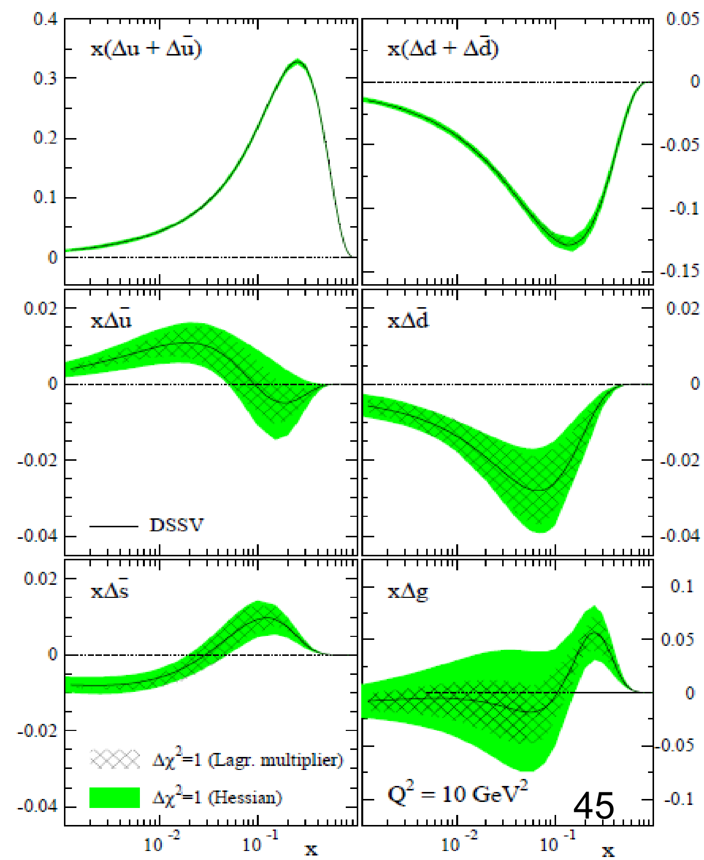
$$\Delta f(x,Q^2) \equiv f^{+}(x,Q^2) - f^{-}(x,Q^2)$$



◆ Double spin asymmetry:

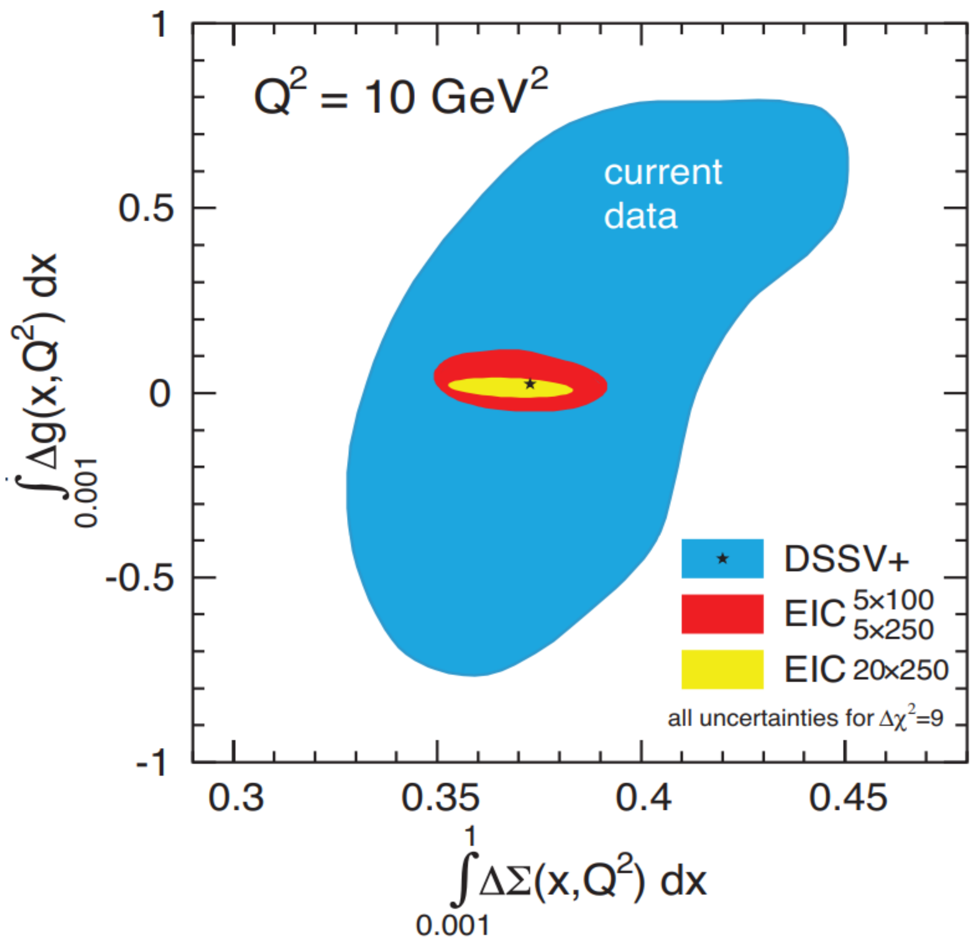
$$\frac{1}{2}\left[\frac{d^2\sigma^{\rightleftarrows}}{dx\,dQ^2} - \frac{d^2\sigma^{\rightrightarrows}}{dx\,dQ^2}\right] \simeq \frac{4\pi\,\alpha^2}{Q^4} y\,(2-y)\,g_1(x,Q^2)$$

$$g_1(x,Q^2) = \frac{1}{2}\sum e_q^2\left[\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2)\right]$$



Modern experimental constraint

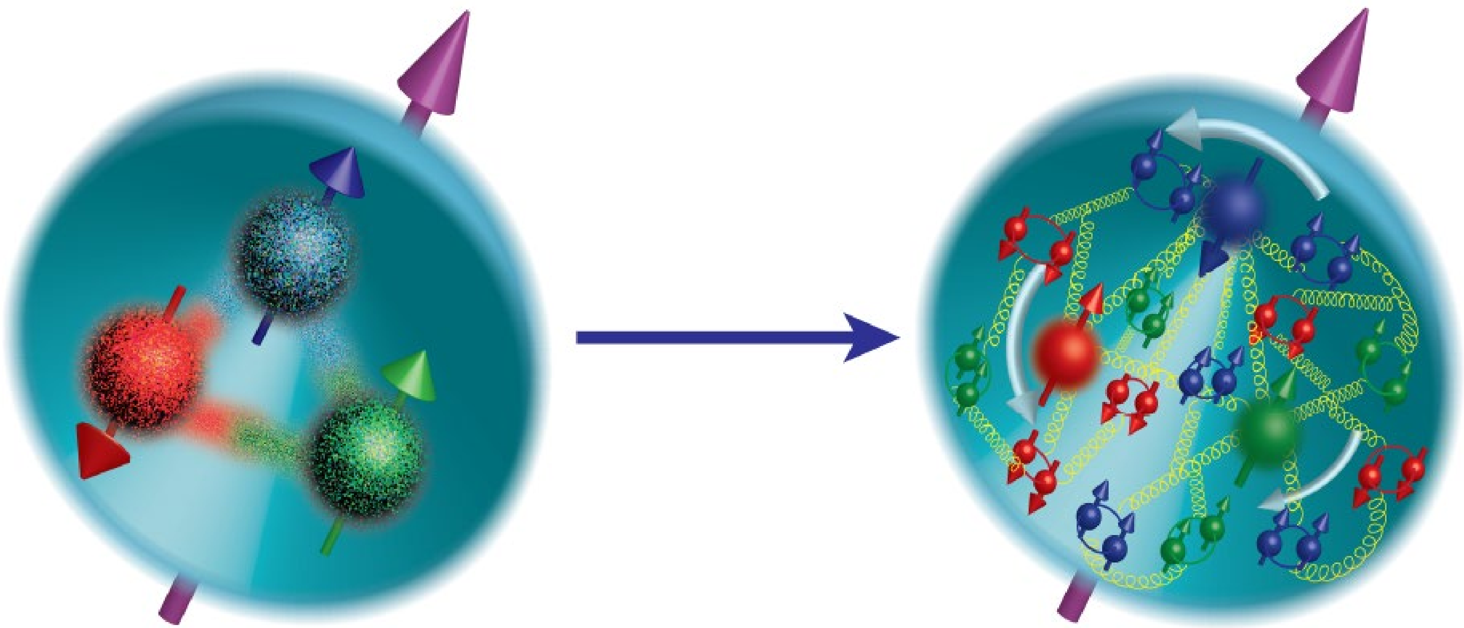
➢ Quark and gluon spin contribution $$\int_0^1 dx\, \Delta q(x)$$



Other sources of angular momentum?

Proton spin sum rule:

$$J = \frac{1}{2}\Delta\Sigma(Q^2) + \mathrm{L}_q(Q^2) + \Delta G(Q^2) + \mathrm{L}_g(Q^2) = \frac{1}{2}$$



Spin in asymptotic limit(Q^2)

- Scale evolution equation

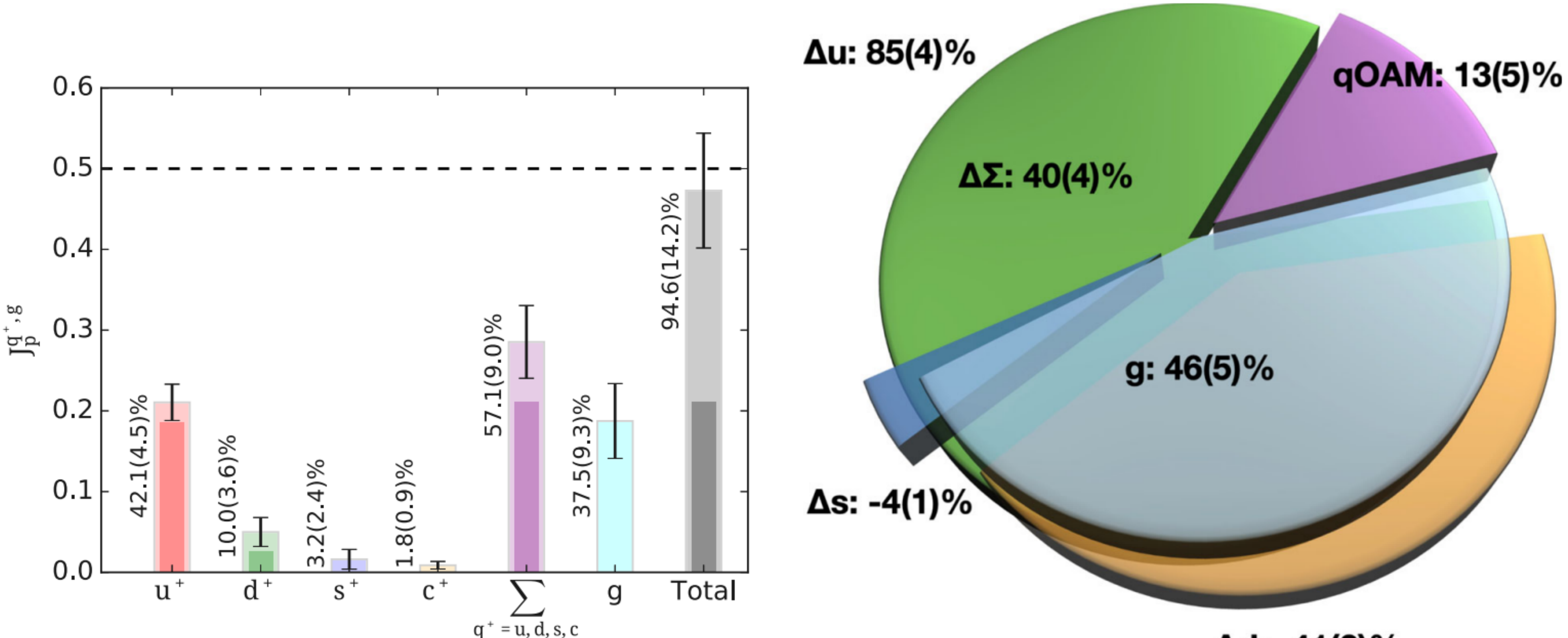
$$\frac{d}{d\,ln\,\mu^2}\begin{pmatrix} J_q(\mu^2) \\ J_g(\mu^2) \end{pmatrix} = \frac{\alpha_s}{2\pi \cdot 9}\begin{pmatrix} -16, 3n_f \\ 16, -3n_f \end{pmatrix}\begin{pmatrix} J_q(\mu^2) \\ J_g(\mu^2) \end{pmatrix}$$

- Asymptotic solution

$$J_q(\infty) = \frac{1}{2}\frac{3n_f}{16+3n_f}, J_g(\infty) = \frac{1}{2}\frac{16}{16+3n_f}$$

➢ Roughly half of the angular momentum is carried by gluons!

Lattice results



(a) (b)

Fig. 3 a Proton spin decomposition in terms of the angular momentum J_q for the u, d and s quarks and the glue angular momentum J_g in Ji's decomposition in the $n_f = 2+1+1$ calculation [74]. **b** Spin decomposition in terms of the quark spin $\Delta\Sigma$ and its flavor contributions Δu, Δd and Δs, the glue J_g, and the quark OAM for the $n_f = 2+1$ case [80]

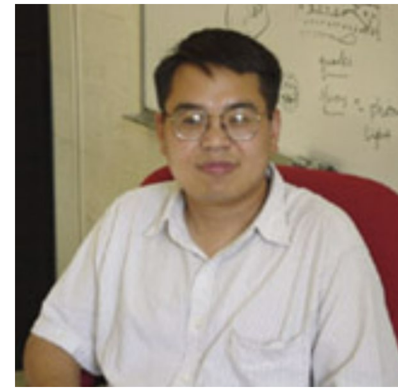
taken from Keh-Fei Liu's paper

How to measure orbital angular momentum?

Ji's sum rule

- The total angular momentum is related to the GPD:

$$J_q = \lim_{t\to 0}\frac{1}{2}\int dx x[H_q(x,t,\xi)+E_q(x,t,\xi)]$$

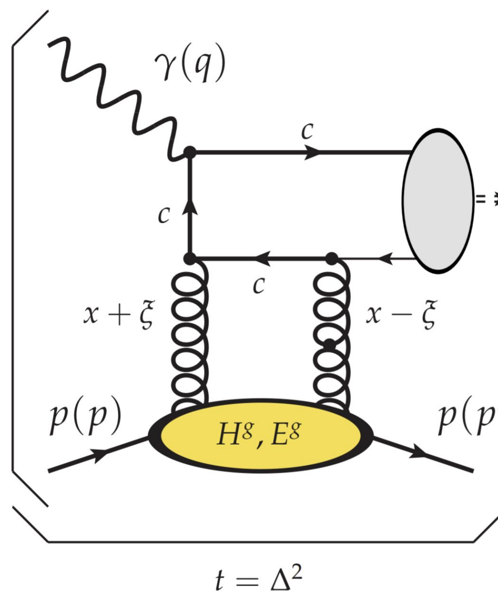


- an analogous relation holds for gluon. Ji, 1997

◆ SSA in exclusive process

$$A_N^{\gamma} = \frac{\frac{1}{2m_N}(1+\xi)|\Delta_T|\sin(\phi_{\vec{\Delta}})\,\Im(\mathcal{H}^g\mathcal{E}^{g\star})}{(1-\xi^2)|\mathcal{H}^g|^2+\frac{\xi^4}{1-\xi^2}|\mathcal{E}^g|^2-2\xi^2\Re(\mathcal{H}^g\mathcal{E}^{g\star})}$$

Koempel, Kroll, Metz, ZJ, 2012



Small x asymptotic behavior

- Never can reach x=0 at any experiment, how to extrapolate down to x=0

- Small x evolution equation for Eg(x)

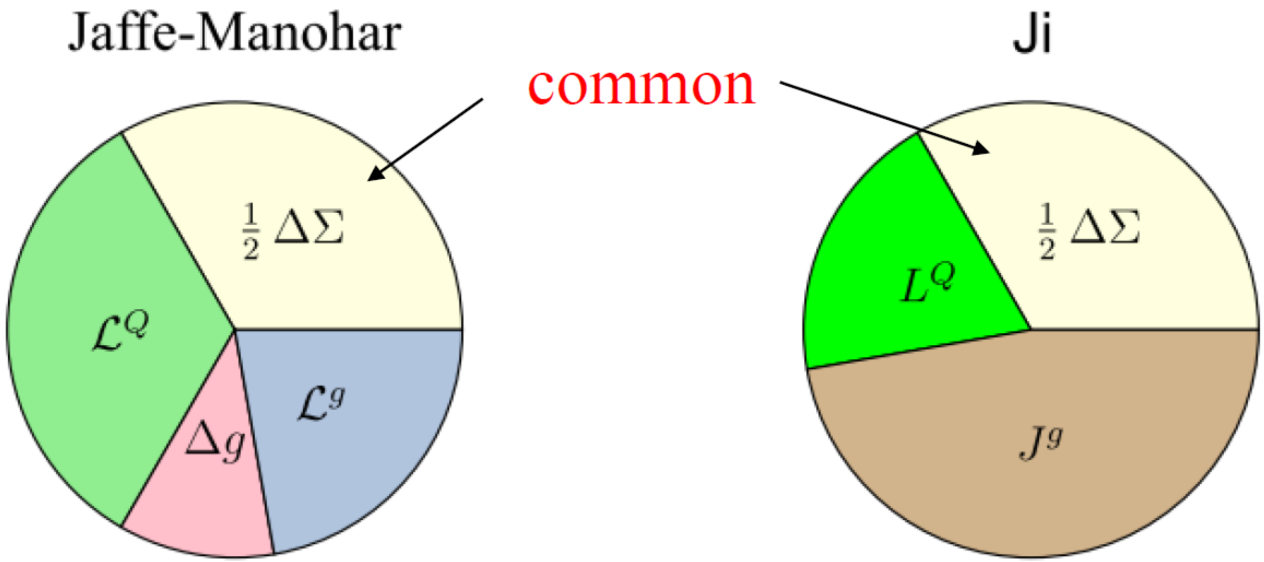
$$\partial_Y \mathcal{E}(k_\perp) = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2 k'_\perp}{(k_\perp - k'_\perp)^2} \left[\mathcal{E}(k'_\perp) - \frac{k_\perp^2}{2k'^2_\perp} \mathcal{E}(k_\perp) \right] - 4\pi^2 \alpha_s^2 \overline{\mathcal{F}}_{1,1}(k_\perp) \mathcal{E}(k_\perp)$$

Hatta, ZJ, 2022

Conclusion: Eg(x) rises as rapidly as the normal unpolarized gluon distribution!

$x^{-0.3}$

Two different spin decompositions



$$
\begin{aligned}
\boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma}\, \psi \, d^3x \\
&+ \int \psi^\dagger \boldsymbol{x} \times \frac{1}{i} \nabla \psi \, d^3x \\
&+ \int \boldsymbol{E}^a \times \boldsymbol{A}^a \, d^3x \\
&+ \int E^{ai} \boldsymbol{x} \times \nabla A^{ai} \, d^3x
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma}\, \psi \, d^3x \\
&+ \int \psi^\dagger \boldsymbol{x} \times \frac{1}{i} \boldsymbol{D} \psi \, d^3x \\
&+ \int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \, d^3x
\end{aligned}
$$

Gauge invariant extensions

- EM gauge potential is separated into the physical one and the pure gauge:

$$A^{\mu} = A^{\mu}_{phys} + A^{\mu}_{pure}$$

Not unique!

Chen, et.al. 2009

- Re-orgainze contributions:

$$\begin{aligned} J_{QCD} &= \int \psi^{\dagger} \frac{1}{2} \Sigma \psi \, d^3x + \int \psi^{\dagger} \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}_{pure}) \, \psi \, d^3x \\ &+ \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x + \int E^{aj} \, (\boldsymbol{x} \times \nabla) \, \boldsymbol{A}^{aj}_{phys} \, d^3x \\ &= \boldsymbol{S}'^q + \boldsymbol{L}'^q + \boldsymbol{S}'^g + \boldsymbol{L}'^g \end{aligned}$$

$$L_Q(\mathrm{JM}) \sim \psi^{\dagger} \boldsymbol{x} \times \boldsymbol{p} \, \psi \qquad L_Q(\mathrm{Ji}) \sim \psi^{\dagger} \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}) \, \psi$$

canonical OAM **dynamical OAM**

and commented on their advantages and disadvantages. There have been many very interesting theoretical developments, but we have concluded that they contain no new important physical implications, and for that reason we have concentrated on experimental tests and measurements only with regard to the canonical and Belinfante versions of the angular momentum.

From a review article by Leader and Lorce, 2013

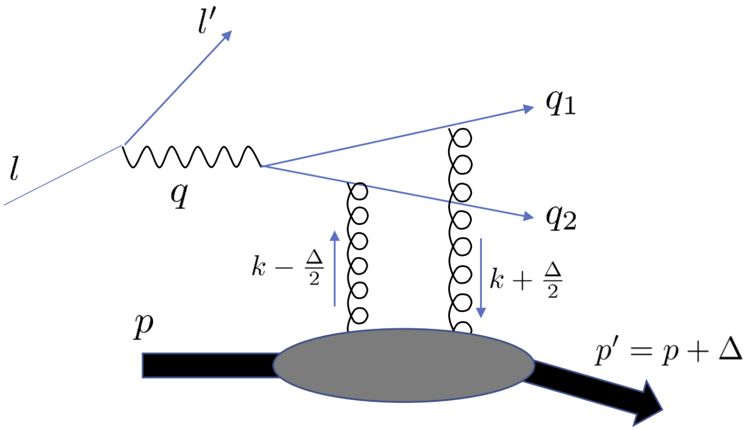
How to probe canonical OAM

- Canonical OAM is related the kt moment of a special parton Wigner distribution

$$L_{can} = -\int dx\, d^2 q_T \, \frac{\boldsymbol{q}_T^2}{M^2} \, F_{1,4}^q(x, 0, q_T^2, 0, 0)$$

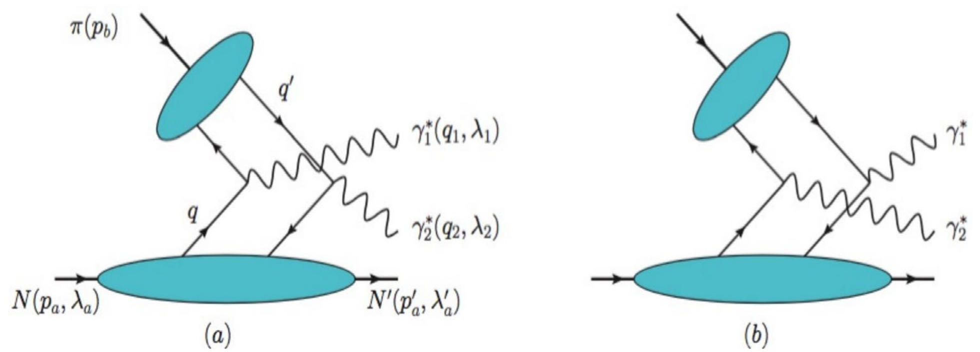
Hatta, 2012
Lorce-Pasquini, 2011

- Diffractive di-jet production



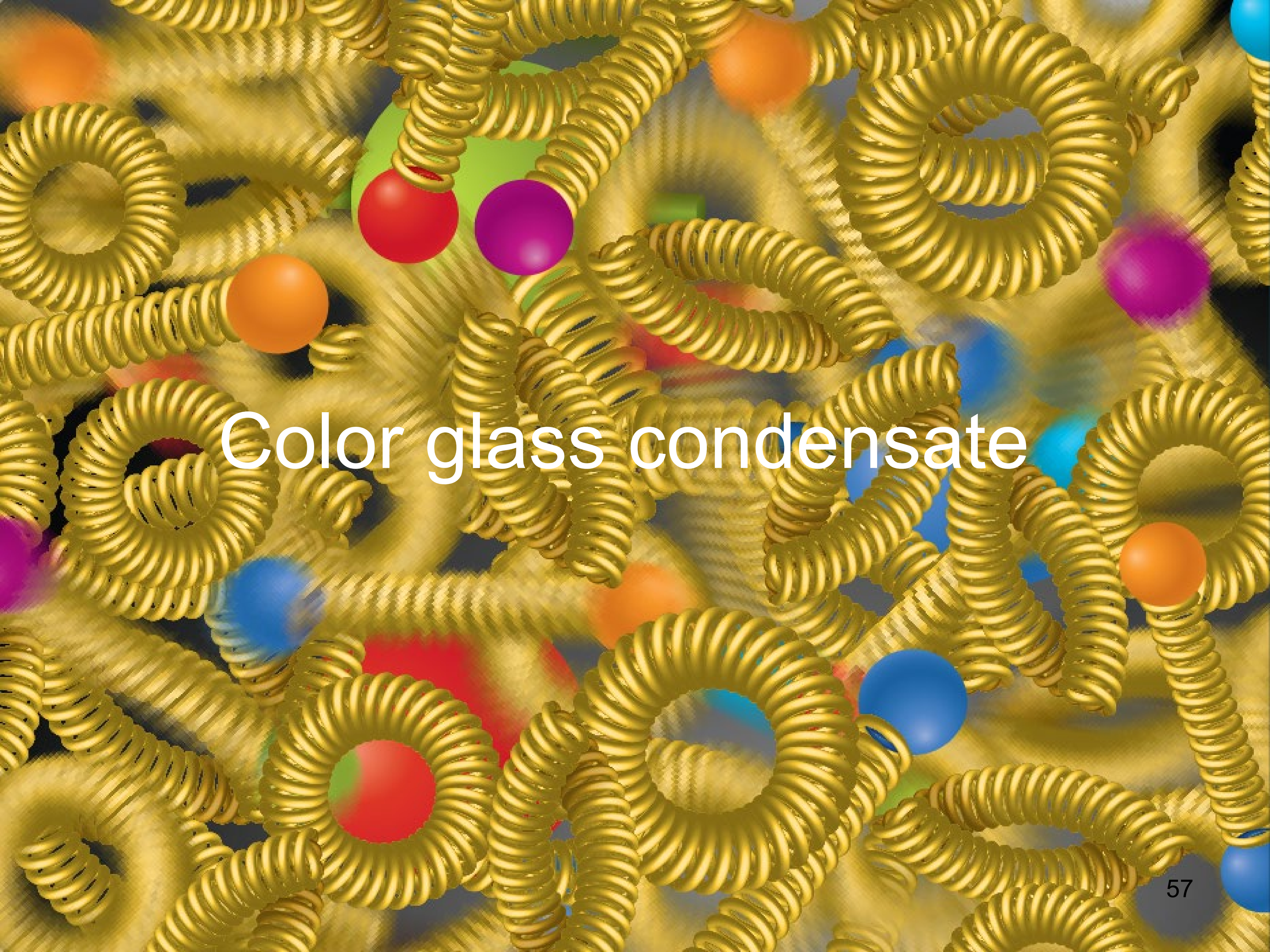
Hatta-Nakagawa-Yuan-Zhao-Xiao, 2017

- Exclusive double DY process



Bhattacharya, Metz, ZJ, 2017

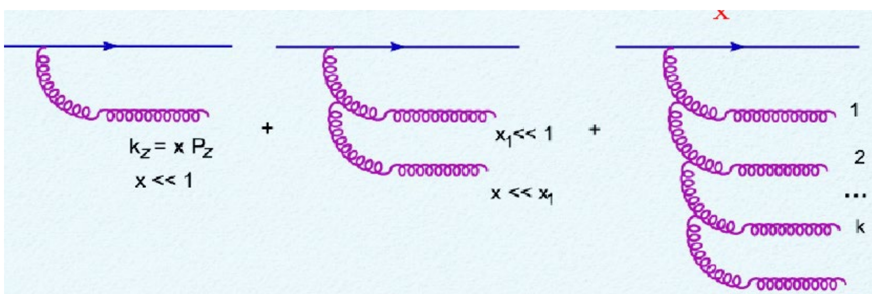
Color glass condensate

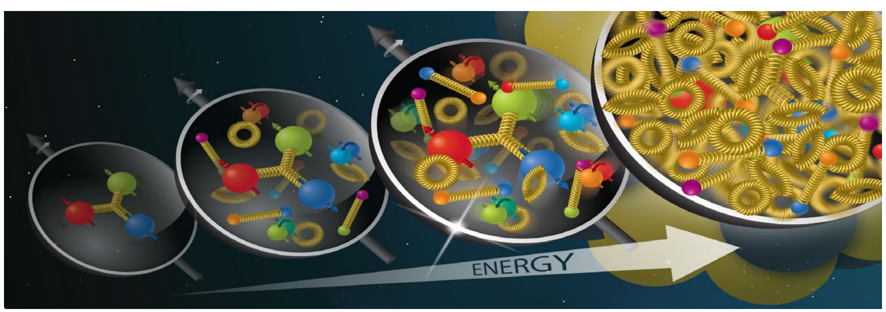


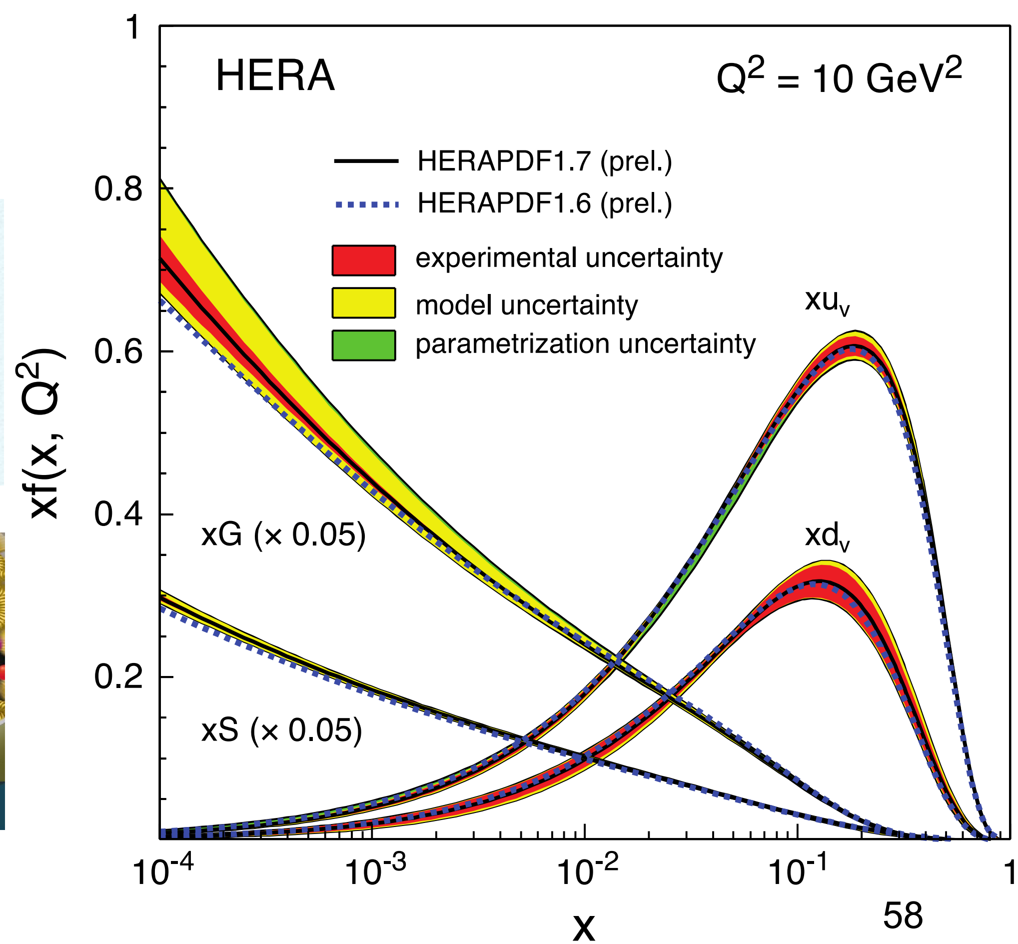
Glouns at small x

- DGLAP splitting function

$$P_{gg}(x) \sim \frac{1}{x} \text{ for } x \to 0$$

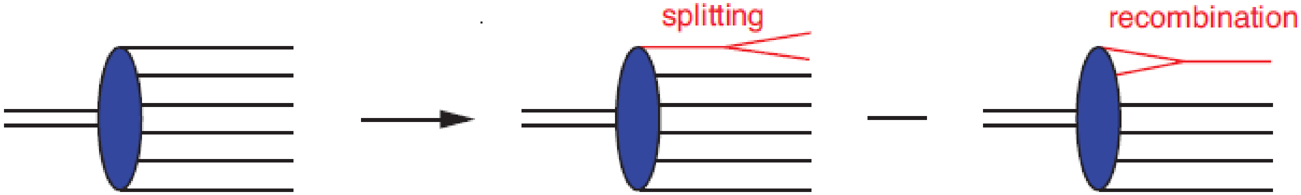




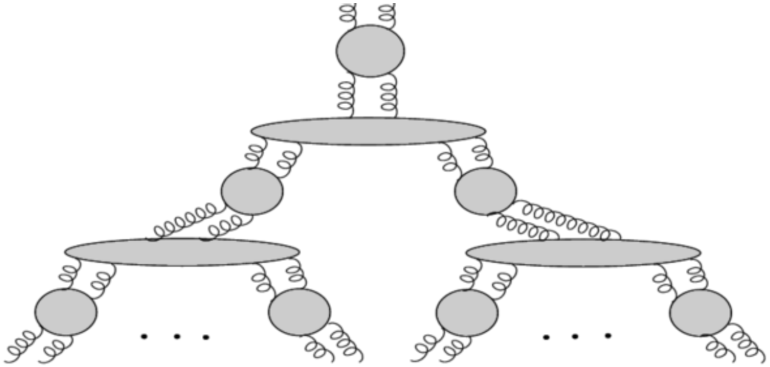


Small x evolution equations

- DGLAP equation $\ln\frac{Q^2}{\mu^2}$
- BFKL(CCFM) equations $\ln\frac{1}{x}$



- GLR-MQ equation



$$\frac{\partial N(\eta,k_\perp)}{\partial \eta}=\frac{\bar{\alpha}_s}{\pi}\left[\int\frac{d^2 l_\perp}{l_\perp^2}N(\eta,k_\perp+l_\perp)-\int_0^{k_\perp}\frac{d^2 l_\perp}{l_\perp^2}N(\eta,k_\perp)\right]-\bar{\alpha}_s N^2(\eta,k_\perp)$$

Small x evolution equations II

- Balitsky-Kovchegov(BK) equation:

Balitsky, 1996
Kovchegov, 1997

$$\partial_Y \mathcal{N}(\mathbf{x},\mathbf{y}) = \frac{\bar{\alpha}}{2\pi}\int d^2z \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\left[\mathcal{N}(\mathbf{x},\mathbf{z}) + \mathcal{N}(\mathbf{z},\mathbf{y}) - \mathcal{N}(\mathbf{x},\mathbf{y}) - \mathcal{N}(\mathbf{x},\mathbf{z})\mathcal{N}(\mathbf{z},\mathbf{y})\right]$$

- Dipole amplitude: $\frac{1}{N_c}\mathrm{Tr}U(b_\perp + r_\perp/2)U^\dagger(b_\perp - r_\perp/2)$
- Resuming gluon merge to all orders
- Meanfield approximation and large Nc approximation

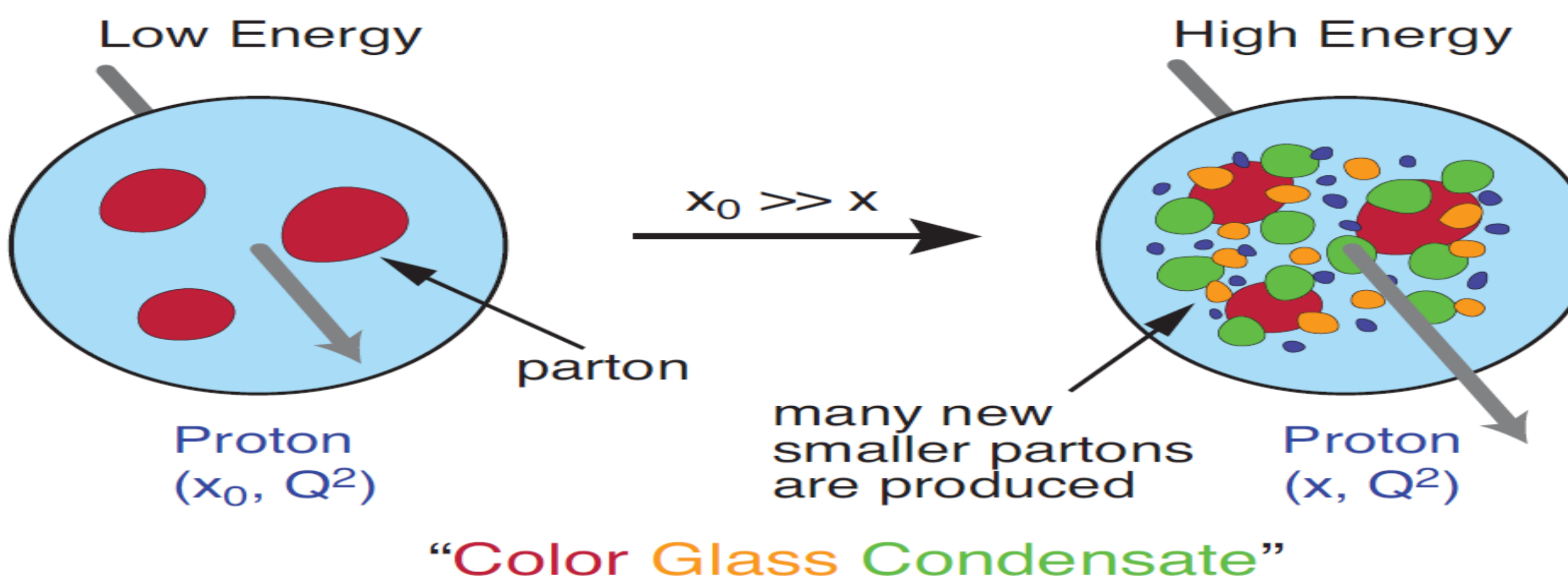
- The complete small x evolution equation:

  Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner(JIMWLK) equation

- Not a close equation, involve multiple point correlation functions

Saturation scale I

- Transverse size of gluon is reversely proportional to its kt
- First partons are produced with relatively large size(small kt).
- When some critical density is reached no more partons of given size can fit in the wave function.Smaller gluon(larger kt) is produced to fit the rest space.
- Average kt increase with increasing number density.

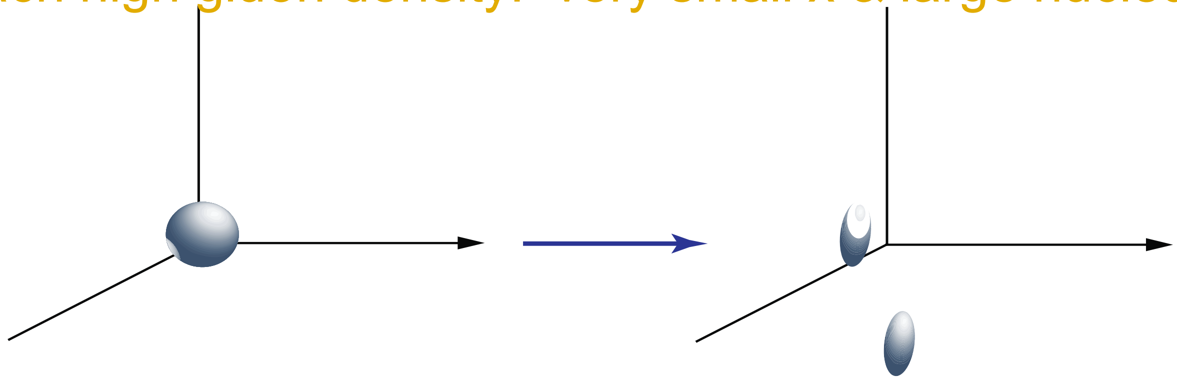


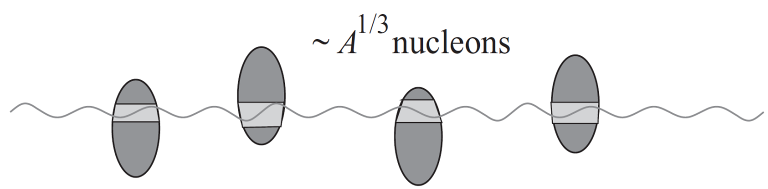
Saturation scale II

- A semi-hard scale Qs emerge when gluon density is very high

  Q_s^2 ∞ gluons per unit transverse area

- To reach high gluon density: very small x & large nucleus





Small x gluons(with long wave length) from different nucleons overlap with each other!

$$\Rightarrow \quad Q_s^2(x) \sim \left(\frac{A}{x}\right)^{1/3}$$

rcBK

Classical gluon fields

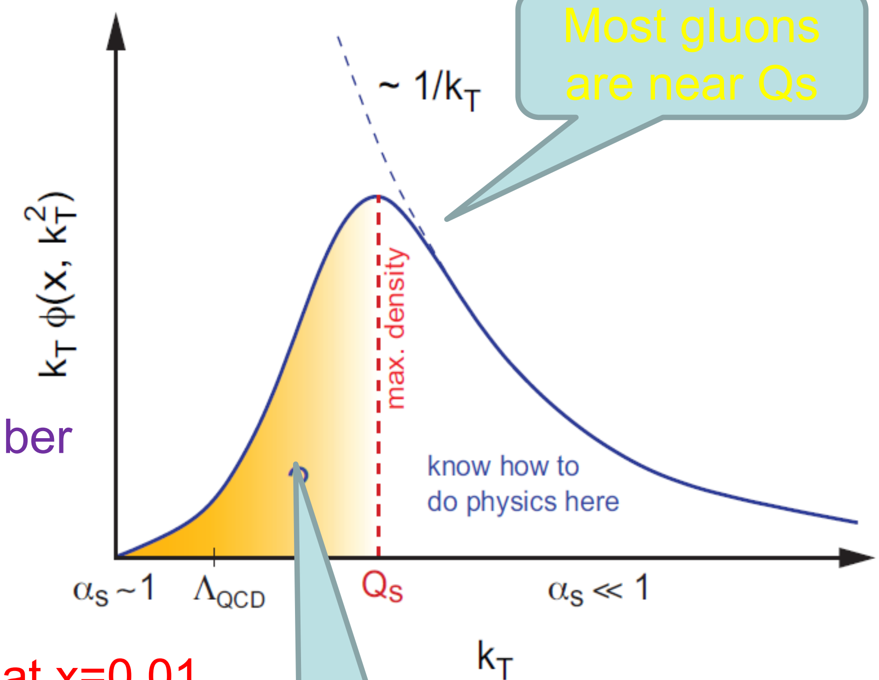
- For $Q_s >> \Lambda_{QCD}$, $\alpha_s(Q_s^2) \ll 1$

  Perturbative treatment is justified!

- For a large nucleus, high occupation number

  A semi-classical treatment is justified

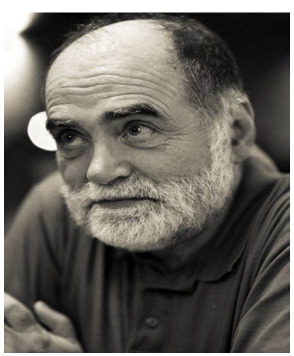
MV model & Glauble-Mueller model applied at x=0.01

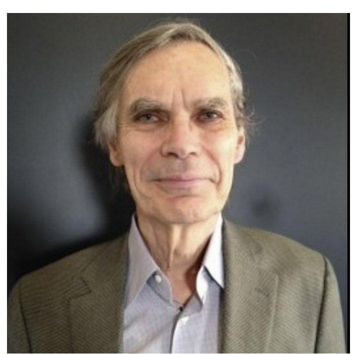


Gluon TMD at small x

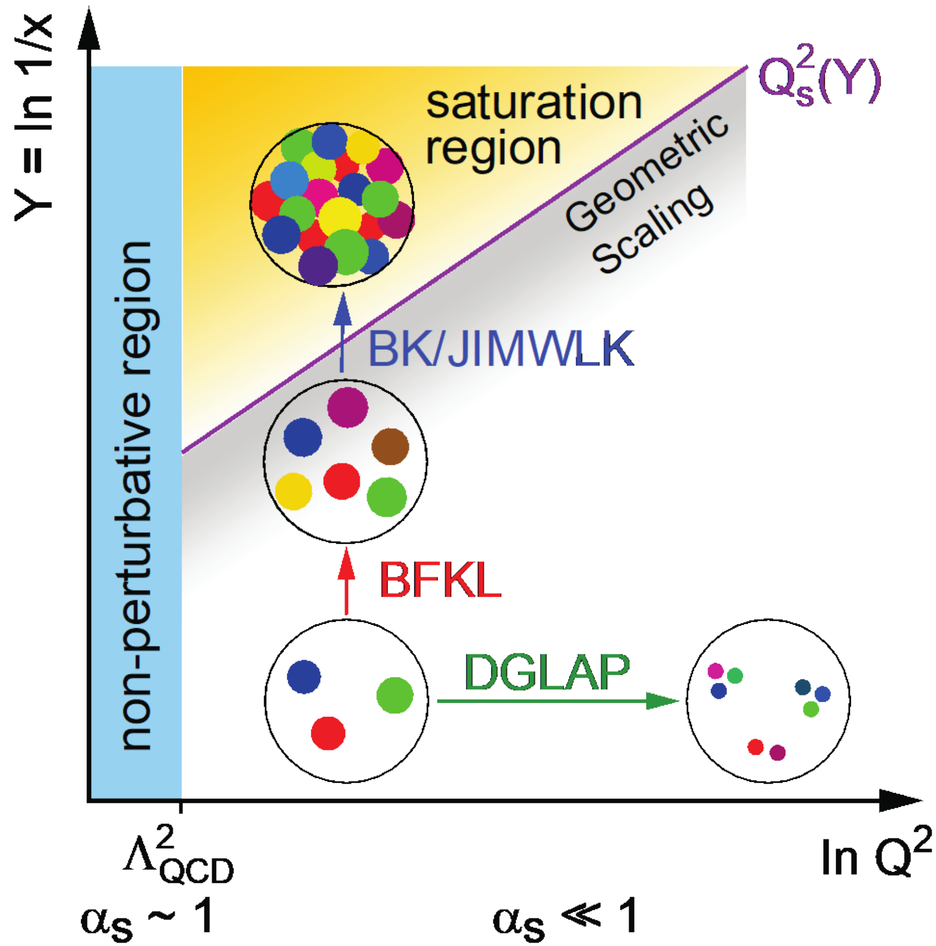
Most gluons are near Qs

Gluon distribution is suppressed at low kt due to saturation effect





Map of High Energy QCD

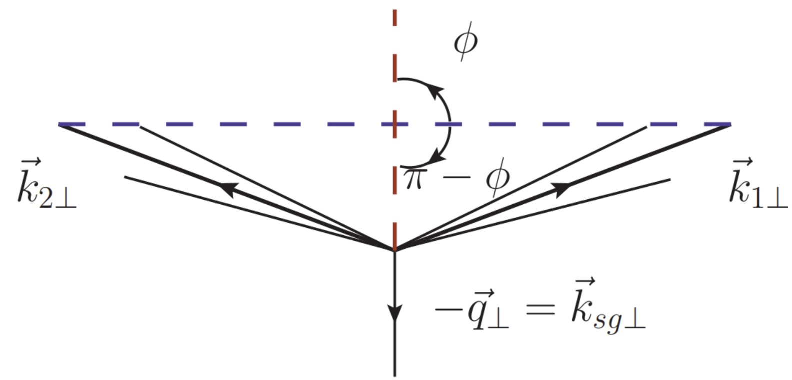


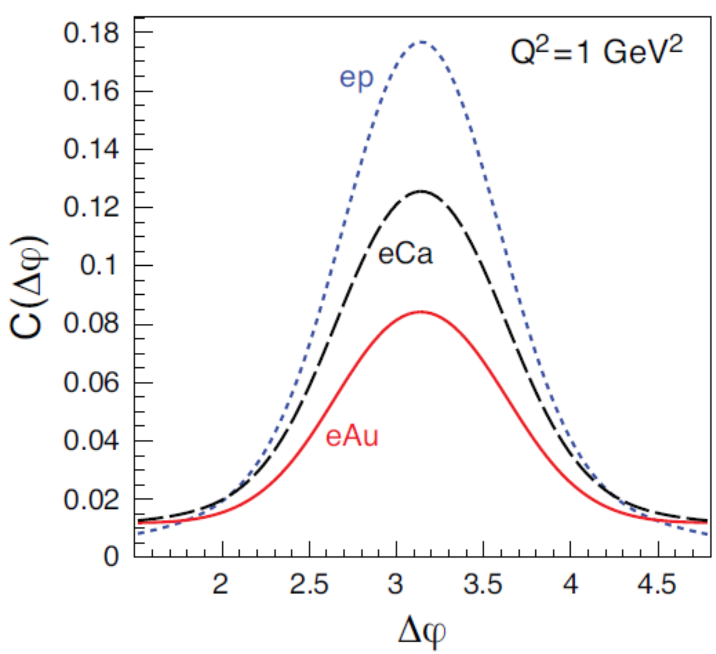
energy

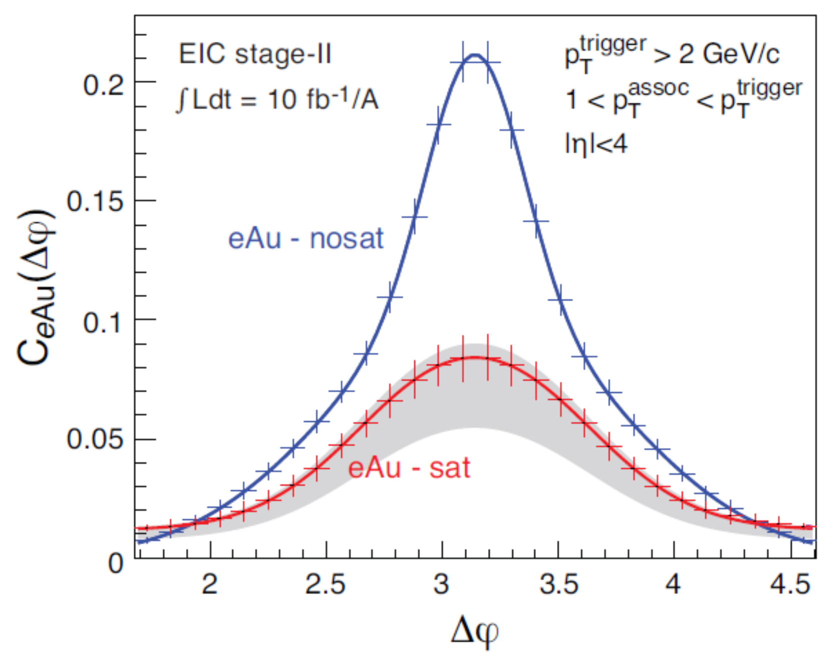
size of gluons

Di-hadron de-correlation

Back to back correlation

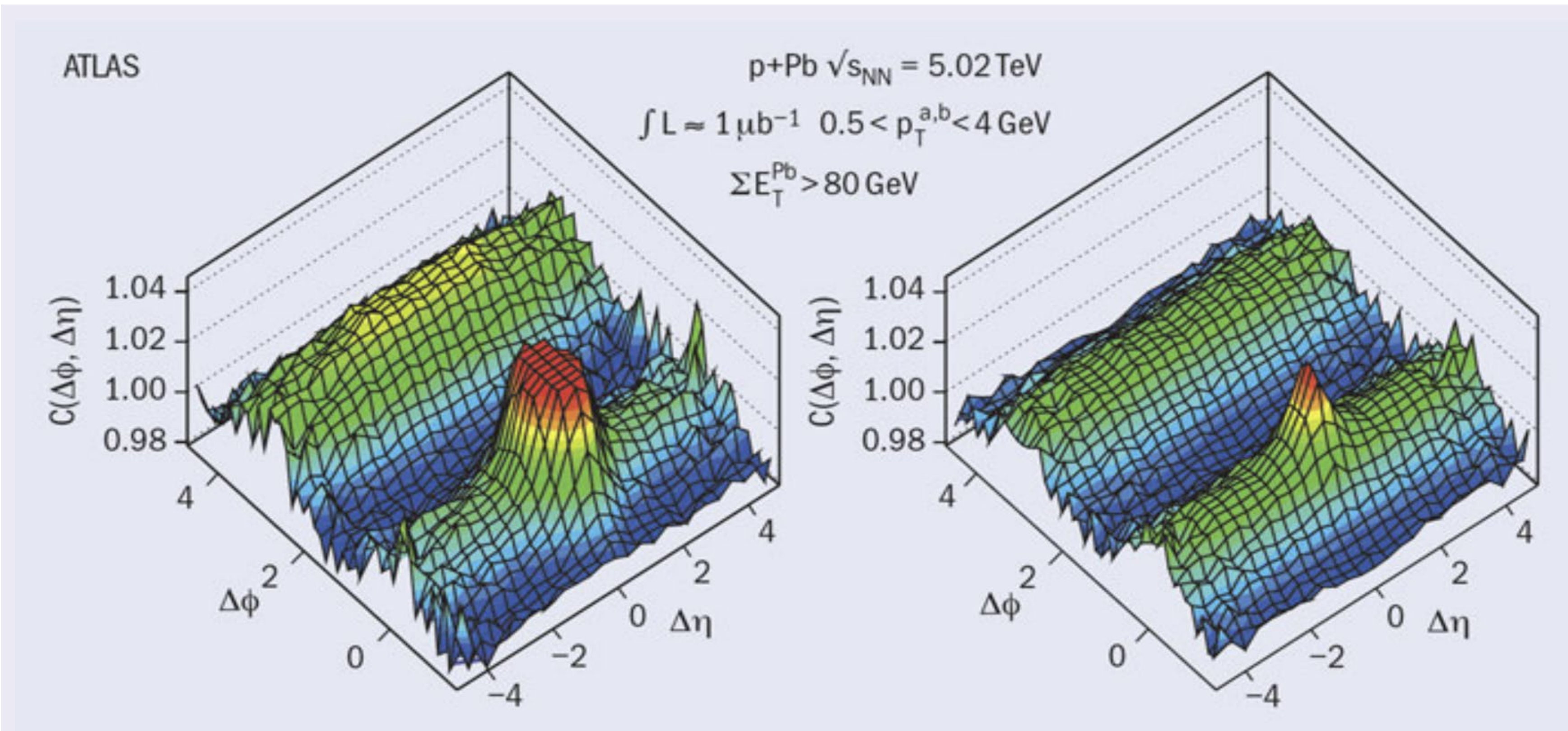




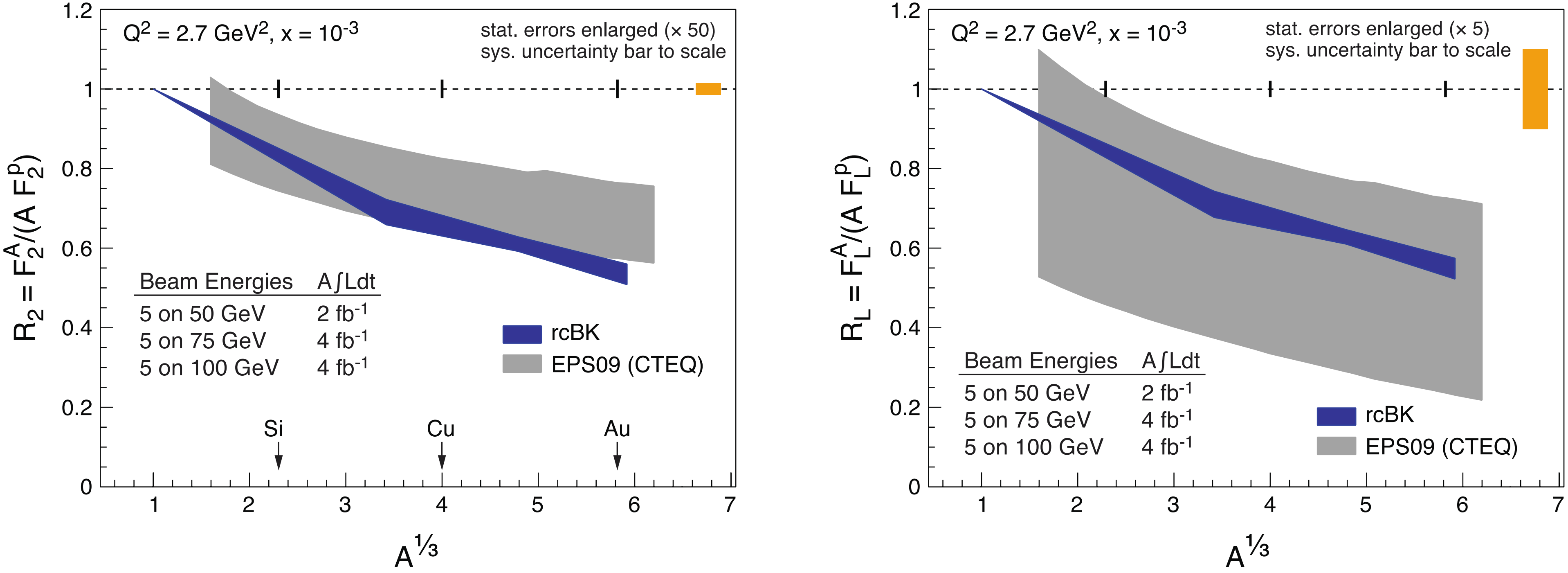


Ridge in heavy ion collisions

- Two particles emission are aligned!



Nuclear suppression



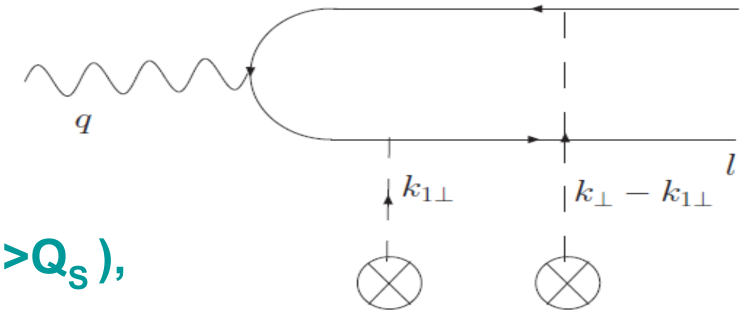
Can be tested at EIC!

Derive the TMD factorization formula I

Starting from the full CGC result,

Balitsky 1996
Gelis, Jalilian-Marian 2003

$$\mathcal{M} = \int d^2x_\perp d^2x_{1\perp} \int \frac{d^2k_{1\perp}}{(2\pi)^2} e^{ik_{1\perp}\cdot x_{1\perp}} e^{i(k_\perp - k_{1\perp})\cdot x_\perp} H(k, k_{1\perp}) \left[U(x_{1\perp})U^\dagger(x_\perp) - 1\right]$$



Taylor expanding the impact factor ($P_T >> Q_S$),

$$H(k, k_{1\perp}) = H(k=0, k_{1\perp}=0) + \frac{H(k_\perp, k_{1\perp})}{\partial k_\perp^i}\Big|_{k_\perp = k_{1\perp} = 0} k_\perp^i + \frac{H(k_\perp, k_{1\perp})}{\partial k_{1\perp}^i}\Big|_{k_\perp = k_{1\perp} = 0} k_{1\perp}^i + \ldots$$

Integrating out k_{1T},

$$\mathcal{M} \approx \int d^2x_\perp e^{ik_\perp \cdot x_\perp} \frac{H(k_\perp, k_{1\perp})}{\partial k_{1\perp}^i}\Big|_{k_\perp = 0, k_{1\perp} = 0} (-i) \left[\left(\partial^i U(x_\perp)\right) U^\dagger(x_\perp) - 1\right]$$

F. Dominguez, B-W. Xiao, F. Yuan 2011
F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan 2011

Derive the TMD factorization formula II

The cross section then reads,

$$d\sigma \quad \propto \quad \frac{H(k_\perp, k_{1\perp})}{\partial k^i_{1\perp}}\Big|_{k_\perp=0, k_{1\perp}=0} \frac{H^*(k_\perp, k'_{1\perp})}{\partial k'^{\,j}_{1\perp}}\Big|_{k_\perp=0, k'_{1\perp}=0}$$
$$\times(-1)\int d^2x_\perp d^2x'_\perp e^{ik_\perp\cdot(x_\perp - x'_\perp)}\langle \mathrm{Tr}[\partial^i U(x_\perp)]U^\dagger(x'_\perp)[\partial^j U(x'_\perp)]U^\dagger(x_\perp)\rangle$$

One can identify,

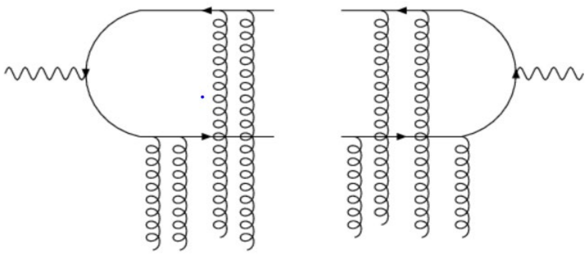
$$M^{ij}_{WW} = -\frac{2}{\alpha_s}\int \frac{d^2x_\perp}{(2\pi)^2}\frac{d^2x'_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot(\vec{x}_\perp - \vec{x}'_\perp)}\langle \mathrm{Tr}[\partial^i U(x_\perp)]U^\dagger(x'_\perp)[\partial^j U(x'_\perp)]U^\dagger(x_\perp)\rangle_x$$
$$= \frac{\delta^{ij}_\perp}{2} x f^g_{1,WW}(x, k_\perp) + \left(\frac{1}{2}\hat{k}^i_\perp \hat{k}^j_\perp - \frac{1}{4}\delta^{ij}_\perp\right) x h^{\perp g}_{1,WW}(x, k_\perp)\,.$$

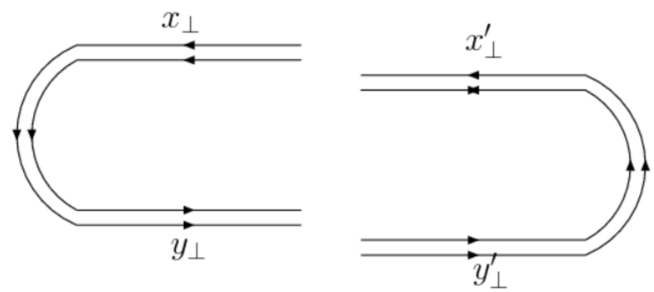
Mulders, Rodrigues, 2001; F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan 2011

CGC	TMD
Derivative of impact factor in k_T	Hard part
Derivative of Wilson lines in x_T	Gluon TMDs

TMD & CGC

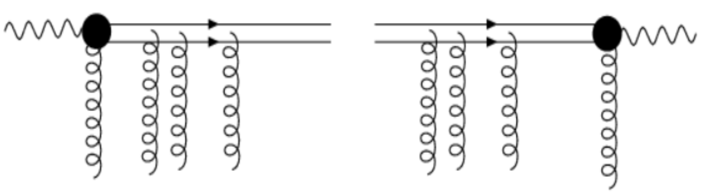
Low jet $P_T \leq K_T$ (only CGC applicable)



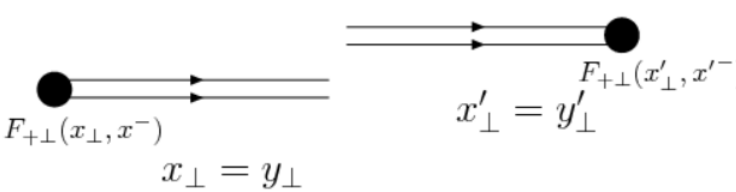


four Wilson lines

High jet $P_T >> K_T$ (both CGC and TMD applicable)



Gluons can not resolve the internal structure of the color dipole system.



Collapse to two semi-finite Wilson lines

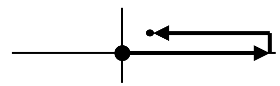
F. Dominguez, B-W. Xiao, F. Yuan 2011
F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan 2011

Gluon TMDs in the MV model

The unpolarized gluon TMDs have been evaluated in the MV model.

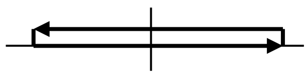
The linearly polarized gluon TMDs in the MV model, **Metz & ZJ, 2011**

Weizsäcker-Williams(WW) distribution:



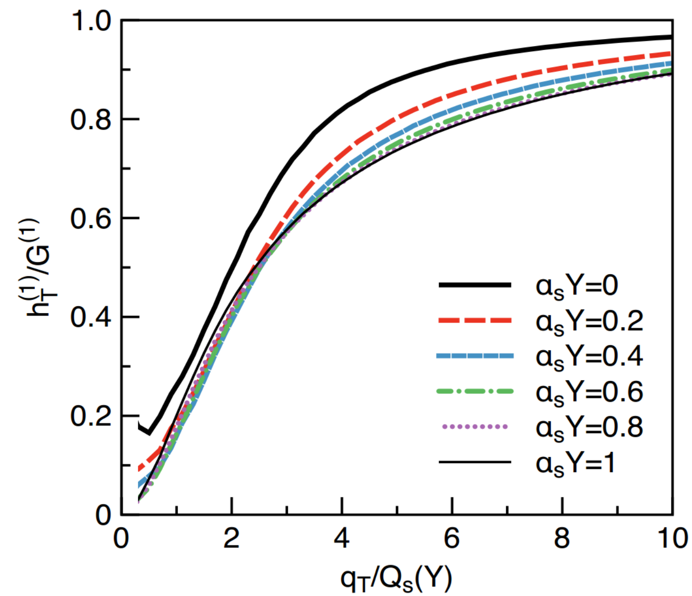
$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{8\pi^3} S_\perp \int d\xi_\perp \frac{K_2(k_\perp \xi_\perp)}{\frac{1}{4\mu_A}\xi_\perp Q_s^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$

Dipole distribution:



$$xh_{1,DP}^{\perp g}(x,k_\perp) = xG_{DP}^{g}(x,k_\perp)$$

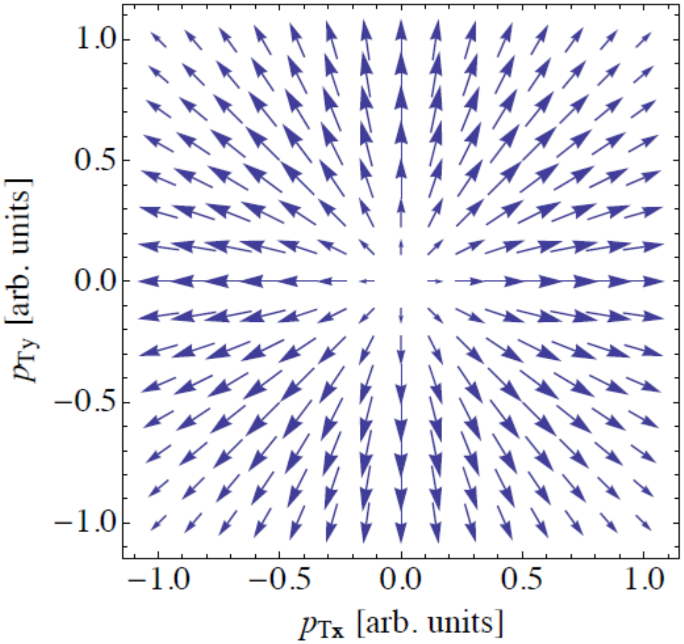
WW type linearly polarized gluon TMD is suppressed in the dense medium region.



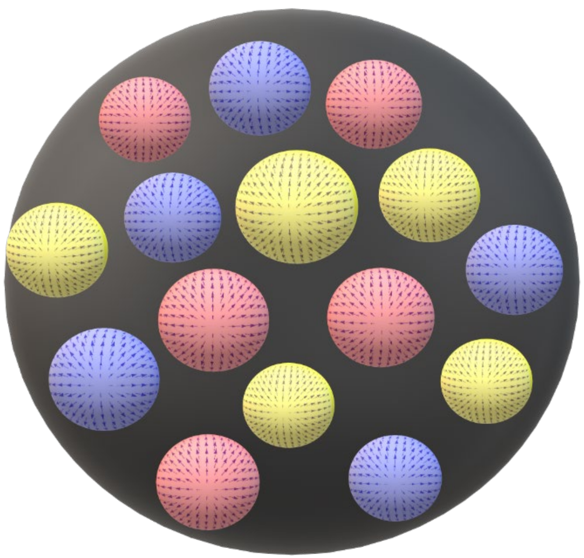
Dumitru, Lappi, Skokov; 2015

Gluon polarization inside a nucleon/nuclei

Transverse momentum space

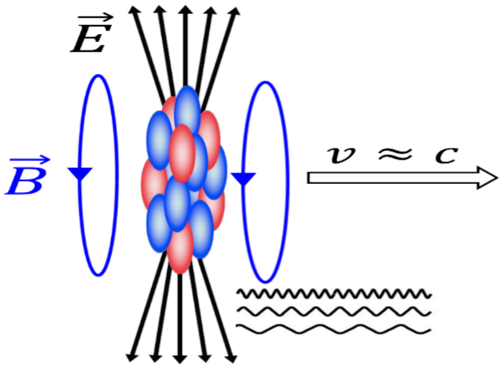


Transverse coordinate space

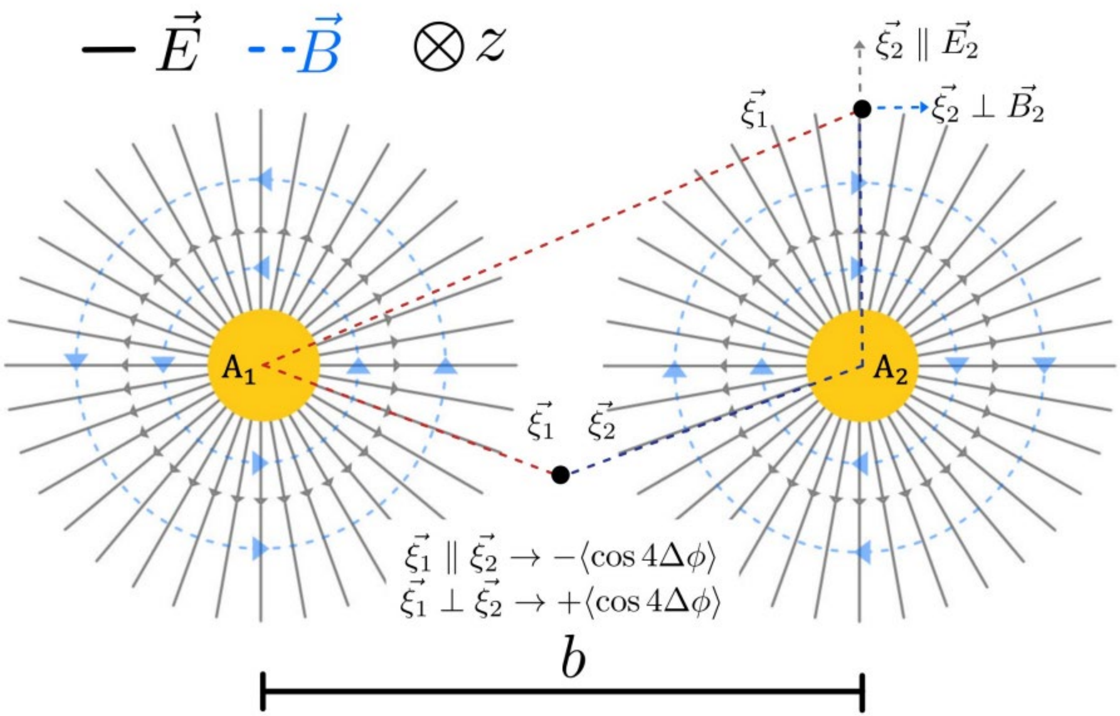


A. Metz, ZJ; 2011

Why study QED in UPC



L. Cong, Y.j. Zhou, ZJ 2019,2020



J.D. Brandenburg, CFNS workshop 2021.04

Coherent photons are linearly polarized!

Cos 4ϕ asymmetry in EM dilepton production

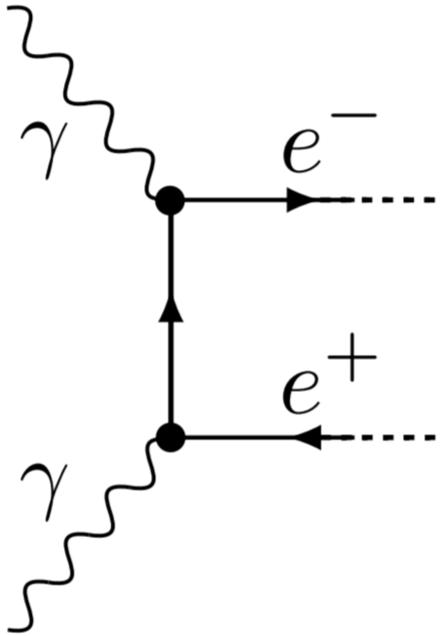
$$\gamma(x_1 P + k_{1\perp}) + \gamma(x_2 \bar{P} + k_{2\perp}) \to l^+(p_1) + l^-(p_2)$$

$$\langle \cos(4\phi) \rangle$$

$$\phi = P_\perp \wedge q_\perp$$

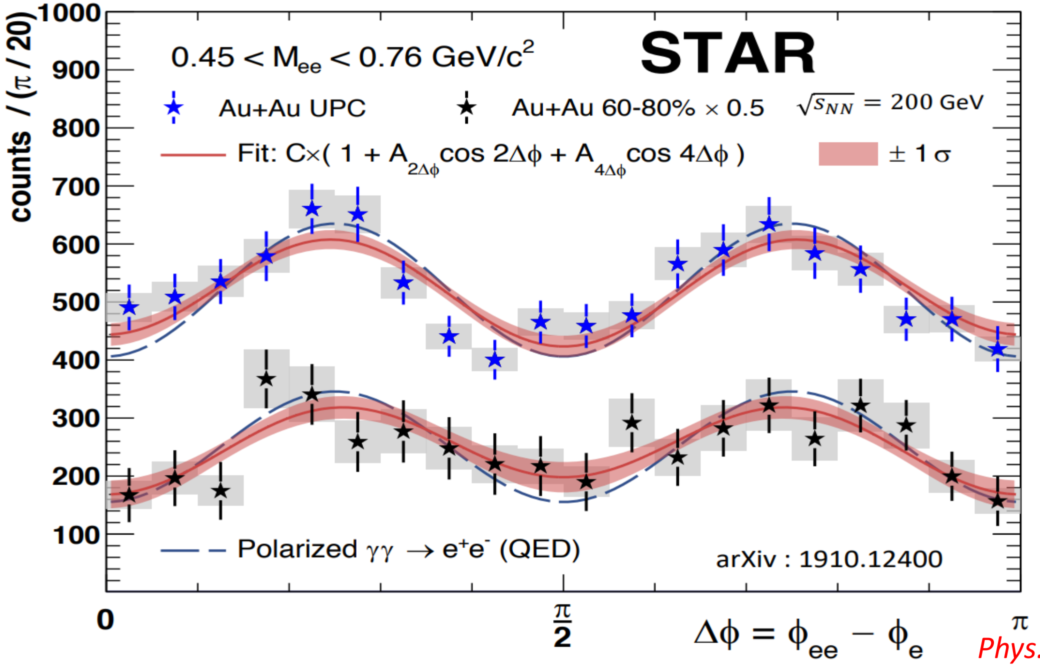
$$P_\perp \equiv (p_{1\perp} - p_{2\perp})/2$$

$$q_\perp \equiv p_{1\perp} + p_{2\perp}$$



correlation limit: $P_\perp \gg q_\perp$

$\tilde{b}_\perp$ dependent $\langle\cos(4\phi)\rangle$ V.S. STAR experiment



Phys.Rev.Lett. 127 (2021) 5, 052302

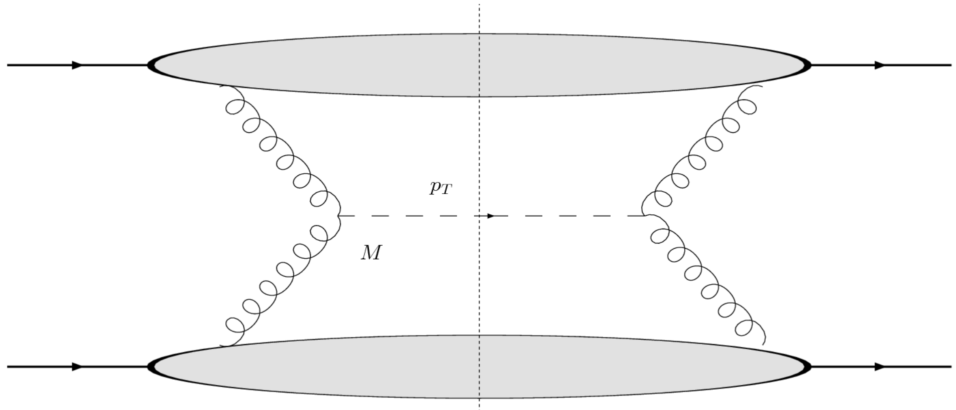
$0.45GeV^2<Q^2<0.76GeV^2$
P_t>200MeV, $|y|<1$, q_t<100MeV

C. Li, JZ and Y. Zhou, 2020

	Measured	QED calculation
Tagged UPC	16.8%±2.5%	16.5%
60%-80%	27%±6%	34.5%

Joint kt & small x resummation

Gluon initiated Drell-Yan process



- $M^2 >> p_T^2$, TMD factorization, $\ln \frac{M^2}{p_T^2}$ resummed by Collins-Soper equation

1982-1983, Collins, Soper

- $S >> M^2$, Kt factorization, $\ln \frac{S}{M^2}$ resummed by BFKL equation

1991, Catani, Ciafaloni and Hautmann
1991, Collins and R. K. Ellis

The overlap region $S >> M^2 >> p_T^2$

An explicit NLO cross section calculation shows that both the large logarithm appear.

2013, Mueller, Xiao, Yuan

Such joint resummation has been also disscussed in other literatures.

2015, Balitsky and Tarasov; 2015 Marzani

In collinear calculation $\ln\frac{M^2}{\mu^2}$ absorbed into PDF.

One natural question:

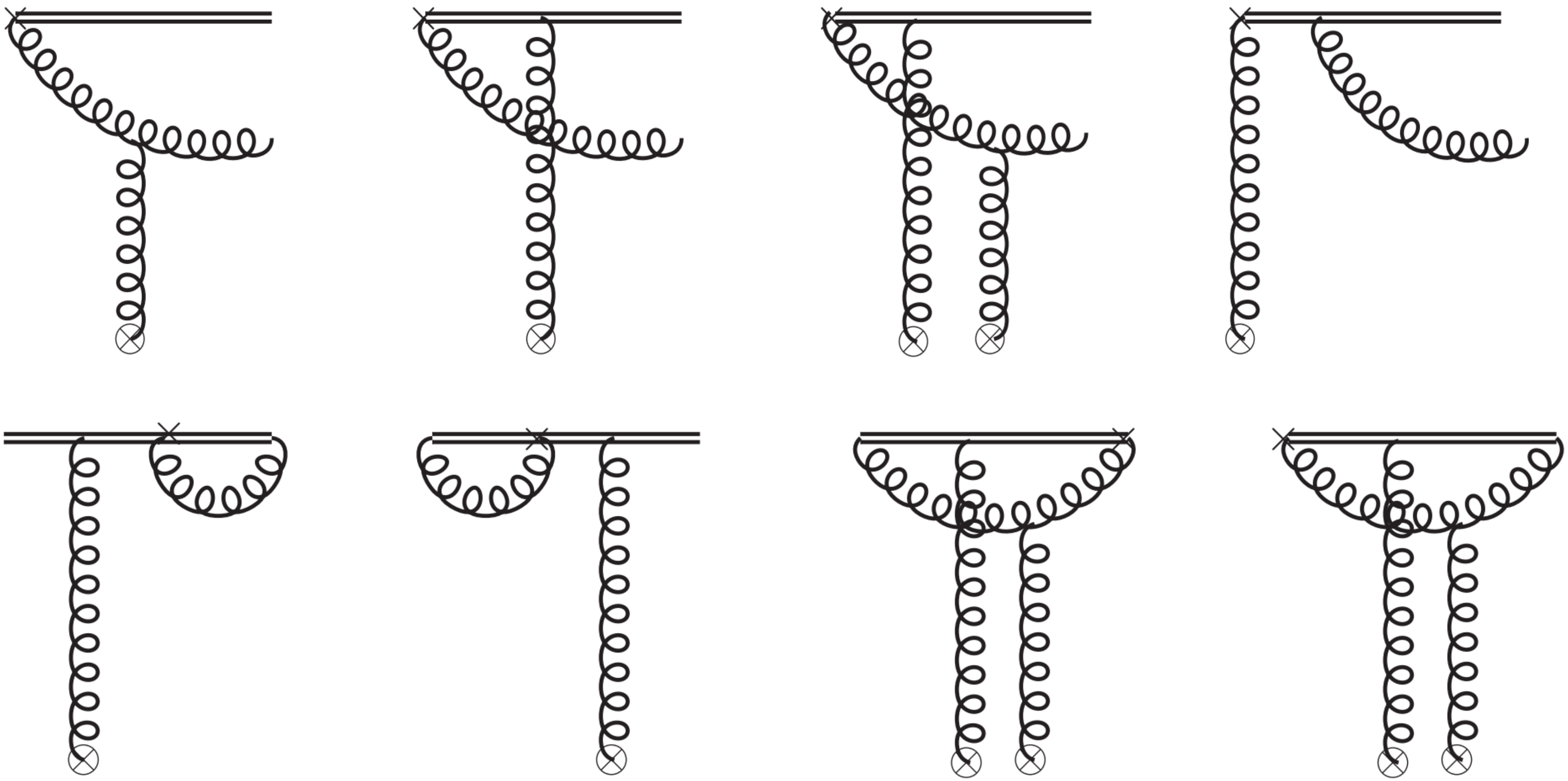
$$\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{L}_\xi^\dagger \mathcal{L}_0 F^{+i}(0)|P\rangle$$

Does it accommodate both type large logarithms?

$$\ln\frac{S}{M^2} \qquad\qquad \ln\frac{M^2}{p_T^2}$$

Small x TMDs in CGC at NLO

Sample diagrams



Xiao, Yuan and ZJ, 2017.

Collinear approach vs CGC I

- TMDs in collinear approach
  collinear divergence DGLAP
- TMDs in CGC,
  rapidity divergence BK or JIMWLK

Collins-Soper light cone divergence appears in both collinear approach and CGC calculation

Match small x TMDs onto two point functions instead of PDFs.

Xiao, Yuan and ZJ, 2017.

Collinear approach vs CGC II

$$\tilde{f}_g^{(sub.)}(x, r_\perp, \zeta_c{}') = e^{-S_{pert}^g(Q, r_\perp)} \sum_i C_{g/i}(\mu_r/\mu) \otimes f_i(x, \mu)$$

Sudakov factor

Hard coefficient

Colliner PDF

$$xG^{(1)}(x, k_\perp, \zeta_c) = -\frac{2}{\alpha_S} \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{ik_\perp \cdot r_\perp} \mathcal{H}^{WW}(\alpha_s(Q)) e^{-\mathcal{S}_{sud}(Q^2, r_\perp^2)} \mathcal{F}_{Y=\ln 1/x}^{WW}(x_\perp, y_\perp$$

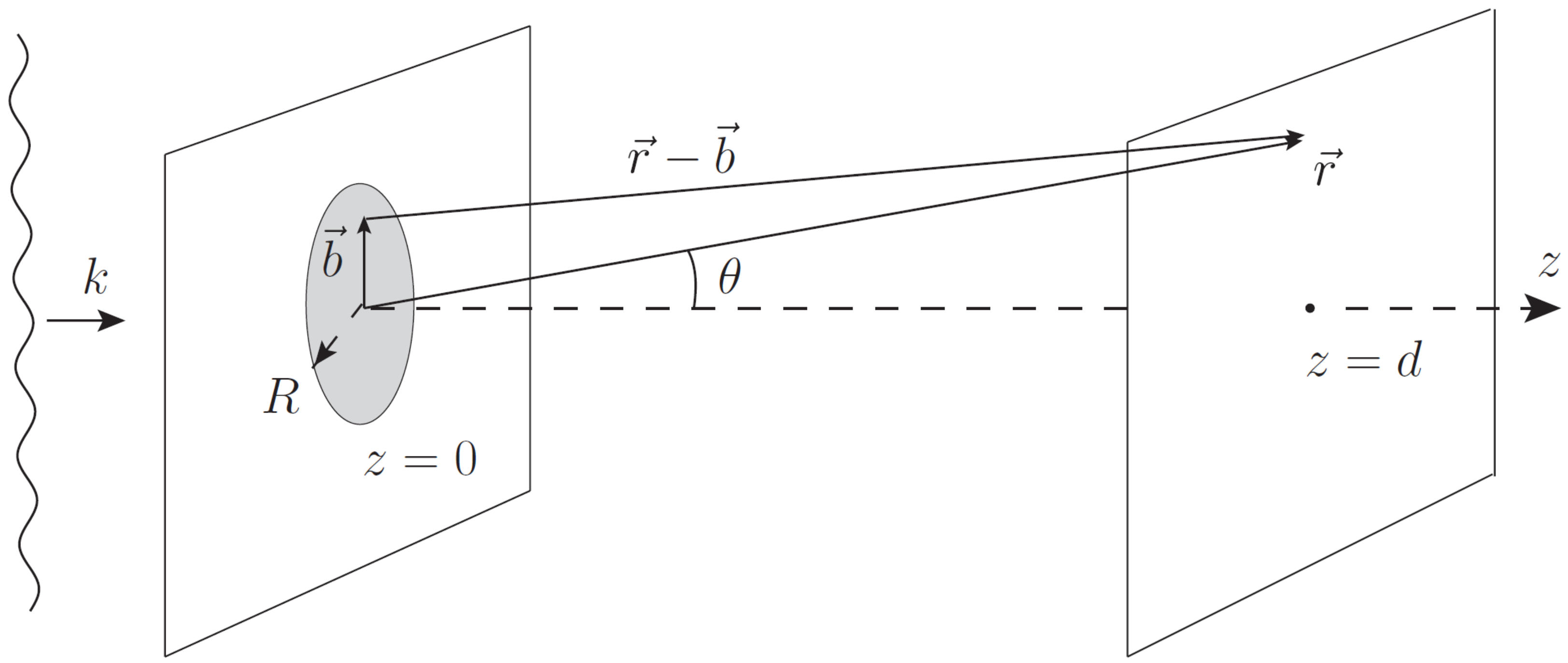
Hard coefficient

Sudakov factor

Two point function

Two step evolution: S → M^2 → k_T^2

Diffraction in optics

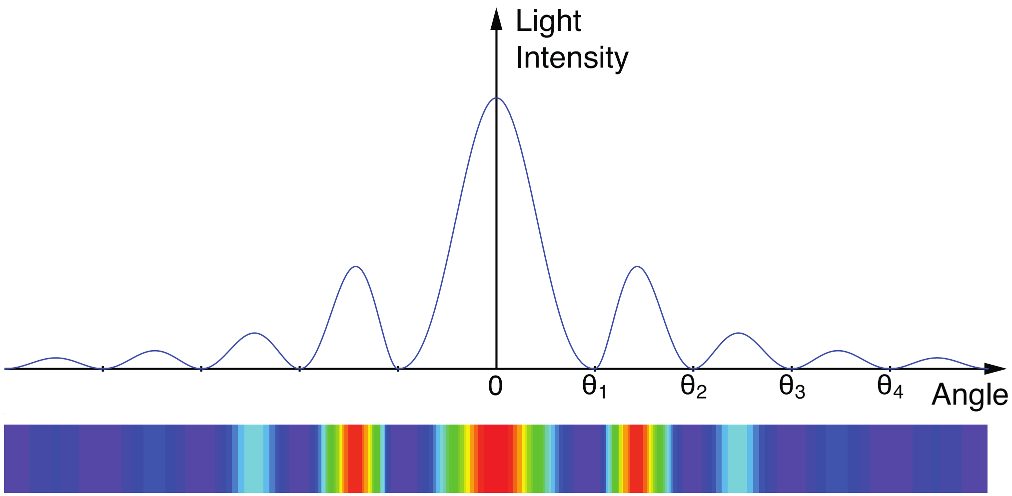


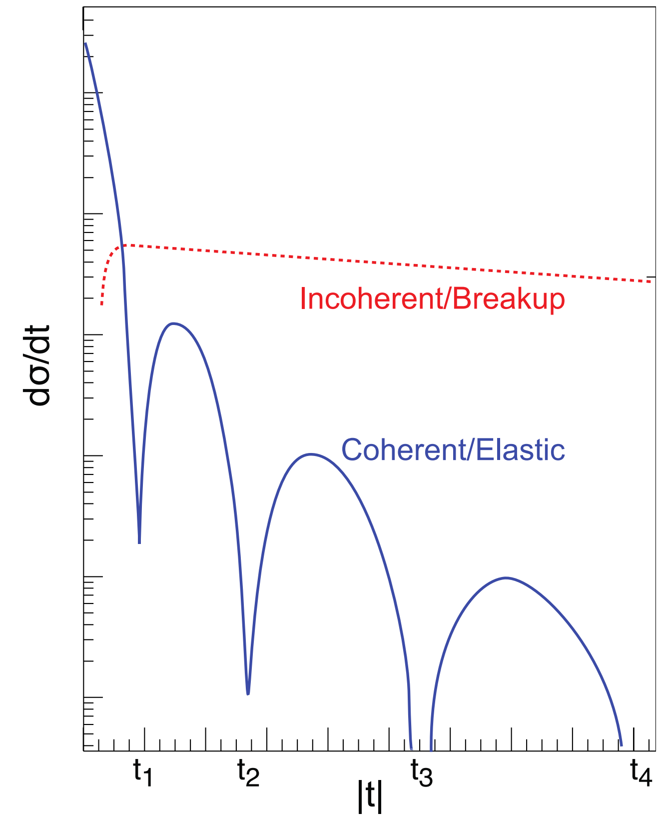
Taken from Yuri's book

- Reconstruct the size R of the obstacle and the optical "blackness" of the obstacle from the diffractive pattern.

Optical Analogy

Diffraction in high energy scattering is not very different from diffraction in optics: both have diffractive maxima and minima:

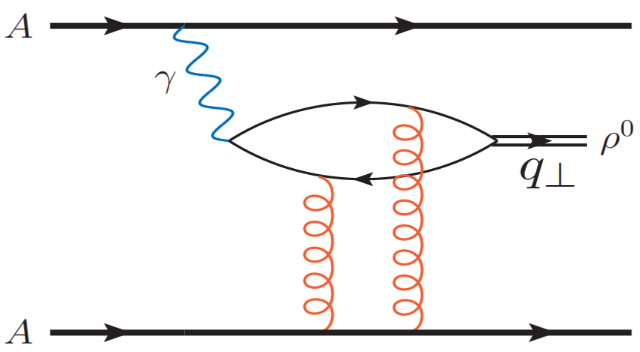




Coherent: target stays intact;
Incoherent: target nucleus breaks up, but nucleons are intact.

Exclusive VM Production off a large nucleus

- Study nuclear geometry
- Test saturation



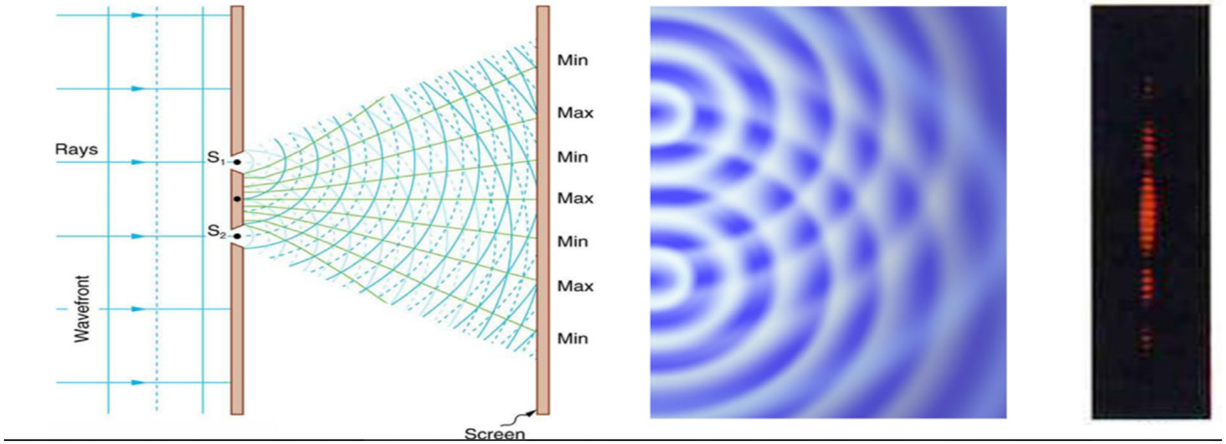
◆ In the black disk limit(very large nucleus & high energy limit), diffraction (elastic scattering) becomes a half of the total cross section

$$\frac{\sigma_{el}^{q\bar{q}A}}{\sigma_{tot}^{q\bar{q}A}} = \frac{\int d^2b\, N^2}{2\int d^2b\, N} \longrightarrow \frac{1}{2}$$

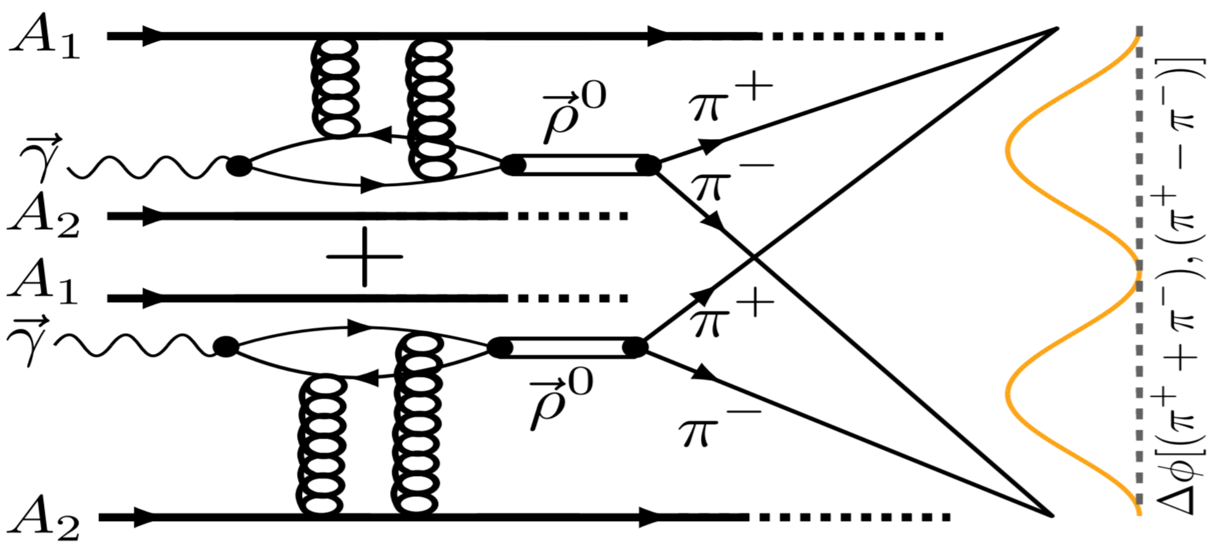
Inelastic scattering cross section=elastic scattering cross section=πR^2

- At EIC energy, in eA collisions, makes up roughly 15% total CS

- Young's double-slit experiment



- double-slit experiment in UPCs



Courtesy of Daniel Brandenburg

Joint $\tilde{b}_\perp$ & $q_\perp$ dependent cross section III

- Full cross section: $k_\perp + \Delta_\perp = k'_\perp + \Delta'_\perp$

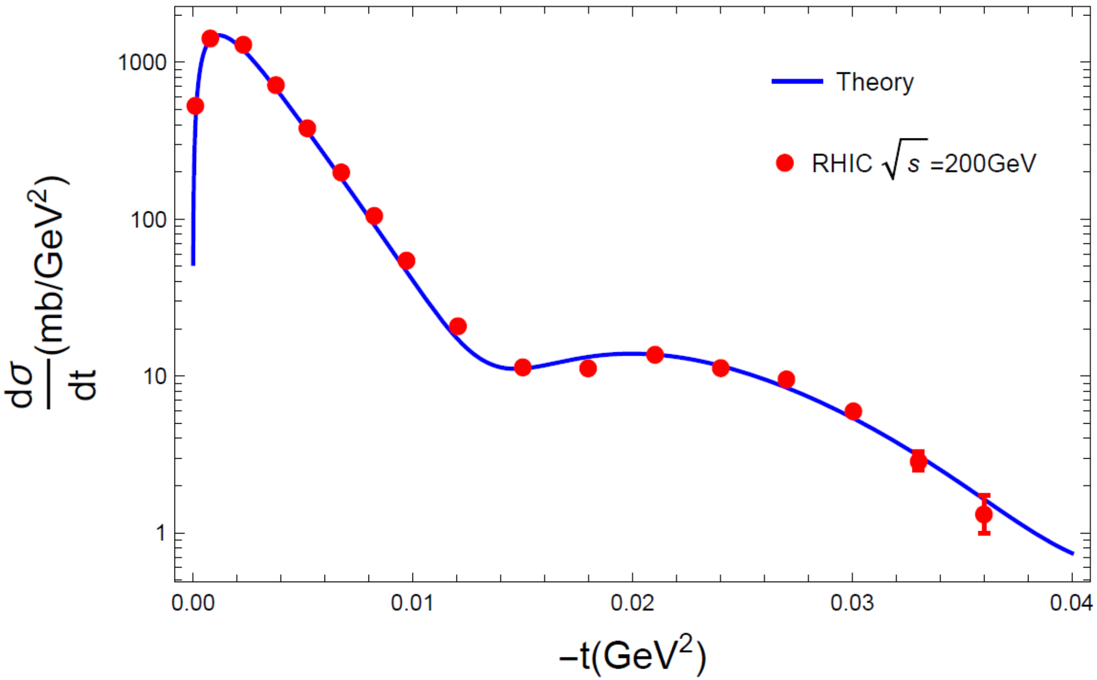
$$\frac{d\sigma}{d^2q_\perp dY d^2\tilde{b}_\perp} = \frac{1}{(2\pi)^4}\int d^2\Delta_\perp d^2k_\perp d^2k'_\perp \delta^2(k_\perp + \Delta_\perp - q_\perp)(\epsilon_\perp^{V*}\cdot\hat{k}_\perp)(\epsilon_\perp^{V}\cdot\hat{k}'_\perp)\Big\{\int d^2b_\perp$$
$$\times e^{i\tilde{b}_\perp\cdot(k'_\perp - k_\perp)}\left[T_A(b_\perp)\mathcal{A}_{in}(Y,\Delta_\perp)\mathcal{A}^*_{in}(Y,\Delta'_\perp)\mathcal{F}(Y,k_\perp)\mathcal{F}(Y,k'_\perp) + (A\leftrightarrow B)\right]$$
$$+\left[e^{i\tilde{b}_\perp\cdot(k'_\perp - k_\perp)}\mathcal{A}_{co}(Y,\Delta_\perp)\mathcal{A}^*_{co}(Y,\Delta'_\perp)\mathcal{F}(Y,k_\perp)\mathcal{F}(Y,k'_\perp)\right]$$
$$+\left[e^{i\tilde{b}_\perp\cdot(\Delta'_\perp - \Delta_\perp)}\mathcal{A}_{co}(-Y,\Delta_\perp)\mathcal{A}^*_{co}(-Y,\Delta'_\perp)\mathcal{F}(-Y,k_\perp)\mathcal{F}(-Y,k'_\perp)\right]$$
$$+\left[e^{i\tilde{b}_\perp\cdot(\Delta'_\perp - k_\perp)}\mathcal{A}_{co}(Y,\Delta_\perp)\mathcal{A}^*_{co}(-Y,\Delta'_\perp)\mathcal{F}(Y,k_\perp)\mathcal{F}(-Y,k'_\perp)\right]$$
$$+\left[e^{i\tilde{b}_\perp\cdot(k'_\perp - \Delta_\perp)}\mathcal{A}_{co}(-Y,\Delta_\perp)\mathcal{A}^*_{co}(Y,\Delta'_\perp)\mathcal{F}(-Y,k_\perp)\mathcal{F}(Y,k'_\perp)\right]\Big\}, \qquad (2.14)$$

H.X. Xing, Z. Zhang, ZJ, Y.J. Zhou, 2020

- EM potential: $\mathcal{F}(Y,k_\perp) = \frac{Z\sqrt{\alpha_e}}{\pi}|k_\perp|\frac{F(k_\perp^2 + x^2M_p^2)}{(k_\perp^2 + x^2M_p^2)}$

VM diffractive pattern

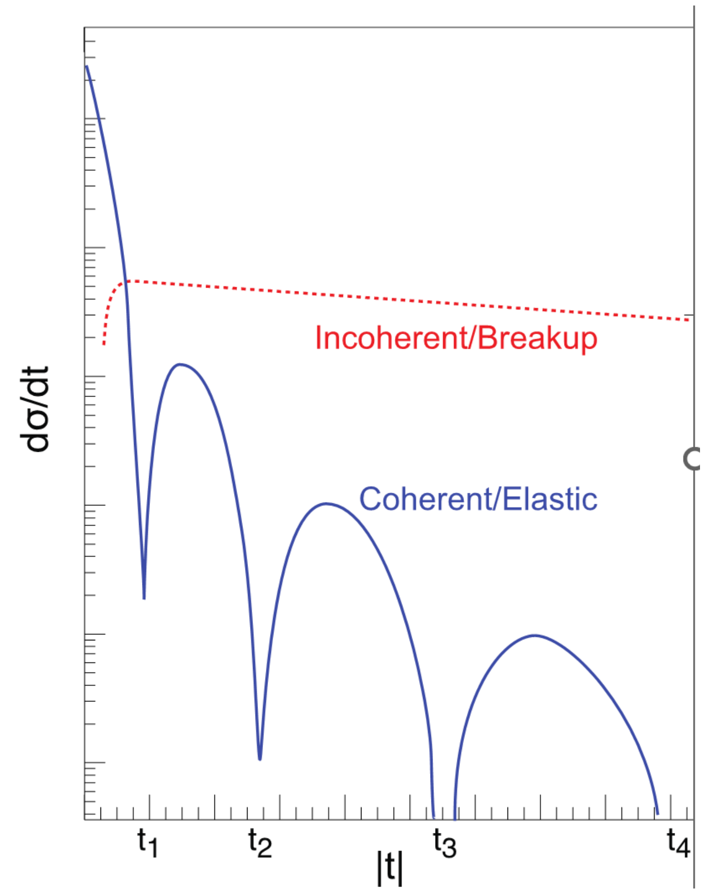
Diffractive VM production in UPC



Xing, Zhang, ZJ, Zhou 2020

- Double slit interference effect
- Smearing caused by finite photon kt

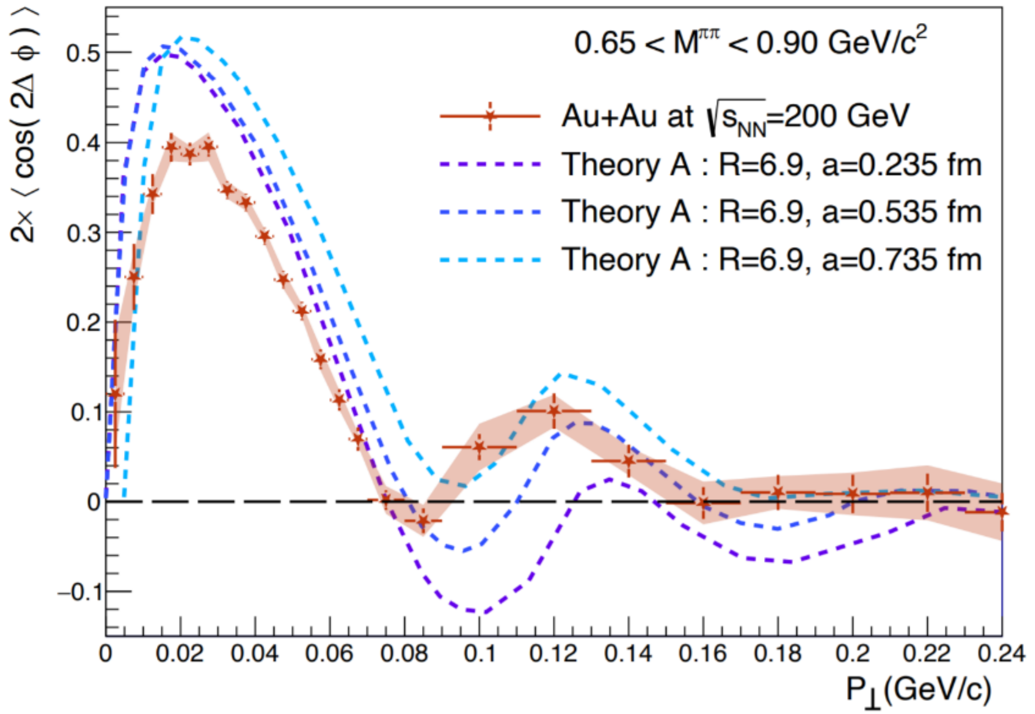
Diffractive VM production in eA



- One slit interference

ρ^0 production in UPCs

Cos2ϕ azimuthal asymmetry



Xing, Zhang, ZJ, Zhou 2020, Zha, Brandenburg, Ruan,Tang,2021

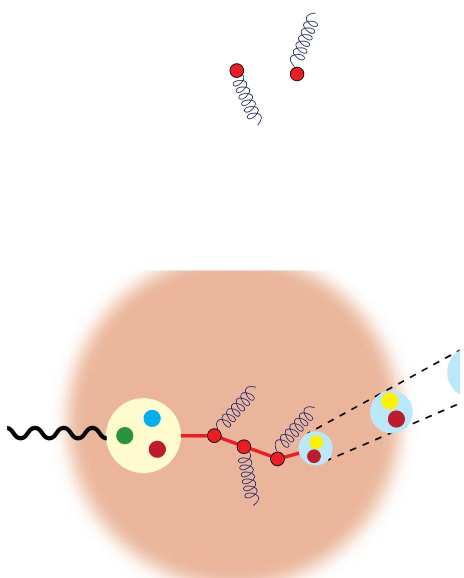
Gold target	Skin depth	Strong interaction radius
Standard value	0.54fm	6.38fm
Fitted to STAR data	0.64fm	6.9fm

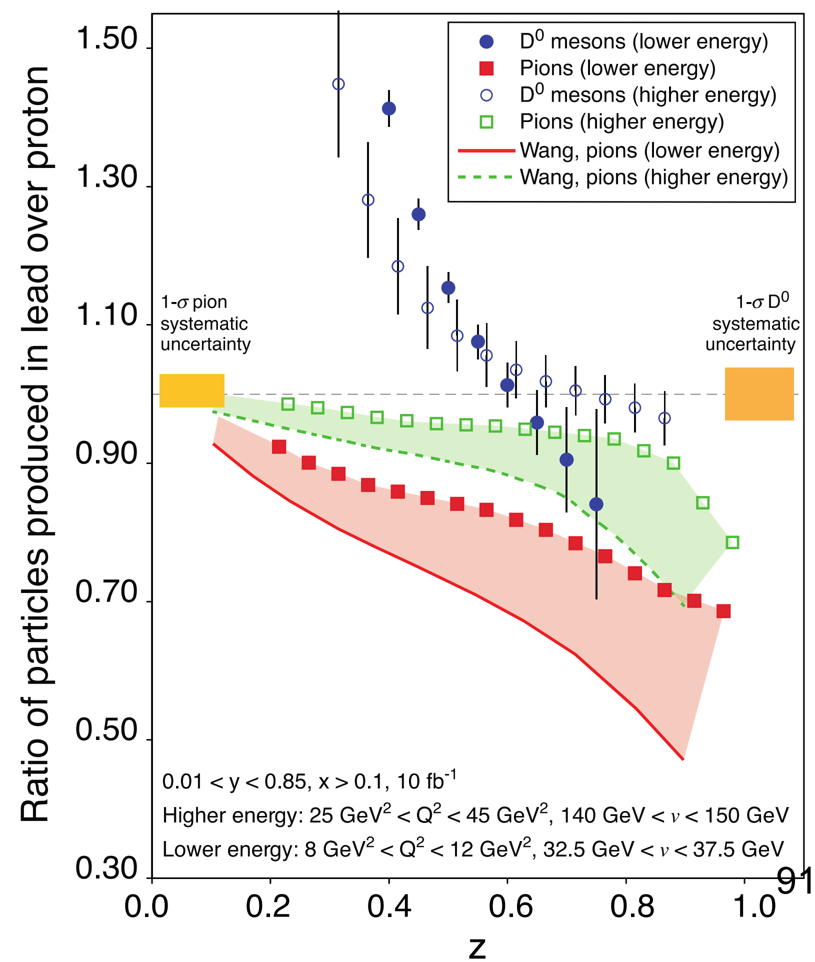
Exploring nucleon structure is great fun!

Look forward to you joining the adventure!

Energy Loss in Cold Nuclear Matter

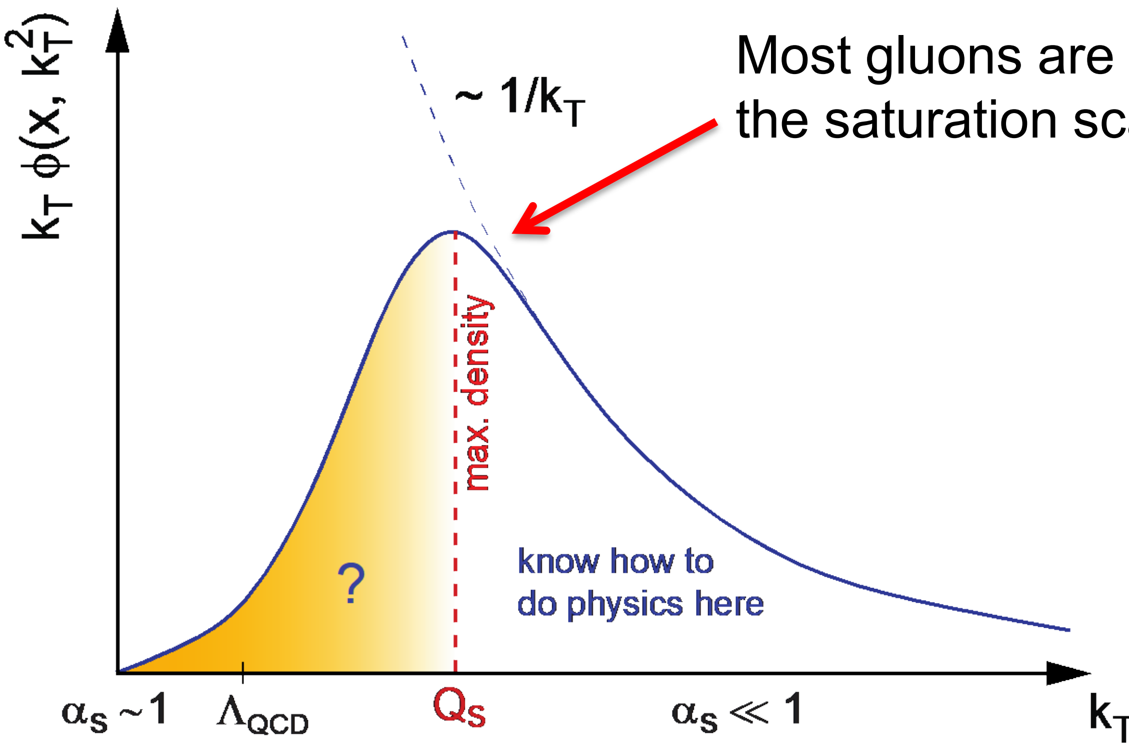
- By studying quark propagation in cold nuclear matter we can learn important information about hadronization and may even measure qhat in the cold nuclear medium:





Typical gluon "size"

Number of gluons (gluon TMD) times the phase space



Most gluons are near the saturation scale Q_s.

Gluon "size" = 1/transverse momentum
= $1/Q_s$

momentum transverse to the beam