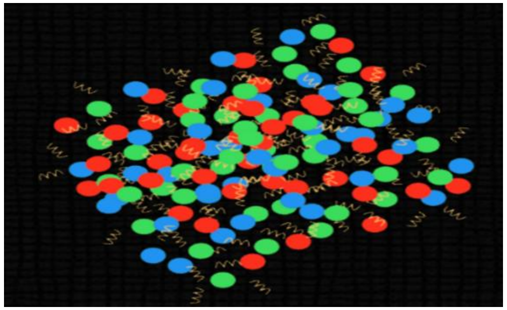
III 演化：量子与经典输运理论

参考林树等人的课程
参考王群等人的工作

Classical Boltzmann equation



$$\left(p^{\mu}\partial_{\mu}^{x} + F^{\mu}\partial_{\mu}^{p}\right)f(x,p) = C$$

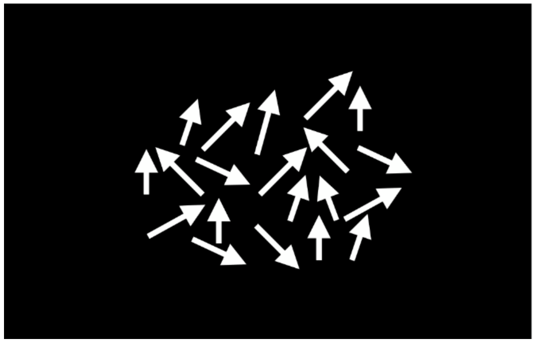
1) A single distribution (particle number) f

2) Quasi-particle (on-shell) approximation:

$$(p^2 - m^2)f(x,p) = 0 \rightarrow f(x,\vec{p}|p_0 = \pm\sqrt{\vec{p}^2 + m^2})$$

Spin

Many quantum anomalies in science are induced by spin, for instance CME and CVE.



Schrodinger equation | *Dirac equation*

Wave function:

scalar $\psi(x)$

spinor $\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$

Probability in coordinate space:

$\psi(x)\psi^*(x)$

$\psi(x)\bar{\psi}(x)$, $a\ 4 \times 4\ matrix$

Probability in phase space (Wigner transformation):

$$f(x,p) = \int d^4y e^{ipy} \psi(x + y/2)\psi^*(x - y/2)$$

$$W(x,p) = \int d^4y e^{ipy} \psi(x + y/2)\bar{\psi}(x - y/2)$$

$Wigner\ function\ with\ 16\ distributions$

Beyond quasi-particle

Particles in medium are in general not quasi-particles (mean field approximation), especially in high energy physics.

1) On-shell → Off-shell constraint:

$$(p^2 - m^2)f(x,p) = 0 \to [(p^2 - m^2) + \hbar\mathcal{A}(p)\,]W(x,p) = 0$$

*2) Physics distributions $W(x,\vec{p})$ are defined in **7D** phase space*
→ Equal-time formalism

$$W(x,p) \to W(x,\vec{p}) = \int dp_0 W(x,p)$$

Covariant and Equal-time Wigner functions

- *Covariant Wigner operator for fermions interacting with a gauge field:*

$$\widehat{W}(x,p)=\int d^4y e^{ipy}\hat{\psi}(x+\frac{y}{2})e^{iq\int_{-1/2}^{1/2}ds\hat{A}(x+sy)y}\hat{\bar{\psi}}(x-\frac{y}{2})$$

gauge link $e^{iq\int_{-1/2}^{1/2}ds\hat{A}(x+sy)y}$ *to guarantee gauge invariance*

Ensemble average → Wigner function

$$W(x,p)=\langle\widehat{W}(x,p)\rangle \qquad (W=\widehat{W}\ in\ quantum\ mechanics)$$

- *Dyson-Schwinger equation for quantum fields* $\hat{\psi}$ *or Dirac equation for wave function* ψ

→ kinetic equations for $W(x,p)$

QED: D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)
QCD: H.-Th.Elze and U.Heinz, Phys. Rep. 183, 81(1989)

- *Problem:*

Initial $W(x,p)$ *is related to the fields* $\hat{\psi}(x)$ *and* $\hat{A}(x)$ *at all times (due to* $\int_{-\infty}^{\infty}dy_0$*)*

→ Causality problem!

Physics distributions are defined through Equal-time Wigner function

$$W_0(x,\vec{p})=\int d^3\vec{y}e^{-i\vec{p}\cdot\vec{y}}\left\langle\hat{\psi}(x+\vec{y}/2)e^{-iq\int_{-1/2}^{1/2}ds\hat{\vec{A}}(x+s\vec{y})\cdot\vec{y}}\hat{\psi}^{+}(x-\vec{y}/2)\right\rangle$$

Dirac-Heisenberg-Wigner equation

Bialynicki-Birula, Gornicki and Rafelski, PRD44, 1825(1991)

- Dirac equation in coordinate space:

$$\left(i\gamma^{\mu}\mathcal{D}_{\mu}-m\right)\psi(x)=0$$

→ DHW equation in phase space:

$$D_t W_0=-\frac{1}{2}\vec{D}\cdot\{\rho_1\vec{\sigma},W_0\}-\frac{i}{\hbar}\left[\rho_1\vec{\sigma}\cdot\vec{P}+\rho_3 m,W_0\right]$$

$$D_t=\partial_t+q\int_{-1/2}^{1/2}ds\vec{E}(\vec{x}+is\hbar\vec{\nabla}_p)\cdot\vec{\nabla}_p,$$

$$\vec{D}=\vec{\nabla}+q\int_{-1/2}^{1/2}ds\vec{B}(\vec{x}+is\hbar\vec{\nabla}_p)\times\vec{\nabla}_p$$

$$\vec{P}=\vec{p}-iq\hbar\int_{-1/2}^{1/2}dss\vec{B}(\vec{x}+is\hbar\vec{\nabla}_p)\times\vec{\nabla}_p$$

- However,

$$W_0(x,\vec{p})=\int dp_0 W(x,p)\gamma_0$$

is not equivalent to $W(x,p)$. We should consider all the moments

$$W_n(x,\vec{p})=\int dp_0 p_0^n W(x,p)\gamma_0 \qquad (n=0,1,2,\ldots)$$

Zhuang and Heinz, Ann.Phys.245, 311(1996)

Only for quasi-particles (on-shell, $p^2-m^2=0,\ p_0=\pm\sqrt{m^2+\vec{p}^2}$),

$$W_n(x,\vec{p})=E_p^n W_0(x,\vec{p}),$$

$W_0(x,\vec{p})$ can completely describe the system.

Equal-time hierarchy I

Zhuang and Heinz, PRD57, 6525(1998)

1) Covariant kinetic equations

$$(\gamma^\mu K_\mu - m)W = 0$$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2} D_\mu \qquad \Pi_\mu = p_\mu - iq\hbar \int_{-1/2}^{1/2} ds s F_{\mu\nu}(x - i\hbar s \partial_p)\partial_p^\nu$$

$$D_\mu = \partial_\mu - q \int_{-\frac{1}{2}}^{\frac{1}{2}} ds F_{\mu\nu}(x - i\hbar s \partial_p)\partial_p^\nu$$

Constraint (with p_μ) and transport (with ∂_μ) equations

$$\begin{cases} [\gamma^\mu(K_\mu + K_\mu^\dagger) - 2m]W(x,p) = 0 \\ \gamma^\mu(K_\mu - K_\mu^\dagger)W(x,p) = 0 \end{cases}$$

2) Equal-time hierarchy

$$\int dp_0 [covariant\ constraint\ and\ transport\ equations] \rightarrow$$

$$\begin{cases} Transport\ equations\ for\ W_0(x,\vec{p}) \quad \rightarrow \quad DHW\ equation \\ Constraint\ equation\ for\ W_1(x,\vec{p}) \end{cases}$$

$$\int dp_0\, p_0 \cdot [covariant\ constraint\ and\ transport\ equations] \rightarrow$$

$$\begin{cases} Transport\ equations\ for\ W_1(x,\vec{p}) \\ Constraint\ equation\ for\ W_2(x,\vec{p}) \end{cases}$$

Equal-time hierarchy II

Zhuang and Heinz, PRD57, 6525(1998)

3) *Spin decomposition*

$$W_0(x,\vec{p}) = \frac{1}{4}[f_0 + \gamma^5 f_1 - i\gamma^0\gamma^5 f_2 + \gamma^0 f_3 + \gamma^5\gamma^0\vec{\gamma}\cdot\vec{g}_0 + \gamma^0\vec{\gamma}\cdot\vec{g}_1 - i\vec{\gamma}\cdot\vec{g}_2 - \gamma^5\vec{\gamma}\cdot\vec{g}_3]$$

D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)

Conservation laws → Physics of the spin components

f_0 : *number density*, $\vec{g}_0$: *spin density*

f_1 : *helicity density*, f_2 : *topologic charge density*, f_3 : *mass density*

$\vec{g}_1$: *number current*, $\vec{g}_3$: *magnetic moment*

4) *Truncating the hierarchy*

spin 1/2 particles: W_0 and W_1 form a closed subgroup

spin 0 particles: W_0, W_1 and W_2 form a closed subgroup

Spin distribution at lowest level

Zhuang and Heinz, PRD53, 2096(1996)

$$W_0(x,\vec{p}) = W_0^{(0)}(x,\vec{p}) + \hbar W_0^{(1)}(x,\vec{p}) + \cdots$$

- Constraint equations → only 4 independent components:
  number density f_0 and spin density $\vec{g}_0$

- Boltzmann equation for number density f_0:

$$\left(D_t + \frac{\vec{p}}{E_p} \cdot \vec{D}\right) f_0 = 0$$

$$D_t = \partial_t + q\vec{E} \cdot \vec{\nabla}_p, \qquad \vec{D} = \vec{\nabla} + q\vec{B} \times \vec{\nabla}_p$$

- Bargmann-Michel-Telegdi equation for spin density $\vec{g}_0$:

$$\left(D_t + \frac{\vec{p}}{E_p} \cdot \vec{D}\right) \vec{g}_0 = \frac{q}{E_p^2}\left[\vec{p} \times \left(\vec{E} \times \vec{g}_0\right) - E_p \vec{B} \times \vec{g}_0\right]$$

the phase-space version of the Bargmann-Michel-Telegdi equation

V.Bargmann, L.Michel and V.Telegdi, PRL2, 435(1959)

电磁场中的动力论方程有许多重要的工作，例如王群，侯德福，王梓岳，黄安平等人的工作

Quark Matter in a Rotation Field

Chen and Zhuang, CPC (2022)

Dirac equation:

$$\left(i\gamma^\mu\partial_\mu - m + \gamma_0\vec{\omega}\cdot\hat{\vec{j}}\right)\psi(x) = 0$$

Covariant Wigner function:

$$W(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{ip\cdot y}\left\langle \psi(x+\frac{y}{2})\bar{\psi}(x-\frac{y}{2})\right\rangle$$

Covariant kinetic equation:

$$\left(\gamma^\mu K_\mu + \frac{\hbar}{2}\gamma^5\gamma^\mu\omega_\mu - m\right)W(x,p) = 0$$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2}D_\mu, \qquad \omega_\mu = (0,\vec{\omega})$$

$$\Pi_\mu = (p_0 + \pi_0, \vec{p}), \qquad \pi_0 = \vec{\omega}\cdot\left(\vec{l} + \frac{\hbar^2}{4}\vec{\nabla}\times\vec{\nabla}_p\right) + \mu_B$$

$$D_\mu = \left(d_t, \vec{\nabla}\right), \qquad d_t = \partial_t - \vec{\omega}\cdot\left(\vec{x}\times\vec{\nabla} + \vec{p}\times\vec{\nabla}_p\right)$$

Classical Transport in a Rotation Field

At order $\hbar^0$, only f_0 (number distribution) and $\vec{g}_0$ (spin distribution) are independent components.

Chen and Zhuang, CPC (2022)

16 transport equations are reduced to

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\nabla - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\nabla_p\right] f_0^{(0)\pm} = 0,$$

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\nabla - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\nabla_p\right] \mathbf{g}_0^{(0)\pm} = -\boldsymbol{\omega}\times\mathbf{g}_0^{(0)\pm}.$$

Coriolis force *spin-rotation coupling*

16 constraint equations are reduced to

$$on-shell\ energy \quad E_p^{\pm} = \pm\varepsilon_p - (\vec{\omega}\cdot\vec{l} + \mu_B), \qquad \varepsilon_p = \sqrt{m^2+p^2}$$

$$f_1^{(0)\pm} = \pm\frac{1}{\epsilon_p}\boldsymbol{p}\cdot\mathbf{g}_0^{(0)\pm},$$

$$f_2^{(0)\pm} = 0,$$

$$f_3^{(0)\pm} = \pm\frac{m}{\epsilon_p}f_0^{(0)\pm},$$

$$\mathbf{g}_1^{(0)\pm} = \pm\frac{\boldsymbol{p}}{\epsilon_p}f_0^{(0)\pm},$$

$$\mathbf{g}_2^{(0)\pm} = \frac{1}{m}\boldsymbol{p}\times\mathbf{g}_0^{(0)\pm},$$

$$\mathbf{g}_3^{(0)\pm} = \pm\frac{1}{m\epsilon_p}\left[\epsilon_p^2\mathbf{g}_0^{(0)\pm} - \boldsymbol{p}(\boldsymbol{p}\cdot\mathbf{g}_0^{(0)\pm})\right].$$

One can systematically calculate the higher order contributions.

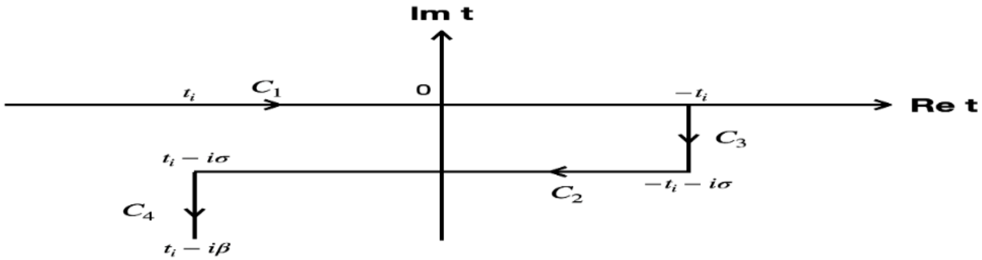
Kadanoff-Baym equations

Dyson-Schwinger equation

$$S(x,y) = S^0(x,y) + \int d^4z d^4w S^0(x,w)\Sigma(w,z)S(z,y)$$

Considering the time order in $W(x,p) = \int d^4y e^{ipy}\, \psi\left(x+\frac{y}{2}\right)\bar{\psi}\left(x-\frac{y}{2}\right)$

→ Schwinger-Keldish time contour



$S^<(x,p) = W(x,p),$ $\quad$ $S^>(x,p)$

Constraint and transport equations with collision terms

$$\{(\gamma^\mu p_\mu - m), S^<\} + \frac{i\hbar}{2}[\gamma^\mu, \nabla_\mu S^<] = \frac{i\hbar}{2}([\Sigma^<, S^>]_* - [\Sigma^>, S^<]_*)$$

$$[(\gamma^\mu p_\mu - m), S^<] + \frac{i\hbar}{2}\{\gamma^\mu, \nabla_\mu S^<\} = \frac{i\hbar}{2}(\{\Sigma^<, S^>\}_* - \{\Sigma^>, S^<\}_*)$$

$$A * B = AB + \frac{i\hbar}{2}[AB]_{P.B.} + \mathcal{O}(\hbar^2)$$

including spin: Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612

- *Spin decomposition for Wigner function, self-energy and collision term*

General collision terms

Wang, Guo and Zhuang, EPJC, (2021)

- **0th order transport**

描述相对论重离子碰撞的许多模型，例如URQMD, AMPT, BAMPS.

$$p\cdot\nabla\mathcal{V}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{A}^{(0)\lambda}} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)}_\nu},$$

$$p\cdot\nabla\mathcal{A}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} + \widehat{\Sigma^{(0)}_{A\mu}p^\nu\mathcal{V}^{(0)}_\nu} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(0)\nu}},$$

$$\widehat{FG} = \bar{F}G - F\bar{G}$$

Local collision term
Dynamical effect ,
e.g. diffusion effect

- **1st order transport**

$$\begin{aligned}
p\cdot\nabla\mathcal{V}^{(1)}_\mu =& + m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(1)}_\mu} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(1)}_\nu} - \frac{p_\nu}{m}\epsilon_{\rho\sigma\alpha\mu}p^\rho\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(1)\sigma}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(0)\sigma\nu}_T\mathcal{A}^{(1)\lambda}} \\
& + m\widehat{\Sigma^{(1)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{V}^{(0)}_\mu} - p_\mu\widehat{\Sigma^{(1)\nu}_A\mathcal{A}^{(0)}_\nu} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(1)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(1)\sigma\nu}_T\mathcal{A}^{(0)\lambda}} \\
& + \frac{1}{2m}p^\nu[\widehat{\Sigma^{(0)}_{T\mu\nu}(p^\alpha\mathcal{V}^{(0)}_\alpha)}]_{\text{P.B.}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{V}^{(0)\nu}}]_{\text{P.B.}} + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu[\widehat{\Sigma^{(0)\alpha}_A\mathcal{V}^{(0)\beta}}]_{\text{P.B.}} \\
& - \frac{1}{2m}p_\nu\Sigma^{(0)}_{T\alpha\mu}\widehat{\nabla^{[\alpha}\mathcal{V}^{(0)\nu]}} + \frac{1}{2m}p_\nu\widehat{\Sigma^{\alpha\nu(0)}_T\nabla_{[\alpha}\mathcal{V}^{(0)}_{\mu]}} + \frac{1}{2}\epsilon_{\beta\nu\lambda\mu}\widehat{\Sigma^{(0)\beta}_A\nabla^\nu\mathcal{V}^{(0)\lambda}} \\
& + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha\Sigma^{(0)\nu}_V)\mathcal{A}^{(0)\beta}} - \frac{1}{2m}p_\mu\widehat{(\nabla^\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\nu} - \frac{1}{2m}\widehat{(p^\nu\nabla_\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\mu} + \frac{p^\nu}{2m}\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha\Sigma^{(0)}_S)\mathcal{A}^{(0)\beta}},
\end{aligned}$$

$$\begin{aligned}
p\cdot\nabla\mathcal{A}^{(1)}_\mu =& + m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{A\mu}\mathcal{V}^{(1)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(1)\lambda}} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(1)\nu}} \\
& + m\widehat{\Sigma^{(1)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{A\mu}\mathcal{V}^{(0)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(1)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(1)}_{A\nu}\mathcal{V}^{(0)\nu}} \\
& - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\widehat{(\nabla^\sigma\Sigma^{(0)\nu}_V)\mathcal{V}^{(0)\rho}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_P\mathcal{V}^{(0)}_\mu}]_{\text{P.B.}} + \frac{1}{2m}p_\mu[\widehat{\Sigma^{(0)}_P(p^\nu\mathcal{V}^{(0)}_\nu)}]_{\text{P.B.}} \\
& + \frac{1}{2}\epsilon_{\mu\sigma\nu\rho}\nabla^\sigma\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)\rho}} + \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}[\widehat{\Sigma^{(0)\nu}_A(p^\alpha\mathcal{A}^{(0)\beta})}]_{\text{P.B.}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)\nu}}]_{\text{P.B.}} + \frac{p_\mu}{2m}[\widehat{\Sigma^{(0)}_{T\rho\nu}(p^\rho\mathcal{A}^{(0)\nu})}]_{\text{P.B}} \\
& - \frac{1}{2m}p_\sigma\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)\nu}}) + \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)}_\sigma}) + \frac{1}{2m}p_\mu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)\nu}}) - \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)}_\mu}).
\end{aligned}$$

- **Nonlocal collision term**
- **Related to spatial derivatives**
- **Correlated transport of V&A**
- **Polarization can be generated in a initially unpolarized syste**

the interaction needs to be specified to calculate the off-diagonal self-energy $\Sigma^>$ & $\Sigma^<$

Near equilibrium state

Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

- *Collision is the driving force for the system to go from non-equilibrium to equilibrium.*

- *Local equilibrium state is determined by detailed balance principle (loss term and gain term cancel to each other).*

- *Global equilibrium state is determined by detailed balance + Killing condition*

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0, \qquad \beta_\mu = u_\mu / T$$

- *Near-equilibrium state can be described by relaxation time approximation (RTA).*

Relaxation Time Approximation (RTA)

Classical RTA: *[J.Anderson and H.Witting, Physica 74, 466(1974)]*

$$p^\mu \partial_\mu f = -p^\mu u_\mu \frac{\delta f}{\tau}, \qquad \delta f = f - f_{eq}$$

Quantum RTA:

$$(\gamma^\mu p_\mu - m)W + \frac{i\hbar}{2}\gamma^\mu \partial_\mu W = -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{\delta W}{\tau}, \qquad \delta W = W - W_{eq}$$

A single time scale to control the relaxation process for all the components.

Motivations:

1) A multi-component kinetic theory needs more time scales to control different degrees of freedom.

2) Calculating the relaxation times in Kadanoff-Baym formalism.

RTA from Kadanoff-Baym equations

Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

Calculating damping and correlation times for different degrees of freedom with Kadanoff-Baym equations.

Method:

1) Quantum kinetic equations in near equilibrium state:

$$W = W_{eq} + \delta W$$

2) Expanding collision terms around the equilibrium state:

$$C(\Sigma, W) = C'(\Sigma_{eq}, W_{eq})\delta W$$

Note: $C\left(\Sigma_{eq}, W_{eq}\right) = 0 \; in\; equilibrium\; state$

Kadanoff-Baym RTA:

Chiral fermions: $f_\pm = \frac{1}{2}(f_V \pm f_A)$

$$\begin{cases} p^\mu \partial_\mu f_+ = -\frac{\delta f_+}{\tau_+} - \frac{\delta f_+ - \delta f_-}{\tau_{+-}} \\ p^\mu \partial_\mu f_- = -\frac{\delta f_-}{\tau_-} - \frac{\delta f_- - \delta f_+}{\tau_{+-}} \end{cases}$$

$\tau_\pm$: $damping\; time\; for\; chiral\; fermions$

$\tau_{\pm-}$: $correlation\; time\; between\; two\; kinds\; of\; fermions$