II 新颖：电磁场与涡旋场中的QCD物质

为什么考虑电磁场与涡旋场？

由于电磁相互作用远比强相互作用弱，一般研究*QCD*物质时不考虑电磁相互作用。

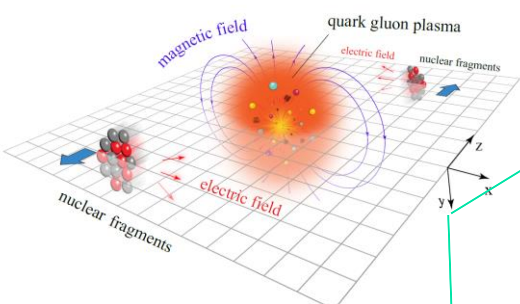
非对心核碰撞使QGP处于自然界最强的电磁场和涡旋场之中：

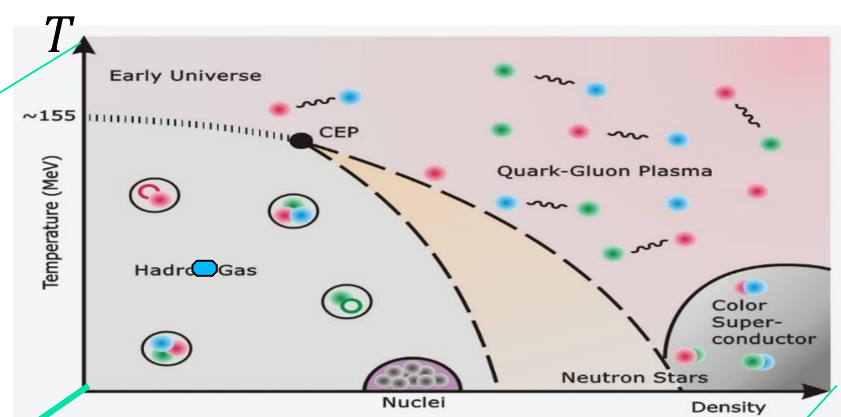
$|eB| \sim 5m_\pi^2$ *at RHIC and* $70m_\pi^2$ *at LHC !*

$\omega \sim 10^{21}/s$ *at RHIC !*

可参考黄旭光等人的工作

其强度已经与强相互作用可比拟。





T

n_B

?

?

E, B, ω

参考黄梅，侯德福等人的课程

3 Questions

1) *Strength:*

E & B breaks down the translation invariance, but T will restore the invariance. Only charged quarks join electromagnetic interaction, but all the partons join thermal motion

→ *Is the E and B field strong enough in comparison with the fireball T?*

$$|eB| \sim 10 m_\pi^2, \qquad T \sim 300 - 500 \text{ MeV}$$

2) *Time:*

The lifetime of external E and B field is very short, we need induced E and B field in QGP

→ *Can the current be completely induced before the disappearance of the external field?*

$$\tau_{ind}, \qquad \tau_B \sim 0.1 \text{ fm}$$

3) *Clear signals:*

If the answers to the above 2 questions are negative, is it possible to have sensitive electromagnetic signals of QCD matter?

Feynman Rules in Magnetic Field

Kostenko and Thompson, Astrophys J. 869, 44(2018), 875, 23(2019).

External lines:

$$[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi = 0$$

$$\psi^\sigma_\mp(x,p) = \begin{cases} e^{-ip\cdot x} u_\sigma(\boldsymbol{x},p) \\ e^{ip\cdot x} v_\sigma(\boldsymbol{x},p) \end{cases}$$

$$u_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z p_n \phi_{n-1} \\ (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \\ -ip_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -p_z(\epsilon_n+m)\phi_n \end{bmatrix}, \qquad v_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -p_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -ip_z(\epsilon_n+m)\phi_n \\ -p_z p_n \phi_{n-1} \\ i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \end{bmatrix},$$

$$u_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ -ip_z p_n \phi_n \\ p_z(\epsilon_n+m)\phi_{n-1} \\ ip_n(\epsilon+\epsilon_n)\phi_n \end{bmatrix} \qquad v_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z(\epsilon_n+m)\phi_{n-1} \\ -p_n(\epsilon+\epsilon_n)\phi_n \\ -i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ p_z p_n \phi_n \end{bmatrix}$$

Quark propagator:

$$G(x'-x) = -i\left(\frac{L}{2\pi\lambda}\right)^2 \int dp_z da \sum_{\sigma,n}\left[\theta(t'-t)u_\sigma(\boldsymbol{x}',p)\bar{u}_\sigma(\boldsymbol{x},p)e^{-ip\cdot(x'-x)} - \theta(t-t')v_\sigma(\boldsymbol{x}',p)\bar{v}_\sigma(\boldsymbol{x},p)e^{ip\cdot(x'-x)}\right]$$

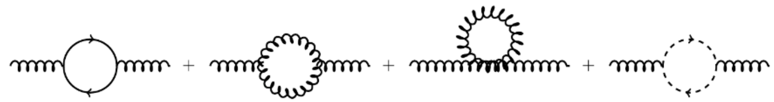
$$G(p) = -\int_0^\infty \frac{dv}{|qB|}\left\{[m+(\gamma\cdot p)_\parallel][1-isgn(q)\gamma_1\gamma_2\tanh(v)] - \frac{(\gamma\cdot p)_\perp}{cosh^2(v)}\right\} e^{-\frac{v}{|qB|}\left[m^2 - p_\parallel^2 + \frac{\tanh(v)}{v}p_\perp^2\right]}$$

Schwinger propagator, 1951

- *no more translation invariance.*
- *the two Schwinger phases for q and $\bar{q}$ are cancelled to each other in loop calculation.*

Gluon Propagator in QCD Matter

Gluon self-energy in magnetic field:



for quark loop

$$\Pi^{||}_{\mu\mu}(T,B) = g^2 T|qB| \sum_{np_z n_1} \frac{(2-\delta_{n_1 0})\,(\delta^{||}_{\mu\mu} + g^{||}_{\mu\mu})(-\omega_n^2 + p_z^2)}{(m^2+\omega_n^2+p_z^2+2n_1|qB|)^2}$$

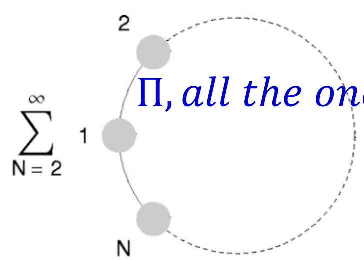
$$\Pi^{\perp}_{\mu\mu}(T,B) = 0$$

Matsubara frequencies $p_0 = i\omega_n = i(2n+1)\pi T$
quark longitudinal momentum p_z
transverse Landau energy $\varepsilon_k = 2n_1|qB|$

for gluon and ghost loops

$$\overline{\Pi}_{\mu\nu}(T,B) = \overline{\Pi}_{\mu\nu}(T,0)$$

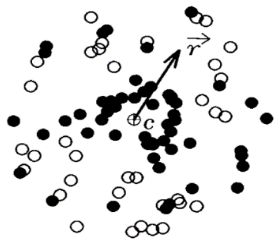
To include non-perturbative effect, we take the summation of ring diagrams
→ gluon propagator and thermodvnamic potential:



$\Pi, all\ the\ one-loop\ polarization$

Color Screening in QCD Matter

Debye screening of a pair of charged particles $q\bar{q}$:



$$\frac{1}{r} \to \frac{1}{r} e^{-m_D r} = \frac{1}{r} e^{-r/r_D}$$

screening mass m_D

screening length r_D

Pole of the propagator at gluon momentum $\left(k_0^2 = 0,\ \vec{k}^2 = -m_D^2\right) \to$ screening mass:

$$m_D^2(T,B) = m_Q^2(T,B) + m_G^2(T),$$

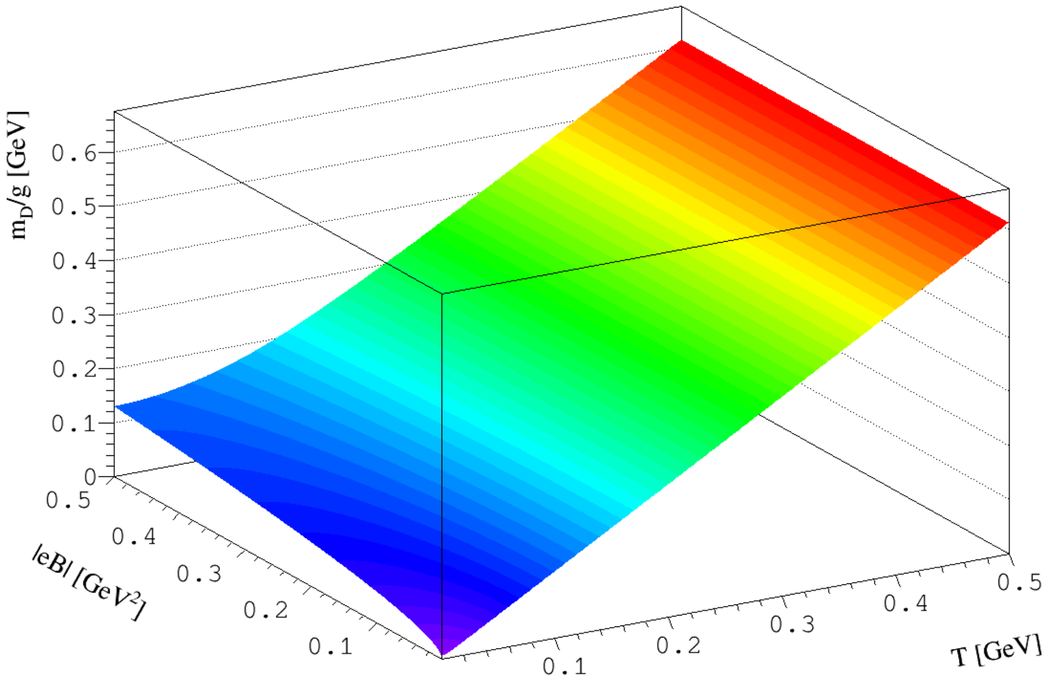
$$m_Q^2(T,B) = -\Pi_{00}^{||}(T,B),$$

$$m_G^2(T) = -\overline{\Pi}_{00}^{||}(T).$$

$$m_Q^2(T,B) = -g^2 T|qB| \sum_{n p_z n_1} \left[(2-\delta_{n_1,0}) \frac{m^2 - \omega_n^2 + p_z^2 + 2n_1|qB|}{(m^2 + \omega_n^2 + p_z^2 + 2n_1|qB|)^2}\right]$$

$$m_G^2(T) = \frac{N_c}{3} g^2 T^2$$

$m_D(T,B)$



Huang, Zhao, Zhuang, PRD107, 114035(2023)

- $m_D(T=0,B) = 0.13$ GeV at $eB = 25m_\pi^2$.
- *the B effect is gradually washed out by thermal motion!*

还可参考侯德福等人的工作

Two time scales

✦ *The induced current needs time* τ_{rel} *to relax from zero to the Ohm's current*

$$\vec{J}_{Ohm} = \sigma_{el}\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

✦ *The lifetime of the external B-field:* τ_B

A complete electromagnetic response of QGP requires

$$\tau_{rel} < \tau_B$$

For normal materials like conductor and EM plasma,

$$\tau_{rel} \ll \tau_B$$

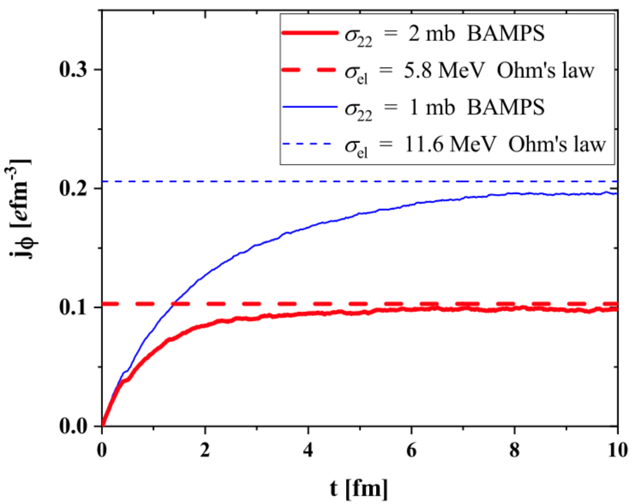
the relaxation time can be neglected.

How is about the relaxation in HIC where τ_B *and* τ_{QGP} *are very short?*

Broken Ohm's Law in HIC

The evolution of the fireball in described by a Boltzmann Approach of MultiParton Scatterings (BAMPS):

$$\left(\partial/\partial t + \vec{p}/E \cdot \vec{\nabla} + \vec{F} \cdot \vec{\nabla}_p\right) f = C_{22} + C_{23}, \qquad Lorentz\ force\ \vec{F} = q\left(\vec{p}/E \times \vec{B} + \vec{E}\right)$$



Wang, Zhao, Greiner, Xu and Zhuang, PRC105, L041901(2022), Letter, Featured in Physics

还可参考黄旭光，王群等人的工作

$\vec{j} < \vec{j}_{Ohm}$, *Ohm's law is broken!*

The electromagnetic response of the hot QCD matter to the fast decay of the external electromagnetic field is incomplete, which strongly suppresses the induced magnetic field.

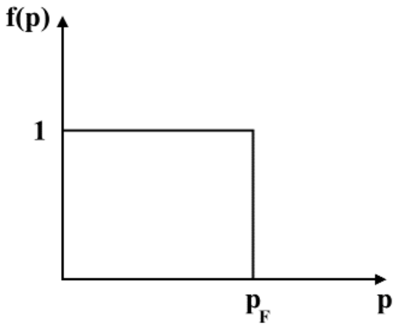
Color Screening in Dense QCD Matter

In the frame of ring diagram summation

$$m_D^2(\mu_f, B) = \frac{g^2}{(2\pi)^2}\sum_{n=0}(2-\delta_{n0})\frac{\mu_f|qB|}{\sqrt{\mu_f^2 - 2n|qB|}}$$

For chiral quarks, the distribution

$$f(p) = \theta(\mu_f - p)$$



the Fermi surface is determined by

$$p_z^2 + 2n|qB| = \mu_f^2$$

when μ_f and Landau levels are equal

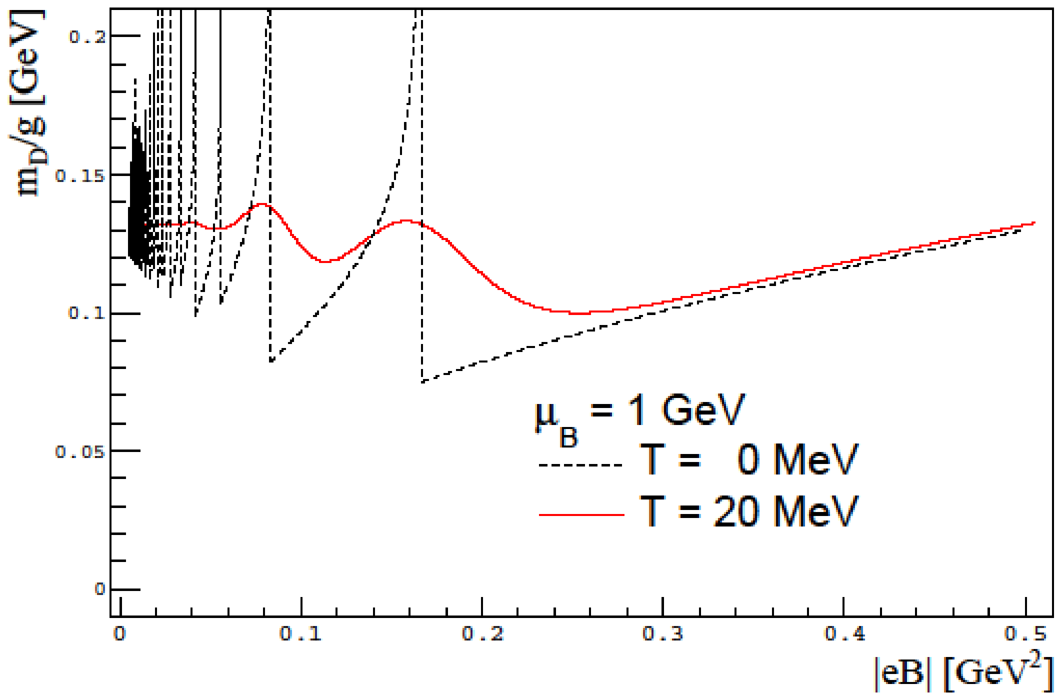
$$\mu_f^2 = 2n|qB|$$

the infrared divergence at the Fermi surface induces a complete screening $m_D \to \infty$, called resonant screening.

The case here is similar to the resonant transmission in Quantum Mechanics.

Resonant Screening

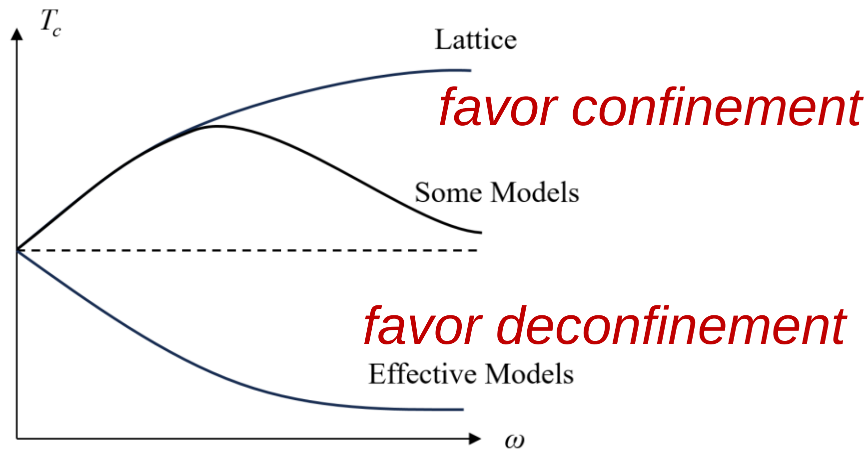
Huang, Zhao and Zhuang, PRD108, L091503(2023), Letter

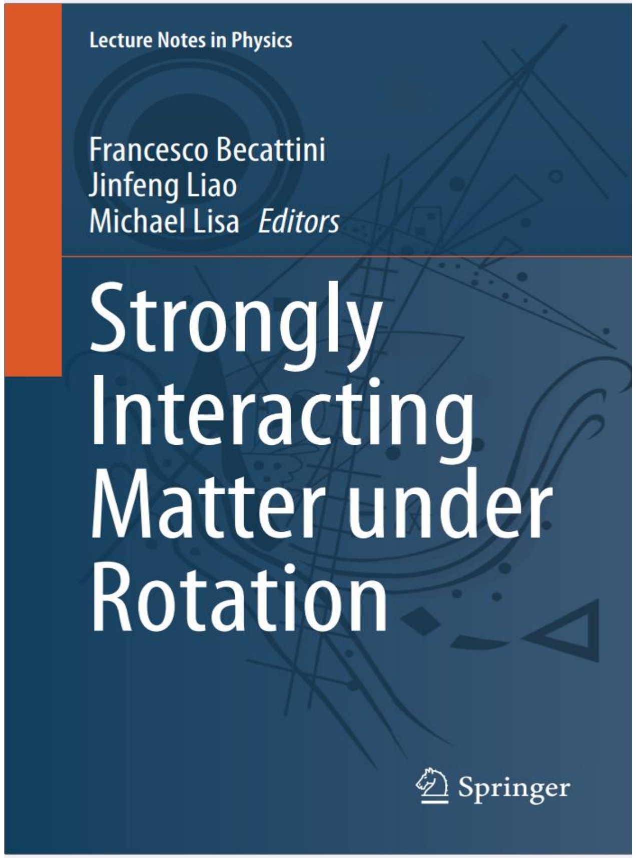


The color interaction between a pair of quarks is completely screened in dense and magnetized QCD matter, when chemical potential matches the Landau levels $\mu_f^2 - 2n|qB| = 0.$

Rotation Puzzle

Does a rotation favor QCD phase transition or not ?





We need serious calculations in QCD.

[参考廖劲峰，侯德福，黄梅等人的课程]()
[参考姜寅，黄旭光等人的工作]()

Workshop on QCD under rotation, Shanghai, 20231111

Gluon Propagator in Curved Space ($\omega \neq 0$)

A rotating system is equivalent to a curved space,
it can be described by a transformation between the flat and curved space.

Flat space: $\bar{x}_\mu$ *Curved space:* x_μ

Transformation: $\begin{cases} t = \bar{t} \\ x = \bar{x}cos(\omega\bar{t}) - \bar{y}sin(\omega\bar{t}) \\ y = \bar{x}sin(\omega\bar{t}) + \bar{y}cos(\omega\bar{t}) \\ z = \bar{z} \end{cases}$

Gluon self-energy:

$$\Pi_{\mu\nu}^{ab}(q,q') = \int \frac{dQ_0}{2\pi} \sum_{\vec{Q}} h_\mu^A(Q|q) h_\nu^B(-Q|q') \bar{\Pi}_{AB}^{ab}(Q)$$

curved space *flat space*

$$h_\mu^A(Q|q) = \int d^4x \sqrt{-det\left(g_{\sigma\rho}(x)\right)} \frac{\partial \bar{x}^A}{\partial x^\mu} e^{i(qx - Q\bar{x})}$$

Causality and Boundary Condition

Causality:

$$\omega R < 1$$

For a rotating quark-gluon plasma created in RHIC,

$R \sim 10\ \mathrm{fm}$

$\omega \lesssim 30\ \mathrm{MeV},$ *smaller in comparison with normal* μ_f *and* T

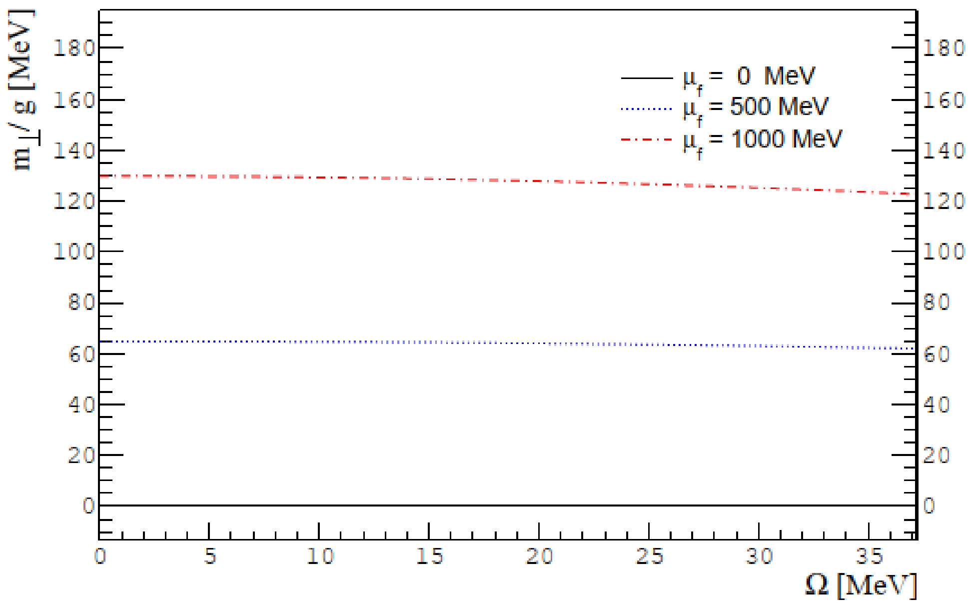
→ Boundary condition:

$$\vec{Q} = \frac{2\pi}{L}\vec{N}, \qquad \vec{N} = (N_1, N_2, N_3)$$

$$\vec{q} = \frac{2\pi}{L}\vec{n}, \qquad \vec{n} = (n_1, n_2, n_3)$$

Gluon Mass at Finite μ_f and ω

*Huang, Chen, Jiang, Zhao and Zhuang, arXiv 2408****



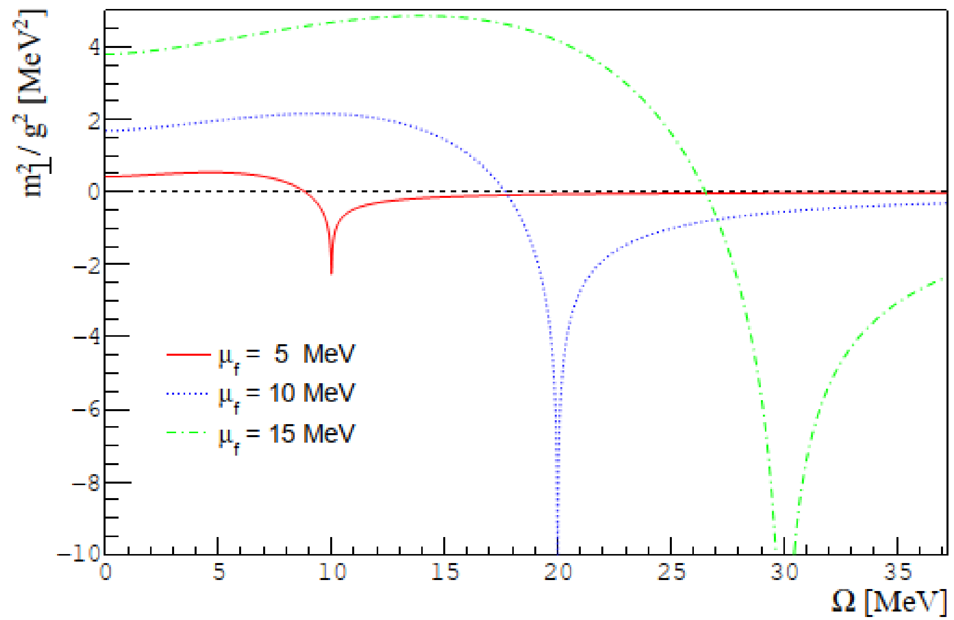
1) m *decreases with* ω*, it means an antiscreening effect.*
The density induced screening effect is partly cancelled by the rotation.
Rotation favors confinement.

2) ω $(\lesssim 30\ \mathrm{MeV})$ *is much smaller than* μ_f $(\sim 500\ \mathrm{MeV})$*, the rotation effect is weak, the maximum cancellation is* $\sim 5\%$*.*

3) There is no rotation effect in vacuum $(\mu_f = 0)$*.*

Rotation Effect at Small μ_f

Huang, Chen, Jiang, Zhao and Zhuang, arXiv 2408***



1) At small μ_f, the rotation effect becomes dominant!

2) The density effect is completely cancelled at a critical rotation $\omega_c = 2\mu_f$, and then the imaginary mass means that gluons can no longer be considered as physical particles (confinement), as suggested by Gribov.

3) At finite T, the divergence is expected to become an oscillation.