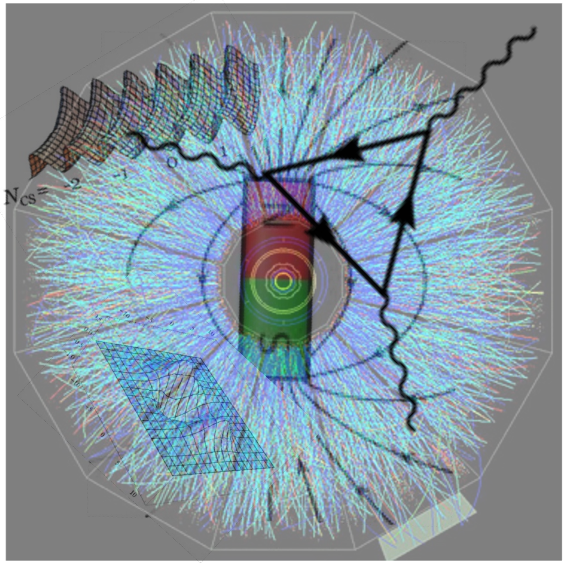
Heavy Ion Collision Theory — Selected Topics (Part I)



Jinfeng Liao

Indiana University, Physics Dept. & CEEM

Plan of the Lecture

- ***Introduction: Why heavy ion collisions?***
- ***What happens in a heavy ion collision?***
- ***Collective flow and hydrodynamics***

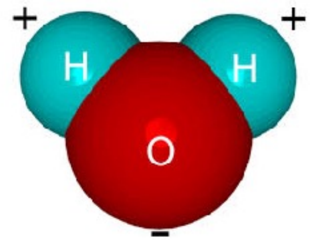
Introduction

Nuclear Physics: Exploring the Heart of Matter

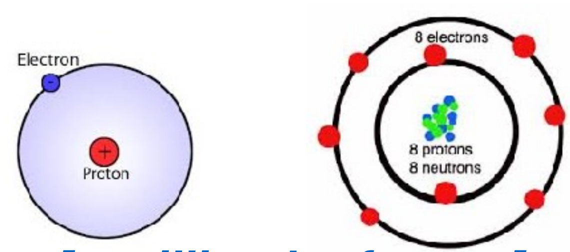
The physical world has a hierarchy of structures.

matter

molecule

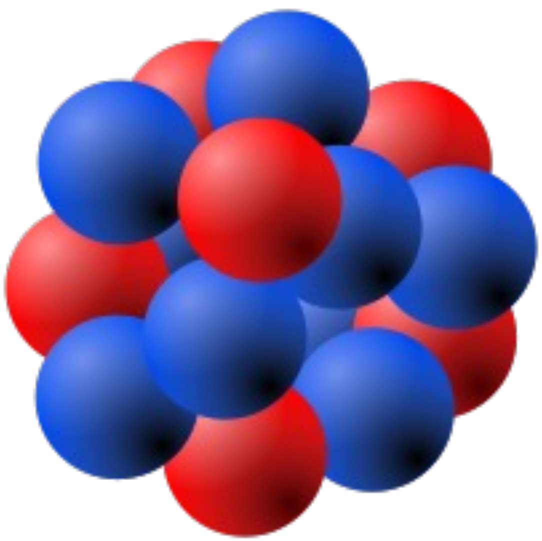


atoms



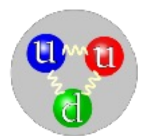
[a millionth of a mm]

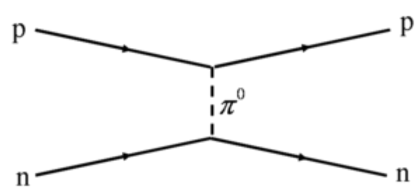
atomic nucleus

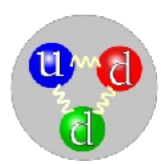


[a trillionth of a mm]

proton



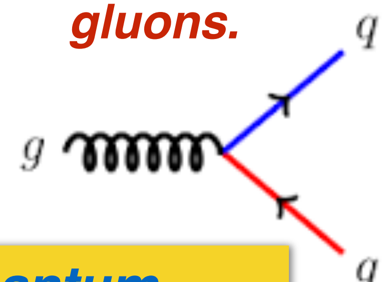




neutron

nuclear force

Most basic entities: quarks and gluons.



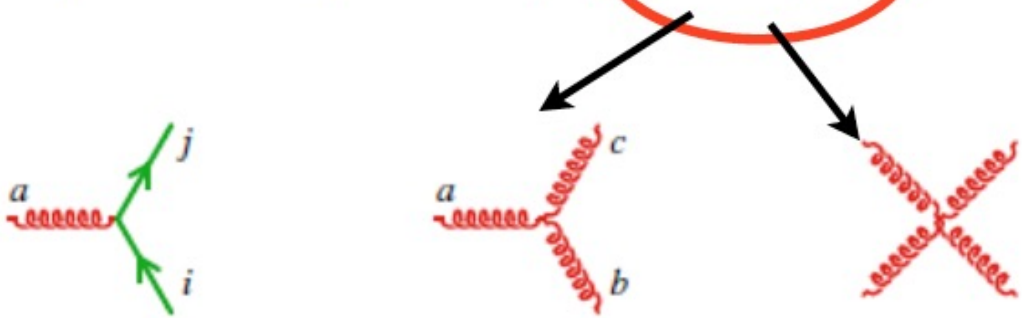
Quantum Chromodynamics (QCD)

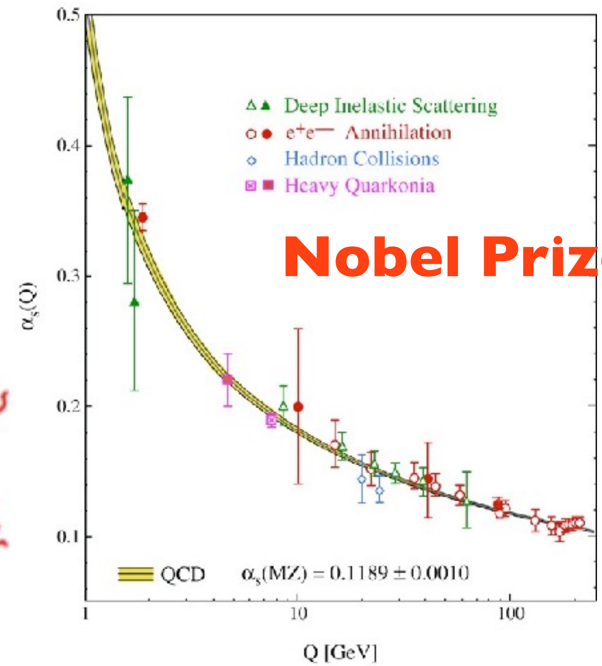
Quantum Chromodynamics (QCD)

The fundamental theory of strong nuclear force:
QCD, a non-Abelian gauge theory of quarks and gluons

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - M - g A\!\!\!/_a G^a)\psi - \frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$





Nobel Prize 2004

Asymptotic Freedom: coupling becomes large at low energy or long distance scale.

$$\Lambda_{QCD} \sim 200\text{MeV} \qquad R \sim 1\,\text{fm}$$

where "quark math" becomes very hard!

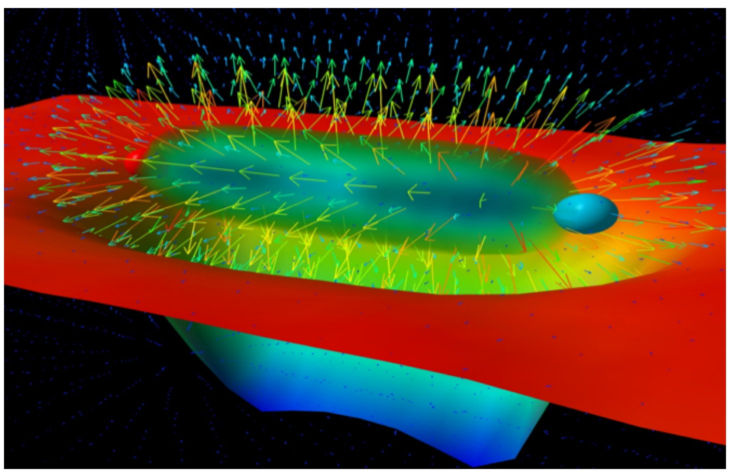
The QCD Vacuum: Confinement

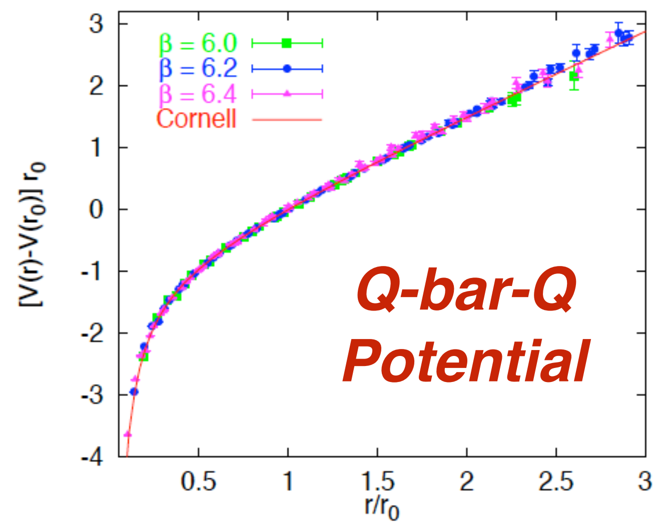
The missing particles: quarks & gluons (in the QCD lagrangian) are not seen in physically observed states.

Free Quark Searches

from Particle Data Book

All searches since 1977 have had negative results.

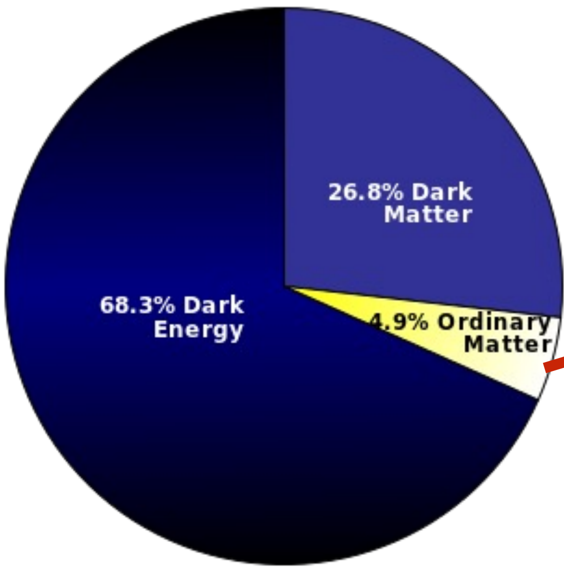


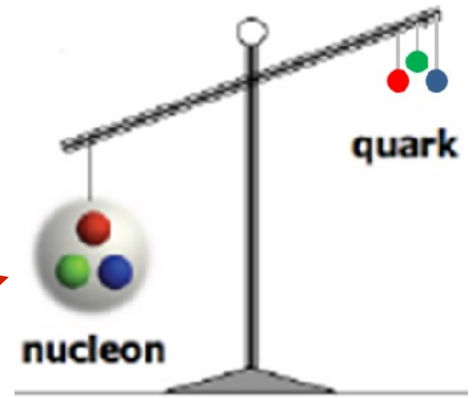


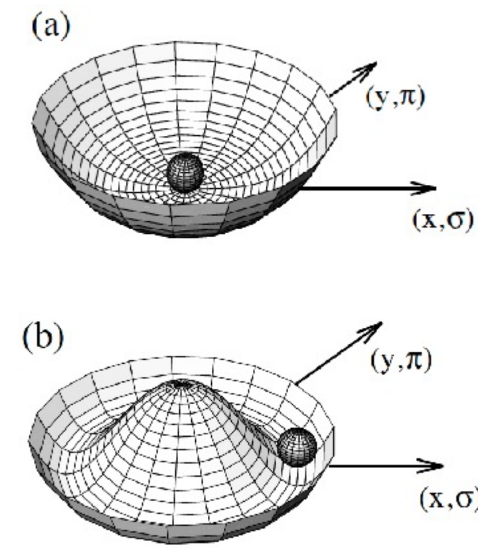
QCD vacuum as "dual superconductor" with dual Meissner effect.

The QCD Vacuum: Chiral Symmetry Breaking

The missing symmetries: while the Lagrangian has (approximate) chiral symmetry, the vacuum and hadron spectrum do not have that.







$$m_\pi \approx 140MeV \;,\; m_n \approx 940MeV$$

QCD vacuum is not empty, but a complicated, nonperturbative, emergent form of condensed matter.

[It accounts for 99% of the mass of our visible matter in universe.]

"Vacuum Engineering"

两大疑难：
丢失的对称性，
看不见的夸克。

通过真空激发来探索！

Abnormal nuclear states and vacuum excitation*†

T. D. Lee

Physics Department, Columbia University, New York, New York 10027

We examine the theoretical possibility that at high densities there may exist a new type of nuclear state in which the nucleon mass is either zero or nearly zero. The related phenomenon of vacuum excitation is also discussed.

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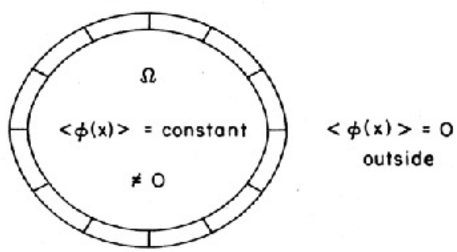
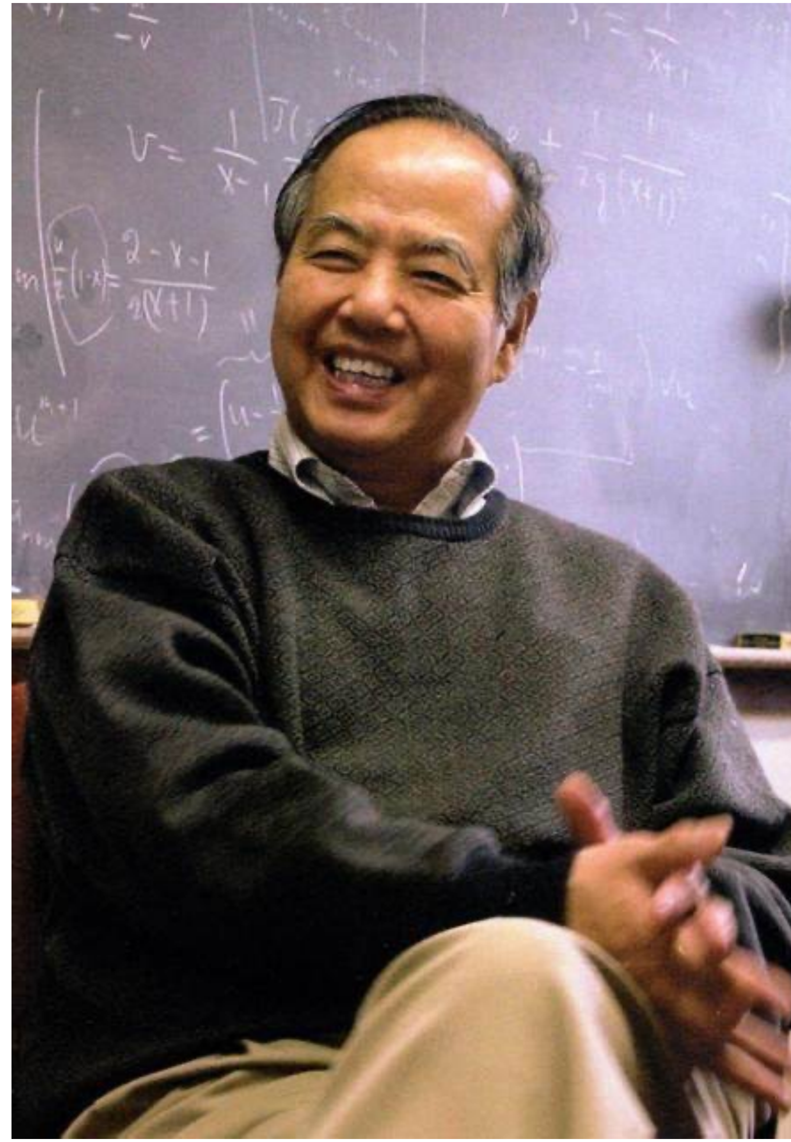


I. INTRODUCTION

Reviews of Modern Physics, Vol. 47, No. 2, April 1975



From Fire to Extreme Temperature

Our heating capability has advanced VERY dramatically…

Laser ignition for fusion

Extreme high energy collider

Curiosity questions that even K-12 kids may wonder about:

*** What is the highest temperature ever?**

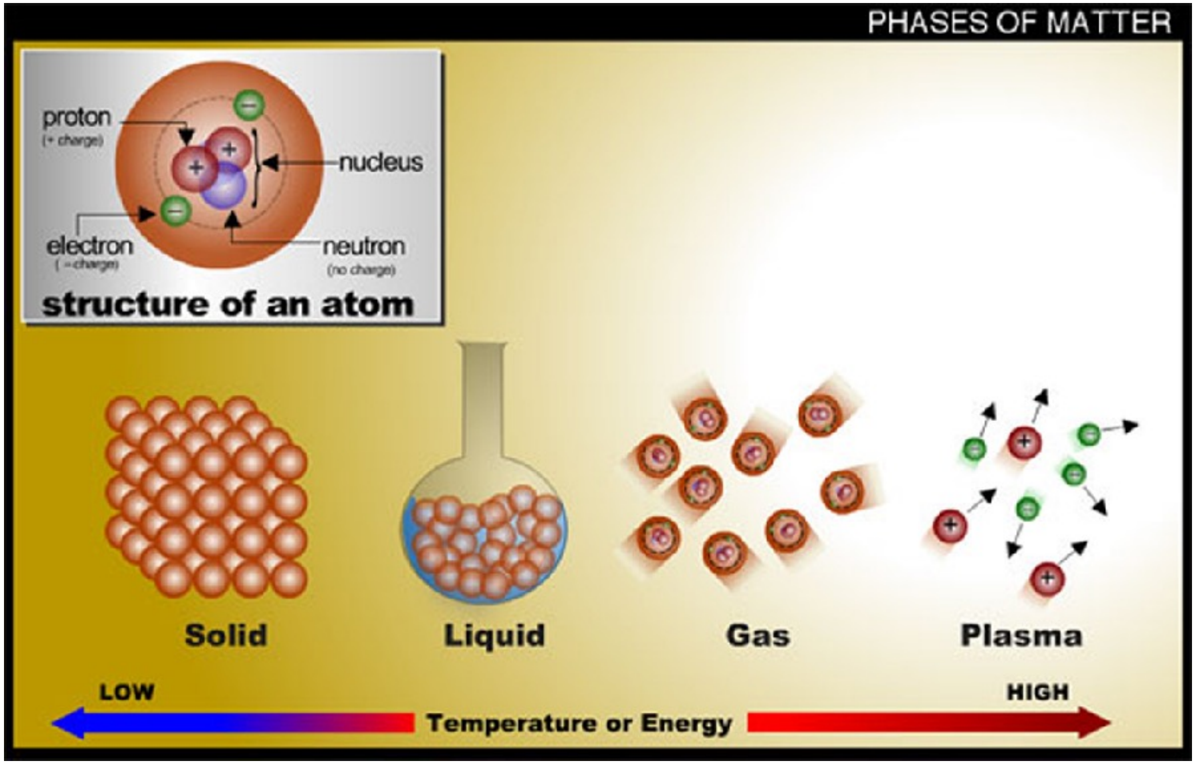
*** What is the highest man-made temperature?**

*** What does the matter look like at such extreme temperature?**

——> a scientific frontier of high energy physics

Heating It Up: Energy Scale Matters

Heating increases temperature and enhances the thermal motion of whatever micro. degrees of freedom: a combat of <u>random thermal motion v.s. ordered structure</u> (with the latter typically due to dynamical interaction)



From NASA

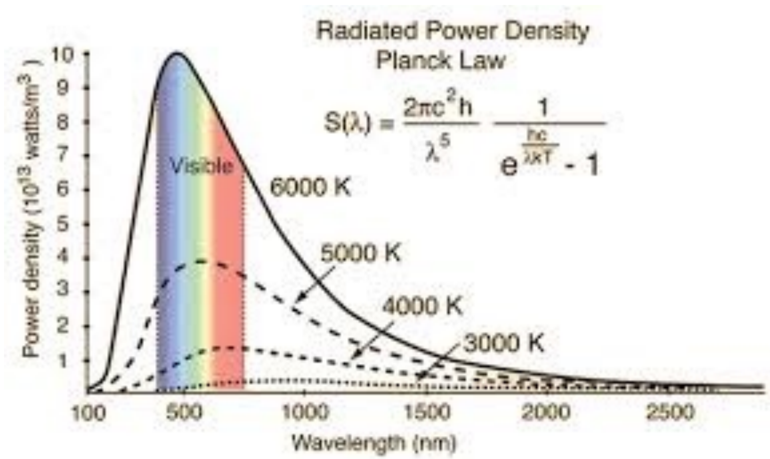
$10^2 \sim 10^3 K$ $\qquad$ $10^4 \sim 10^5 K$

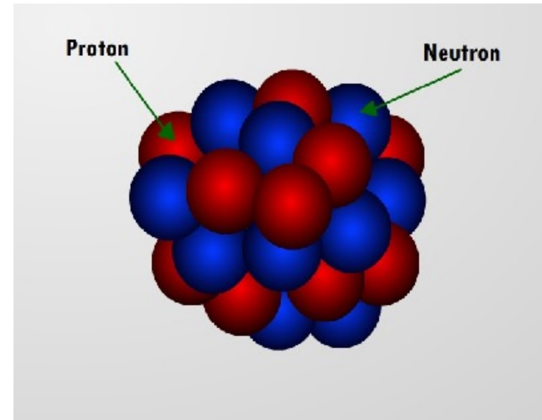
$0.01 \sim 0.1\,\mathrm{eV}$ $\qquad$ $1 \sim 10\, eV$

Inner electrons "peeled off" ~ keV ~ 10^7K

Heating It Up: Energy Scale Matters

What's coming up next upon further heating?





Emissions!!
massless photons — all the time!
What's next?

$e^{-M_e/T}$ $M_e \sim 0.5\mathrm{MeV}$

Ions (nuclei) —
when do they break up?
Nuclear binding energy:
$\sim MeV$

Getting to ~ MeV ~ 10 Million Kelvin

—> Need to know quantum field theory (relativity+QM)

—> Need to know nuclear physics

Heating It Up: Energy Scale Matters

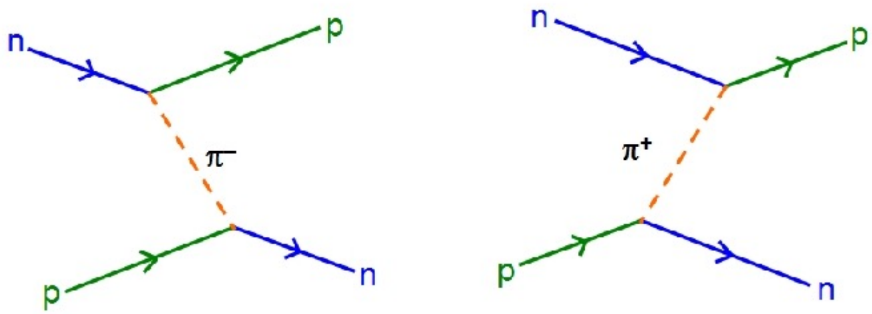
Again, what's coming up next?
— what is the next massive particle?
— what is behind nuclear force?
Hadrons! Specifically, Pions!

$$\pi^{\pm}, \pi^0$$

$$M_\pi \sim 140\text{MeV} \sim 10^{12} K$$

$$R_{nuc} \sim 1\,\text{fm} \sim 200\text{MeV}$$

Yukawa



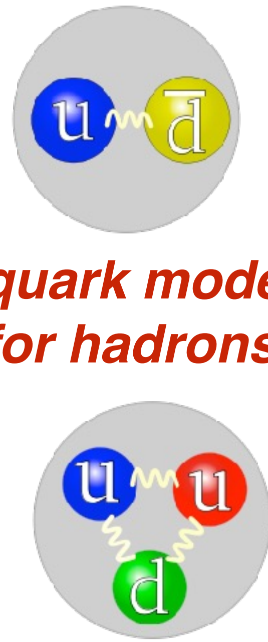
What do we expect next?

*** Heating toward T ~ M_pi , many pions are thermally produced.**
*** Repeating the same story of atoms at nuclear level?**
*** Many more hadron types, to be produced sequentially?**
*** Maybe hadrons to be broken up?**

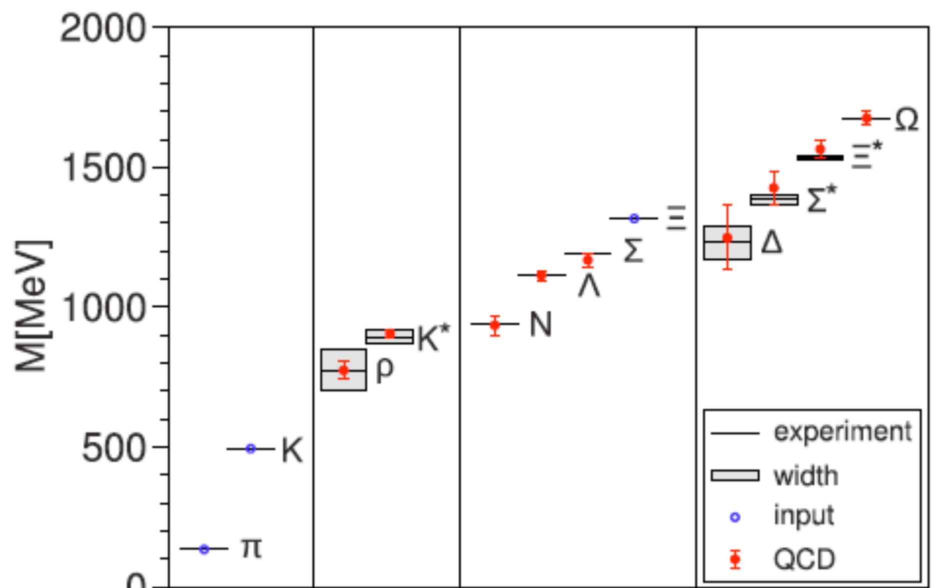
Heating It Up: The “Weird” Hadrons

As it turns out, there are thousands different types of hadrons…

Gell-mann



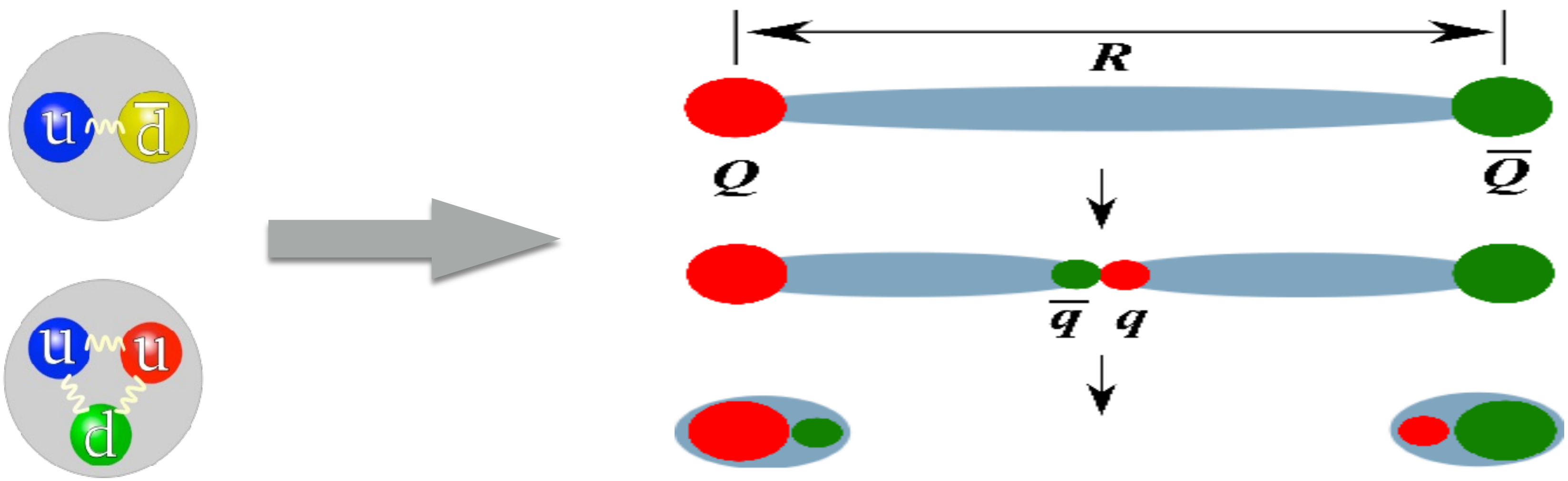
quark model for hadrons



Simply “atomic physics” for hadrons based on quarks?
The answer turns out to be no.

Heating It Up: The “Weird” Hadrons

Surprisingly, hadrons seem to be unbreakable!



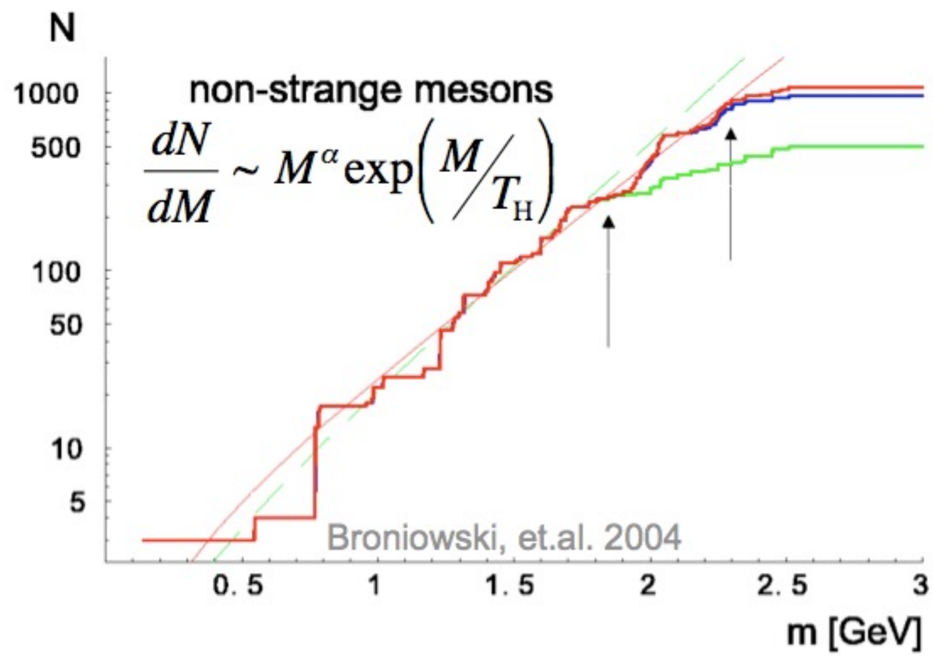
Upon injection of energy, a highly excited hadron becomes STRING like, and eventually breaks into more hadrons (not quarks)!

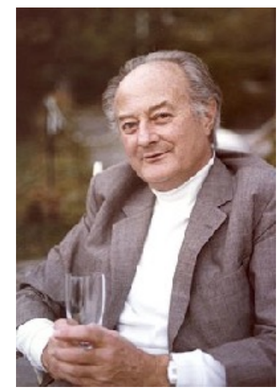
Heating It Up: The "Weird" Hadrons

So… are we going to stay with hadrons despite how hot we heat up matter?

The answer is NO! Surprisingly, there is a predicted limiting temperature for hadrons.

# of hadron types grows exponentially with mass!





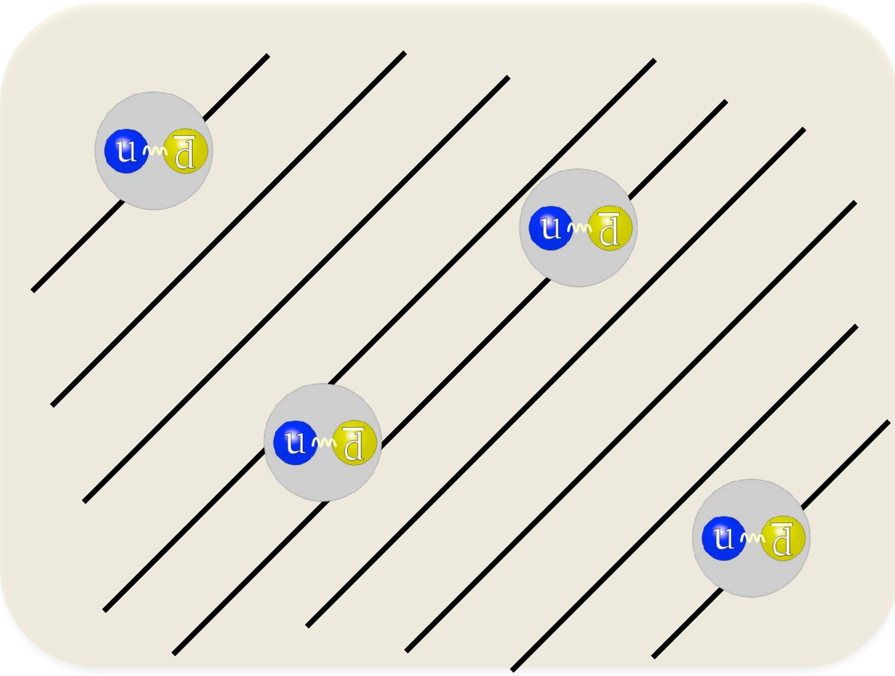
Hagedorn

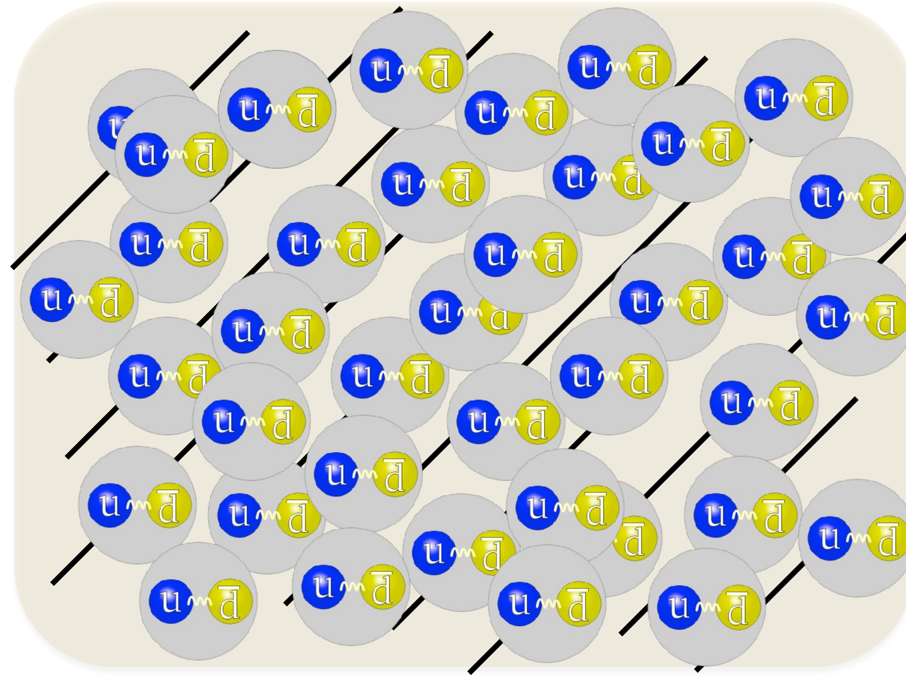
$$\epsilon \sim \int dM\, M^{\beta}\, e^{\frac{M}{T_H}}\, e^{-\frac{M}{T}}$$

This statistical sum diverges for T > T_H ~ 160MeV !

Exercise! **WHAT IS THE MATTER BEYOND THAT?**

Heating Nuclear Matter Up (or Compressing It)





It appears most certain that when the system is hot/dense enough:

(1) Individual hadrons will lose their identities —> quarks/gluons

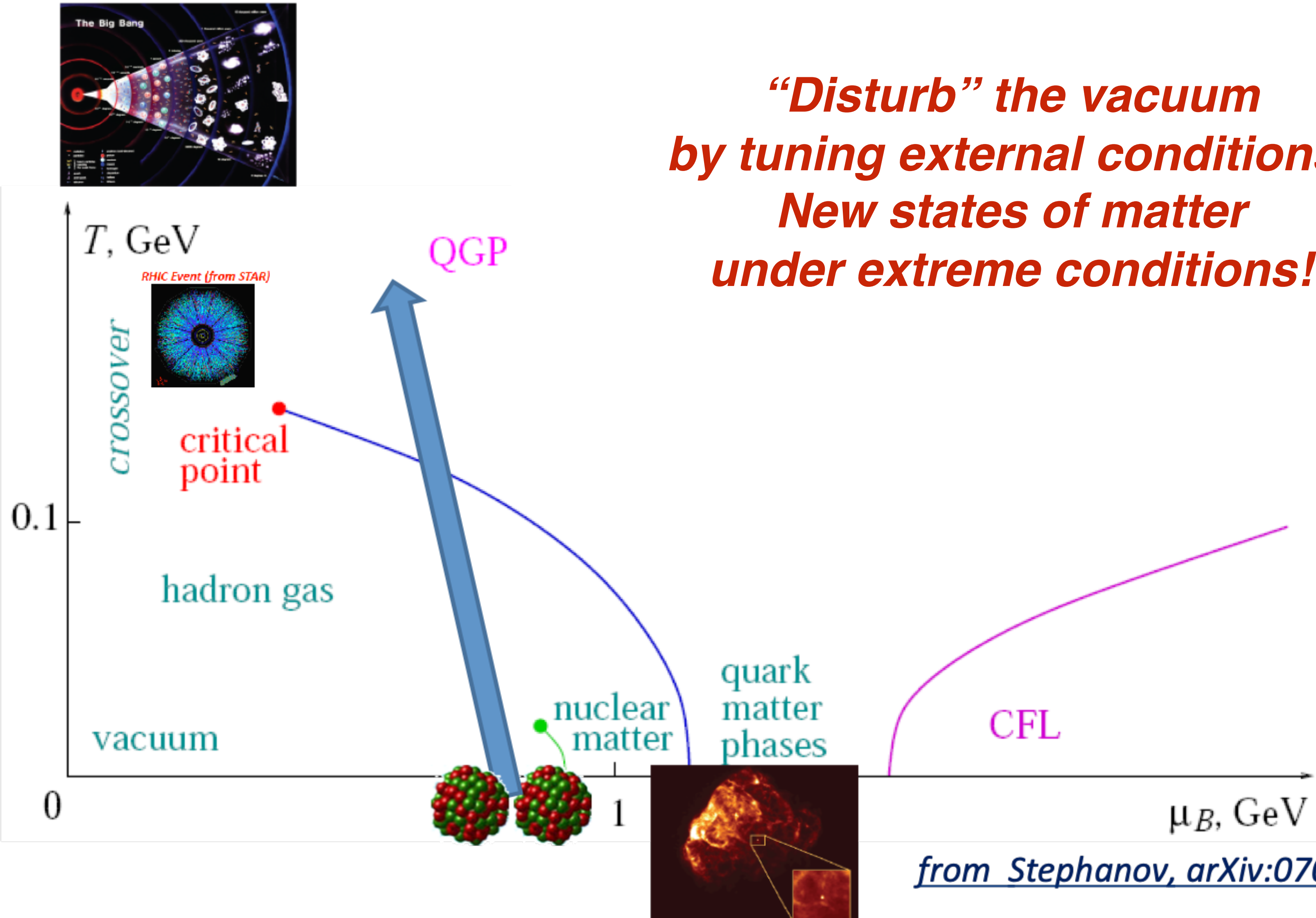
(2) The vacuum ordered structure would be destroyed.

—> Must be a distinctive new phase of nuclear matter !

[Early ideas:T.D.Lee, Wick, Collins, Perry, McLerran, Shuryak, Kapusta, Itoh, …]

Condensed Matter Physics of QCD

"Disturb" the vacuum
by tuning external conditions:
New states of matter
under extreme conditions!

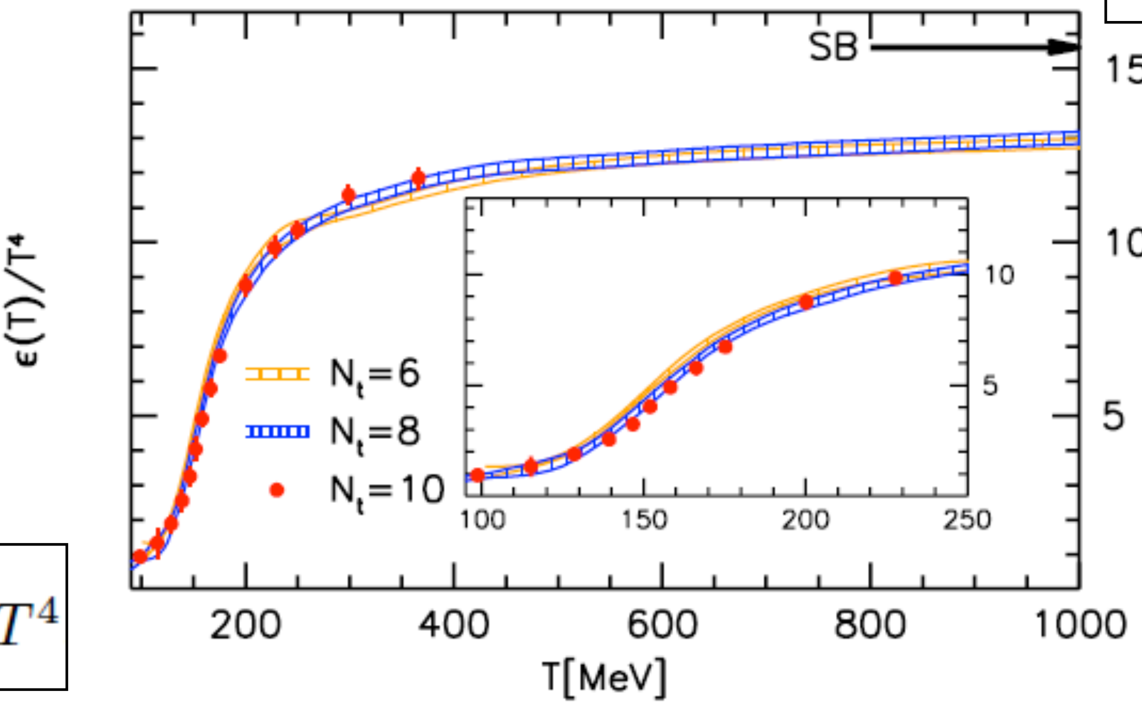


from Stephanov, arXiv:0701002

Answer from Lattice QCD

from Lattice QCD (Wuppertal-Budapest)

RHIC LHC



$$\epsilon = 47.5 \times \frac{\pi^2}{30} T^4$$

free QGP

a relativistic pion gas

$$\epsilon = 3 \times \frac{\pi^2}{30} T^4$$

More precisely, Hadron Resonance Gas

Exercise!

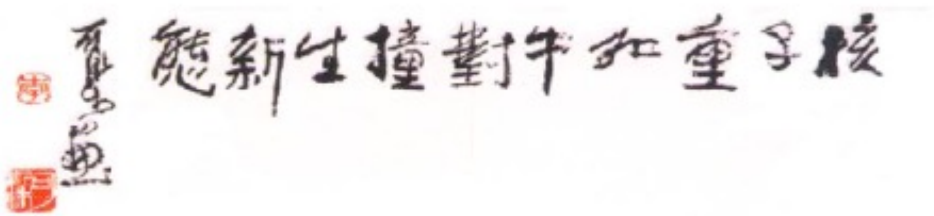
****** Two benchmarks at low/high T***

****** A transition regime in the middle***

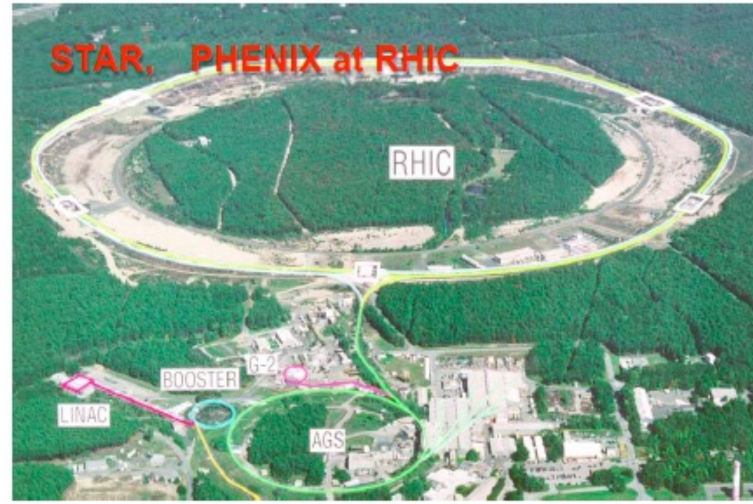
****** Crossover (instead of a phase transition)***

Little Bangs in Heavy Ion Collisions (HIC)

Quark Gluon Plasma (QGP):
A New phase of matter



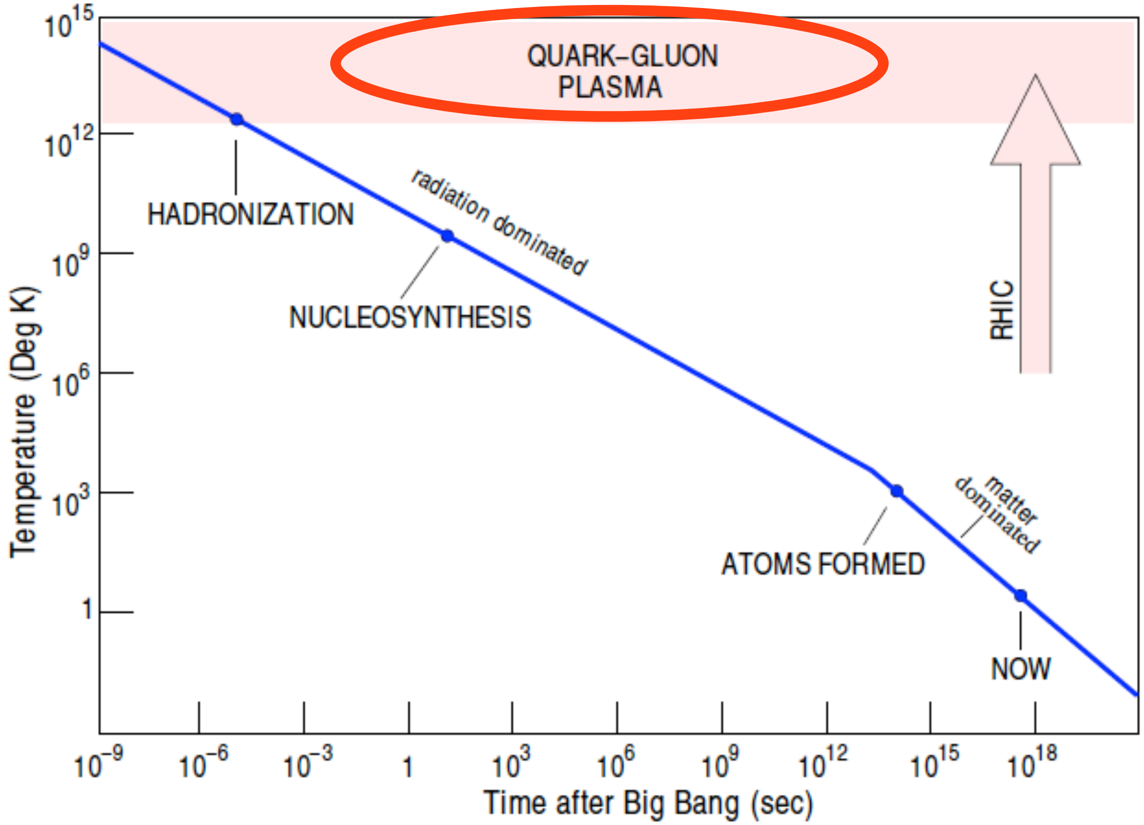
An artistic presentation:
"nuclei as heavy as bulls,
colliding into new phase of matter"



our most powerful
heating machine ever

QGP: An Old Phase of Matter

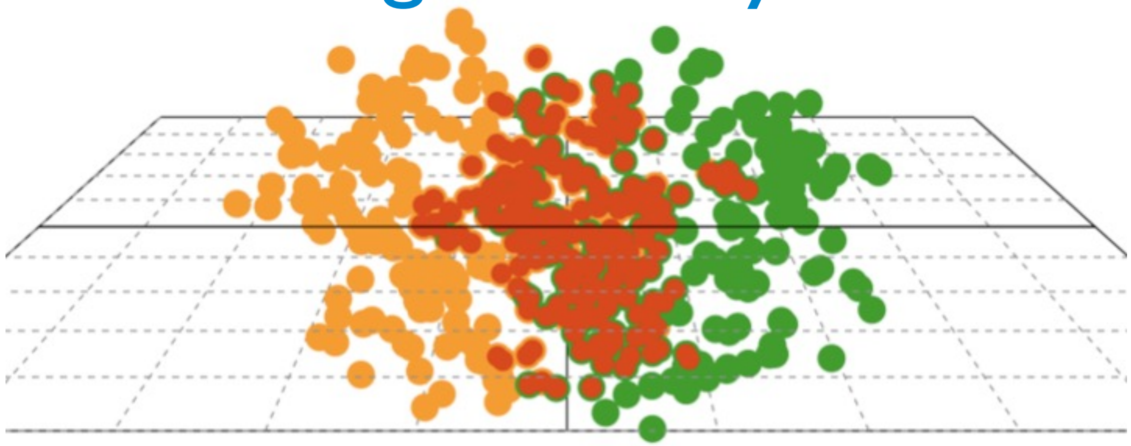
The highest ever temperature was in the beginning of universe.
The QGP temperature was available back then.



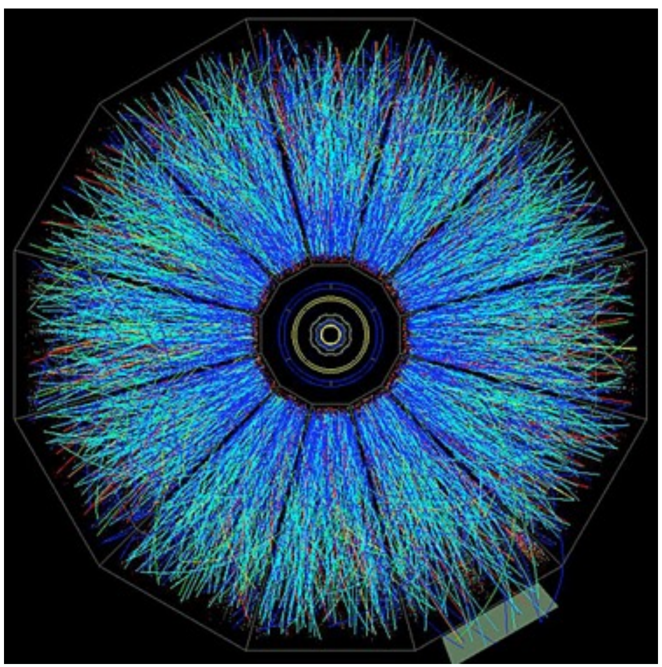
Heavy ion collision is the only laboratory experiment
that helps answer:
What was it like in the baby Universe?

Basics of Heavy Ion Collisions

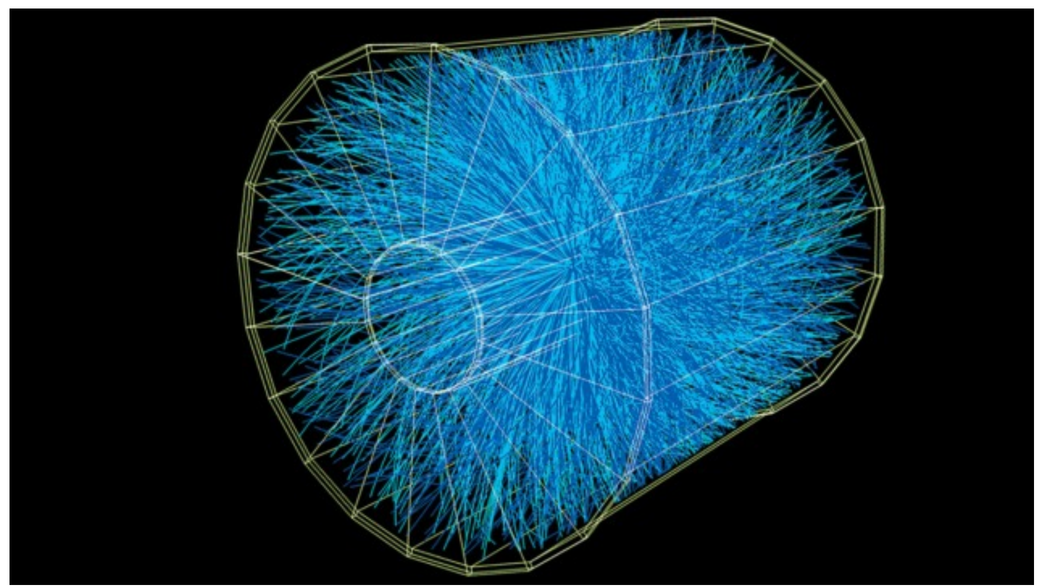
Little Bang in Heavy Ion Collisions (HIC)



Is QGP created?
Just how hot?
How do we know?

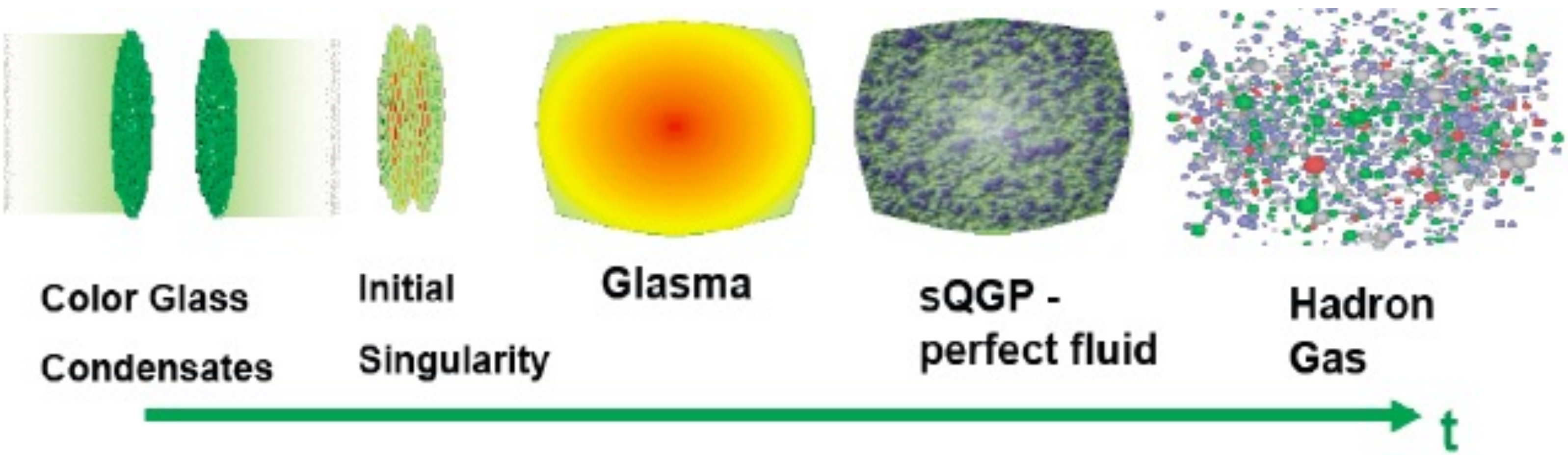


from STAR @ RHIC



from ALICE @ LHC

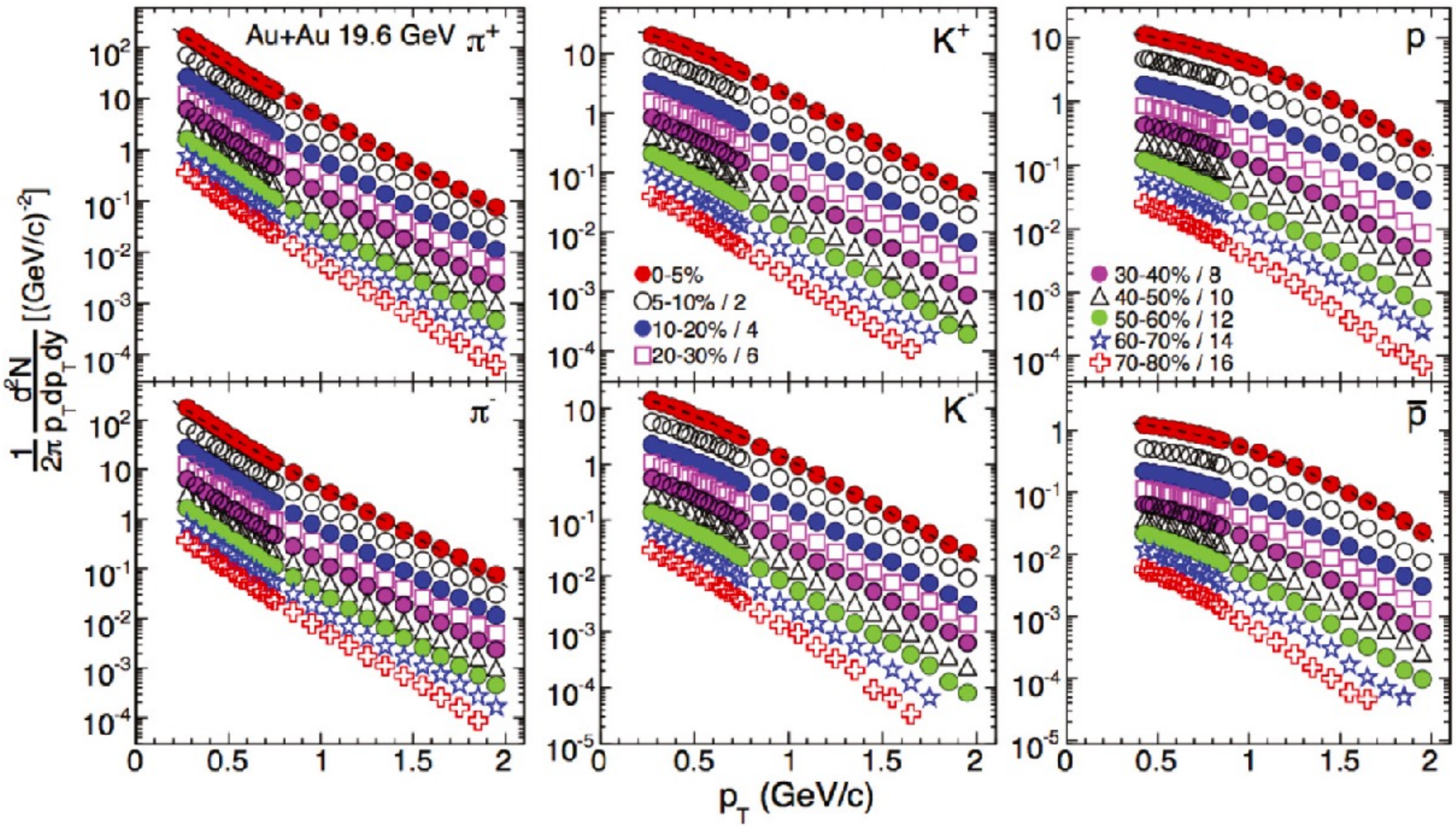
Some Basics of Heavy Ion Collisions



To give some ideas (taking Gold-Gold 200GeV at RHIC as example):

- 197 (79p+118n) nucleons colliding with 197 nucleons
  (Nuclei A as a handle)
- 100GeV/nucleon, 200GeV N-N C.M. energy, 42mb x-section
  (Collision Energy as a handle)
- 39TeV in, about 28TeV left in the middle → creating **~7500** particles
- **We observe the final state hadrons' identity and 3-momentum**
- Estimated initial temperature ~300MeV **(Trillion Kelvin)** > Tc ~170MeV
- Estimated initial energy density 5-10GeV/fm^3 > H.G. threshold 1GeV/fm^3

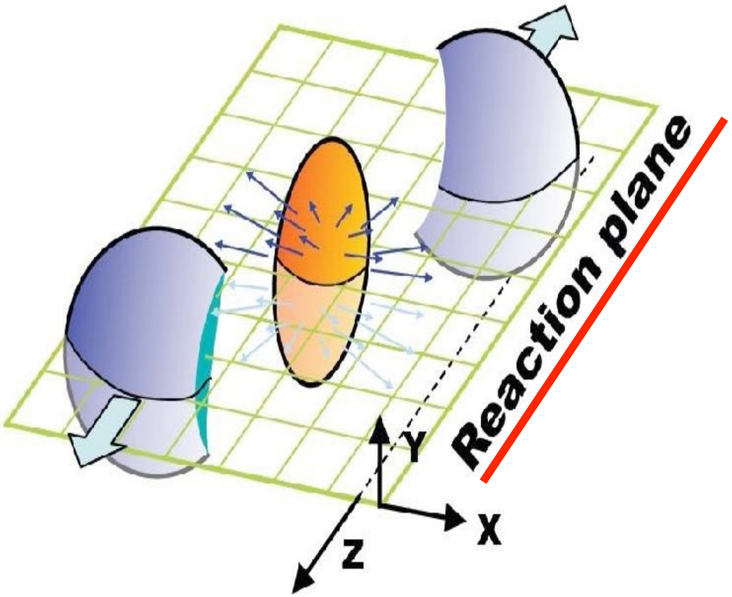
What You Can Actually "See"…

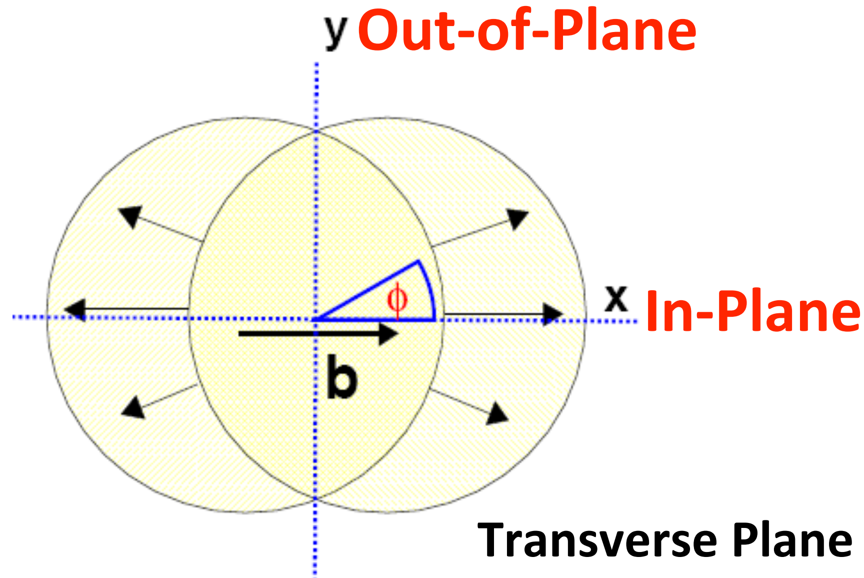


Detectors simply count many "particles" along with their p_T and y values

HIC Systematics

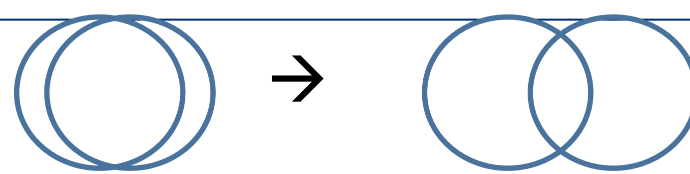
Centrality:





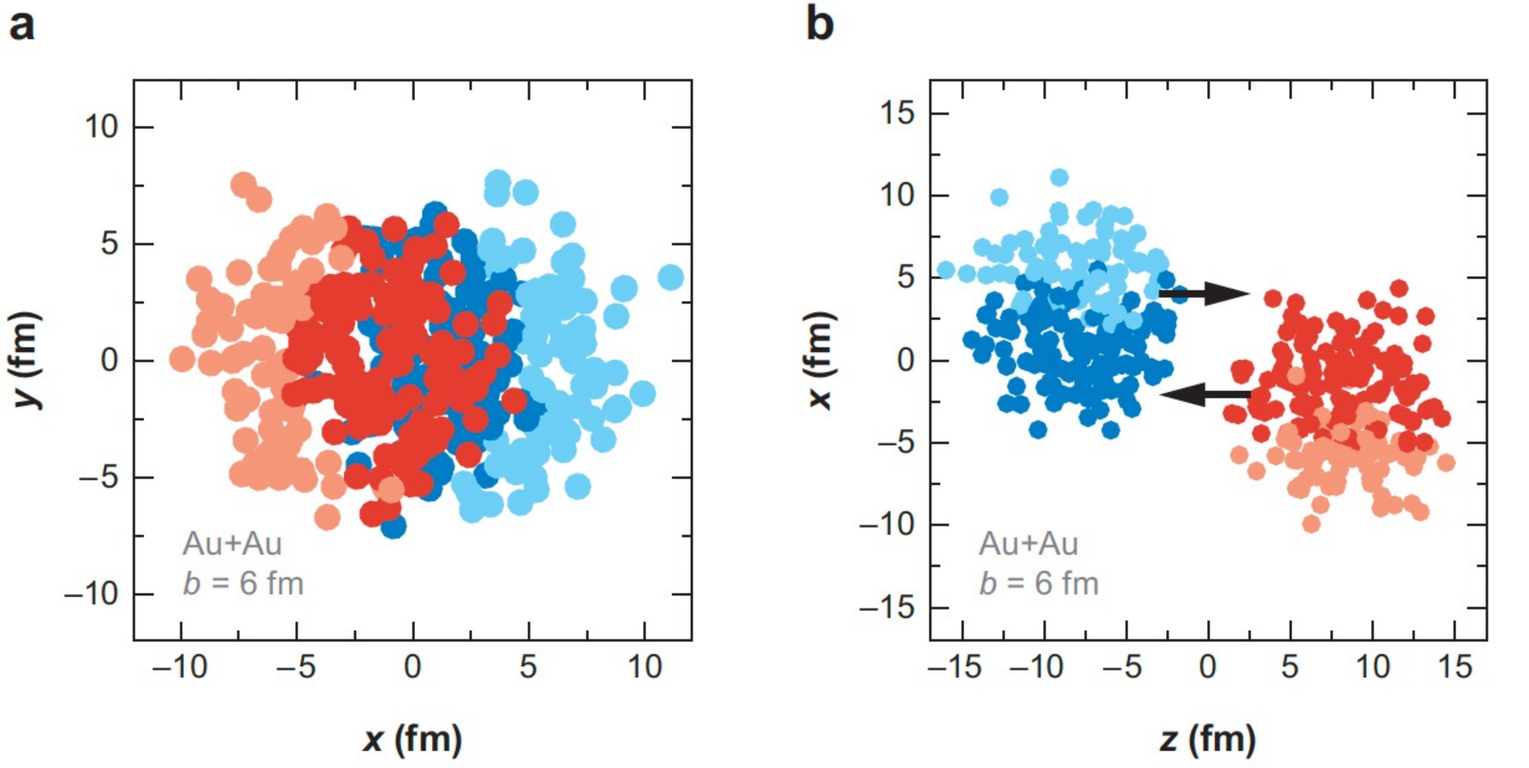
Centrality:	*(most) central*	→	*(most) peripheral*
Impact parameter b:	*(very) small*	→	*(very) large*
Initial geo. anisotropy:	*(very) small*	→	*(very) large*
Final hadron multiplicity:	*high*	→	*low (exp. classification)*

Fireball geometry from initial overlap: crucial !



Glauber Model

Connecting pre-collision nucleons within the atomic nucleus with initial condition upon collision: Which nucleons will collide? Which nucleons will not?



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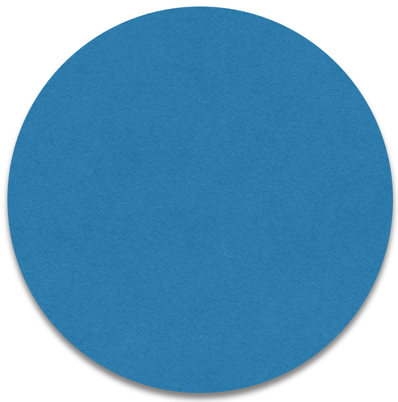
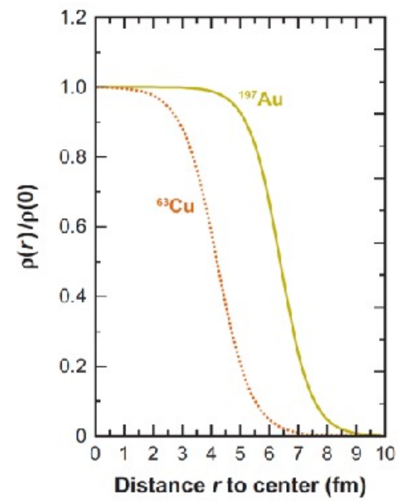
Glauber Model

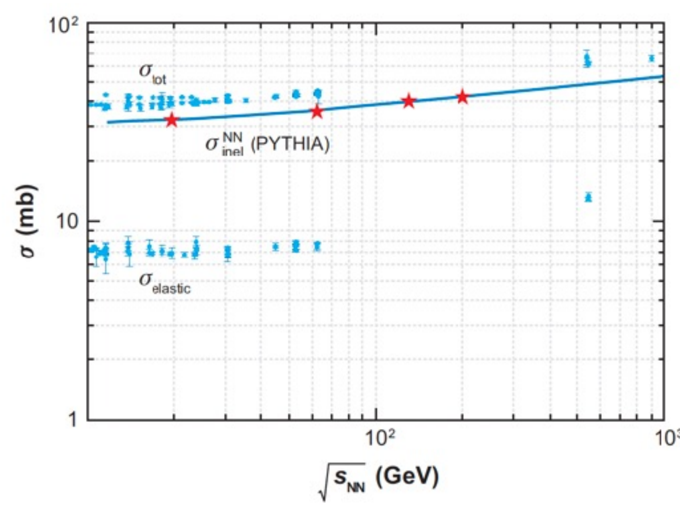
Initial nucleon distributions within an atomic nucleus is an important input !!

$$\rho(r) = \rho_0 \frac{1 + w(r/R)^2}{1 + \exp\left(\frac{r-R}{a}\right)},$$

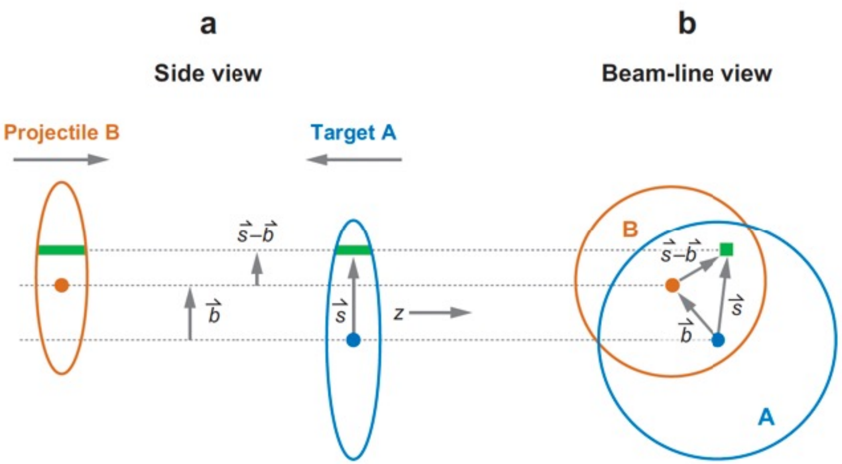




Nucleon-nucleon cross section



Glauber Model



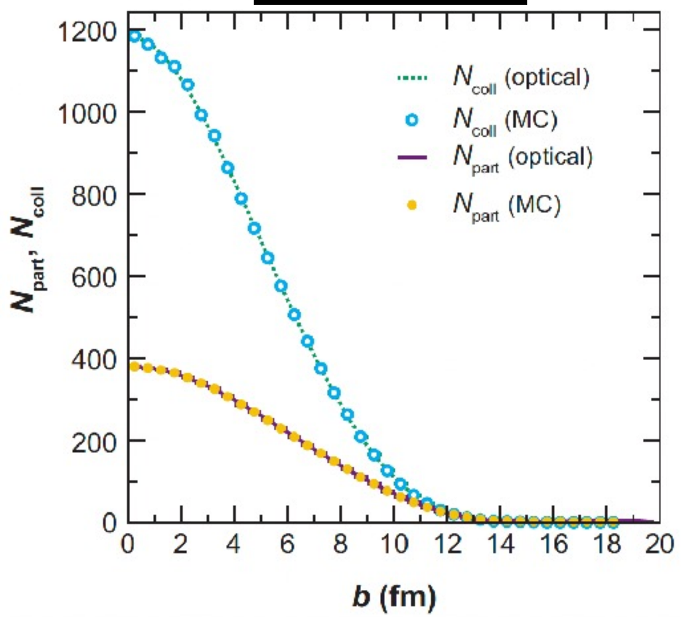
$$\hat{T}_A(\mathbf{s}) = \int \hat{\rho}_A(\mathbf{s}, z_A) dz_A$$

$$\hat{T}_{AB}(\mathbf{b}) = \int \hat{T}_A(\mathbf{s}) \hat{T}_B(\mathbf{s}-\mathbf{b}) d^2 s .$$

Exercise!

$$N_{part}(\mathbf{b}) = A \int \hat{T}_A(\mathbf{s}) \left\{ 1 - \left[1 - \hat{T}_B(\mathbf{s}-\mathbf{b}) \sigma_{inel}^{NN} \right]^B \right\} d^2 s + B \int \hat{T}_B(\mathbf{s}-\mathbf{b}) \left\{ 1 - \left[1 - \hat{T}_A(\mathbf{s}) \sigma_{inel}^{NN} \right]^A \right\} d^2 s$$

$$N_{coll}(b) = \sum_{n=1}^{AB} n P(n, b) = AB \hat{T}_{AB}(b) \sigma_{inel}^{NN},$$



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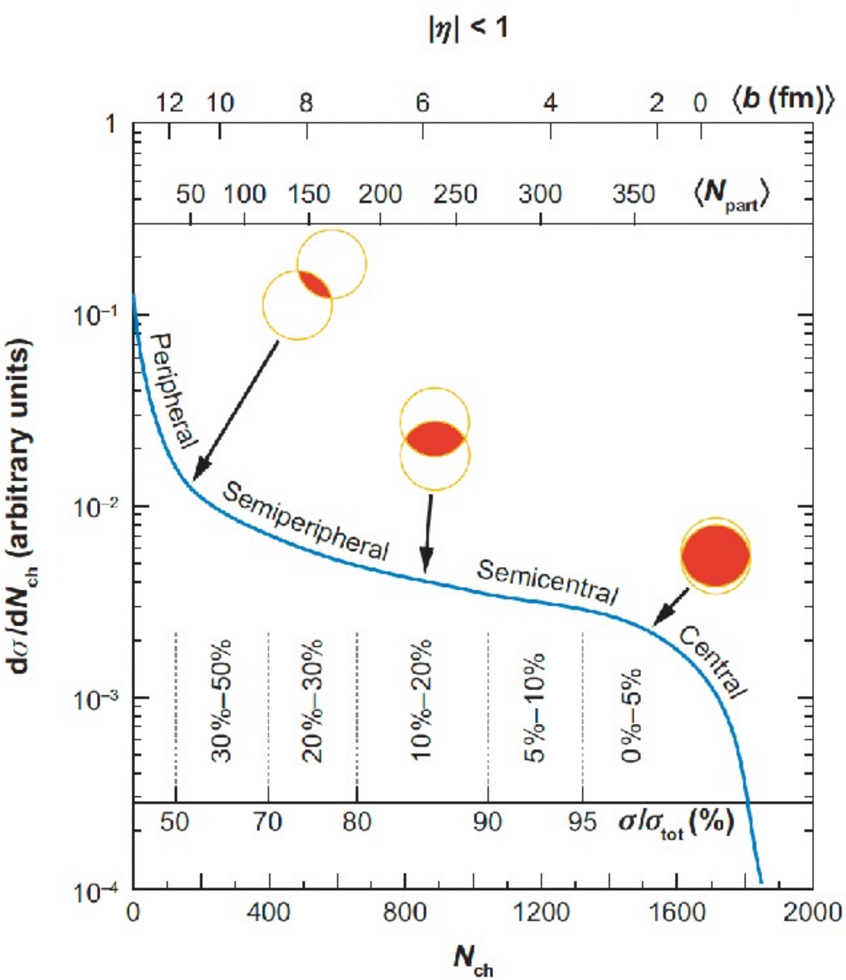
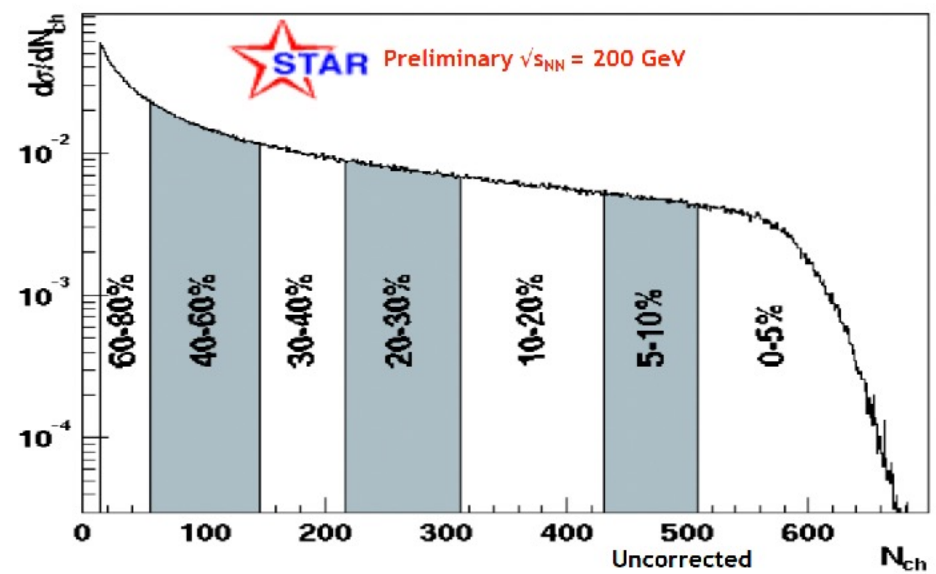
Glauber Model





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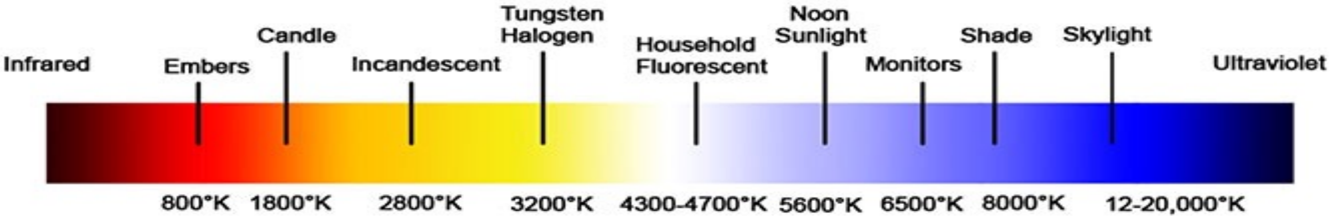
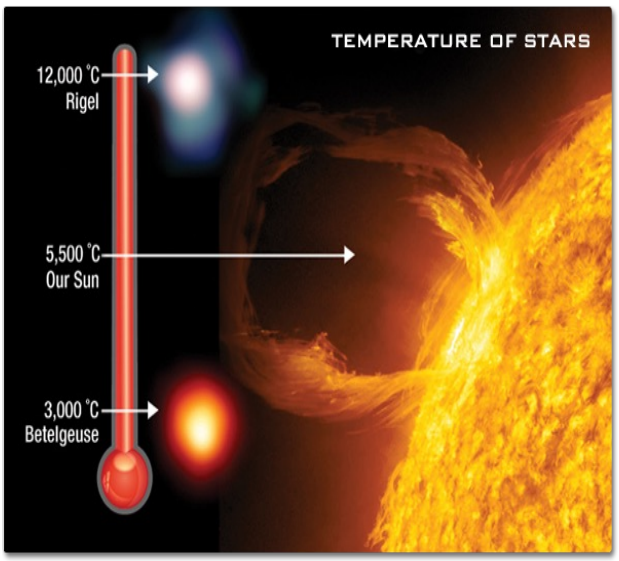
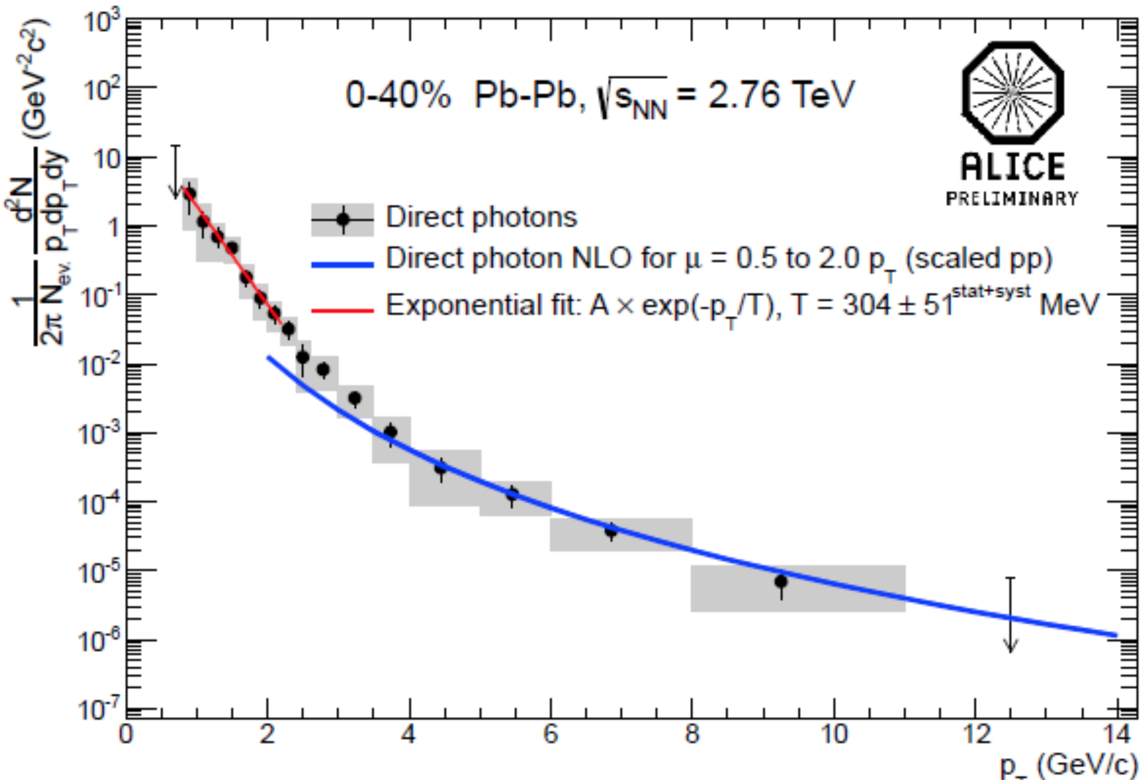
QGP Shining Bright!



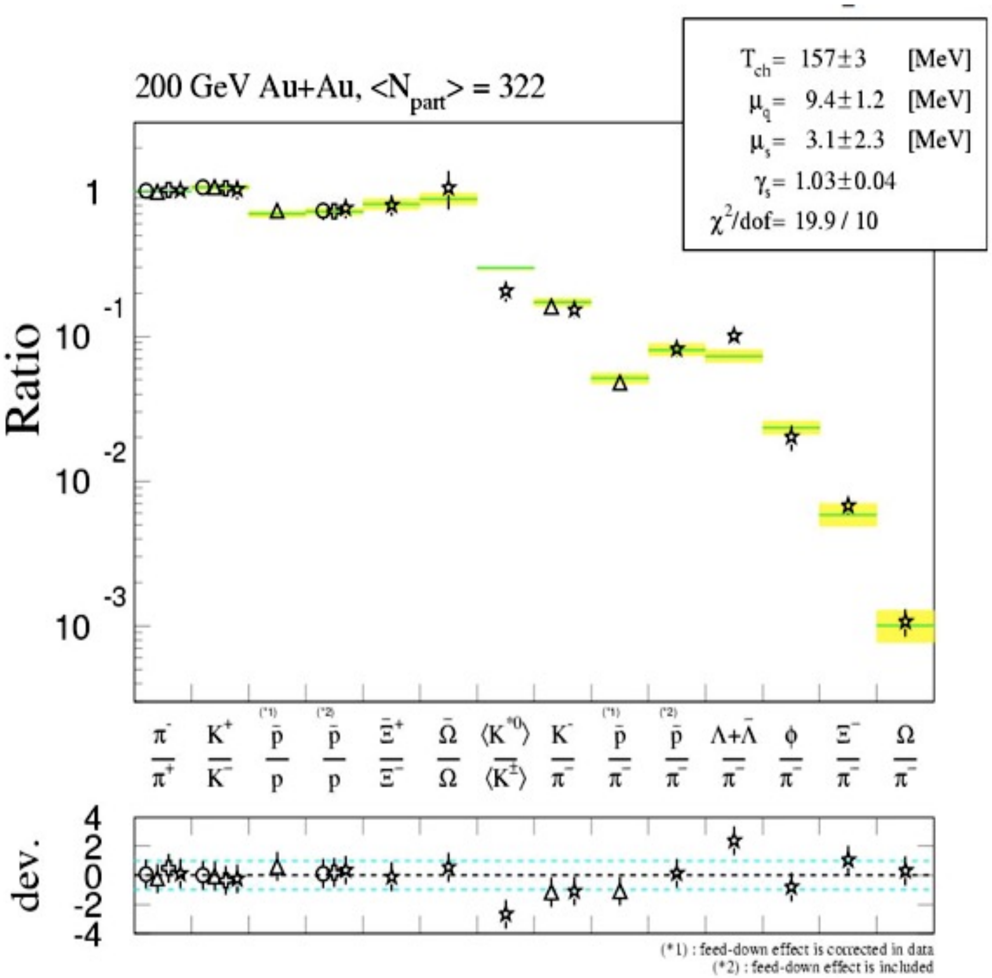




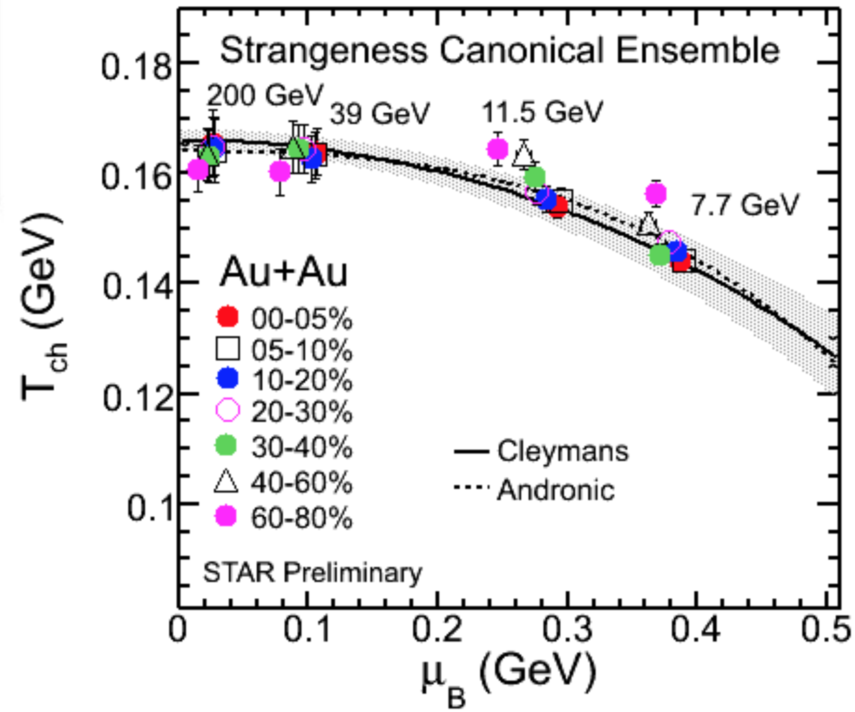
QGP is hot stuff: about trillion degrees !
Official Guinness World Record:
the highest man-made temperature!

$$n \propto \frac{1}{e^{\frac{E}{T}} - 1} \approx e^{-\frac{p_T}{T}}$$

QGP Thermally Produces Hadrons



From STAR

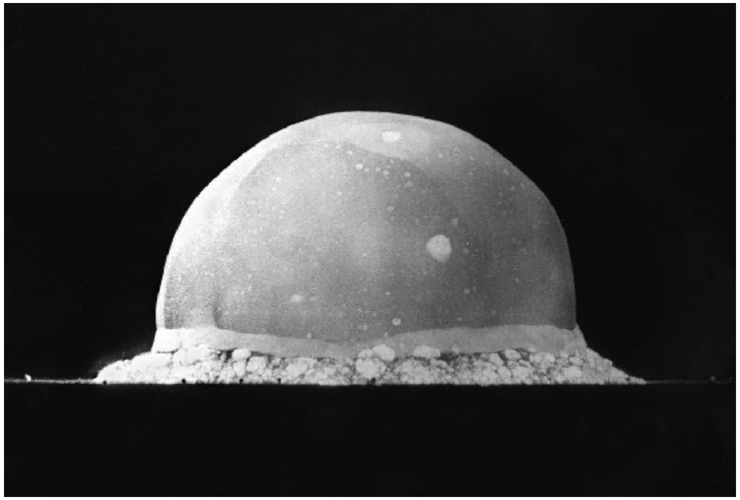


QGP is hot stuff!

$$n \propto \int d^3\vec{p}\, e^{-\frac{\sqrt{\vec{p}^2+m^2}-Q_i\mu_i}{T}}$$

Exercise!

Being Explosive!



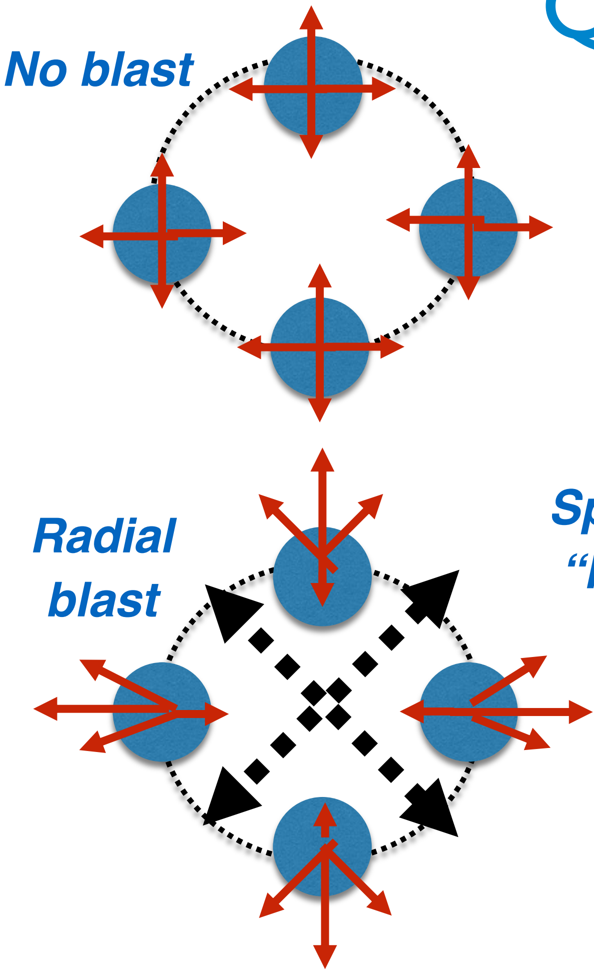
From Wikipedia about Trinity Bomb:

Fifty beryllium-copper diaphragm microphones were also used to record the pressure of the blast wave. These were supplemented by mechanical pressure gauges.[104] These indicated a blast energy of 9.9 kilotons of TNT (41 TJ) ± 0.1 kilotons of TNT (0.42 TJ). With only one of the mechanical pressure gauges working correctly that indicated 10 kilotons of TNT (42 TJ). [105]

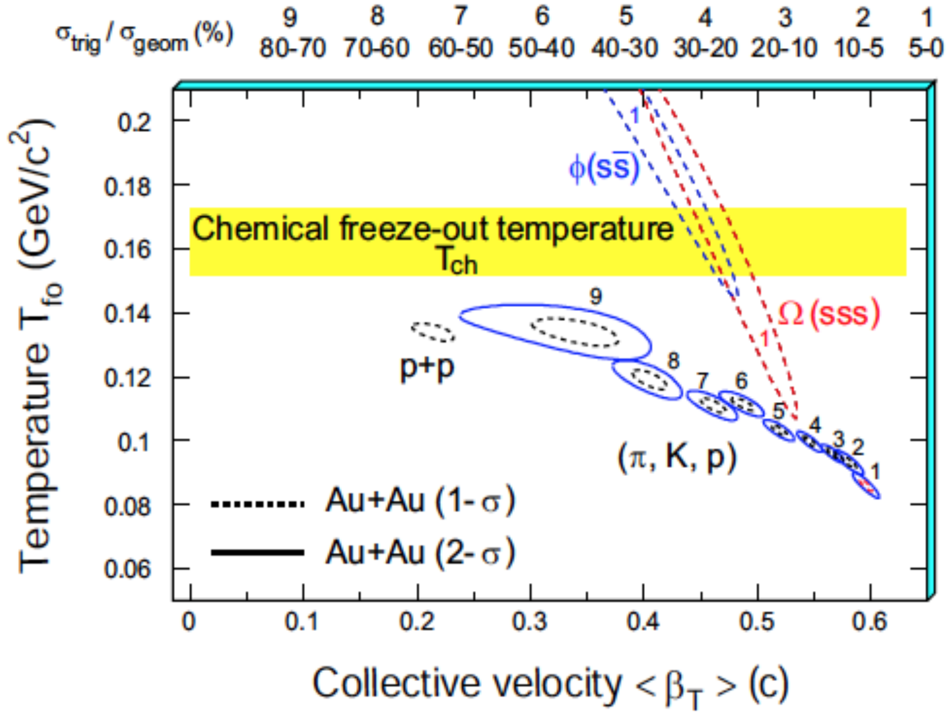
Fermi prepared his own experiment to measure the energy that was released as blast. He later recalled that:

About 40 seconds after the explosion the air blast reached me. I tried to estimate its strength by dropping from about six feet small pieces of paper before, during, and after the passage of the blast wave. Since, at the time, there was no wind I could observe very distinctly and actually measure the displacement of the pieces of paper that were in the process of falling while the blast was passing. The shift was about 2 1/2 meters, which, at the time, I estimated to correspond to the blast that would be produced by ten thousand tons of T.N.T.[106]

QGP Blasting Out!



Spectra with "blue shift"



Strong blast wave seen in final hadrons distributions

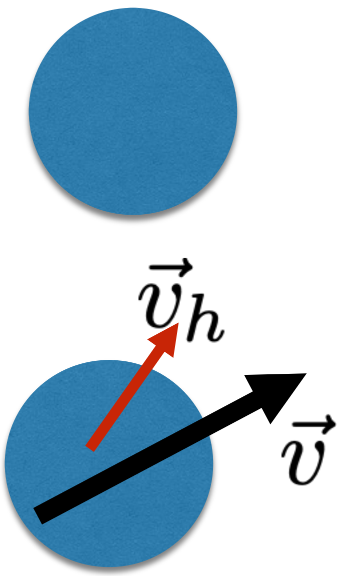
—> highly explosive

—> high initial energy density & pressure gradient!

$$\epsilon_{in} \sim 20GeV/fm^3 \gg 1 \sim 2GeV/fm^3$$

Blast Wave Model

Boosted thermal distribution:



$$f_{F/B} = \frac{1}{e^{E/T} \pm 1}$$

Exercise!

$$f_{F/B} = \frac{1}{e^{P \cdot u/T} \pm 1} \sim e^{-E\gamma(1 - \vec{v} \cdot \vec{v}_h)}$$

$$P^\mu = (E, \vec{p}) = E(1, \vec{v}_h)$$

$$u^\mu = \gamma(1, \vec{v}) \ , \ \gamma = \frac{1}{\sqrt{1 - \vec{v}^2}}$$

Cooper-Frye freeze-out: converting moving fluid into hadrons

$$E\frac{d^3 N_s}{dp^3} = g_s \int_\Sigma d\Sigma_\mu P^\mu f_{F/B}\left(\frac{P \cdot u}{T(X)}\right)$$

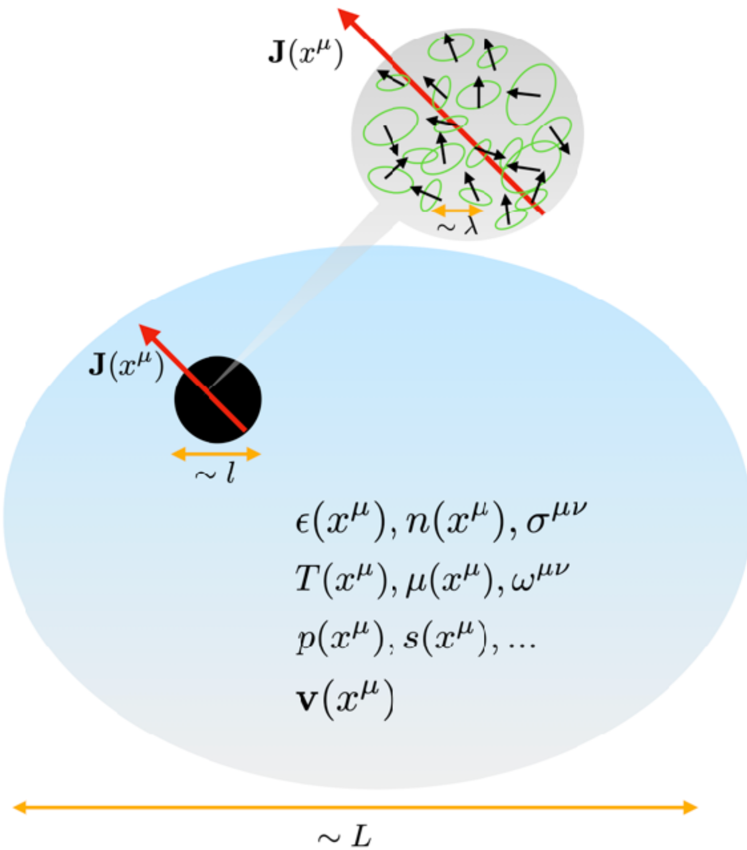
(integrating over freeze-out surface)

Collective Flow and Hydrodynamics

Hydrodynamics

Hydrodynamics: an effective theory of* many-body *systems at the long-time, large-distance limit —> only conservation laws!

Hydrodynamics ~ non-static "thermodynamics"



$\lambda \ll l \ll L$, a coarse-graining process

A key quantity: dissipative length/time scale: e.g. relaxation time, mean-free-path in kinetic regime

$$l \sim \tau \sim \frac{1}{n\sigma}$$

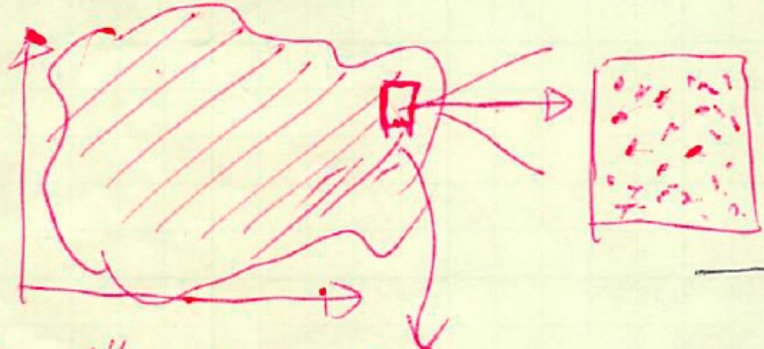
Hydrodynamics = Macroscopic conservation laws

See e.g. Landau and Lifshitz

NR Hydrodynamics

[*] Nonrelativistic Fluid Mechanics Equations

- Continuum matter: separation between microscopic/macroscopic scales



e.g. a drop of water containing $\sim 10^{22}$ molecules

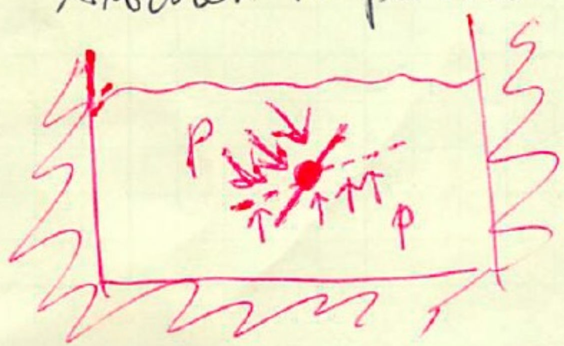
— we call it "coarse-graining"

"local": $\rho(x)$, $p(x)$ ...

— In F.M. we deal with fluid field variables rather than individual particles

- An obvious variable: mass density $\rho(x)$ ⟨e.g. water v.s. vapor⟩

- Another important variable: pressure $p(x)$



$$p = \frac{\text{Force}}{\text{Area}}$$

① Pressure is normal to "probe" surface

② Pressure is independent of orientation.

NR Hydrodynamics

⟨continued⟩

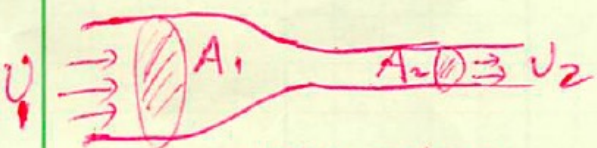
- The continuity Equation: conservation of mass ⟨energy in N.R.⟩

— velocity field for describing flow

$$\vec{v}(\vec{x}, t) = (v_x, v_y, v_z)$$

— Mass is conserved!

— A simple example



water pipe

$$(v_1 \Delta t) A_1 = (v_2 \Delta t) A_2$$

$$\Rightarrow v_1 A_1 = v_2 A_2$$

NR Hydrodynamics

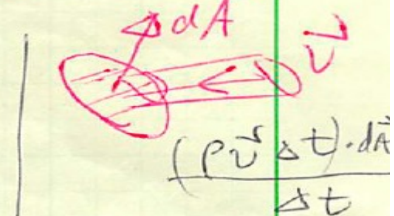
— A more general treatment:

① consider a closed volume V, counting the mass inside

$$\Rightarrow \int_V d^3\vec{x}\,\rho(\vec{x},t) \Rightarrow \frac{d}{dt}\int_V d^3\vec{x}\,\rho = \int_V d^3\vec{x}\,\frac{\partial\rho}{\partial t}$$

② $$\int_V d^3\vec{x}\,\frac{\partial\rho}{\partial t} = -\int_A d\vec{A}\cdot(\rho\vec{v})$$

③ Divergence theorem: $$\int_A d\vec{A}\cdot(\rho\vec{v}) = \int_V d^3\vec{x}\,\vec{\nabla}\cdot(\rho\vec{v})$$



$$\Rightarrow \int_V d^3\vec{x}\left[\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v})\right] = 0$$ for any volume element

$$\Rightarrow \boxed{\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0}$$ Continuity Equation

↑ mass current density $\vec{j}_m = \rho\vec{v}$

— $$\frac{\partial\rho}{\partial t} + (\vec{\nabla}\rho)\cdot\vec{v} + \rho\,\vec{\nabla}\cdot\vec{v} = 0$$

$$\Rightarrow \frac{\hat{d}\rho}{dt} = \left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\rho \Rightarrow \frac{\hat{d}\rho}{dt} + \rho\,\vec{\nabla}\cdot\vec{v} = 0$$

— Incompressible flow: $\vec{\nabla}\cdot\vec{v} = 0$ with $\frac{\hat{d}\rho}{dt} = \frac{\partial\rho}{\partial t} = 0$

NR Hydrodynamics

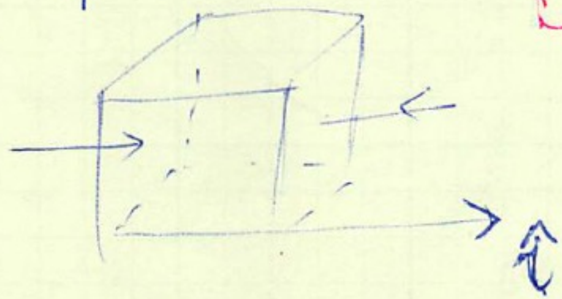
- Conservation of Momentum: Stress Tensor & Euler's Theorem

— consider total momentum enclosed in the volume V and see how it changes

$$\frac{d}{dt}\int_V d^3\vec{x}\,(\rho \vec{v}_i) = \int_V d^3\vec{x}\,\frac{\partial}{\partial t}(\rho \vec{v}_i)$$

— Now we consider factors that cause change of momentum: forces! Transport!

① pressure force $-\int_A d\vec{A}_i \cdot p(\vec{x})$



⟨like surface force!⟩

② bulk volume force (e.g. from gravity)

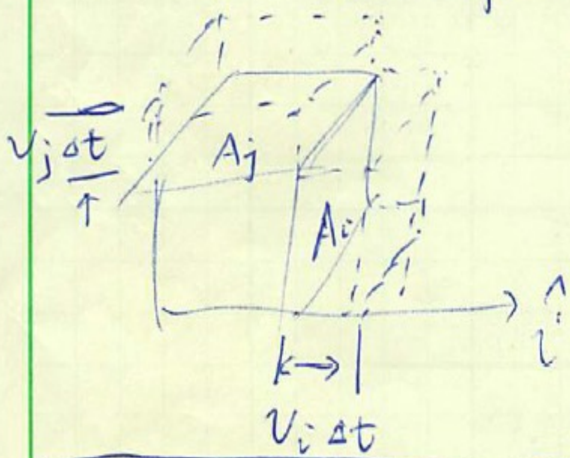
$\Rightarrow \int_V d^3\vec{x}\,\rho f_i$

$\vec{f}$: force per unit mass

⟨e.g. Earth gravity $\vec{f} = \vec{g}$⟩

NR Hydrodynamics

③ Transport



$$-\int_A (d\vec{A}\cdot\vec{v})(\rho \vec{v}_i)$$

momentum flowing out!

— Putting all pieces together

$$\frac{d}{dt}\int_V d^3\vec{x}\,\rho v_i = \int_V d^3\vec{x}\,\frac{\partial}{\partial t}(\rho v_i)$$

$$= -\int_A d\vec{A}_i\, p - \int_A dA_j v_j (\rho v_i) + \int_V d^3\vec{x}\, f_i \rho$$

$$\Rightarrow \int_V d^3\vec{x}\,\partial_t(\rho v_i) = -\int_A dA_j T_{ij} + \int_V d^3\vec{x}\,\rho f_i$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho v_i) + \sum_{j=1}^{3}\frac{\partial T_{ij}}{\partial x_j} = \rho f_i \quad (i=1,2,3)$$

Stress Tensor

$$T_{ji} = T_{ij} \equiv p\delta_{ij} + \rho v_i v_j$$

$$\begin{pmatrix} p+\rho v_1^2 & \rho v_1 v_2 & \rho v_1 v_3 \\ \rho v_1 v_2 & p+\rho v_2^2 & \rho v_2 v_3 \\ \rho v_1 v_3 & \rho v_2 v_3 & p+\rho v_3^2 \end{pmatrix}$$

NR Hydrodynamics

<continued>

- Summarizing N.R. F.M. equations $\left\{ \begin{array}{l} \rho(\vec{x},t) \\ p(\vec{x},t) \\ v_i(\vec{x},t) \\ i=1,2,3 \end{array} \right.$

① Continuity Eq. $\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$, where $\vec{\nabla}\cdot(\rho\vec{v}) = \sum_{j=1}^{3} \frac{\partial(\rho v_j)}{\partial x_j}$ <conservation of mass>

② Euler's Eq. $(i=1,2,3)$ $\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^{3} \frac{\partial T_{ij}}{\partial x_j} = \rho f_i$ <conservation of momentum>

$\Updownarrow$

$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \vec{f} \iff \boxed{\frac{\hat{d} v_i}{dt} = \frac{\partial v_i}{\partial t} + (\vec{v}\cdot\vec{\nabla}) v_i = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + f_i}$

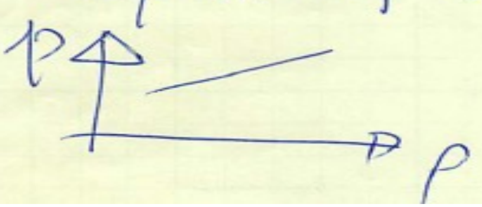
<Euler's Eq: 2nd Newton Law for fluid cells!!>

<Pressure gradient drives flow!!>

③ closing the Eqns: 5 variables v.s. 4 F.M. equations

$\Rightarrow$ Need one more relation!!

Equation of state: $\rho = \rho(p)$ or $p = p(\rho)$



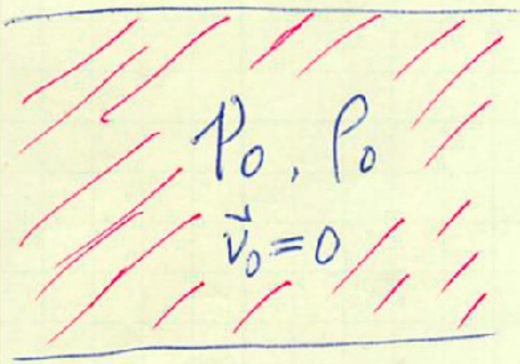
<Underlying assumption: locally true everywhere; local thermal equilibrium!>

NR Hydrodynamics: Sound Wave

- Physics of sound waves: propagation of the small density/pressure fluctuations across a fluid.

  <deviation from equilibrium>

- Consider a uniform, stationary fluid.



— Adding "disturbance"

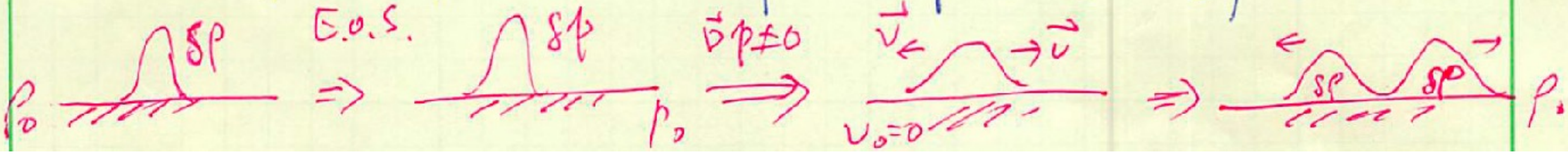
$$\Rightarrow \rho = \rho_0 + \delta\rho \,, \quad p = p_0 + \delta p$$

$$\vec{v} = 0 + \vec{v}$$

small fluctuations

$\Rightarrow$ Linearized F.M. equations.

— physical picture: $\delta\rho \Rightarrow \delta p \Rightarrow \vec{v} \Rightarrow \delta\rho \Rightarrow \ldots\ldots$



NR Hydrodynamics: Sound Wave

- Linearized continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = \frac{\partial(\rho_0+\delta\rho)}{\partial t} + \vec{\nabla}\cdot\left[(\rho_0+\delta\rho)\vec{v}\right] = 0$$

To linear order of "fluctuations" $(\delta\rho, \delta P, \vec{v})$

$$\Rightarrow \boxed{\frac{\partial(\delta\rho)}{\partial t} + \rho_0 \vec{\nabla}\cdot\vec{v} = 0}$$

ρ_0 is constant from background fluid!

Note: $\partial_t P_0 = 0$, $\partial_t \rho_0 = 0$, $\vec{\nabla}\rho_0 = 0$

- Linearized Euler's equations (ignoring external force!!)

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \vec{f} \Rightarrow \frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho_0+\delta\rho}\vec{\nabla}(P_0+\delta P)$$

To linear order of "fluctuations"

$$\boxed{\frac{\partial\vec{v}}{\partial t} = -\frac{1}{\rho_0}\vec{\nabla}(\delta P)}$$

$\Rightarrow$ In component:

$$\begin{cases} \dfrac{\partial v_1}{\partial t} = -\dfrac{1}{\rho_0}\dfrac{\partial\delta P}{\partial x} \\ \dfrac{\partial v_2}{\partial t} = -\dfrac{1}{\rho_0}\partial\delta P/\partial y \\ \partial v_3/\partial t = -\dfrac{1}{\rho_0}\partial\delta P/\partial z . \end{cases}$$

ρ_0 is constant from background fluid.

NR Hydrodynamics: Sound Wave

- Summarizing linearized equations for sound wave

$$\frac{\partial(\delta\rho)}{\partial t} + \rho_0 \vec{\nabla}\cdot\vec{v} = 0 \qquad \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_0}\vec{\nabla}(\delta p)$$

— Now: Equation of state! ⇒ δp and $\delta\rho$ are NOT independent!

$$p = p_0 + \delta p = p(\rho_0 + \delta\rho) = p_0 + \left(\frac{\partial p}{\partial \rho}\right)\delta\rho + \ldots\ldots$$

⇒ To linear order: $\delta p = \left(\frac{\partial p}{\partial \rho}\right)\cdot\delta\rho = c_s^2\cdot(\delta\rho)$

where the constant $c_s^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)\Big|_0$ from E.O.S. !

<This constant depends on the matter properties>

$$\Rightarrow \begin{cases} \dfrac{\partial(\delta\rho)}{\partial t} + \rho_0 \vec{\nabla}\cdot\vec{v} = 0 & (1) \\ c_s^2 \vec{\nabla}(\delta\rho) + \rho_0 \dfrac{\partial \vec{v}}{\partial t} = 0 & (2) \end{cases}$$

Now: $\partial_t (1) - \vec{\nabla}\cdot(2)$

$$\Rightarrow \boxed{\frac{\partial^2(\delta\rho)}{\partial t^2} - c_s^2 \vec{\nabla}^2(\delta\rho) = 0}$$

This is a typical wave equation!!

NR Hydrodynamics: Sound Wave

— 1D version (e.g. propagating along $\hat{x}$ direction)

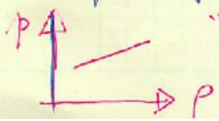
$$\frac{\partial^2 \delta\rho}{\partial t^2} - c_s^2 \frac{\partial^2 \delta\rho}{\partial x^2} = 0 \Rightarrow \delta\rho = A\cos(\omega t - kx - \phi)$$

$$\Rightarrow \boxed{\omega^2 - c_s^2 k^2 = 0}$$

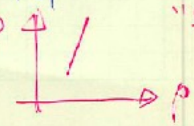
This is sound wave dispersion, with c_s the speed of sound!

— More general: $\delta\rho = A\cos(\omega t - \vec{k}\cdot\vec{x} - \phi)$ $\Rightarrow \omega^2 - c_s^2|\vec{k}|^2 = 0$

— Speed of sound $c_s = \sqrt{\partial p/\partial\rho}$



"soft E.O.S" small c_s! <e.g. gas>



"steep E.O.S" large c_s! <e.g. liquid, solid>

☒ Another example: hydrostatics

$$\begin{cases} \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0 \\ \frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \vec{f} \end{cases}$$

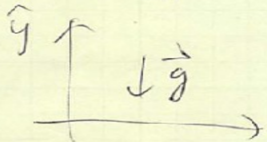
"statics" $\Rightarrow \vec{v} = 0$

$$\Rightarrow \begin{cases} \frac{\partial\rho}{\partial t} = 0 \\ -\frac{1}{\rho}\vec{\nabla}p + \vec{f} = 0 \end{cases}$$

E.O.S. $\frac{\partial\rho}{\partial t} = \frac{1}{c_s^2}\frac{\partial p}{\partial t} = 0$

$$\Rightarrow \boxed{c_s^2 \frac{\vec{\nabla}\rho}{\rho} = \vec{f}}$$

$\Rightarrow$ Application to gravity $\vec{f} = -g\,\hat{y}$



$$\Rightarrow \partial_x\rho = \partial_z\rho = 0$$

$$c_s^2 \frac{1}{\rho}\frac{\partial\rho}{\partial y} = -g$$

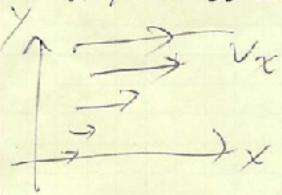
$$\Rightarrow \boxed{\rho = \rho_0 e^{-\frac{gy}{c_s^2}}}$$

<More nontrivial: application to star structure; even neutron star, TOV Eq.>

NR Viscous Hydrodynamics

[*] Viscous fluid

- The concept of viscosity: internal frition inside fluid



$\frac{\partial v_x}{\partial y} \neq 0 \Rightarrow$ drag between layers

Newton: $F_x = -\eta A \frac{\partial v_x}{\partial y}$

$\Rightarrow$ It must affect Euler's equations in some way

- Euler's Eqs. (ignore external forces)

$$\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^{3} \frac{\partial T_{ij}}{\partial x_j} = 0$$

Ideal fluid: $T^0_{ij} = p\delta_{ij} + \rho v_i v_j$

Viscous fluid T_{ij}

$\Rightarrow T^0_{ij} - \delta T_{ij}$

Concept of gradient expansion:

$$\hat{O}(1) + \hat{O}(l/L) + \hat{O}(l^2/L^2) + ...$$

- Determining viscous tensor: the 2nd law of thermodynamics

$\Rightarrow$ entropy only grows <in dissipative processes!>

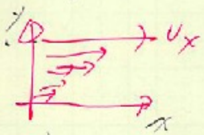
$$\Rightarrow \delta T_{ij} = \eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\,\vec{\nabla}\cdot\vec{v}\right) + \zeta\,\delta_{ij}(\vec{\nabla}\cdot\vec{v})$$

<Navier-Stokes>

— coming from gradient of velocity ("friction")

— η, ζ: shear and bulk viscosity

— e.g.:



$\delta T_{xy} = \delta T_{yx} = \frac{\partial v_x}{\partial y}$

NR Viscous Hydrodynamics

- Viscous hydro equation

$$\Rightarrow \frac{\partial(\rho v_i)}{\partial t} + \sum_j \frac{\partial T_{ij}}{\partial x_j} = \rho f_i \qquad \longrightarrow \quad T_{ij} = T^0_{ij} + \delta T_{ij}$$

$$\Rightarrow \underbrace{\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = \vec{f} - \frac{1}{\rho}\vec{\nabla} p}_{\text{Euler's Eqns. (ideal fluid)}} + \underbrace{\frac{\eta}{\rho}\vec{\nabla}^2\vec{v} + \frac{1}{\rho}\left(\zeta + \frac{1}{3}\eta\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{v})}_{\text{viscous correction}}$$

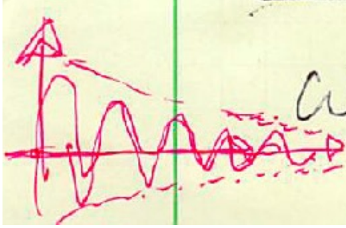
— Incompressible flow: $\vec{\nabla}\cdot\vec{v} = 0$

$$\Rightarrow \frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = \vec{f} - \frac{1}{\rho}\vec{\nabla} p + \left(\frac{\eta}{\rho}\right)\vec{\nabla}^2\vec{v}$$

(Navier–Stokes Equation)

$$\left\langle \eta_{water} \sim 100\, \eta_{air}, \quad \left(\frac{\eta}{\rho}\right)_{water} \sim \frac{1}{10}\left(\frac{\eta}{\rho}\right)_{air} \right\rangle$$

— Sound wave in viscous fluid



$$\omega \simeq c_s k - \frac{i}{2} k^2 \left(\frac{\frac{4}{3}\eta + \zeta}{\rho_0}\right)$$

dissipative term

$\Rightarrow$ sound damping!!

Relativistic Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0,$$ **Energy-momentum conservation**

$$\partial_\mu N^\mu = 0,$$ **Charge conservation**

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \widetilde{T}^{\mu\nu}$$

$$N^\mu = n u^\mu + \widetilde{N}^\mu\,,$$

$$\epsilon = -p + Ts + \mu n$$

& Equation of State (EOS)

$$\partial_\mu S^\mu \geq 0.$$

For ideal hydro:

$$T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu},\; N^\mu_{(0)} = n u^\mu.$$

For Navier-Stokes hydro (Eckart frame):

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi)\,\Delta^{\mu\nu} + 2u^{(\mu}q^{\nu)} + \pi^{\mu\nu},$$
$$N^\mu = n u^\mu,$$

$$\Pi = -\zeta\theta,$$
$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle},$$
$$q^\mu = \lambda T\left(\frac{\nabla^\mu T}{T} - Du^\mu\right)$$

Transport coefficients:
shear/bulk viscosity, heat conductivity, …

Example: Isotropic Expansion

$$\tau = \sqrt{t^2 - z^2}, \quad \eta = \frac{1}{2}\ln\frac{t+z}{t-z}, \qquad t = \tau\cosh\eta, \quad z = \tau\sinh\eta,$$

$$\rho = \sqrt{x^2 + y^2}, \quad \phi = \frac{1}{2i}\ln\frac{x+y\cdot i}{x-y\cdot i}, \qquad x = \rho\cos\phi, \quad y = \rho\sin\phi.$$

$$T^{mn}{}_{;n} = 0, \qquad T^{mn} = (\epsilon + p)u^m u^n - p g^{mn},$$

$$p = \nu(\epsilon + p).$$

Exercise!

$$u^\tau = \gamma(\cosh\eta - v_z\sinh\eta), \quad u^\eta = \frac{\gamma}{\tau}(v_z\cosh\eta - \sinh\eta),$$

$$u^\rho = \gamma(v_x\cos\phi + v_y\sin\phi), \quad u^\phi = \frac{\gamma}{\rho}(v_y\cos\phi - v_x\sin\phi).$$

1+1D expansion: Bjorken flow

$$p_{Bj.} = \frac{\text{constant}}{\tau^{1/(1-\nu)}},$$

$$u_{Bj.} = (1, 0, 0, 0).$$

1+3D expansion: Hubble flow

$$p_{\text{Hu.}} = \frac{\text{constant}}{(\tau^2 - \rho^2)^{\frac{3}{2(1-\nu)}}},$$

$$u_{\text{Hu.}} = \gamma\left(\frac{1}{\cosh\eta}, 0, \frac{\rho}{\tau}, 0\right),$$

$$\gamma = \frac{\cosh\eta}{\sqrt{1 - (\rho/\tau)^2\cosh^2\eta}},$$

Example: Sound Wave

NR Navier-Stokes:

$$(\partial_t + \vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla} p}{\rho} + \frac{\eta}{\rho}\nabla_j \vec{\Sigma}^{ji},$$

R Navier-Stokes:

$$\gamma^2(\partial_t + \vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{w/c^2}\left(\vec{\nabla} p + \frac{\vec{v}}{c}\partial_0 p\right) + \frac{\eta}{w/c^2}\partial_\nu \vec{\Sigma}^{\nu i},$$

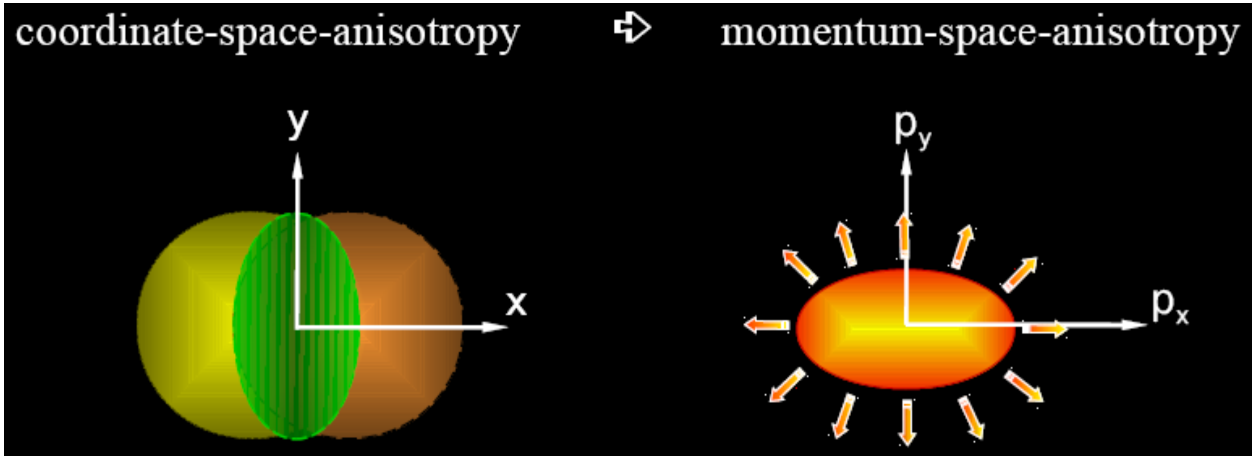
Exercise!

$$\omega = c_s k - \frac{i}{2}k^2 \times \begin{cases} \dfrac{\frac{4}{3}\eta}{w/c^2}, & \text{R fluid,} \\ \dfrac{\frac{4}{3}\eta}{\rho}, & \text{NR fluid.} \end{cases}$$

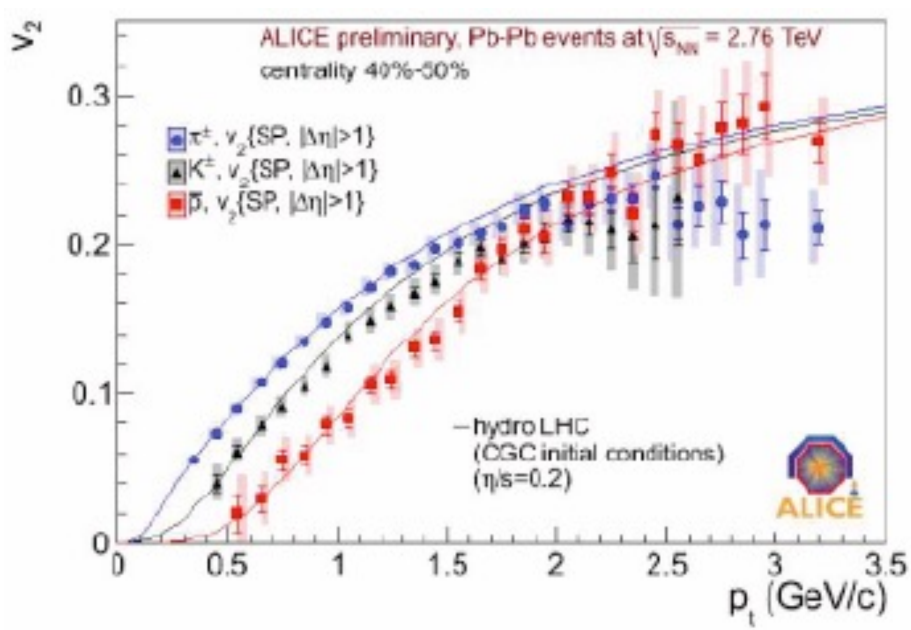
$$c_s = \begin{cases} \sqrt{\dfrac{\partial P}{\partial(\epsilon/c^2)}}, & \text{R fluid,} \\ \sqrt{\dfrac{\partial P}{\partial \rho}}, & \text{NR fluid.} \end{cases}$$

Sound propagation is very sensitive to ratio of shear viscosity over density

Anisotropic Blast: Elliptic Flow



$$\frac{dN}{dP_t\, d\phi} = \frac{dN}{dP_t}\left[1 + 2\, v_2\,(P_t)\cos(2\,\phi) + \ldots\right]$$

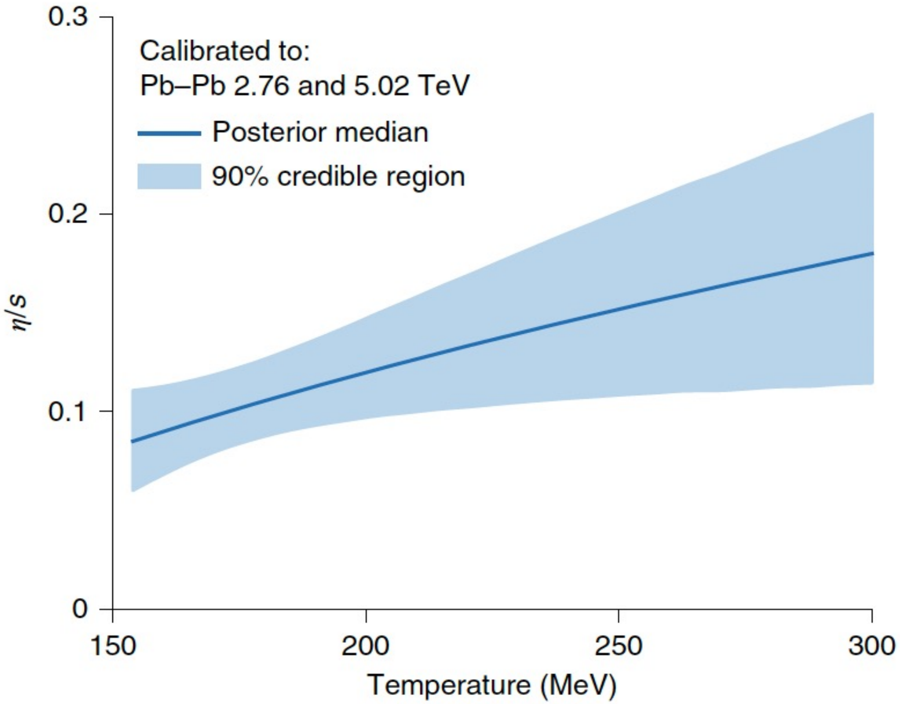


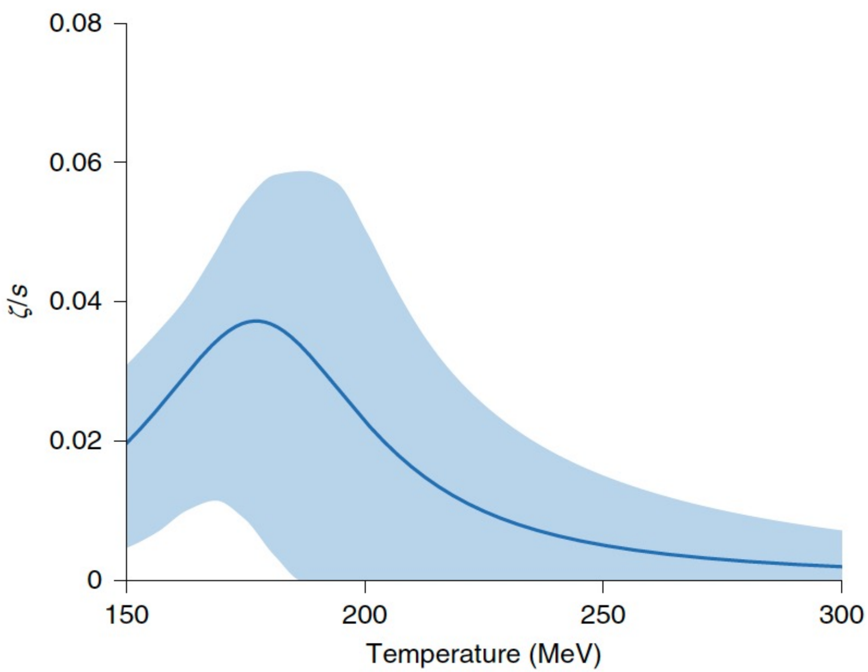
relativistic hydrodynamics @ 1~ 10 fm scale.

This response is very sensitive to fluid dissipation

$$1 \leq 4\pi(\eta/s)_{QGP} \leq 2.5$$

Extracting Transport Coefficients





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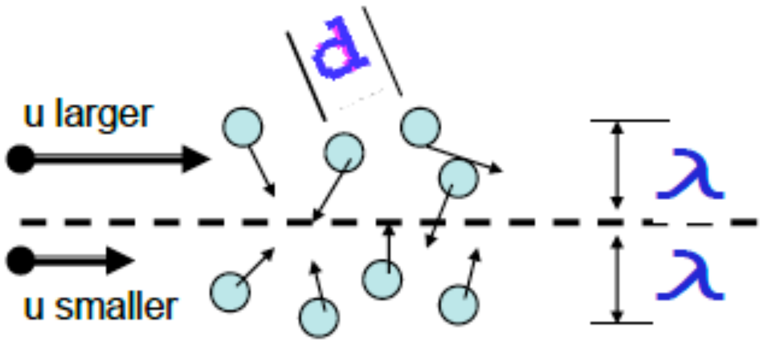
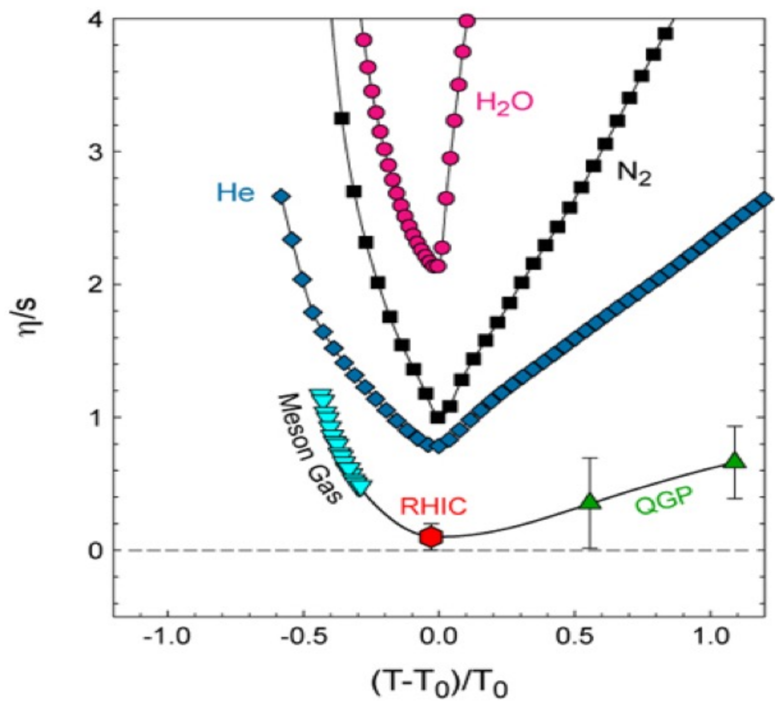
QGP: Nearly Perfect Quantum Liquid



$$\eta \sim \rho\, v_T\, \lambda \sim n\, p_T\, \lambda$$

$$s \sim n$$

$$\eta/s \sim p_T \lambda \sim \lambda/\lambda_{dB}$$



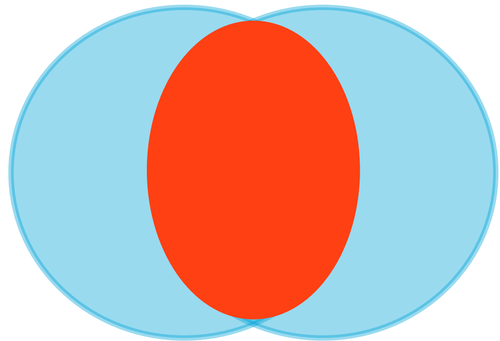
QGP is a strongly coupled quantum liquid:

$$\lambda_{M.F.P.} \sim \lambda_{de\ Broglie}$$

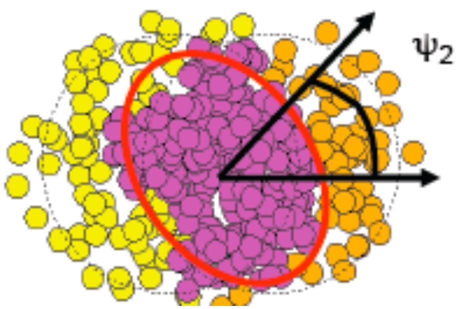
It has nearly perfect fluidity:
less dissipative than known substance;
very close to conjectured lower bound.

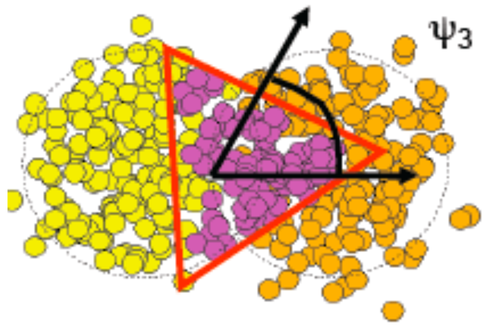
Fluctuating Initial Condition (I.C.)

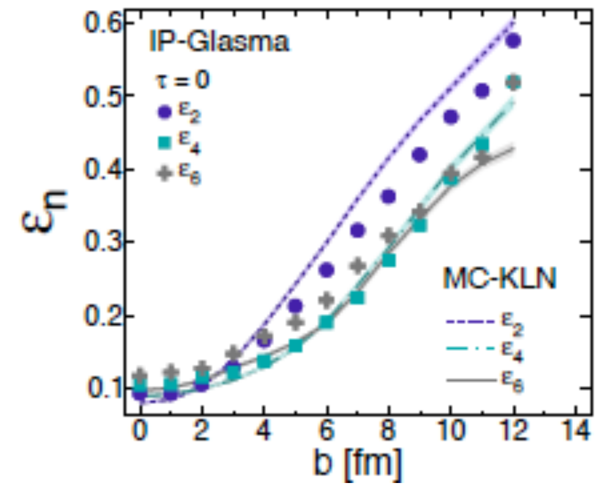
The initial condition used to be like this ...

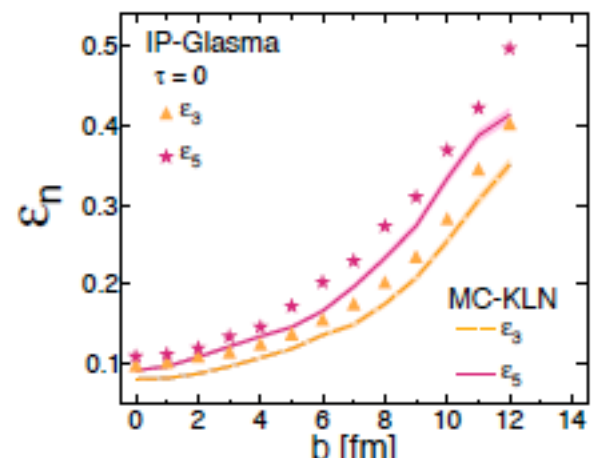


We now know it is actually like this:



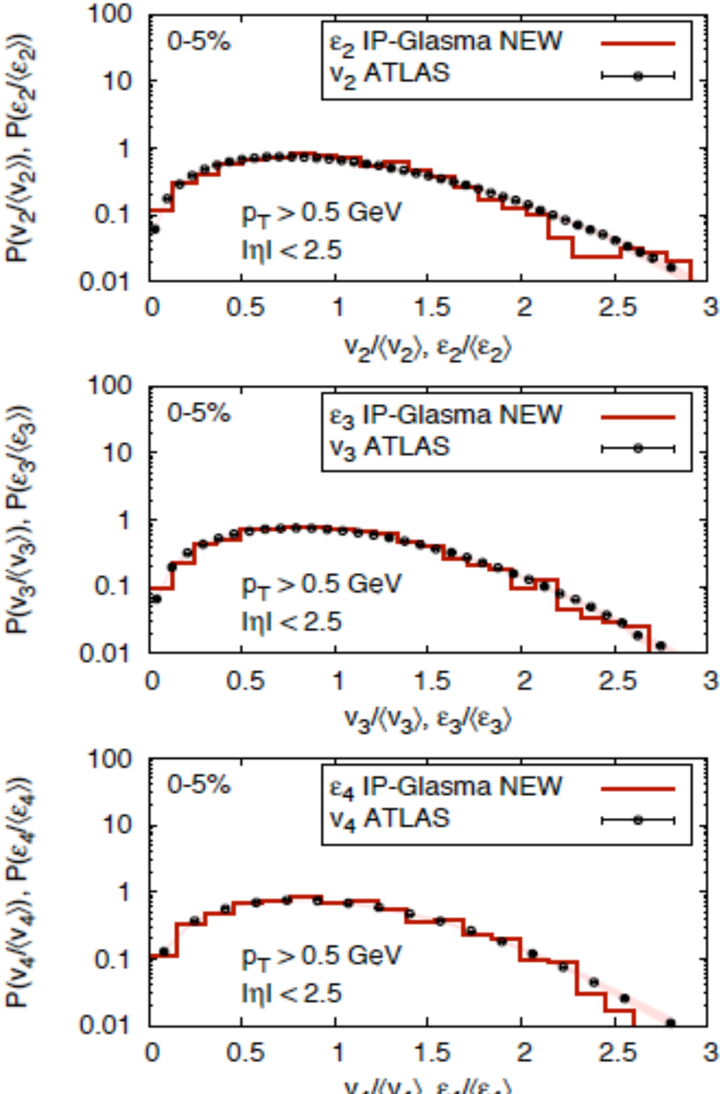






Mapping from I.C. to Final Observables

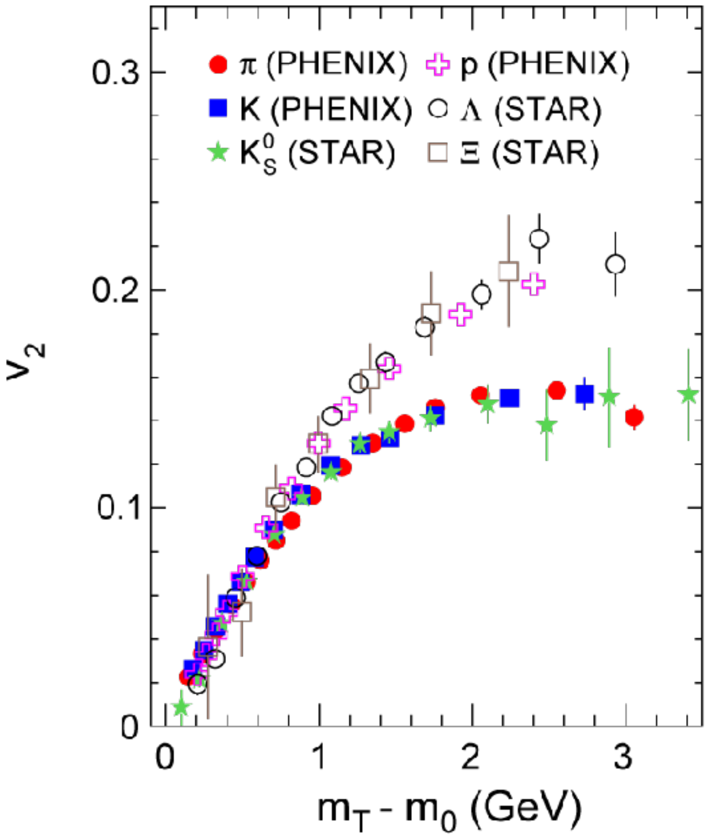
$$\epsilon_n \xrightarrow[\text{expansion}]{\text{hydrodynamic}} v_n$$



Not only for the mean value, but for the whole distribution, response can be established between I.C. and observables!

computation from IP-Glasma, arXiv:1312.5588 (see more centralities there)

Partonic Collectivity

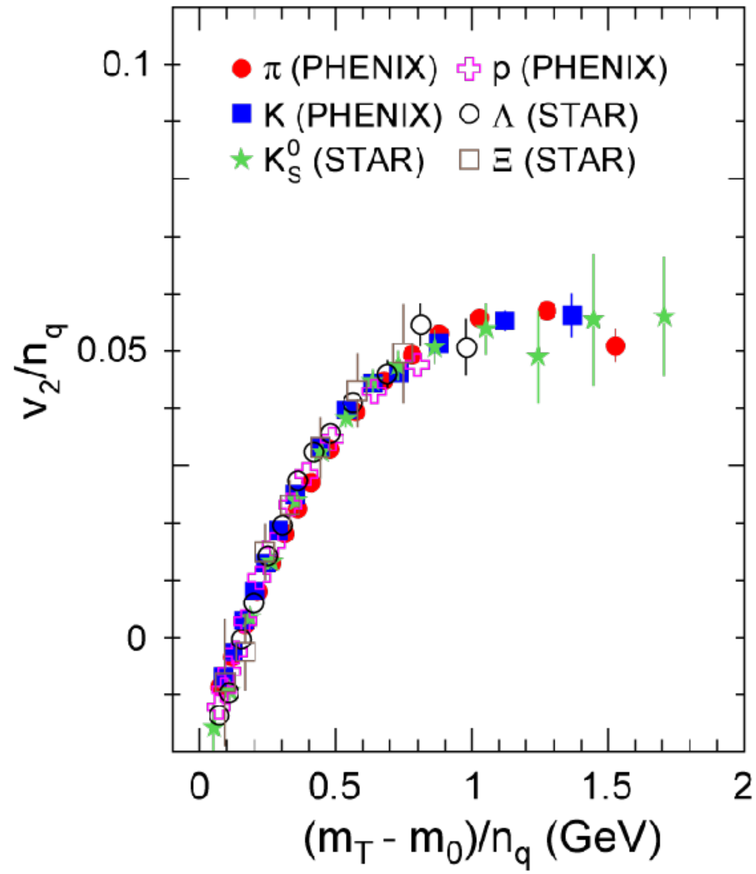


$p_T \rightarrow p_T / n$

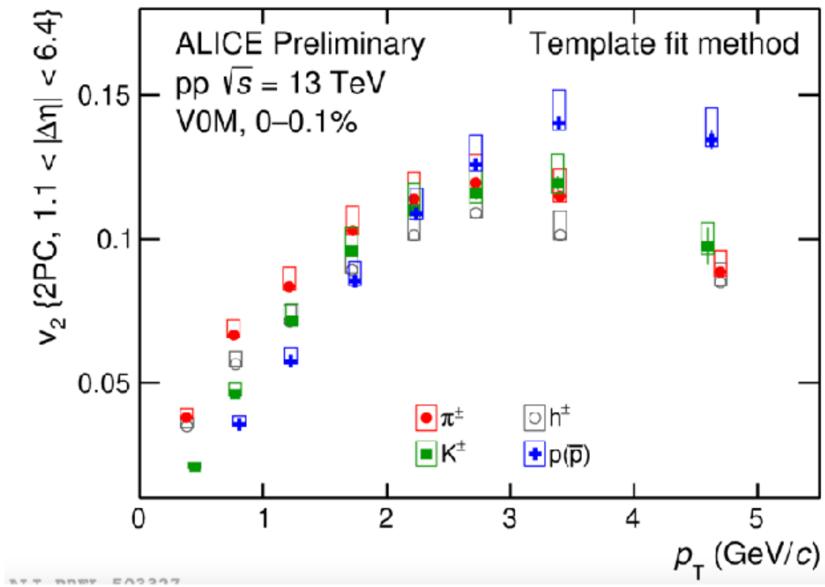
$v_2 \rightarrow v_2 / n$

n = (2, 3) for (meson, baryon)

From H. Caines's talk



Small Systems



Keep in mind the condition for hydrodynamic behavior:

$\lambda \ll l \ll L$, a coarse-graining process

Hydrodynamics on its edge!!

Pushing the limit of the smallest QGP droplet!!