Lect3. Transports of QCD matter

侯德富

华中师范大学粒子物理研究所

Outlines

- **Introduction and motivation**
- **Viscosity from Kubo formula (QFT)**
- **Viscosity from kinetic theory (Boltzmann Eq)**
- **Viscosity from AdS/CFT**
- **Anomalous transports**
- **Summary**

Motivations

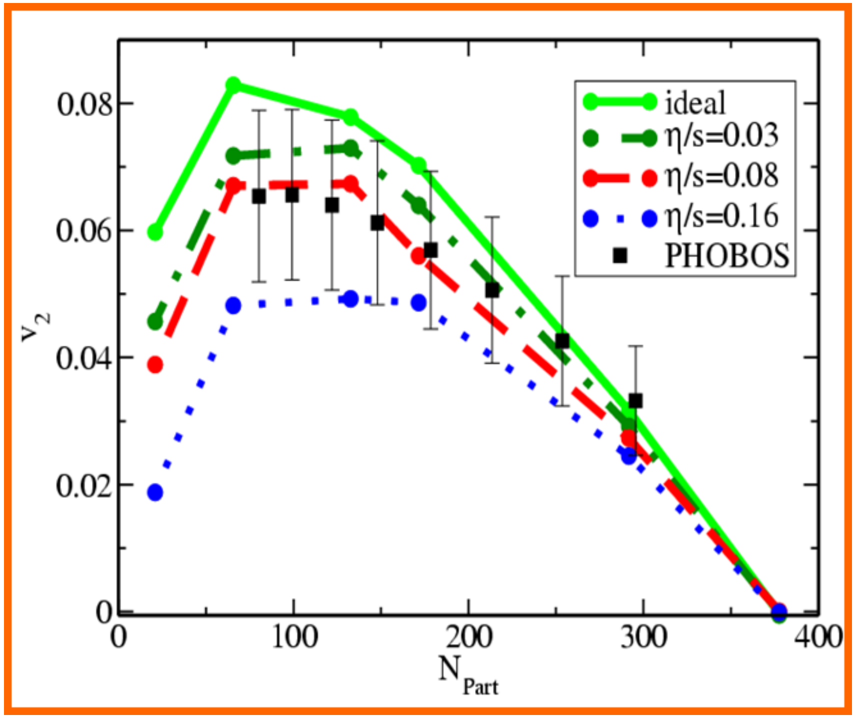
Experiments aspect:

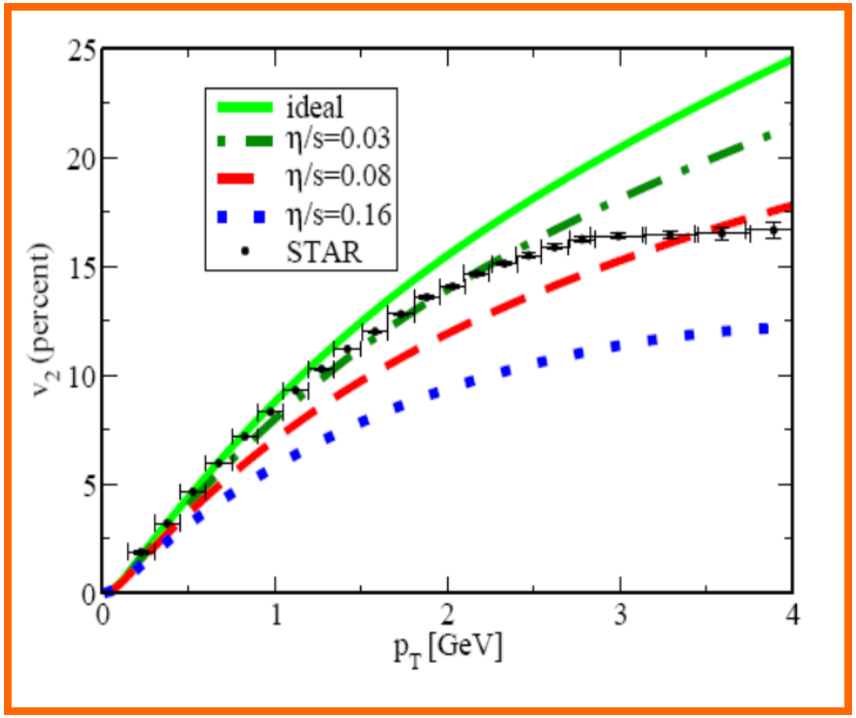
@ RHIC

- **Robust collective flows, well described by ideal hydro with Lattice-based EoS. This indicates very strong interaction even at early time => sQGP**

- **sQGP seems to be the almost perfect fluid known $\eta/s >= .1-.2 << 1$**

Study of dissipative effects on $<v_2>$





P. Romatschke, PRL99 (07)

Theoretic aspect:

- To calculate Trsp. Coefs. in FT in highly nontrivial

  (nonperturbative ladder resummation)   (c around 5)

$$\frac{\eta}{s} = \frac{C}{g^4 \ln \frac{1}{g}}$$

- String theory method: **AdS/CFT (D.Son et al 2003)**

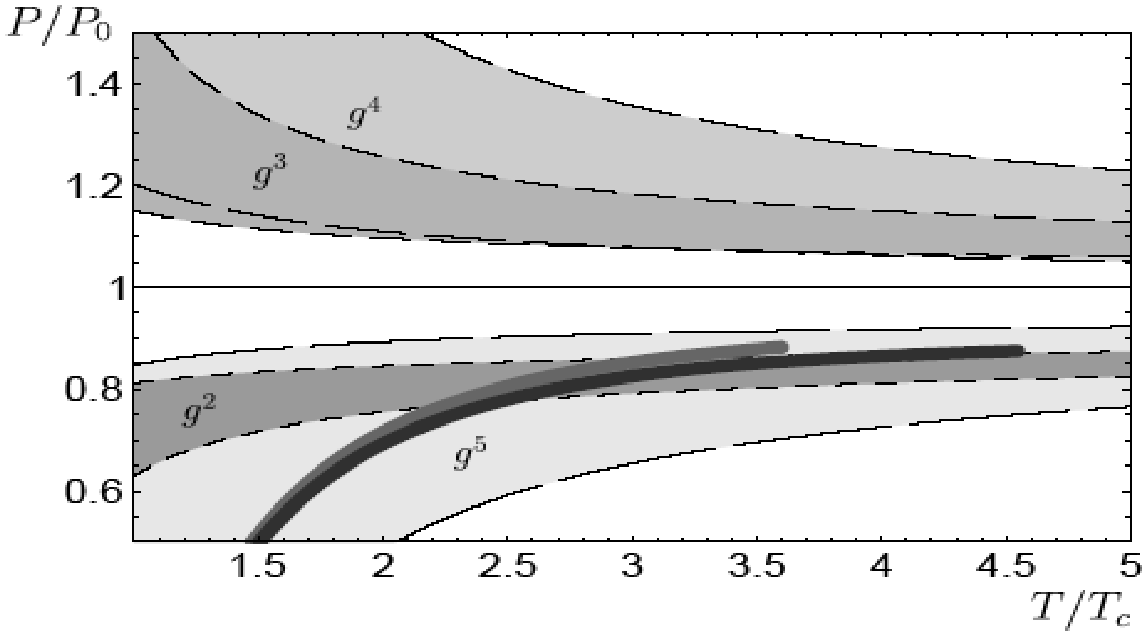
$$\eta/s = 1/4\pi$$

. **Kinetic theory + uncertainty principle (Gyulassy)**

$$\frac{\eta}{s} \geq \frac{1}{12\pi}. \qquad \eta = \frac{1}{3} n m \bar{v} l$$

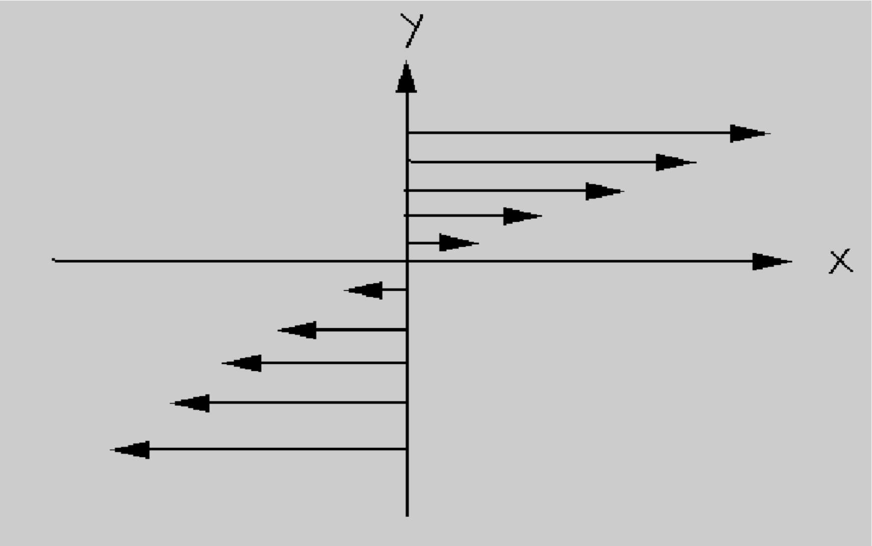
Main obstacle for theory

- QCD in nonperturbative regime (T~200Mev)
- Pertburb. Expansion of QCD is not well behaved for realistic T



- For thermodyn.,one can use lattice and resummation techniques
- Kinetic coefficients are difficult to extract from lattice

Shear Viscosity



$$\frac{F_x}{A} = -\eta \frac{\partial v_x}{\partial y}$$

- Noneq. system $\rightarrow$ gradients of $T, u \rightarrow$ ThermD forces $\rightarrow \delta T_{\mu\nu}$ characterized by trsp. coefs:

$$
\begin{aligned}
T^{\mu\nu} &= \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + P^\mu u^\nu + P^\nu u^\mu + \pi^{\mu\nu} \\
\Delta_{\mu\nu} &= g_{\mu\nu} - u_\mu u_\nu .
\end{aligned}
$$

ϵ: energy density

p: pressure

P_μ: heat current

$\pi_{\mu\nu}$: viscous shear stress

u_μ :four-velocity, $u^\mu(x)u_\mu(x) = 1$.

- Expc.value of $\pi_{\mu\nu}$ after gradient expansion in u^μ,

$$\delta\langle\pi_{\mu\nu}\rangle = \eta^{(1)} H_{\mu\nu} + \eta^{(2)} H^{T2}_{\mu\nu} + \cdots$$

$$H_{\mu\nu} = \partial_\mu u_\nu + \partial_\nu u_\nu - \frac{2}{3}\Delta_{\mu\nu}\Delta_{\rho\sigma}\partial^\rho u^\sigma$$

$$H^{T2}_{\mu\nu} := H_{\mu\rho}H^{\rho}{}_{\nu} - \frac{1}{3}\Delta_{\mu\nu}H_{\rho\sigma}H^{\rho\sigma}$$

- $\eta^{(1)}$: shear viscosity
- $\eta^{(2)}$: quadratic shear viscous response

Viscosity from Kubo formula

$$\delta\langle\pi_{\mu\nu}\rangle^l = \frac{H_{\mu\nu}}{10}\int d^3x' \int_{-\infty}^{t} dt' e^{\epsilon(t'-t)} \int_{-\infty}^{t'} dt'' D_R(x,x')$$

where

$$D_R(x,x') = -i\theta(t-t'') < [\pi_{ij}(x,t), \pi_{ji}(x',t'')] >$$

Extracting the shear viscosity in momentum space:

$$\eta^{(1)} = \frac{1}{10}\frac{d}{dq_0}\mathrm{Im}[\lim_{\vec{q}\to 0} D_R(Q)]|_{q_0=0}\,.$$

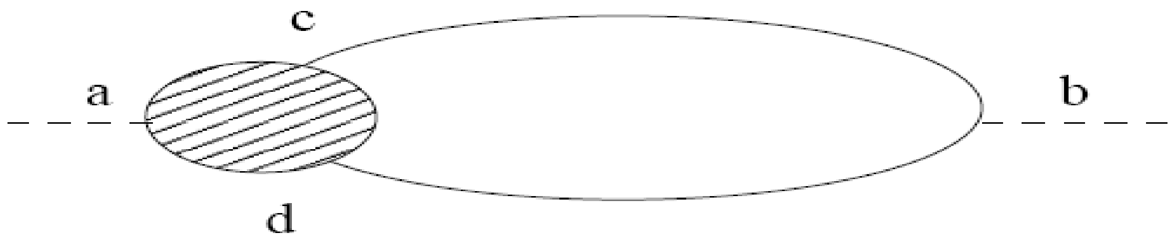
Nonlinear Response

$$\delta\langle\pi_{\mu\nu}(x,t)\rangle^q = \eta^{(2)} H^{T2}_{\mu\nu}$$

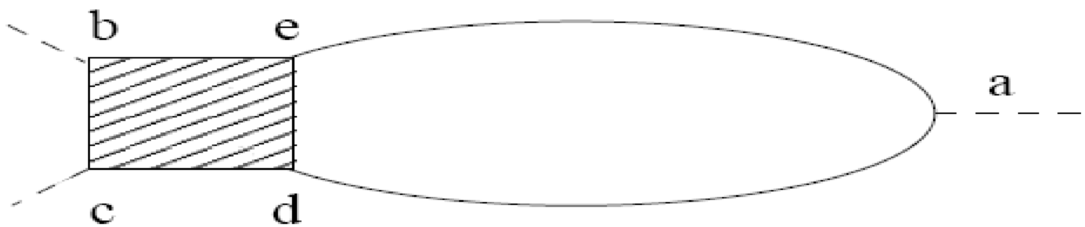
$$\eta^{(2)} \qquad = \frac{3}{70}\frac{d}{dq_0}\frac{d}{dq_0'} \mathrm{Re}\,[\lim_{\vec{q}\to 0} G_{R1}(-Q-Q',Q,Q')]|_{q_0=q_0'=0}$$

- G_{R1} *is a 3-point retarded FCN*
- Nonlinear Kubo formula -new result

[*Carrington, Hou, Kobes , PRD* **64** *(2001)030114*]



(a)



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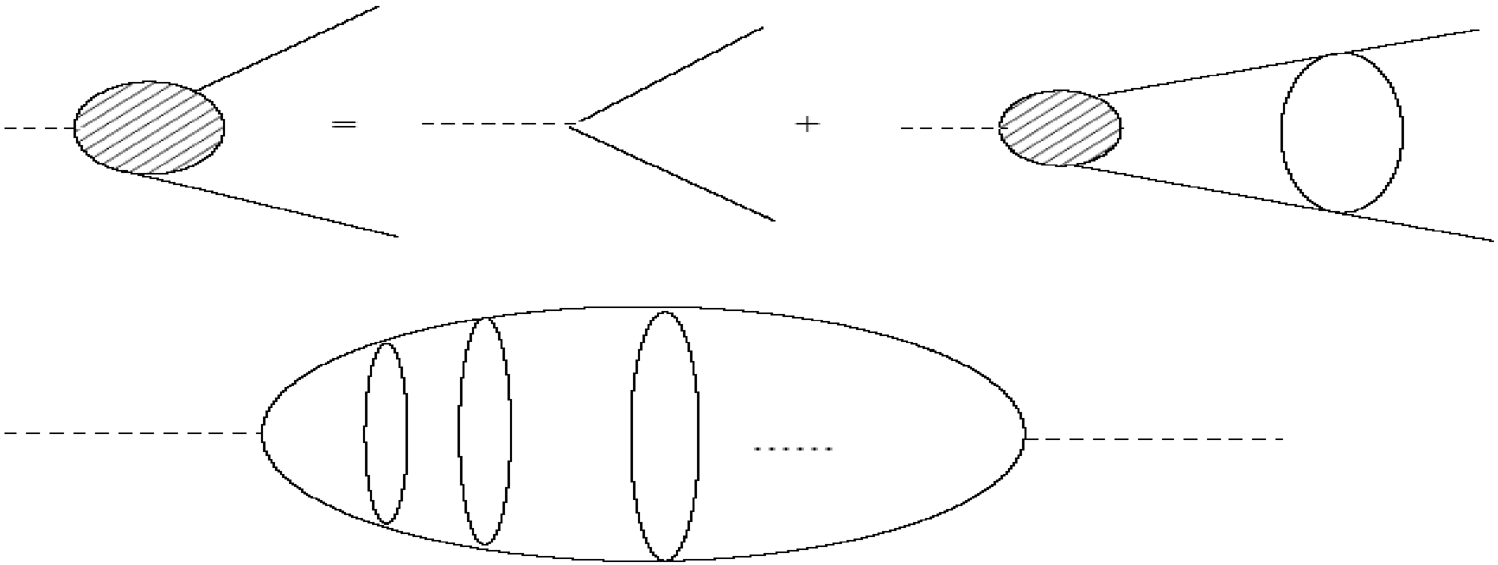
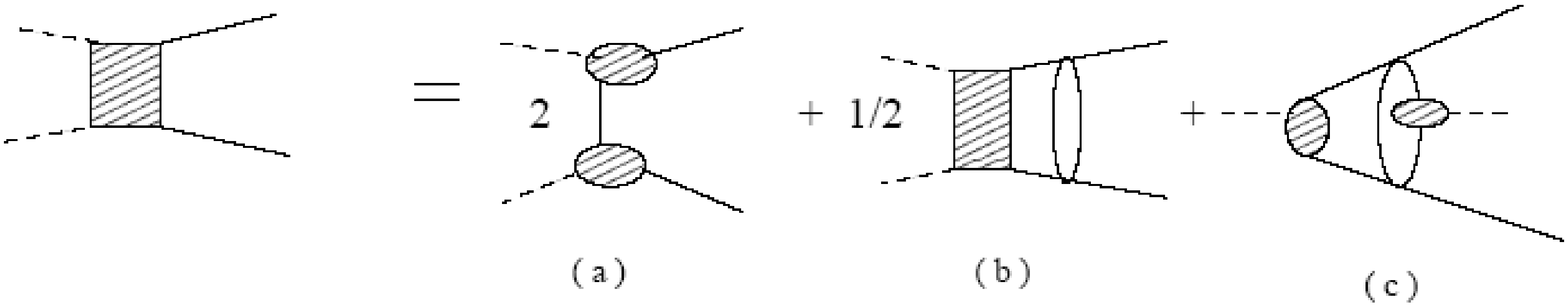


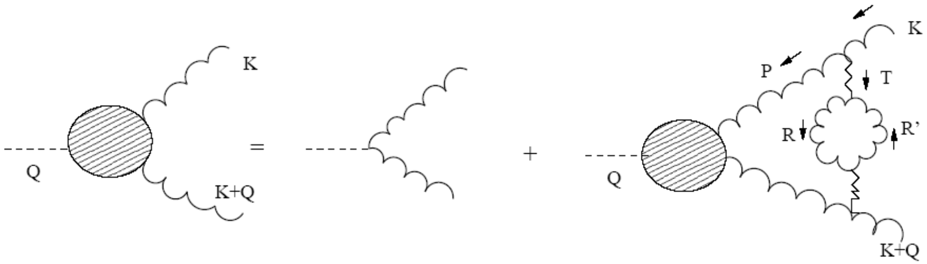
• **The integral equation for ladder resummation has exactly the same form as the linearized Boltzmann equation**

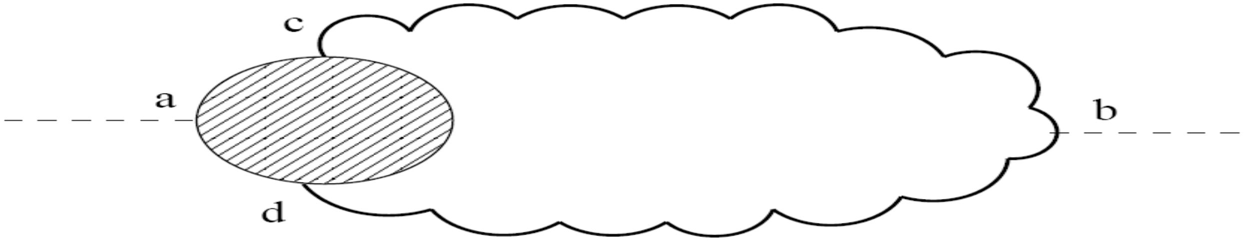
S. Jeon, PRD 52; Carrington, Hou, Kobes, PRD61



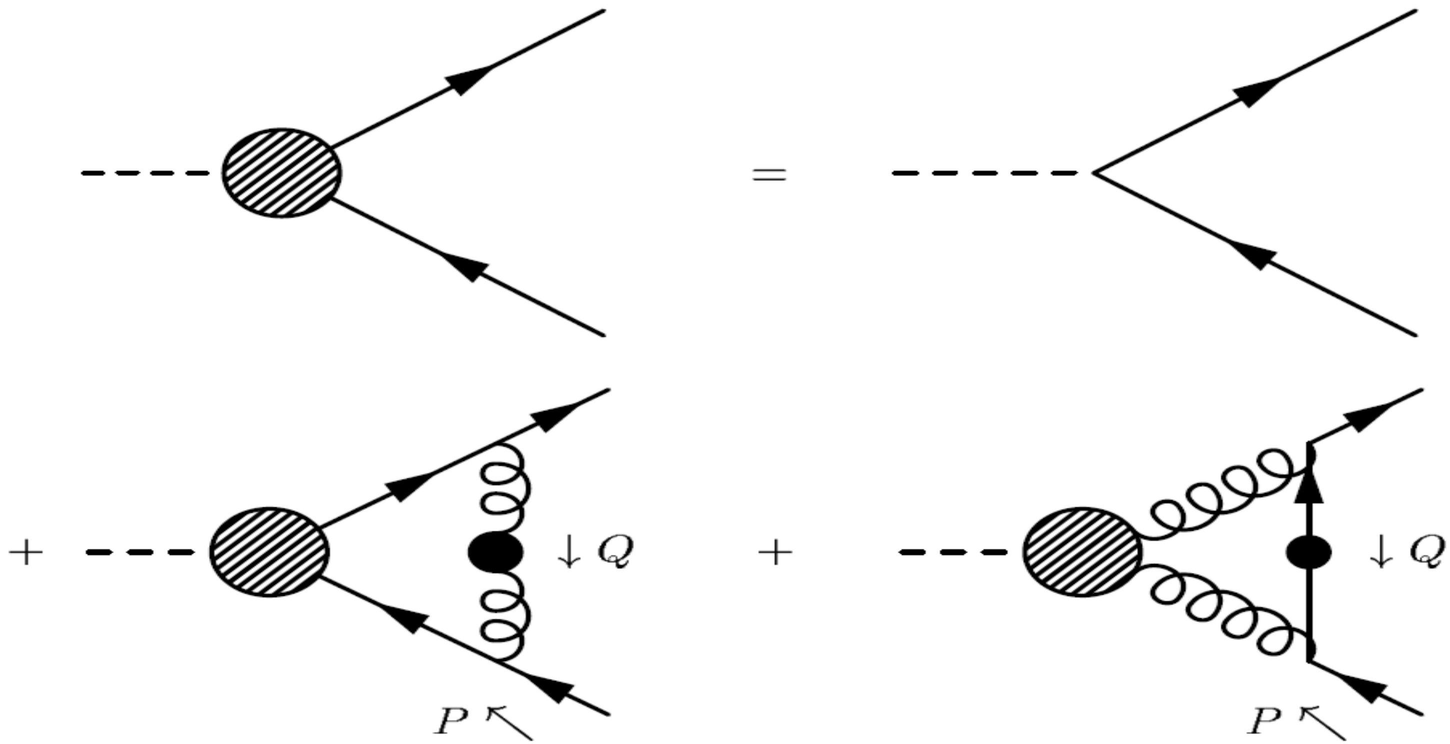
• The integral equation for extended ladder resummation has exactly the same form as the 2nd order of BLZ EQN

Carrington, Hou, Kobes, PRD64 (2001)





Hou, hep-ph/0501284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Viscosity from kinetics theory

- Most of previous calculations are from BLZ Eqs

  [Baym et al (1990)], [Arnold, Moore, Yaffe, 2000(2001)], [Heiselberg (1994)]

- Blz Eqn. valid : $\lambda_{mf} \gg \lambda_{comp}$ (dilute sys.)

- For hot QCD: $O(1/g^2 T \ln(1/g)) >> O(1/gT)$. if $g << 1 \to$ trsp. theory works well

Expand $f(x, \underline{k})$ around its local EQ form,

$$f^{(0)} = \frac{1}{e^{\beta(x) u_\mu(x) \underline{k}^\mu} - 1} := n_k \, ; \quad N_k := 1 + 2n_k$$

By gradient expansion in local rest frame $\vec{u}(x) = 0$:

$$f = f^{(0)} + f^{(1)} + f^{(2)} + \cdots$$

$$f^{(1)} := -n_k(1+n_k)\phi_k\,; \quad \phi_k = \beta\frac{1}{2}B_{ij}(\underline{k})H_{ij}$$

$$f^{(2)} := n_k(1+n_k)N_k\theta_k\,; \quad \theta_k := \beta^2\frac{1}{4}C_{ijlm}(\underline{k})H_{ij}H_{lm}\,.$$

The viscous shear stress tensor is given by

$$\langle\pi_{ij}\rangle = \int \frac{d^3k}{(2\pi)^3 2\omega_k} f\left(k_i k_j - \frac{1}{3}\delta_{ij}k^2\right).$$

Using above expansion of f and rotational invariance we obtain,

$$\delta\langle\pi_{ij}\rangle = -\frac{\beta}{15}\int\frac{d^3k}{(2\pi)^3 2\omega_k} n_k(1+n_k)k^2 B(\underline{k})\, H_{ij} + \frac{2\beta^2}{105}\int\frac{d^3k}{(2\pi)^3 2\omega_k}[n_k(1+n_k)N_k]k^2 C(\underline{k}) H_{ij}^{T2}\,.$$

Comparing with (3) we have,

$$\eta^{(1)} = \frac{\beta}{15}\int\frac{d^3k}{(2\pi)^3 2\omega_k} n_k(1+n_k)k^2 B(\underline{k})$$

$$\eta^{(2)} = \frac{2\beta^2}{105}\int\frac{d^3k}{(2\pi)^3 2\omega_k}[n_k(1+n_k)N_k]k^2 C(\underline{k}).$$

Viscosity of hot QCD at finite density

Fluctuation of distribution $f_s = n_s + \delta f_s$ (s: species)

$$\delta f_s = n_s(p)[1 \pm n_s(p)]\varphi_s(\mathbf{x};\mathbf{p})$$

$$\varphi_s(\mathbf{x};\mathbf{p}) = \beta^2 I_{ij}(\hat{\mathbf{p}}) X_{ij}(\mathbf{x}) \chi_s(p)$$

$$\delta\langle T_{ij}\rangle = \int_{\mathbf{p}} \frac{p_i p_j}{p_0}\left(g_f \delta f + g_{\bar{f}}\delta\bar{f} + g_b \delta b\right) = \delta\langle \pi_{ij}\rangle = -\eta(\partial_i u_j + \partial_j u_i - \tfrac{2}{3}\delta_{ij}\partial_k u^k)$$

Boltzmann Equation $\left(\frac{\partial}{\partial t} + \hat{\mathbf{p}}\cdot\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}_{\mathbf{ext}}\cdot\frac{\partial}{\partial \mathbf{p}}\right) f_s(t,\mathbf{x};\mathbf{p}) = -(\mathcal{C} f_s)(t,\mathbf{x};\mathbf{p})$

Recast the Boltzmann equation $S^s_{ij}(\mathbf{p}) = (\mathcal{C}\chi^s_{ij})(\mathbf{p})$

$$\begin{aligned} S^s_{ij}(\mathbf{p}) &\equiv -Tp\, n_s(p)[1 \pm n_s(p)] I_{ij}(\hat{\mathbf{p}}) \\ \chi^s_{ij}(\mathbf{p}) &\equiv I_{ij}(\hat{\mathbf{p}})\chi_s(p). \end{aligned}$$

P.Arnold, G.D.Moore and G.Yaffe,

JHEP 0011(00)001

Shear viscosity

With a definition of inner product and expanded distribution functions,

$$\eta = \frac{2}{15} Q_{max}\Big|_{\chi=\chi_{max}}$$

where $$Q[\chi] \equiv (\chi_{ij}, S_{ij}) - \frac{1}{2}(\chi_{ij}, \mathcal{C}_{ij})$$

$$(\chi_{ij}, S_{ij}) = -\beta^2 \sum_a \int_{\mathbf{p}} \mathbf{p}\ f_0^a(\mathbf{p})[1 \pm f_0^a(\mathbf{p})]\chi^a(p)$$

$$\begin{aligned}(\chi_{ij}, \mathcal{C}\chi_{ij}) &= \frac{\beta^2}{8} \int_{\mathbf{p},\mathbf{k},\mathbf{k},\mathbf{k}'} \sum_{abcd} |M^{ab}_{cd}|^2\ (2\pi)^4 \delta^{(4)}(P + K - P' - K') \\ &\times\ n_a(\mathbf{p}) n_b(\mathbf{k})[1 \pm n_c(\mathbf{p}')][1 \pm n_d(\mathbf{k}')] \\ &\times\ \left[\chi^a_{ij}(\mathbf{p}) + \chi^b_{ij}(\mathbf{k}) - \chi^c_{ij}(\mathbf{p}') - \chi^d_{ij}(\mathbf{k}')\right]^2,\end{aligned}$$

Collision terms

Performing the integral over dk' with the help of $\delta^3(\mathbf{p}+\mathbf{k}+\mathbf{p}'+\mathbf{k}')$

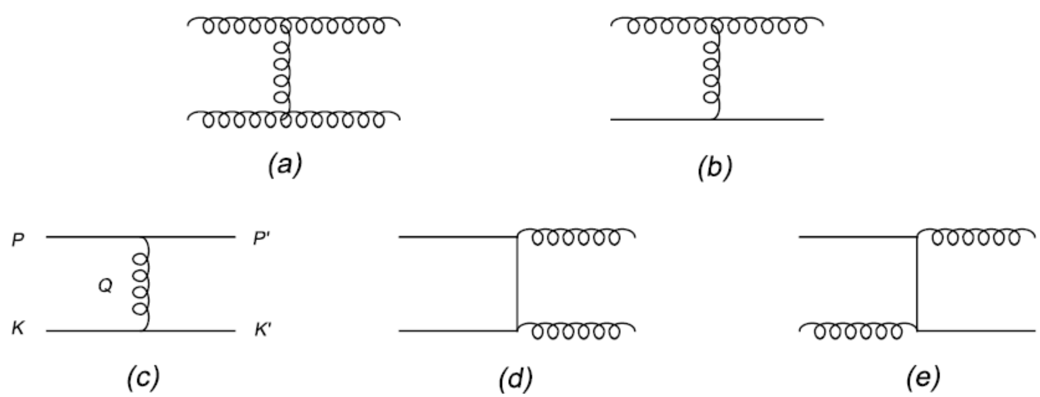
$$(\chi_{ij}, \mathcal{C}\chi_{ij}) = \frac{\beta^3}{(4\pi)^6}\int_0^\infty dq \int_{-q}^{q} d\omega \int_0^\infty dp \int_0^\infty dk \int_0^{2\pi} d\phi$$

$$\times \sum_{abcd} |\mathcal{M}^{ab}_{cd}|^2 n_a(p) n_b(k)[1 \pm n_c(p)][1 \pm n_d(k)] \left[\chi^a_{ij}(\mathbf{p}) + \chi^b_{ij}(\mathbf{k}) - \chi^c_{ij}(\mathbf{p}') - \chi^d_{ij}(\mathbf{k}')\right]^2$$

Scattering amplitude

Distribution function term

\chi term



Matrix Element

Processes	Matrix elements
Fig.1(a)	$16g^4 d_A C_A^2 \left(3 - \frac{su}{t^2} - \frac{st}{u^2} - \frac{tu}{s^2}\right)$
Fig.1(b)	$8g^4 d_F C_F C_A \left(\frac{s^2+u^2}{t^2}\right)$
Fig.1(c)	$8g^4 \frac{d_F^2 C_F^2}{d_A} \left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2}\right)$
Fig.1(d)	$8g^4 d_F C_F^2 \left(\frac{t}{u} + \frac{u}{t}\right)$
Fig.1(e)	$-8g^4 d_F C_F^2 \left(\frac{s}{u} + \frac{u}{s}\right)$

$$\left(\frac{s^2+t^2}{u^2}\right)_{u-ch} = \left(\frac{s^2+u^2}{t^2}\right)_{t-ch} \approx \frac{8p^2k^2}{q^4}(1-\cos\phi)^2 \qquad \left(\frac{t}{u}\right)_{u-ch} = \left(\frac{u}{t}\right)_{t-ch} \approx \frac{2pk}{q^2}(1-\cos\phi)$$

- Variation method gives

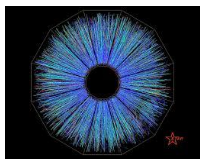
$$\eta \approx \frac{1.09T^3}{\alpha_s^2 \ln \alpha_s^{-1}} \left(1 + 0.25\frac{\mu^2}{T^2}\right)$$

Liu, Hou, Li EPJC 45(2006)

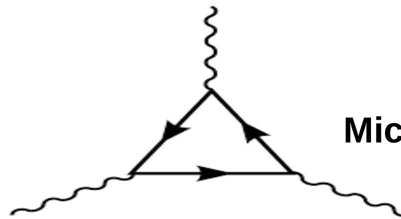
Anomalous Transports

What are their effects to the QCD transports

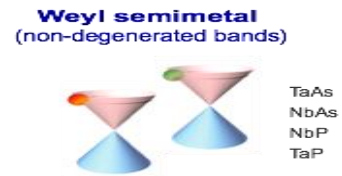
Micro-quantum anomaly + B/ Ω → macro-transport (CME/CVE)



Search in HIC



Micro



B Ω

$$\vec{\mathrm{J}} = \sigma_5 \mu_5 \vec{\mathrm{B}}$$

Macro

Nature Phys.12（2016）

Science350（2015）413

Astrophysics, cosmology

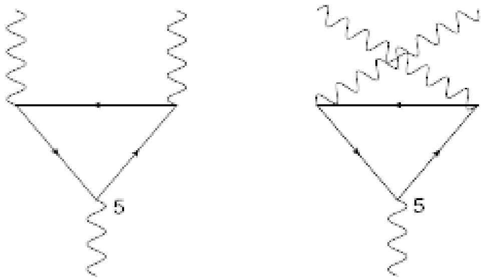
Kharzeev,Liao,Voloshin,Wang: Prog.Part.Nucl.Phys.2016

The relation of CME current to chiral anomaly

The CME current

$$J_i(p) = \eta\mu_5 K_{ij}(p) A_j(p) + \mathcal{O}(\mu_5^3)$$

- In terms of the AVV three point function $\Lambda_{\mu\nu\rho}(Q_1, Q_2)$



$$Q_1 = (\mathbf{q}, i(\omega + \frac{k_0}{2})),$$

$$Q_2 = (-\mathbf{q}, i(-\omega + \frac{k_0}{2}))$$

- the coefficient

$$K_{ij}(q) = \Lambda_{ij4}(q, -q) = -i \lim_{k_0 \to 0} \frac{1}{k_0} (Q_1 + Q_2)_\rho \Lambda_{ij\rho}(Q_1, Q_2)$$

- the chiral anomaly

$$(Q_1 + Q_2)_\rho \Lambda_{\mu\nu\rho}(Q_1, Q_2) = -i \frac{e^2}{2\pi^2} \epsilon_{\mu\nu\alpha\beta} Q_{1\alpha} Q_{2\beta}$$

Universal to all orders of coupling， all temperature & chemical potential .Necessary to explain $\pi^0 \to 2\gamma$

It follows that

$$K_{ij}(q) = i\frac{e^2}{2\pi^2}\epsilon_{ikj}q_k$$

Then the CME current

$$\mathbf{J} = \frac{e^2}{2\pi^2}\mu_5\mathbf{B} \tag{1}$$

There are, however, two shortcomings in the above establishment

1. distinction between chiral anomaly at the operator level and its matrix element

   only the former one is free from radiative corrections.

2. the constant μ_5 limit in eq.(1) becomes subtle at finite temperature

$$\lim_{k_0\to 0}\lim_{\mathbf{k}\to 0} \neq \lim_{\mathbf{k}\to 0}\lim_{k_0\to 0} \tag{2}$$

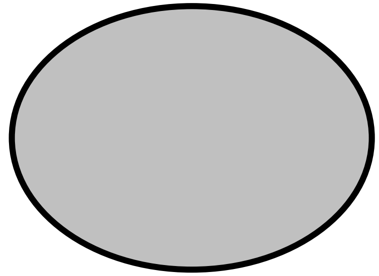
note that in the limiting process $\lim_{\mathbf{k}\to 0}\lim_{k_0\to 0}$, *the relation of CME current to chiral anomaly becomes unclear.*

$$\mu_5\left(\mathbf{k}, k_0\right)$$

$$\Downarrow$$

Hou, Ren, Liu JHEP 05(2011)046

$$\mathbf{J}\left(\mathbf{q}+\frac{1}{2}\mathbf{k},\ \omega+\frac{1}{2}k_0\right) \Leftarrow$$



$$\Leftarrow \mathbf{B}\left(\mathbf{q}-\frac{1}{2}\mathbf{k}, \omega-\frac{k_0}{2}\right)$$

Constant μ_5 *, non-constant* **B:** $\mathbf{k} = k_0 = 0$

$$\mathrm{limit}_{\mathbf{q}\to 0}\mathrm{limit}_{\omega\to 0} \Rightarrow \mathbf{J} = \eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

$$\mathrm{limit}_{\omega\to 0}\mathrm{limit}_{q\to 0} \Rightarrow \mathbf{J} = \frac{1}{3}\times\eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

Artifact of one-loop approximation. The ambiguity disappears with higher order corrections. (*Satow & Yee*)

Constant ***B****, non-constant* $\mu_5(\mathbf{k}, k_0)$

$$\Downarrow$$

$$\mathrm{limit}_{\mathbf{k}\to 0}\,\mathrm{limit}_{k_0\to 0} \Rightarrow \qquad \mathbf{J} = 0$$

$$\mathrm{limit}_{k_0\to 0}\,\mathrm{limit}_{\mathbf{k}\to 0} \Rightarrow \qquad \mathbf{J} = \eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

Follows from the EM gauge invariance and the non-renormalization of the axial anomaly. Valid to all orders!

<u>with T=0 and $\mu = 0$</u>: relativistic invariance requires the two limit orders are equivalent:

- **The electric current extracted from the wigner function**

$$J_\mu(x) = ie\int \frac{d^4 p}{(2\pi)^4} \mathrm{tr} W(x,p)\gamma_\mu$$

$$= ie\int d^4 y \delta^4(y) U(x_+, x_-) < \overline{\psi}(x_+)\gamma_\mu \psi(x_-) >$$

$$= \lim_{y\to 0} J_\mu(x,y)$$

$$J_\mu(x,y) = ieU(x_+, x_-) < \overline{\psi}(x_+)\gamma_\mu \psi(x_-) >$$

For a constant μ_5

$$J_\mu = \eta \frac{e^2}{2\pi^2}\mu_5 B_\mu \qquad \text{with } B_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\lambda} u_\nu F_{\rho\lambda}$$

$$u_\mu = \text{fluid velocity}$$

For a non-constant μ_5, it is problematic because of UV divergence with the limit $y \to 0$

The regulated Wigner function

a robust regularization scheme has to be introduced to the underlying field theory before defining the wigner function.

e.g. If the underlying field theory is regularized by PV scheme

$$L = -\overline{\psi}\gamma_\mu(\partial_\mu - ieA_\mu - i\gamma_5 A_{5\mu})\psi$$

$$J_\mu(x) = i\int \frac{d^4 p}{(2\pi)^4} \mathrm{tr} W(x,p)\gamma_\mu$$

$$= i \lim_{y\to 0} U(x_+, x_-) < \overline{\psi}(x_+)\gamma_\mu \psi(x_-) >$$

⟵ PV regulator should be included in it

$$J_\mu(x) = -ie\frac{1}{2}\left[\mathrm{Tr}\gamma_\mu \mathcal{S}_0(x,x) - \sum_s C_s \mathrm{Tr}\gamma_\mu \mathcal{S}_s(x,x)\right]$$

$$J_\mu(x) = e^2 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} e^{i(q_1+q_2)\cdot x} \Lambda_{\mu\rho\lambda}(q_1, q_2) A_\rho(q_1) A_{5\lambda}(q_2)$$

gives CME current :

$$\lim_{q_{20}\to 0} \lim_{\vec{q}_2\to 0} \Lambda_{ij4}(q_1, q_2) = -\frac{i}{2\pi^2} \epsilon_{ikj} q_{1k}$$

CME current canceled at thermal equilibrium.

$$\lim_{\vec{q}_2\to 0} \lim_{q_{20}\to 0} \Lambda_{ij4}(q_1, q_2) = \frac{2f(0)-1}{2\pi^2} \epsilon_{ikj} q_{1k} + O(q_1^2)$$

Wu,Hou, Ren, PRD 2017

Phenomenological implications of the subtleties regarding the order of limits

Axial charge generated via toplogical fluctuations dictated by the stochastic Eq with a white noise

$$\left(\frac{\partial}{\partial t} - D\nabla^2 + \frac{1}{\tau}\right) n_5 = g(x)$$

In Momentum sapce $$n_5(k) = \frac{g(k)}{-ik_0 + D\vec{k}^2 - \frac{1}{\tau}}$$

Corresponding an axial potential

$$A_{5\mu}(k) = -i\delta_{\mu 4}\frac{n_5(k)}{\chi(k)}$$

Average current vanishes, the correlation funct. $< J_i(x)J_j(y) >$ **is dominated by diffusion pole**

$$-iq_{20} + D\vec{q}_2^{\,2} + \frac{1}{\tau} = 0$$

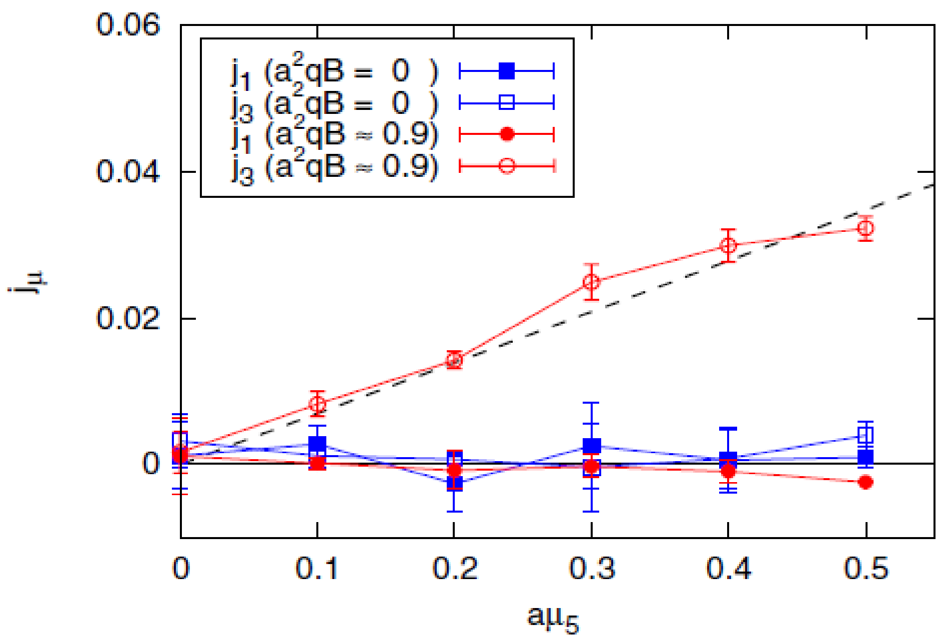
$$\frac{|q_{20}|}{|\vec{q}_2|} = D|\vec{q}_2| + \frac{1}{|\vec{q}_2|\tau} \geq \sqrt{\frac{D}{\tau}}.$$

If $\sqrt{D/\tau} >> 1$ **the homog. \mu_5 is a good approximation and classic form of CME current emerges ---Noneq. Phenom.**

Towards equilibrium, $\tau \to \infty,$ $\frac{|q_{20}|}{|\vec{q}_2|} = D|\vec{q}_2| \sim \frac{D}{|\vec{x} - \vec{y}|} \to 0$

Inverse limit-order prevails, and CME current disappears,

CME on Lattice



Yamamoto,PRL(2011)

using lattice QCD with Wilson term

$$
\begin{aligned}
I = &-\sum_x \sum_\mu \frac{1}{2a}\left[\bar{\psi}(x)\left(\frac{1}{i}\gamma_\mu - r\right)U_\mu(x)\psi(x+a_\mu)\right.\\
&\left.-\ \bar{\psi}(x+a_\mu)\left(\frac{1}{i}\gamma_\mu + r\right)U_\mu^\dagger(x)\psi(x)\right]\\
&-\sum_x M\bar{\psi}(x)\psi(x) + \cdots
\end{aligned}
$$

Karsten and Smit (1981)

$$J_i(p) = -\Pi_{ij}(p) A_j(p)$$

One-loop self-energy on lattice of size $N_s^3 \times N_t$

$$\Pi_{ij}^{(1)}(p) = \mathcal{I} \sum_k \epsilon_{ikj} p_k + \mathcal{O}(a)$$

CME vanishes at continu. limit .

At zero temperature

$$\Pi_{ij}(q) \equiv \Lambda_{ij4}(q)$$
$$= -\lim_{q_4 \to 0} \frac{1}{q_4} \sum_\rho \frac{2}{a} \sin \frac{1}{2} a (Q_1 + Q_2)_\rho \Lambda_{ij\rho}(Q_1, Q_2)$$

$$\Pi_{ij}(q) = \frac{e^2}{2\pi^2} \sum_k \epsilon_{ijk} q_k$$

Feng, Hou, Liu, Ren, Wu, PRD95,(2017)

umerical calculations

Lattice size	$\mathcal{I}$
$N_s = 6, N_t = 4$	1.347×10^{-2}
$N_s = 12, N_t = 4$	2.439×10^{-4}
$N_s = 20, N_t = 4$	8.886×10^{-7}
$N_s = 50, N_t = 8$	4.512×10^{-9}

nalytical calculations(In the limit $N_s \to \infty$)

$$\mathcal{I} = 12 \frac{1}{N_t} \sum_{l_4} \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{\mathcal{N}(l)}{\left[\sin^2 l + \mathcal{M}^2(l)\right]^3} = 0$$

The chiral vortical effect (CVE)

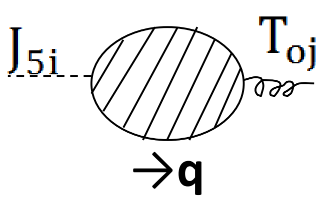
Under B & vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{v}$

$$\vec{J}_{\rm em} = \sigma^B_{\rm em}\vec{B} + \sigma^V_{\rm em}\vec{\omega},$$
$$\vec{J}_{\rm b} = \sigma^B_{\rm b}\vec{B} + \sigma^V_{\rm b}\vec{\omega},$$
$$\vec{J}_{5} = \sigma^B_{5}\vec{B} + \sigma^V_{5}\vec{\omega},$$

Son & Surowka

$\boldsymbol{\sigma}^{\rm B}_{\rm em}, \boldsymbol{\sigma}^{\rm B}_{\rm b}, \boldsymbol{\sigma}^{\rm B}_{5} \rightarrow \mathrm{CME}$ $\quad$ $\boldsymbol{\sigma}^{\rm v}_{\rm em}, \boldsymbol{\sigma}^{\rm v}_{\rm b}, \boldsymbol{\sigma}^{\rm v}_{5} \rightarrow \mathrm{CVE}$

Kubo formula：



$\xrightarrow{\vec{\mathbf{q}} \to 0}$ $\sigma^{\rm v}_{5}\varepsilon_{ijk}q_k$

Higher order correction to CVE

Field Theoretic Formulation:

QED Lagrangian density

$$\mathcal{L} = -\frac{1}{4e^2}V^{\mu\nu}V_{\mu\nu} - i\bar{\psi}\gamma^\mu D_\mu\psi + \frac{1}{2}h^{\mu\nu}T_{\mu\nu} + A^\mu J_{5\mu}$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$D_\mu = \partial_\mu - iV_\mu$$

$$T_{\mu\nu} = V_\mu{}^\rho\, V_{\nu\rho} - \frac{1}{4}g_{\mu\nu}V^{\rho\lambda}V_{\rho\lambda} + \frac{1}{4}\left(-D_\mu\bar{\psi}\gamma_\nu\psi - D_\nu\bar{\psi}\gamma_\mu\psi + \bar{\psi}\gamma_\mu D_\nu\psi + \bar{\psi}\gamma_\nu D_\mu\psi\right)$$

$$J_5^\mu = i\bar{\psi}\gamma_\mu\gamma_5\psi$$

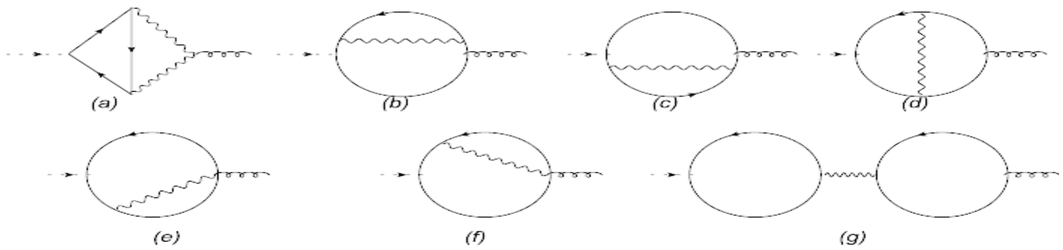
Anomalous Ward identity

$$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2\sqrt{-g}}\epsilon^{\mu\nu\rho\lambda}V_{\mu\nu}V_{\rho\lambda}$$

Kubo formula for CVE

$$\mathcal{G}_{ij}(\vec{q}) = -\int_0^\infty dt \int d^3\vec{r}\, e^{-i\vec{q}\cdot\vec{r}}\, \frac{\mathrm{Tr}e^{-\beta H}\left[J_{5i}(\vec{r},t), T_{0j}(0,0)\right]}{\mathrm{Tr}e^{-\beta H}} \underset{\vec{q}\to 0}{\longrightarrow} \sigma_V\epsilon_{ijk}q_k$$

Hou,Liu,Ren, PRD86 (2012) 121703®



$$\xi_5 = \frac{\mu_5^2}{2\pi^2} + cT^2$$

Are there any corrections from higher orders ?

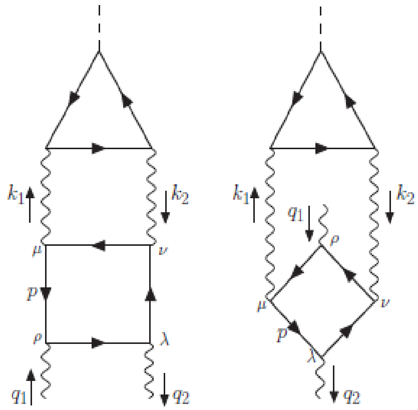
S. Golkar and D. T. Son, arXiv:1207.5806 ,JHEP02(2015)169: No (Yes)

$$c = \frac{1}{12} + \frac{N_c^2 - 1}{2N_c}\frac{g_0^2}{48\pi^2} \xrightarrow[N_c \to \infty]{} \frac{1}{12} + \frac{\lambda}{96\pi^2} \qquad c = \frac{1}{12} + \frac{e_0^2}{48\pi^2}$$

QED radiative corrections to CME

Feng, Hou, Ren   PRD 99   (2019)

Radiative corrections from photon-photon rescattering



Photon rescattering contribution to the AVV function

Ansel'm and Ioganson (1989')

- The anomalous Ward identity

$$(Q_1 + Q_2)_\rho \Lambda_{\mu\nu\rho}(Q_1, Q_2) = -i\frac{e^2}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}Q_{1\alpha}Q_{2\beta} \times \left(1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{k^2}\right)$$

- The kernel of CME current becomes

$$K_{ij}(q) = i\frac{e^2}{2\pi^2}\mu_5\epsilon_{ikj}q_k\left(1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{k^2}\right)$$

- Likewisely, the same diagrams with two internal photons replaced by two gluons may also contribute to CME.

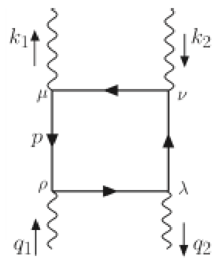
Radiative corrections at finite temperature

At finite temperature, the results correspond to the two limits

$$\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}, \qquad \lim_{\mathbf{Q}\to 0}\lim_{Q_0\to 0}$$

may be different.

- The order $\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}$
  1. The AVV triangle in real-time formulation(e.g., CTP) is diagonal, i.e., only 111 and 222 components survive.
  2. For static external momenta, the real-time formulation reduces to Mastubara one.



The photon box is free from IR singularity, thus

$$\Gamma_{mnij}(0,0;k) = \frac{\partial}{\partial q_l}\Gamma_{mnij}(q,q;k)\Big|_{q=0} = 0$$

the three-loop diagrams would be at order of $\mathcal{O}(\mathbf{q}^2)$ and thus do not contribute to CME.

3-loop radiation correction to CME

Feng, Hou, Ren PRD99 (2019)

- the kernel of CME current

$$K_{ij}(\mathbf{q}) = i\frac{e^2}{2\pi^2}F_s\left(\frac{|\mathbf{q}|}{T}\right)\epsilon_{ikj}q_j$$

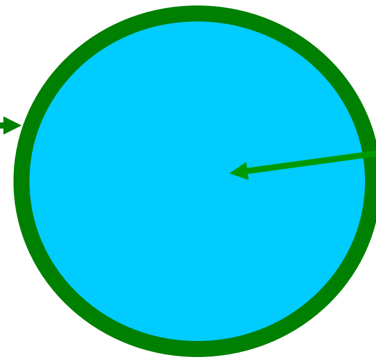
1. In low temperature limit($T << |\mathbf{q}|$): $F_s(|\mathbf{q}|/T) \to 1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{q^2}$
2. At finite temperature(T>$|\mathbf{q}|$): for $\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}$, $F_s(|\mathbf{q}|/T) \to 1$
   for $\lim_{\mathbf{Q}\to 0}\lim_{Q_0\to 0}$, $F_s(|\mathbf{q}|/T) \to 0$

If the two internal photons are replaced by gluons

$$F_s(|\mathbf{q}|/T) \to 1 - \frac{3g^4}{32\pi^4}\log\frac{\Lambda^2}{q^2}.$$

Field Theory = **Gravity Theory**

Gauge Theories
QCD



Quantum Gravity
String theory

Holography

the large N limit
Supersymmetric Yang Mills
↓ N large
Gravitational theory in 10 dimensions

Calculations → Correlation functions
Quark-antiquark potential

Computing transport coefficients from AdS/CFT

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using AdS/CFT

Universality of shear viscosity

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w) dw^\mu dw^\nu$$

$$\left.\begin{aligned} \eta &= \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle \\ \sigma_{abs} &= -\frac{16\pi G}{\omega} \,\mathrm{Im}\, G^R(\omega) \\ &= \frac{8\pi G}{\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle \end{aligned}\right\} \quad \eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

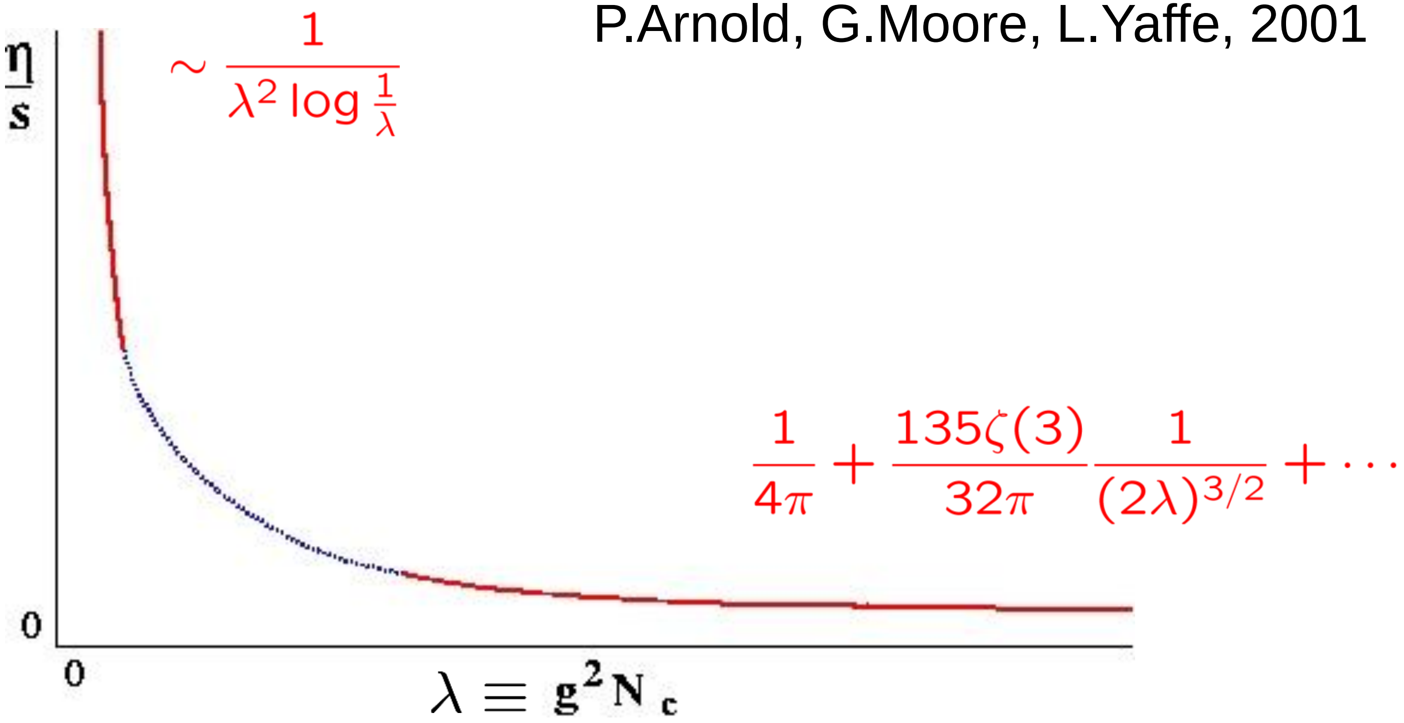
Since the entropy (density) is $s = A_H / 4G$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

D. Son, P. Kovtun, A.S., hep-th/0405231

Shear viscosity in $\mathcal{N} = 4$ SYM

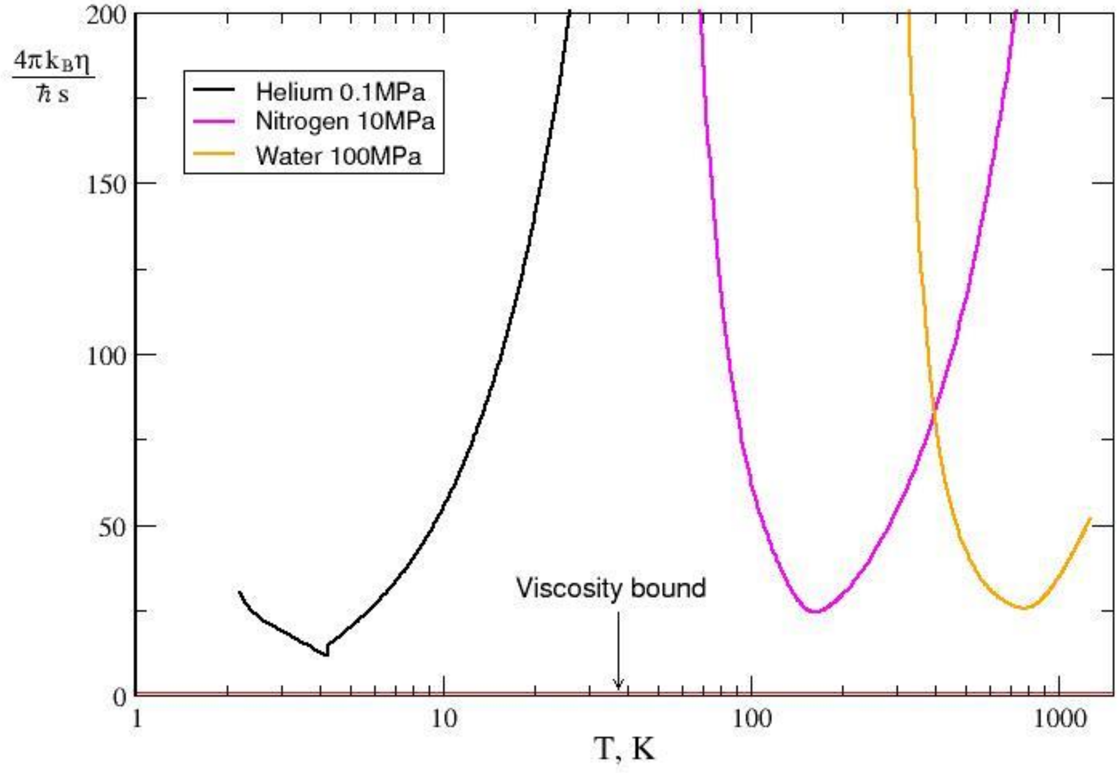
P.Arnold, G.Moore, L.Yaffe, 2001



Correction to $1/4\pi$: A.Buchel, J.Liu, A.S., hep-th/0406264

A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

Universality of η/s

Theorem:

For any thermal gauge theory (with zero chemical potential), the ratio of shear viscosity to entropy density is equal to $1/4\pi$ ***in the regime described by a corresponding dual gravity theory***

$$(\text{e.g. at } \; g^2_{YM} N_c = \infty \, , N_c = \infty \; \text{ in } \; \mathcal{N} = 4 \; \text{SYM})$$

Remark:

Gravity dual to QCD (if it exists at all) is currently unknown.

Possible Mechanisms for Low viscosity

- Large cross-section, strong coupling

- Anomalous viscosity: turbulence

M. Asakawa, S.A. Bass, B.M., hep-ph/0603092, PRL
See Abe & Niu (1980) for effect in EM plasmas

Take moments of $\left[\frac{\partial}{\partial t} + \frac{p}{E_p}\cdot\nabla_r - \nabla_p \cdot D(p)\cdot\nabla_p\right]\bar{f}(r,p,t) = C\left[\bar{f}\right]$ with p_z^2

$$\frac{1}{\eta} = O(1)\frac{N_c}{N_c^2-1}\,\frac{g^2\left\langle B^2\right\rangle \tau_m}{sT^3} + O\left(10^{-2}\right)\frac{g^4 \ln g^{-1}}{T^3} \equiv \frac{1}{\eta_A} + \frac{1}{\eta_C}$$

M. Asakawa, S.A. Bass, B.M., hep-ph/0603092

See Abe & Niu (1980) for effect in EM plasmas

Low viscosity due to Anderson Local.

- AL effect renders infinite reduces viscosity significantly even at weak coupling

- Mechanism:coherent backscattering (CBS) effect

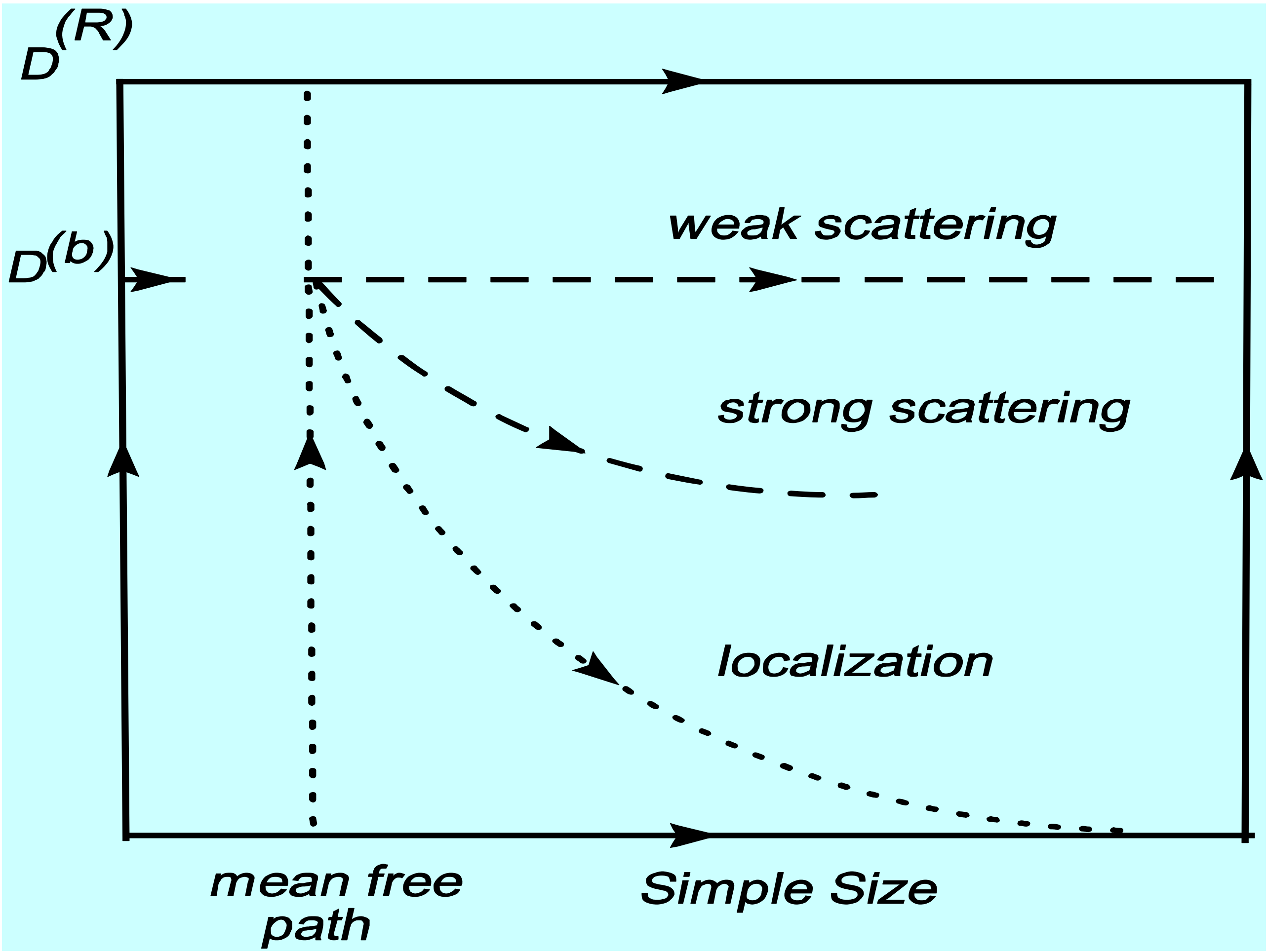
Ginaaki, Hou , Ren PRD 77(2008)

Weak Localization (WL)

- Anderson proposed ('58) that electronic diffusion can vanish in a random potential (AL)
- Experiments detected (Ishimaru 1984,Wolf Maret 1985)
- Mechanism:coherent backscattering (CBS) effect

after a wave is multiply scattered many times， its phase coherence is preserved in the backscattering direction，the probability of back scattering is enhenced via constructive interference

Renormalized diffusion

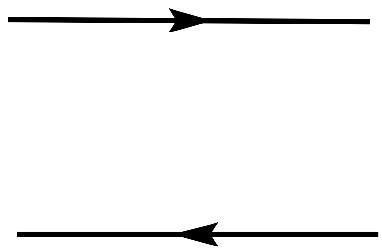
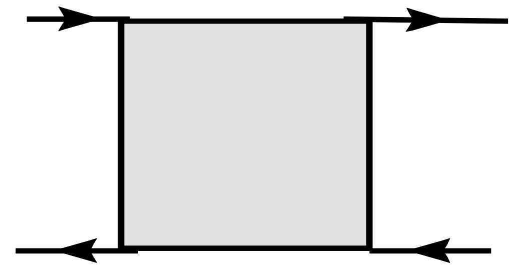


$$\eta = \frac{1}{10} \lim_{\omega \to 0} \lim_{\vec{q} \to 0} \mathrm{Im} \frac{\chi(\vec{q}, \omega)}{\omega}$$

$$\chi(\vec{q}, \omega) = \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3p'}{(2\pi)^3} I_{ij}(\vec{p}, \vec{q}) I_{ij}(\vec{p}', -\vec{q})$$

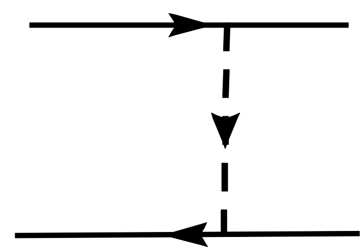
$$\times \int_{-\infty}^{\infty} \frac{dp_0}{2\pi i} \{[n(p_0) - n(p_0 + \omega)]\Phi^{RA}(P, P', Q) - n(p_0)\Phi^{RR}(P, P', Q) + n(p_0 + \omega)\Phi^{AA}(P, P', Q)\}$$

$$= \chi^{RA}(\vec{q}, \omega) + \chi^{RR}(\vec{q}, \omega) + \chi^{AA}(\vec{q}, \omega)$$

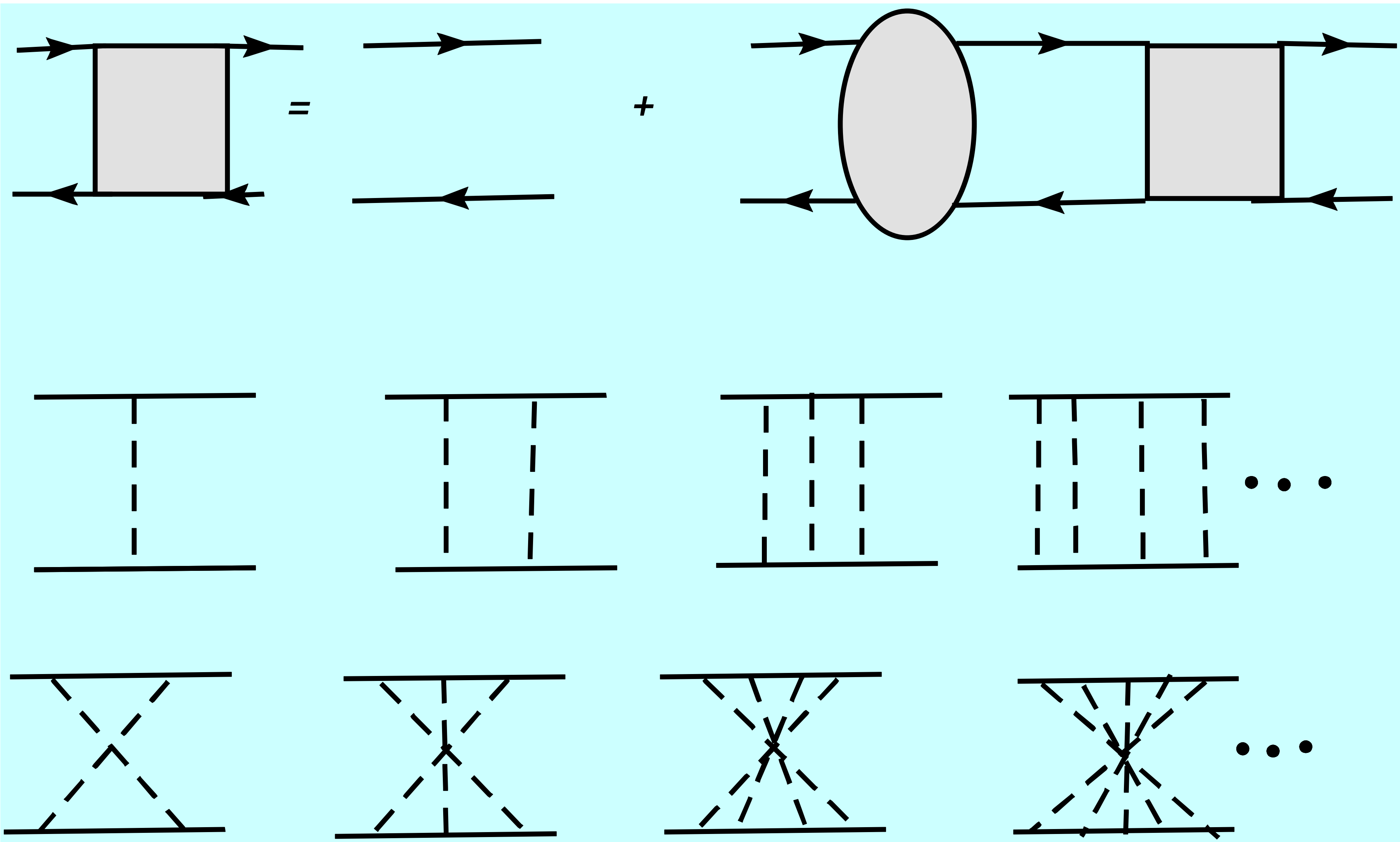
$\phi^{RA} =$  $+$ 

 $= D^{R}(P)$

 $= D^{A}(P)$

 $= -U(P-P')$

BS Eq. In Diagrams



Viscosity with random medium

System: quasi-particles in random potential

Candidate disorder in sQGP ?

1. The islands of heavy state; bound states (Shuryak);
2. The reminiscent of confinement vaccum, say the domain structure of 't Hooft's monopole condensation;
3. The disoriented chiral condensate (DCC);
4. CGC

$$\frac{1}{D} = \frac{1}{D_0} + \frac{24\pi\gamma^2}{p_0} \int_{|\vec{p}+\vec{p}'|<k_c} \frac{d^3\vec{p}'}{(2\pi)^3} \frac{\hat{p}\cdot\hat{p}'}{i\omega - D(\vec{p}+\vec{p}')^2}. \tag{24}$$

$$\xi = \lim_{\omega\to 0} \sqrt{i\frac{D}{\omega}},$$

the onset of WL is characterized by $\xi \neq 0$, which implies

$$\lim_{\omega\to 0} D(\omega) = 0,$$

Summary

Approches to calculate transport coefficients

- **Kubo formula: via correlation functions of currents**
- **Transport theory: Boltzmann Eqs. (for weak scattering)**
- **ADS/CFT(strongly coupled)**
- **Lattice calculation (noisy)**

- **Thanks**

$$
\begin{aligned}
\epsilon &= u_\alpha u_\beta T^{\alpha\beta} \\
p &= -\frac{1}{3}\Delta_{\alpha\beta} T^{\alpha\beta} \\
P_\mu &= \Delta_{\mu\alpha} u_\beta T^{\alpha\beta} \\
\pi_{\mu\nu} &= (\Delta_{\mu\alpha}\Delta_{\nu\beta} - \frac{1}{3}\Delta_{\nu\mu}\Delta_{\alpha\beta}) T^{\alpha\beta}.
\end{aligned}
$$