Visita por el mundo de los hadrones exóticos: el comienzo de una nueva tabla periódica hadrónica.

Exotic hadrons, paving the path to a new hadron periodic Table

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Constituents of matter, quarks, leptons, gauge bosons

Mesons and baryons

Quarks models, effective theories

Chiral Lagrangians, dynamically generated states

Exotic mesons and baryons

Compact tetraquarks or pentaquarks and molecular states

$X_0(2866)$, $T_{cc}(3875)$, P_c states

Multimeson molecules. The beginning of new periodic Table

Components of matter

Quarks

Generation	Name	Symbol	Antiparticle	Spin	Charge (e)	Mass (MeV/c^2) [5]
1	up	u	$\bar{u}$	½	$+2/3$	$2.2^{+0.6}_{-0.4}$
	down	d	$\bar{d}$	½	$-1/3$	$4.6^{+0.5}_{-0.4}$
2	charm	c	$\bar{c}$	½	$+2/3$	1280 ± 30
	strange	s	$\bar{s}$	½	$-1/3$	96^{+8}_{-4}
3	top	t	$\bar{t}$	½	$+2/3$	173 100 ± 600
	bottom	b	$\bar{b}$	½	$-1/3$	4180^{+40}_{-30}

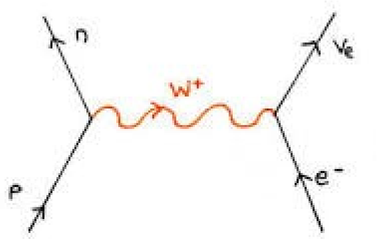
Leptons

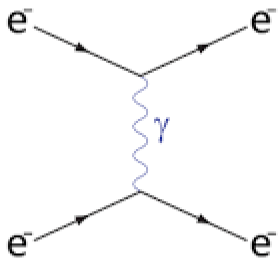
Generation	Name	Symbol	Antiparticle	Spin	Charge (e)	Mass (MeV/c^2) [5]
1	electron	e^-	e^+	$\frac{1}{2}$	−1	0.511[note 1]
	electron neutrino	ν_e	$\bar{\nu}_e$	$\frac{1}{2}$	0	< 0.0000022
2	muon	μ^-	μ^+	$\frac{1}{2}$	−1	105.7[note 2]
	muon neutrino	ν_μ	$\bar{\nu}_\mu$	$\frac{1}{2}$	0	< 0.170
3	tau	τ^-	τ^+	$\frac{1}{2}$	−1	1 776.86 ± 0.12
	tau neutrino	ν_τ	$\bar{\nu}_\tau$	$\frac{1}{2}$	0	< 15.5

Gauge bosons

Name	Symbol	Antiparticle	Spin	Charge (*e*)	Mass (GeV/c^2) [5]	Interaction mediated	Observed
photon	γ	self	1	0	0	electromagnetism	Yes
W boson	W^-	W^+	1	±1	80.385 ±0.015	weak interaction	Yes
Z boson	Z	self	1	0	91.1875 ±0.0021	weak interaction	Yes
gluon	g	self	1	0	0	strong interaction	Yes
Higgs boson	H^0	self	0	0	125.09 ±0.24	mass	Yes

The interaction of quarks, or matter, is supposed to proceed via the exchange of a gauge boson





There is violation of energy in the vertex : $\Delta E \, \Delta t = h/2\pi$ → $\Delta l = c \, \Delta t$ → the weak interaction is of very short range
→ the electromagnetic Interaction is of very long range

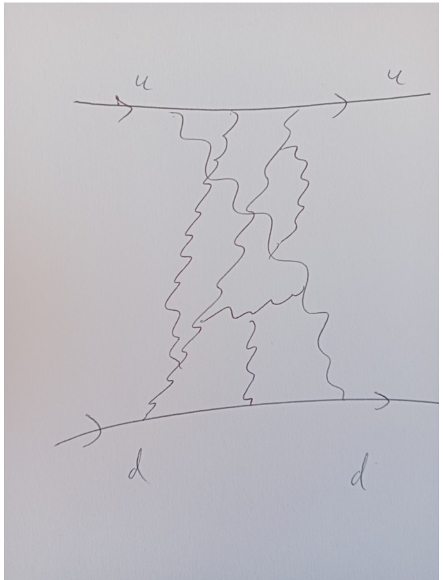
The gluon is the mediator of the strong interaction and should be of long range.
But it is very special. The gluons also interact with themselves

$$\mathscr{L}_{classic} = \sum_{f=1}^{N_f} \bar{\psi}_f (i\gamma^\mu D_\mu - m_f)\psi_f - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}{}_a$$

$$\begin{aligned} D_\mu &= \partial_\mu + igT_a A^a_\mu, \\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^a{}_{bc} A^b_\mu A^c_\nu \end{aligned}$$

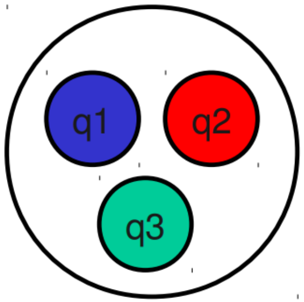
$$[T_a, T_b] = i f_{abc} T^c$$

T_a generators of the SU(3) group of color



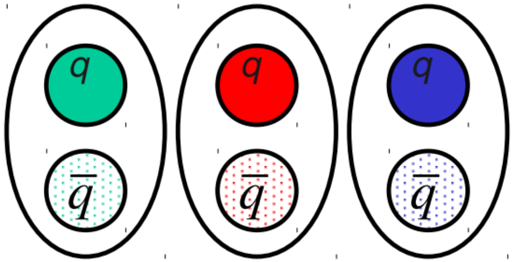
As a consequence, the QCD interaction becomes of short range. Very difficult to solve at low energies, But this is where most of the current world stands.

Color of Hadrons



BARYONS

RED + BLUE + GREEN = “WHITE”
or “COLORLESS”



MESONS

GREEN + ANTIGREEN = “COLORLESS”
RED + ANTIRED = “COLORLESS”
BLUE + ANTIBLUE = “COLORLESS”

A meson can be any one of these combinations !

Hadrons observed in nature are colorless (but there constituents are not)

Obtained using effective interaction between quarks

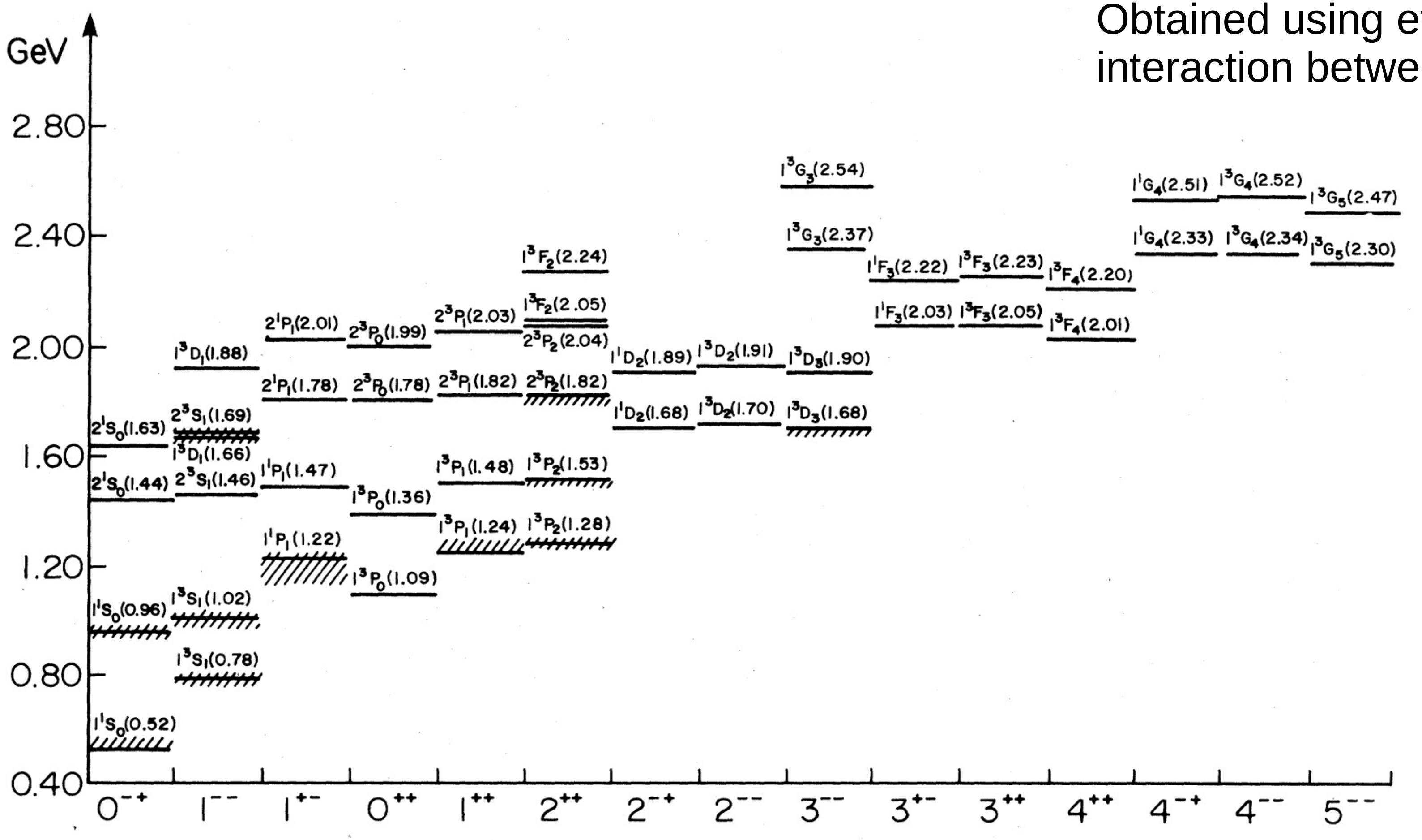


FIG. 5. The isoscalar mesons (mainly $u\bar{u}$, $d\bar{d}$, $s\bar{s}$). The legend is as for Fig. 3. Significant spectroscopic mixings in this sector are given in Table III. The comparison of the 0^{-+} isoscalars with experiment requires special consideration: see Sec. V A. For the E meson see Ref. 9.

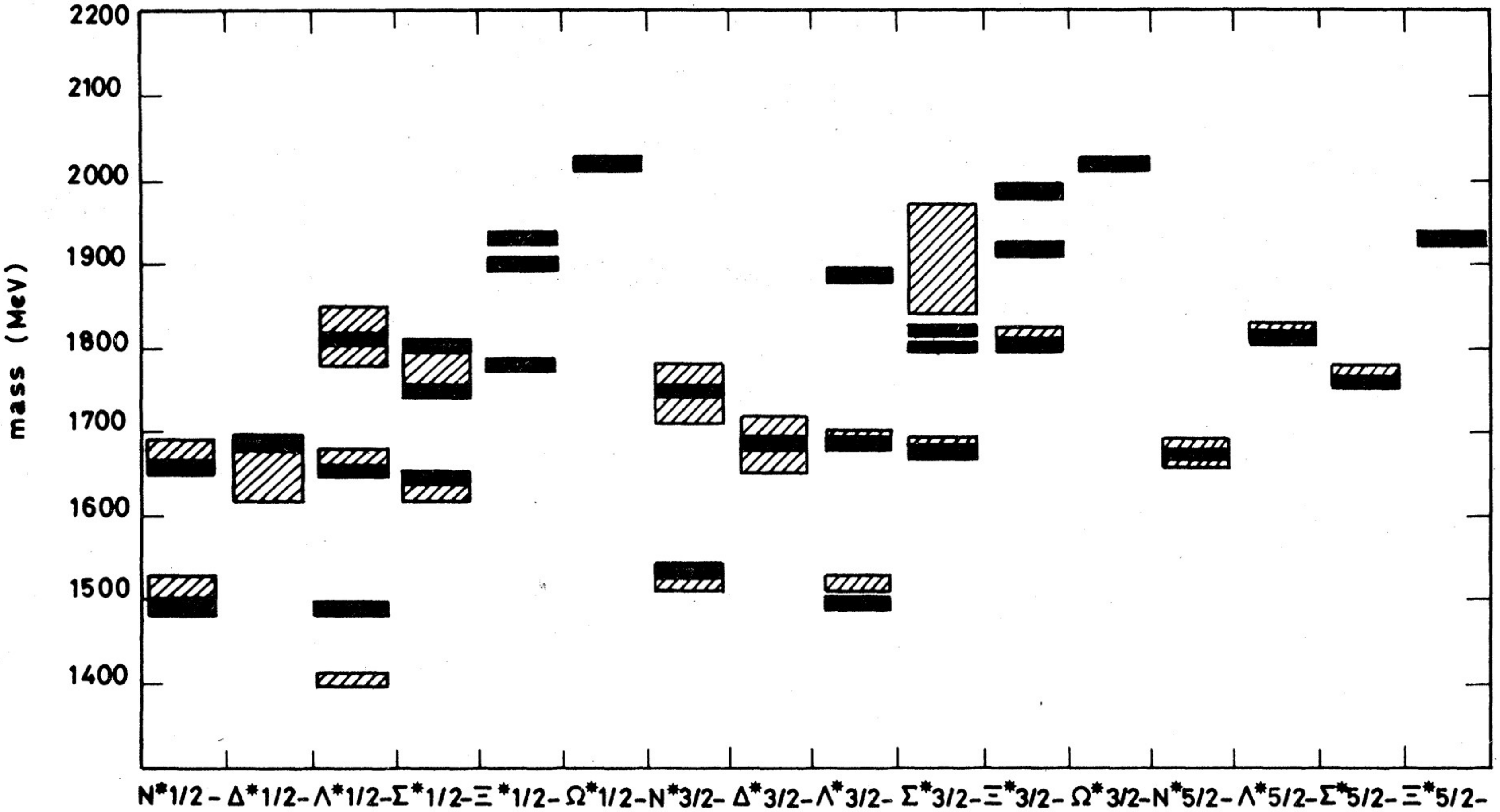
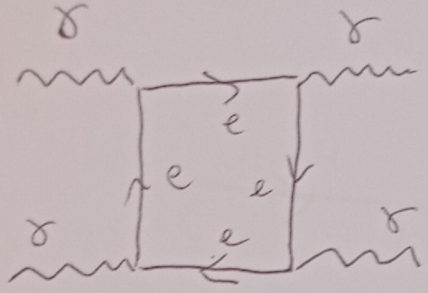


FIG. 1. Comparison of the predicted and observed spectrum of negative-parity baryons. The shaded regions correspond to the likely mass values of resonances; the solid bars are the predictions of the text, corresponding to the parameters $m_0 = 1610$ MeV, $\omega = 520$ MeV, $x = 0.6$, $\Delta m = 280$ MeV, and $\delta = 300$ MeV.

Effective field theories.
Example γγ → γγ



standard mechanism in QED

After integrating over the electron loop, the amplitude is of the type

$$\mathcal{L}_{eff} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a}{m_e^4} \left(F^{\mu\nu} F_{\mu\nu} \right)^2 + \frac{b}{m_e^4} F^{\mu\nu} F_{\nu\sigma} F^{\sigma\rho} F_{\rho\mu} + O(F^6/m_e^8)$$

$a = -\frac{\alpha^2}{36}$; $b = \frac{7\alpha^2}{90}$. We can make an effective theory at low energies where only the photon field appears, and the electron field has disappeared (integrated out).

Effective theories for the interaction of hadrons.

Weinberg had the wisdom to propose an effective theory to describe the interaction at low energies between hadrons, eliminating the quarks and considering only the hadrons as elementary fields: Chiral Lagrangians

$$\mathcal{L}_2 = \frac{1}{12 f^2}\langle(\partial_\mu \Phi \Phi - \Phi \partial_\mu \Phi)^2 + M\Phi^4\rangle$$

Meson-Meson

$$\Phi \equiv \frac{\boldsymbol{\lambda}}{\sqrt{2}}\boldsymbol{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix},$$

Meson baryon Lagrangian

$$L_1^{(B)} = \langle \bar{B} i\gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B} B \rangle$$

$$\nabla_\mu B = \partial_\mu B + [\Gamma_\mu, B] \ ,$$

$$\Gamma_\mu = \tfrac{1}{2}(u^+ \partial_\mu u + u \partial_\mu u^+)$$

$$U = u^2 = \exp(i\sqrt{2}\Phi / f)$$

$$u_\mu = i u^+ \partial_\mu U u^+ \ .$$

$$L_1^{(B)} = \langle \bar{B} i\gamma^\mu \frac{1}{4f^2} [(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi) B - B(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi)] \rangle$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

With these Lagrangians one can do perturbation theory → chiral perturbation theory

However, one can use the amplitudes obtained and consider them as the potential to be used I in the Shroedinger equations (Lippmann Schwinger equation, Bethe Salpeter equation) →

Chiral unitary theory

$$H\Psi = (H_0 + V)\Psi = E\Psi \Rightarrow (E - H_0)\Psi = V\Psi$$

$$(E - H_0)\Phi = 0$$

$$\Psi = \Phi + \frac{1}{E - H_0} V\Psi \Rightarrow \Psi = \Phi + \frac{1}{E - H_0} T\Phi$$

$$T\Phi \equiv V\Psi.$$

T= V+ VGT

$$T = V + V \frac{1}{E - H} V.$$

In coupled channels

T=(1-VG)$^{-1}$ V

T has a pole for eigenstates of H

We are familiar with nuclei: The smallest one, the deuteron. n p proton bound state.

In principle, there are 6 quarks, but in the deuteron the n p keep their identity
→ molecule of two baryons, not a 6 quark bag.

What happens with mesons? Something similar:

One can look for poles of the T matrix → bound states or resonances

Chiral unitary approach with meson-meson: ππ, K Kbar, ηη → $f_0(500)$,$f_0(980)$
πη, K Kbar → $a_0(980)$
πK, ηK → $K_0^*(700)$

With pseudoscalar-vector → axial vector mesons: $a_1(1260)$, $b_1(1235)$, $h_1(1270)$, $h_1(1380)$, $f_1(1285)$, $f_1(1420)$, $K_1(1270)$

With vector vector → $f_2(1270)$, $f_0(1370)$, $f_0(1710)$ …….$a_0(1780)$

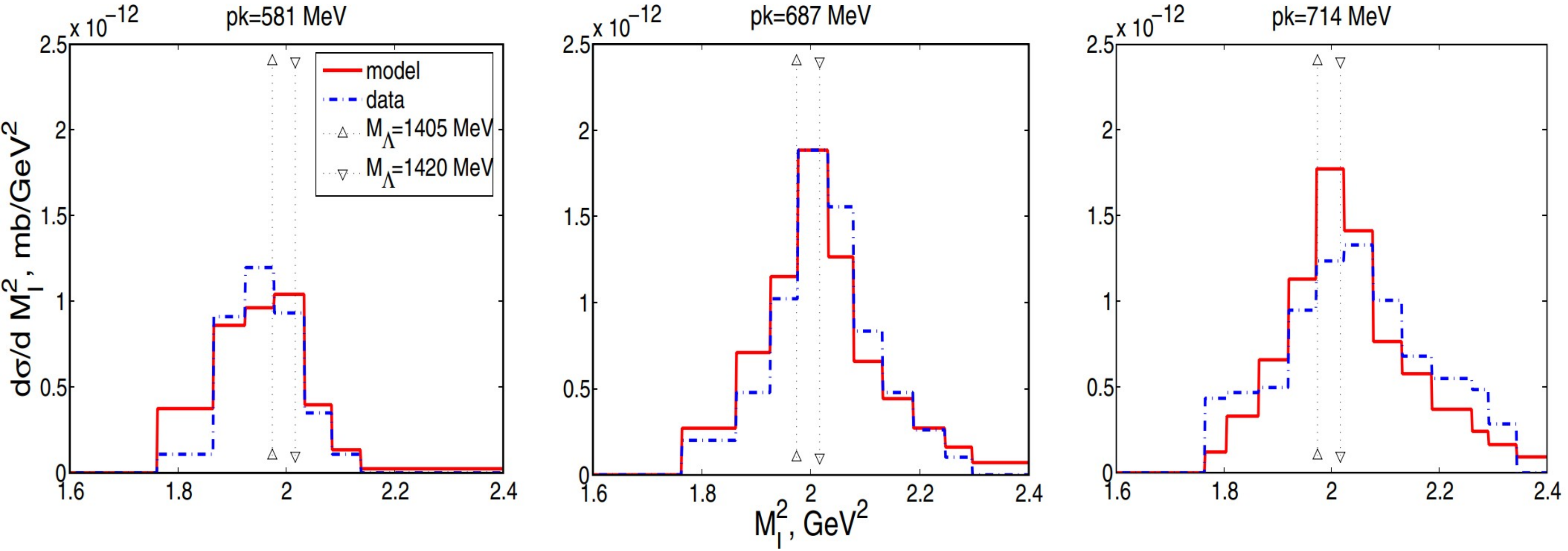
The $a_0(1780)$ was predicted in L.S. Geng and E. O. Phys.Rev.D 79 (2009) 074009

Has been found in BESIII , Phys Rev Lett 129,182001 (2022)

Meson baryon interaction: if one takes the coupled channels

$K^- p \quad \bar{K}^0 n \quad \pi^0 \Lambda \quad \pi^0 \Sigma^0 \quad \eta \Lambda \quad \eta \Sigma^0 \quad \pi^+ \Sigma^- \quad \pi^- \Sigma^+ \quad K^+ \Xi^- \quad K^0 \Xi^0$

Then one finds two poles, corresponding to two states of the Λ(1405)



$\pi^0 \pi^0 \Sigma^0$ reaction

$\pi^0 \Sigma^0$

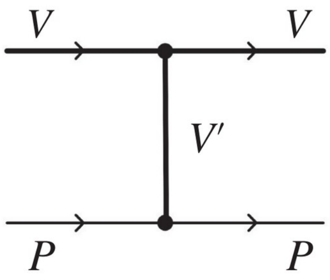
The PDG recently introduced two Λ(1405) states:

Λ(1380) 1/2⁻

Λ(1405) 1/2⁻

VP INTERACTION IN THE LOCAL HIDDEN GAUGE APPROACH

Bando et al Phys Rep. 164



$$\mathcal{L}_{VVV} = ig\langle (V_\mu \partial_\nu V^\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle$$

$$g = M_V/2f \; (M_V \approx 800 \text{ MeV}, \; f = 93 \text{ MeV})$$

$$\mathcal{L}_{VPP} = -ig\langle V^\mu [P, \partial_\mu P] \rangle$$

Neglecting the k/M_V

$$\varepsilon_1(\mathbf{k}) = (0, 1, 0, 0)$$

$$\varepsilon_2(\mathbf{k}) = (0, 0, 1, 0)$$

$$\varepsilon_3(\mathbf{k}) = (|\mathbf{k}|, 0, 0, \omega_\mathbf{k})/m_W$$

$$-it = -g(V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu)_{ij} V^\nu_{ji} \frac{i}{q^2 - M_V^2} V^{\nu'}_{lm} [P, \partial_{\nu'} P]_{ml}$$

$$\sum_{pol} \epsilon^\nu_{ji} \epsilon^{\nu'}_{lm} = \left(-g^{\nu\nu'} + \frac{q^\nu q^{\nu'}}{M_V^2} \right) \delta_{jl} \delta_{im}$$

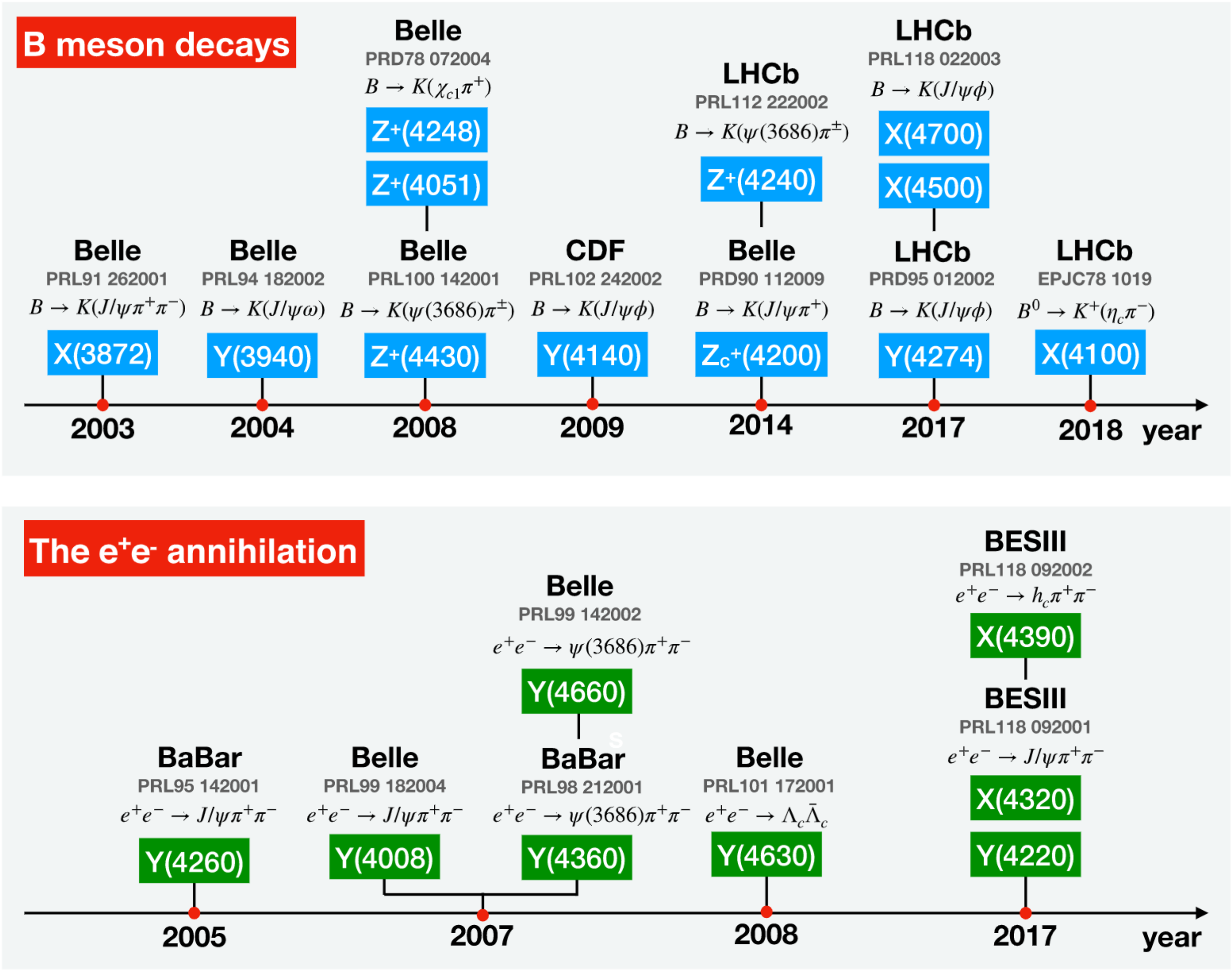
$$-it = -i \frac{g^2}{M_V^2} \langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu)[P, \partial^\nu P] \rangle$$

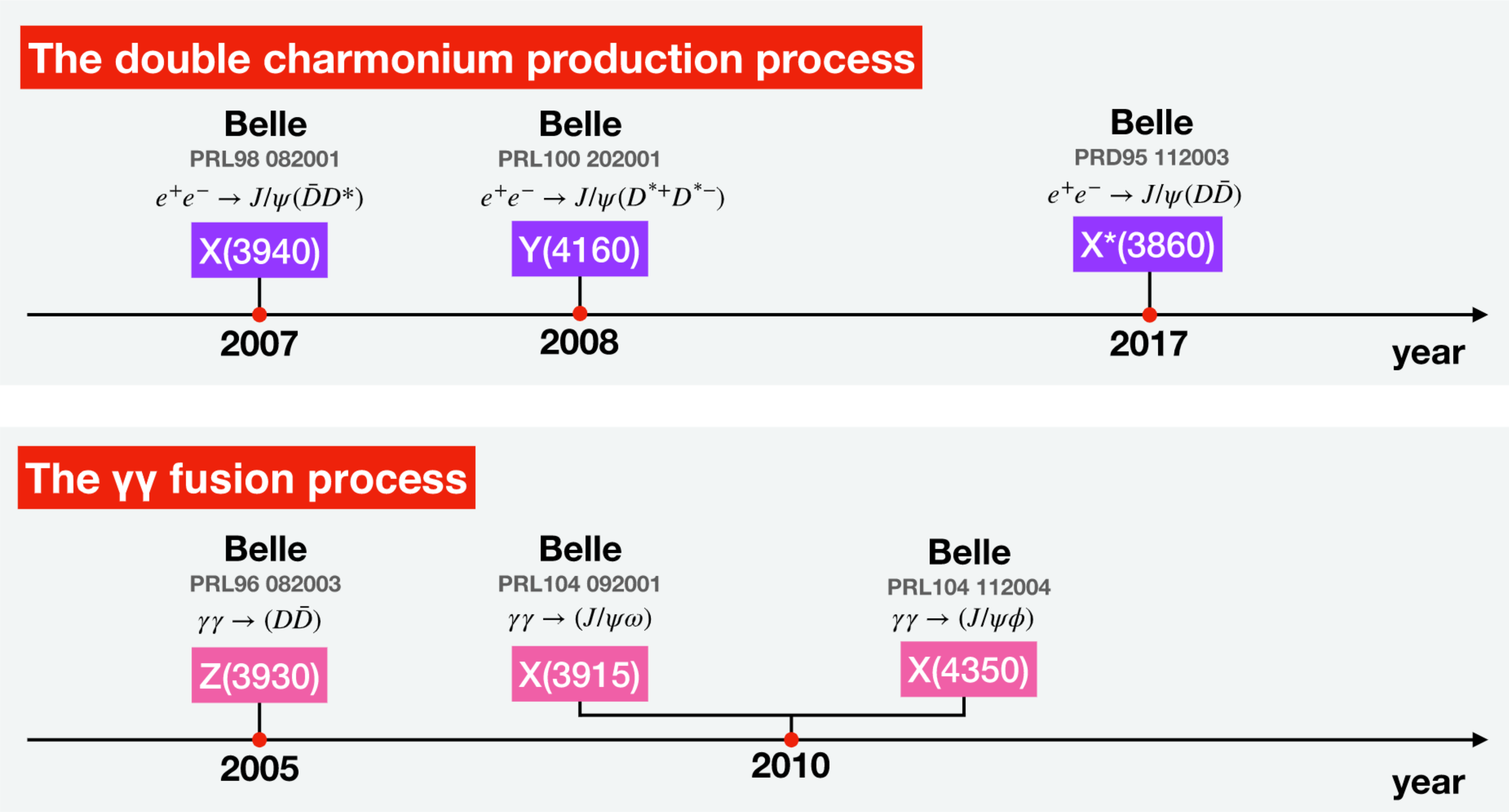
$$\mathcal{L} = -\frac{1}{4f^2} \langle [V^\mu, \partial_\nu V^\mu][P, \partial^\nu P] \rangle$$

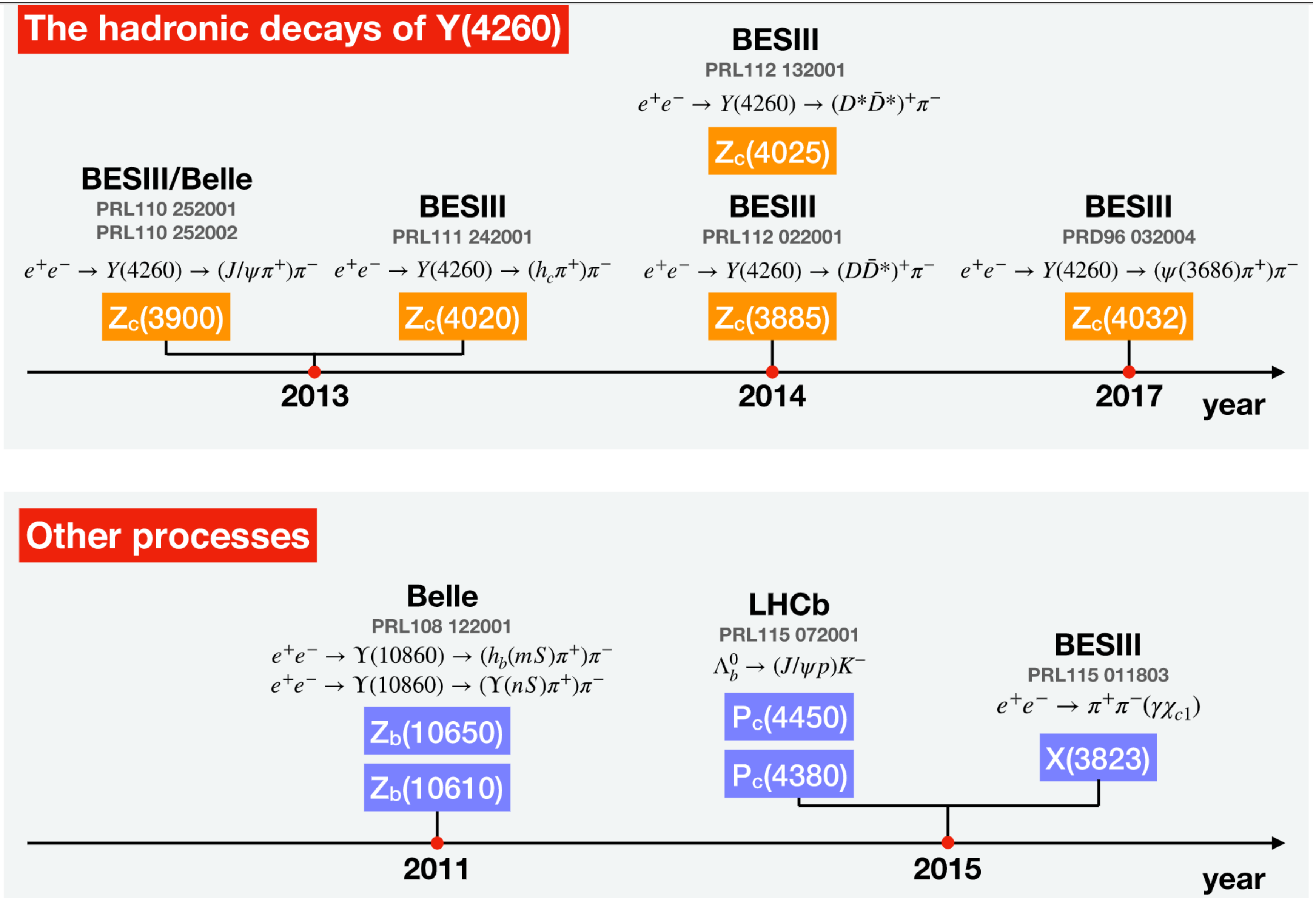
Chiral Lagrangian of M. C. Birse, Z. Phys. A 355, 231 (1996)

For PP interaction general prove in De Rafael et al. Phys Lett B 223, 425 (1989)

Exotic Hadrons



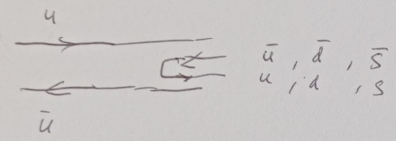

The double charmonium production process
Belle
PRL98 082001
$e^+e^- \to J/\psi(\bar{D}D^*)$
X(3940)
2007
Belle
PRL100 202001
$e^+e^- \to J/\psi(D^{*+}D^{*-})$
Y(4160)
2008
Belle
PRD95 112003
$e^+e^- \to J/\psi(D\bar{D})$
X*(3860)
2017
year
The γγ fusion process
Belle
PRL96 082003
$\gamma\gamma \to (D\bar{D})$
Z(3930)
2005
Belle
PRL104 092001
$\gamma\gamma \to (J/\psi\omega)$
X(3915)
Belle
PRL104 112004
$\gamma\gamma \to (J/\psi\phi)$
X(4350)
2010
year


The hadronic decays of Y(4260)
BESIII
PRL112 132001
$e^+e^- \to Y(4260) \to (D^*\bar{D}^*)^+\pi^-$
$Z_c(4025)$
BESIII/Belle
PRL110 252001
PRL110 252002
$e^+e^- \to Y(4260) \to (J/\psi\pi^+)\pi^-$
$Z_c(3900)$
BESIII
PRL111 242001
$e^+e^- \to Y(4260) \to (h_c\pi^+)\pi^-$
$Z_c(4020)$
BESIII
PRL112 022001
$e^+e^- \to Y(4260) \to (D\bar{D}^*)^+\pi^-$
$Z_c(3885)$
BESIII
PRD96 032004
$e^+e^- \to Y(4260) \to (\psi(3686)\pi^+)\pi^-$
$Z_c(4032)$
2013
2014
2017
year
Other processes
Belle
PRL108 122001
$e^+e^- \to \Upsilon(10860) \to (h_b(mS)\pi^+)\pi^-$
$e^+e^- \to \Upsilon(10860) \to (\Upsilon(nS)\pi^+)\pi^-$
$Z_b(10650)$
$Z_b(10610)$
LHCb
PRL115 072001
$\Lambda_b^0 \to (J/\psi p)K^-$
$P_c(4450)$
$P_c(4380)$
BESIII
PRL115 011803
$e^+e^- \to \pi^+\pi^-(\gamma\chi_{c1})$
X(3823)
2011
2015
year

X(3872) seen in $J/\psi\, \pi\pi$, $J/\psi\, \pi\pi\pi$ ...

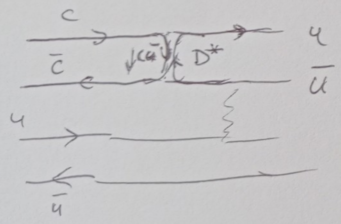
$$I^G(J^{PC}) = 0^+(1^{++})$$

n Exotic: Could be $q\bar{q}$ → But then decay width very large

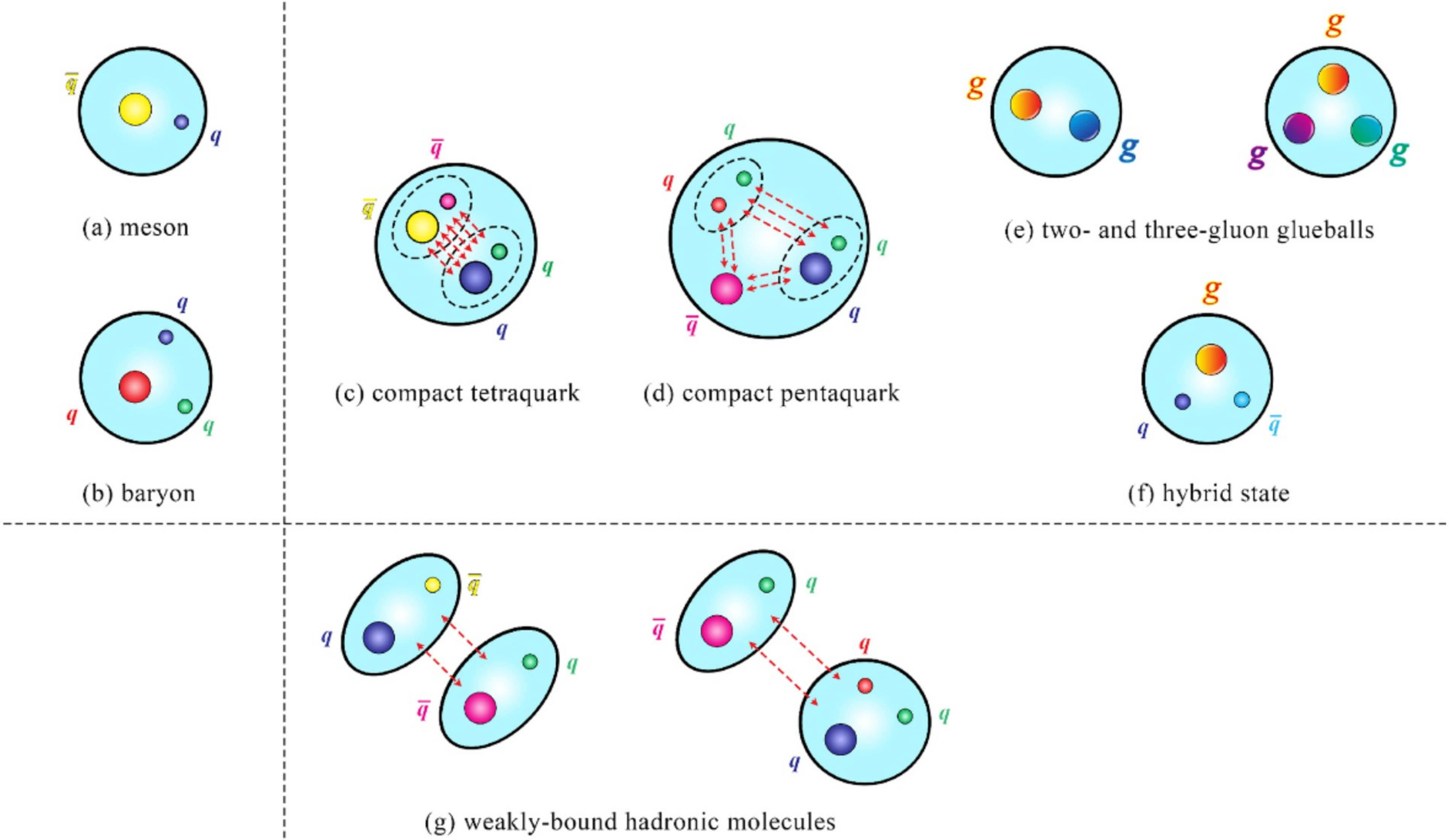


⇒ $\pi\pi, \eta\eta, \pi\eta, K\bar{K}$: Much energy available for the decay → Γ ↑↑

Exotic : $c\bar{c}\, u\bar{u}$ ...



$\sim \frac{1}{m_{D^*}^2}$ is suppressed . The decay is suppresed because one must convert $c\bar{c}$ into light quarks.

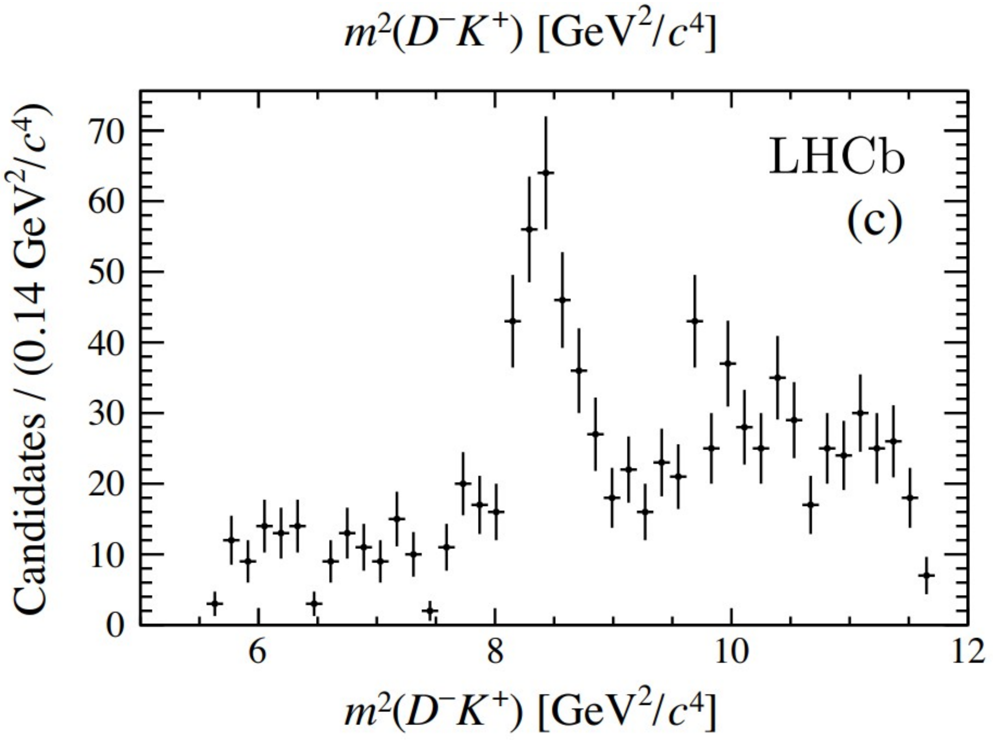


(a) meson
(b) baryon
(c) compact tetraquark
(d) compact pentaquark
(e) two- and three-gluon glueballs
(f) hybrid state
(g) weakly-bound hadronic molecules

LHCb : PHYS. REV. D 102, 112003 (2020)

$B^+ \to D^+ D^- K^+$ decay

LHCb,   PHYSICAL REVIEW D 102, 112003 (2020)



$X_0(2866)$ in the D- K+
invariant mass distribution

Quark content:

cbar sbar q q

It is necessarilly exotic since it has
two open quarks . Cannot be q qbar

New interpretation for the Ds2*(2573) and the prediction of novel exotic charmed mesons

R. Molina, T. Branz, E. Oset, PHYSICAL REVIEW D 82, 014010 (2010)

State predicted of D* K*bar nature.   This contains c s quarks and is exotic

The local hidden gauge   for VV interaction has an extra contact term

$$\mathcal{L}_{\mathrm{VVVV}} = \frac{1}{2} g^2 \langle [V_\mu, V_\nu] V^\mu V^\nu \rangle$$

Spin projection operators

$$V_\mu = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$

$$\mathcal{P}^{(0)} = \frac{1}{3} \epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu$$

$$\mathcal{P}^{(1)} = \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu - \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu)$$

$$\mathcal{P}^{(2)} = \{\frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu) - \frac{1}{3} \epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu\}$$

TABLE XI. Amplitudes for $C = 1$, $S = -1$ and $I = 0$.

J	Amplitude	Contact	V exchange	~Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$4g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1 + p_3).(p_2 + p_4)$	$-9.9g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1 + p_3).(p_2 + p_4)$	$-10.2g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-2g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1 + p_3).(p_2 + p_4)$	$-15.9g^2$

TABLE XII. Amplitudes for $C = 1$, $S = -1$ and $I = 1$.

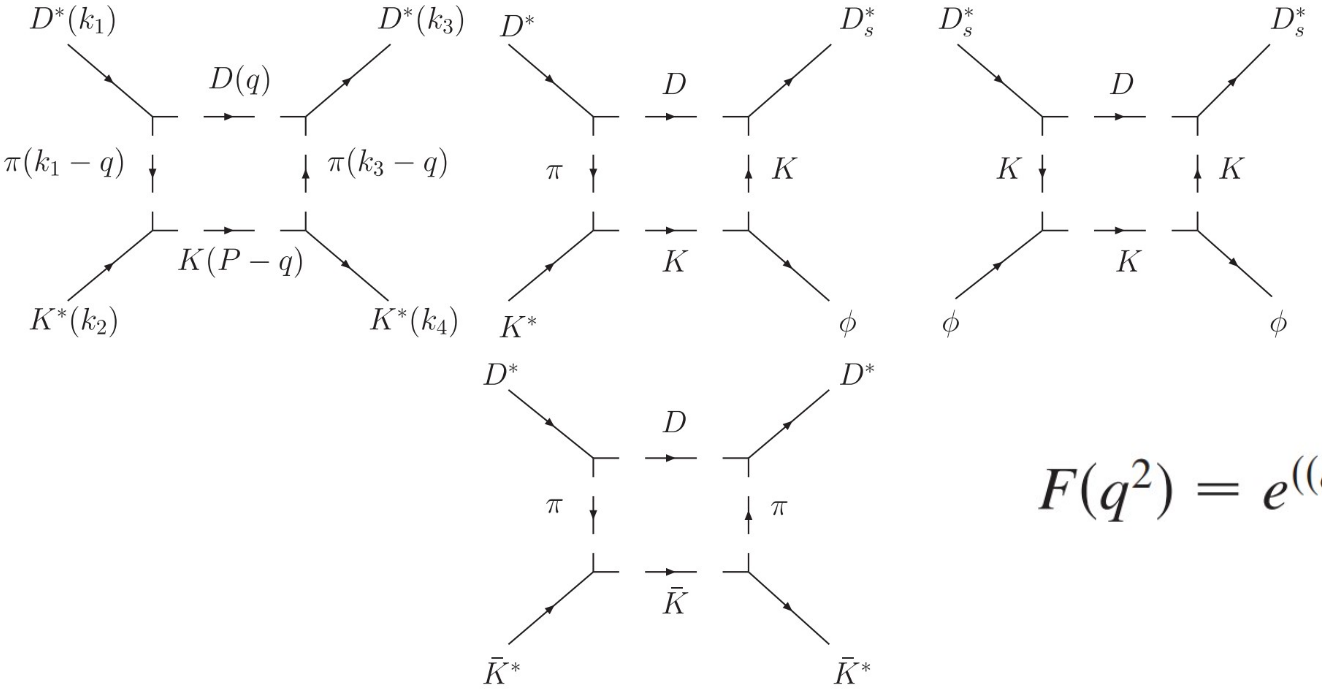
J	Amplitude	Contact	V exchange	~Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-4g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{g^2}{2}(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1 + p_3).(p_2 + p_4)$	$9.7g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{g^2}{2}(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1 + p_3).(p_2 + p_4)$	$9.9g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$2g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{g^2}{2}(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1 + p_3).(p_2 + p_4)$	$15.7g^2$

$$T = (\hat{1} - VG)^{-1}V.$$

$$G_i = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - M_1^2 + i\epsilon} \frac{1}{(P-q)^2 - M_2^2 + i\epsilon}$$

G is regularized either with a cutoff in the three momentum or dimensional regularization, with qmax, or a subtraction constant α.

Decay terms, added to V and iterated in the Bethe Salpeter equation.
Through its imaginary part they provide the decay to DKbar



$$F(q^2) = e^{((q^0)^2-|\vec{q}|^2)/\Lambda^2}$$

TABLE VI. $C = 1$; $S = -1$; $I = 0$. Mass and width for the states with $J = 0$ and 2.

$I[J^P]$	$\sqrt{s}_{\text{pole}}$ (MeV)	Model	Γ (MeV)
$0[0^+]$	2848	A, $\Lambda = 1400$ MeV	23
		A, $\Lambda = 1500$ MeV	30
		B, $\Lambda = 1000$ MeV	25
		B, $\Lambda = 1200$ MeV	59
$0[1^+]$	2839	Convolution	3
$0[2^+]$	2733	A, $\Lambda = 1400$ MeV	11
		A, $\Lambda = 1500$ MeV	14
		B, $\Lambda = 1000$ MeV	22
		B, $\Lambda = 1200$ MeV	36

R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 125, 242001 (2020)

R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 102, 112003 (2020)

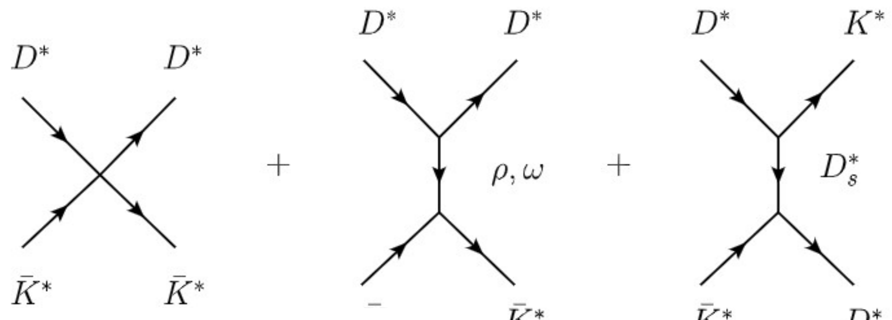
$$X_0(2866) : M = 2866 \pm 7 \quad \text{and} \quad \Gamma = 57.2 \pm 12.9\,\text{MeV},$$

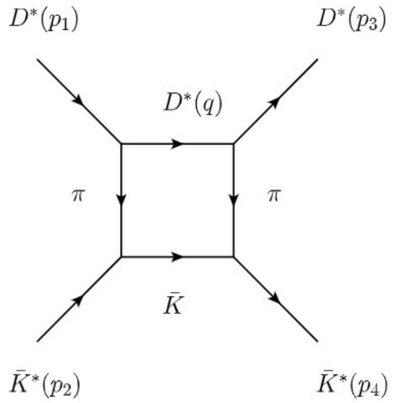
$$X_1(2900) : M = 2904 \pm 5 \quad \text{and} \quad \Gamma = 110.3 \pm 11.5\,\text{MeV}$$

Decaying to DKbar

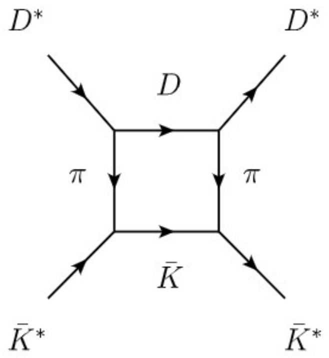
The state predicted corresponds to the $X_0(2866)$

Revision to the light of experimental results   R. Molina, E. O. Phys.Lett.B 811 (2020) 135870





Decay mode



$$\mathcal{L} = \frac{iG'}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \delta_\mu V_\nu \delta_\alpha V_\beta P \rangle$$

with $G' = \frac{3g'}{4\pi^2 f}$; $g' = -\frac{G_V m_\rho}{\sqrt{2} f^2}$, $G_V \simeq 55$ MeV,

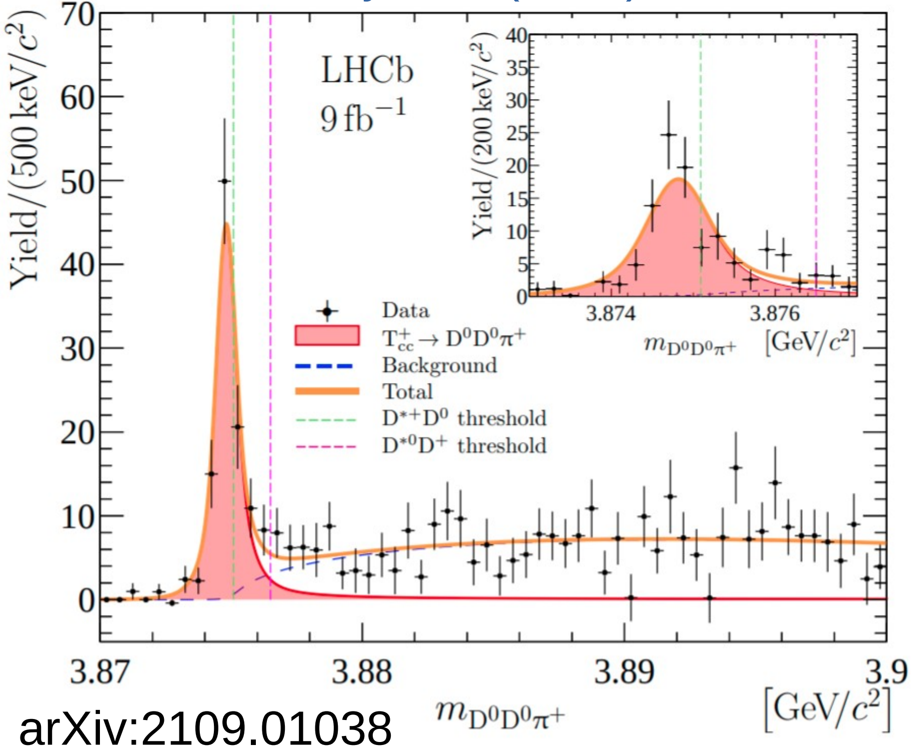
q_{max} is chosen to fit the exact mass
Λ to get the precise width of X_0

$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$0(2^+)$	2775	38	$D^*\bar{K}^*$	?
$0(1^+)$	2861	20	$D^*\bar{K}^*$	?
$0(0^+)$	2866	57	$D^*\bar{K}^*$	$X_0(2866)$

No D Kbar decay
No D* Kbar decay

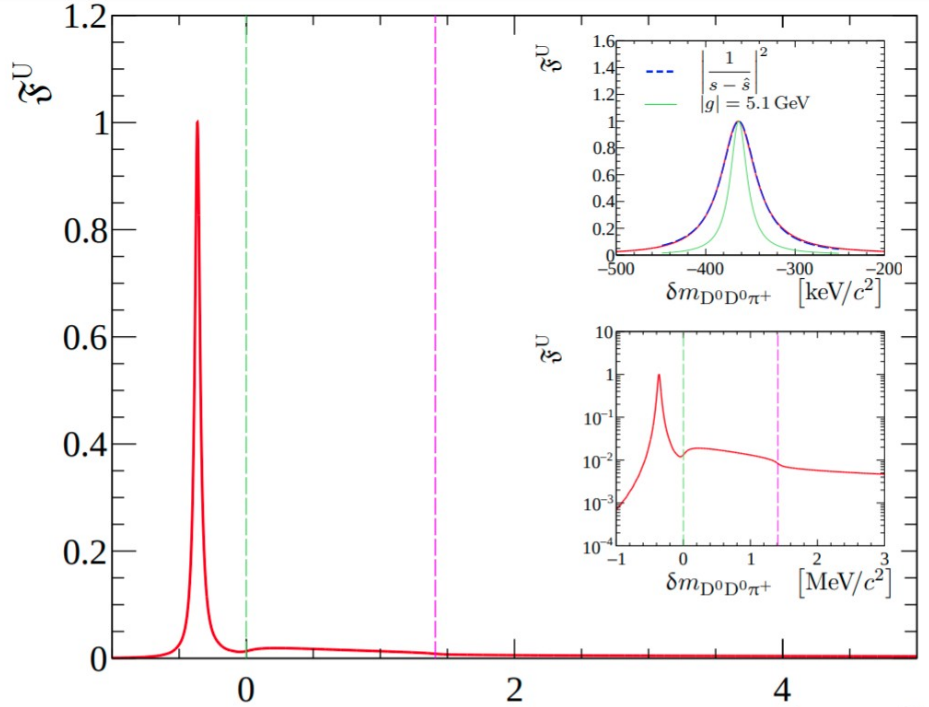
The Tcc discovery by the LHCb collaboration

Nature Phys. 18 (2022) 7, 751



arXiv:2109.01038

Nature Commun. 13 (2022) 1, 3351



arXiv:2109.01056 [

Spectra without correction by experimental resolution

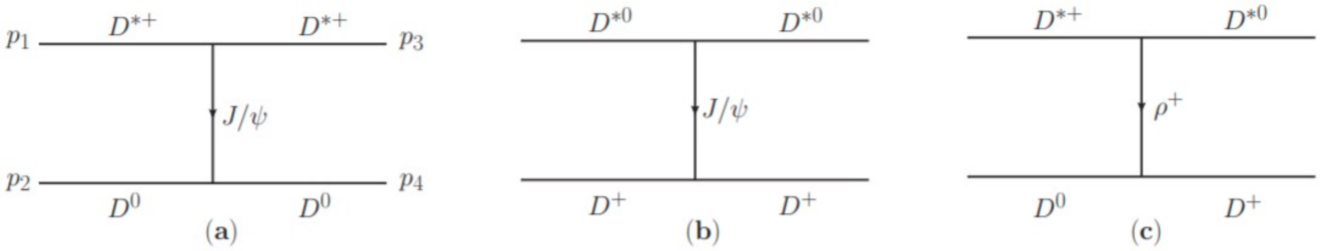
$$m_{\rm exp} = 3875.09 \text{ MeV} + \delta m_{\rm exp},$$

$$\delta m_{\rm exp} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}. \quad \Gamma = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

Spectra corrected by resolution and analyzed with a unitary amplitude

$$\delta m_{\rm exp} = -360 \pm 40^{+4}_{-0} \text{ keV} \qquad \Gamma = 48 \pm 2^{+0}_{-14} \text{ keV}.$$

A. Feijoo, W.H. Liang, Eulogio Oset, Phys.Rev.D 104 (2021) 11, 114015



$$\mathcal{L}_{VPP} = -ig\,\langle [P, \partial_\mu P] V^\mu \rangle,$$
$$\mathcal{L}_{VVV} = ig\,\langle (V^\nu \partial_\mu V_\nu - \partial_\mu V^\nu V_\nu) V^\mu \rangle,$$
$$g = \frac{M_V}{2\,f}, \quad (M_V = 800\ \text{MeV},\ f = 93\ \text{MeV}).$$

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}$$

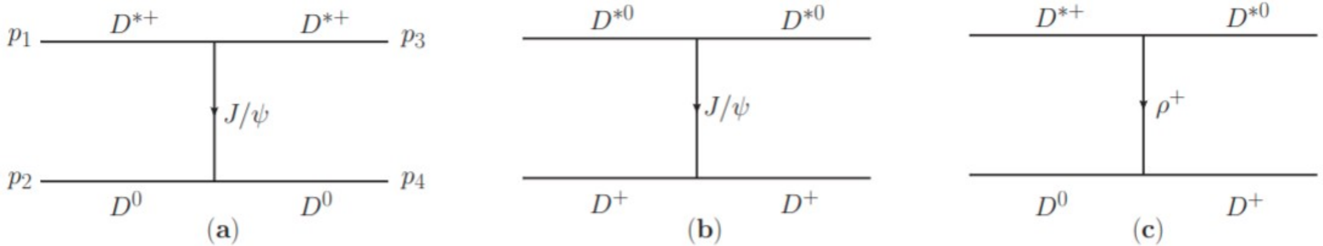
$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$

$D^{*+}D^0, D^{*0}D^+$ the 1, 2 channels, the interaction that we obtain is

$$V_{ij} = C_{ij}\, g^2\, (p_1 + p_3)\cdot(p_2 + p_4)\, \vec{\epsilon}\cdot\vec{\epsilon}\,'$$
$$\rightarrow C_{ij}\, g^2\, \frac{1}{2}[3s - (M^2 + m^2 + M'^2 + m'^2) - \frac{1}{s}(M^2 - m^2)(M'^2 - m'^2)]\, \vec{\epsilon}\cdot\vec{\epsilon}\,',$$

$$C_{ij} = \begin{pmatrix} \frac{1}{M^2_{J/\psi}} & \frac{1}{m^2_\rho} \\ \frac{1}{m^2_\rho} & \frac{1}{M^2_{J/\psi}} \end{pmatrix}$$

$$T = [1 - VG]^{-1}\, V,$$



$$|D^*D, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+),$$
$$|D^*D, I=1, I_3=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+),$$

$$C_{00} = \frac{1}{M^2_{J/\psi}} - \frac{1}{m^2_\rho};\quad C_{11} = \frac{1}{M^2_{J/\psi}} + \frac{1}{m^2_\rho};\quad C_{01} = 0;$$

There is attraction in I=0, repulsion in I=1, but due to different masses there is a bit of isospin breaking

Convolution of the G function:
Origin of the width.

Spectral function
Mass distribution

$$\mathrm{Im}[D(s_V)] = \mathrm{Im}\Big(\frac{1}{s_V - M_V^2 + iM_V\Gamma_V}\Big)$$

$$G(\sqrt{s}, M_k, m_k) = \frac{\int_{(M_V-2\Gamma_V)^2}^{(M_V+2\Gamma_V)^2} ds_V\, G(\sqrt{s}, \sqrt{s_V}, m_k) \times \mathrm{Im}[D(s_V)]}{\int_{(M_V-2\Gamma_V)^2}^{(M_V+2\Gamma_V)^2} ds_V\, \mathrm{Im}[D(s_V)]}$$

$$G_l = i \int \frac{d^4q}{(2\pi)^4} \frac{M_l}{E_l(\mathbf{q})} \frac{1}{k^0 + p^0 - q^0 - E_l(\mathbf{q}) + i\epsilon}$$

$$\Gamma_{D^{*+}}(M_{\rm inv}) = \Gamma(D^{*+}) \left(\frac{m_{D^{*+}}}{M_{\rm inv}}\right)^2 \cdot \left[\frac{2}{3}\left(\frac{p_\pi}{p_{\pi,\rm on}}\right)^3 + \frac{1}{3}\left(\frac{p'_\pi}{p'_{\pi,\rm on}}\right)^3\right]$$

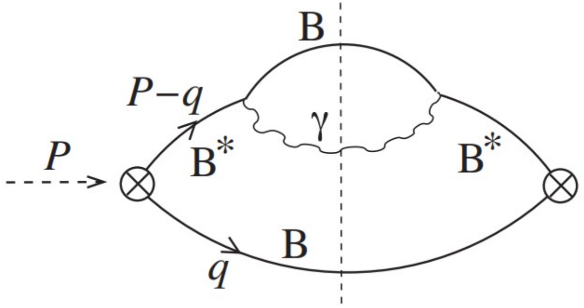
where p_π is the π^+ momentum in $D^{*+} \to D^0\pi^+$ decay

$p'_\pi, p'_{\pi,\rm on}$ are the same magnitudes for $D^{*+} \to D^+\pi^0$.

$$\Gamma_{D^{*0}}(M_{\rm inv}) = \Gamma(D^{*0}) \left(\frac{m_{D^{*0}}}{M_{\rm inv}}\right)^2 \cdot \left[0.647\left(\frac{p_\pi}{p_{\pi,\rm on}}\right)^3 + 0.353\right]$$

$D^{*0} \to D^0\pi^0 \qquad D^{*0} \to D^0\gamma$

Alternative method including vector selfenergy



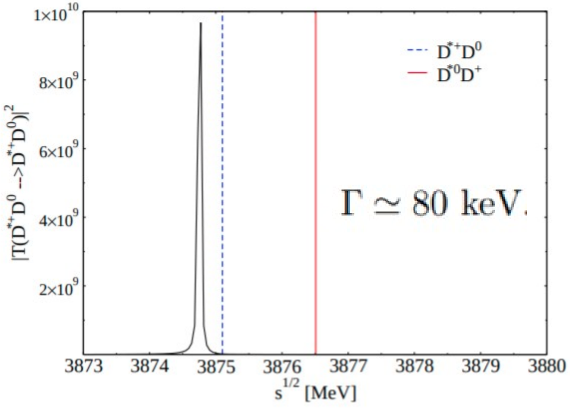
$$G(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_B^2 + i\epsilon}\frac{1}{(P-q)^2 - m_{B^*}^2 + i\sqrt{(P-q)^2}\Gamma_{B^*}((P-q)^2)}$$

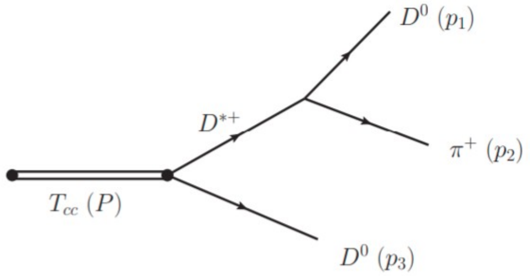
$$\Gamma_{B^*}(s') = \Gamma_{B^*}(m_{B^*}^2)\frac{m_{B^*}^2}{s'}\left(\frac{p_\gamma(s')}{p_\gamma(m_{B^*}^2)}\right)^3\Theta(\sqrt{s'} - m_B)$$

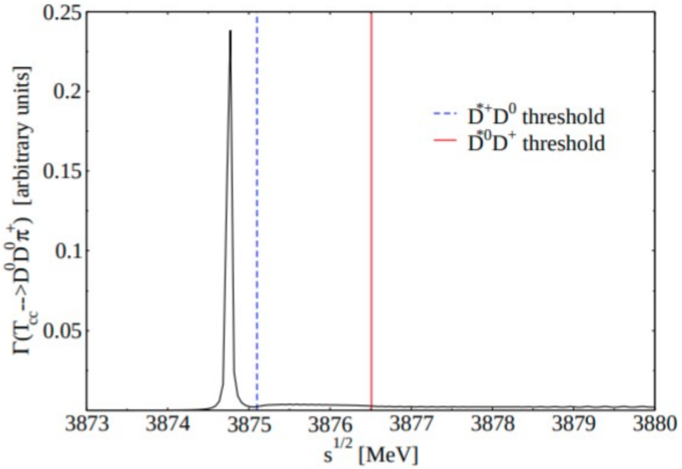
$$G(s) \simeq \int_0^{q_{\max}} dq\,\frac{q^2}{4\pi^2}\frac{\omega_B + \omega_{B^*}}{\omega_B\omega_{B^*}}\frac{1}{\sqrt{s} + \omega_B + \omega_{B^*}} \times \frac{1}{\sqrt{s} - \omega_B - \omega_{B^*} + i\frac{\sqrt{s'}}{2\omega_{B^*}}\Gamma_{B^*}(s')},$$

$\omega_{B(B^*)} = \sqrt{\vec{q}^2 + m_{B(B^*)}^2}$ and $s' = (\sqrt{s} - \omega_B)^2 - \vec{q}^2$.

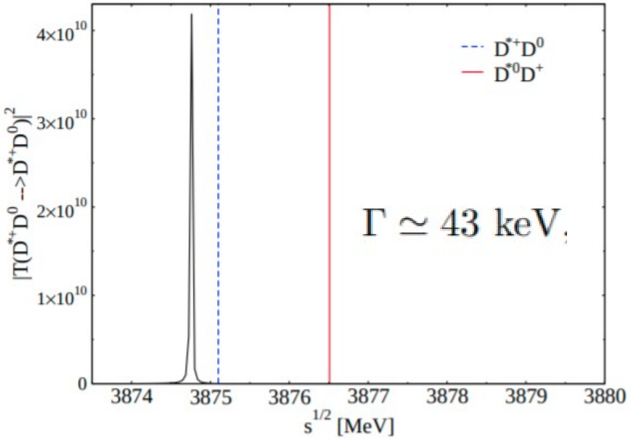
With mass of experimental raw data

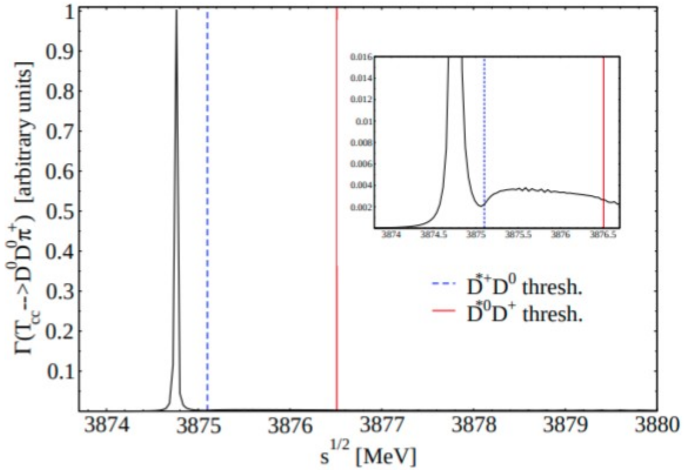






With mass from unitary reanalysis of LHCb data , Mikhasenko

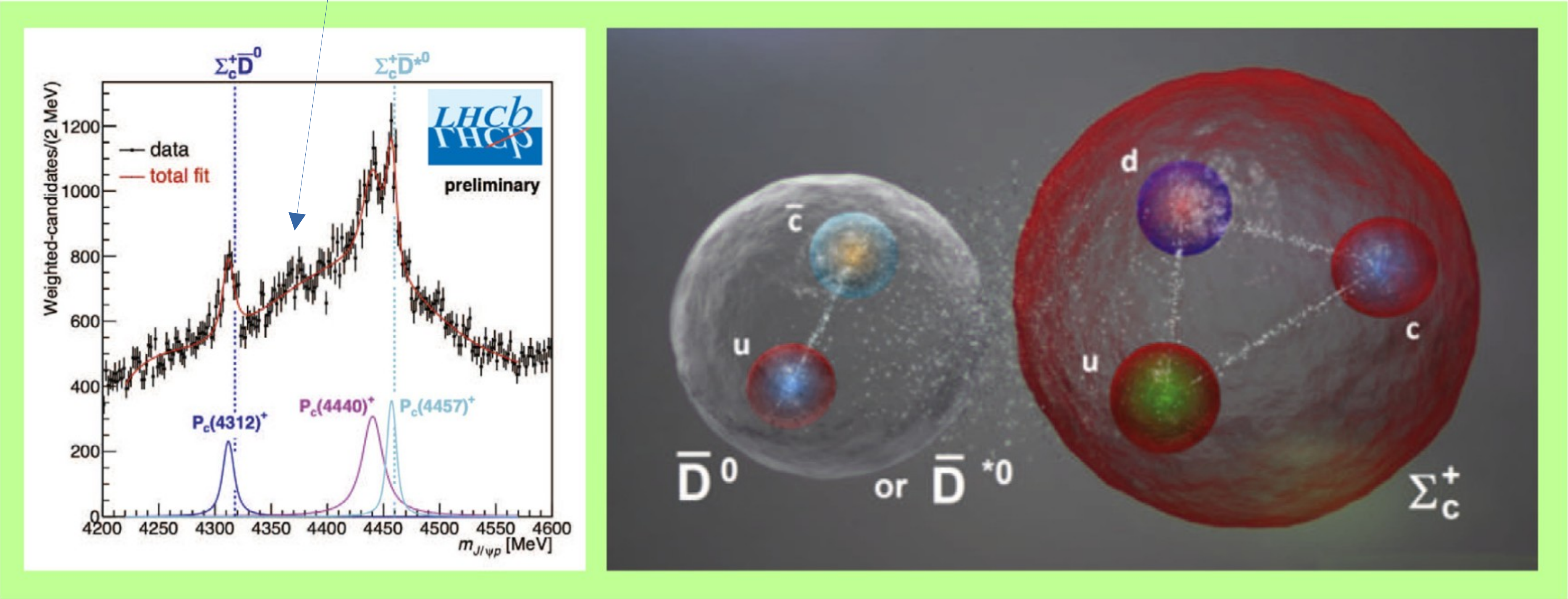




LHCb Phys.Rev.Lett. 115 (2015) 072001 Phys.Rev.Lett. 122 (2019) 22, 222001

J.~J.~Wu, R.~Molina, E.~Oset and B.~S.~Zou, %``Prediction of narrow N^* and Λ^* resonances with hidden charm above 4 GeV,'' Phys. Rev. Lett. 105, 232001 (2010)

$\bar{D}\Sigma_c^*$?



States of three or more mesons

A.~Martinez Torres, K.~P.~Khemchandani, L.~Roca and E.~Oset
“Few-body systems consisting of mesons," Few Body Syst. 61 35 (2020)

Table 1 Few-body systems studied in the literature involving one, two or more mesons

Components	States generated	Method u
$\bar{K}NN$	$\bar{K}$ bound states	F V FCA
$2PN$	$1/2^+$ Σ, Λ excited $1/2^+$ N^* states $(N^*(1920))$	χF χF
$\pi\pi N$	$N^*(1710)$	χF
$K\bar{K}N$	$N^*(1920)$	V, FCA
$KK\bar{K}$	$K(1460)$	χF CS F
$\pi K\bar{K}, \pi\pi\eta$	$\pi(1300)$, $f_0(1790)$	χF
$\phi K\bar{K}, \phi\pi\pi$	$\phi(2170)$	χF
$\pi\rho\Delta$	$\Delta_{5/2^+}(2000)$	FCA
$\pi\bar{K}K^*$	$\pi_1(1600)$	FCA
$\eta\bar{K}K^*$	$0(1^-)$ state around 1700 MeV	FCA
$\rho K\bar{K}$	$\rho(1700)$	FCA
Multi-ρ	$f_2(1270)$, $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$, $f_6(2510)$	FCA
K^* multi-ρ	$K_2^*(1430)$, $K_3^*(1780)$ $K_4^*(2045)$, $K_5^*(2380)$, K_6^*	FCA
PVV	$\pi_2(1670)$, $\eta_2(1645)$, $K_2^*(1770)$	FC
K multi-ρ	several K^* states	FCA
DNN	D bound state	FCA V
NDK, $ND\bar{K}$, $ND\bar{D}$	bound states of 3050, 3150, 4400 MeV	FC
DDK, $DD_s\eta$, $DD_s\pi$	$I = 1/2$ state around 4140 MeV	χF
DDK	Bound state, $B \simeq 70$ MeV	GE
$DDDK$	Bound state, $B \simeq 90 - 110$ MeV	GE
$J/\psi K\bar{K}$	$Y(4260)$	χF
$KD\bar{D}^*$	K^* Bound states	FCA
$DK\bar{K}$	D-like state at 2900 MeV	QSR, χF FC
$\rho D\bar{D}$	$I = 0$, 1 states 4200–4300 MeV	FCA
$\rho B^*\bar{B}^*$	$J = 3$ state at 10950 MeV	FCA
$D^{(*)}B^{(*)}\bar{B}^{(*)}$	Several bound states	FCA
$BD\bar{D}$, BDD	$BD\bar{D}$ bound state $\sim$ 8950 MeV	FCA
BB^*B^*, $B^*B^*B^*$	Bound $C = 3$ meson	F
DD^*K, $BB^*\bar{K}$	Bound states 4318 MeV, 11014 MeV	BO
$\bar{K}^*B\bar{B}$, $\bar{K}^*B^*\bar{B}^*$	Several bound states	FCA
BBB^*	Probable bound state	BO
D multi-ρ	Seven D^* states	FCA

P Pseudoscalar, *F* Faddeev, *FCA* Fixed center approximation, χ *F* Chiral Faddeev, *V* Variational, *GE* Gaussian expansion, *QSR* QCD sum rules, *BO* Born–Oppenheimer, *CS* Complex scaling

States of three or more mesons

The fixed center approximation to Faddeev equations

$$T_1 = t_1 + t_1 G_0 T_2,$$

$$T_2 = t_2 + t_2 G_0 T_1,$$

$$T = T_1 + T_2,$$

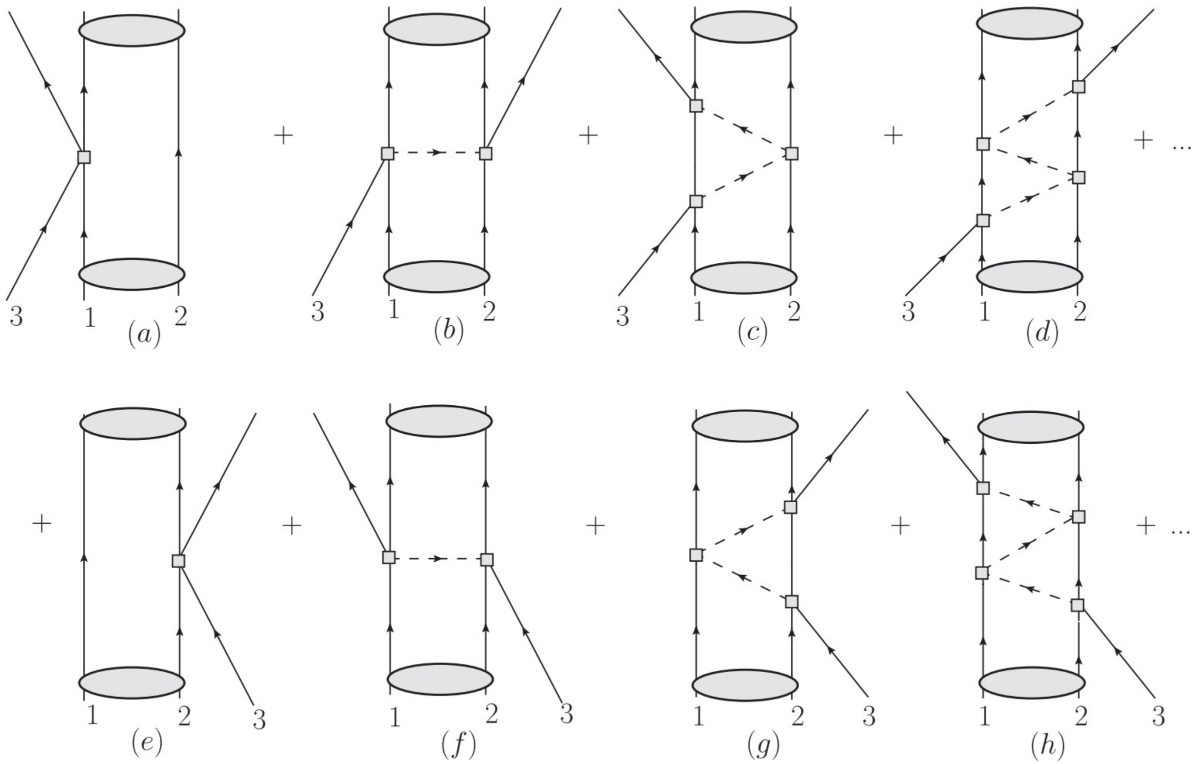


FIG. 1. Diagrammatic representation of the FCA to Faddeev equations.

Multimeson states

L.~Roca and E.~Oset,
%``A description of the f2(1270), rho3(1690), f4(2050), rho5(2350) and f6(2510) resonances as multi-rho(770) states," Phys. Rev. D 82, 054013 (2010)

TABLE I. Results for the masses of the dynamically generated states.

n_ρ		Mass, PDG [25]	Mass, only single scatt.	Mass, full model	$E(n_\rho)$
2	$f_2(1270)$	1275 ± 1	1275	1285	133
3	$\rho_3(1690)$	1689 ± 2	1753	1698	209
4	$f_4(2050)$	2018 ± 11	2224	2051	263
5	$\rho_5(2350)$	2330 ± 35	2690	2330–2366	302–309
6	$f_6(2510)$	2465 ± 50	3155	2607–2633	337–341

J.~Yamagata-Sekihara, L.~Roca and E.~Oset,
%``On the nature of the $K^*_2(1430)$, $K^*_3(1780)$, $K^*_4(2045)$, $K^*_5(2380)$ and K^*6 as K^* - multi-ρ states," Phys. Rev. D 82, 094017 (2010)

TABLE II. Results for the masses of the dynamically generated states. (All units are MeV.)

Generated resonance	Amplitude	Mass, PDG [26]	Mass only single scatt.	Mass full model
$K_2^*(1430)$	ρK^*	1429 ± 1.4	$\cdots$	1430
$K_3^*(1780)$	$K^* f_2$	1776 ± 7	1930	1790
$K_4^*(2045)$	$f_2 K_2^*$	2045 ± 9	2466	2114
$K_5^*(2380)$	$K^* f_4$	$2382 \pm 14 \pm 19$	2736	2310
K_6^*	$K_2^* f_4 - f_2 K_4^*$	$\cdots$	3073–3310	2661–2698

Main difference between nuclei and meson aggregates → Baryonic number conservation

There is no meson number conservation.

But in strong interaction there is FLAVOR CONSERVATION

This means we can construct meson aggregates with different flavors that cannot decay to a system with smaller number of mesons

Example : c c s s qbar q bar qbar qbar (q= u, d quarks) has 4 mesons and cannot decay to a system with less than 4 mesons

This makes these systems similar to ordinary nuclei : One can create many new system classified by NUMBER OF OPEN FLAVOR (quarks that their corresponding antiquark is not present in the system).

T.~W.~Wu, Y.~W.~Pan, M.~Z.~Liu and L.~S.~Geng,
%``Multi-hadron molecules: status and prospect,''
Sci. Bull. \textbf{67}, 1735-1738 (2022)

"P. W. Anderson once said, "more is different", which could also be true in hadron physics. Studies of multi-hadron molecules have just started and are in an infant stage, compared with the studies of multi-nucleon states (nuclei) and of two-body hadronic molecules." Much progress is expected in the coming years.

Correlation functions for the $D_{s0}(2317)$ and N*(1535) : the inverse problem

E. Oset, Natsumi Ikeno, Genaro Toledo, Raquel Molina, Chu Wen Xiao and Wei Hong Liang

IFIC, Departamento de Fisica Teorica, Universidad de Valencia

Construction of correlation functions

The channels in $D_{s0}(2317)$ production

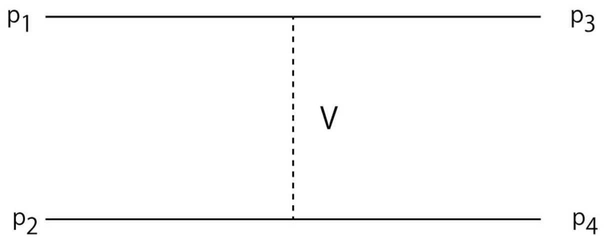
The channels in the N*(1535) production

The inverse problem of getting information from the correlation functions

Discussion on experimental extraction of scattering parameters

The $D_{s0}(2317)$ state

$$D^0 K^+, \quad D^+ K^0, \quad \text{and} \quad D_s^+ \eta$$



$$V_{ij} = C_{ij}\, g^2 (p_1 + p_3) \cdot (p_2 + p_4);$$

$$T = [1 - VG]^{-1} V$$

$$g = \frac{M_V}{2f}, \quad M_V = 800 \text{ MeV}, \quad f = 93 \text{ MeV},$$

$$G_i(s) = \int_{|\mathbf{q}|<q_{\max}} \frac{d^3 q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1 \omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon}$$

$$C_{ij} = \begin{pmatrix} -\frac{1}{2}\left(\frac{1}{M_\rho^2} + \frac{1}{M_\omega^2}\right) & -\frac{1}{M_\rho^2} & \frac{2}{\sqrt{3}} \frac{1}{M_{K^*}^2} \\ & -\frac{1}{2}\left(\frac{1}{M_\rho^2} + \frac{1}{M_\omega^2}\right) & \frac{2}{\sqrt{3}} \frac{1}{M_{K^*}^2} \\ & & 0 \end{pmatrix}$$

Ikeno, Toledo, E. O. PLB 847, 138281

$$(p_1 + p_3) \cdot (p_2 + p_4) \to \frac{1}{2}\Big[3s - (M^2 + m^2 + M'^2 + m'^2) - \frac{1}{s}(M^2 - m^2)(M'^2 - m'^2)\Big],$$

Projection in s-wave

Correlation functions

$$C(\boldsymbol{p}) = \int d^3\boldsymbol{r} S_{12}(\boldsymbol{r}) |\psi(\boldsymbol{r},\boldsymbol{p})|^2 \qquad S_{12}(r) = \frac{1}{(\sqrt{4\pi})^3 R^3} \exp\left(-\frac{r^2}{4R^2}\right)$$

Modified Kookin Pratt formalism

I.~Vidana, A.~Feijoo, M.~Albaladejo, J.~Nieves and E.~Oset Phys.Lett.B 846 (2023) 138201

$$C_{D^0K^+}(p_{K^+}) = 1+4\pi \int_0^{+\infty} dr r^2 S_{12}(r)\, \theta(q_{\max} - p_{K^+}) \left\{ \left| j_0(p_{K^+} r) + T_{11}(\sqrt{s})\, \widetilde{G}^{(1)}(s,r) \right|^2 + \omega_2 \left| T_{21}(\sqrt{s})\, \widetilde{G}^{(2)}(s,r) \right|^2 + \omega_3 \left| T_{31}(\sqrt{s})\, \widetilde{G}^{(3)}(s,r) \right|^2 - j_0^2(p_{K^+} r) \right.$$

$$C_{D^+K^0}(p_{K^0}) = 1+4\pi \int_0^{+\infty} dr r^2 S_{12}(r)\, \theta(q_{\max} - p_{K^0}) \left\{ \left| j_0(p_{K^0} r) + T_{22}(\sqrt{s})\, \widetilde{G}^{(2)}(s,r) \right|^2 + \omega_1 \left| T_{12}(\sqrt{s})\, \widetilde{G}^{(1)}(s,r) \right|^2 + \omega_3 \left| T_{32}(\sqrt{s})\, \widetilde{G}^{(3)}(s,r) \right|^2 - j_0^2(p_{K^0} r) \right\}$$

$$H\Psi = (H_0 + V)\Psi = E\Psi \Rightarrow (E - H_0)\Psi = V\Psi$$

$$(E - H_0)\Phi = 0$$

$$\Psi = \Phi + \frac{1}{E - H_0} V\Psi \Rightarrow \Psi = \Phi + \frac{1}{E - H_0} T\Phi$$

$$V(\vec{p}, \vec{p}\,') = V\theta(q_{\text{max}} - |\vec{p}\,|)\theta(q_{\text{max}} - |\vec{p}\,'|)$$

$$T(E; \vec{p}, \vec{p}\,') = T(E)\theta(q_{\text{max}} - |\vec{p}\,|)\theta(q_{\text{max}} - |\vec{p}\,'|)$$

$$\Psi(\vec{r}, \vec{p}\,) = e^{i\vec{p}\cdot\vec{r}} + \theta(q_{\text{max}} - |\vec{p}\,|)\, T(E) \times \int_{|\vec{q}\,|<q_{\text{max}}} \frac{d^3\vec{q}\; e^{i\vec{q}\cdot\vec{r}}}{E - \omega_1(q) - \omega_2(q) + i\eta}$$

$$e^{i\vec{q}\cdot\vec{r}} = 4\pi \sum_{\ell=0}^{+\infty} i^{\ell} j_{\ell}(qr) \sum_{m=-\ell}^{+\ell} Y^{*}_{\ell m}(\hat{q}) Y_{\ell m}(\hat{r})$$

$$\Psi(\vec{r}, \vec{p}\,) = e^{i\vec{p}\cdot\vec{r}} + f_E(r)$$

$$|\Psi(\vec{r}, \vec{p}\,)|^2 = 1 + |f_E(r)|^2 + 2\mathrm{Re}\left[e^{i\vec{p}\cdot\vec{r}} f^{*}_E(r)\right]$$

$$\int d^3\vec{r}\, S_{12}(r) |\Psi(\vec{r}, \vec{p}\,)|^2 = 1 + 4\pi \int_0^{+\infty} dr r^2\, S_{12}(r)$$

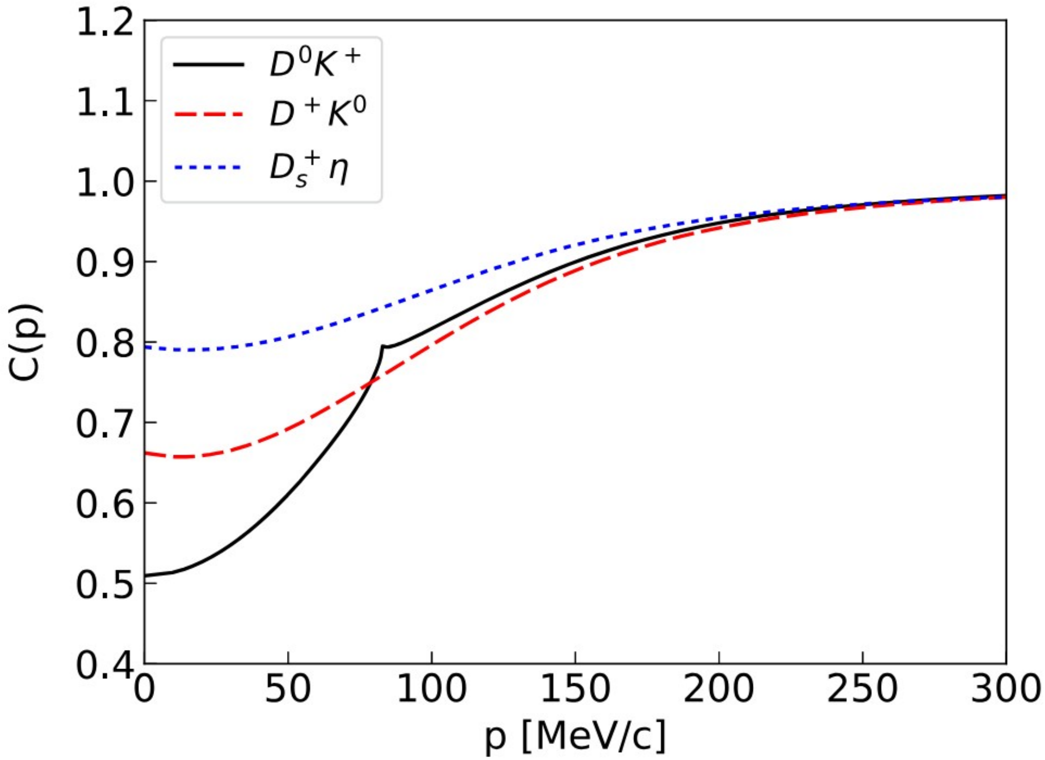
$$\times \left\{ |j_0(pr) + f_E(r)|^2 - j_0^2(pr) \right\}$$

$$C_{D_s\eta}(p_\eta) = 1+4\pi \int_0^{+\infty} dr r^2 S_{12}(r)\, \theta(q_{\max} - p_\eta)$$

$$\Big\{ \left| j_0(p_\eta r) + T_{33}(\sqrt{s})\, \widetilde{G}^{(3)}(s,r) \right|^2$$

$$+\, \omega_1 \left| T_{13}(\sqrt{s})\, \widetilde{G}^{(1)}(s,r) \right|^2$$

$$+\, \omega_2 \left| T_{23}(\sqrt{s})\, \widetilde{G}^{(2)}(s,r) \right|^2 - j_0^2(p_\eta r) \Big\}$$



C(p) constructed with R=1m

$$\widetilde{G}^{(i)}(s,r) = \int_{q<q_{\max}} \frac{d^3q}{(2\pi)^3}\, \frac{\omega_1^{(i)}(q) + \omega_2^{(i)}(q)}{2\omega_1^{(i)}(q)\, \omega_2^{i}(q)} \cdot \frac{j_0(qr)}{s - \left[\omega_1^{(i)}(q) + \omega_2^{(i)}(q)\right]^2 + i\epsilon}$$

Inverse problem

$$|DK, I=0\rangle = \frac{1}{\sqrt{2}}(D^+K^0 + D^0K^+)$$

$$|DK, I=1, I_3=0\rangle = \frac{1}{\sqrt{2}}(D^+K^0 - D^0K^+)$$

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ & V_{22} & V_{23} \\ & & 0 \end{pmatrix}$$

we will assume that the potential has isospin symmetry

we impose that $\langle I=0|V|I=1\rangle = 0$

$$V_{11} = V_{22}, \quad V_{13} = V_{23},$$

$$\langle DK, I=0|V|DK, I=0\rangle = V_{11} + V_{12}$$

$$\langle DK, I=1|V|DK, I=1\rangle = V_{11} - V_{12}$$

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ & V_{11} & V_{13} \\ & & 0 \end{pmatrix}$$

$$V_{11} = V'_{11} + \frac{\alpha}{M_V^2}(s - \bar{s}),$$

$$V_{12} = V'_{12} + \frac{\beta}{M_V^2}(s - \bar{s}),$$

$$V_{13} = V'_{13} + \frac{\gamma}{M_V^2}(s - \bar{s}),$$

Free parameters $V'_{11}, V'_{12}, V'_{13}, \alpha, \beta, \gamma, q_{\max}$, and R

We do many fits to the data with the resampling technique to evaluate errors in the observables, assuming errors in the correlation functions of the order of 0,02

$$q_{\rm max} = 689.03 \pm 103.37 \text{ MeV}$$

$$R = 0.984 \pm 0.040 \text{ fm}.$$

We get a pole at

$$E = 2314.2 \pm 21.0 \text{ MeV}$$

$$T_{ij} \sim \frac{g_i g_j}{s - s_0}$$

$$g_1^2 = \lim_{s \to s_0} (s - s_0)\, T_{11}; \quad g_j = g_1 \lim_{s \to s_0} \frac{T_{1j}}{T_{11}}$$

$$P_i = -g_i^2 \frac{\partial G_i}{\partial s}\Big|_{s=s_0}$$

$$T \equiv -8\pi\sqrt{s} f^{QM} \approx -8\pi\sqrt{s} \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik}$$

$$-\frac{1}{a} = -8\pi\sqrt{s} T^{-1}\big|_{s=s_{\rm th}},$$

$$r_0 = \frac{\partial}{\partial k^2} 2(-8\pi\sqrt{s}\, T^{-1} + ik)$$

$$= \frac{\sqrt{s}}{\mu} \frac{\partial}{\partial s} 2(-8\pi\sqrt{s} T^{-1} + ik)\big|_{s=s_{\rm th}}$$

Table 1

Values of the couplings, probabilities, scattering lengths, and effective ranges.

channel i	$1: D^0 K^+$	$2: D^+ K^0$	$3: D_s^+ \eta$
g_i [MeV]	8556.08 ± 2707.16	8571.21 ± 2710.52	-6161.84 ± 6307.93
P_i	0.357 ± 0.133	0.306 ± 0.119	0.083 ± 0.070
a_i [fm]	0.720 ± 0.131	$(0.518 \pm 0.051) - i\,(0.120 \pm 0.030)$	$(0.213 \pm 0.014) - i\,(0.054 \pm 0.025)$
$r_{0,i}$ [fm]	-2.479 ± 0.824	$(-0.162 \pm 0.778) - i\,(2.520 \pm 0.329)$	$(-0.165 \pm 1.677) - i\,(0.171 \pm 0.663)$

The probabilities are similar as in the lattice work, A. Martínez Torres, E. Oset, S. Prelovsek, A. Ramos, J. High Energy Phys. 05 (2015)

Table 2

Same as Table 1 except with the use of the two correlation functions of $D^0 K^+$ and $D^0 K^0$.

channel i	$1: D^0 K^+$	$2: D^+ K^0$	$3: D_s^+ \eta$
g_i [MeV]	7773.42 ± 3462.55	7789.64 ± 3483.53	-5716.45 ± 5659.24
P_i	0.353 ± 0.198	0.301 ± 0.184	0.080 ± 0.134
a_i [fm]	0.707 ± 0.060	$(0.504 \pm 0.034) - i\,(0.110 \pm 0.015)$	$(0.259 \pm 0.067) - i\,(0.055 \pm 0.036)$
$r_{0,i}$ [fm]	-3.139 ± 1.299	$(-0.665 \pm 1.020) - i\,(2.386 \pm 0.341)$	$(0.336 \pm 0.858) - i\,(0.081 \pm 0.447)$

The equal couplings for $D^0 K^+$ and D^+K^0 indicate that we have a D K isospin I=0 state

The chiral unitary approach for the $N^*(1535)$

Kaiser, Siegel and Weise
Phys.Lett.B 362 (1995) 23

$$K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \pi^+ n, \pi^0 p, \eta p$$

$$V_{ij} = -\frac{1}{4f^2} C_{ij}(k^0 + k'^0); \qquad f = 93 \text{ MeV} \qquad T = [1 - VG]^{-1} V$$

TABLE I. C_{ij} coefficients of Eq. (3).

C_{ij}	$K^0\Sigma^+$	$K^+\Sigma^0$	$K^+\Lambda$	$\pi^+ n$	$\pi^0 p$	ηp
$K^0\Sigma^+$	1	$\sqrt{2}$	0	0	$\frac{1}{\sqrt{2}}$	$-\sqrt{\frac{3}{2}}$
$K^+\Sigma^0$		0	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$K^+\Lambda$			0	$-\sqrt{\frac{3}{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{3}{2}$
$\pi^+ n$				1	$\sqrt{2}$	0
$\pi^0 p$					0	0
ηp						0

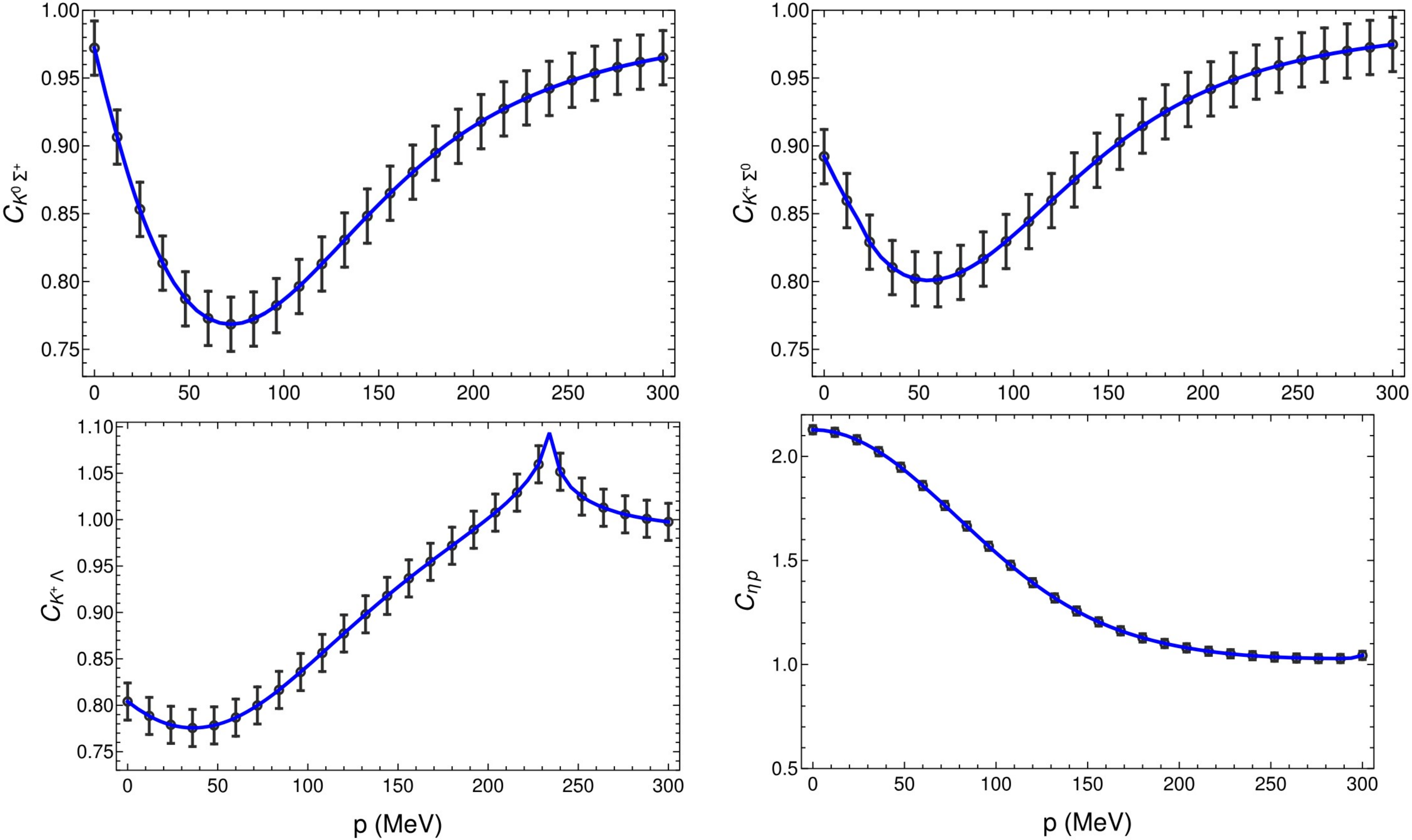
Correlation functions: Molina, Xiao, Liang, E. O. PRD 109, 054002

$$C_{K^0\Sigma^+}(p_{K^0}) = 1 + 4\pi\theta(q_{\max} - p_{K^0}) \int drr^2 S_{12}(r) \cdot \{|j_0(p_{K^0}r) + T_{K^0\Sigma^+,K^0\Sigma^+}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2$$
$$+ |T_{K^+\Sigma^0,K^0\Sigma^+}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2 + |T_{K^+\Lambda,K^0\Sigma^+}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2$$
$$+ |T_{\eta p,K^0\Sigma^+}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^0}r)\},$$

$$C_{K^+\Sigma^0}(p_{K^+}) = 1 + 4\pi\theta(q_{\max} - p_{K^+}) \int drr^2 S_{12}(r) \cdot \{|j_0(p_{K^+}r) + T_{K^+\Sigma^0,K^+\Sigma^0}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2$$
$$+ |T_{K^0\Sigma^+,K^+\Sigma^0}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Lambda,K^+\Sigma^0}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2$$
$$+ |T_{\eta p,K^+\Sigma^0}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^+}r)\},$$

$$C_{K^+\Lambda}(p_{K^+}) = 1 + 4\pi\theta(q_{\max} - p_{K^+}) \int drr^2 S_{12}(r) \cdot \{|j_0(p_{K^+}r) + T_{K^+\Lambda,K^+\Lambda}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2$$
$$+ |T_{K^0\Sigma^+,K^+\Lambda}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Sigma^0,K^+\Lambda}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2$$
$$+ |T_{\eta p,K^+\Lambda}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^+}r)\},$$

$$C_{\eta p}(p_\eta) = 1 + 4\pi\theta(q_{\max} - p_\eta) \int drr^2 S_{12}(r) \cdot \{|j_0(p_\eta r) + T_{\eta p,\eta p}(E)\tilde{G}^{(\eta p)}(r;E)|^2$$
$$+ |T_{K^0\Sigma^+,\eta p}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Sigma^0,\eta p}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2 + |T_{K^+\Lambda,\eta p}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2 - j_0^2(p_\eta r)\}$$



R=1fm

Isospin symmetry

$$\left|K\Sigma, I=\frac{1}{2}, I_3=\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}K^0\Sigma^+ + \sqrt{\frac{1}{3}}K^+\Sigma^0,$$

$$(K^+, K^0), (-\pi^+, \pi^0, \pi^-), (-\Sigma^+, \Sigma^0, \Sigma^-)$$

$$\left|K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\right\rangle = -\sqrt{\frac{1}{3}}K^0\Sigma^+ + \sqrt{\frac{2}{3}}K^+\Sigma^0.$$

$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\right|V\left|K\Sigma, I=\frac{1}{2}, I_3=\frac{1}{2}\right\rangle = 0,$$

$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\right|V|K^+\Lambda\rangle = 0,$$

$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\right|V|\eta p\rangle = 0.$$

channels $K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \eta p$

$$V_{ij} = -\frac{1}{4f^2}\tilde{C}_{ij}(k^0 + k'^0)$$

$$V_{ij} = \begin{pmatrix} V_{11} & \sqrt{2}(V_{11}-V_{22}) & V_{13} & V_{14} \\ & V_{22} & \frac{1}{\sqrt{2}}V_{13} & \frac{1}{\sqrt{2}}V_{14} \\ & & V_{33} & V_{34} \\ & & & V_{44} \end{pmatrix}$$

7 Cij free parameters plus qmax, and R

$q_{\max}$ (MeV)	R (fm)
637 ± 72	1.02 ± 0.02

TABLE III. Scattering lengths for channel i (in units of fm).

a_1 $(0.46 \pm 0.04) - (0.64 \pm 0.03)i$	a_2 $(0.32 \pm 0.01) - (0.35 \pm 0.02)i$
a_3 $(0.30 \pm 0.02) - (0.22 \pm 0.04)i$	a_4 $(-0.780 \pm 0.013) + (0 \pm 0)i$

$$\mathcal{P}_1 \simeq 0.12 - 0.23i, \qquad \mathcal{P}_2 \simeq 0.06 - 0.12i,$$
$$\mathcal{P}_3 \simeq 0.22 - 0.28i, \qquad \mathcal{P}_4 \simeq -0.34 - 0.24i$$

$$|\mathcal{P}_1| = 0.26, \qquad |\mathcal{P}_2| = 0.13,$$
$$|\mathcal{P}_3| = 0.35, \qquad |\mathcal{P}_4| = 0.42.$$

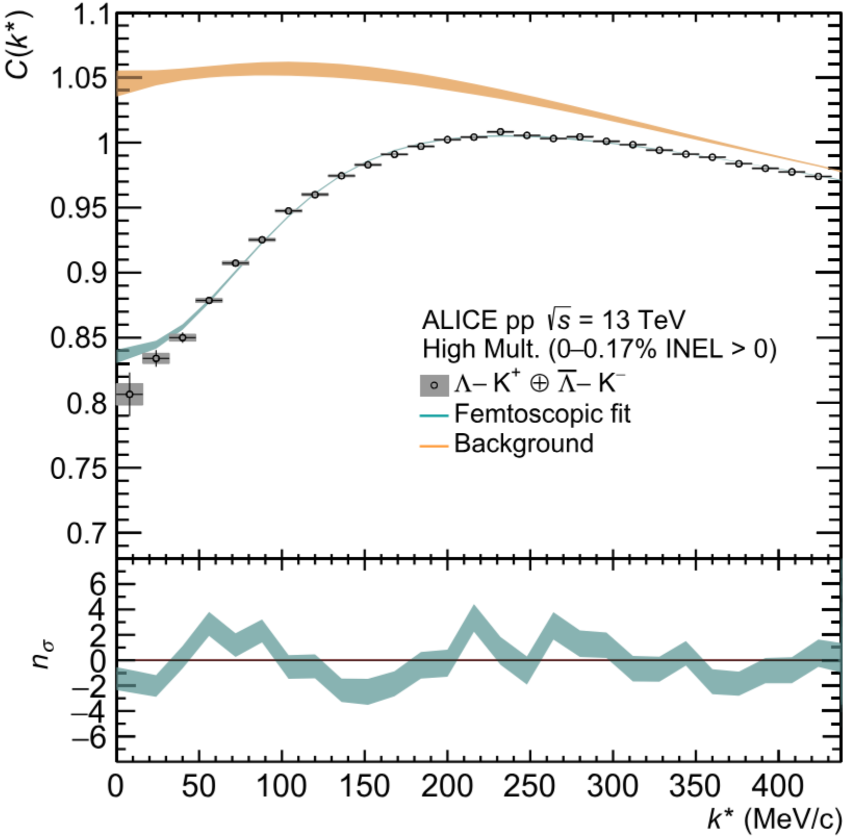
TABLE IV. Effective range parameters for channel i (in units of fm).

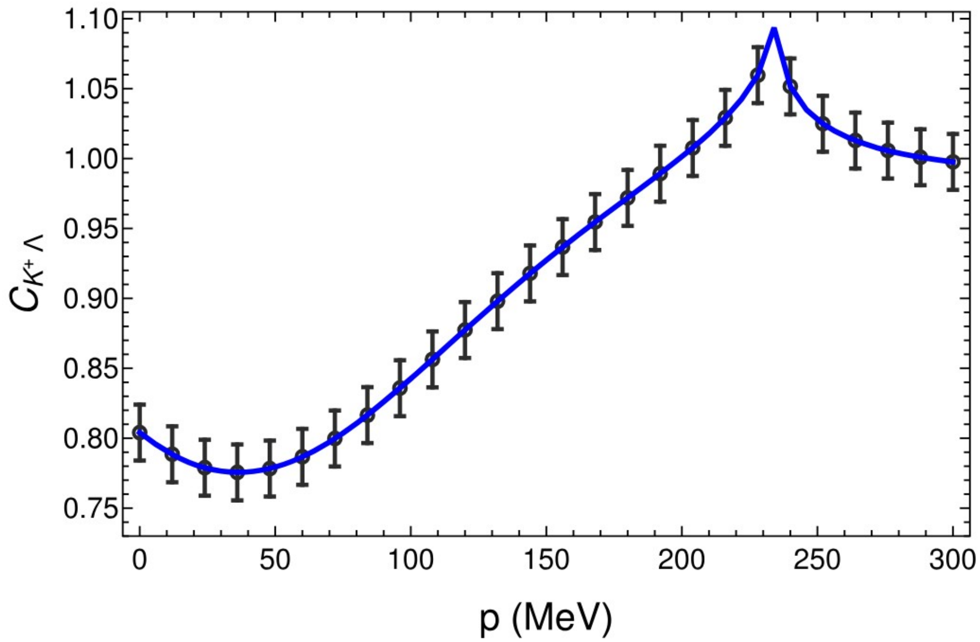
r_1	r_2	r_3	r_4
$(-1.1 \pm 0.2) - (2.7 \pm 0.2)i$	$(-6.2 \pm 1.4) + (8.8 \pm 0.5)i$	$(-2.8 \pm 0.3) - (0.3 \pm 0.6)i$	-1.48 ± 0.13

The couplings g1, g2 indicate I=1/2 state

TABLE V. Pole position and couplings (in units of MeV).

$\sqrt{s_p}$ $(1515 \pm 6) - (89 \pm 9)i$	g_1 $(3.7 \pm 0.3) - (1.04 \pm 0.13)i$	g_2 $(2.6 \pm 0.2) - (0.74 \pm 0.10)i$
	g_3 $(3.6 \pm 0.2) - (0.28 \pm 0.05)i$	g_4 $(-2.68 \pm 0.13) + (1.4 \pm 0.2)i$





Experimental analysis of a, r, done with single channel

MESSAGE: the analysis must be done with coupled channels.

	Pair	Λ–K^+
$-a_3$	$\Re f_0$ (fm)	$-0.61 \pm 0.03(\text{stat}) \pm 0.03(\text{syst})$
	$\Im f_0$ (fm)	$0.23 \pm 0.06(\text{stat}) \pm 0.04(\text{syst})$
r_0	d_0 (fm)	$0.80 \pm 0.19(\text{stat}) \pm 0.18(\text{syst})$

a_3

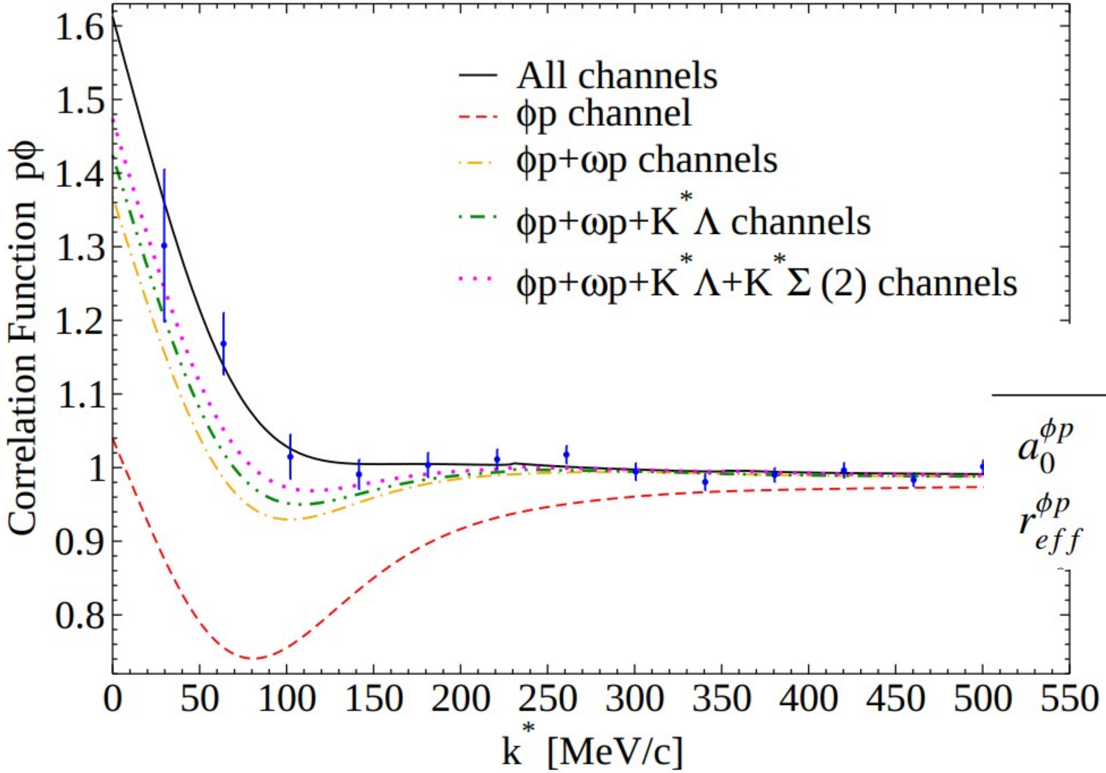
$(0.30\pm0.02)-(0.22\pm0.04)i$

r_3

$(-2.8 \pm 0.3) - (0.3 \pm 0.6)i$

Relevance of coupled channel analysis stressed in a recent paper

A.~Feijoo, M.~Korwieser and L.~Fabbietti,
%``Relevance of the coupled channels in the ϕp and ρ^0p Correlation Functions,''
[arXiv:2407.01128 [hep-ph]].



(from single channel analysis)

$$a_0^{\phi p} = (0.85 \pm 0.48) + i\,(0.16 \pm 0.19)\,\text{fm}$$

$$r_{eff}^{\phi p} = 7.85 \pm 1.80\,\text{fm}.$$

(coupled channels)

	Pure theoretical	**Bootstrap**
$a_0^{\phi p}$	$0.272 + i\,0.189$	$(-0.034 \pm 0.035) + i\,(0.57 \pm 0.09)$
$r_{eff}^{\phi p}$	$-7.20 - i\,0.09$	$(-8.06 \pm 2.57) + i\,(0.05 \pm 0.53)$

Conclusions

We explore the inverse problem of getting a,r_0, bound states associated, molecular probabilites

From the correlation functions of D^0K^+, D^+K^0, and $D_s^+\eta$
we find the existence of the $D_{s0}(2013)$ state

From the correlation functions of the channels $K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \eta p$
we find the existence of the N*(1535) state

a, r_0 for all the channels are obtained with high precision.

ONE MUST AVOID USING SINGLE CHANNEL ANALYSIS TO DETERMINE a AND r_0