Dense QCD Matter and Astrophysical Implications

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The University of Tokyo

— Fudan Summer School 2024 —

Lecture Plan

[Basic Course]

1. **Foundation of Nuclear Physics**

2. **Foundation of Neutron Stars**

[Advanced Course]

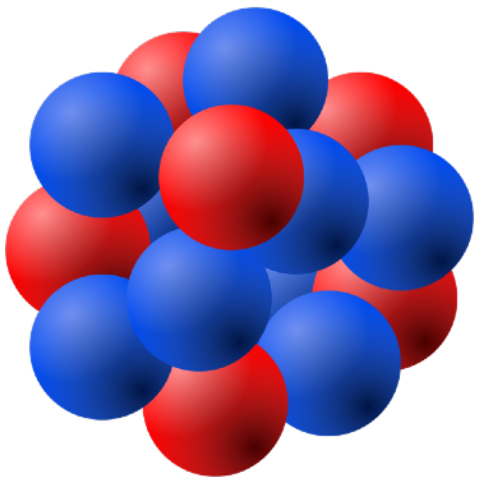
3. **Exotic Phases and Phase Transitions**

4. **Constraing the EOS**

Foundation of Nuclear Physics

Nuclear Matter

The (stable) densest matter on the Earth



[Nuclei from Wikipedia]

Heavy nuclei have an almost constant density.

$$n_{\mathrm{sat}} = 0.16\ (\mathrm{nucleon})\ \mathrm{fm}^{-3}$$

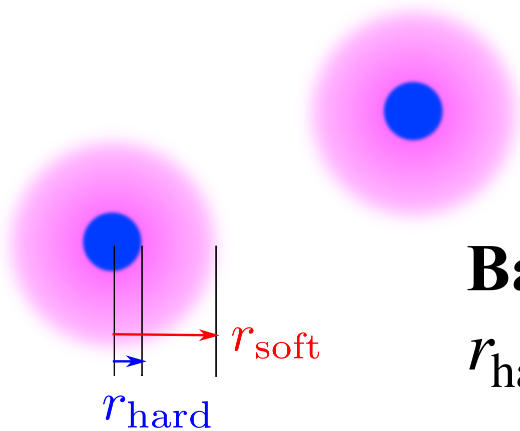
called "saturation density" in nuclear physics

In astrophysics, the rest-mass density is used:

$$\rho_{\mathrm{sat}} = 2.6 \times 10^{14}\ \mathrm{g\,cm}^{-3}$$

Nuclear Matter

How dense (dilute) is it?



Interaction Cloud Size

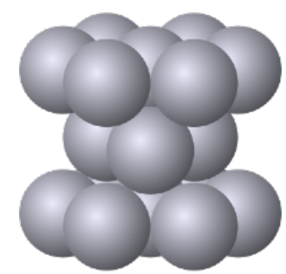
$r_{\mathrm{soft}} \sim 1/(2m_\pi) \sim 0.7\ \mathrm{fm}$

Baryon Number Distribution Size

$r_{\mathrm{hard}} \sim 0.5\ \mathrm{fm}$

Closest Packed State (hcp/fcc) Filling rate ~ 74%

$$0.74 \times \left(\frac{4\pi}{3} r_{\mathrm{hard}}^3\right)^{-1} \approx 1.4\ \mathrm{fm}^{-3} \approx \underline{8.3\, n_{\mathrm{sat}}}$$



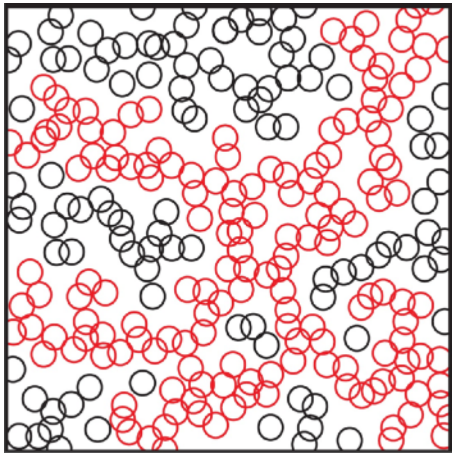
Nuclear matter cannot exist at this density!

Nuclear Matter

How dense (dilute) is it?

Percolation transition?

3D critical filling density ~ 34%



(From Wikipedia)

Interaction-mediated Percolation

$$0.34 \times \left(\frac{4\pi}{3} r_{\text{soft}}^3\right)^{-1} \approx 0.24\ \text{fm}^{-3} \approx \underline{1.5\, n_{\text{sat}}}$$

Standard nuclear-physics calculations may break down at this density due to the lack of multi-body interactions.

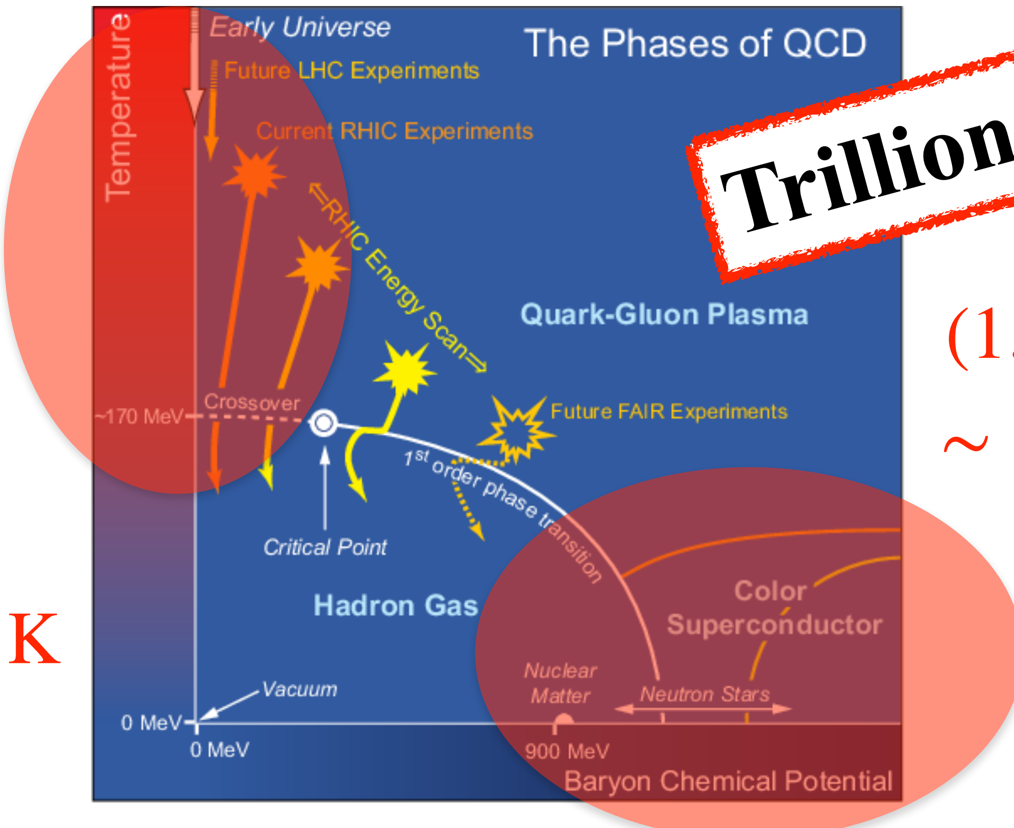
See: Fukushima-Kojo-Weise (2020) for more details.

Phase Diagram

QCD Phase Diagram

Quark-
-Gluon-
-Plasma

Λ_{QCD}
$\sim$ Trillion K



Trillion Physics

$(1.5\text{-}8)n_{\rm sat}$
$\sim$ Trillion $\mathrm{kg\,cm^{-3}}$

[Nuclear Science NSAC Long Range Plan]

Phase Diagram

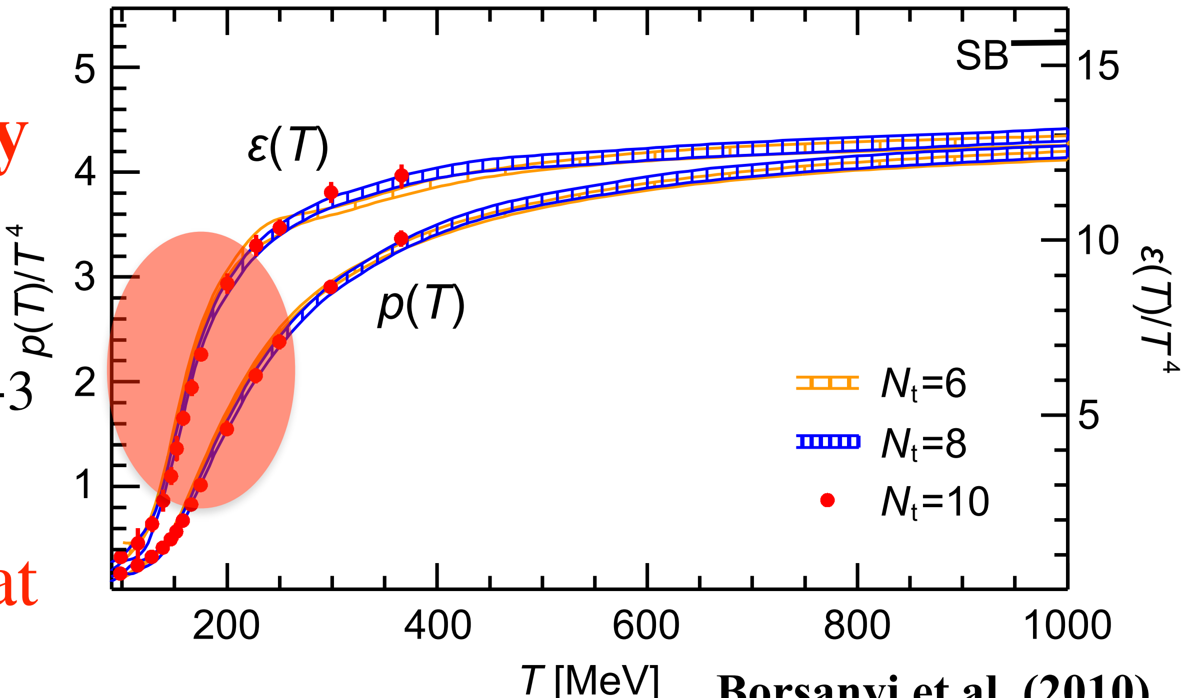
One neutron energy $E_N = M_n c^2 \simeq 939\,\mathrm{MeV}$

Saturation energy $\varepsilon_{\mathrm{sat}} \simeq 150\,\mathrm{MeV\,fm^{-3}}$

Transition energy to a QGP

$200\text{-}500\,\mathrm{MeV\,fm^{-3}}$

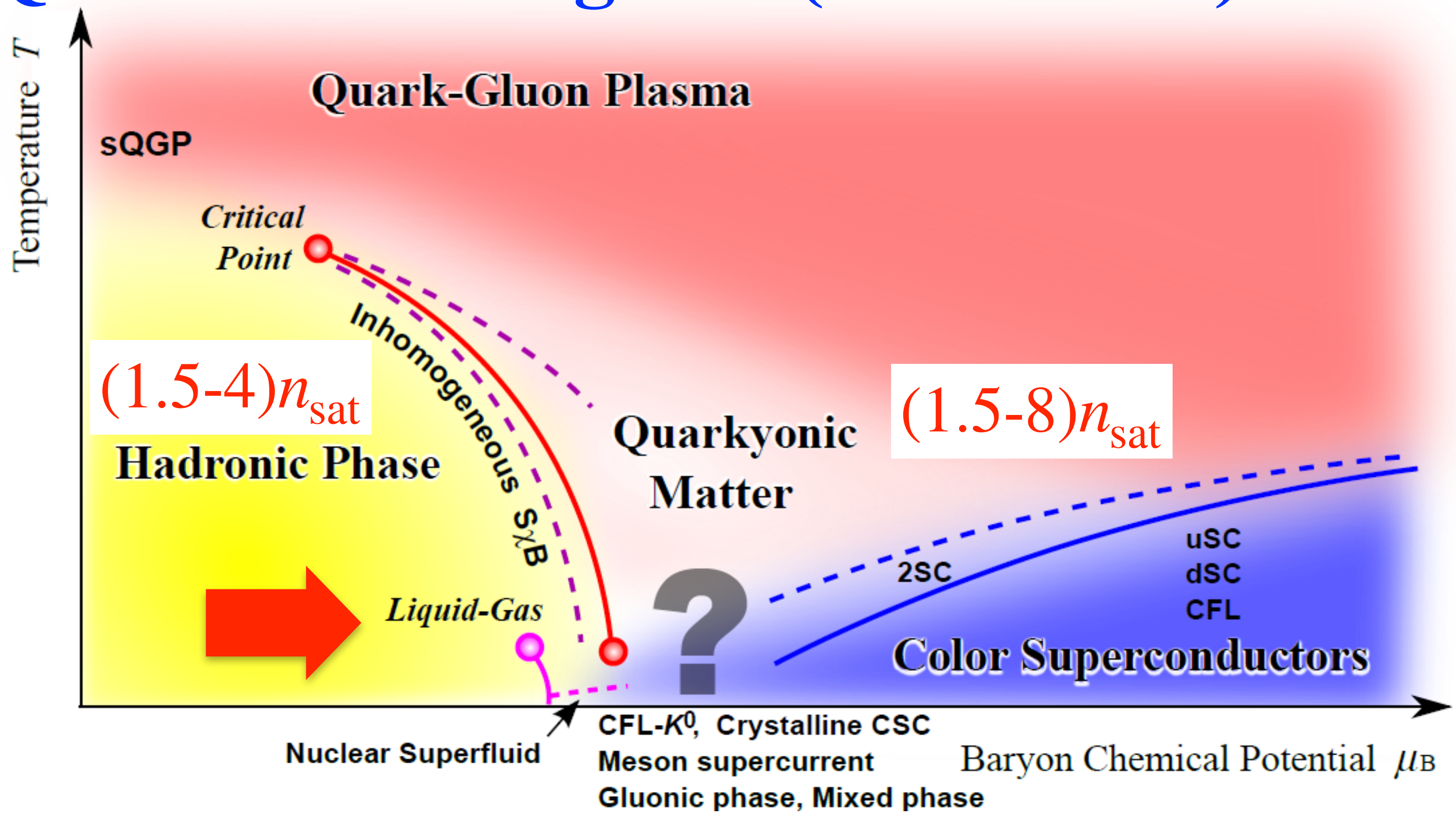
$\sim\ 1.5 - 4 \times \varepsilon_{\mathrm{sat}}$



Borsanyi et al. (2010)

Phase Diagram

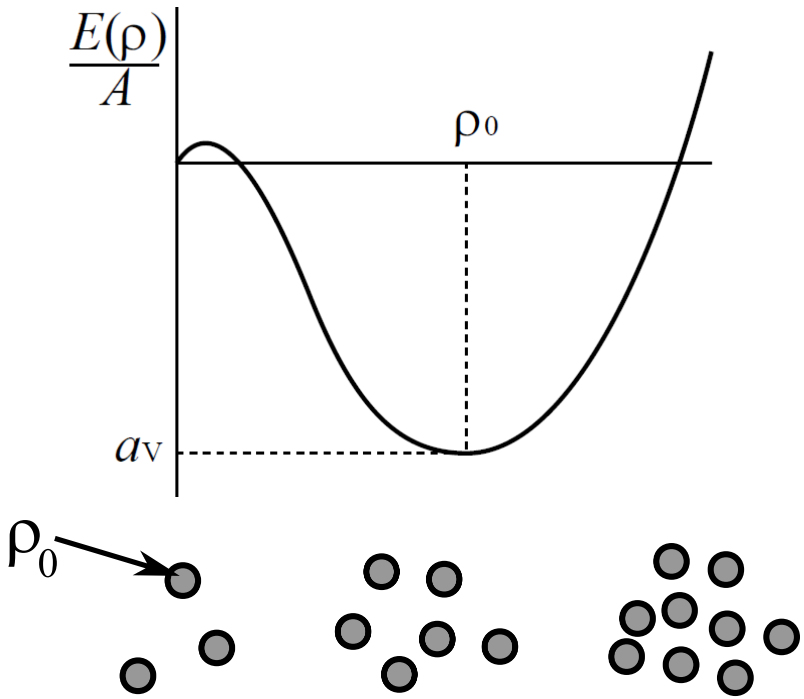
QCD Phase Diagram (structured)



Fukushima-Hatsuda (2010); see also 50 Years of QCD Chap.7 (2023)

Nuclear Liquid-Gas Transition

Nuclear Saturation



Self-bound fermionic systems have a preferred density.
Diluteness is realized as a "mixed phase" of nuclei.

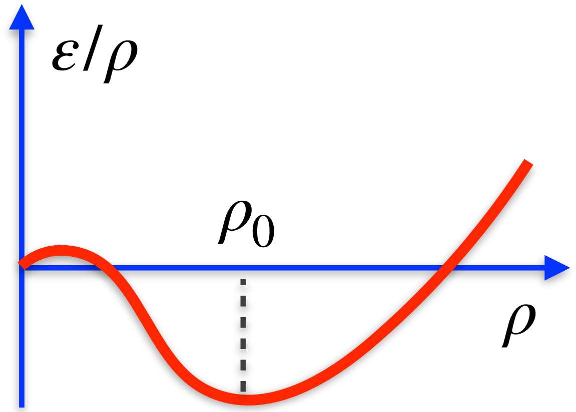
This is how we can live!

Nuclear Liquid-Gas Transition

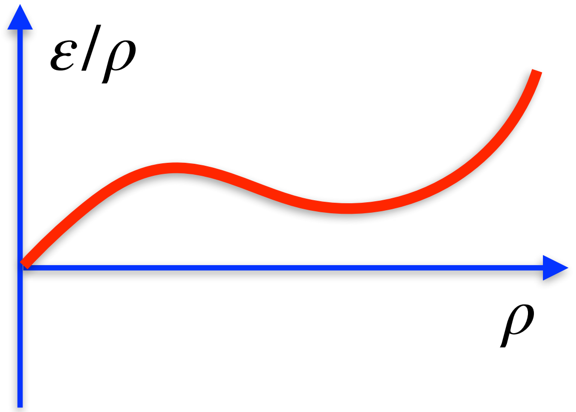
1st-order Phase Transition

$$\frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\bigg|_{\text{gas}} - \frac{\varepsilon}{\rho}\bigg|_{\text{liquid}}\right) = \frac{p_{\text{gas}} - p_{\text{liquid}}}{\rho^2} = 0$$

Metastability **1st-order PT**



Nuclear Matter



Meta-stable? Quark Matter?

Nuclear Liquid-Gas Transition

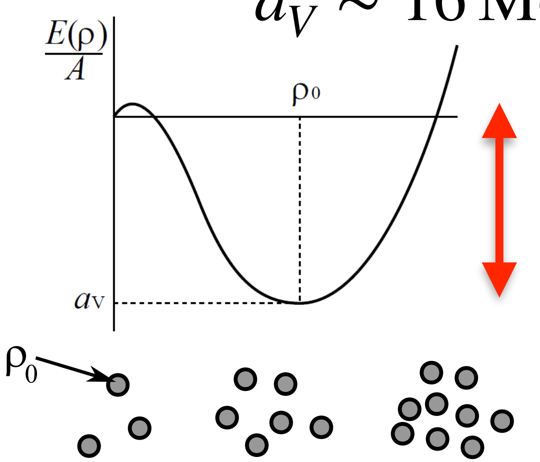
Mass Formula

$$m(Z,N)c^2 = Am_N c^2 - B(Z,N)$$

$$B(Z,N) = \underbrace{a_V A}_{\text{Volume}} - \underbrace{a_S A^{2/3}}_{\text{Surface}} - \underbrace{a_C \frac{Z^2}{A^{1/3}}}_{\text{Coulomb}} - \underbrace{a_{\text{sym}} \frac{(N-Z)^2}{A}}_{\text{Symmetry}}$$

$a_V \sim 16\,\text{MeV}$

$a_{\text{sym}} \sim 24\,\text{MeV}$



For $N = Z$ and $A \to \infty$

$B/A \sim a_V \sim 16\,\text{MeV}$

Nuclear Stability

Mass Formula

$$m(Z,N)c^2 = Am_N c^2 - B(Z,N)$$

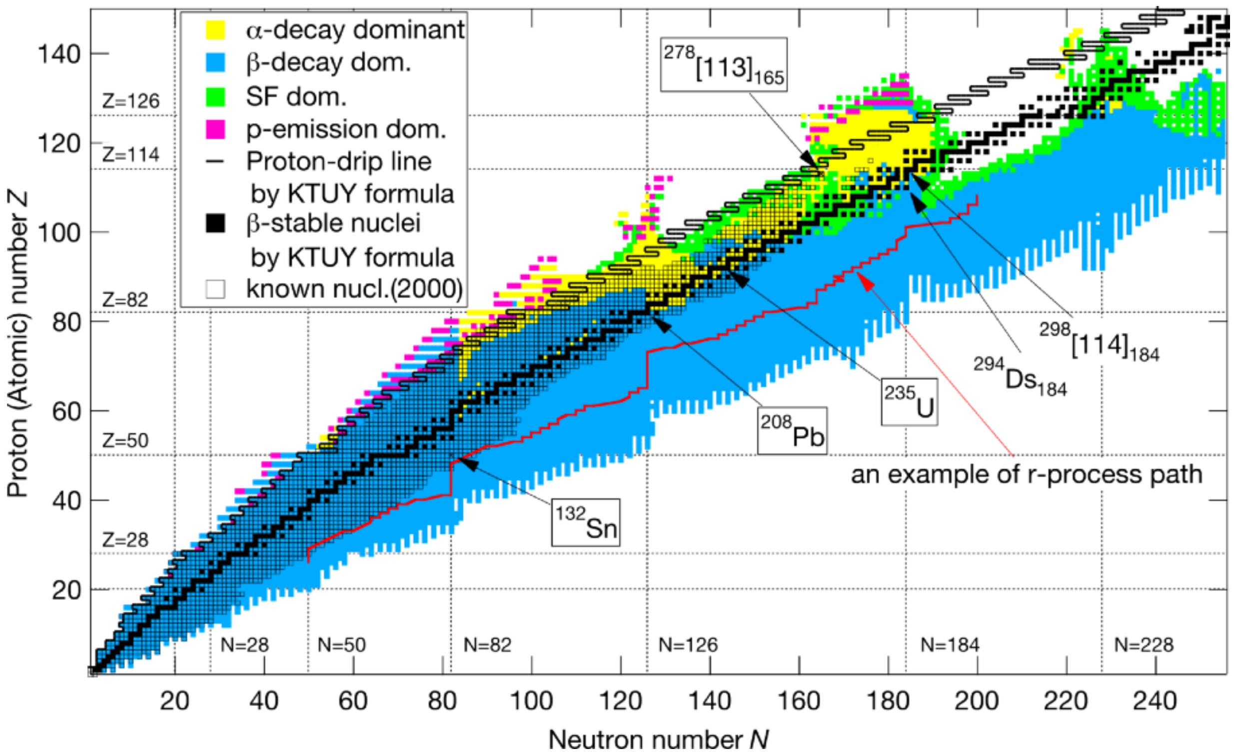
$$B(Z,N) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathsf{sym}} \frac{(N-Z)^2}{A}$$

For small A, the symmetric energy makes $N \sim Z$, while the Coulomb energy leads to $N \gg Z$ for large A.

If A is too large, heavy nuclei break up to smaller nuclei (nuclear fission).

Nuclear Stability

Nuclear Chart



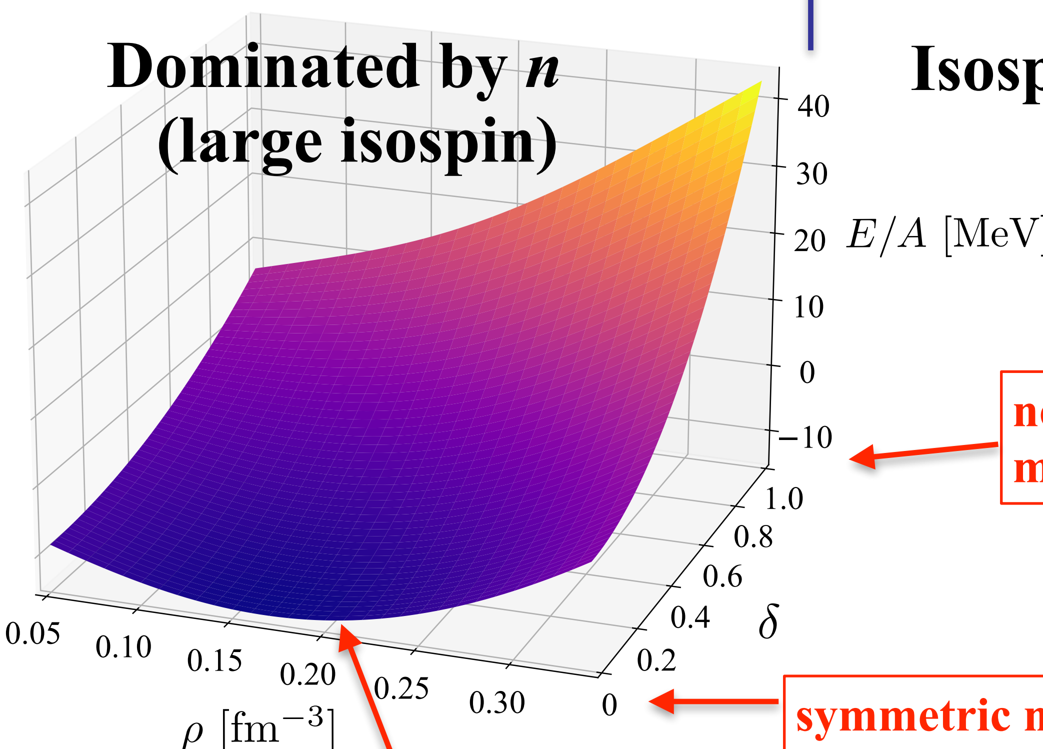
Along the stable (black) curve (Heisenberg's valley), the slope is getting smaller than 1.

[From Wikipedia]

Nuclear Stability

Neutron Star

Dominated by *n*
(large isospin)



saturation density
(1st-order transition)

neutron matter

symmetric nuclear matter (HIC)

Heavy-Ion Collision

Isospin fixed by *p*/*n* ratio
(small isospin)

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$\rho = \rho_n + \rho_p$$

Nuclear Stability

Gigantic nuclei stabilized by Gravity?

B.1 We assume that $N = A$ and $Z = 0$ is realized for sufficiently large A and Eq. (1) is not modified. The binding energy due to gravity is 1.5 pt

$$B_{\text{grav}} = \frac{3}{5}\frac{GM^2}{R},$$

where $M = m_N A$ and $R = \gamma A^{1/3}$ with $\gamma \simeq 1.1 \times 10^{-15}\ \mathrm{m} = 1.1\ \mathrm{fm}$ are the mass and the size of the nucleus, respectively.
For $B_{\text{grav}} = a_{\text{grav}} A^{5/3}$, obtain a_{grav} in the MeV unit up to the first significant digit. Then, ignoring the surface term, estimate A_c up to the first significant digit. In the calculation, use $m_N c^2 \simeq 939\ \mathrm{MeV}$ and $G = \hbar c / M_P^2$ where $M_P c^2 \simeq 1.22 \times 10^{22}\ \mathrm{MeV}$ and $\hbar c \simeq 197\ \mathrm{MeV \cdot fm}$.

Theory

INTERNATIONAL PHYSICS OLYMPIAD
2023 TOKYO JAPAN

https://international-physics-olympiad2023-tokyo.jp/theoretical-exam/

Nuclear Stability

$$B_{\text{grav}} = a_{\text{grav}} A^{5/3} \qquad G = \hbar c / M_{\text{Planck}}^2$$

$$a_{\text{grav}} = \frac{3}{5} \frac{G(AM_n)^2}{1.2A^{1/3}} \cdot A^{-5/3} = 7 \times 10^{-37}\,\text{MeV}$$

Stability Condition

$$(\text{B.E}) = (a_V - a_{\text{sym}})A + a_{\text{grav}} A^{5/3} > 0$$

“Radius” of This Gigantic Nuclei

$$A_c = \left(\frac{a_{\text{sym}} - a_V}{a_{\text{grav}}} \right)^{3/2} \simeq 3.8 \times 10^{55} \;\rightarrow\; R = 1.2A^{1/3}\,\text{fm} \simeq \underline{4\,\text{km}}$$

Foundation of Neutron Stars

Neutron Star

Lev Landau and the conception of neutron stars

Yakovlev-Haensel-Baym-Pethick (2012)

White dwarfs have the mass limit (Chandrasekhar mass limit)

Heavier?
Protons + Electrons?

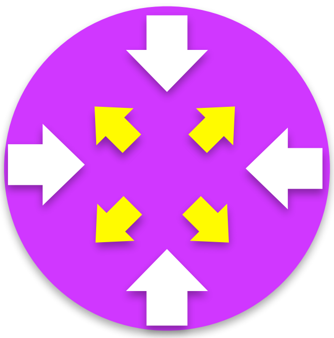
February 1931

Chadwick's discovery of neutrons

January 1932

Neutron Star

Force Balance



Gravitational force is supported by the pressure from inside.

Hydrostatic condition for $r \sim r + dr$

$$\frac{dp(r)}{dr} = -G\frac{M(r)}{r^2}\varepsilon(r) \qquad (2)$$

$M(r)$ **represents the integrated mass in** r**-sphere.**

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r) \qquad (3)$$

(In Newtonian gravity)

Neutron Star

$$\frac{dp(r)}{dr} = -G\frac{M(r)\,\varepsilon(r)}{r^2}$$

Tolman-Oppenheimer-Volkoff Eq.

General Relativistic extension $\longrightarrow$

$$\frac{dp(r)}{dr} = -G\frac{M\varepsilon}{r^2}\left(1+\frac{p}{\varepsilon}\right)\left(1+\frac{4\pi r^3 p}{M}\right)\left(1-\frac{2GM}{r}\right)^{-1}$$

One condition still missing…

A relation between p and ε $\longrightarrow$ **Equation of State (EOS)**

$\boxed{\text{Initial}}$

$$r = 0$$

$$\varepsilon(r=0) = \varepsilon_c$$

free parameter

$$p(r=0) = p_c = p(\varepsilon_c)$$

$\boxed{\text{Final}}$

$$r = R$$

$$p(r=R) = 0$$

$$M = \int dr\, 4\pi r^2 \varepsilon(r)$$

Neutron Star

To solve Eqs. (2) and (3), we need the equation of state showing the relation between pressure P and mass density ρ, which is assumed to be given by the following polytropic relation for arbitrary r in terms of a positive number n,

$$P = K\,\rho^{1+1/n}, \qquad (4)$$

Polytoropic EOS

where K is a positive constant independent of ρ.
Pressure distribution inside a star can be calculated from the differential equation with the following boundary condition, $\rho(r=0)=\rho_0$ with a parameter ρ_0.
By eliminating P from Eqs. (2) and (4) and replacing the reduced mass density $\rho/\rho_0 = \psi^n$ and the reduced length $r/r_0 = \xi$ in terms of a certain scale r_0, we obtain

$$\xi^2 \frac{d\psi}{d\xi} = -\frac{GM}{(n+1) r_0 K \rho_0^{1/n}}.$$

By taking a derivative of this equation with respect to ξ, eliminating M by using Eq. (3), and choosing an appropriate value of r_0, we obtain a second-order differential equation,

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\psi}{d\xi}\right) = -\psi^n\,.$$

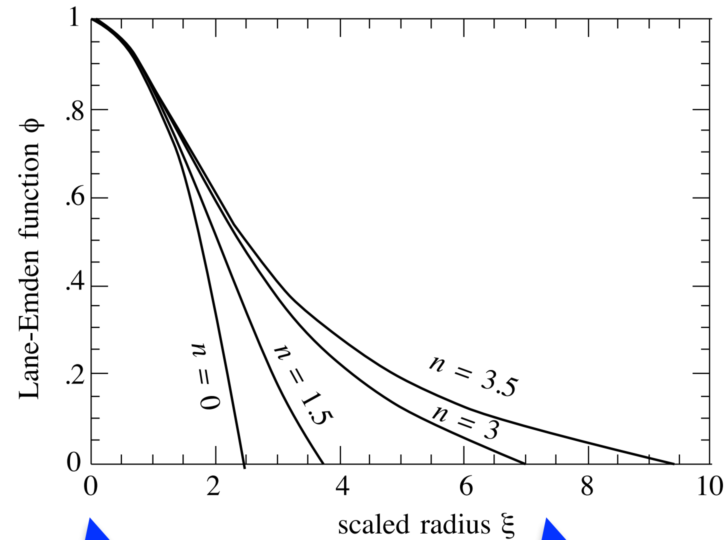
Lane-Emden Eq.

Determine r_0 in terms of ρ_0, n, K, and G.

Neutron Star

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\psi}{d\xi}\right) = -\,\psi^n$$

$$\psi^n = \rho/\rho_0$$



Center

Surface

$$M(R) = -\,4\pi r_0^3 \rho_0 \xi_c^2 \psi'(\xi_c)$$

$$\sim \rho_0^{(3-n)/(2n)} \;\rightarrow \text{(Const.) for } n = 3 \;\; \text{(relativistic EOS)}$$

This upper limit of *M* is the Chandrasekhar limit.

Neutron Star

Much simpler argument for the maximum mass

$$p \sim \hbar n^{1/3} \;\to\; E_{\rm kin} \sim Apc \sim \frac{\hbar c A^{4/3}}{R}$$

$$E_{\rm total}(R) \sim \frac{\hbar c A^{4/3}}{R} - G\frac{A^2 M_n^2}{R}$$

If A is too large, the system collapses to $R \to 0$.

($R \to \infty$ is stabilized by $E_{\rm kin} \sim 1/R^2$ in the non-rela. limit.)

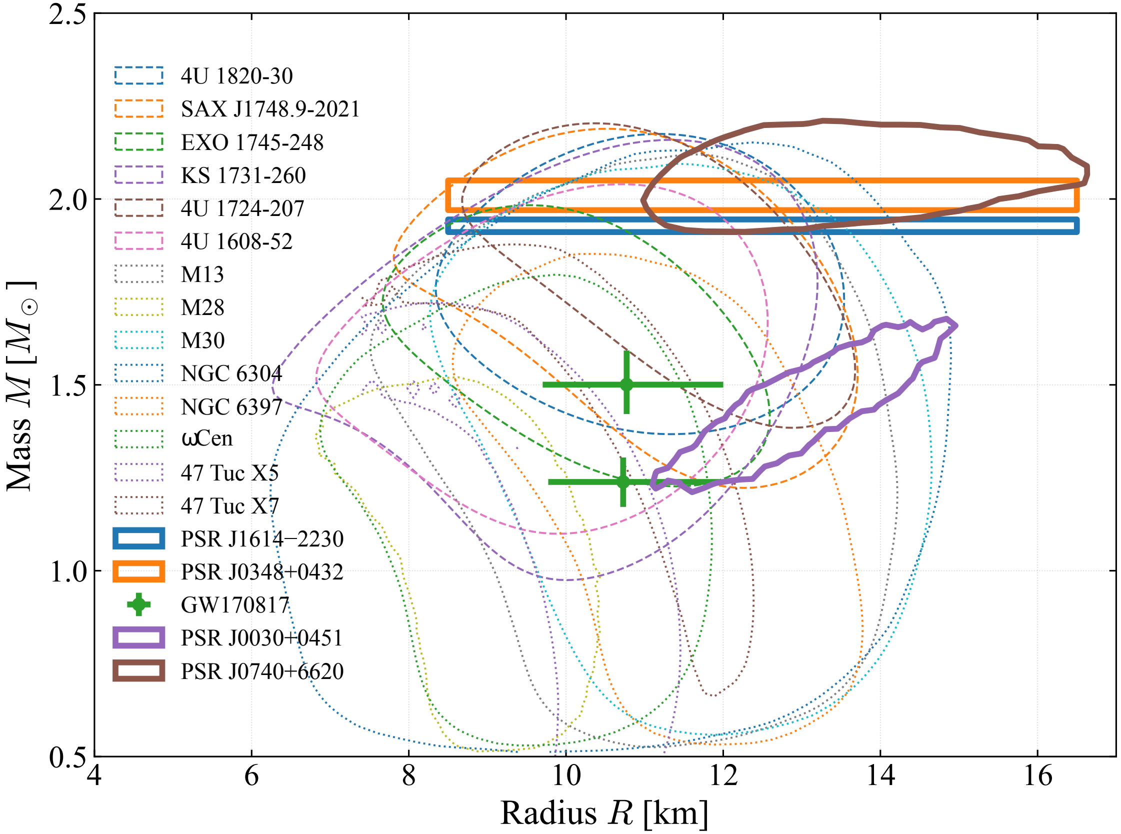
The order-estimated critical mass:

$$M_{\rm c} \sim \left(\frac{M_{\rm Planck}}{M_n}\right)^3 M_n \simeq \underline{2M_\odot}$$

Neutron Star

Compilation of the observed data (68% Credible)

Fujimoto-Fukushima-Kamata-Murase (2024)



Maximum Mass Constraint

Shapiro time delay

C.2 Under the influence of the gravitational potential ϕ we have time delays discussed in **C.1** and the effective speed of light is changed. When $\phi(r=\infty)=0$, in the region where $\phi \neq 0$, the effective speed of light, c_{eff}, observed at the infinity can be given up to the first order in ϕ/c^2 as 1.4pt

$$c_{\text{eff}} \approx \left(1+\frac{2\phi}{c^2}\right) c$$

including the effect of space distortion. We note that the light path can be approximated as a straight line.

As shown in Fig. 2, we take the x-axis along the light path from the neutron star **N** to the Earth **E** and place $x=0$ at the point where the White Dwarf **W** is the closest to the light path. Let $x_N\ (<0)$ be the x-coordinate of **N**, $x_E\ (>0)$ be that of **E**, and d be the distance between **W** and the light path.

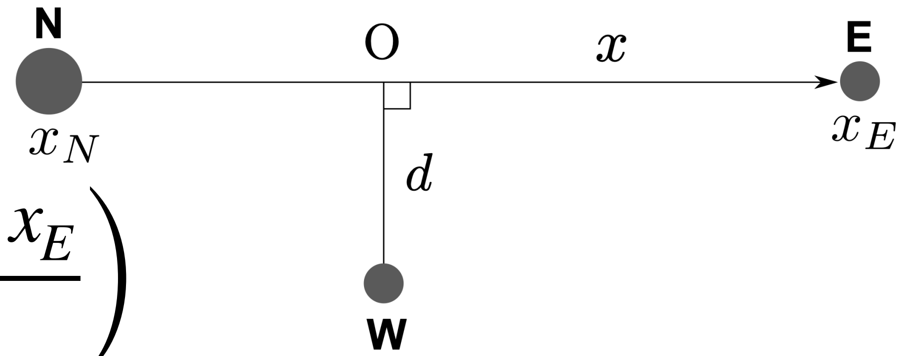
Estimate the changes of the arrival time Δt caused by the White Dwarf with mass M and express the answer in a simple form disregarding higher order terms of the following small quantities: $d/|x_N| \ll 1$, $d/x_E \ll 1$, and $GM/(c^2 d) \ll 1$. If necessary, the following integration formula can be used.

$$\int \frac{dx}{\sqrt{x^2+d^2}} = \frac{1}{2}\log\left(\frac{\sqrt{x^2+d^2}+x}{\sqrt{x^2+d^2}-x}\right)+C,$$

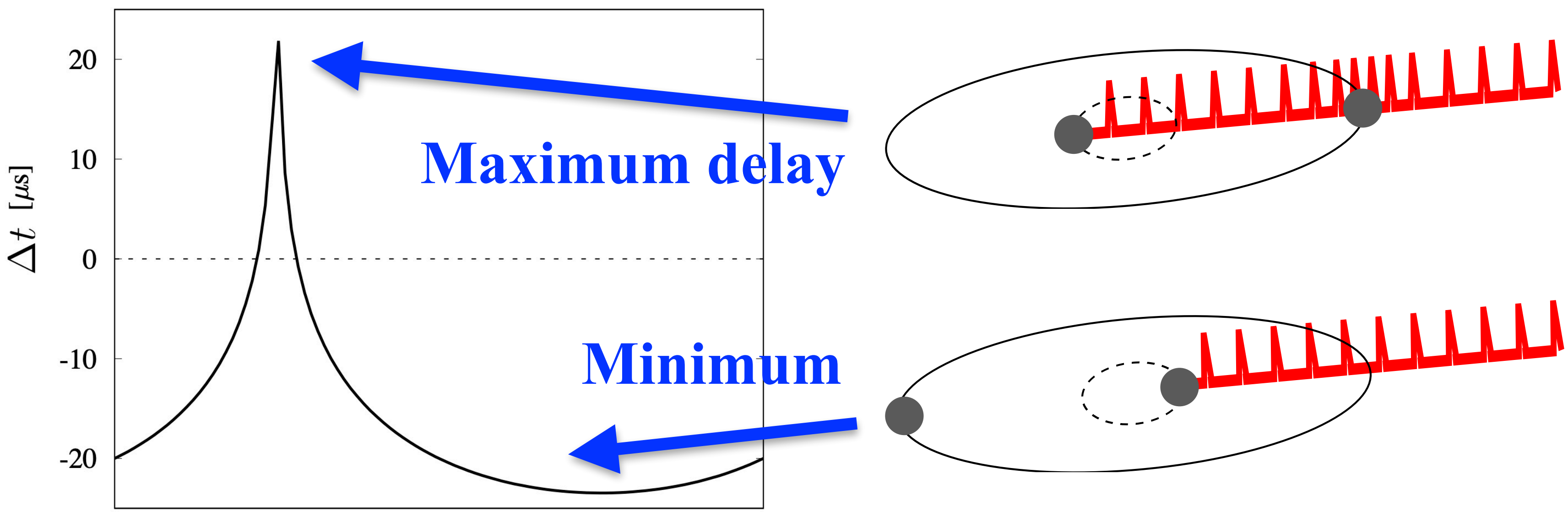
where C is an integration constant.

Maximum Mass Constraint

Shapiro time delay

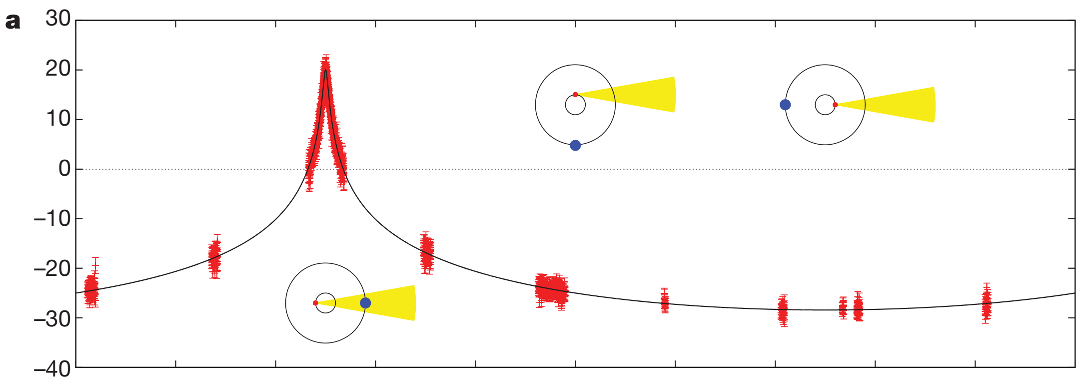


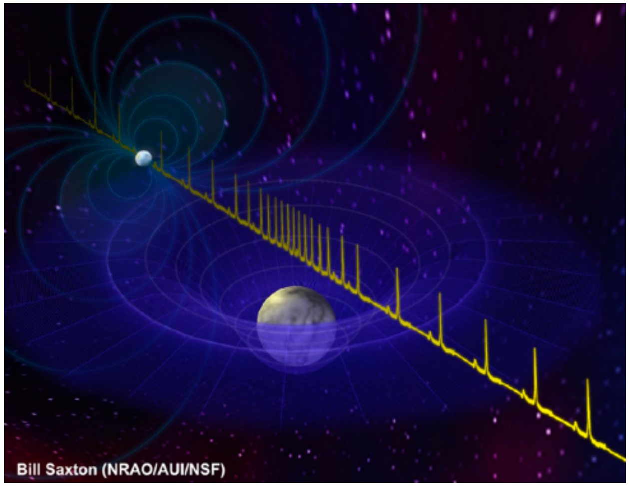
$$\Delta t \approx 2 \times \frac{GM}{c^3} \ln\left(\frac{4\,|x_N|\,x_E}{d^2}\right)$$

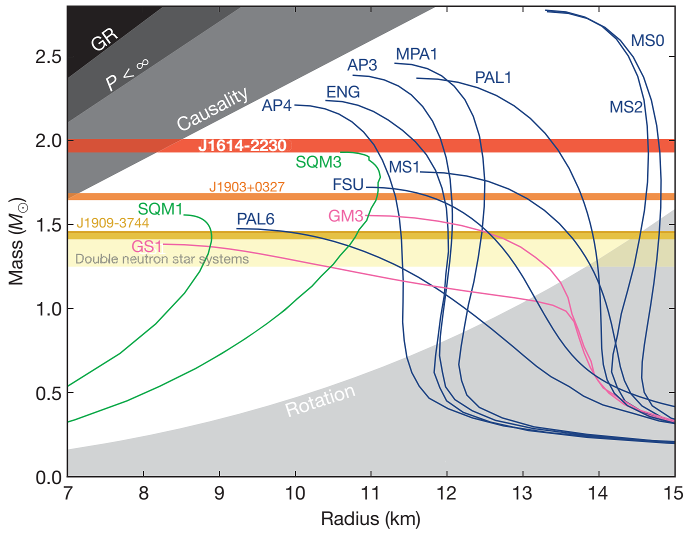


Maximum Mass Constraint

Demorest et al. (2010-2016)



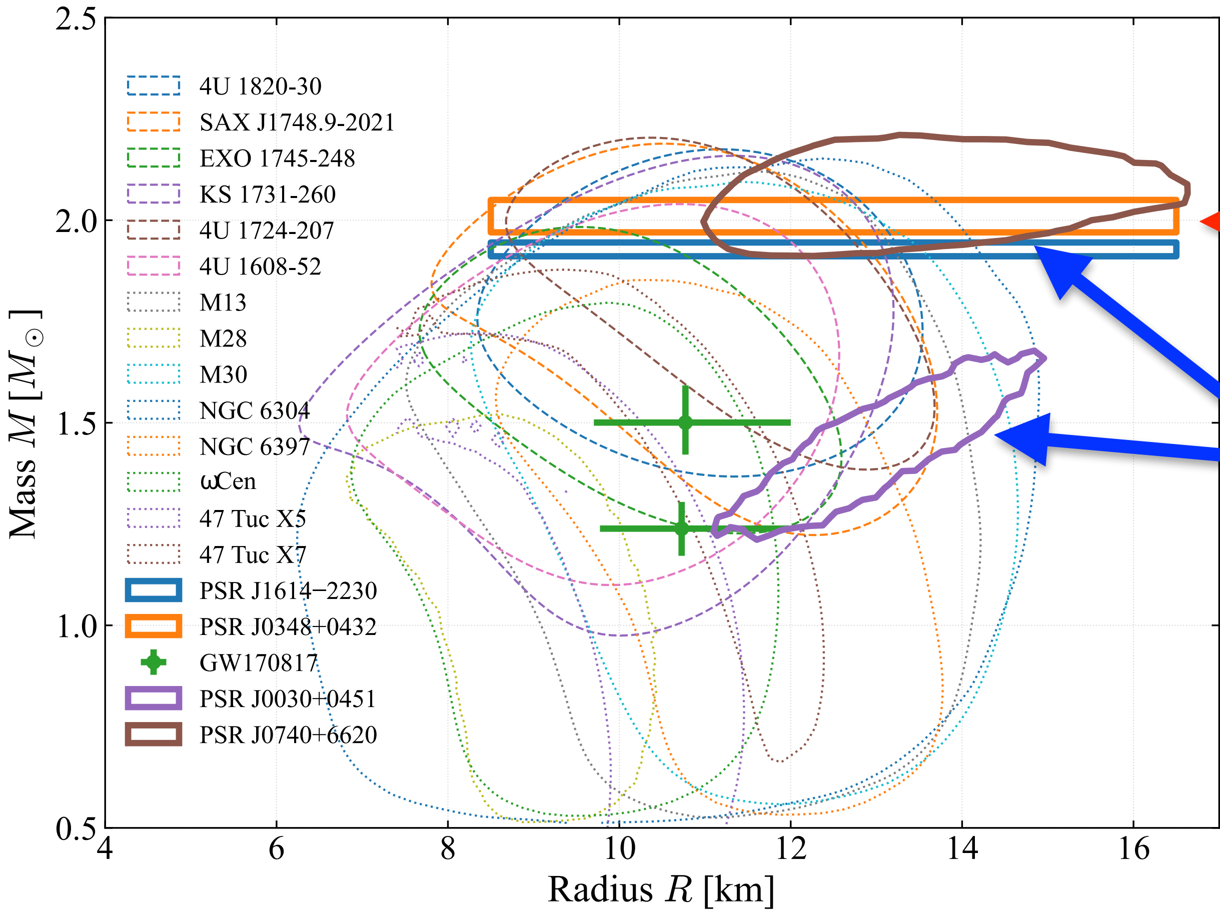




$$1.928(17) M_\odot$$

$$(\mathrm{PSR\,J1614\text{-}2230})$$

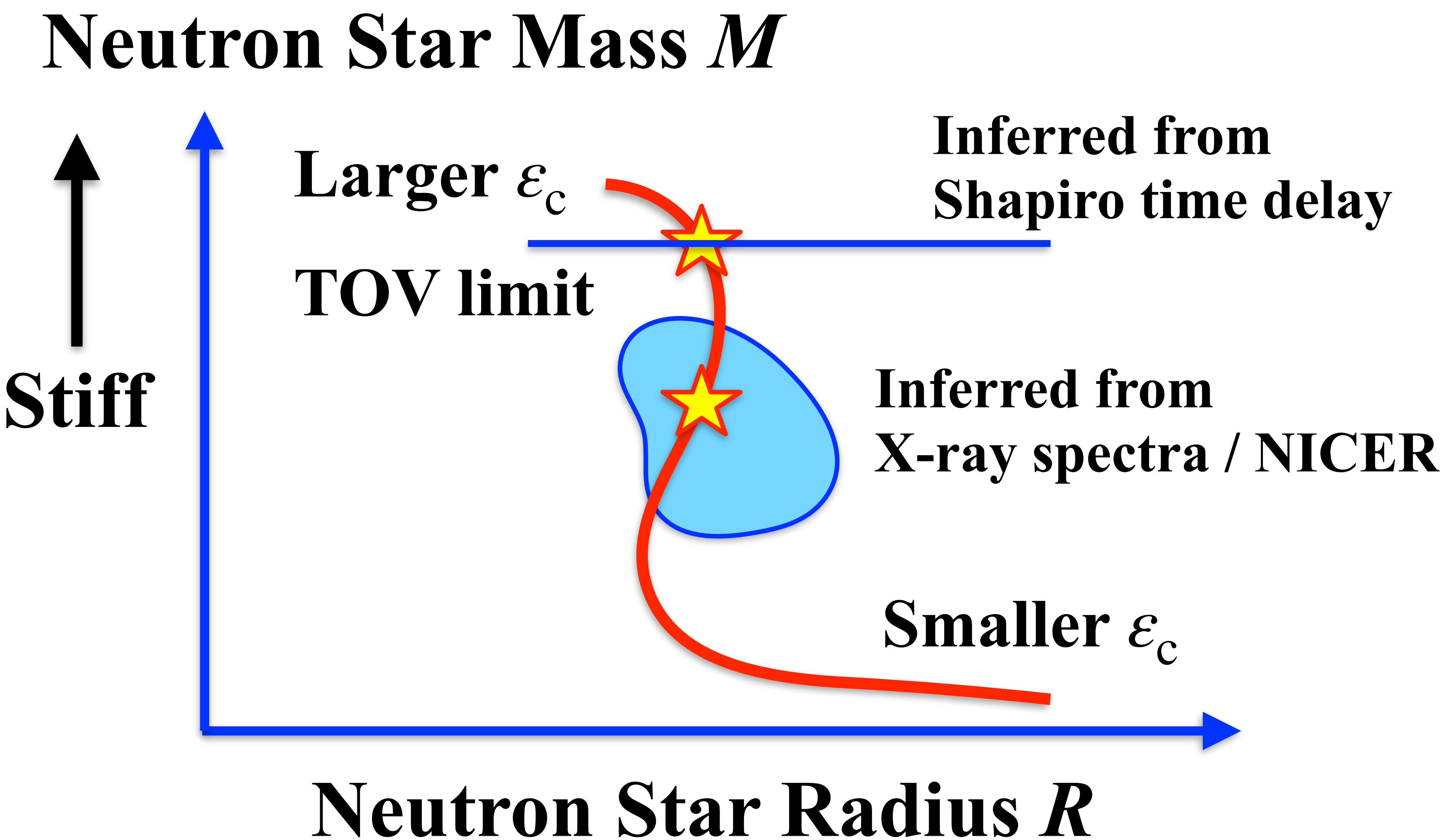
Maximum Mass Constraint



EOS must be "stiff" enough to reach here.

Lensing and timing (NICER) constraining the gravity strength and the radius!

EOS


Neutron Star Mass M
Stiff
Larger ε_c
TOV limit
Inferred from
Shapiro time delay
Inferred from
X-ray spectra / NICER
Smaller ε_c
Neutron Star Radius R

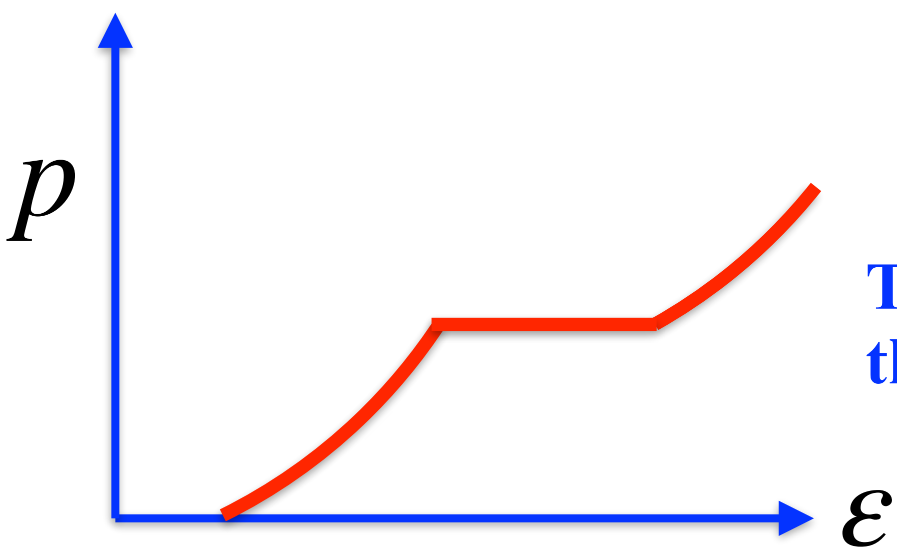
EOS

Mathematically proven:

$$p = p(\varepsilon) \longleftrightarrow M = M(R)$$

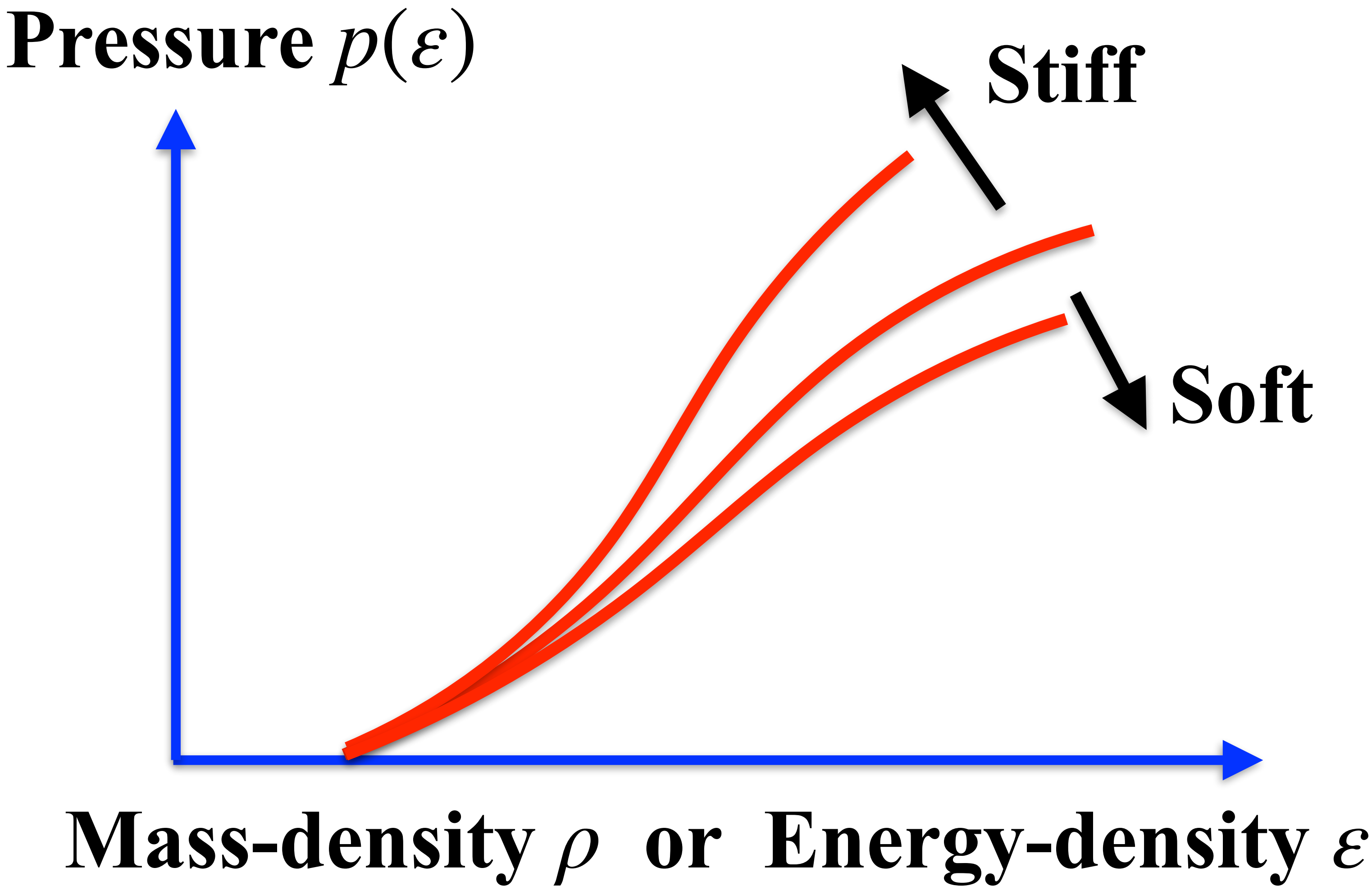
One-to-one Correspondence through TOV eq.

Lindblom (1992)



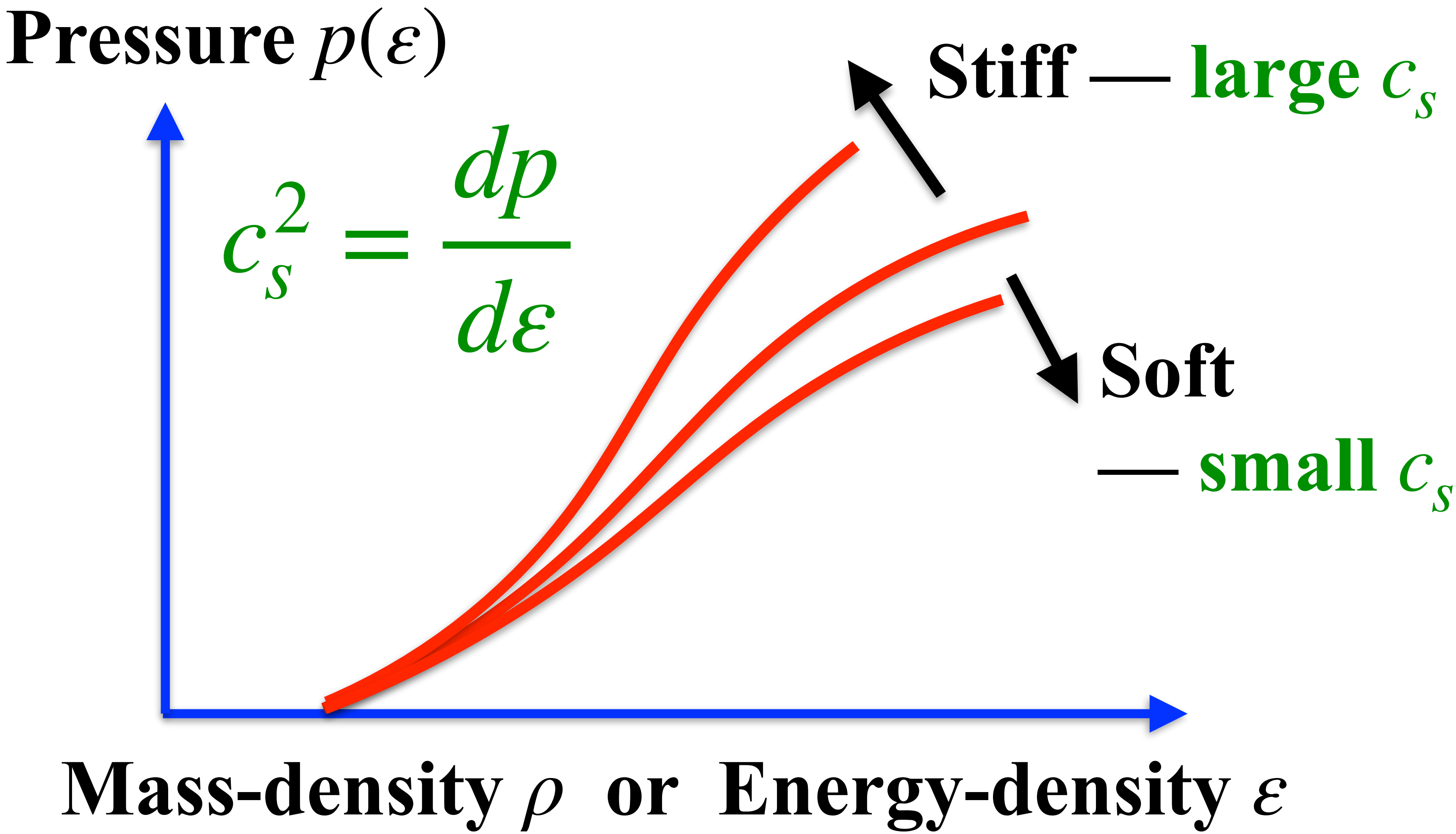
This is the case even with the 1st-order phase transition.

EOS

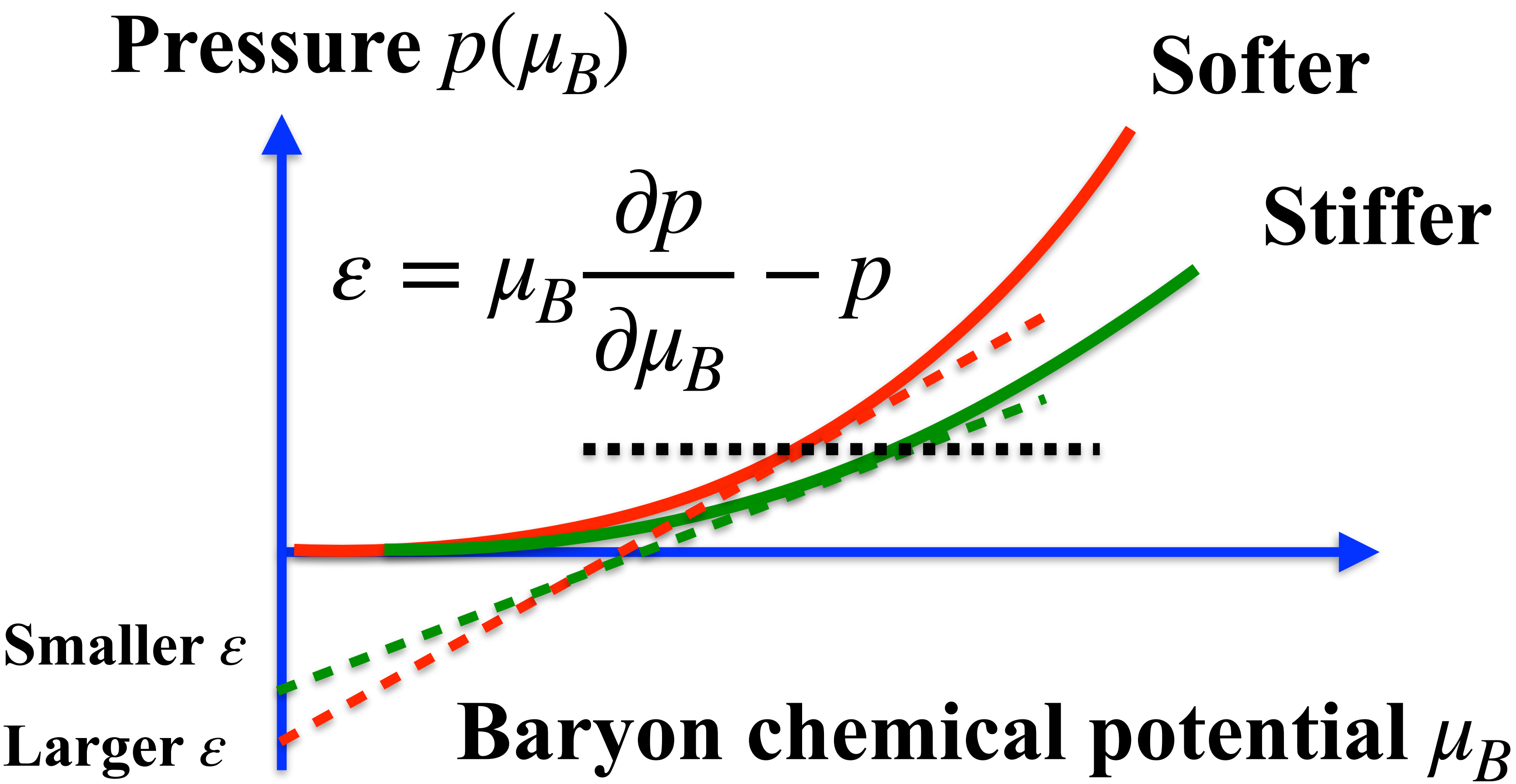


Pressure $p(\varepsilon)$
Stiff
Soft
Mass-density ρ or Energy-density ε

EOS


Pressure $p(\varepsilon)$
$c_s^2 = \frac{dp}{d\varepsilon}$
Stiff — large c_s
Soft
— small c_s
Mass-density ρ or Energy-density ε

EOS


Pressure $p(\mu_B)$
Softer
Stiffer
$\varepsilon = \mu_B \frac{\partial p}{\partial \mu_B} - p$
Smaller ε
Larger ε
Baryon chemical potential μ_B

EOS

Very useful formula:

$$p(\mu) = p_0 + n_0 \int_{\mu_0}^{\mu} d\mu' \exp\left[\int_{\mu_0}^{\mu'} d\mu'' \frac{1}{\mu'' c_s^2(\mu'')} \right]$$

If the sound speed is larger (stiffer), the pressure is suppressed.

Derive this by yourself!

[Hint] $$c_s^2(\mu) = \frac{\partial p}{\partial \mu} \left(\frac{\partial \varepsilon}{\partial \mu} \right)^{-1} = \frac{n}{\mu(\partial n / \partial \mu)}$$

EOS

Structures of pQCD on ε-p

LO: $p \sim \# \mu_B^4$ **(massless case)** $\rightarrow \varepsilon = \frac{1}{3} p$

NLO: $p \sim (\# + \alpha_s \#) \mu_B^4$ $\rightarrow \varepsilon = \frac{1}{3} p$ **(unchanged!)**

N²LO: $p \sim (\# + \alpha_s \# + \alpha_s^2 \# + \# \alpha_s^2 \ln \mu_B^2 / \mu_0^2) \mu_B^4$

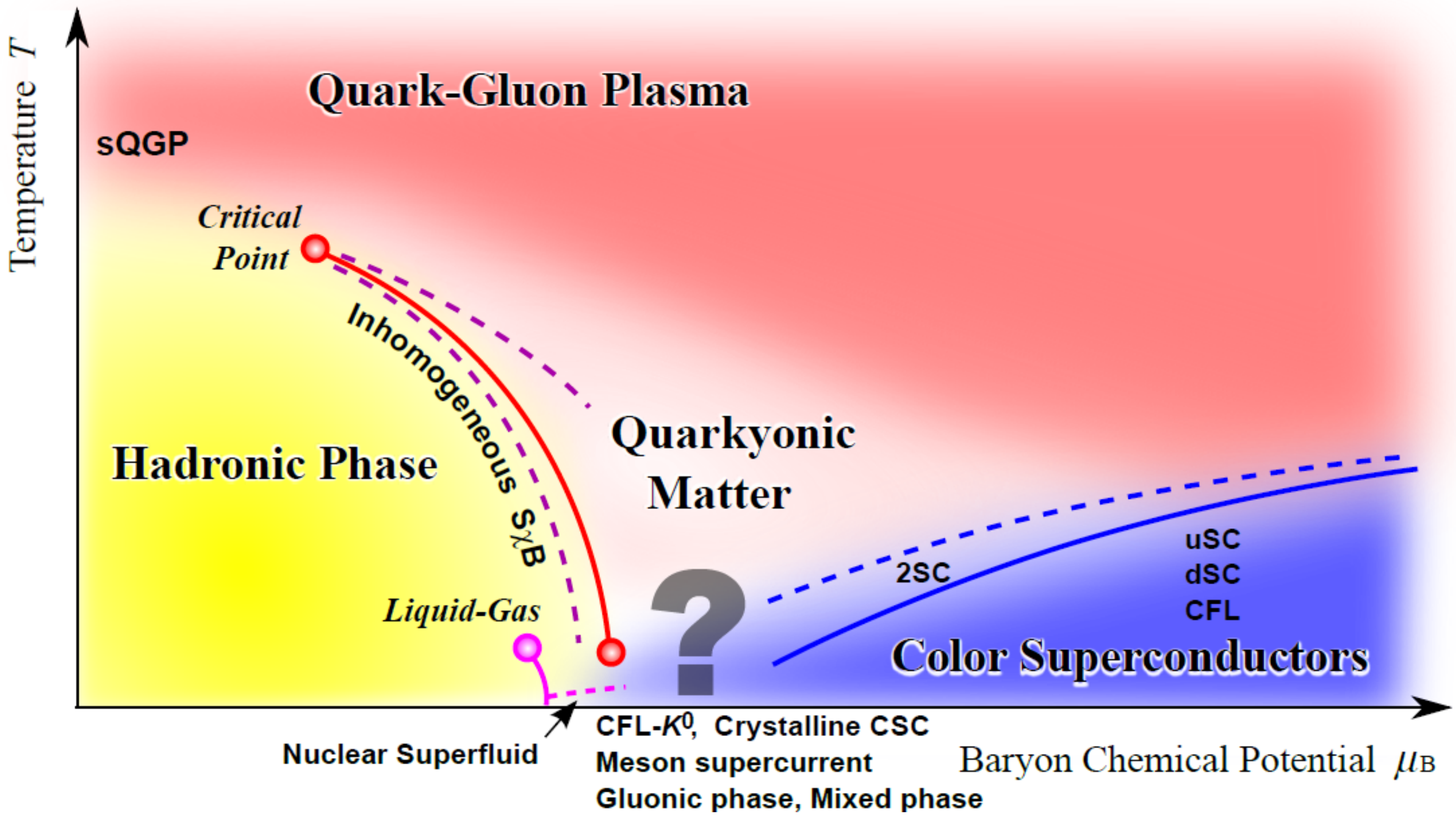
Conformality broken

Running Coupling

$\sim \ln(X^2 \mu_q^2 / \Lambda_{\overline{\mathrm{MS}}}^2)$

Exotic Phases and Phase Transitions

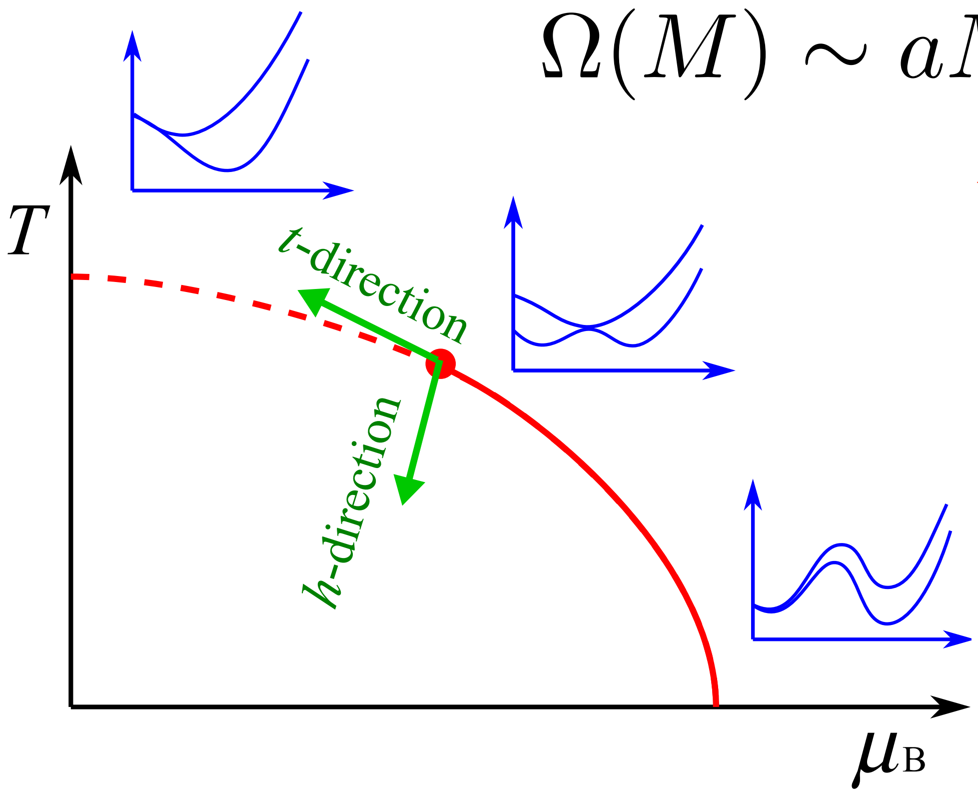
QCD Phases



Fukushima-Hatsuda (2010); see also 50 Years of QCD Chap.7 (2023)

QCD Phases

QCD Critical Point



$$\Omega(M) \sim aM^2 + bM^4 + \underline{cM^6}$$

Assumed to be positive for stability

$a = 0$: 2-nd order

$a = b = 0$: Tricritical

QCD Phases

In the massless NJL model **Nickel (2008)**

$$\Omega(M) \sim aM^2 + bM^4 + cM^6$$

Inhomogeneous condensates induce $\partial M \neq 0$

$$\Omega(M, q) \to aM^2 + bM^4 + cM^6 + dq^2 M^2 + \cdots$$

Spatial inhomogeneity occurs for *d* < 0 (Lifshitz point)

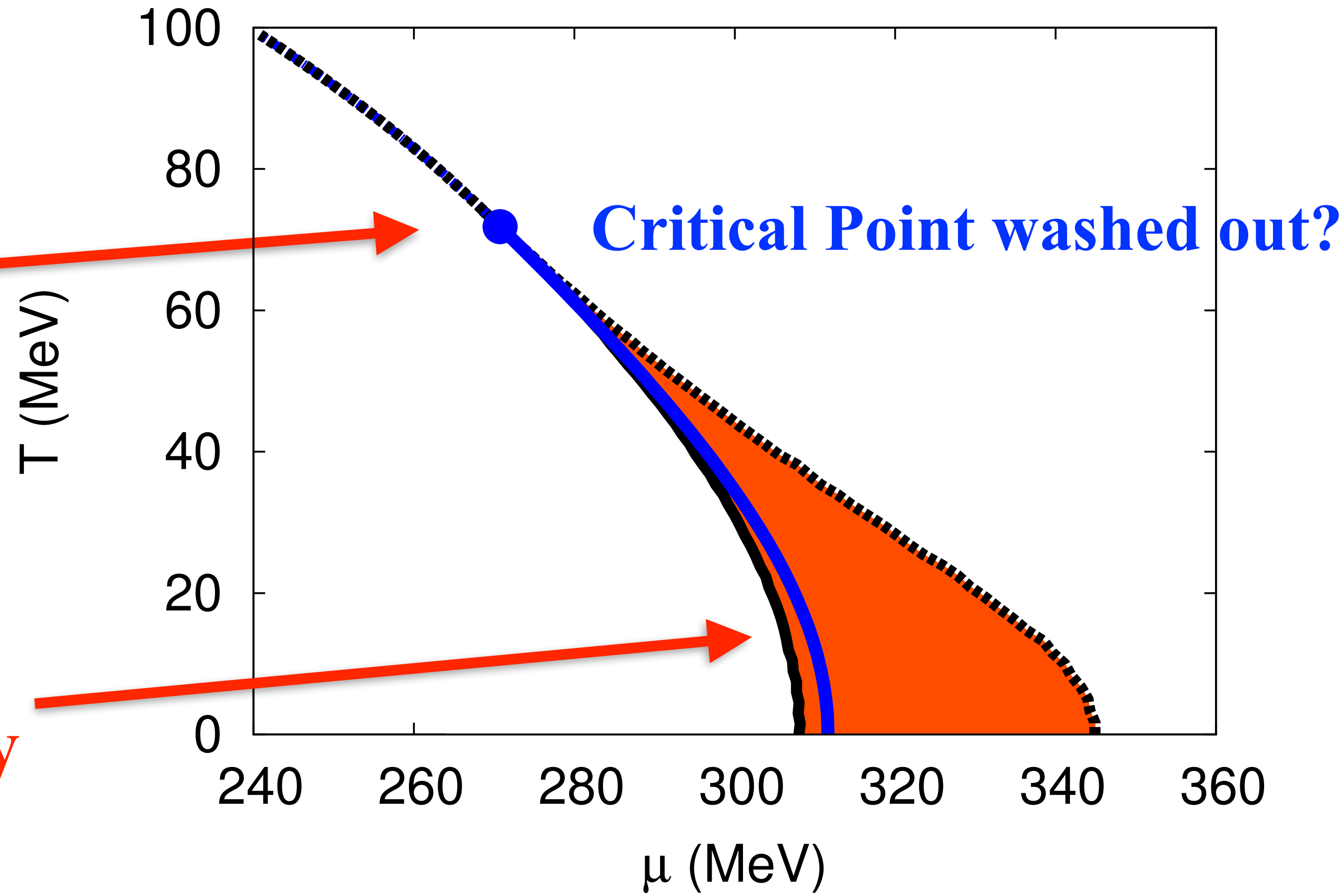
It happens to result in $b \propto d$!

→ Lifshitz point and QCD CP coincide!

QCD Phases

"Mean-Field" Phase Diagram **Review: Buballa-Carignano (2014)**

Lifshitz point



Chiral Density Wave

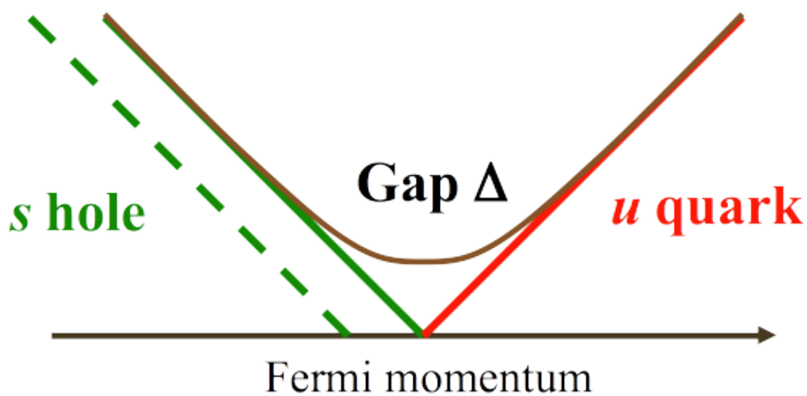
QCD Phases

High density limit where pQCD should work:

→ Color Super Conductivity

Fermi Surface $\mu_q \sim 500\ \mathrm{MeV} \ \to\ \rho \sim 10\rho_0$

Attractive Force $\mathbf{3} \times \mathbf{3} \to \bar{\mathbf{3}}$



$$\sqrt{p_F^2 + m_s^2} = \mu_q$$

$$\to\ p_F \simeq \mu_q - \frac{m_s^2}{2\mu_q}$$

Gap and Fermi surface mismatch are of the same order

QCD Phases

Color Interaction

$$(t^a)_{ij}(t^a)_{kl} = -\frac{N_c+1}{4N_c}(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}) + \frac{N_c-1}{4N_c}(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})$$

Color Triplet
(antisymmetric)
Attractive

Color Sextet
(symmetric)
Repulsive

Dominant

Only this channel considered
(flavor) (spin) (~~orbital~~)
should be symmetric

Always mixed with triplet
No new physics brought in
Harmlessly neglected

QCD Phases

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \otimes \mathbf{6}$$

Quantum numbers and operators			
J^P	Color	Flavor	Operator
0^+	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\psi}_C \gamma_5 \psi$, $\bar{\psi}_C \gamma_0 \gamma_5 \psi$
1^+	$\bar{\mathbf{3}}$	$\mathbf{6}$	$\bar{\psi}_C \gamma_i \psi$, $\bar{\psi}_C \sigma_{0i} \psi$
0^-	$\bar{\mathbf{3}}$	$\mathbf{6}$	$\bar{\psi}_C \psi$, $\bar{\psi}_C \gamma_0 \psi$
1^-	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\psi}_C \gamma_i \gamma_5 \psi$, $\bar{\psi}_C \sigma_{ij} \psi$

QCD Phases

Spin-dependent Part **Breit Interaction**

$$H_{\text{color-spin}} = \alpha_s \sum_{i \neq j} M_{ij} \underbrace{(\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j)}_{\text{color}} \underbrace{(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)}_{\text{spin}}$$

> spin-singlet (antisymmetric) + flavor triplet (antisymmetric)

$$(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)|\mathbf{0}\rangle = -(3/4)|\mathbf{0}\rangle$$

$\boxed{\textbf{Good Diquark}}$

> spin-triplet (symmetric) + flavor sextet (symmetric)

$$(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)|\mathbf{1}\rangle = +(1/4)|\mathbf{1}\rangle$$

$\boxed{\textbf{Bad Diquark}}$

QCD Phases

(Good) Diquark Condensate

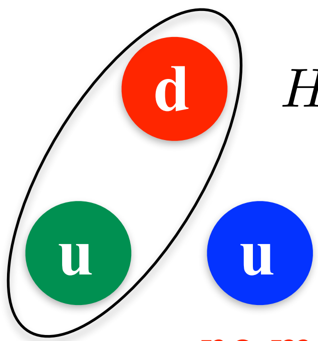
$$\Delta_{\alpha i} \;\propto\; \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk}\langle\bar{\psi}_{\beta j} i\gamma^5 C \bar{\psi}^T_{\gamma k}\rangle$$

Color-Flavor Locking Ansatz

Δ_{ud}	Δ_{ds}	Δ_{su}
up-down	**down-strange**	**strange-up**
up-down	**down-strange**	**strange-up**

QCD Phases

$\boxed{N : S = 1/2}$



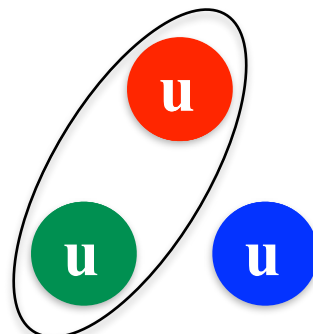
$$H_{\text{color-spin}} = -\frac{3}{4}C$$

$S = 0$

no more s-s int.

$$H_{\text{color-spin}} = -\frac{3}{4}C$$

$\boxed{\Delta : S = 3/2}$



$$H_{\text{color-spin}} = +\frac{1}{4}C$$

$S = 1$

$\boldsymbol{S} \cdot \boldsymbol{s} = 1/2$

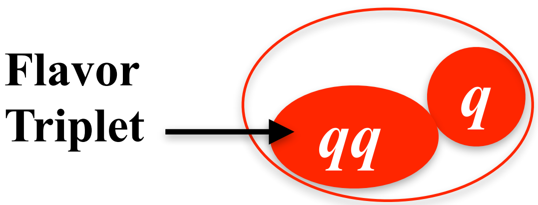
$$H_{\text{color-spin}} = +\frac{3}{4}C$$

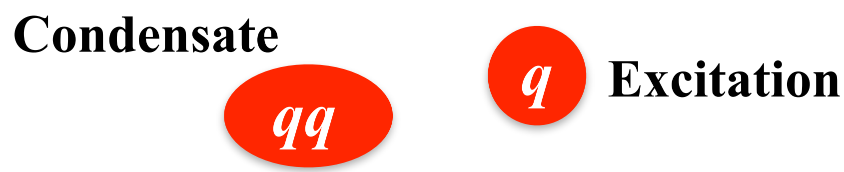
$$m_{\text{bad}} - m_{\text{good}} \approx \frac{2}{3}(M_{\Delta} - M_N)$$ confirmed in lattice QCD

QCD Phases

Matching of Symmetry Breaking Patterns

Baryons: 8+1 (low-lying) **Quarks: 3color × 3flavor = 9**





$\langle ud \rangle \ \langle ds \rangle \ \langle su \rangle$ **Diquark condensates break chiral symmetry in the same way as the hadronic phase.**

Diquarks realize duality between baryons and quarks!

Dense QCD may have more stringent duality than crossover at high *T*…

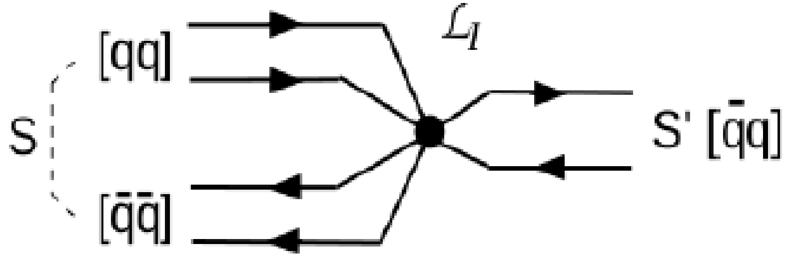
QCD Phases

U(1)$_A$ breaking interaction

$$\det \bar{\psi}_{Lj}\psi_{Ri} + \det \bar{\psi}_{Rj}\psi_{Li}$$

$$\to \det R_{im}\bar{\psi}_{Ln}\psi_{Rm}L^{\dagger}_{nj} + \det L_{im}\bar{\psi}_{Rn}\psi_{Lm}R^{\dagger}_{nj}$$

For N_f =3, this is a six point interaction:

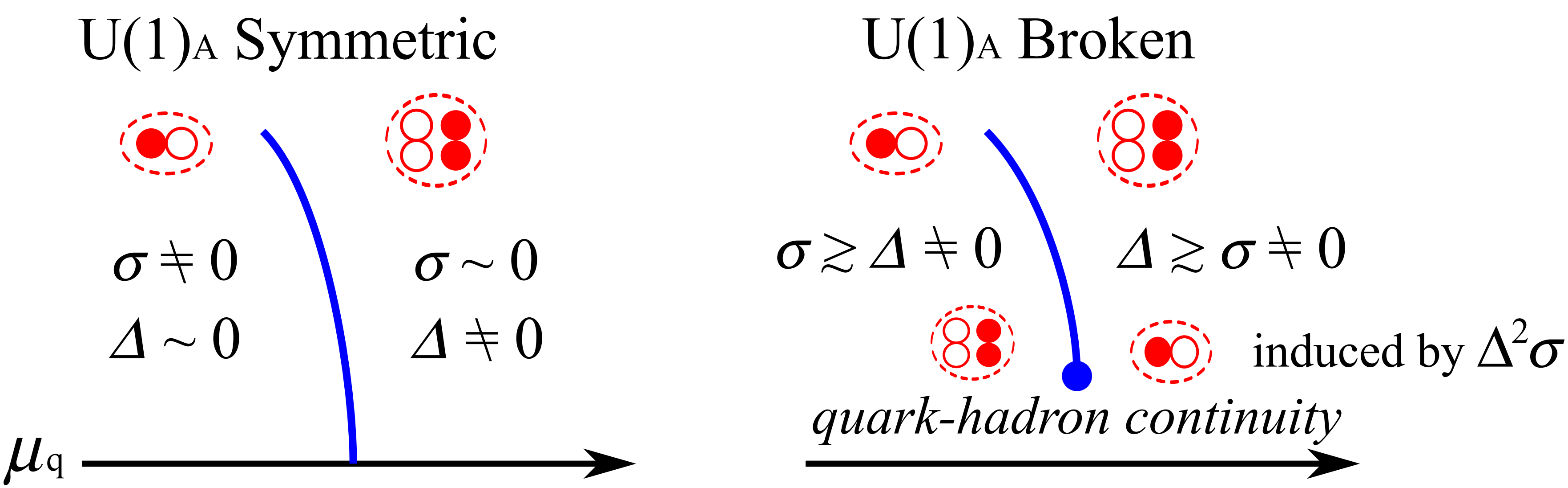


't Hooft-Isidori-Maiani-
-Polosa-Riquer (2008)

$$\sim \langle\psi\psi\rangle\langle\bar{\psi}\bar{\psi}\rangle\langle\bar{\psi}\psi\rangle$$

Anomaly induces a mixing between mesons and diquarks

QCD Phases



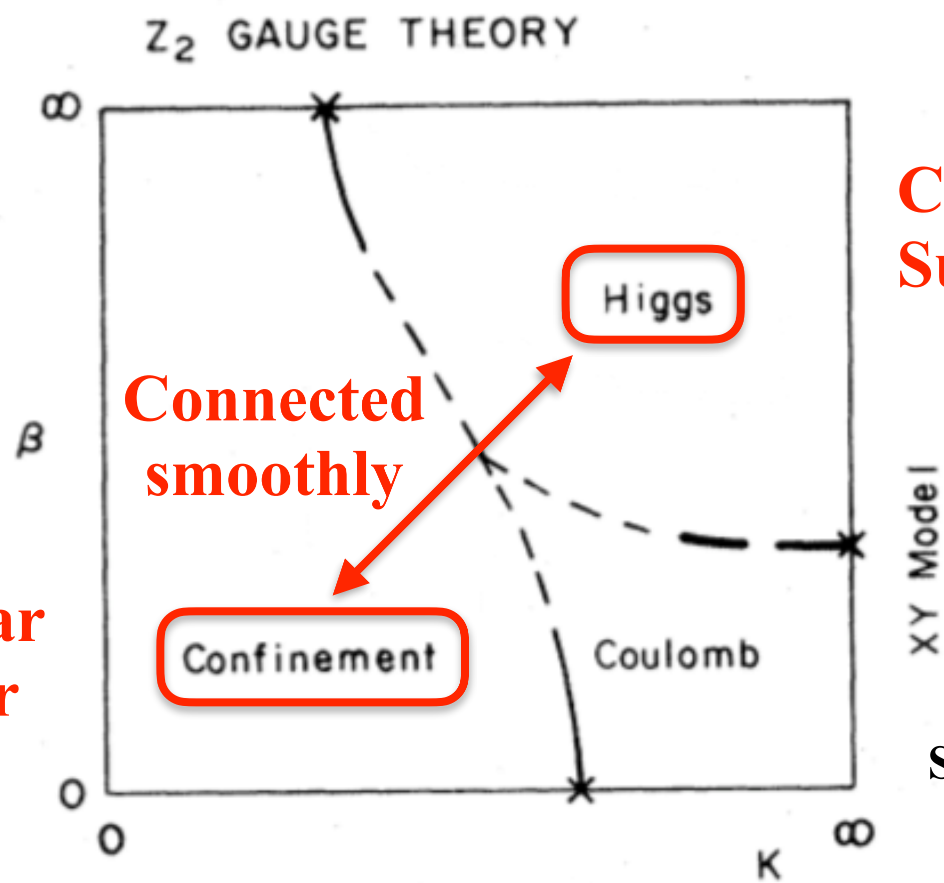
No phase transition because $\sim \Delta\Delta^* M$

Hatsuda-Tachibana-
-Yamamoto-Baym (2006)

U(1)A breaking interaction

QCD Phases

Fradkin-Shenker (1979)



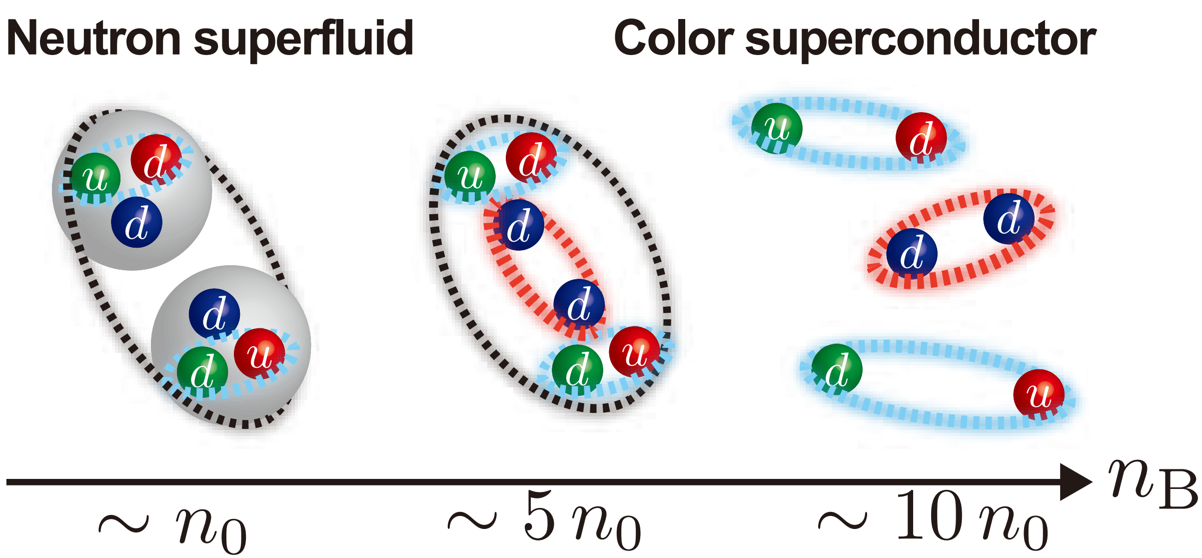
Color Superconductor

Nuclear Matter

Schaefer-Wilczek, PRL (1999)

QCD Phases

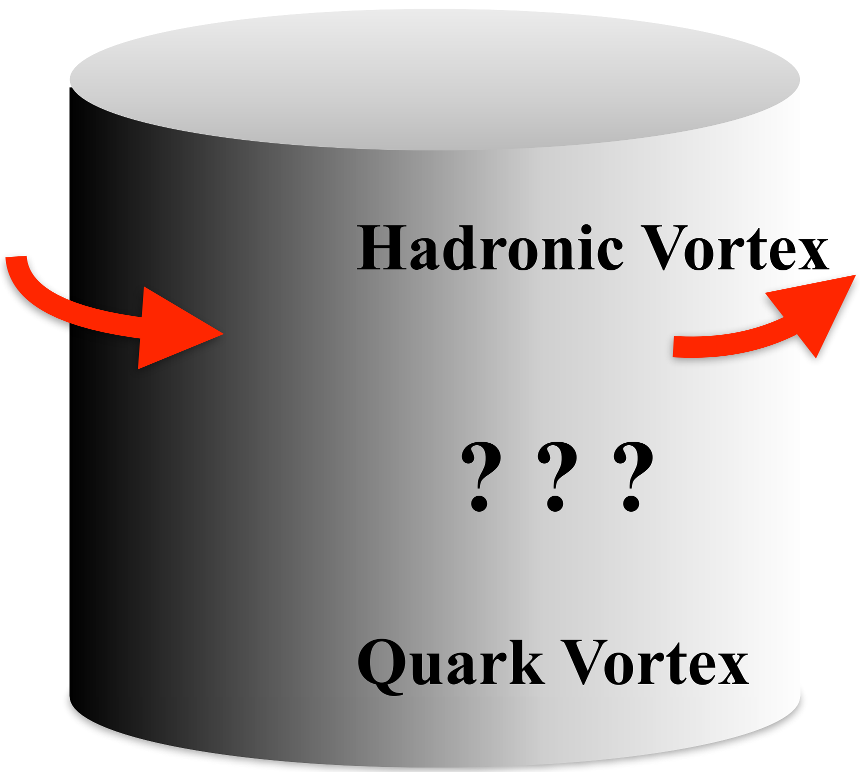
Intuitive picture **Fujimoto-Fukushima-Weise (2020)**



No change in global symmetry for the 2-flavor case.
No need to have a phase transition → Crossover ?

Controversy

Thinking Experiment



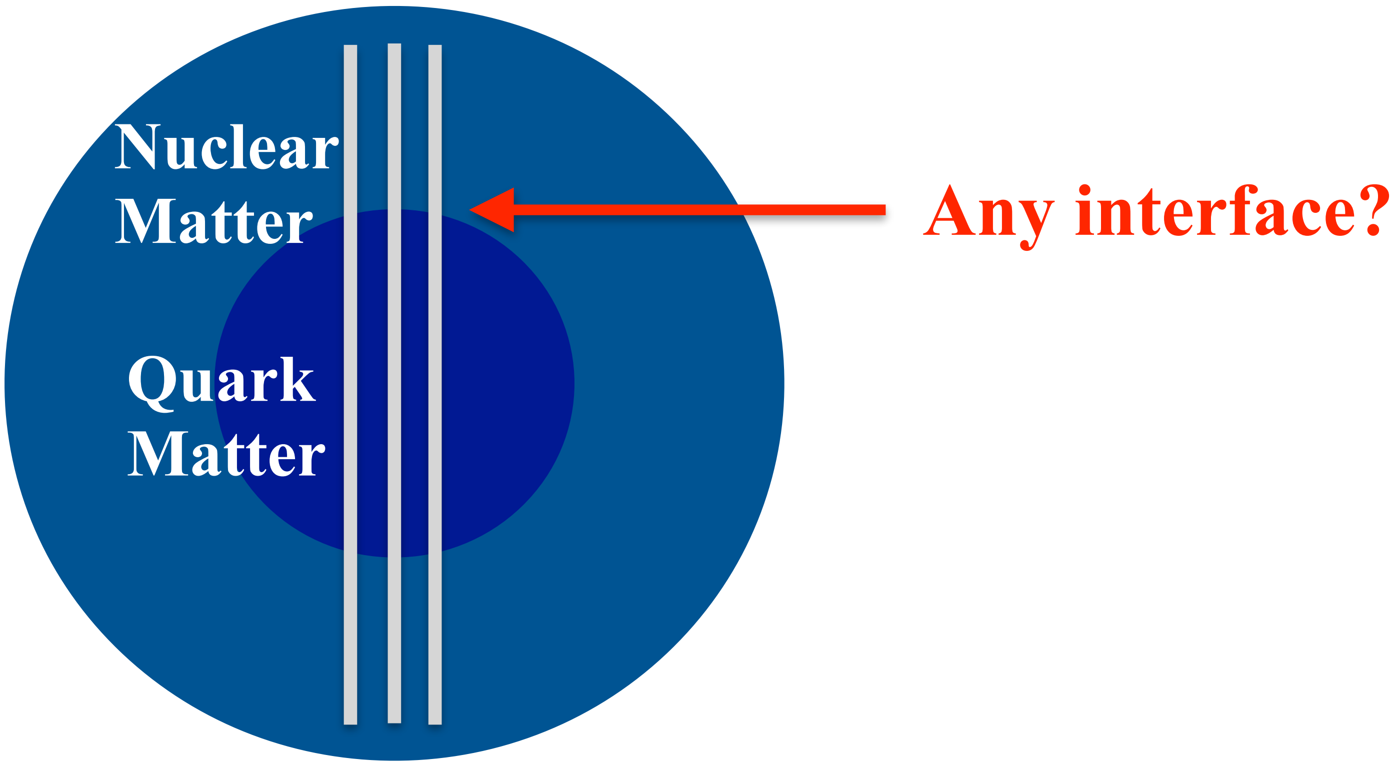
Rotate the bucket filled with quarks

Upper part : Hadronic Vortex
Lower part : Quark Vortex

How can they be connected?

Controversy

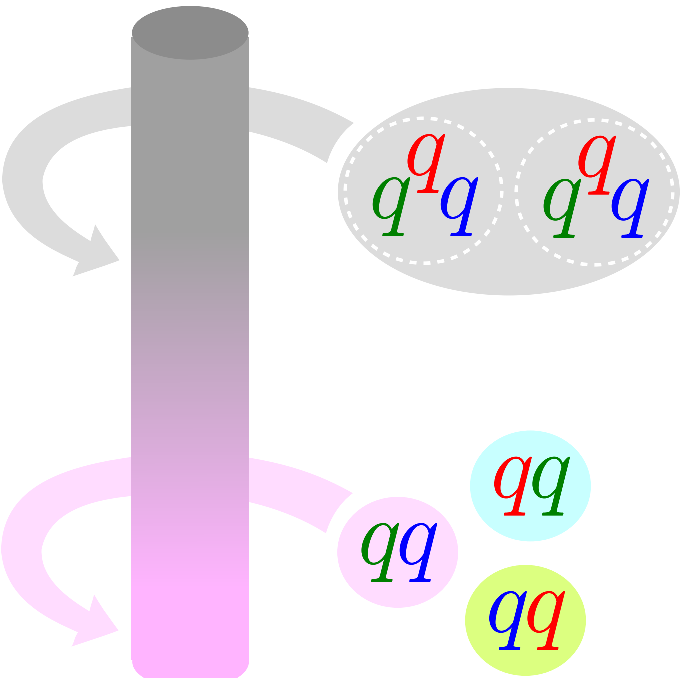
Superfluid vortices pinned in the NS cores



Controversy

Vortex Continuity Scenario

Alford-Baym-Fukushima-Hatsuda-Tachibana (2018)



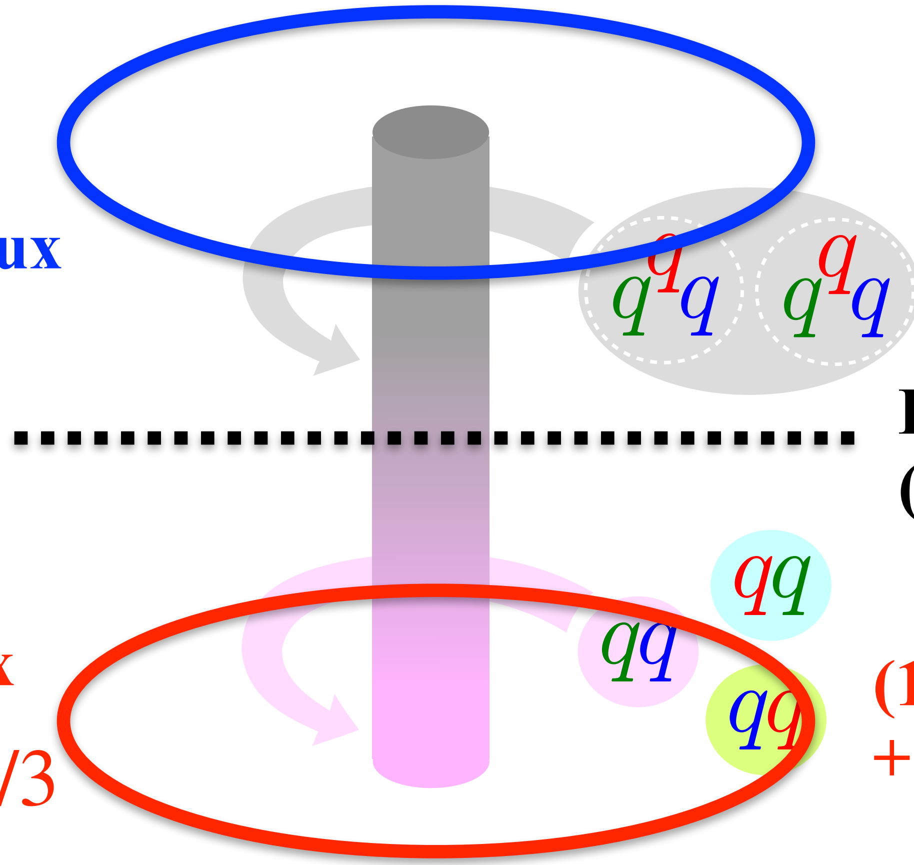
We proposed a scenario of the vortex continuity, but…

Controversy

Topological 1st-order PT ? **Cherman-Sen-Yaffe (2018)**

No magnetic flux

$$e^{i\Phi} = 1$$



Discontinuous (Interface)

Quantized flux

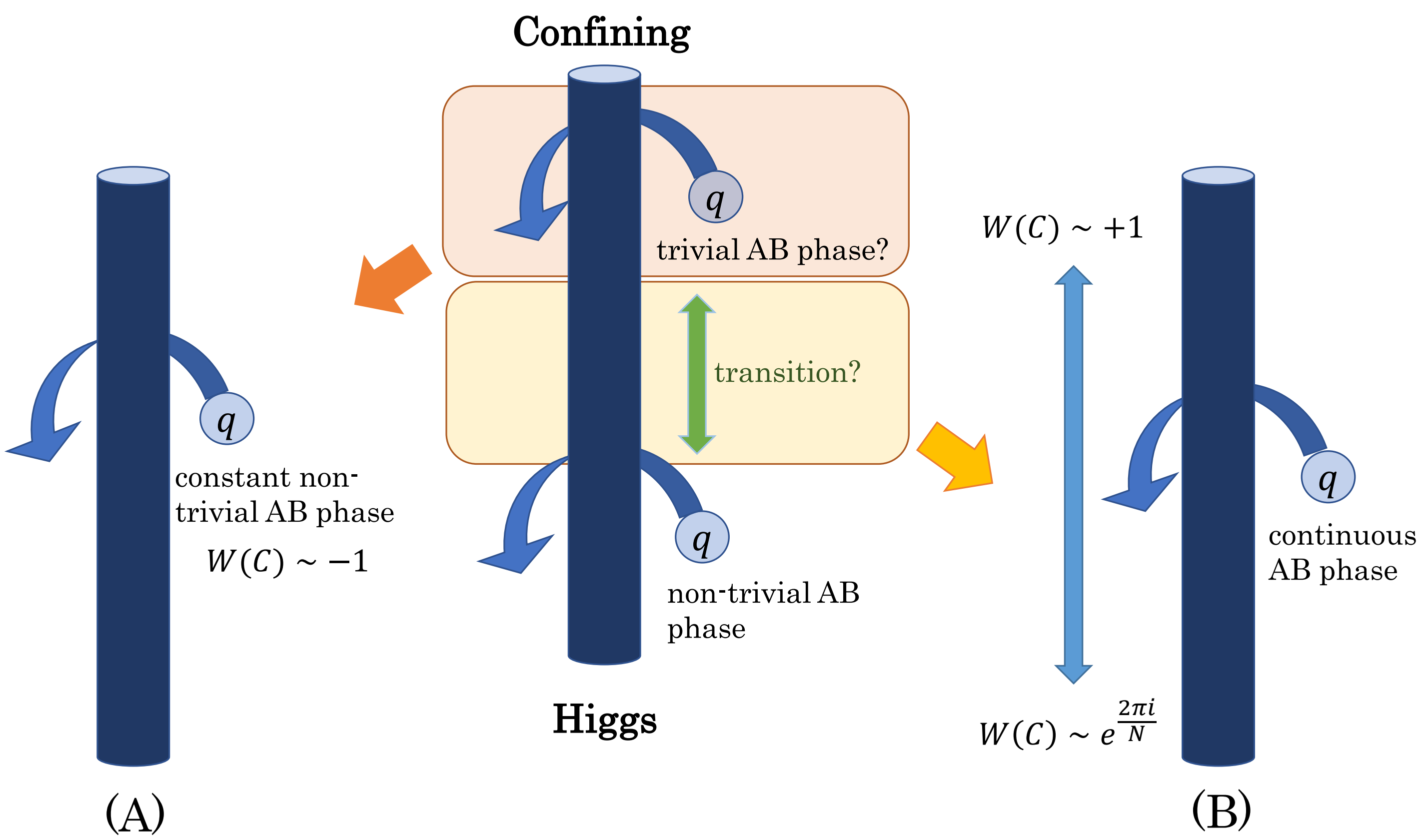
$$e^{i\Phi} = e^{i2\pi/3}$$

(1/3) QED + (2/3) QCD

Controversy

Continuity still alive…

Hayashi (2024)

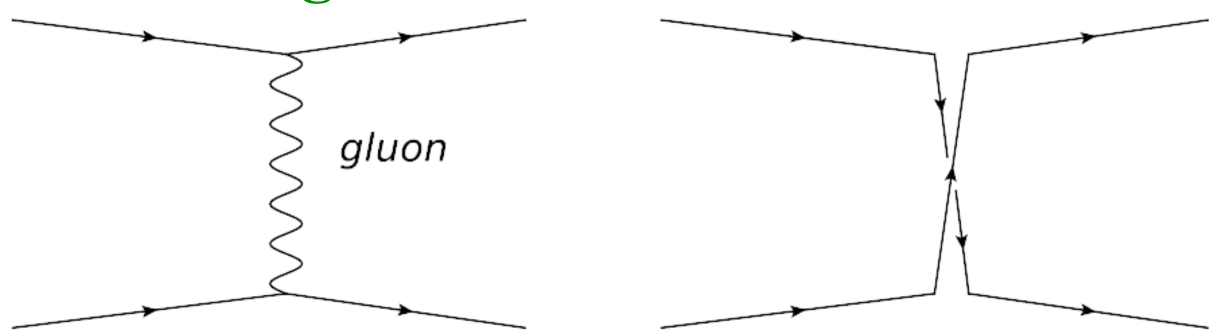


Another Exotica from Large Nc

Quarks **spin-1/2 (fermions)** **6 flavors** N_c **colors**

(transform in the SU(N_c) fundamental rep.)

quark **red** / **green** / **blue**

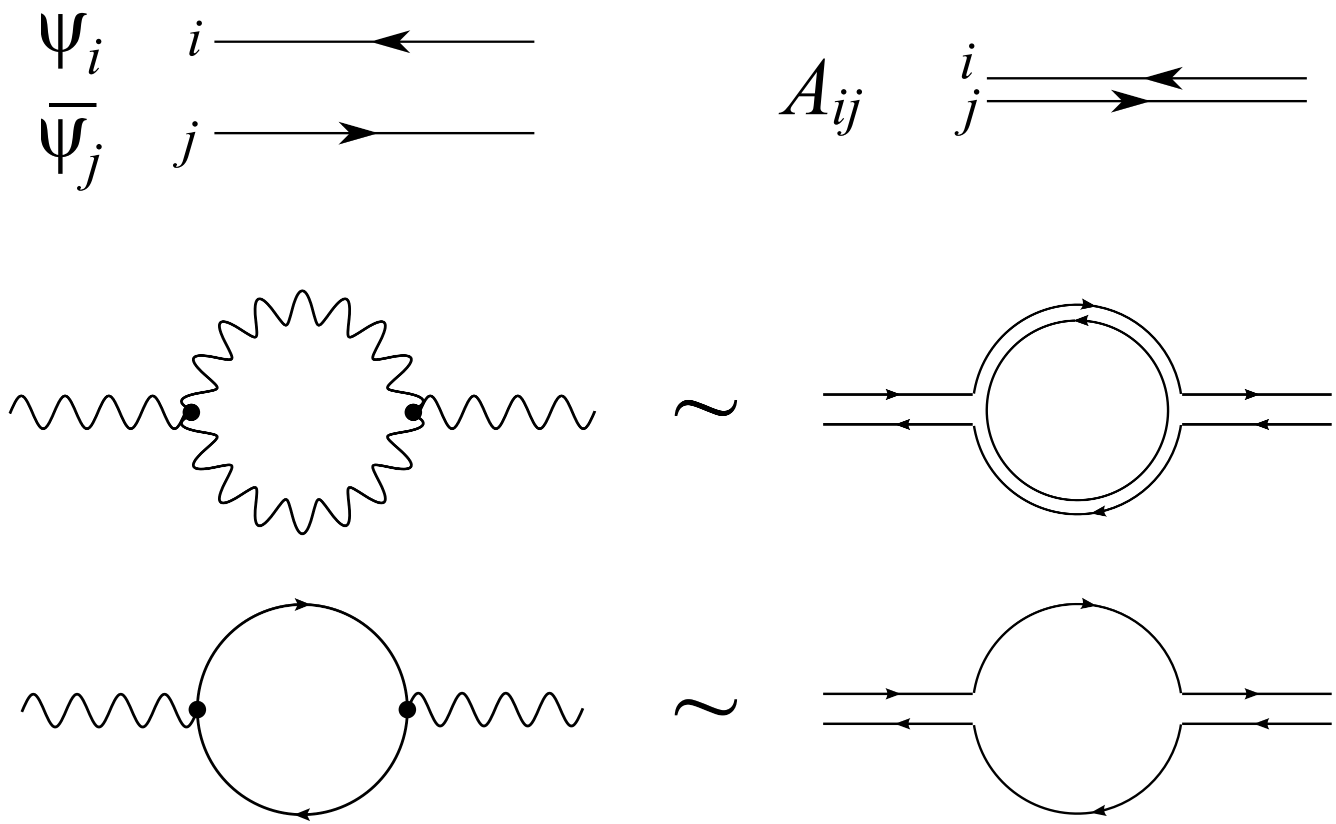


Gluons **spin-1 (bosons)** $N_c^2 - 1$ **colors**

$r\bar{r}$ $r\bar{g}$ $r\bar{b}$ $g\bar{r}$ $g\bar{g}$ $g\bar{b}$ $b\bar{r}$ $b\bar{g}$ $b\bar{b}$ - $(r\bar{r}+g\bar{g}+b\bar{b})$

Another Exotica from Large Nc

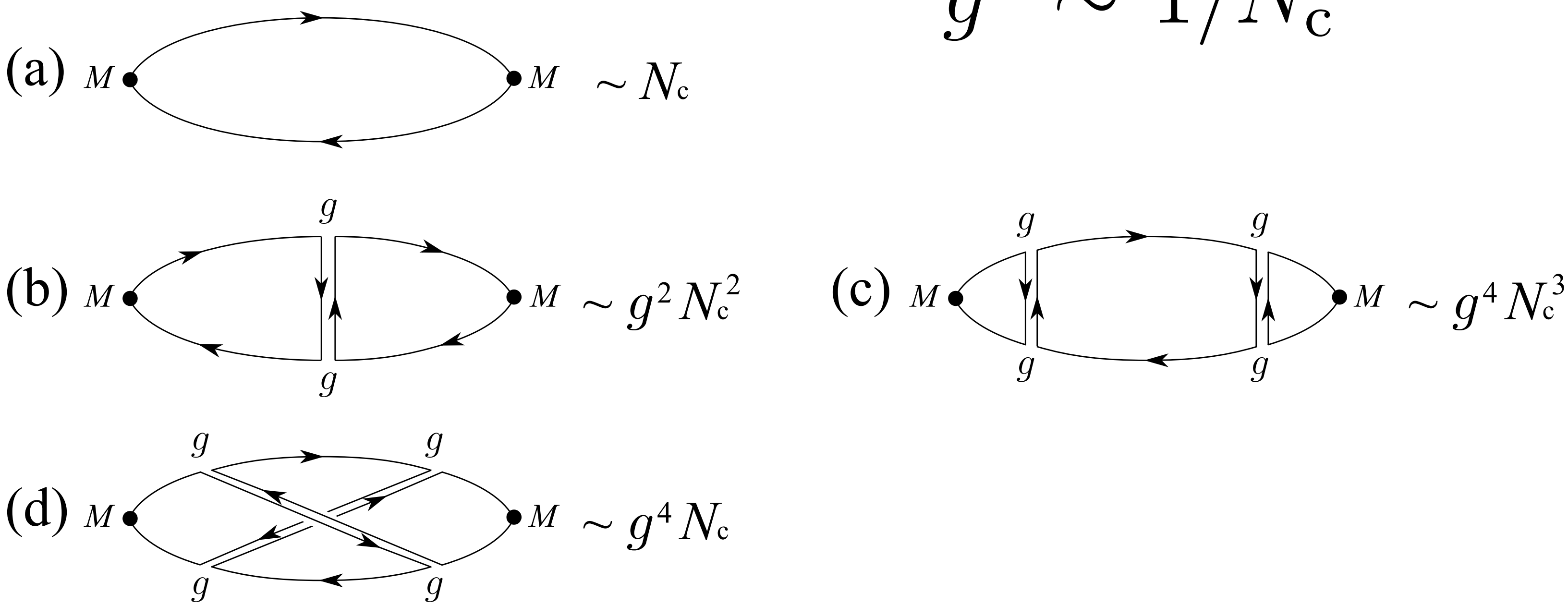
Theoretical Preparation: Large-Nc Counting



Another Exotica from Large Nc

Theoretical Preparation: Large-Nc Counting

$$g^2 \sim 1/N_c$$

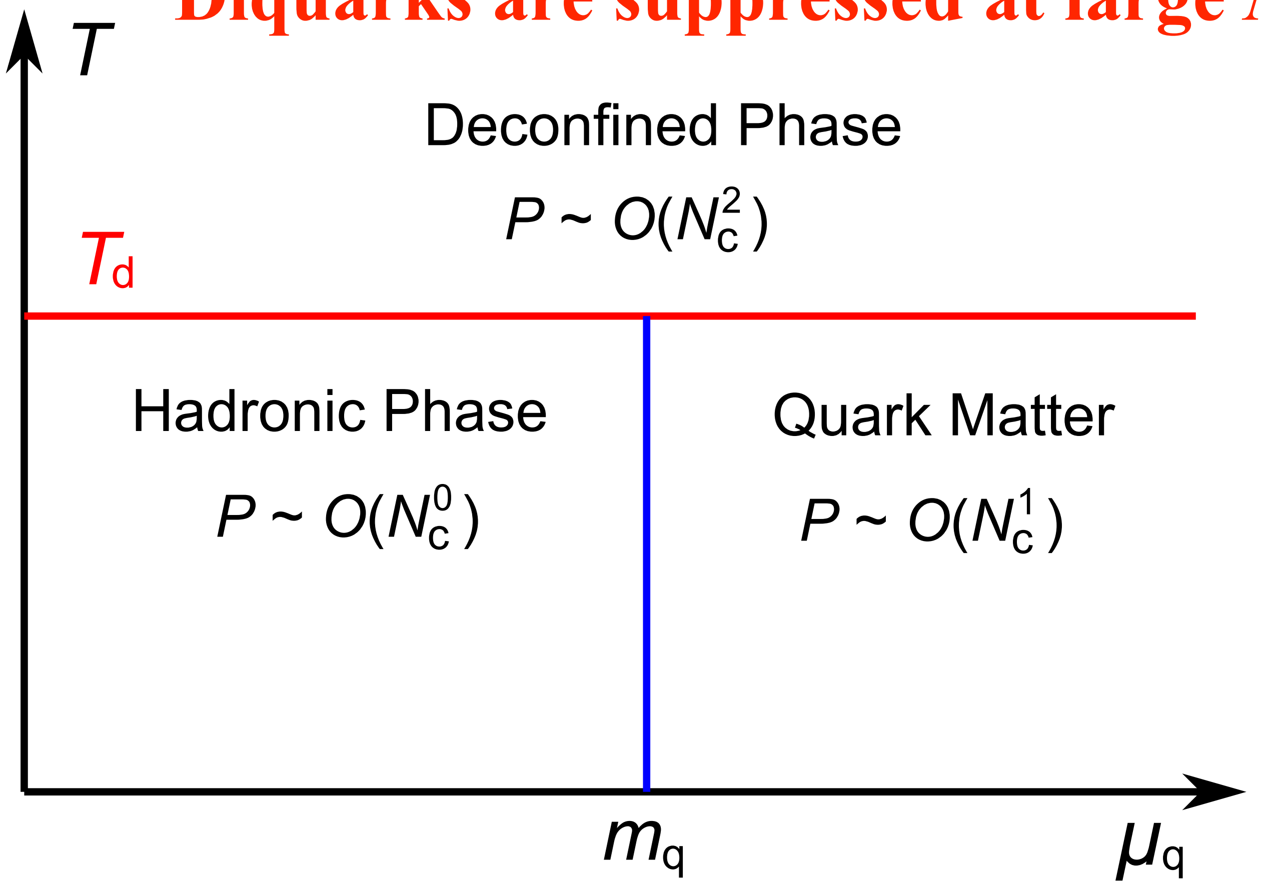


Non-planar diagrams and quark loops suppressed!

Another Exotica from Large Nc

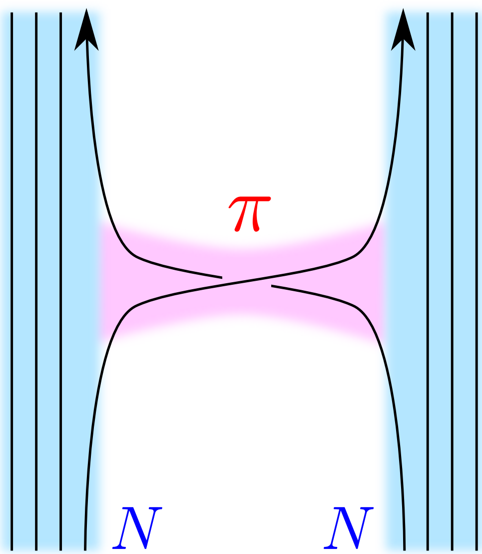
Strongly Interacting Baryons ~ Free Quarks

Diquarks are suppressed at large *N*c.



Another Exotica from Large Nc

This is NOT the end of the story!



If there are infinitely many quarks, mesons do not interact, but baryons do interact very strongly!

Pressure of Quark Matter

Kinetic Energy ~ O(N_c)

Pressure of Baryonic Matter

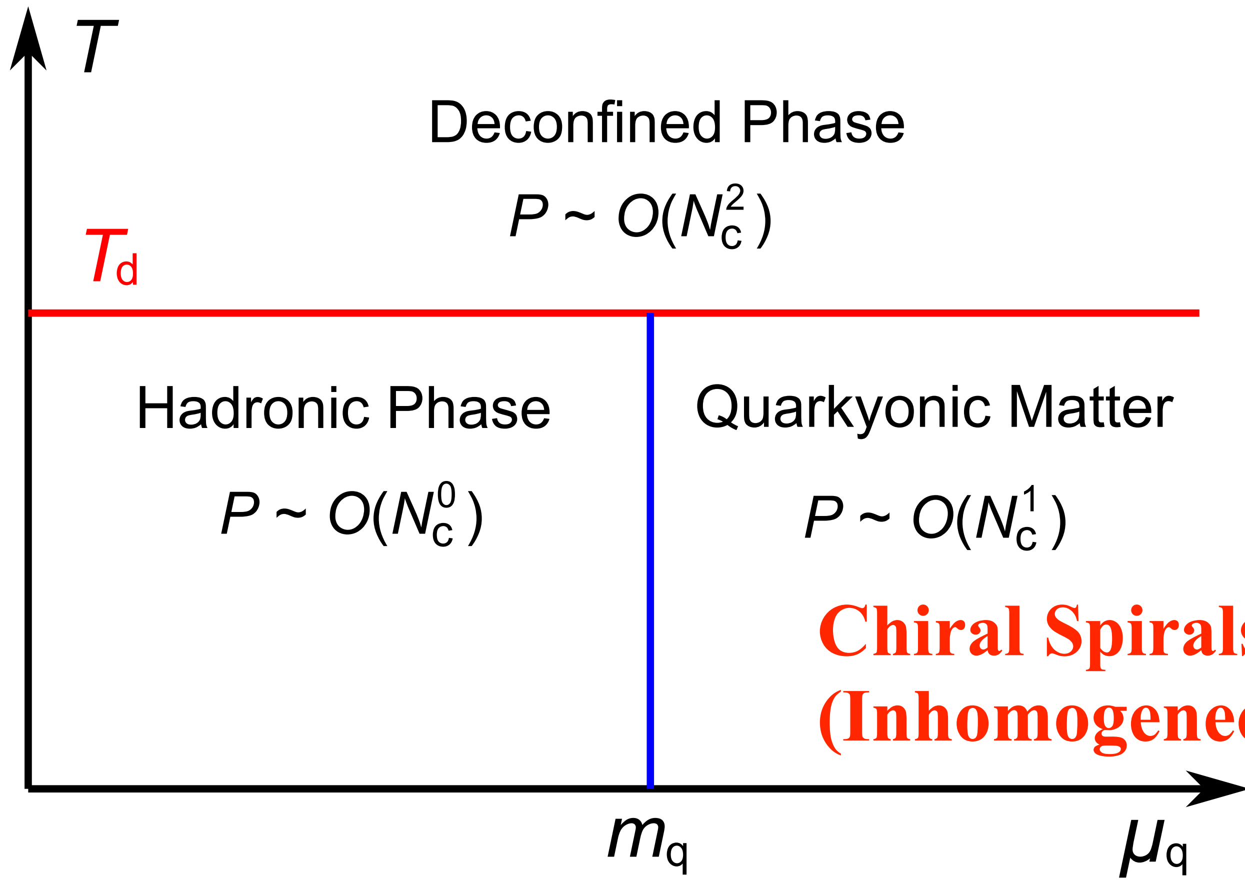
Interaction Energy ~ O(N_c)

Quarkyonic

McLerran-Pisarski (2008)

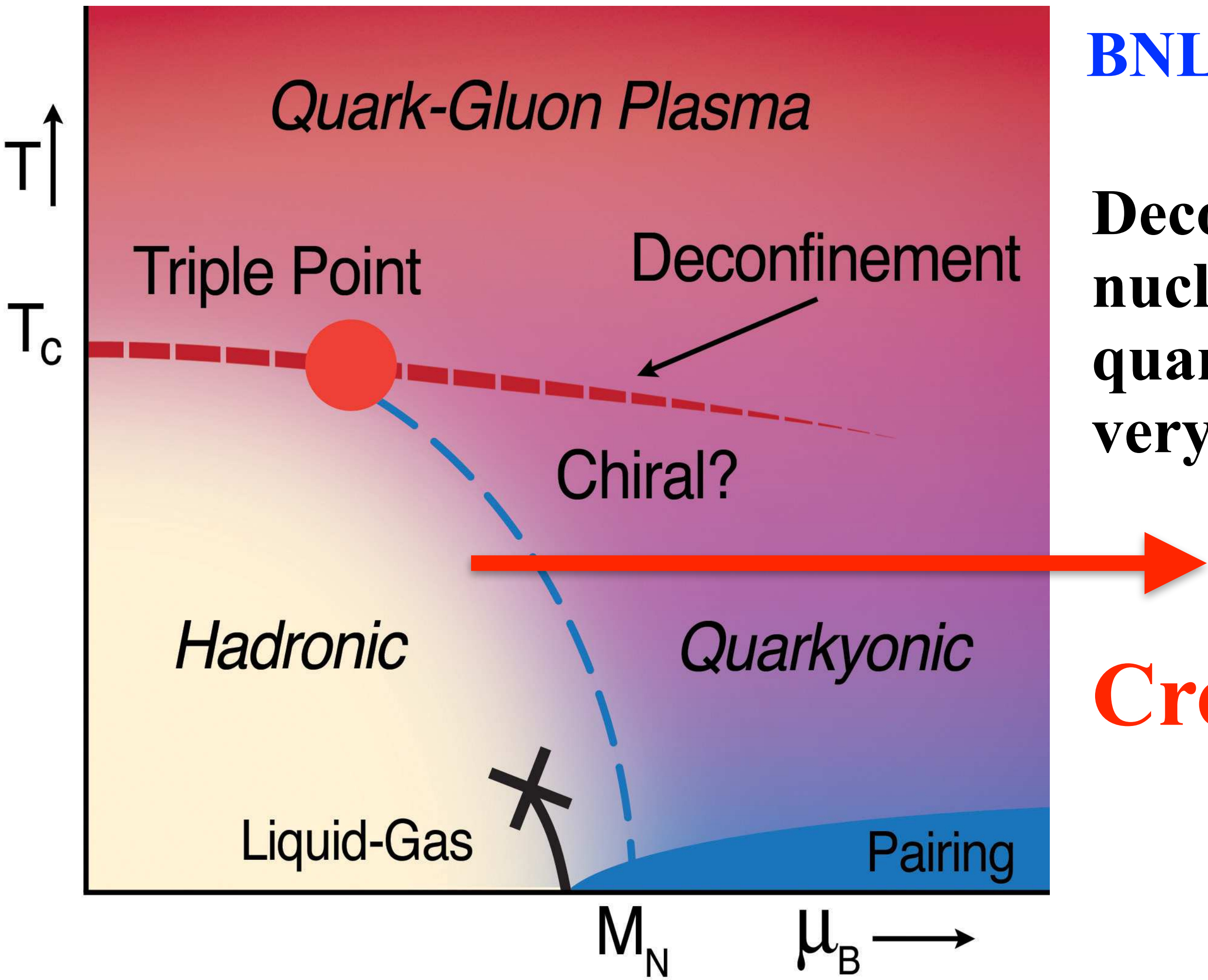
Another Exotica from Large Nc

Strongly Interacting Baryons ~ Free Quarks



Chiral Spirals (Inhomogeneous!)

Another Exotica from Large Nc

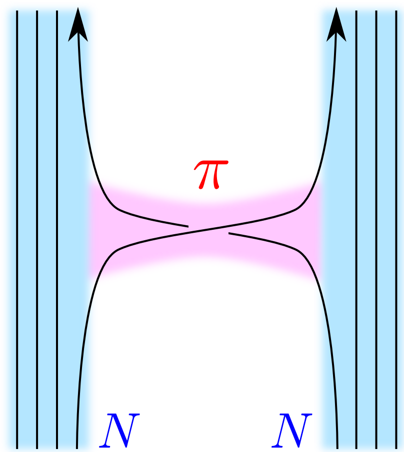


BNL Phase Diagram

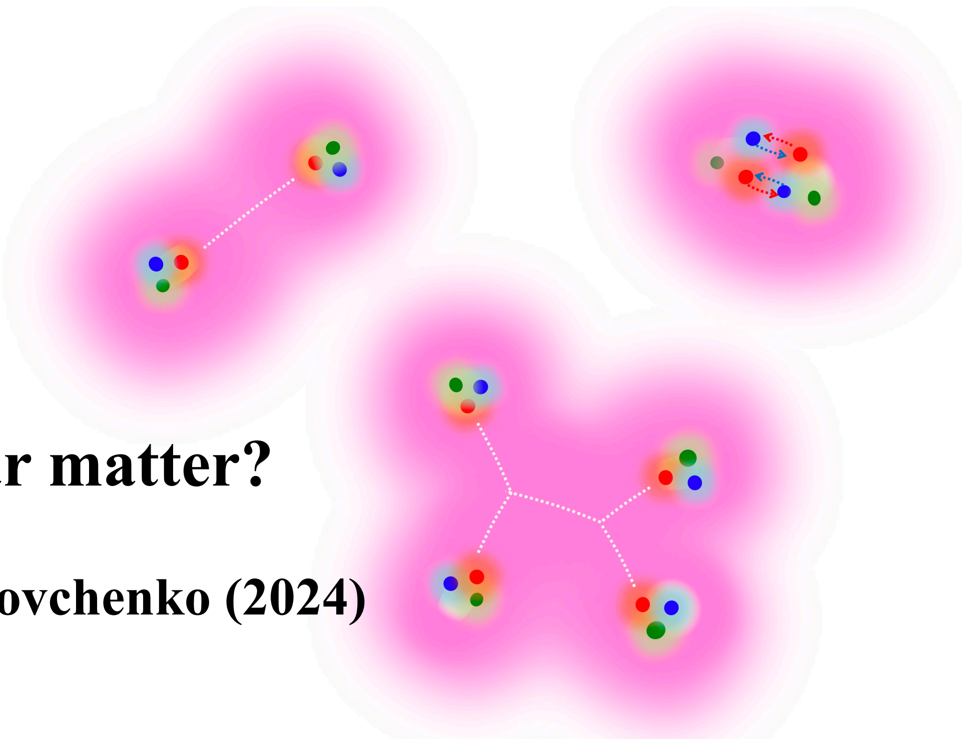
Deconfinement from nuclear d.o.f. to quark d.o.f. occurs very smoothly.

Crossover ?

Another Exotica from Large Nc



Remember that the pion (interaction) percolation starts from $\sim 1.5\, n_{\text{sat}}$



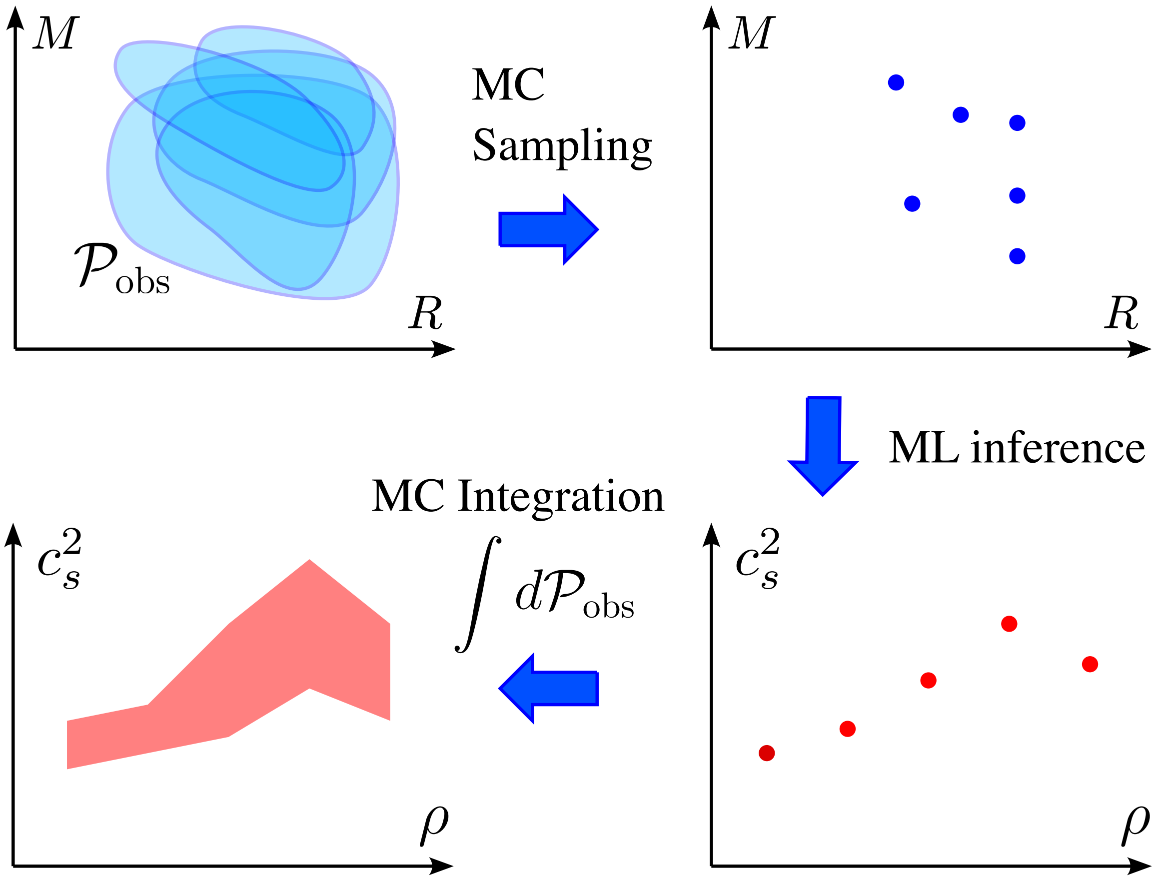
Quarkyonic regime may start near normal nuclear matter?

See: Koch-McLerran-Miller-Vovchenko (2024)

Constraing the EOS

EOS Inference Program

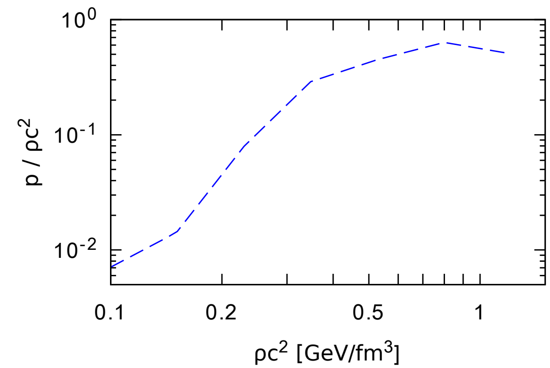
Fujimoto-Fukushima-Kamata-Murase (2018-2024)



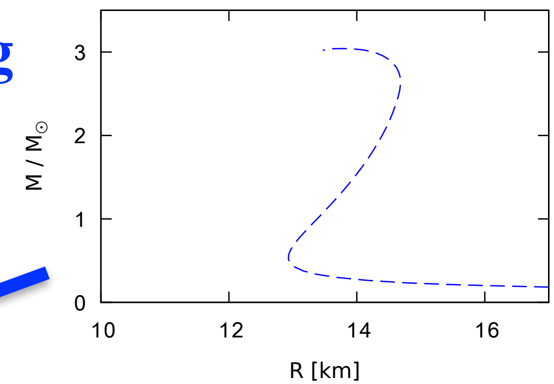
EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)

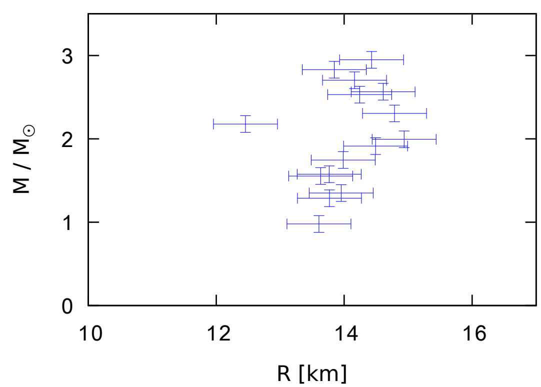
Proof of principle



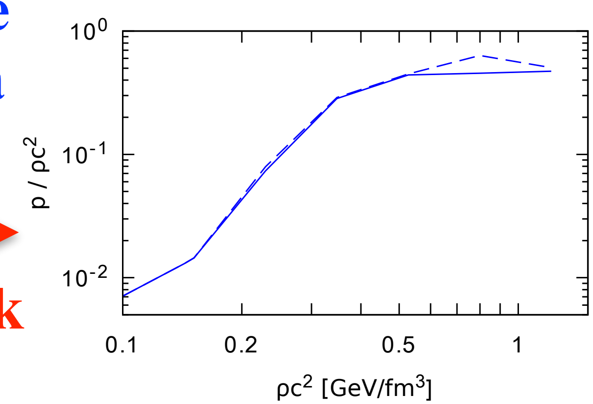
Corresponding M-R relation



Mimic the astro data



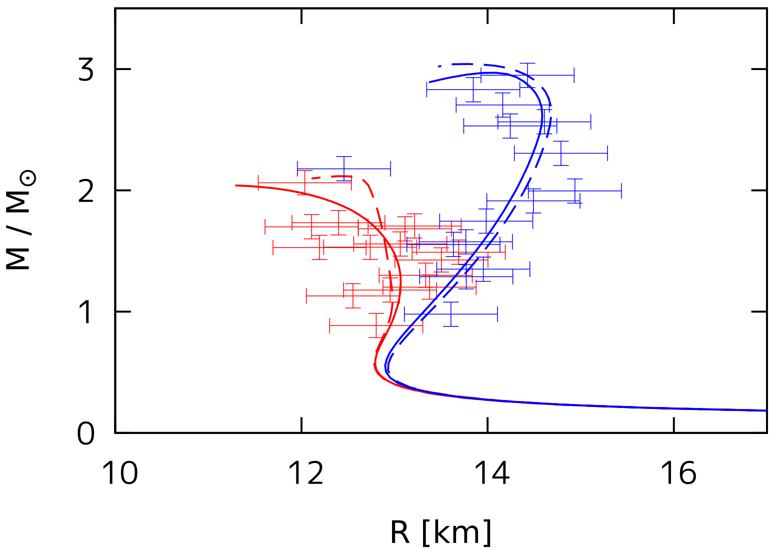
Validity check

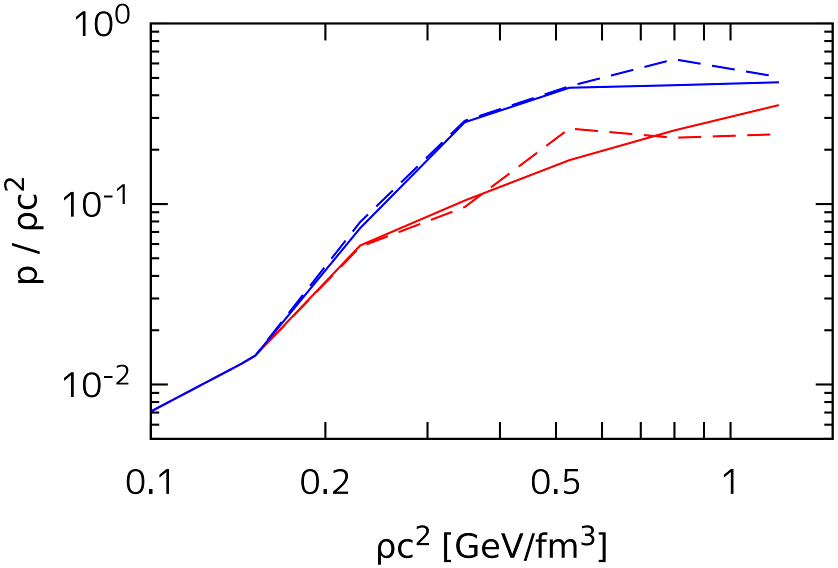


EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)

Machine learning shows amazing performance!

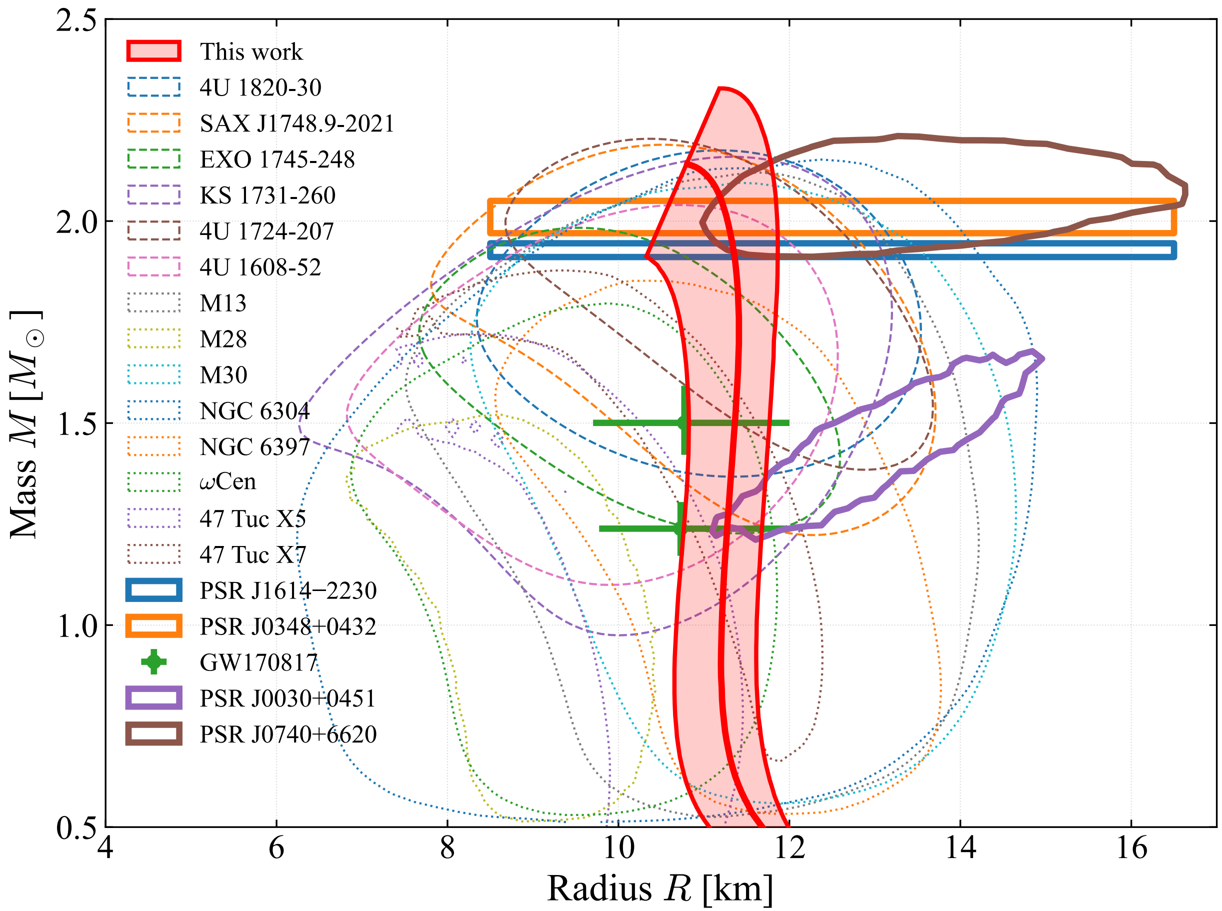




Overfitting is miraculously avoided!

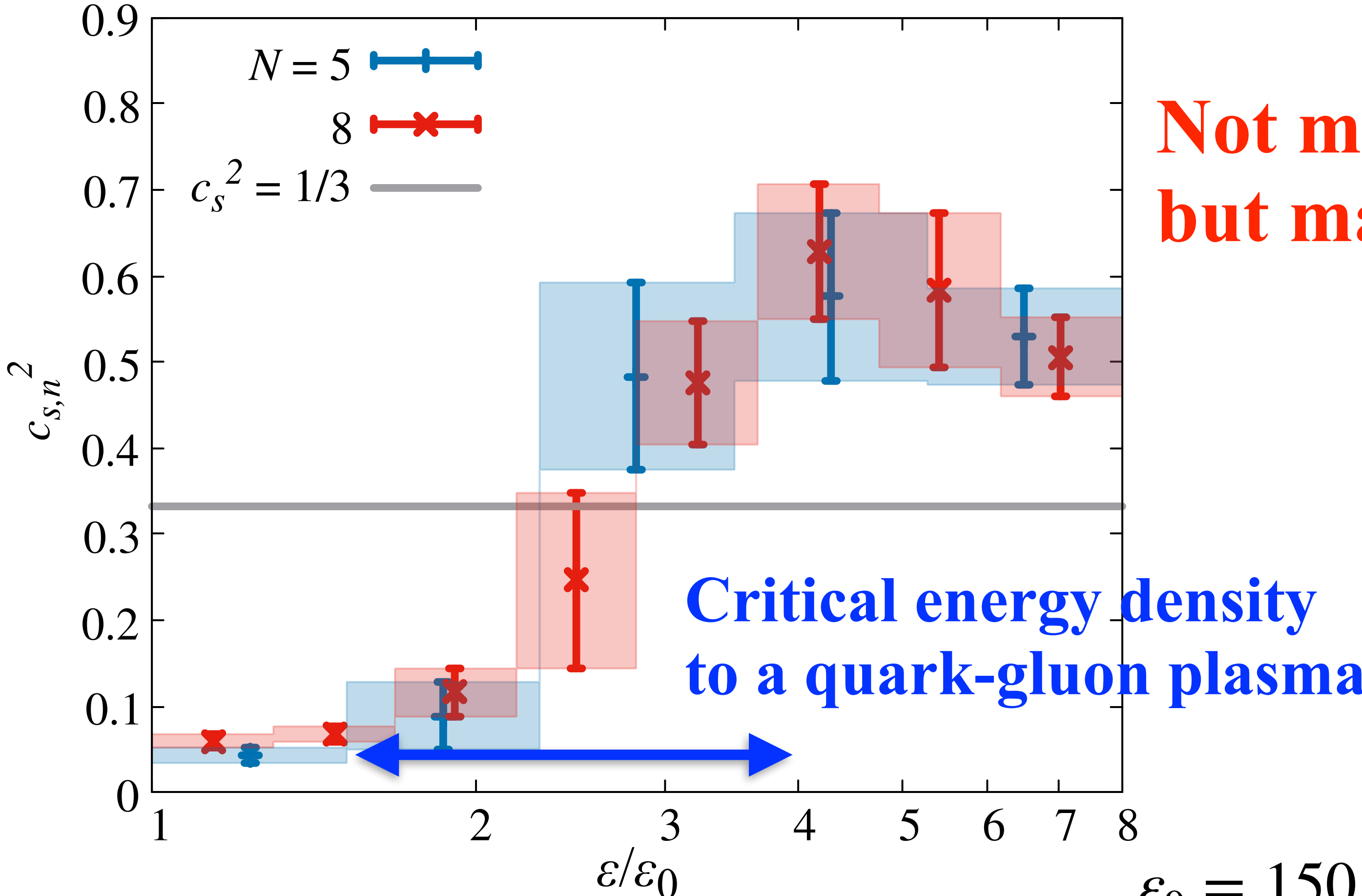
EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)



EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)

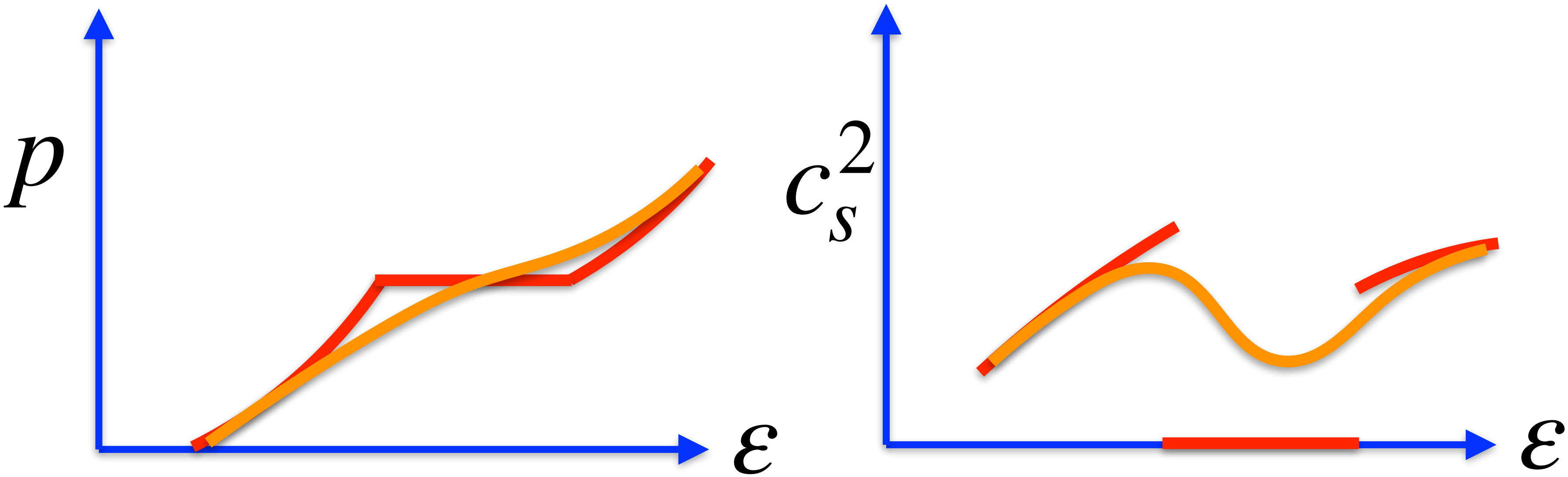


Not minimum,
but maximum!?

$\varepsilon_0 = 150\,\mathrm{MeV\ fm^{-3}}$

Interpretation

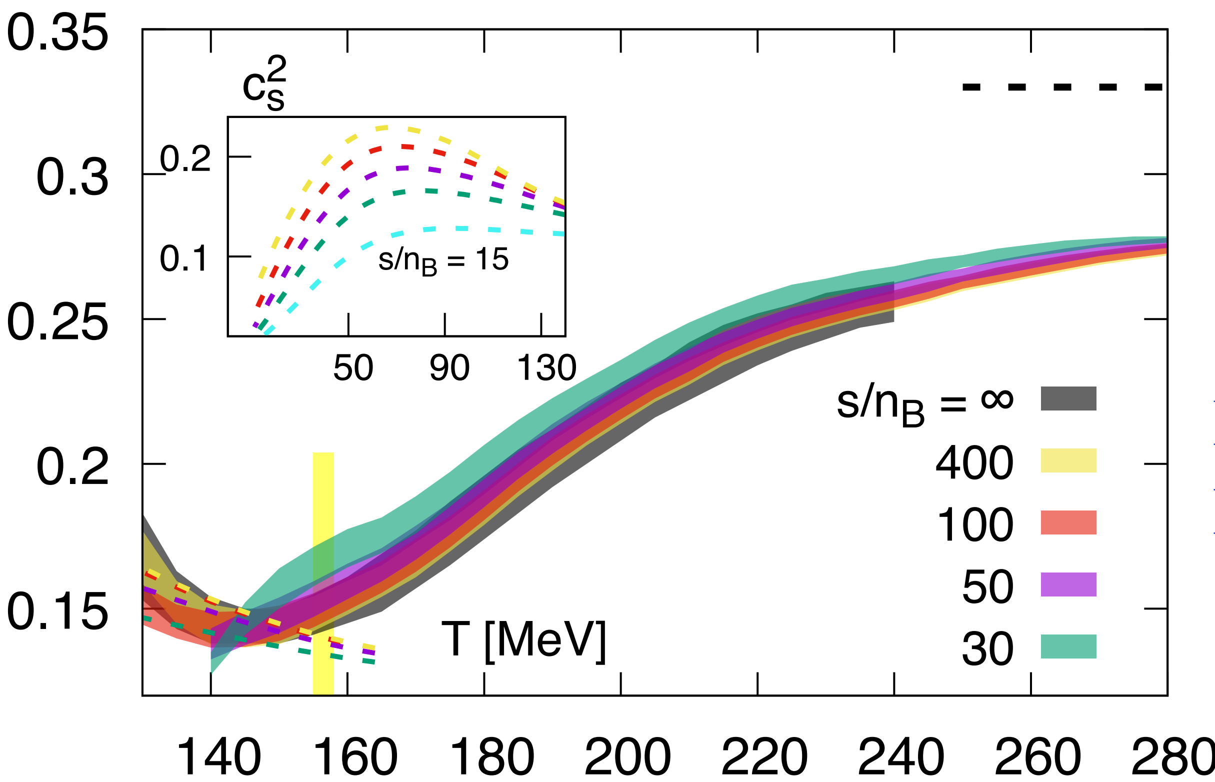
[1st-order-like EoS]



Phase transition is manifested by a minimum in the speed of sound.

Interpretation

[High-Temperature QCD — QGP Crossover]



HotQCD Collab. (2212.09043)

Minimum around phase transition

Interpretation

Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

Measure of conformality: $\Delta = \frac{1}{3} - \frac{p}{\varepsilon}$

Gavai-Gupta-Mukherjee (2004)

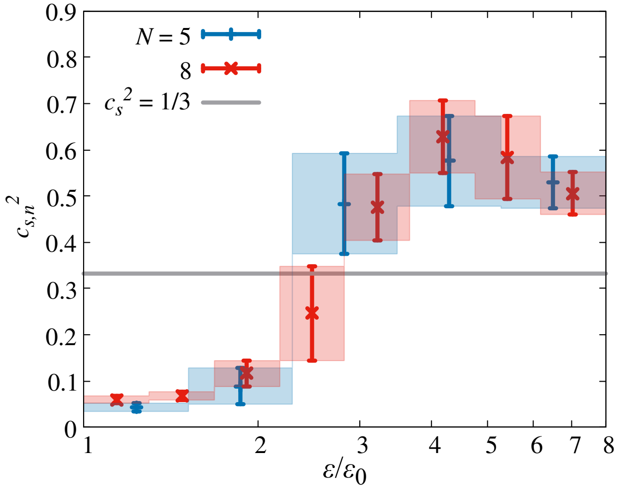
$$c_s^2 = \frac{dp}{d\varepsilon} = c^2_{s,\,\mathrm{deriv}} + c^2_{s,\,\mathrm{non-deriv}}$$

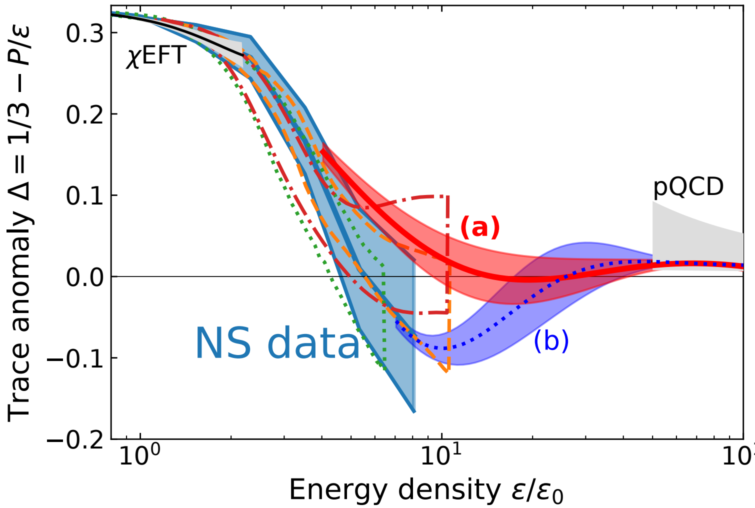
$$c^2_{s,\,\mathrm{deriv}} = -\varepsilon \frac{d\Delta}{d\varepsilon} \qquad c^2_{s,\,\mathrm{non-deriv}} = \frac{1}{3} - \Delta$$

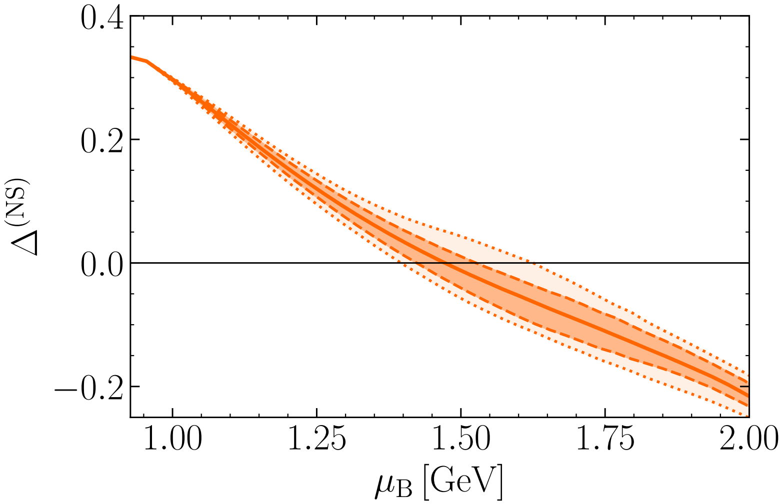
Derivative **Non-Derivative**

Dominant at high density making a peak!

Interpretation







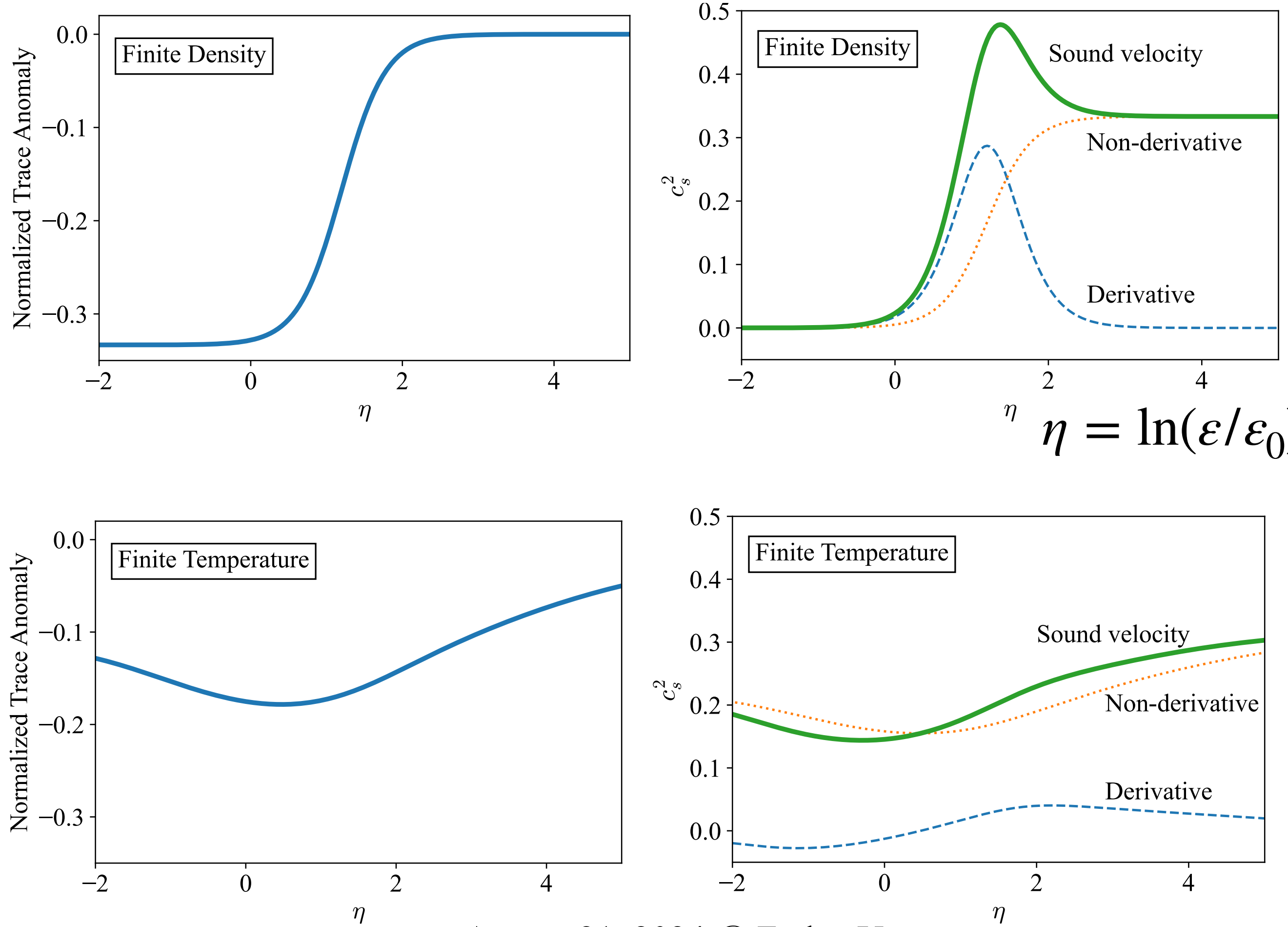
Brandes-Fukushima-Iida-Yu (2024)

Newer analysis suggests that the trace anomaly goes negative!

Interpretation

Derivative contribution makes a peak structure!

Sign Flipped $-\Delta$

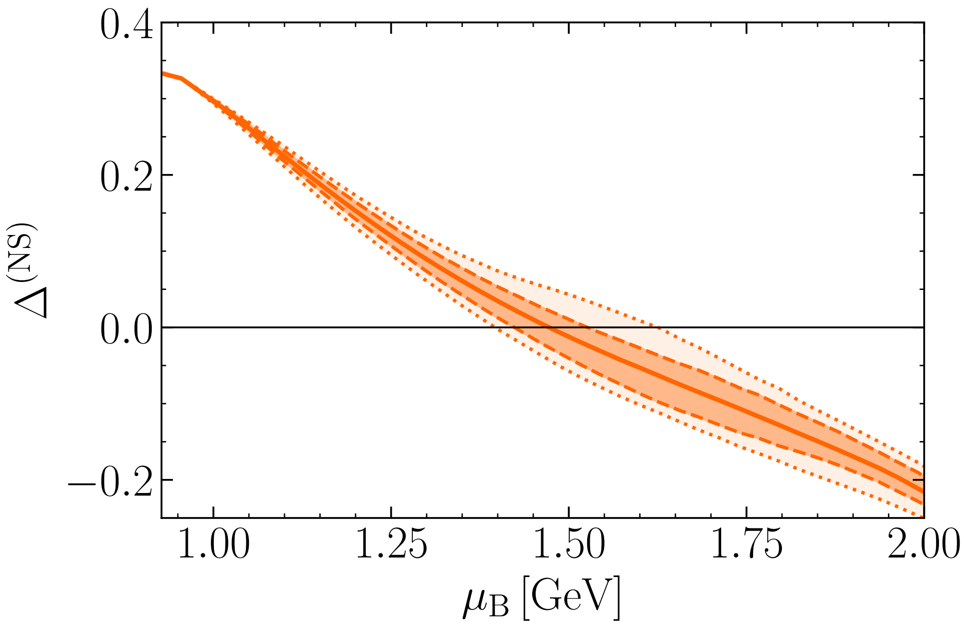


$\eta = \ln(\varepsilon/\varepsilon_0)$

Speed of Sound

Interpretation

Interesting question… $\Delta < 0$???



$$\Delta \propto \varepsilon - 3p$$

$$\propto \frac{d}{d\mu}\left(\frac{p}{\mu^4}\right)$$

Thermodynamic degrees of freedom

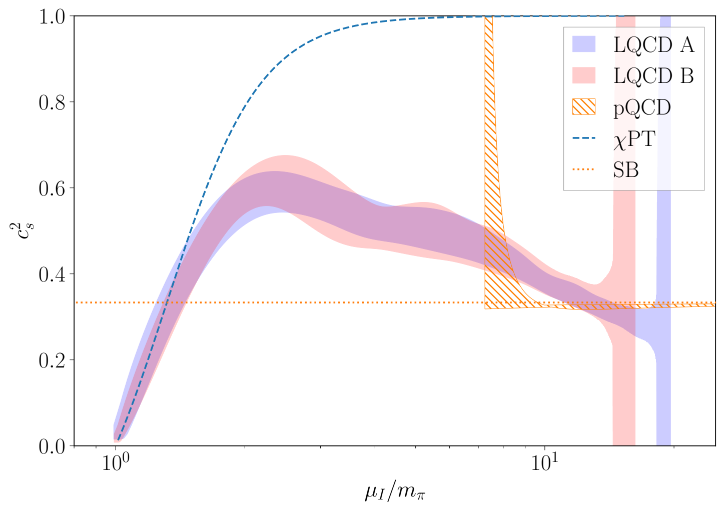
Negative trace anomaly implies the presence of "condensates"!?

Interpretation

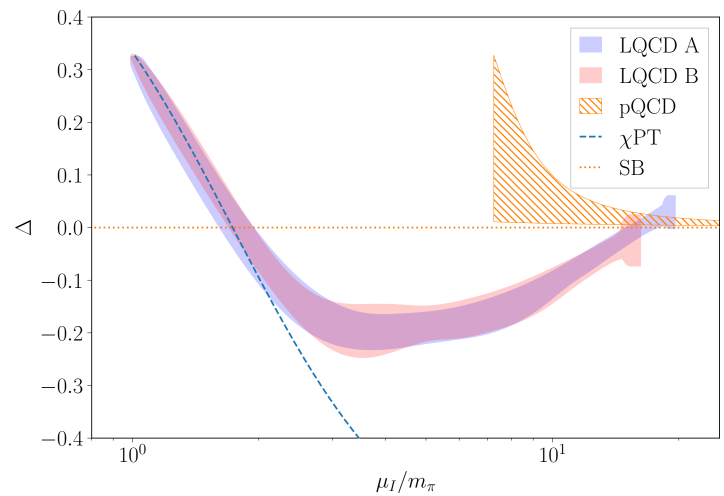
Lesson from high-isospin matter

Abbott+ (2023)

[Speed of sound peak]



[Negative trace anomaly]



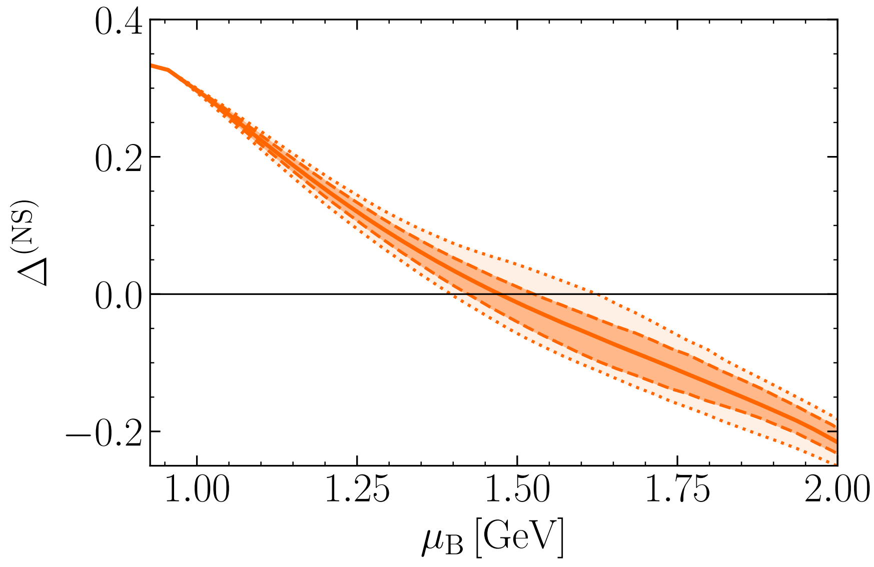
$$p_{\chi\mathrm{PT}} = \frac{f_\pi^2 \mu_I^2}{2}\left(1 - \frac{m_\pi^2}{\mu_I^2}\right)^2$$

No loops, just condensates

Interpretation

Brandes-Fukushima-Iida-Yu (2024)

Assume a general Ginzburg-Landau potential for “some” bosonic condensates to fit the NS trace anomaly behavior: 50 - 300 MeV



cf. Kurkela-Rajagopal-Steinhorst (2024)

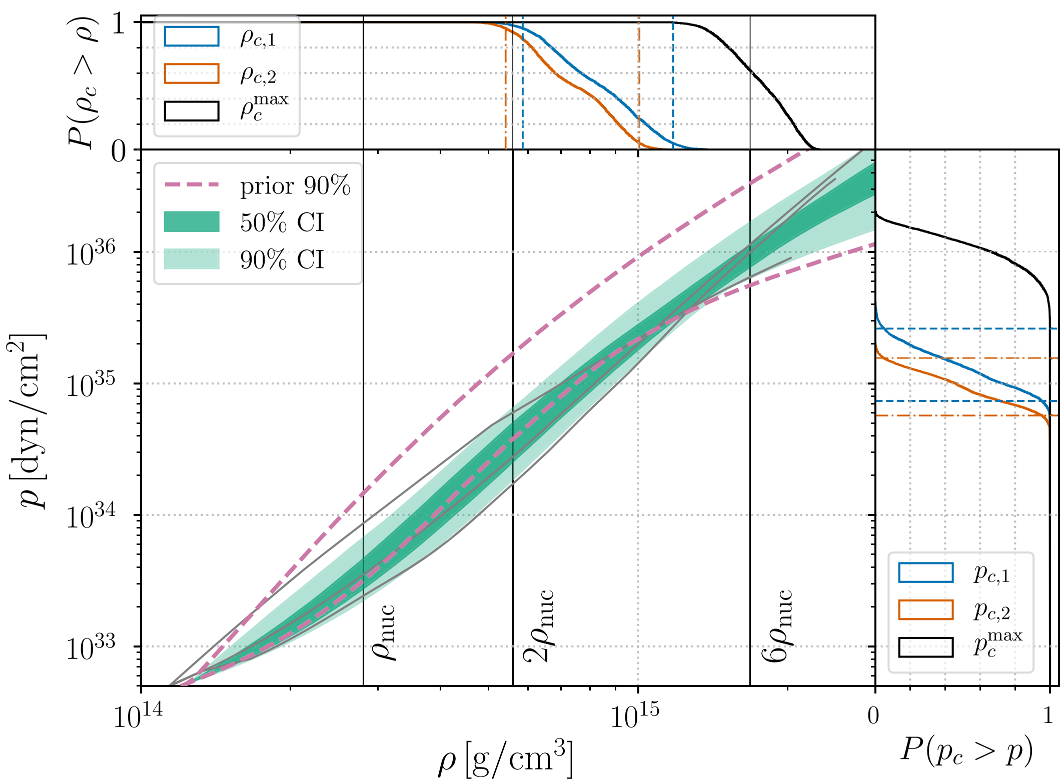
$\Delta_{\mathrm{pair}} < 457\,\mathrm{MeV}$ (216 MeV)
for $\mu_B = 2.6\,\mathrm{GeV}$

Maybe 2SC vs. CFL??
Different condensates??

PT and GW

Gravitational waves from the binary NS merger

GW170817 (2017 August 17)

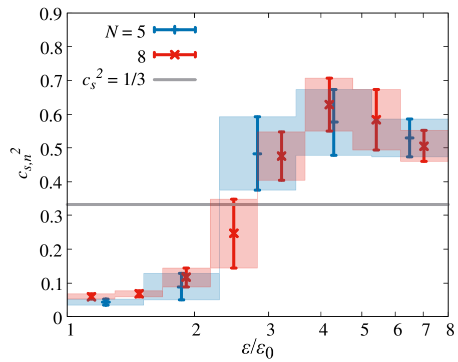


LIGO/Virgo (2018)

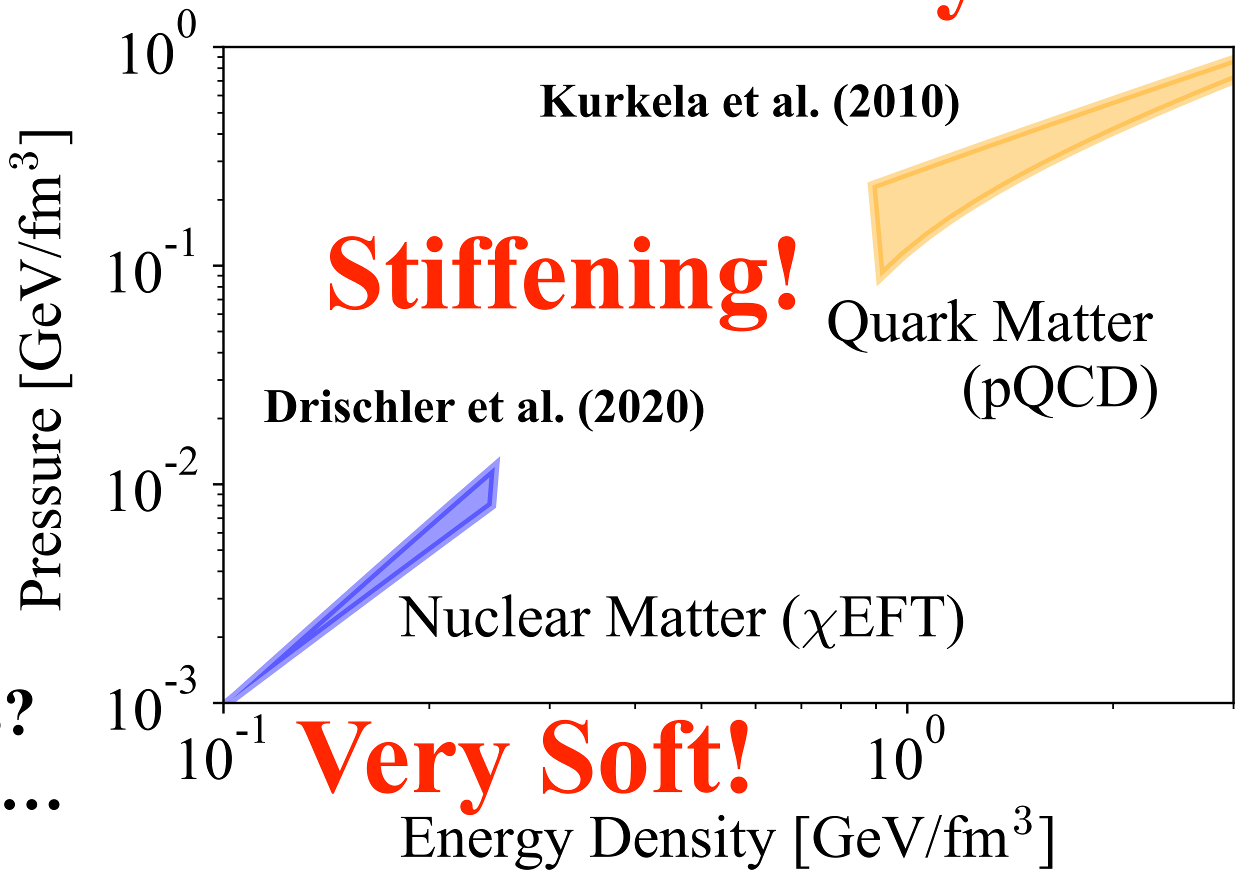
Softer than H4?
WFF1 okay?
APR4 preferred?

Favors soft EOS
at low density
(high density regions really constrained?)

PT and GW



Very Soft!

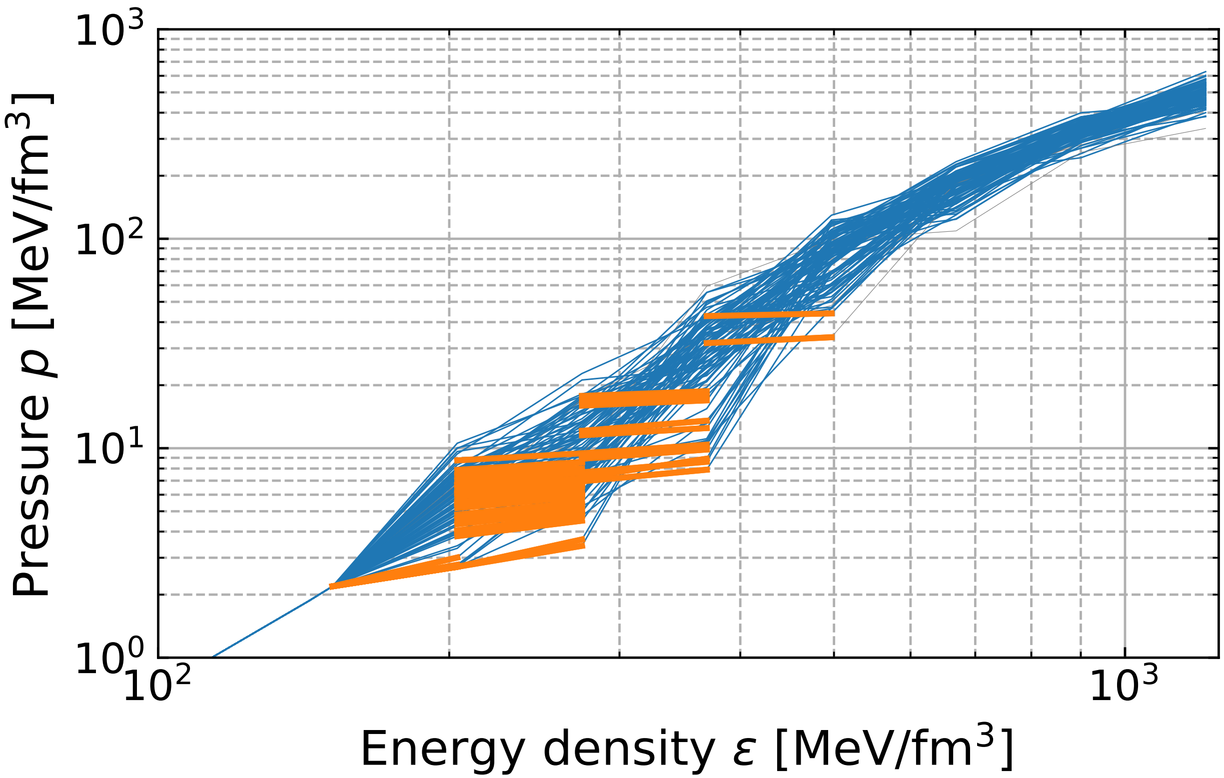


Necessary and likely behavior but how to explain it from nuclear physics? A big challenge…

PT and GW

1st-order phase transition is NOT excluded…

ML inferred EoSs: Fujimoto-Fukushima-Murase (2021)

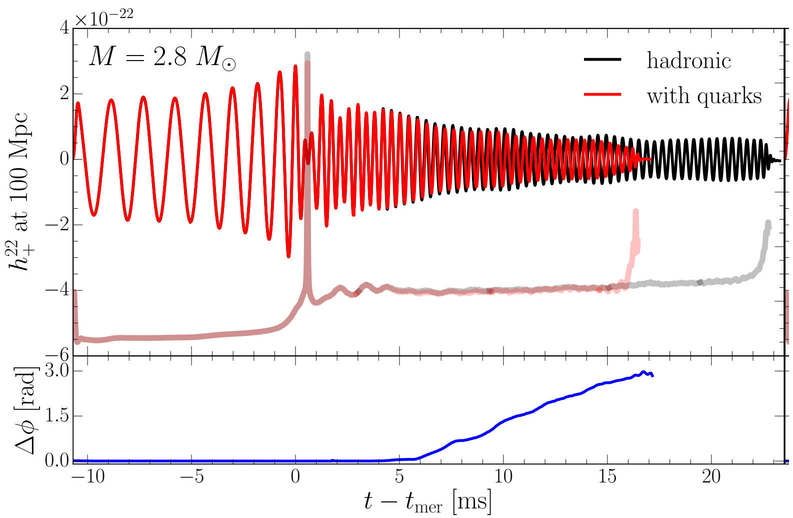


PT and GW

Can we see the phase transition with the GW signal?

Most-Papenfort-Dexheimer-Hanauske-Schramm-Stocker-Rezzolla (2018)

$\mathrm{CMF_Q}$: EOS with a strong-1st PT to Quark Matter (3~4 times n_{sat})
$\mathrm{CMF_H}$: EOS without quarks

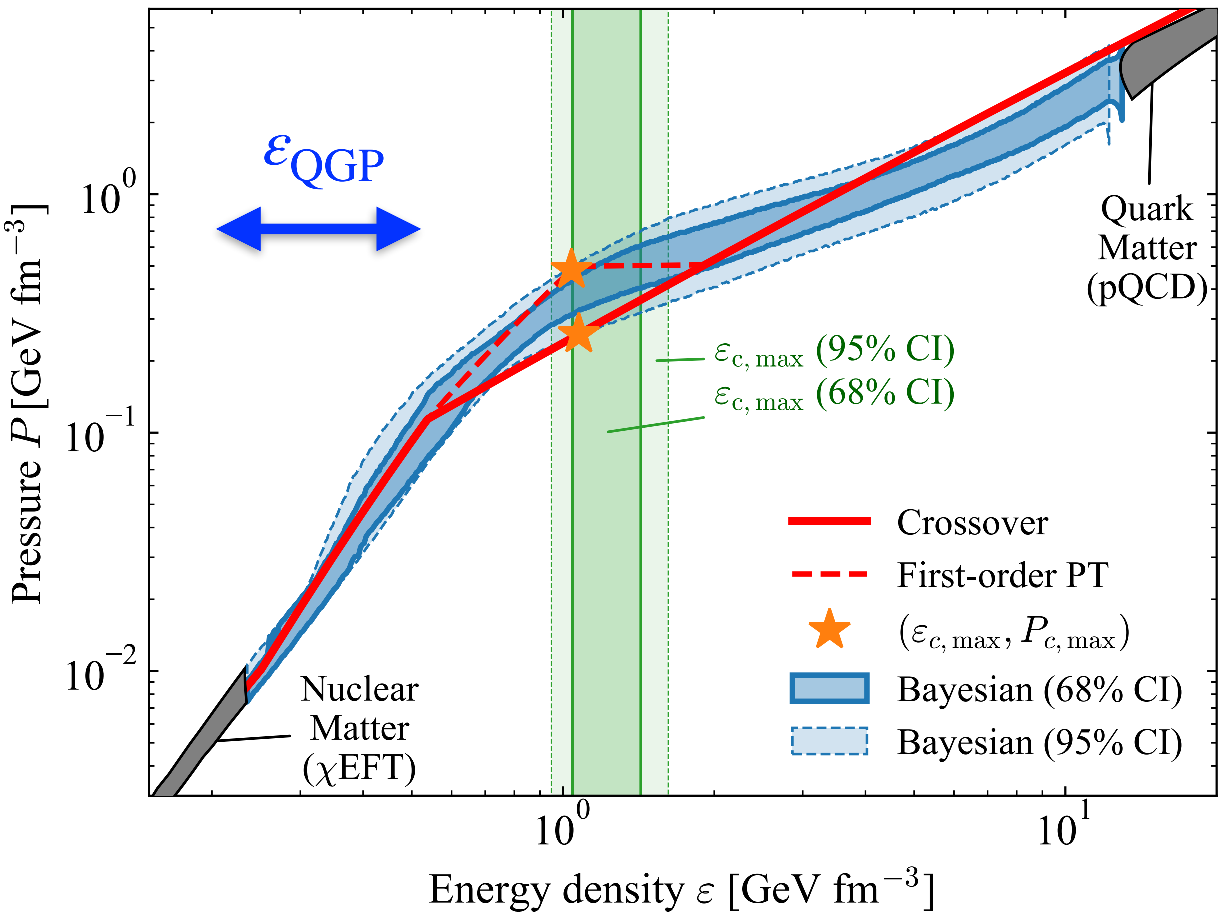


Quark matter shortens the lifetime of post-merger supramassive/hypermassive (uniform / differential) neutron star.

What if the transition is only a smooth crossover?

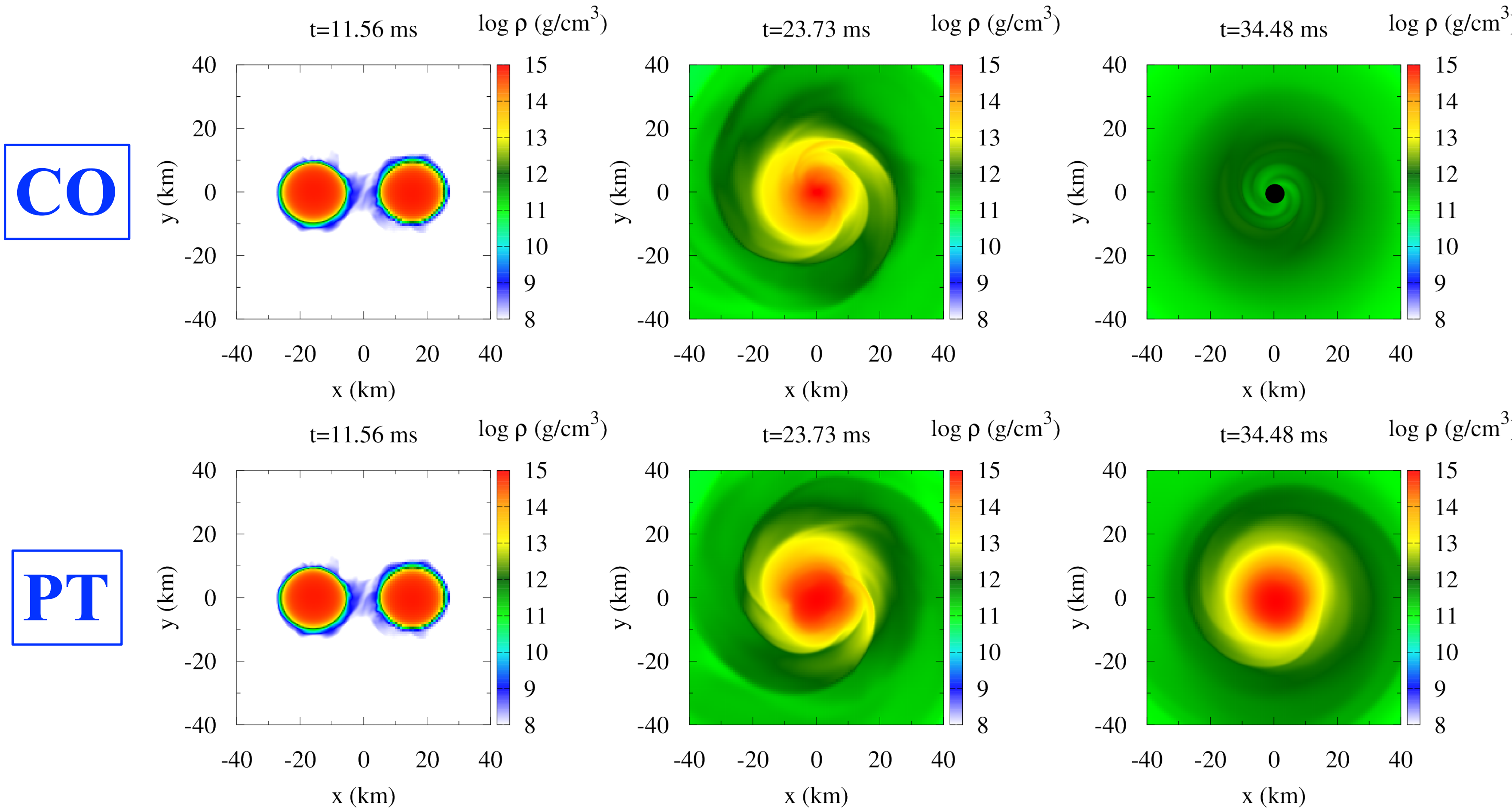
PT and GW

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)



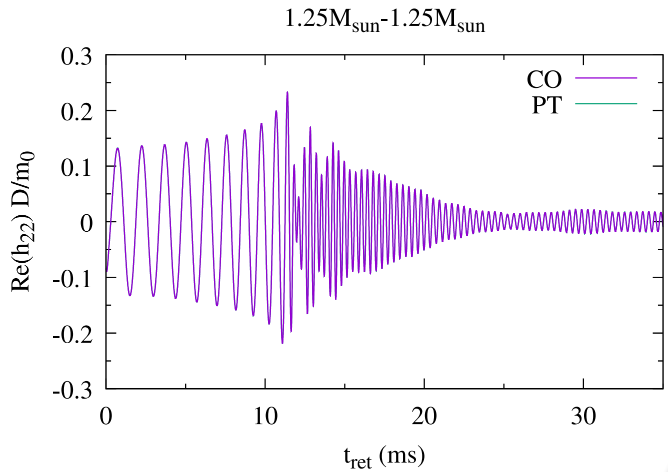
PT and GW

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)



PT and GW

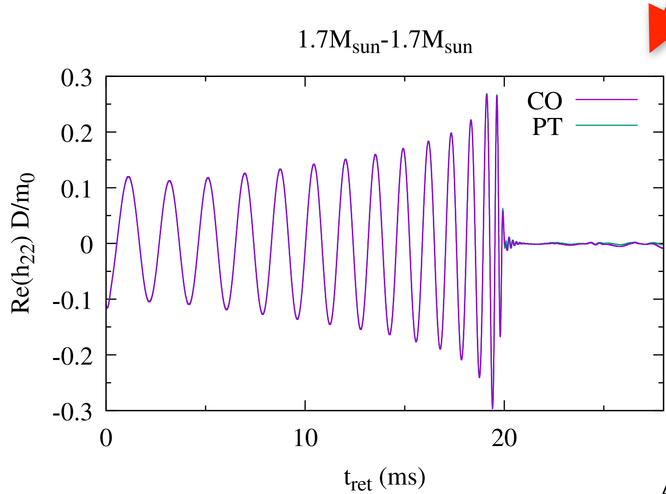
Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)

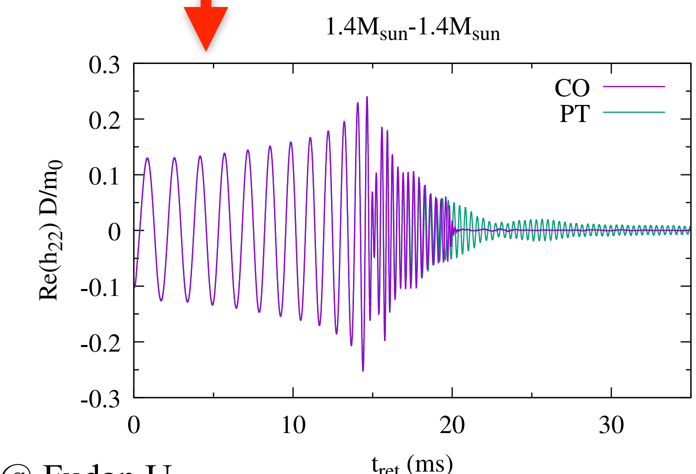


Light System (No BH)

Heavy System (Both BH)

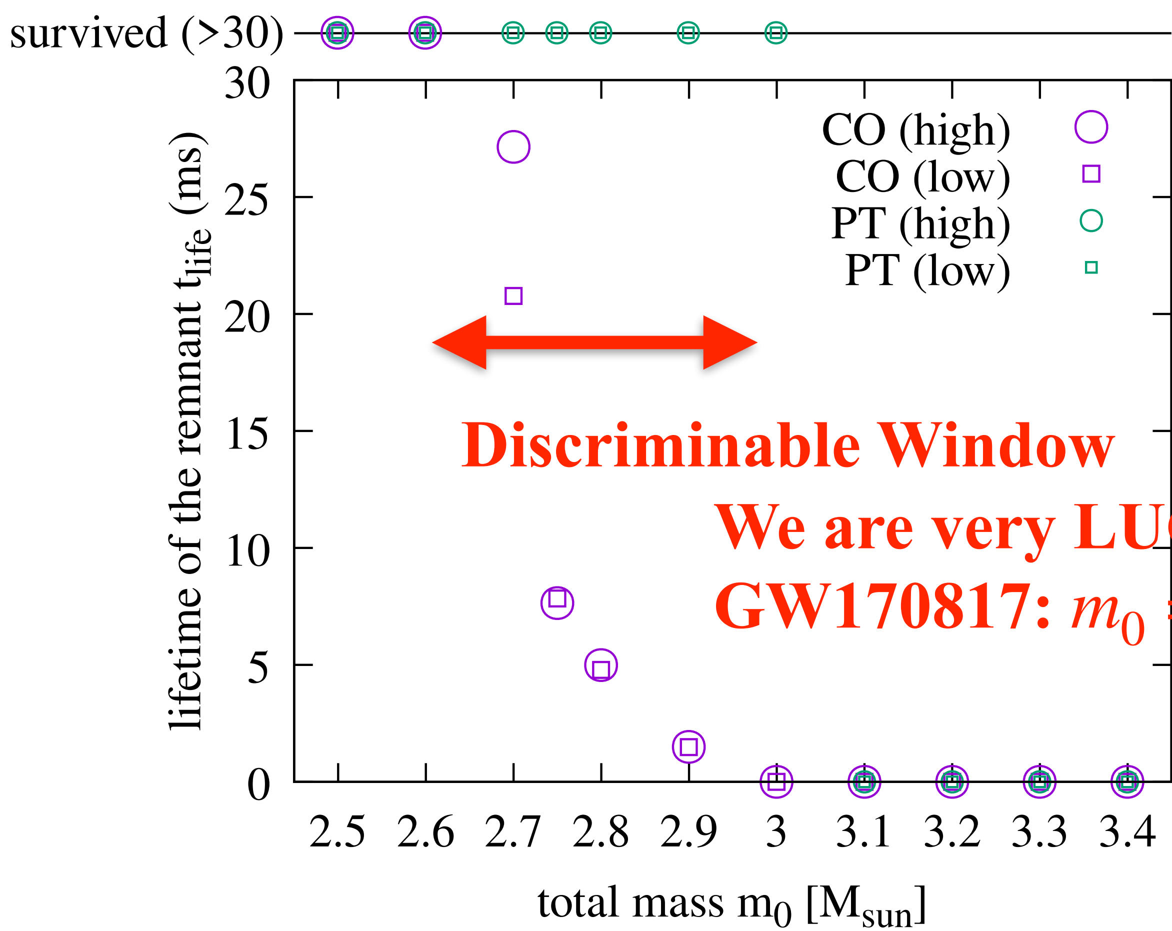
Like GW170817 (Discriminable!)





PT and GW

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)



Discriminable Window

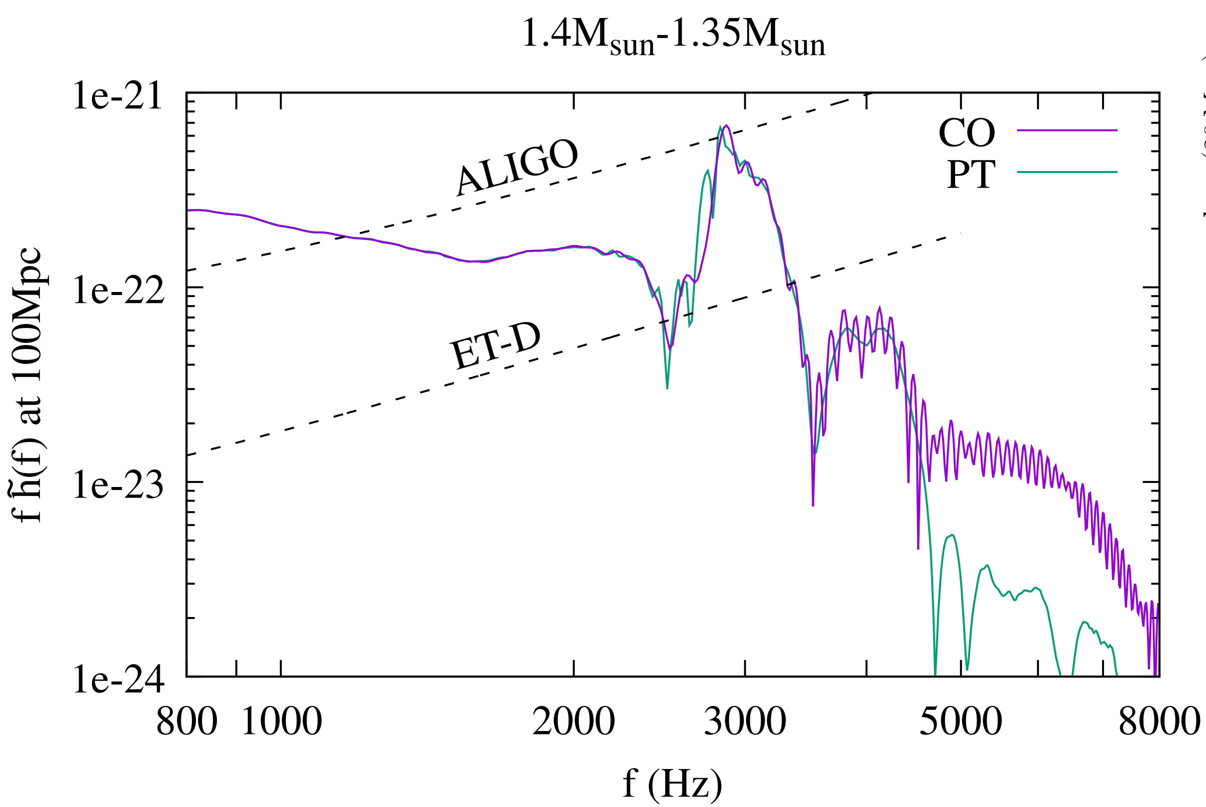
We are very LUCKY!

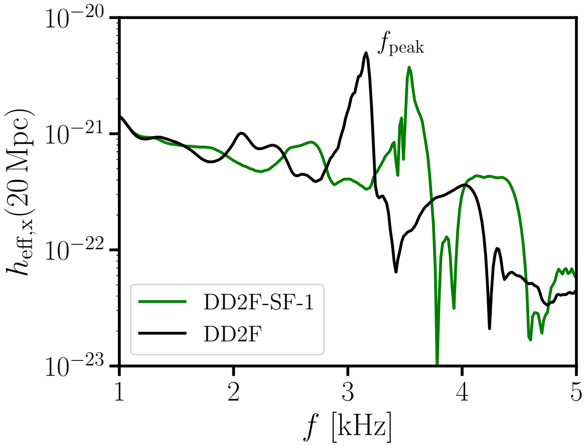
GW170817: $m_0 = 2.75 M_\odot$

PT and GW

Post-merger stage is very challenging to see:

↓ Our crossover case… no difference?



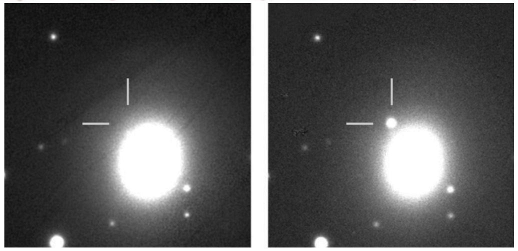


↑ Strong 1st-order PT scenario

Bausewein+ (2018)

Electromagnetic Counterpart

Kilonova brightness:
ejected mass $> 0.05 M_\odot$



AT 2017 gfo

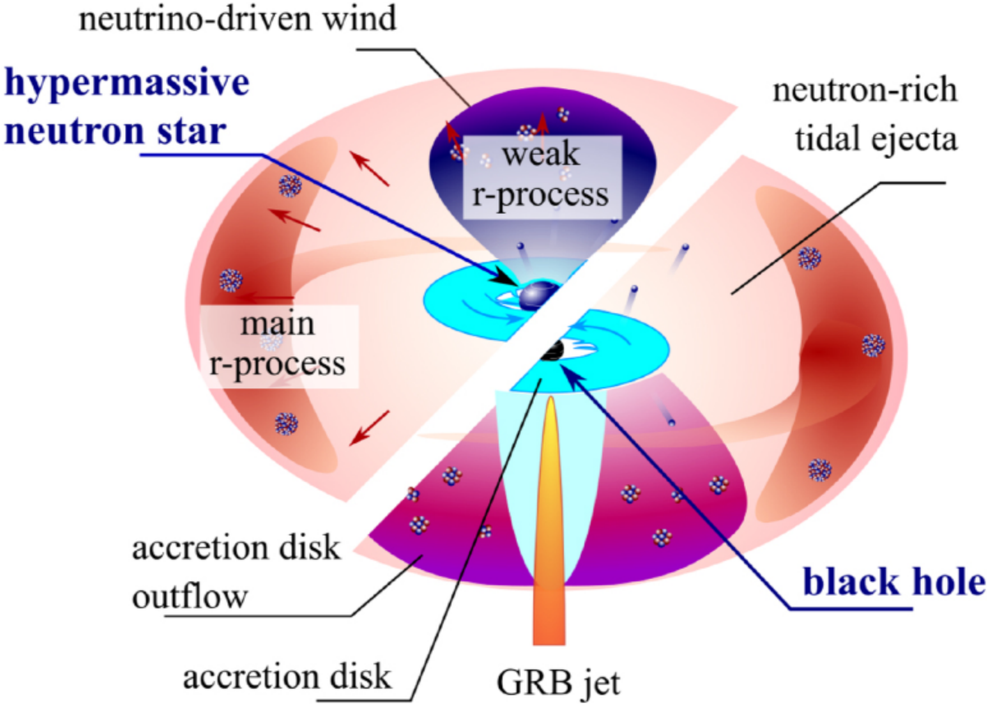
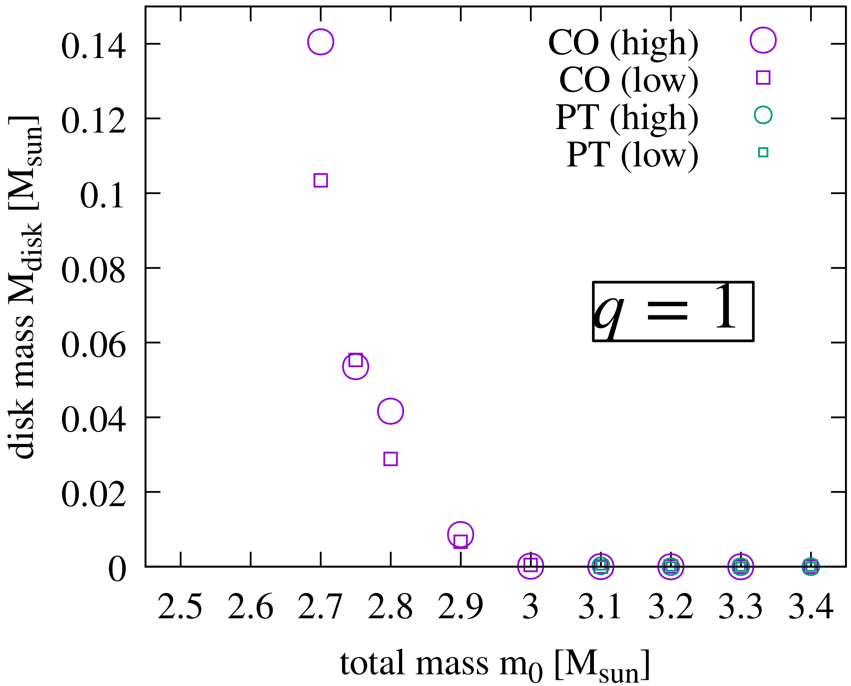
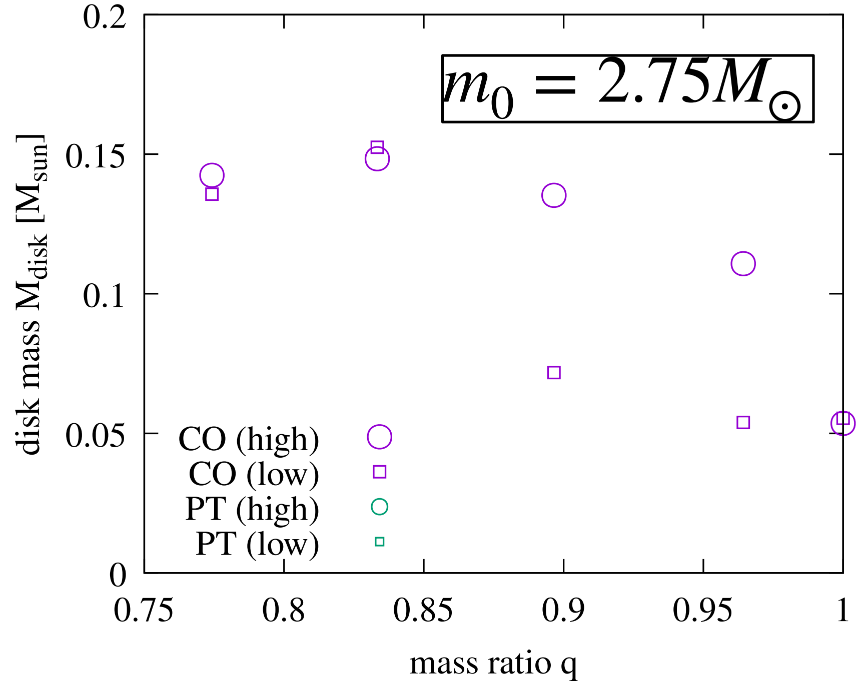


Illustration from Korobkin+ (2021)

Brightness and "color" depend on the EOS and the total mass.

Electromagnetic Counterpart





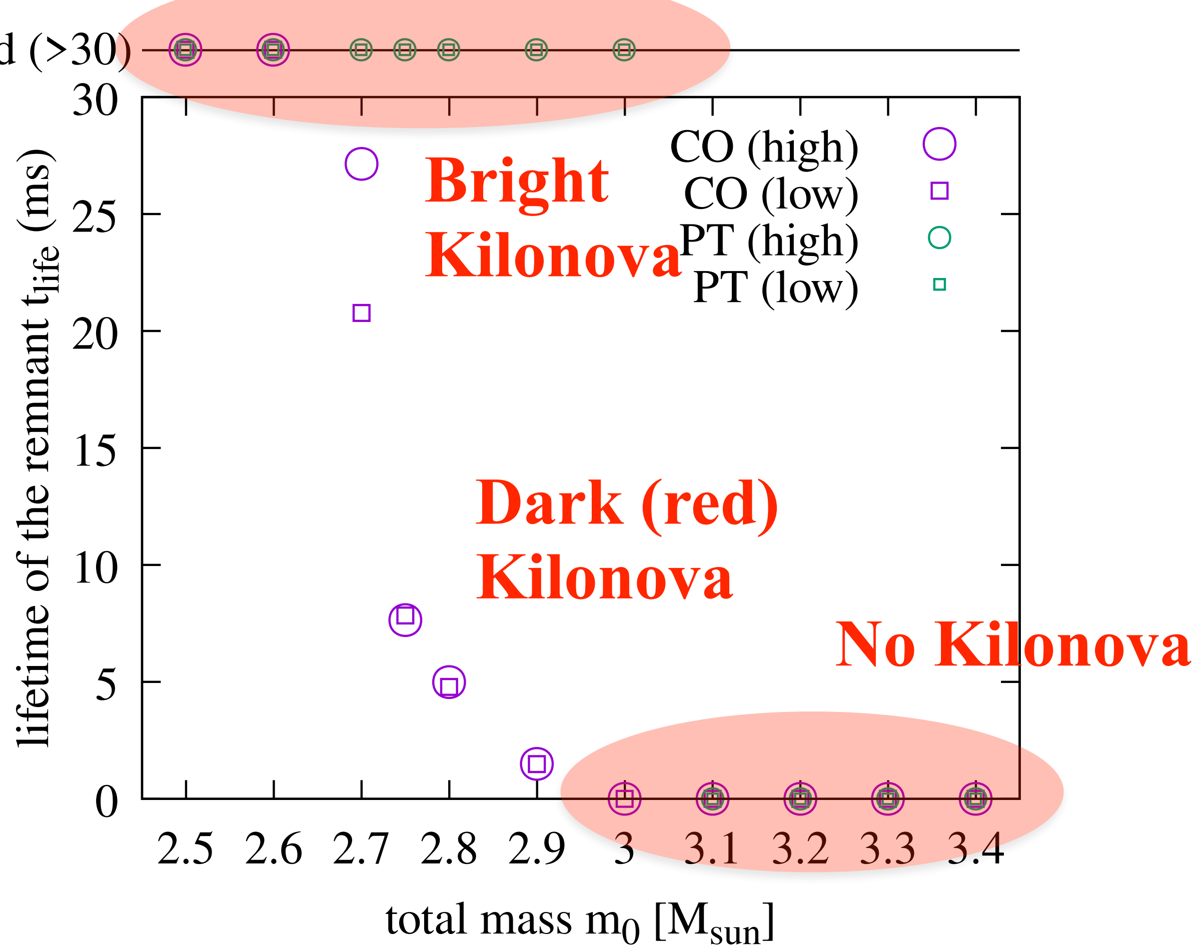
This situation (1.375+1.375) is already ruled out.

Asymmetric mass system can still be consistent.

Consistency with the kilonova tells us a lot!

Electromagnetic Counterpart

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)



In the near future, more data should be accumulated, and then, we can say much more!

Summary

- Speed of sound at high density may increase above the conformal value. This could be confirmed by the heavy-ion collision.
- Trace anomaly is going negative and it implies the presence of some condensates. Color-super?
- QCD phase transition is detectable through the GW signal even if it is a smooth crossover.
- GW170817 was such a fine-tuned event!!!