Nuclear Science
Computing Center at CCNU

Lattice QCD prediction of Pion & Kaon electromagnetic form factors at very large Q^2:

Testing Factorization in Exclusive Processes

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in collaboration with
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arXiv: 2404.04412

第六届重味物理与量子色动力学研讨会
April 19-23, 2024@青岛

Hadron Electromagnetic (EM) form factor

Robert Hofstadter
Nobel Laureate,1961

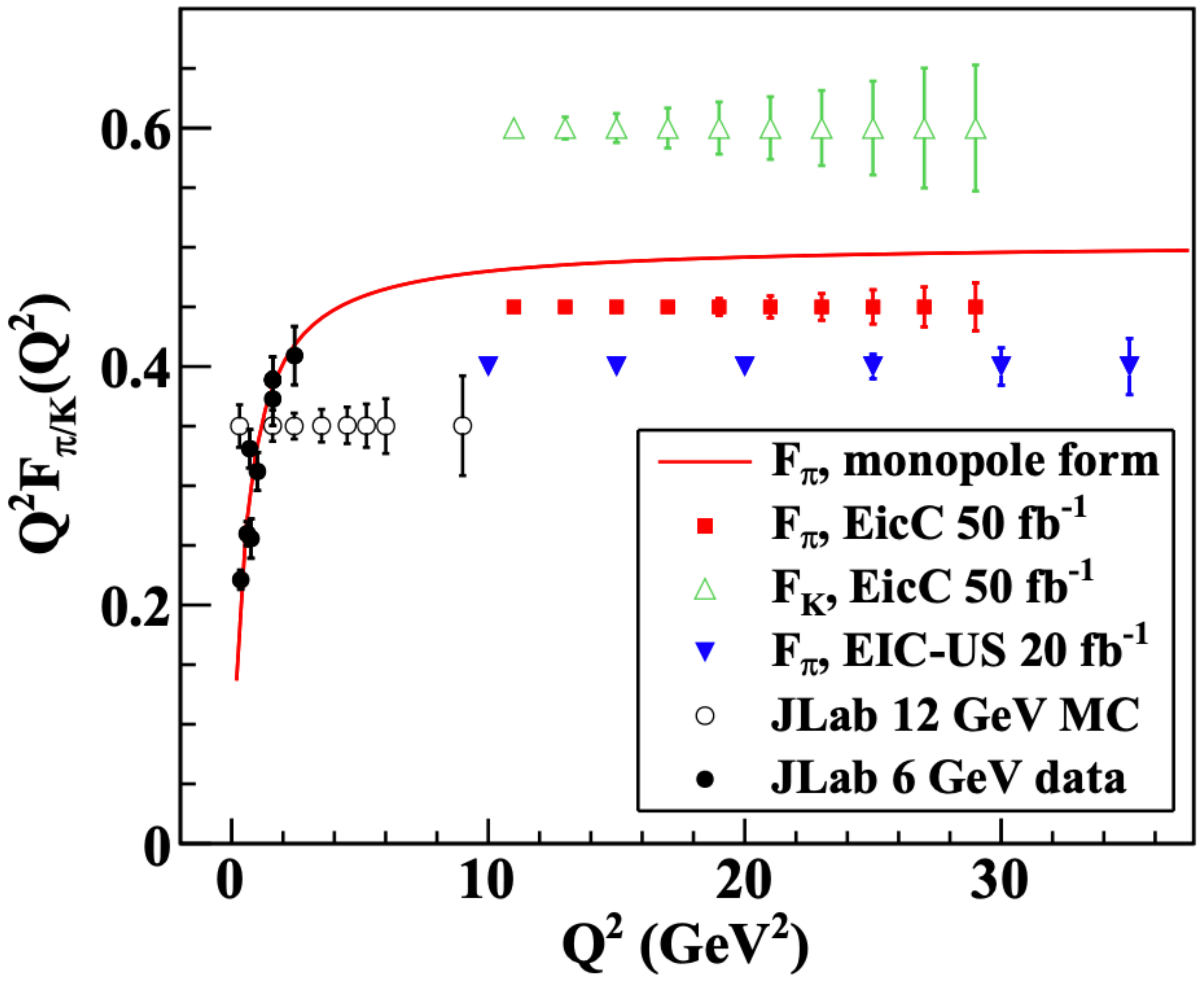
$$< H(P_1)|J_\mu|H(P_2) > = (P_1 + P_2)_\mu F_H(Q^2)$$

- Insights into hadron structure, i.e. on the charge distribution

- Together with PDF produces General Parton Distribution (GPD), i.e. a 3-d image of hadron

- Experiments: Jlab, EiC, EicC

- pQCD, BSE, DSE, lattice QCD...



EicC white paper, arXiv:2102.09222

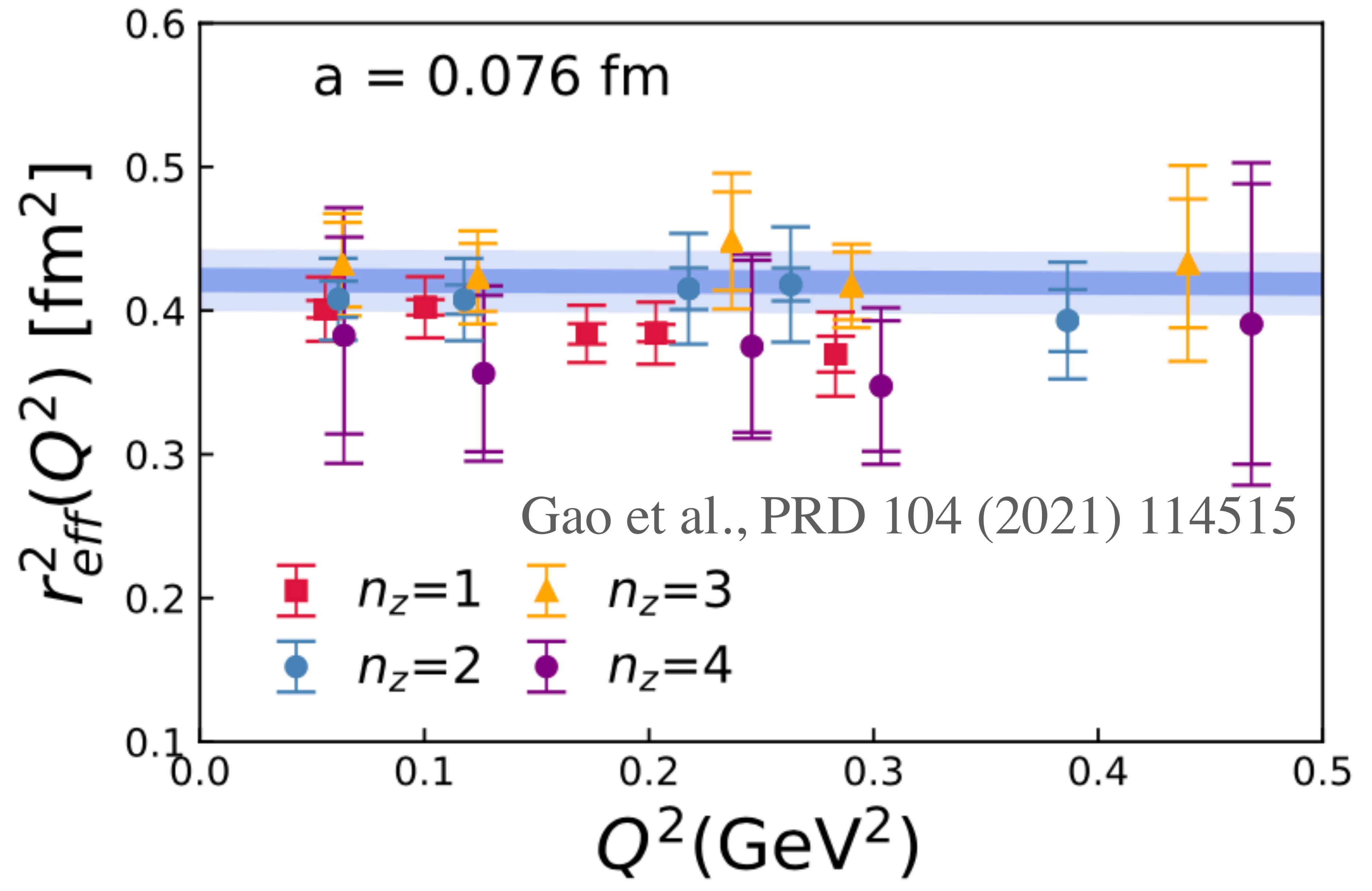
Pion/kaon EM form factors

Small Q^2 limit: hadronic picture

- Vector Meson Dominance -> Charge radius

$$r^2_{eff}(Q^2) = \frac{6(1/F_\pi(Q^2) - 1)}{Q^2}.$$

$$\langle r_\pi^2 \rangle = 0.42(2)\ \text{fm}^2, \langle r_\pi^2 \rangle_{PDG} = 0.434(5)\ \text{fm}^2$$



Large Q^2 limit: partonic picture

$$Q^2 F_M(Q^2) \approx 16\pi \alpha_s(Q^2)\, f_M^2\, \omega_M^2(Q^2), \quad \omega_M^2(Q^2) = e_{\bar{q}}\omega_{\bar{q}}^2(Q^2) + e_u\omega_u^2(Q^2)$$

Lepage & Brodsky, 79', 80'
Efremov & Radyushkin 80'

$$\omega_f = \frac{1}{3}\int_0^1 dx q_f(x) \phi_M(x, Q^2)$$

leading-twist parton distribution amplitude (DA)

- Asymptotic DA: $\phi_M(x, Q^2 \to \infty) = 6x(1-x)$
- DA from LQCD: pion & kaon etc.

张其安, 12:00, April 21
华俊, 17:10, April 21

J. Hua et al. [LPC], Phys.Rev.Lett. 129 (2022) 13
Gao et al., PRD 106 (2022) 074505, G. Bali et al., JHEP 08 (2019) 065, …

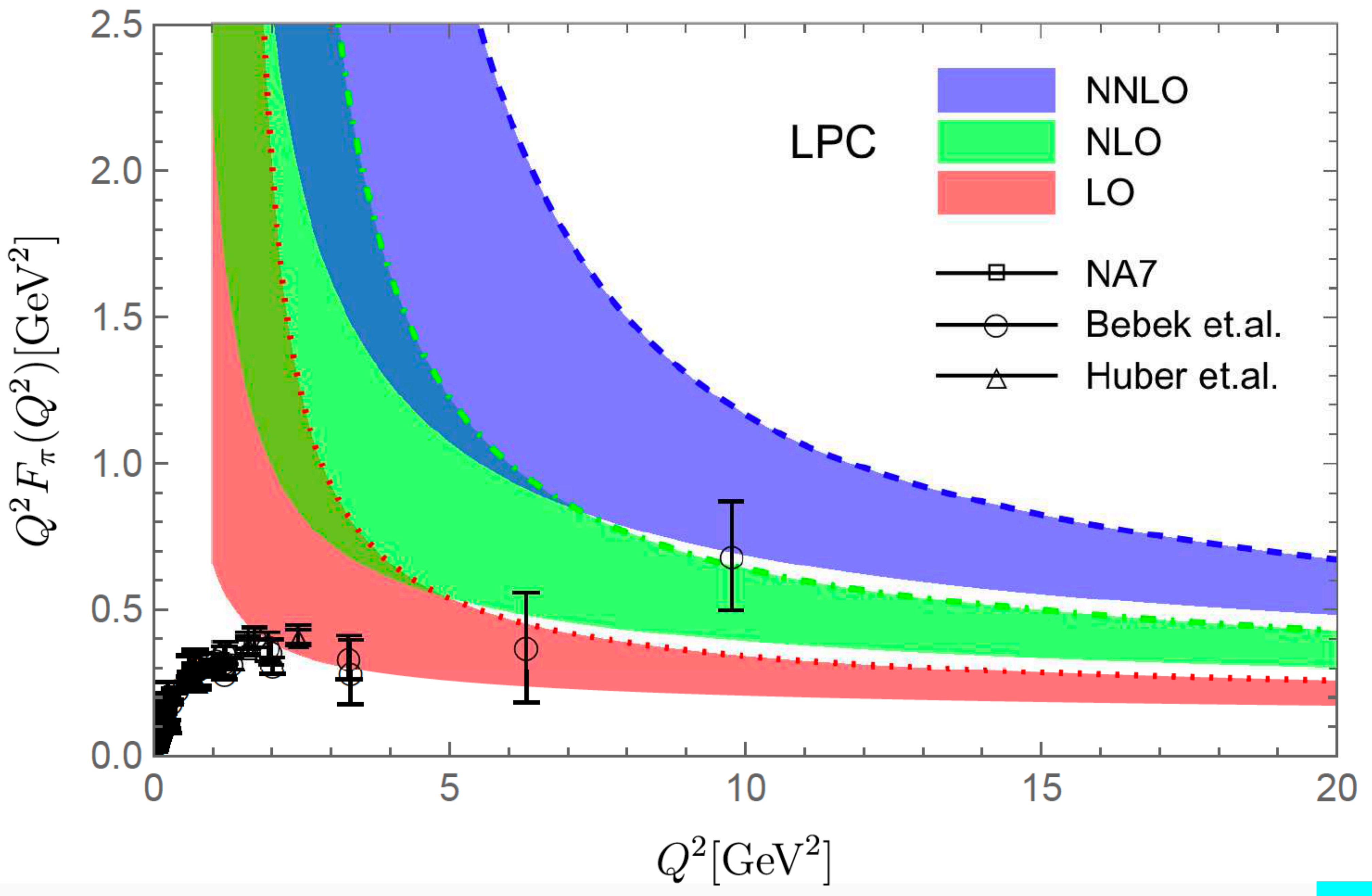
QCD factorization for hard exclusive processes

At leading twist, the collinear factorization of EM form factors

Farrar & Jackson, PRL 79'
Lepage & Brodsky, PRD 80'
Efremov & Radyushkin, PLB 80'

$$F_M(Q^2) = \int_0^1 \int_0^1 dxdy\ \phi_M^*(y, \mu_F^2)\ T_H(x, y, Q^2, \mu_R^2, \mu_F^2)\ \phi_M(x, \mu_F^2)$$

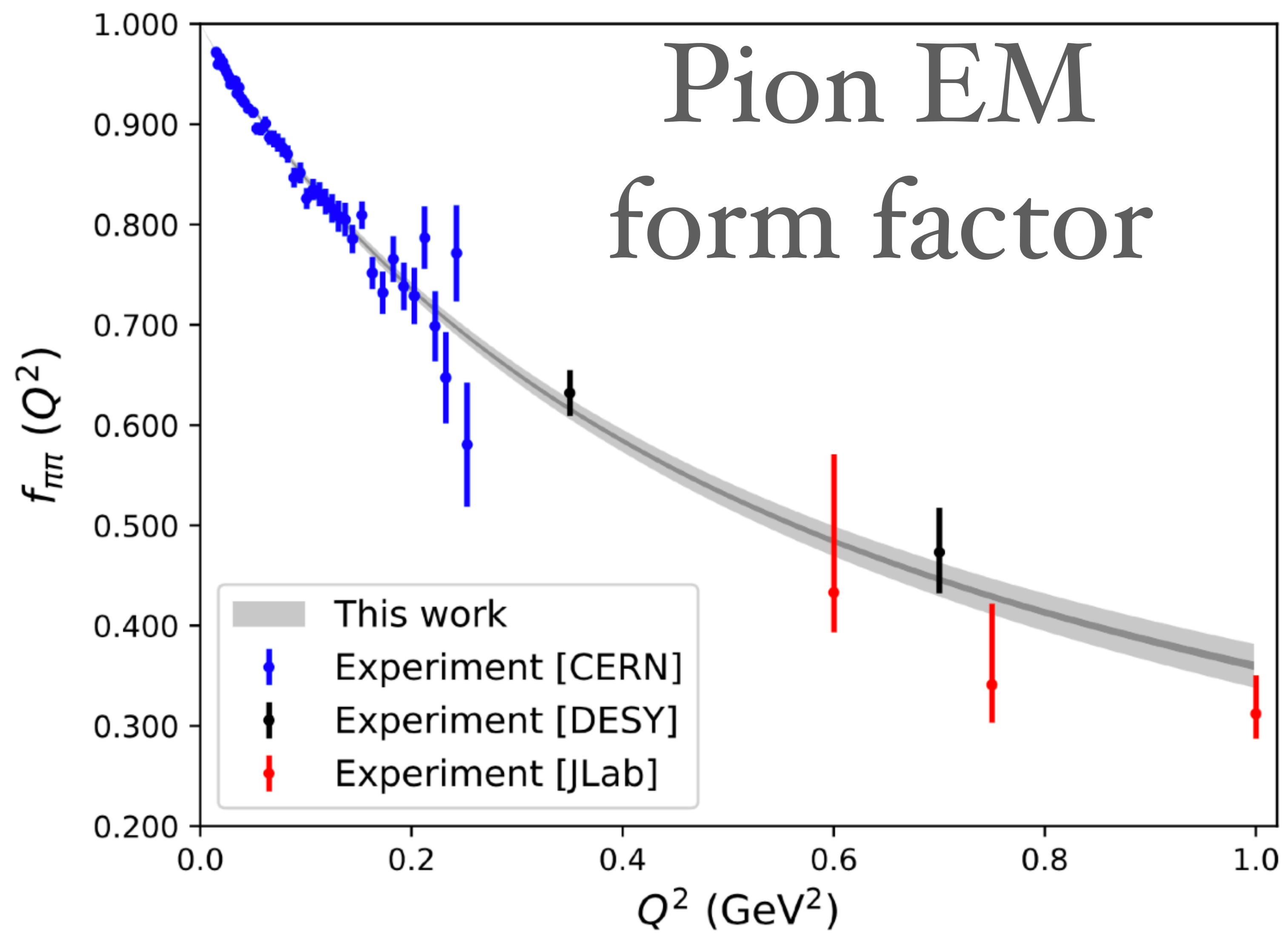
DA: Non-perturbative physics — Hard-process kernel obtained in pQCD — DA: Non-perturbative physics



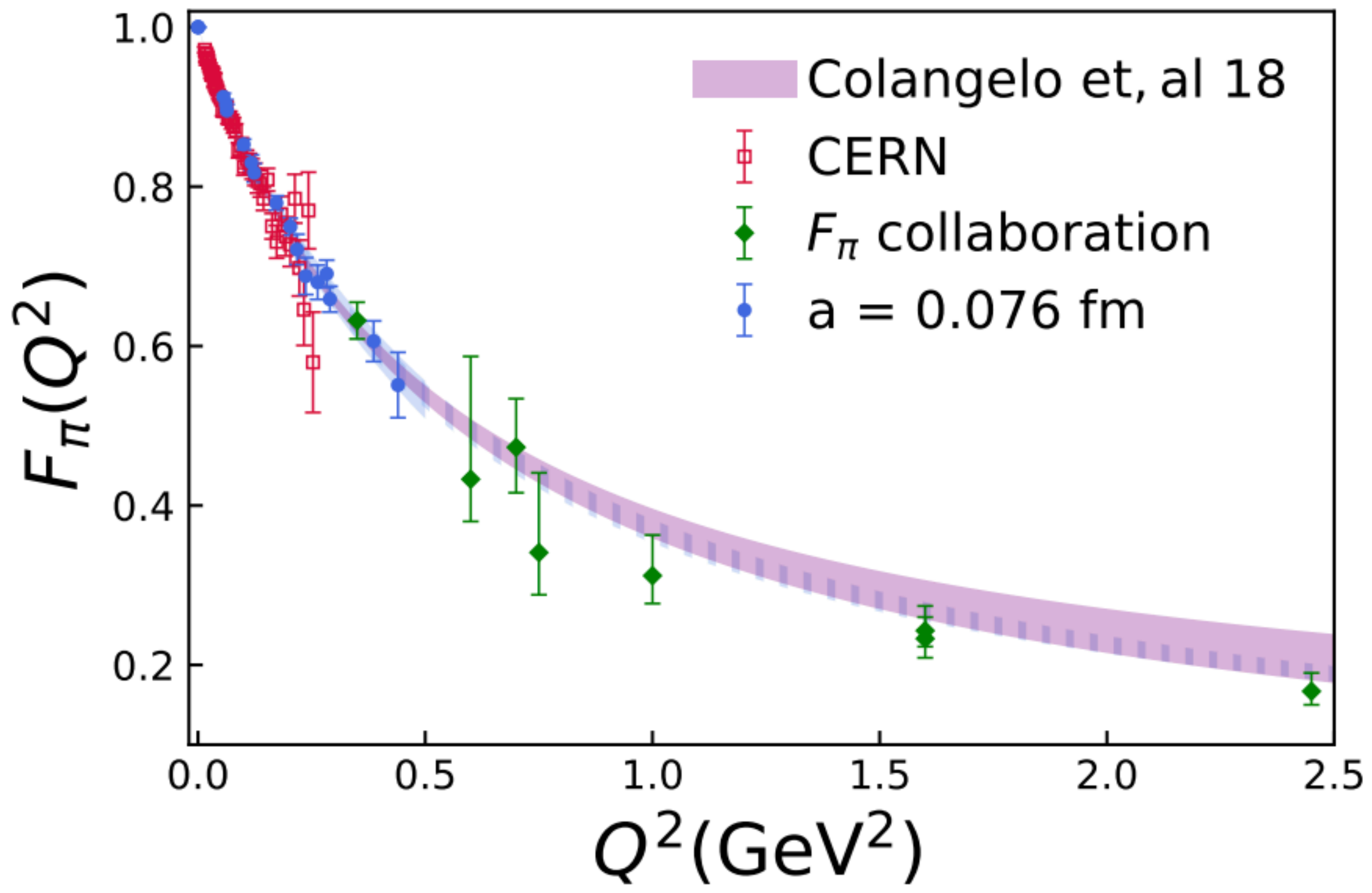
Chen, Chen, Feng & Jia, arXiv:2312.17228 陈龙斌, 16:50, April 21

- NA7: $Q^2 \lesssim 0.25$ GeV^2, elastic scattering of pion from atomic electron NPB 277 (1986)168
- Huber et al. (Jlab F_π collaboration): $Q^2 \lesssim 2.5$ GeV^2, PRC 78 (2008) 045203
- Bebek et al. (Cornell): $Q^2 \lesssim 10$ GeV^2, large statistical and systematic uncertainties PRD 17 (1978) 1693

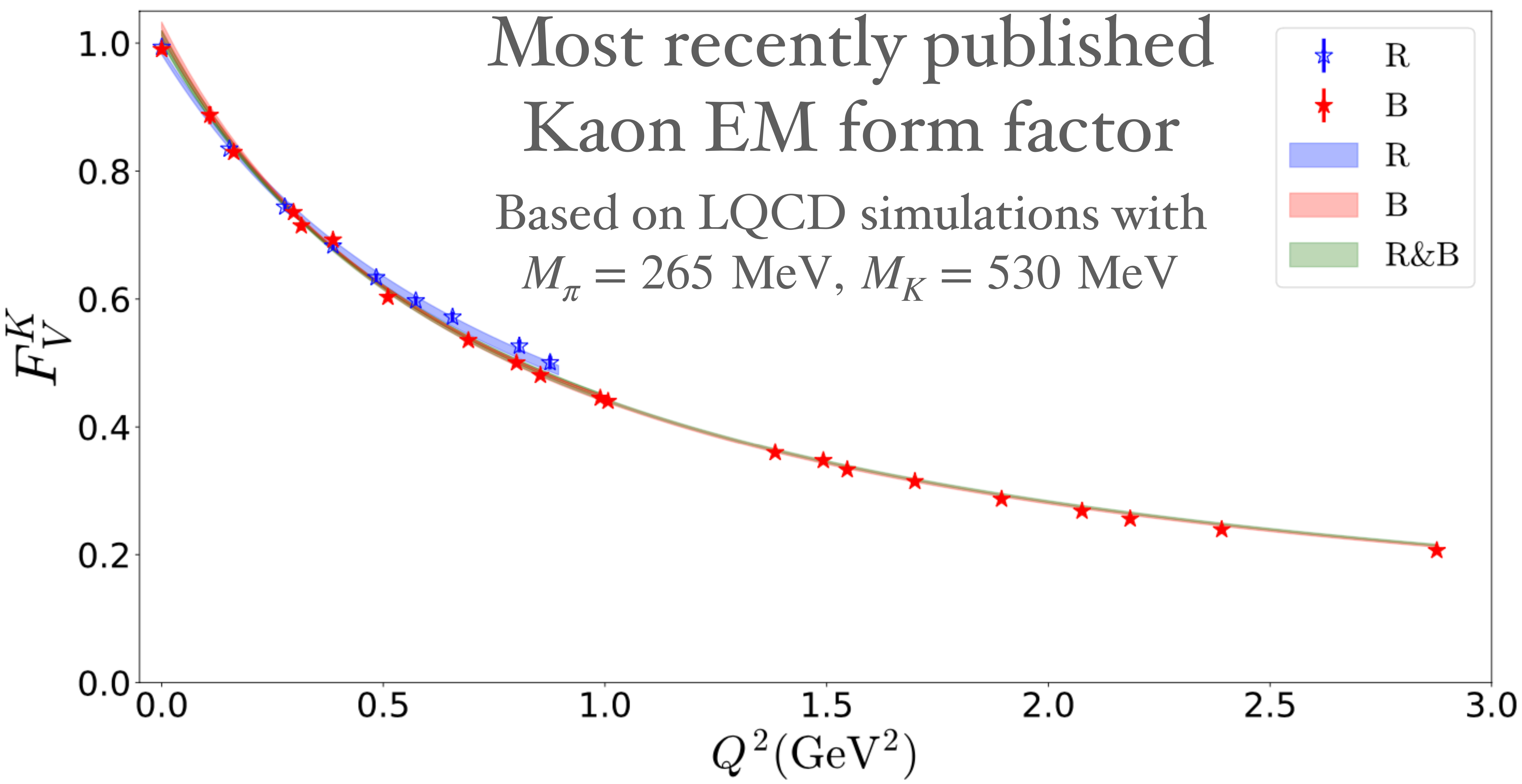
Current status: pion/kaon EM form factors from Lattice QCD



高翔 et al.Tsinghua-BNL-ANL, PRD 104 (2021) 114515



G. Wang et al,, [χQCD], PRD 104 (2021) 074502

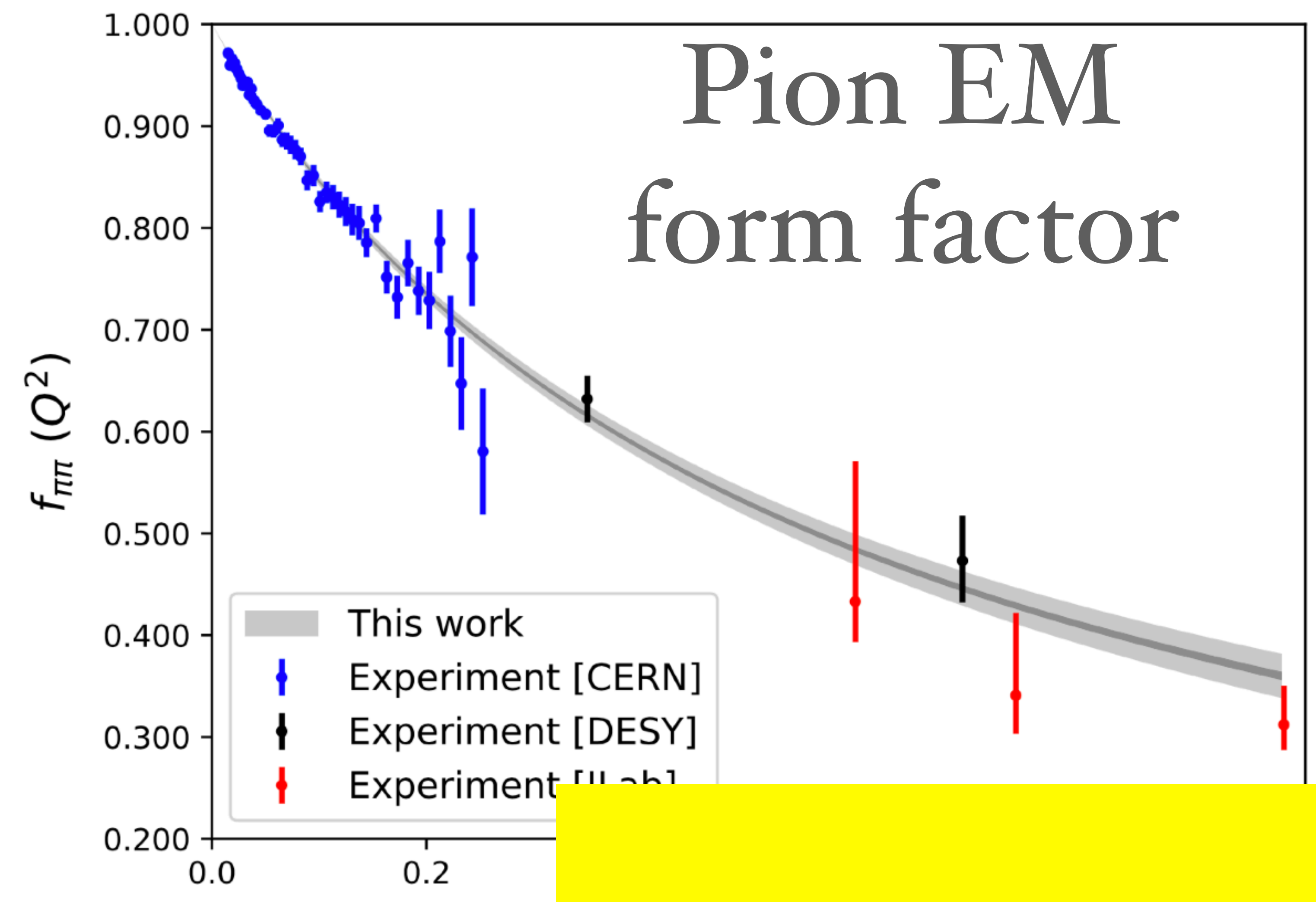


Alexandrou et al., [ETMC], Phys.Rev.D 105 (2022) 5, 054502

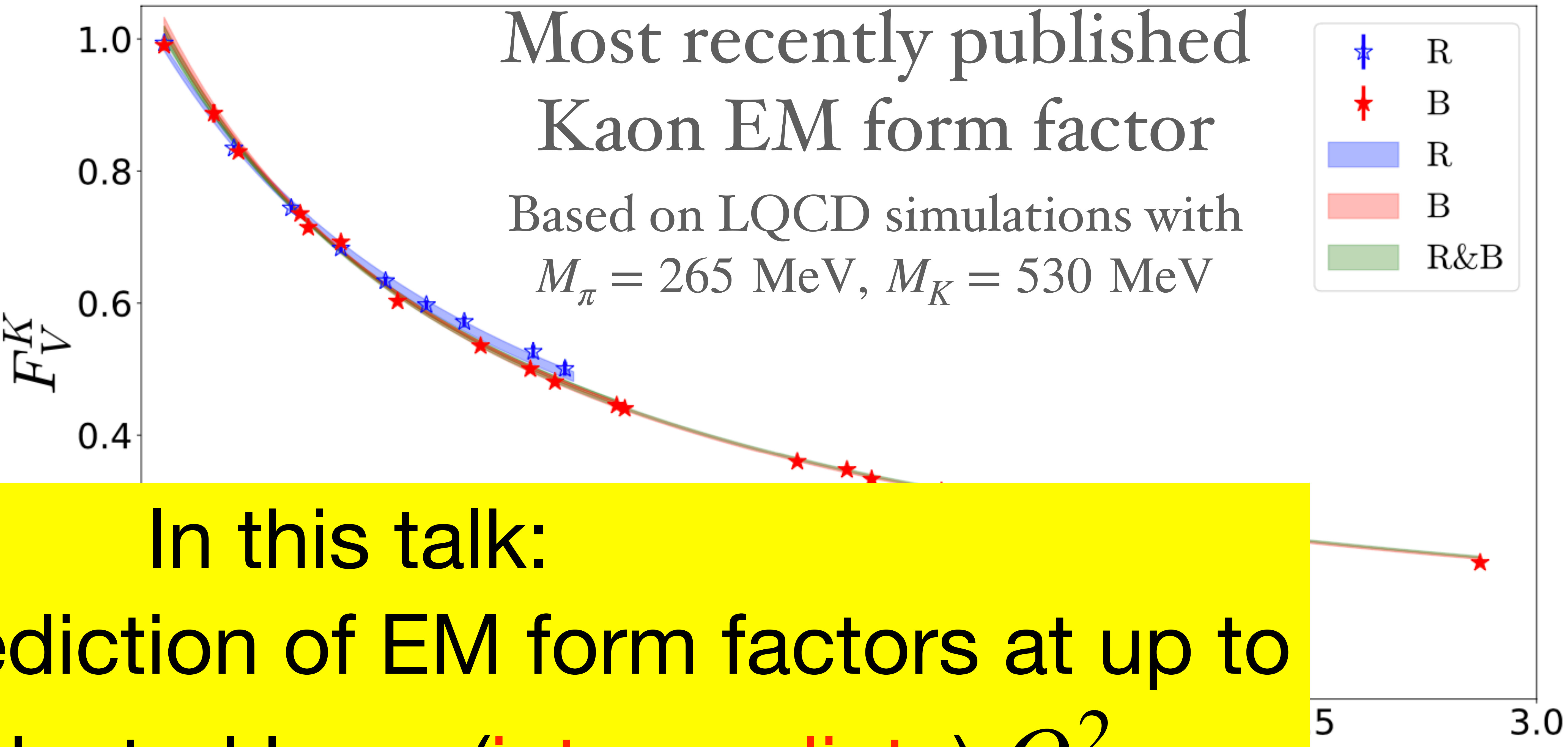
Many computations on the pion form factor, but much less on kaon

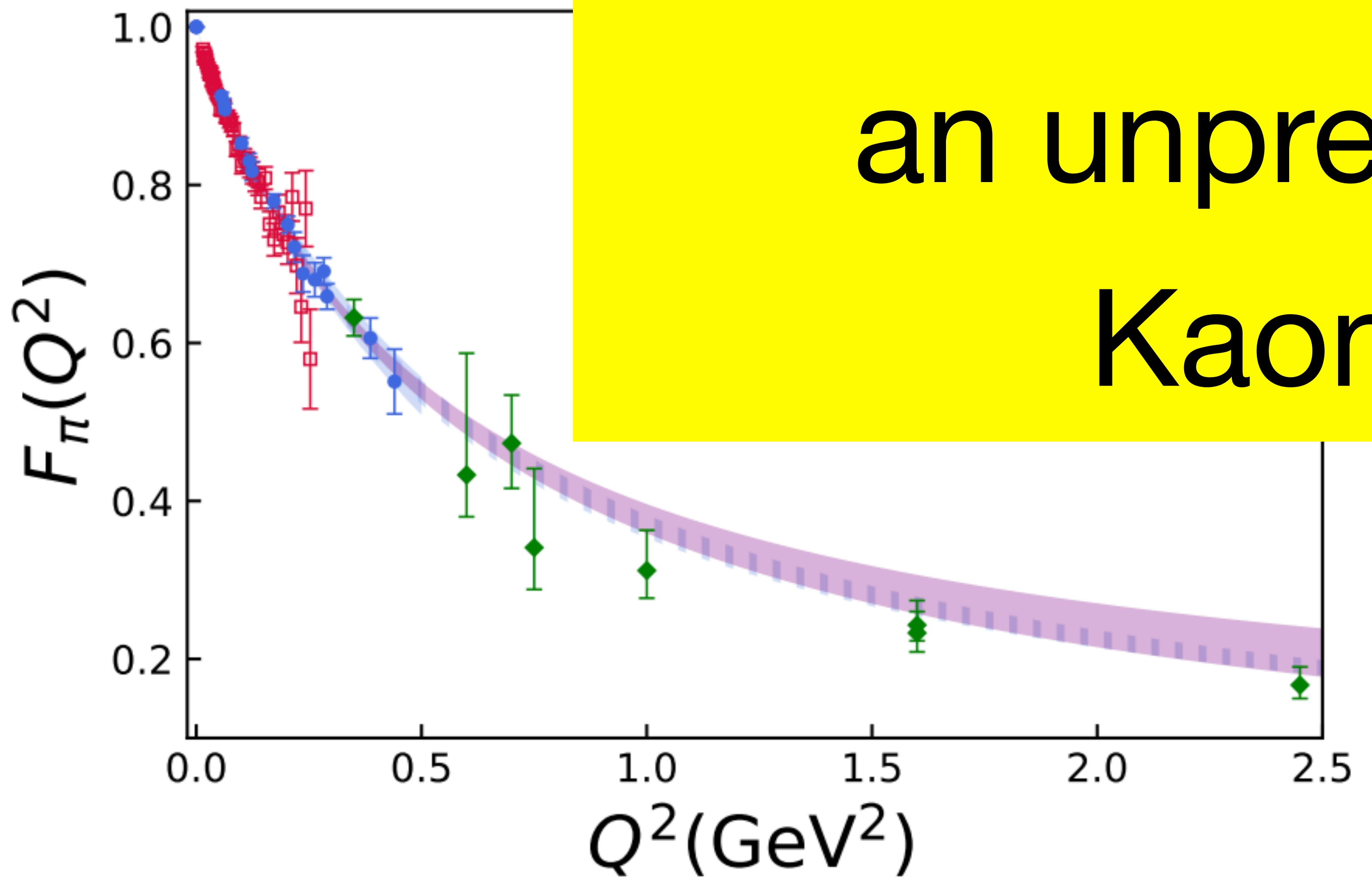
Mostly restricted to $Q^2 \lesssim 3$ GeV^2

Current status: pion/kaon EM form factors from Lattice QCD



高翔 et al.Tsinghua-BN





In this talk:
Lattice QCD prediction of EM form factors at up to an unprecedented large (intermediate) Q^2:
Kaon ~28 GeV^2 , Pion ~ 10 GeV^2

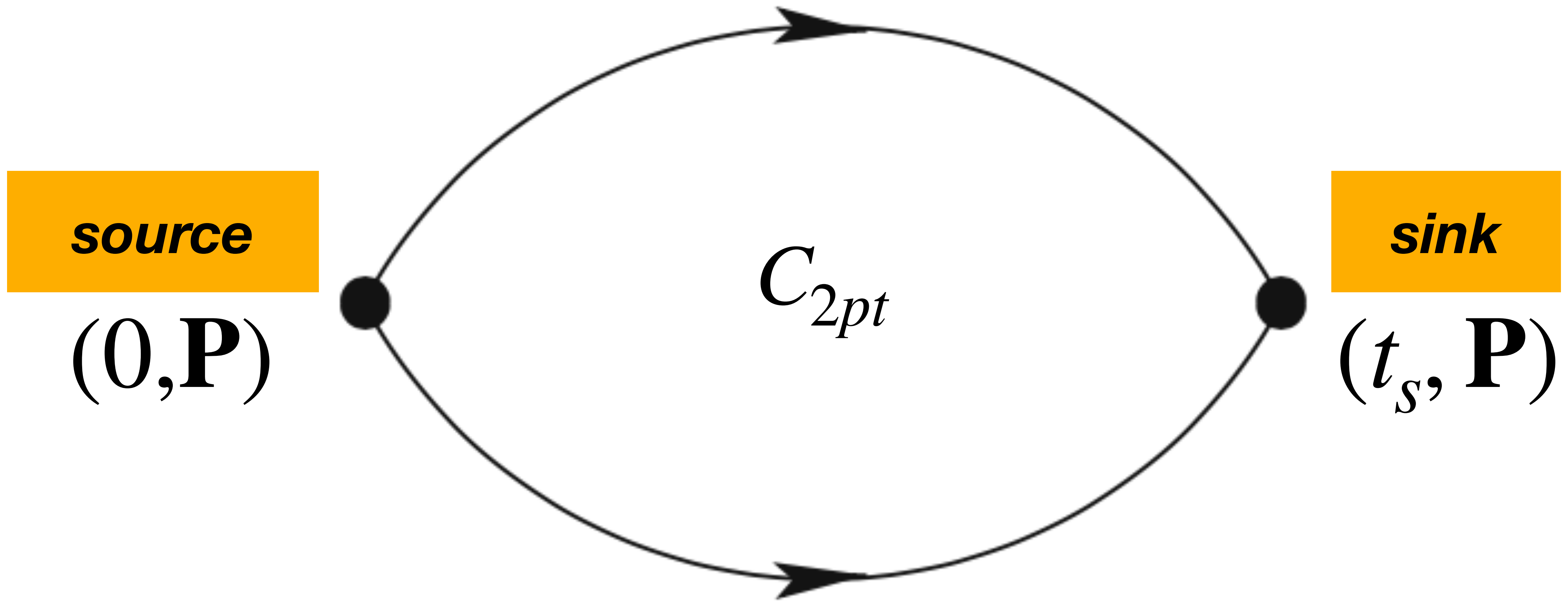
Many computations on the pion form factor, but much less on kaon

Mostly restricted to $Q^2 \lesssim 3$ GeV^2

G. Wang et al,, [χQCD], PRD 104 (2021) 074502

Kaon at nonzero momentum

- Two point kaon correlation function



$$C_{2\mathrm{pt}}(\mathbf{P}, t_s) = \left\langle [K(\mathbf{P}, t_s)][K(\mathbf{P},0)]^\dagger \right\rangle$$

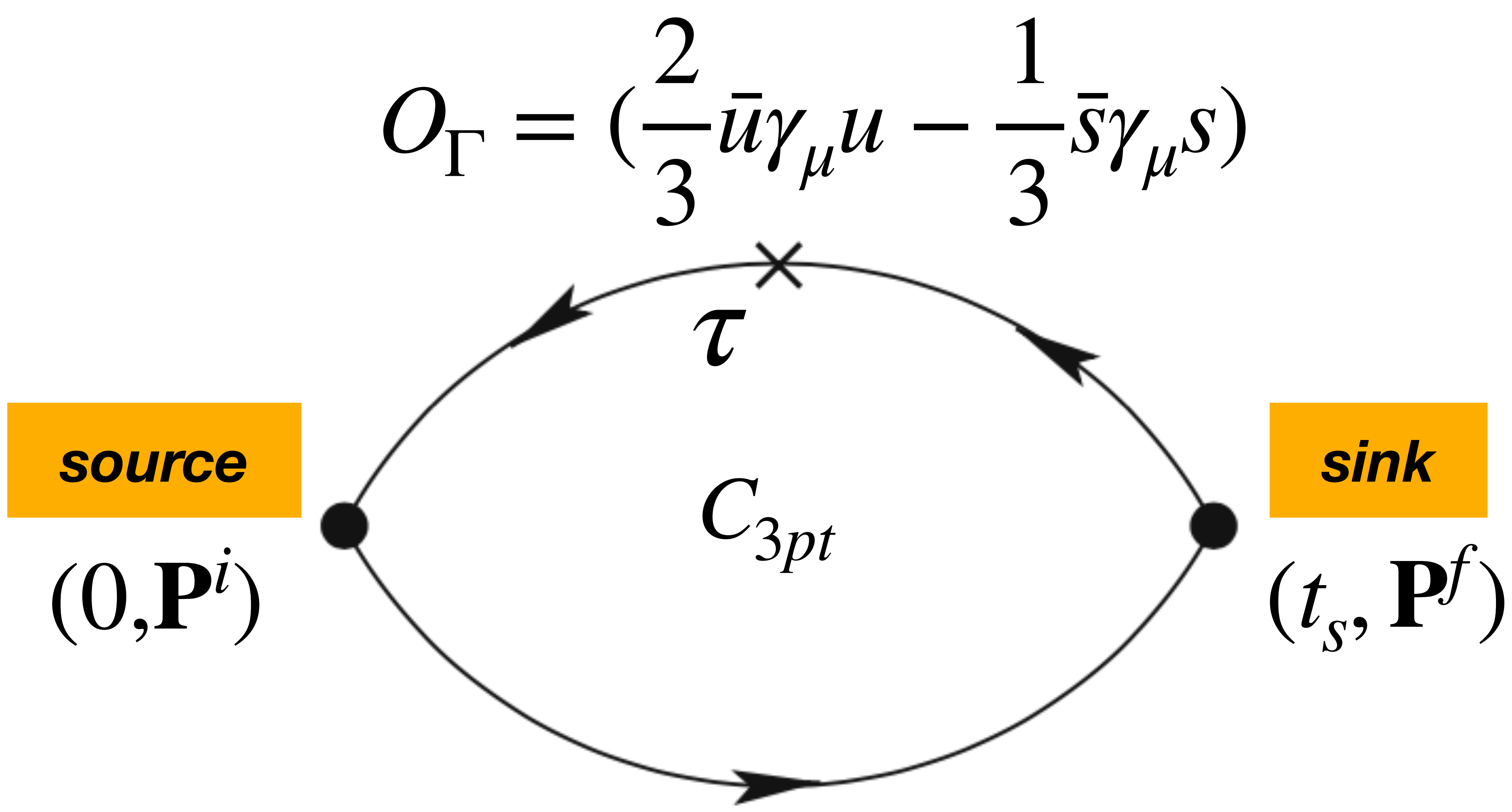
$$K(\mathbf{P}, t) = \sum_{\mathbf{x}} \bar{s}(\mathbf{x}, t)\gamma_5 u(\mathbf{x}, t) e^{-i\mathbf{P}\cdot\mathbf{x}}$$

$$\mathbf{P} = \frac{2\pi}{N_\sigma}\mathbf{n}\, a^{-1}$$

- Determine energy of states from the energy decomposition:

$$C_{2\mathrm{pt}}(\mathbf{P}, t_s) = \sum_{n=0}^{N_{\mathrm{state}}-1} |\langle \Omega | K_S | n; \mathbf{P} \rangle|^2 (e^{-E_n t_s} + e^{-E_n(aL_t - t_s)})$$

Three point correlation function



$$C_{3\text{pt}}(\mathbf{P}^f, \mathbf{P}^i, \tau, t_s) = \left\langle [K_{S^f}(\mathbf{P}^f, t_s)] O_\Gamma(\mathbf{q}, \tau) [K_{S^i}(\mathbf{P}^i, 0)]^\dagger \right\rangle$$

$$\mathbf{P}^i = \mathbf{P}^f - \mathbf{q}$$

$$Q^2 = -(\mathbf{P}^i - \mathbf{P}^f)^2$$

$$C_{3\text{pt}}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) = \sum_{m,n} \langle \Omega | K_{S^f} | m; \mathbf{P}^f \rangle \langle m; \mathbf{P}^f | O_\Gamma | n; \mathbf{P}^i \rangle \langle n; \mathbf{P}^i | K^\dagger_{S^i} | \Omega \rangle \times e^{-(t_s - \tau) E^f_m} e^{-\tau E^i_n}$$

EM form factor:

Bare matrix element of kaon ground state $F^B(Q^2) = \langle 0; \mathbf{P}^f | O_\Gamma | 0; \mathbf{P}^i \rangle$

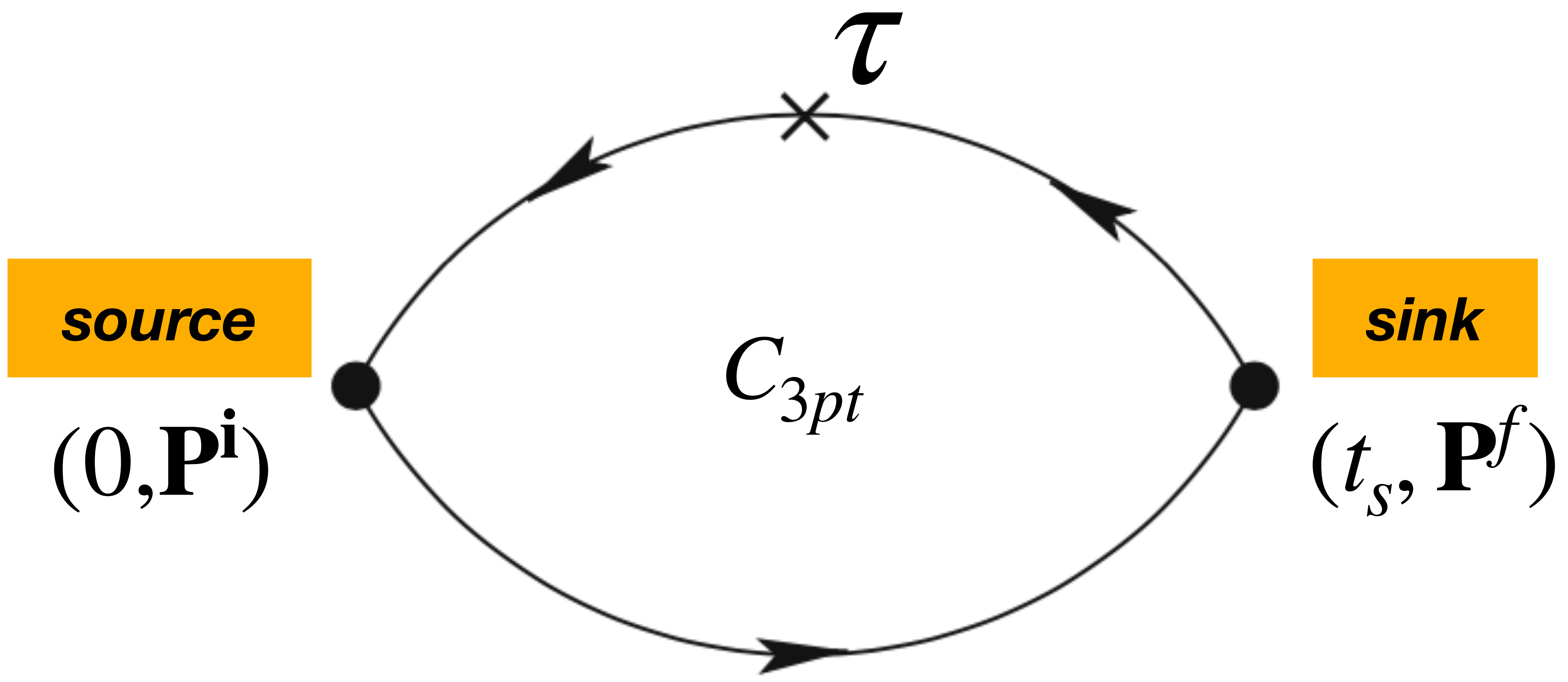
Extraction of bare form factor

- Construct the ratio between 3 and 2-pt corr.:

$$R^{fi}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) \equiv \frac{2\sqrt{E_0^f E_0^i}}{E_0^f + E_0^i} \frac{C_{3pt}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s)}{C_{2pt}(t_s, \mathbf{P}^f)} \times \left[\frac{C_{2pt}(t_s - \tau, \mathbf{P}^i) C_{2pt}(\tau, \mathbf{P}^f) C_{2pt}(t_s, \mathbf{P}^f)}{C_{2pt}(t_s - \tau, \mathbf{P}^f) C_{2pt}(\tau, \mathbf{P}^i) C_{2pt}(t_s, \mathbf{P}^i)} \right]^{1/2}$$

- Bare form factor:

$$F^B(Q^2) = \lim_{\tau \to \infty, t_s \to \infty} R^{fi}(\mathbf{P}^f, \mathbf{P}^i, \tau, t_s)$$



- Form factor: $F(Q^2) = F^B \times Z_V$

Lattice setup

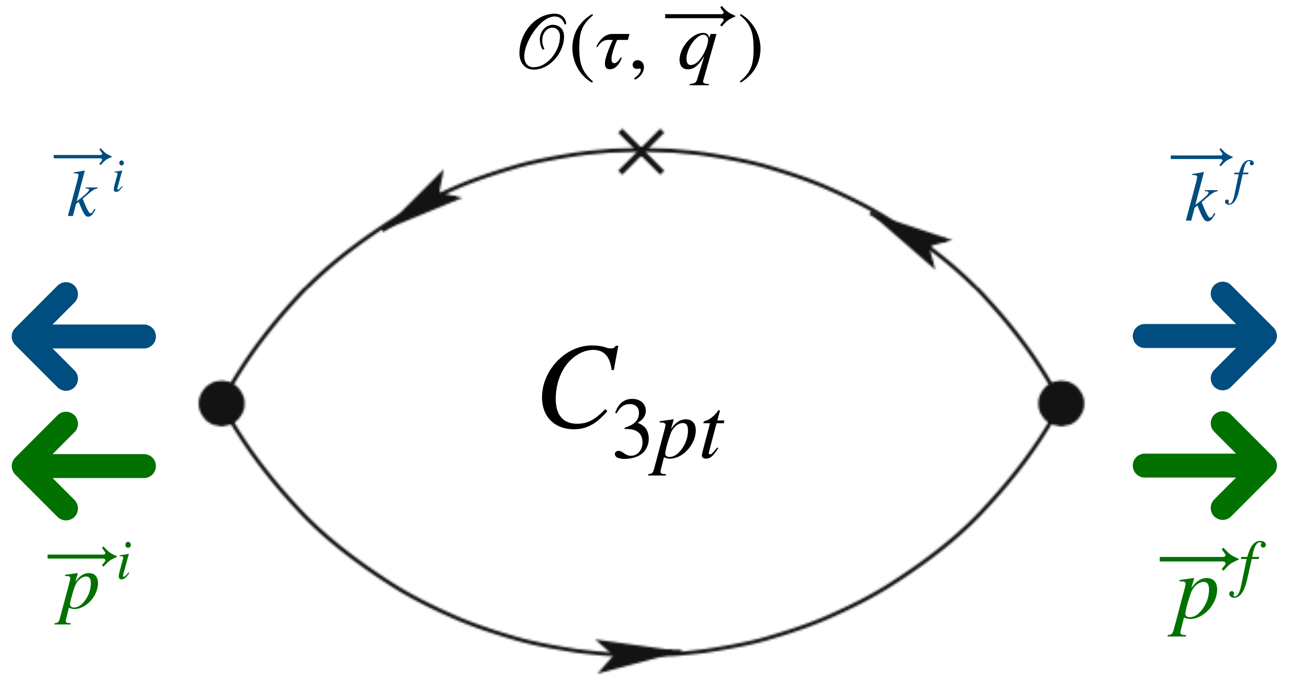
- N_f=2+1 QCD on 64^3x64 lattices with a=0.076 & 0.04 fm ([HotQCD] configurations)

  Sea quark: Highly Improved Staggered Quark (HISQ) action
  Valence quark: Wilson-Clover action

- At the physical point: $M_{\pi^+} = 140$ MeV, $M_{K^+} = 497$ MeV

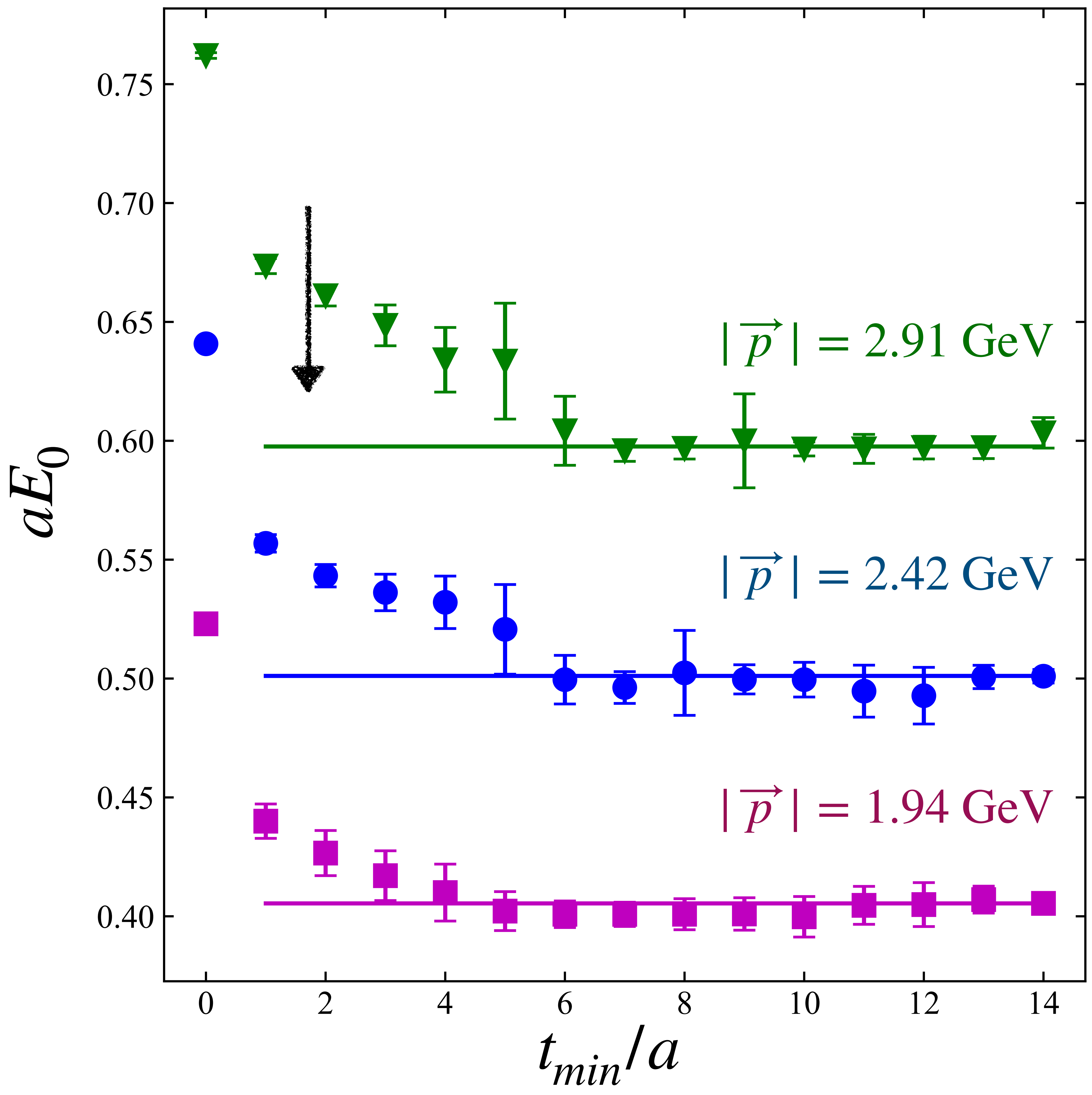
- Boost smearing with the corresponding signs of the quark momenta at source & sink

  - Pion: up to 10 GeV^2 with $a = 0.076$ fm
  - Kaon: up to 28 GeV^2 with $a = 0.076$ & 0.04 fm

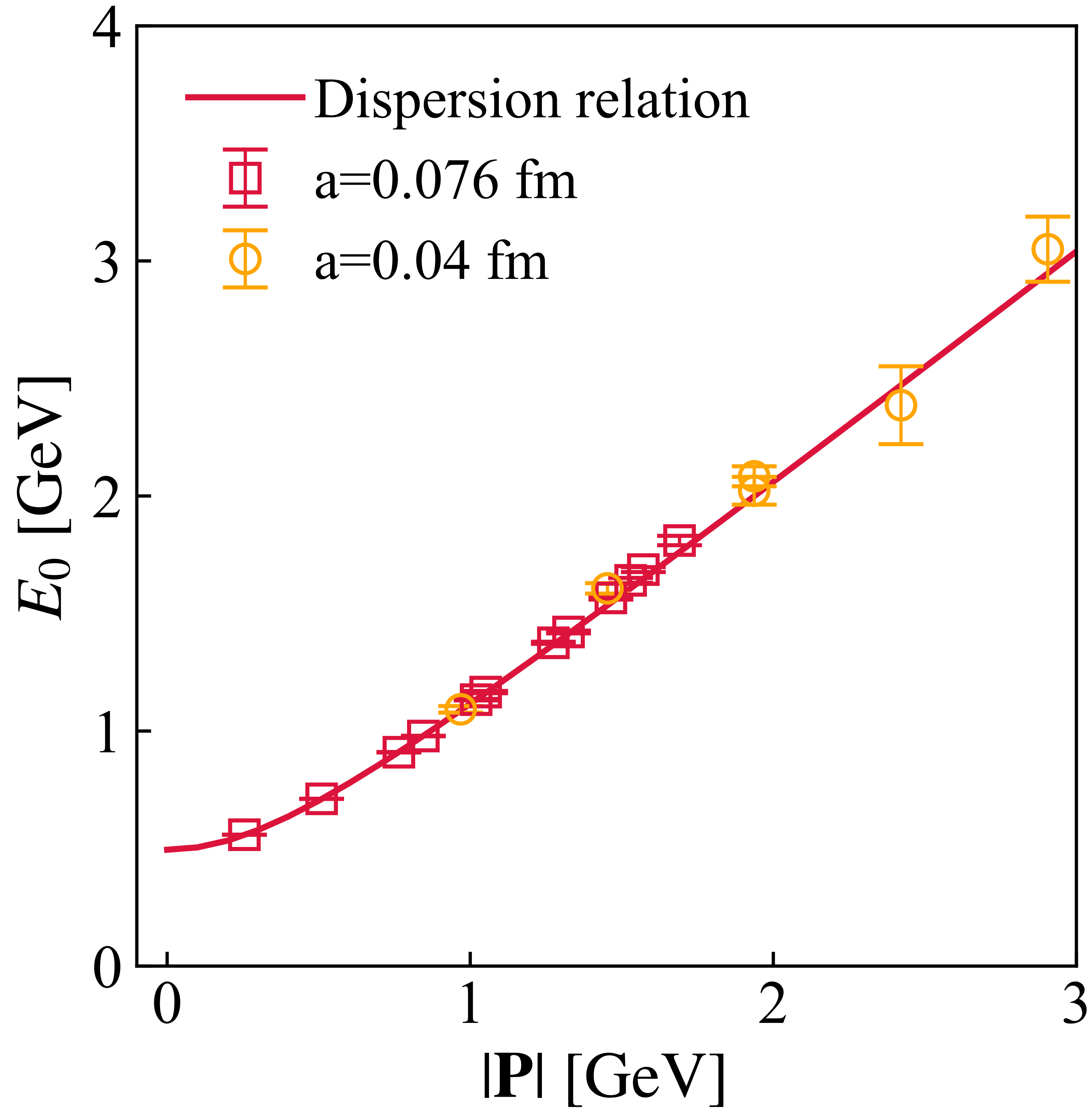


Kaon at large momentum

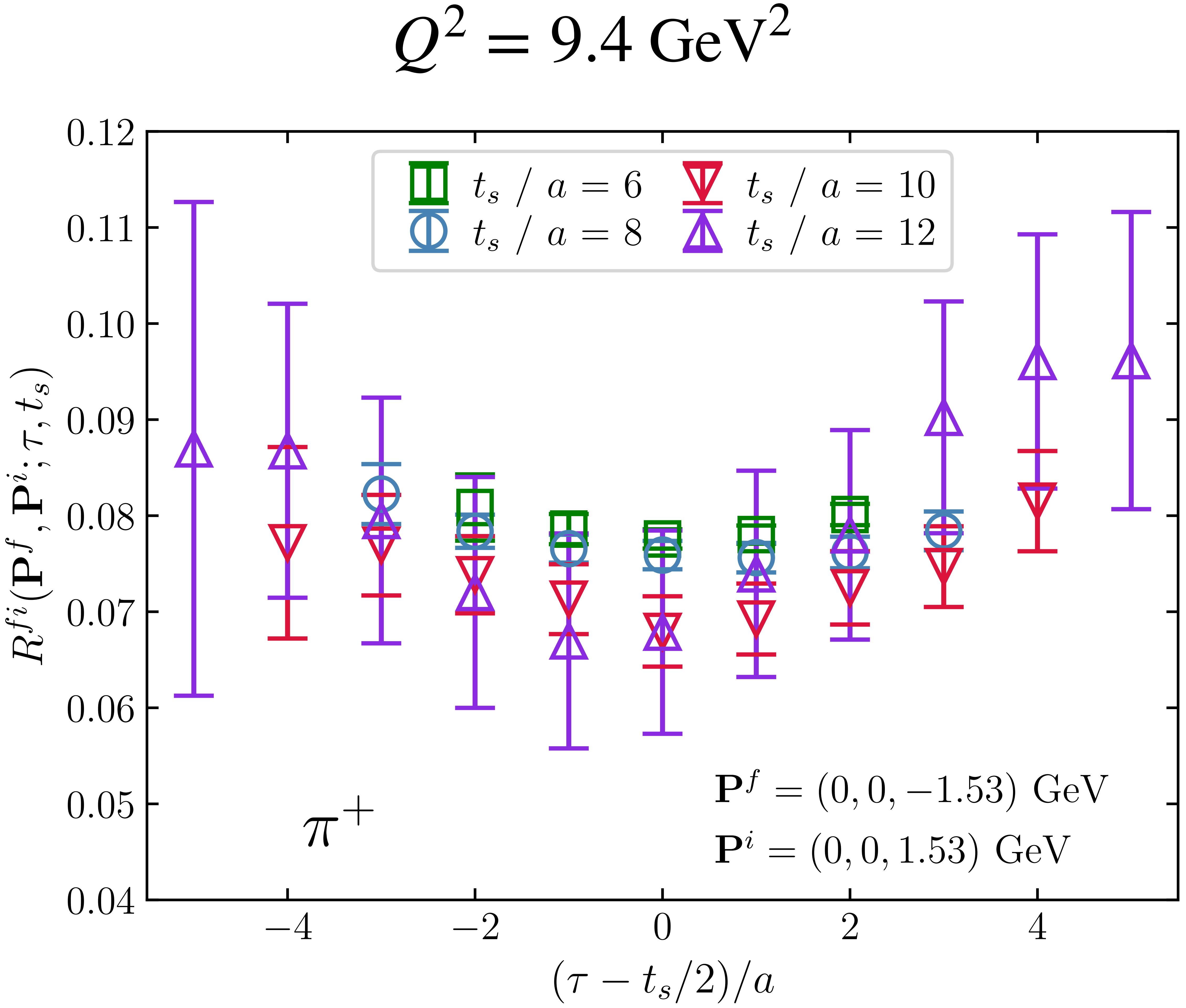
Dispersion relation $E_0(\vec{p}) = \sqrt{m_{K^+}^2 + \vec{p}^2}$

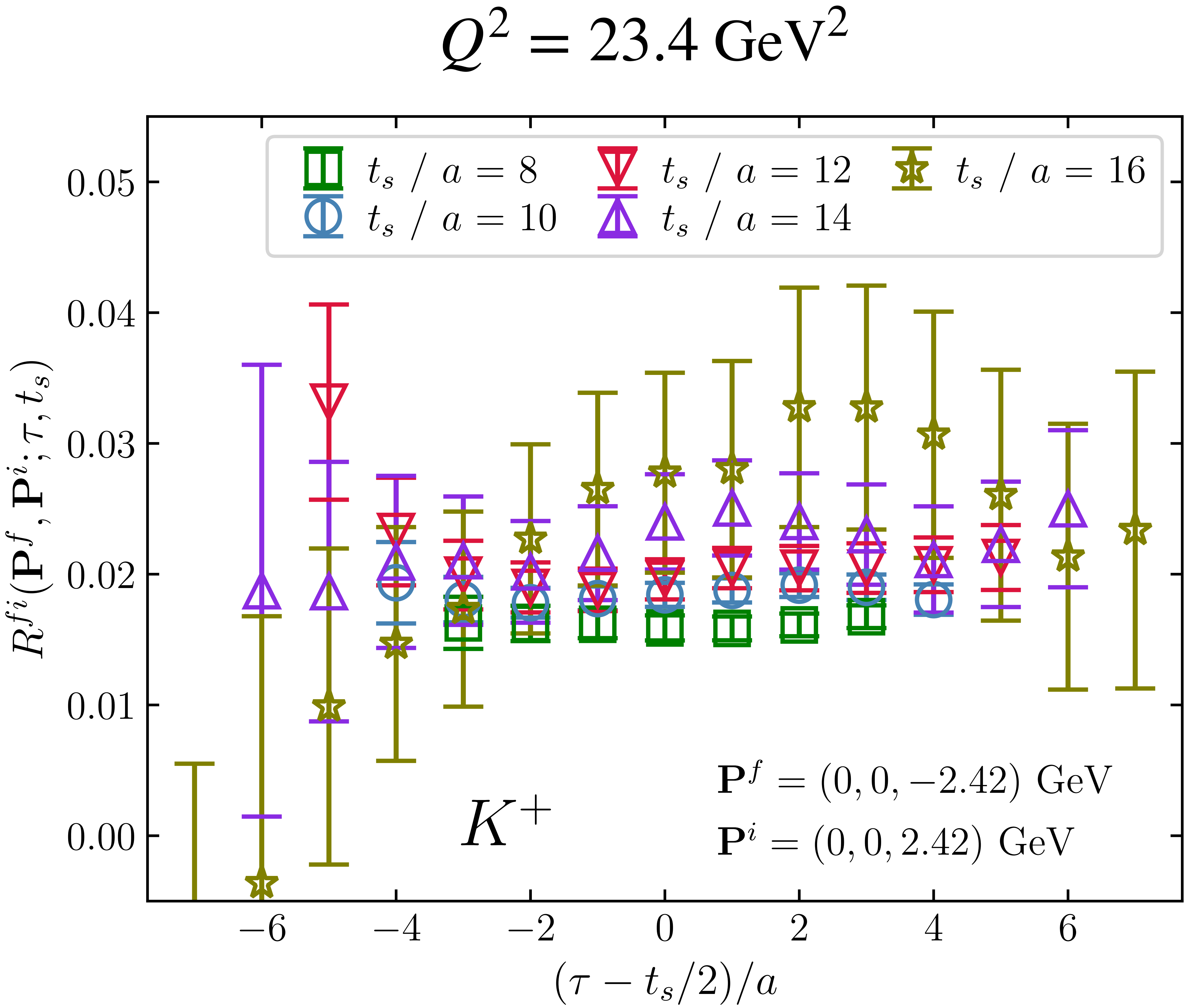


P up to ~3GeV



Lattice data of $R^{fi} \sim C_{3pt}/C_{2pt}$

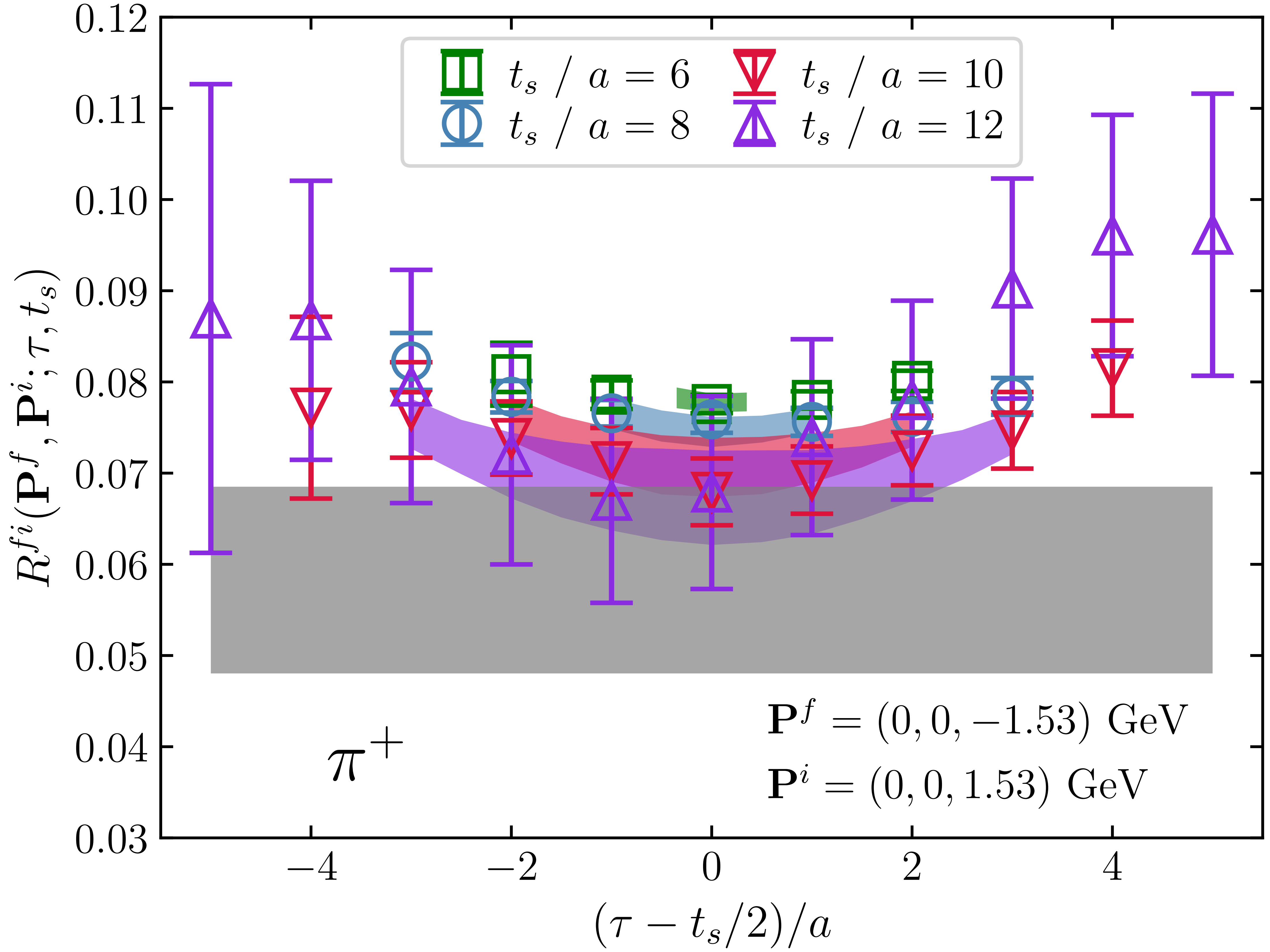




Extraction of the form factor

$$N_{state}=2:\ R^{fi}(\tau,t_s)=\left(\underbrace{\mathcal{O}_{00}}_{F^B}+\frac{A_1}{A_0}\mathcal{O}_{11}e^{-t_s\Delta E}+\sqrt{\frac{A_1}{A_0}}\mathcal{O}_{01}e^{-\tau\Delta E}+\sqrt{\frac{A_1}{A_0}}\mathcal{O}_{10}e^{-(t_s-\tau)\Delta E}\right)\Big/\left(1+\frac{A_1}{A_0}e^{-t_s\Delta E}\right),\ \Delta E=E_1-E_0$$

$Q^2 = 9.4\ \mathrm{GeV}^2$



- Use the values of energy E_n and amplitude A_n extracted from C_{2pt}
- Perform a 4-parameter fit to the ratio R^{fi} to extract F^B

Extraction of the form factor

$$N_{state} = 2\colon R^{fi}(\tau, t_s) = \left(\underbrace{\mathcal{O}_{00}}_{F^B} + \frac{A_1}{A_0}\mathcal{O}_{11}e^{-t_s\Delta E} + \sqrt{\frac{A_1}{A_0}}\mathcal{O}_{01}e^{-\tau\Delta E} + \sqrt{\frac{A_1}{A_0}}\mathcal{O}_{10}e^{-(t_s-\tau)\Delta E}\right) \Big/ \left(1 + \frac{A_1}{A_0}e^{-t_s\Delta E}\right),\ \Delta E = E_1 - E_0$$

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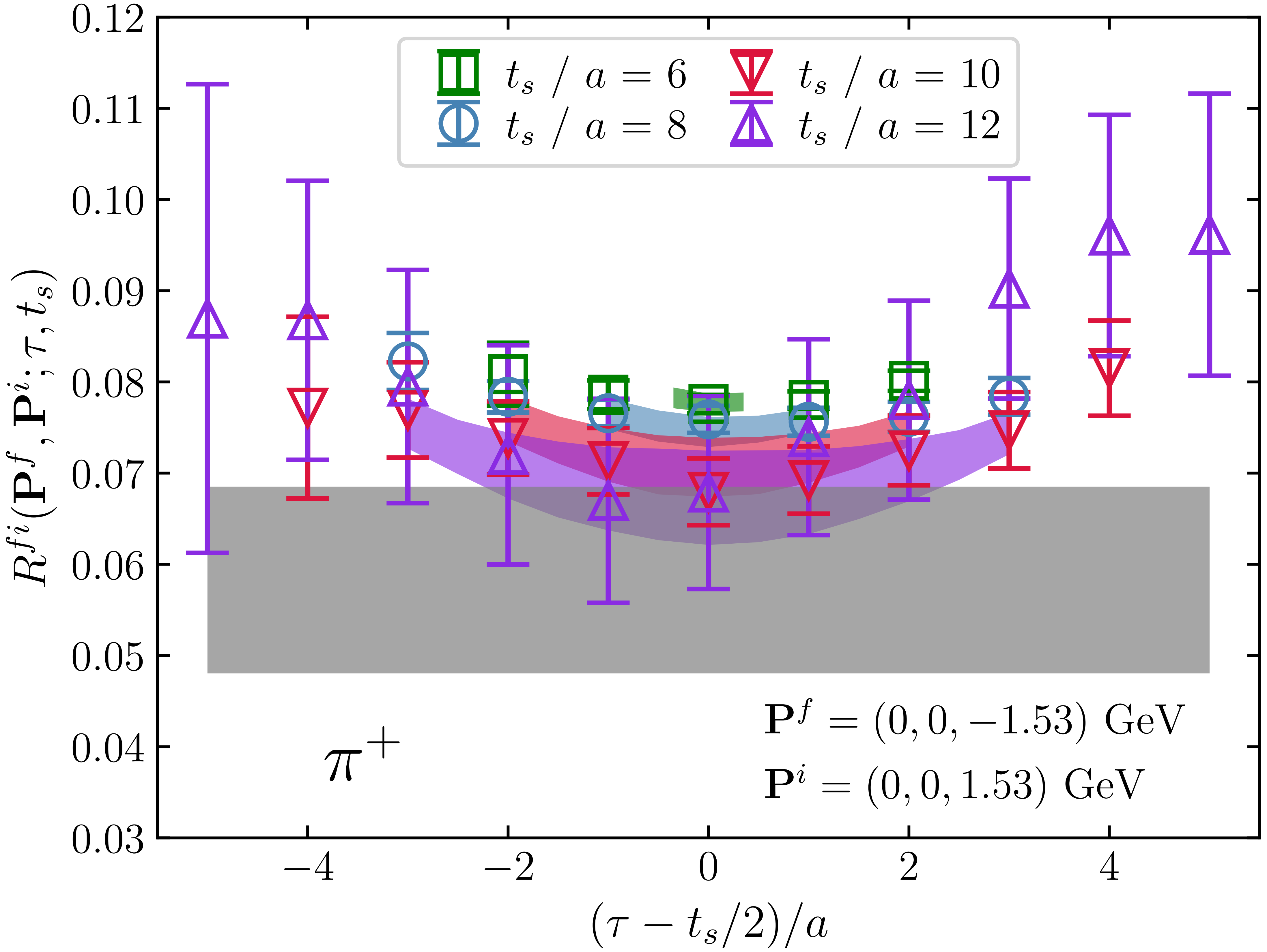
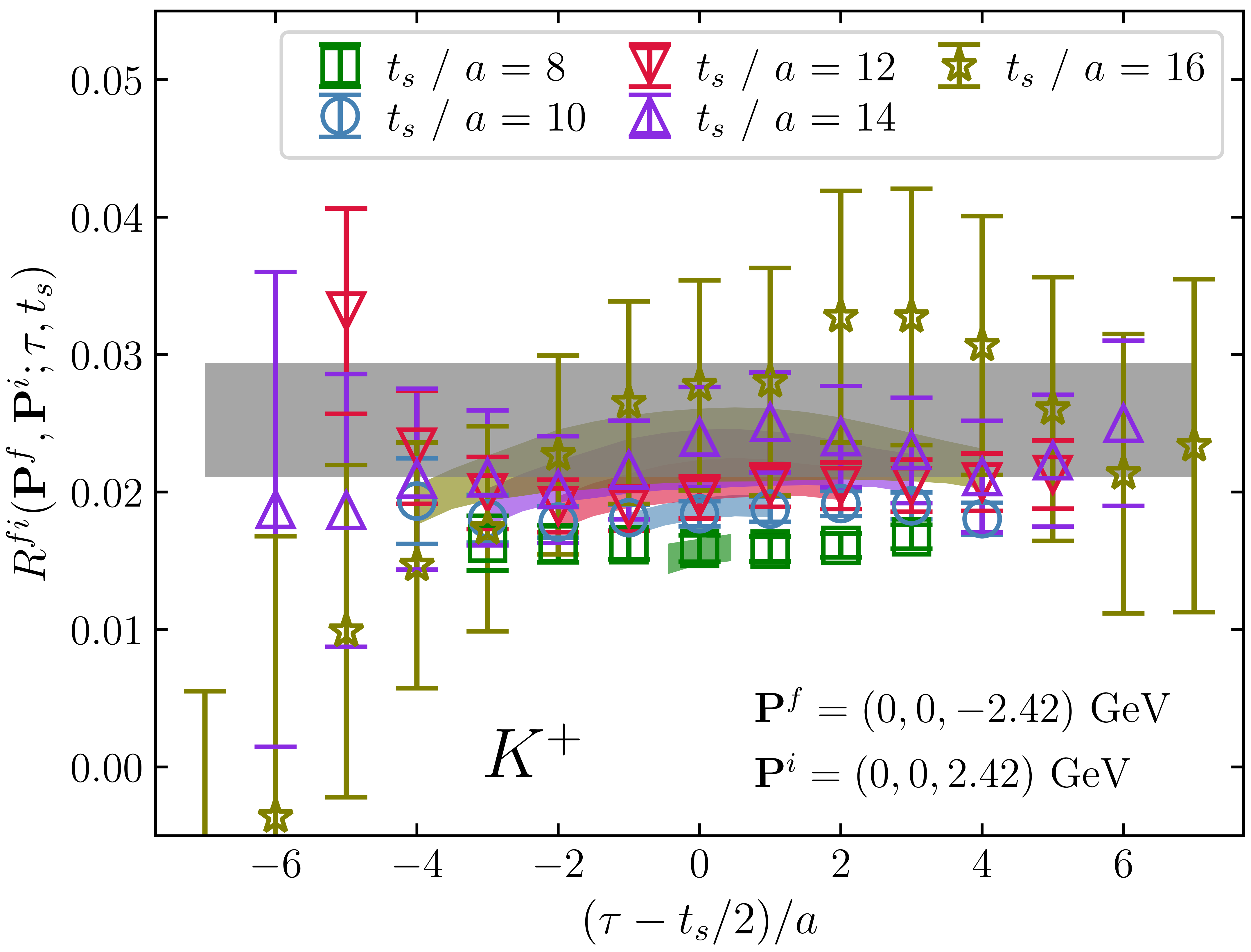
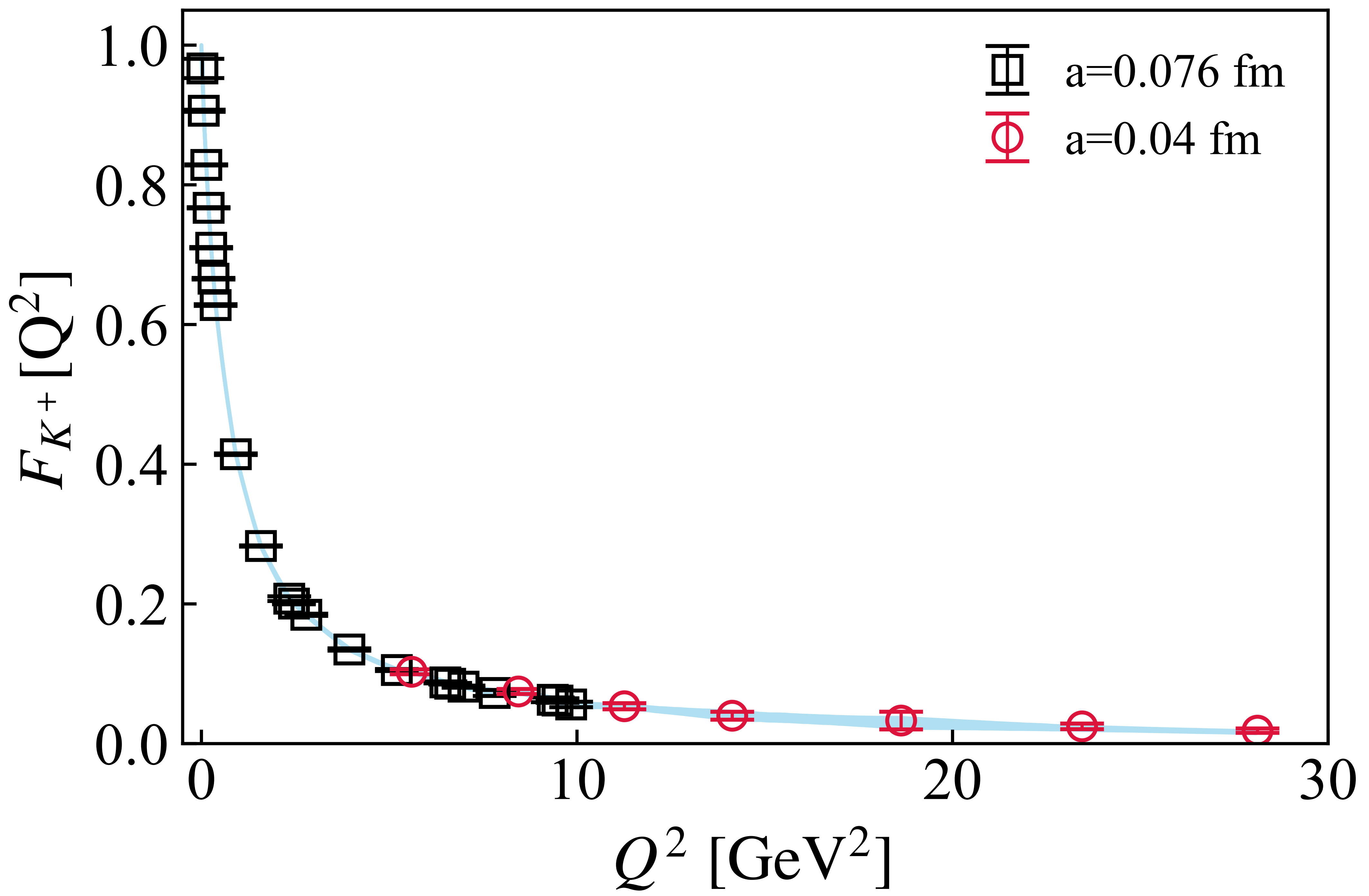
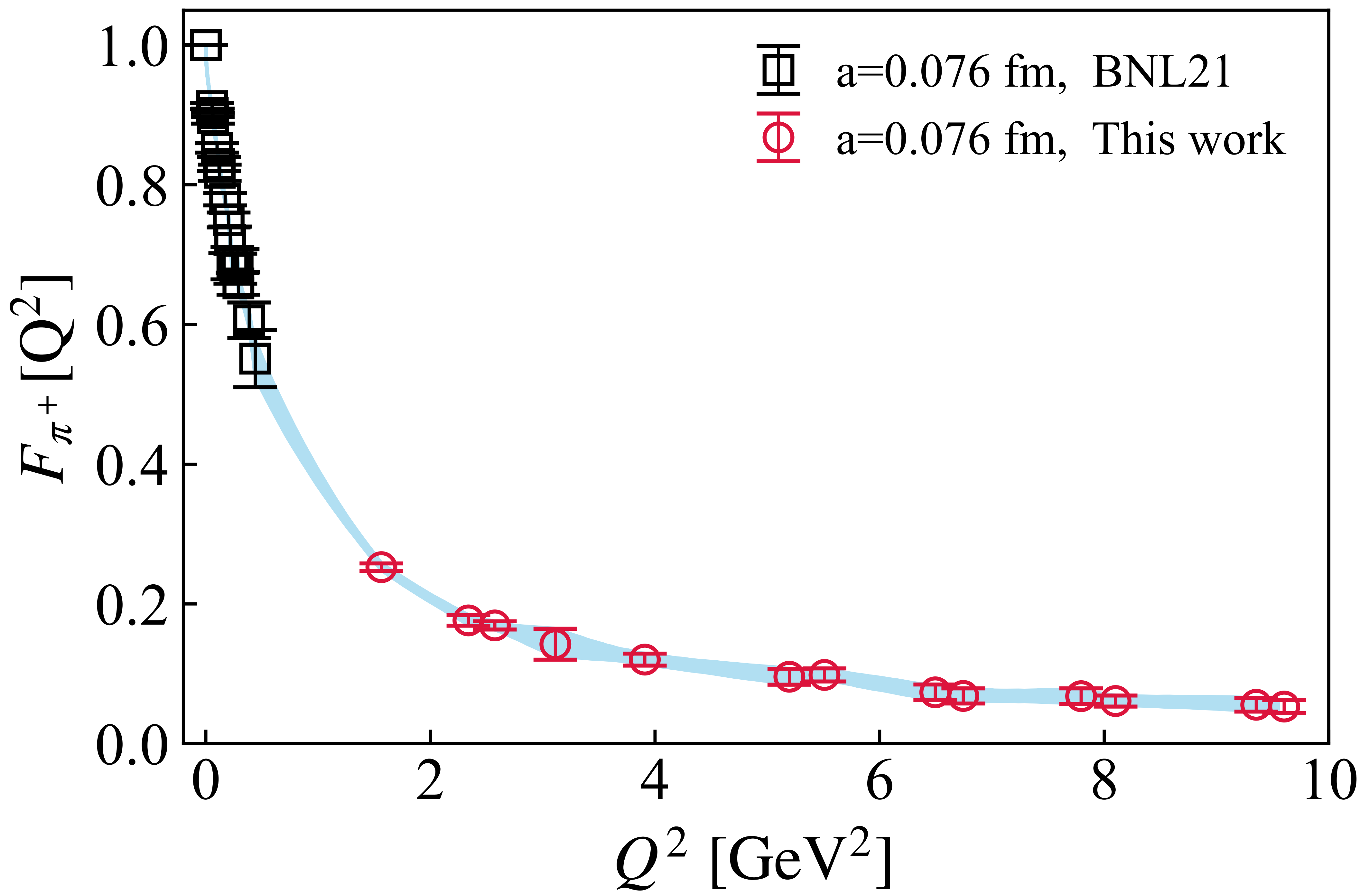


$Q^2 = 23.4\ \mathrm{GeV}^2$



Renormalized Pion and Kaon form factors

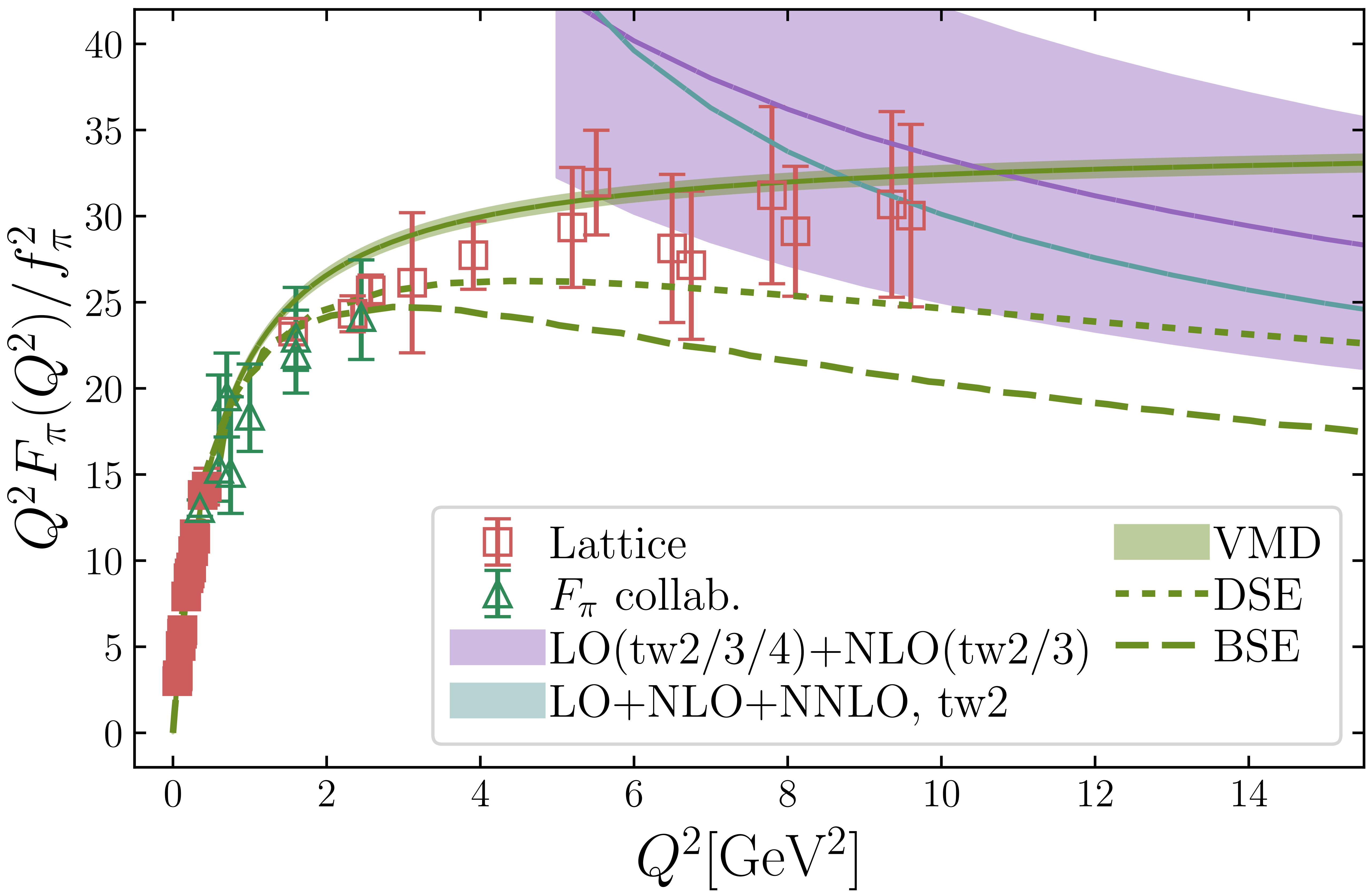




$$F_M(Q^2 \to \infty) = 8\pi\alpha_s(Q^2) f_M^2/Q^2$$

$$Q^2 F_M(Q^2)/f_M^2 = 8\pi\alpha_s(Q^2)$$

Pion form factor up to $Q^2 \sim 10$ GeV^2



- Blue band: collinear factorization, Chen et al., 2312.17228

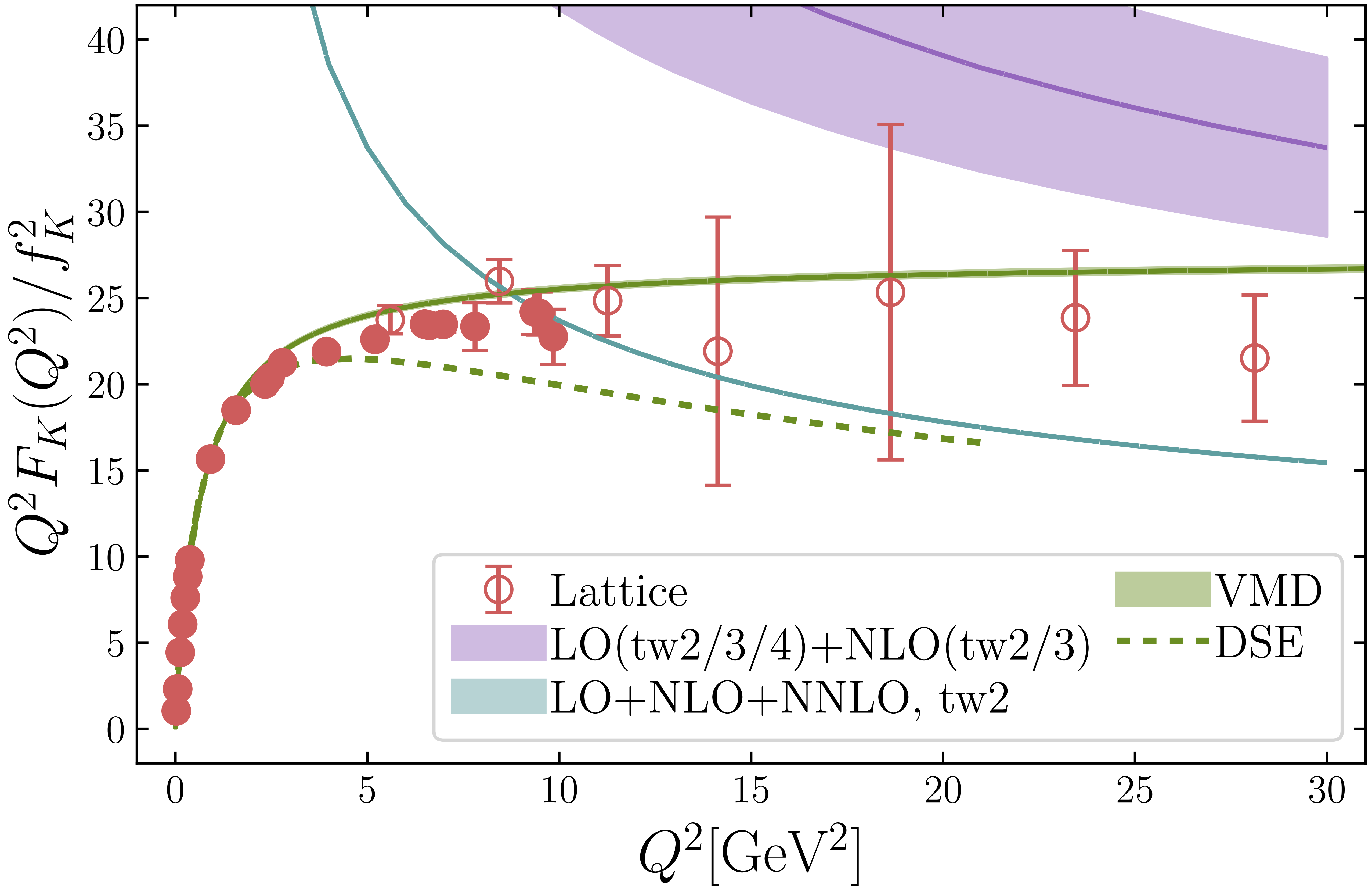
陈龙斌, 16:50, April 21

- Purple band: k_T factorization, Cheng et al., PRD 19', EPJC23'

- DSE: Gao et al., PRD 96(2017)034024
- BSE: Ydrefors et al., PLB 820 (2021)136494
- VMD: $F_\pi(Q^2) = 1/(1 + Q^2/M^2)$, Gao et al., 2102.06047

LO asymptotic result: $Q^2F_{\pi^+}(Q^2)/f_\pi^2 \simeq 8.6$

Kaon form factor up to $Q^2 \sim 28$ GeV^2



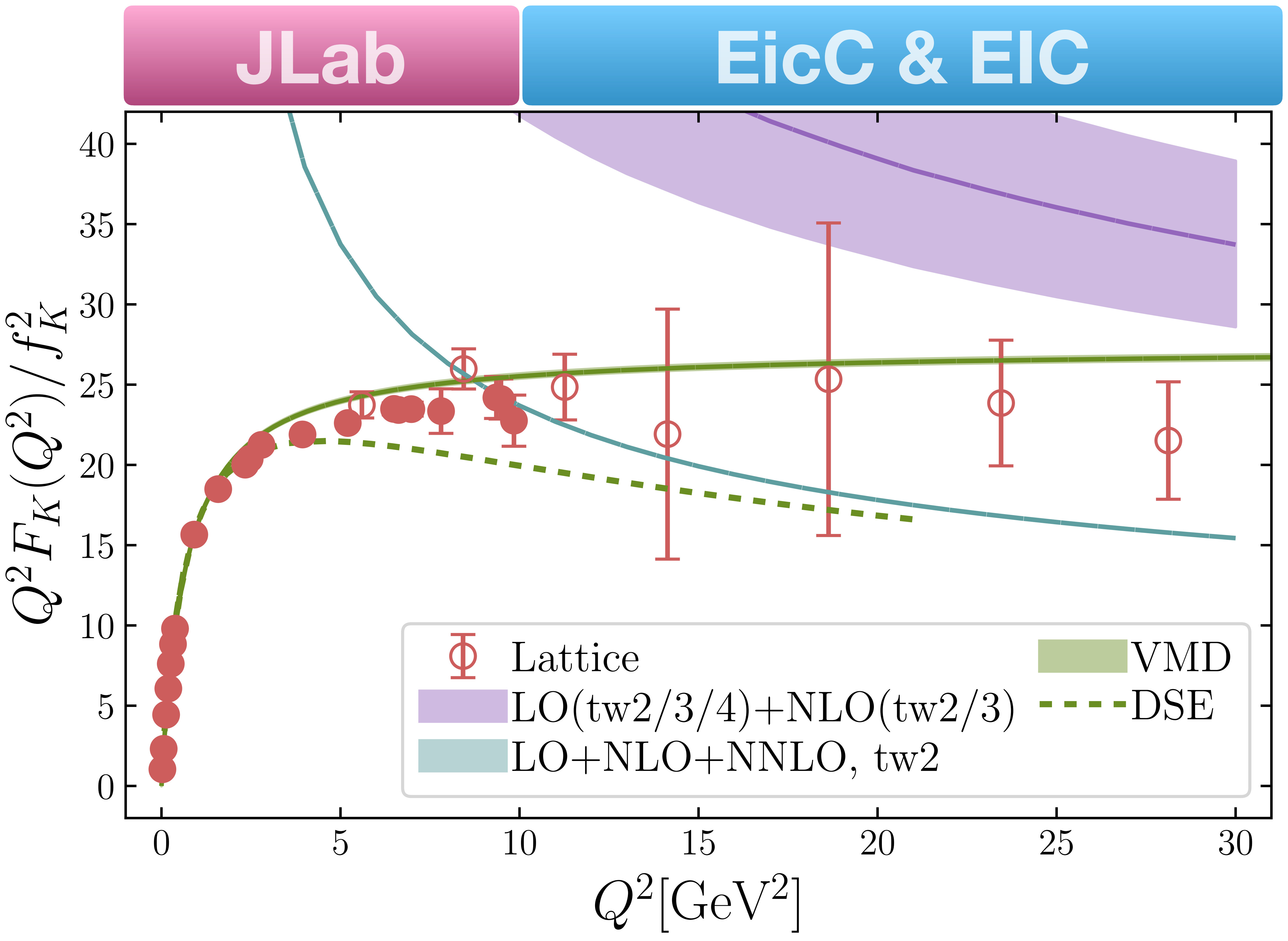
- Blue band: collinear factorization, Chen et al., 2312.17228
- Purple band: k_T factorization, Cheng, priv. com.
- DSE: Gao et al., PRD 96(2017)034024
- VMD: $F_{K^+}(Q^2) = \sum_{v=\rho,\phi,\omega} c_v/(1+Q^2/m_v^2)$

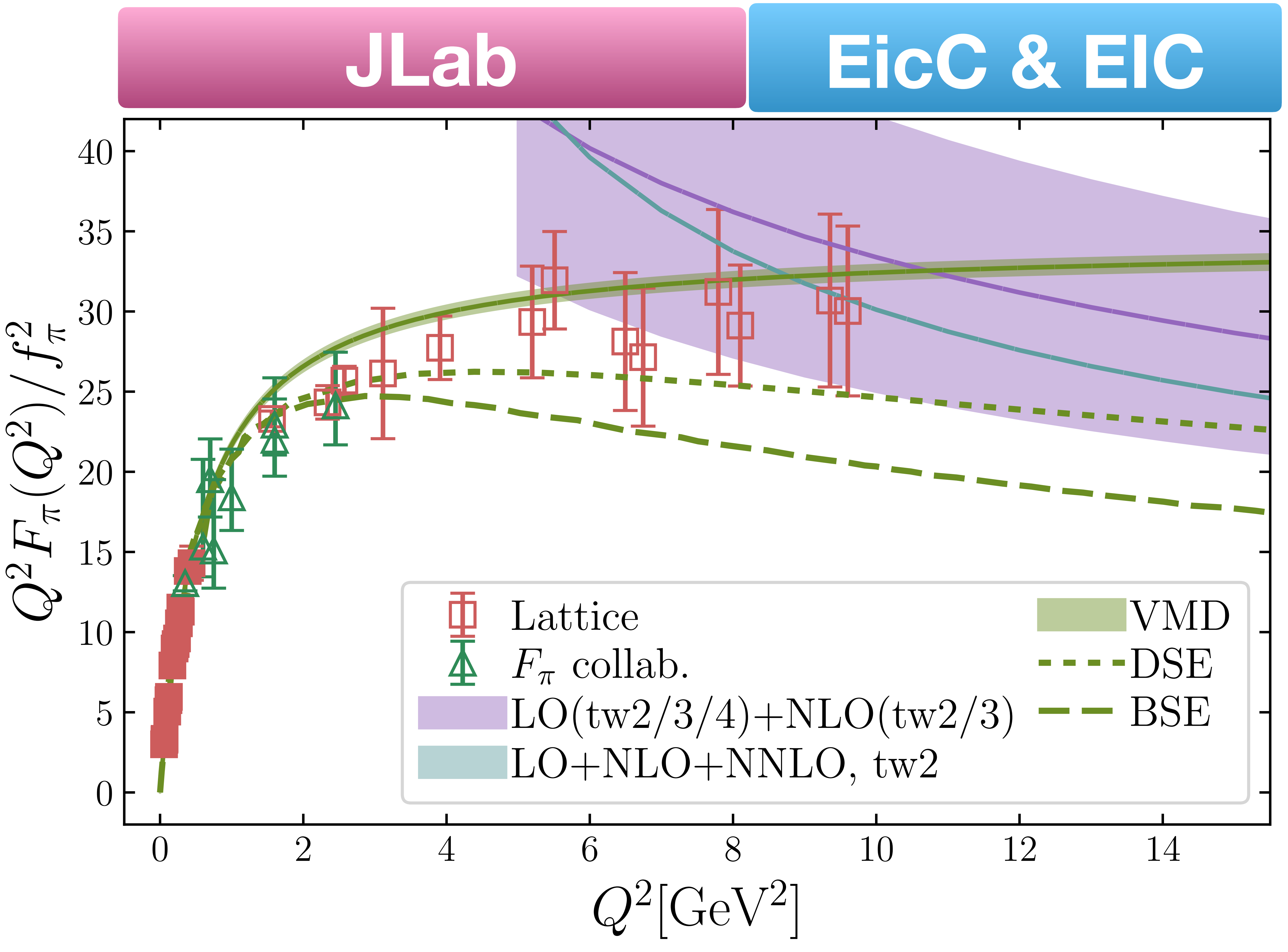
fit in low $Q^2 \lesssim 0.4$ GeV^2, resulting $\langle r_K^2 \rangle = 0.360(2)\text{fm}^2$

Consistent with $\langle r_K^2 \rangle = 0.359(3)\text{fm}^2$ Stamen et al., EPJC 82(2022)432

Summary

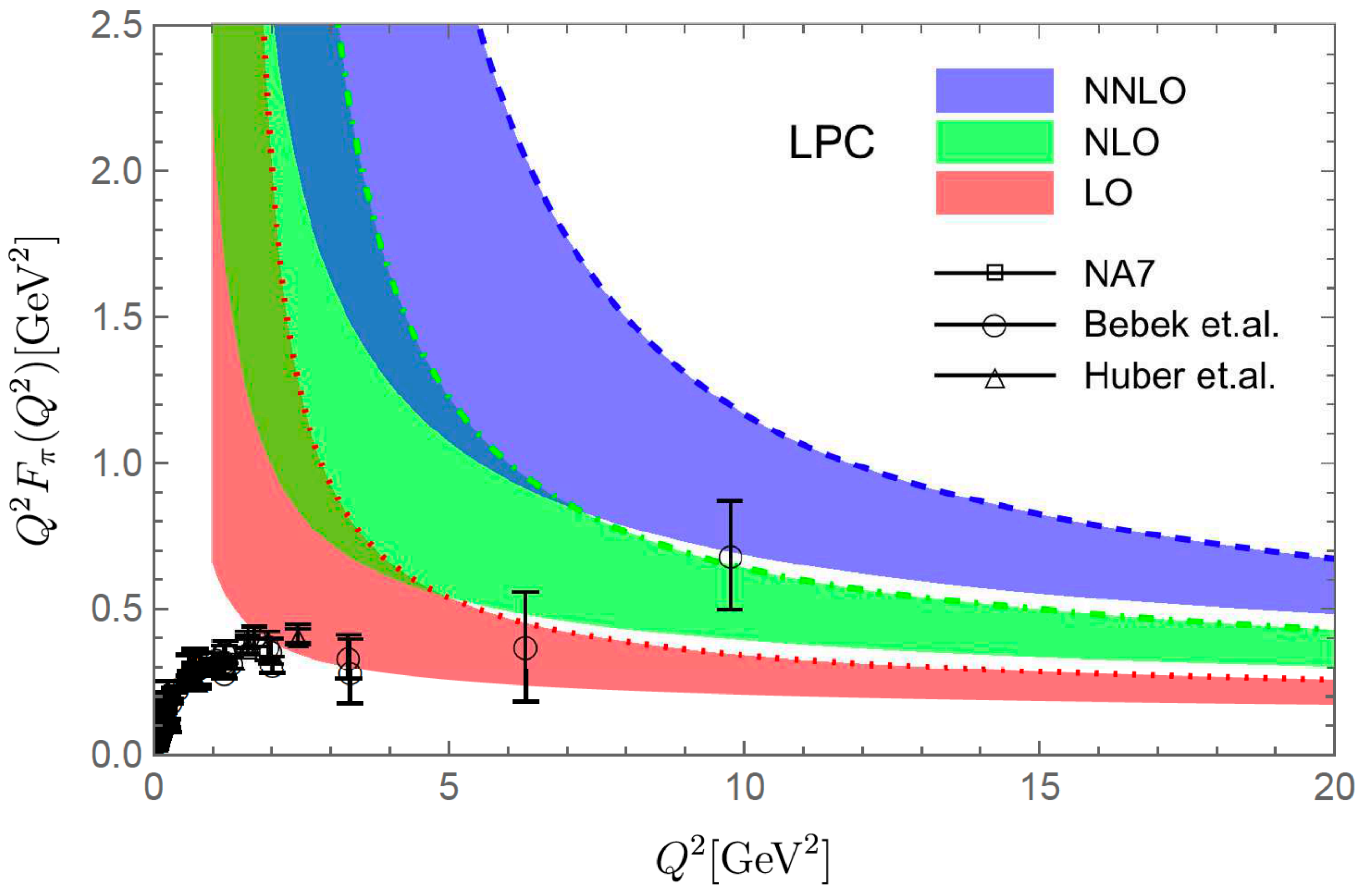
☑ A first LQCD prediction of Kaon and Pion electromagnetic form factors with Q^2 up to ~28 and 10 GeV^2, respectively

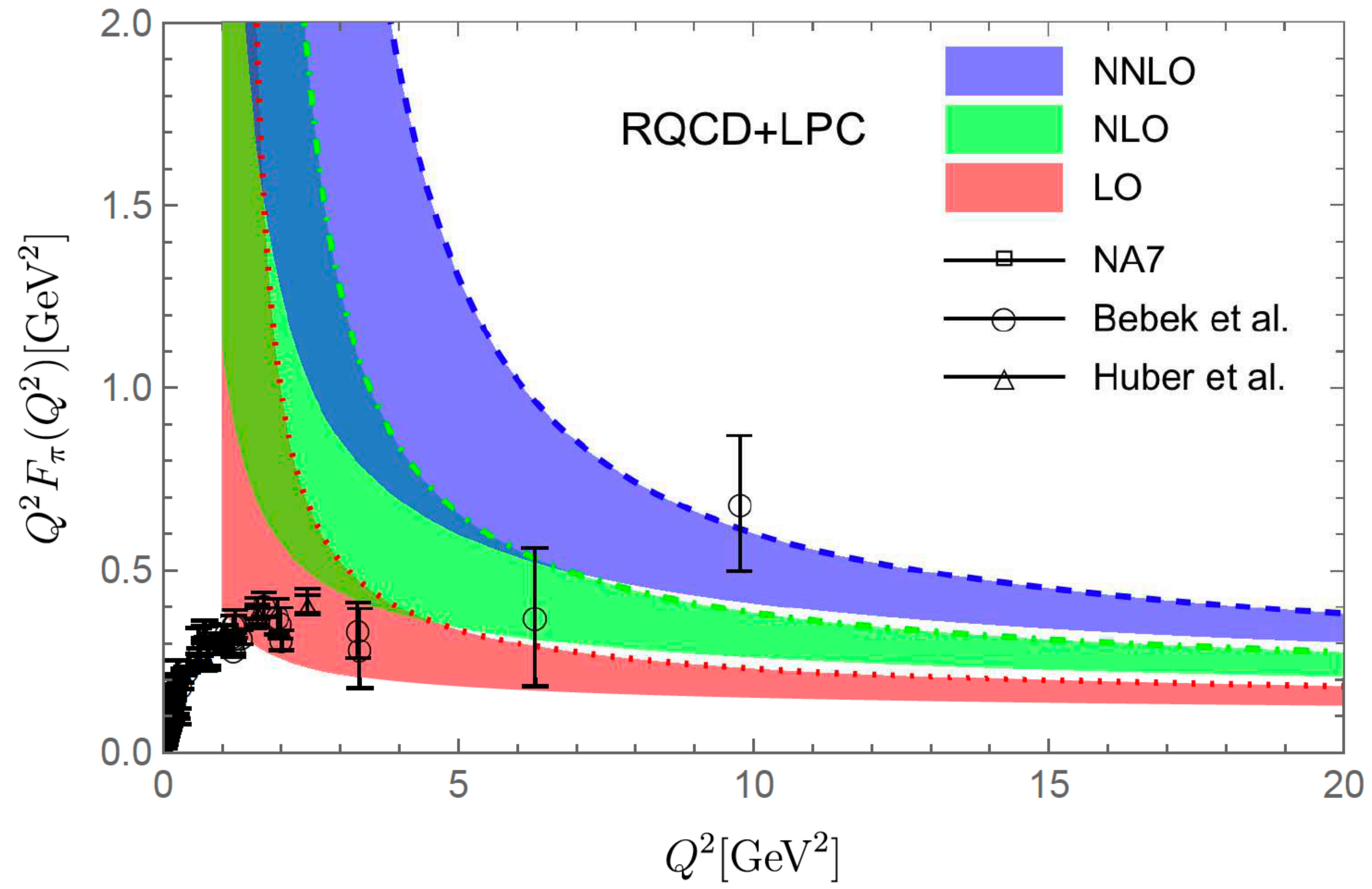




Backup

Impact of Gegenbauer moments from DAs





Chen, Chen, Feng & Jia, arXiv:2312.17228

RQCD: $a_2(2\ \mathrm{GeV}) = 0.116^{+0.019}_{-0.020}$

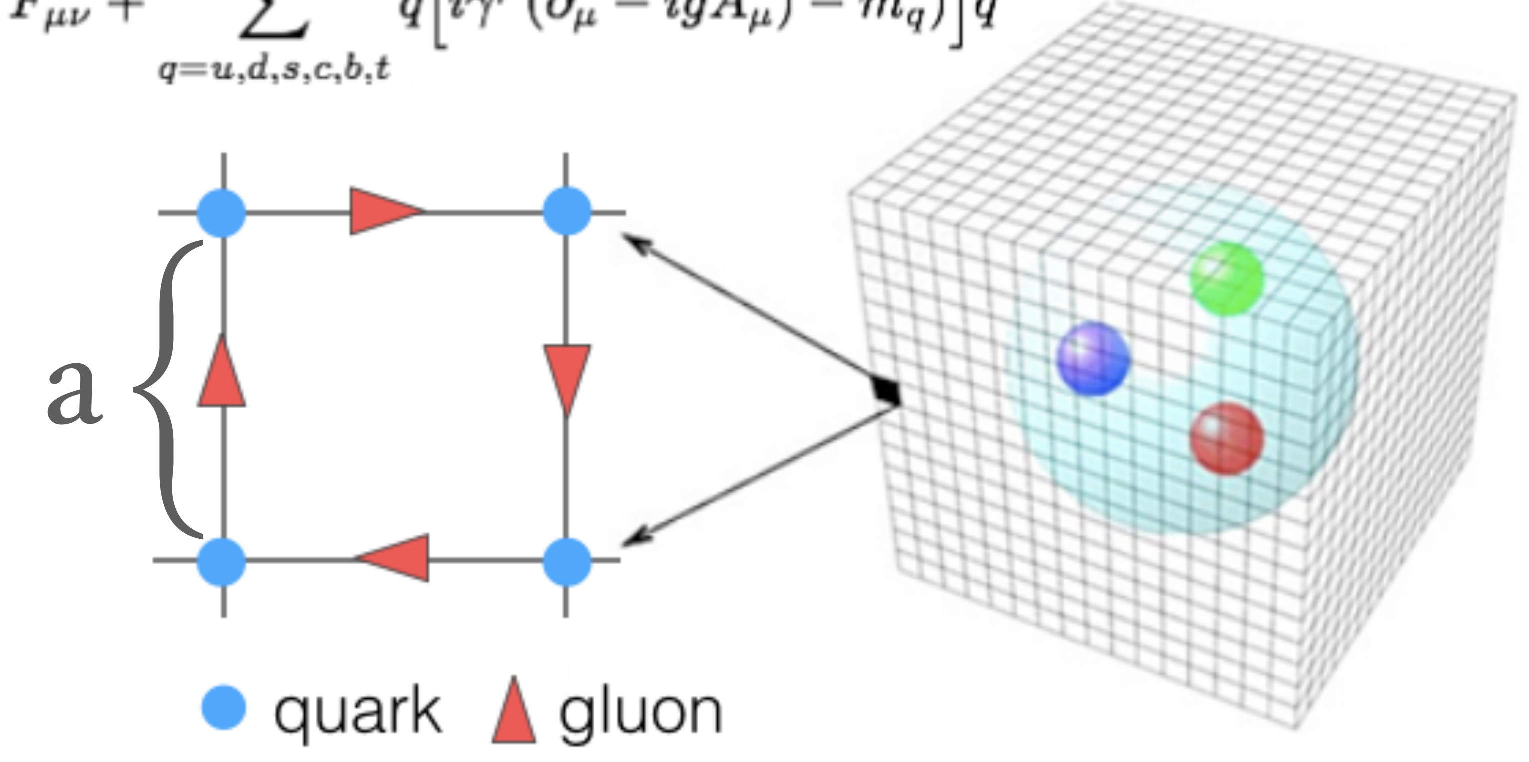
LPC: $a_2(2\ \mathrm{GeV}) = 0.258 \pm 0.087, \quad a_4(2\ \mathrm{GeV}) = 0.122 \pm 0.056, \quad a_6(2\ \mathrm{GeV}) = 0.068 \pm 0.038.$

In this work, pion: $a_2 = 0.196(32)$, $a_4 = 0.085(26)$, $a_6 = 0.056(15)$

Lattice QCD

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\right]q$$



- Lattice simulations of QCD give first principle results
- But need to have control of

		"Goal"
✤ Thermodynamic limit	$V = 2 \sim 4$ fm	$V \to \infty$
✤ Continuum limit	$a = 0.1 \sim 0.04$ fm	$a \to 0$
✤ Chiral extrapolation	$M_\pi \sim 500 \to 200$MeV	$M_\pi = 140$ MeV (Physical Point)
✤ Statistical errors	$N_{conf} \sim \mathcal{O}(1000)$	$N_{conf} \to \infty$

- Fast computers and algorithms are essential

EM form factor of Kaon: $< K(P_1)|J_\mu|K(P_2) > = (P_1 + P_2)_\mu F_K(Q^2)$