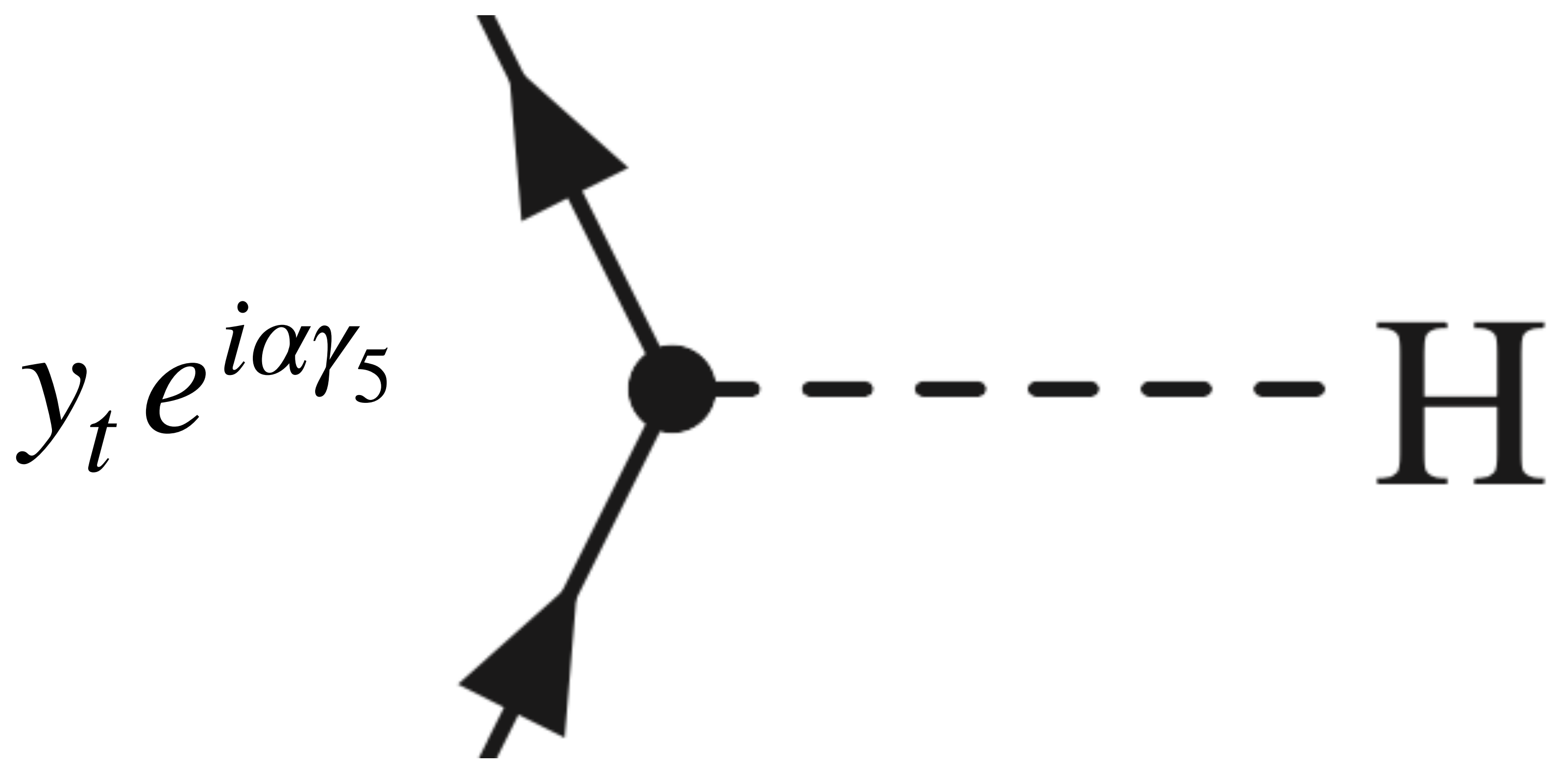
Towards NNLO calculation for high energy production of tTH

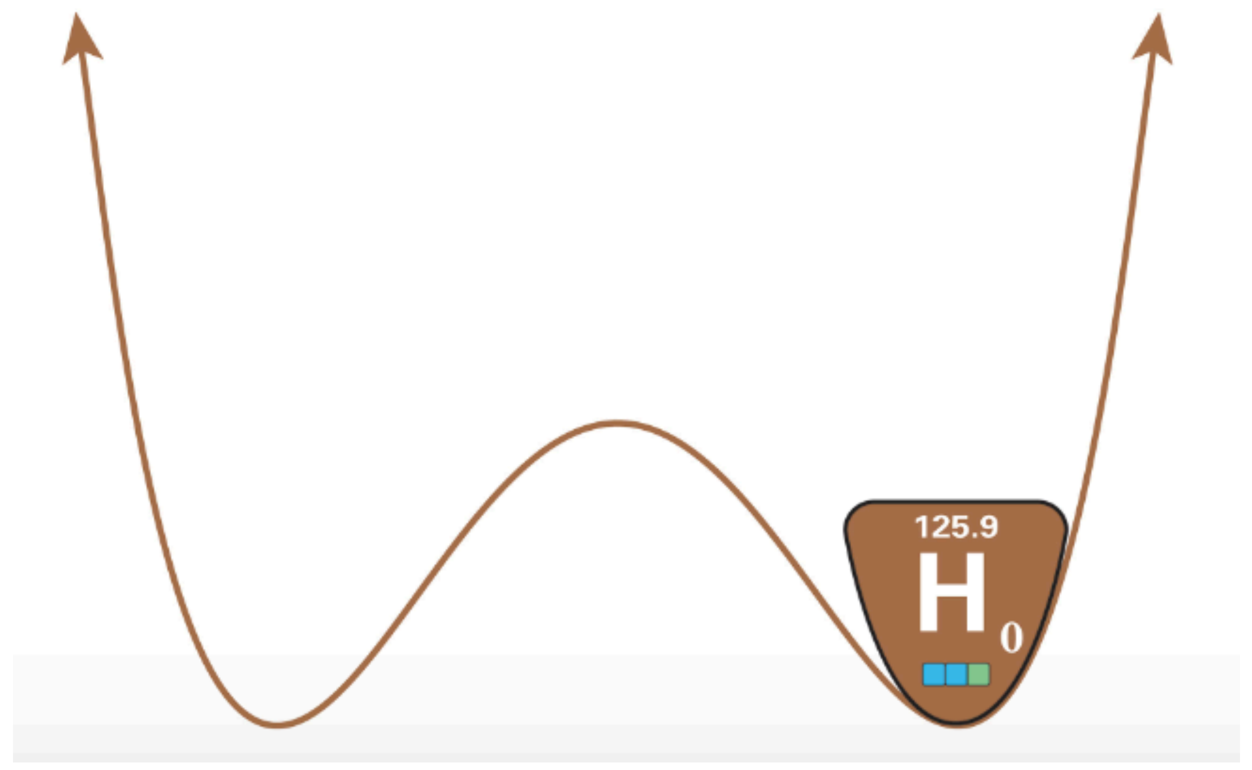
Li Lin Yang
Zhejiang University

The top quark Yukawa coupling

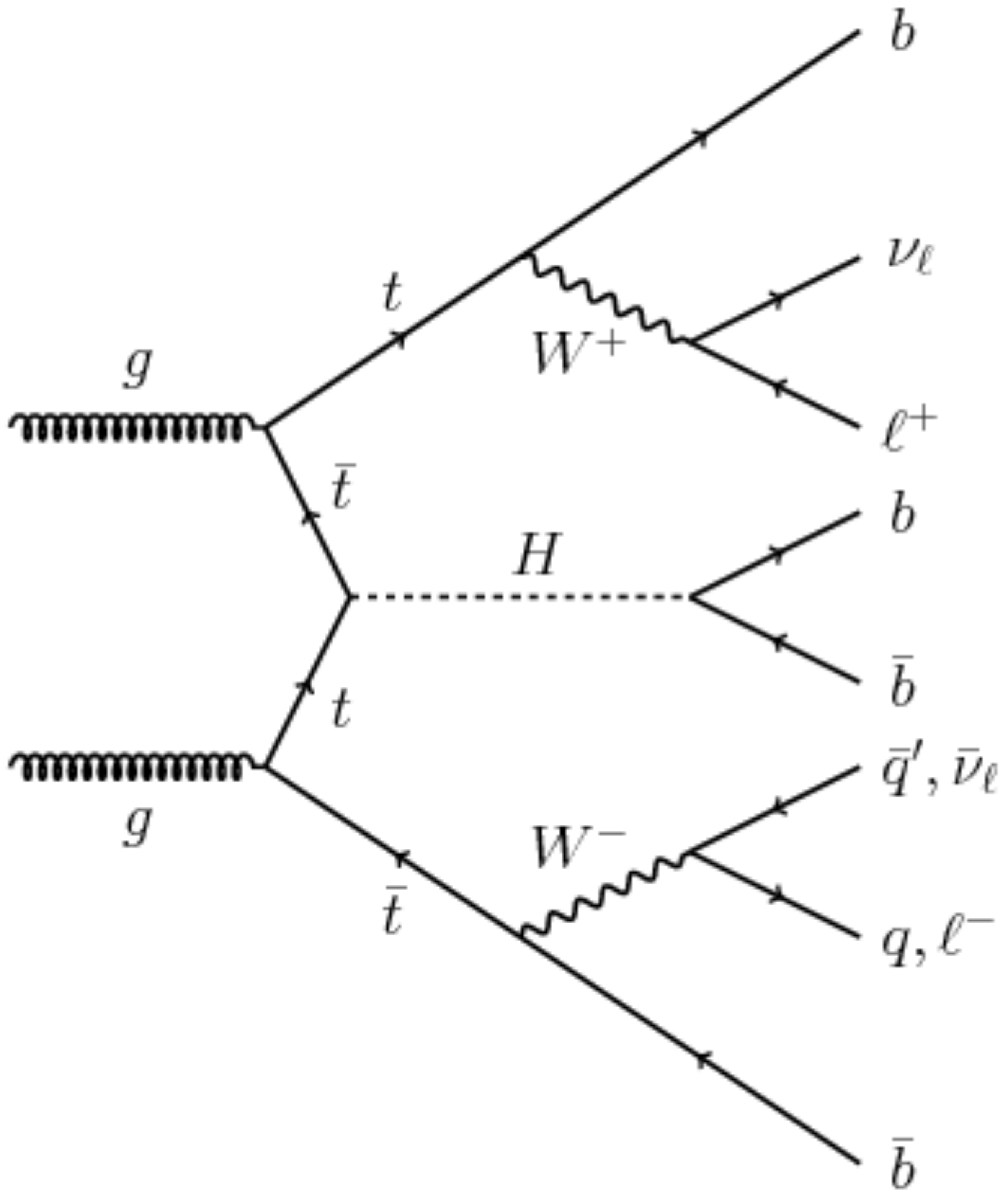


Relevant for

- Origin of masses of fundamental fermions
- Matter-anti-matter asymmetry (possible source of CP violation)
- Higgs effective potential (vacuum stability)

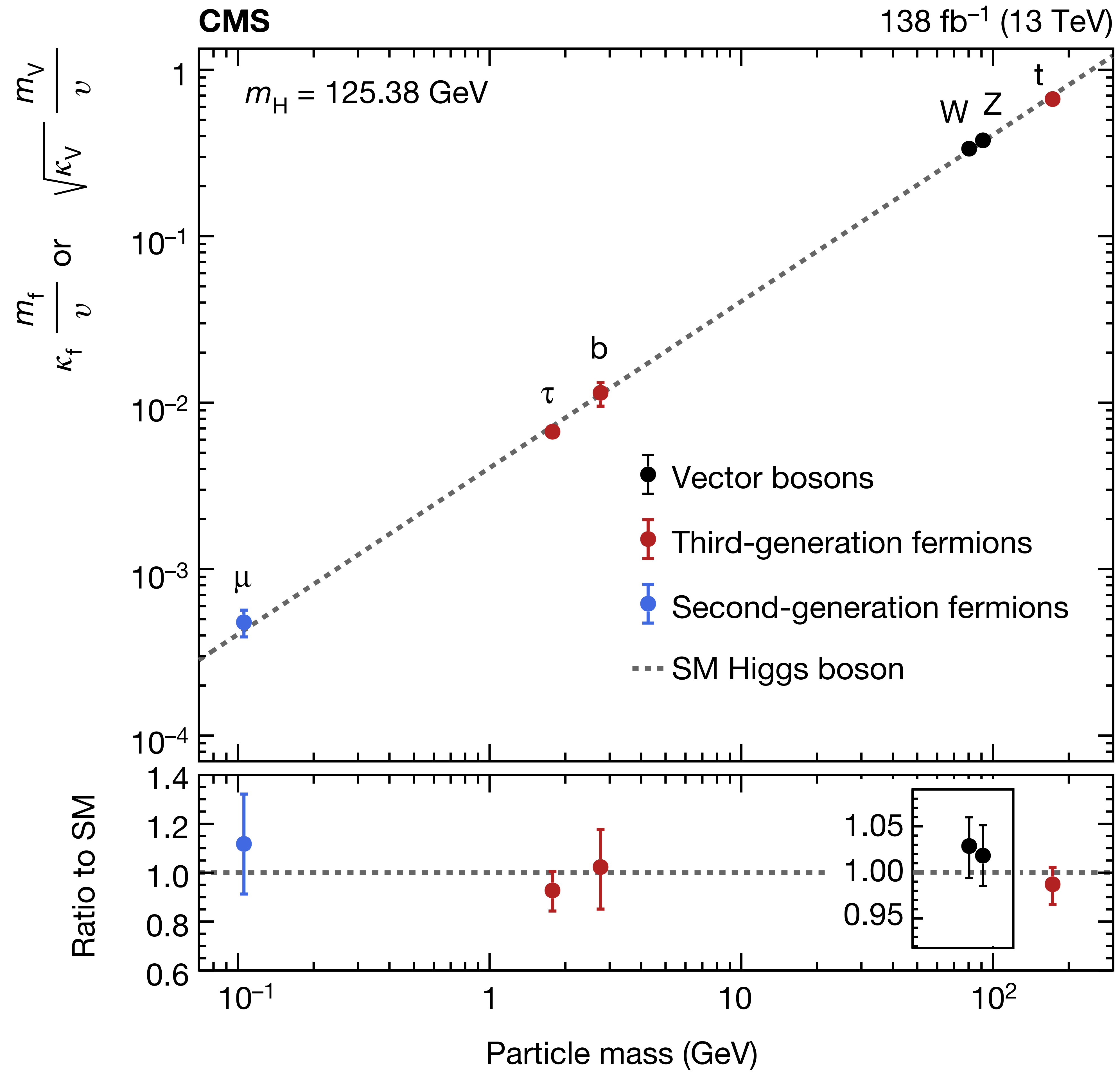


Associated tTH production



- Direct probe of top quark Yukawa coupling
- Observed in 2018 by ATLAS and CMS
- CP structure probed in 2020

The need for precision



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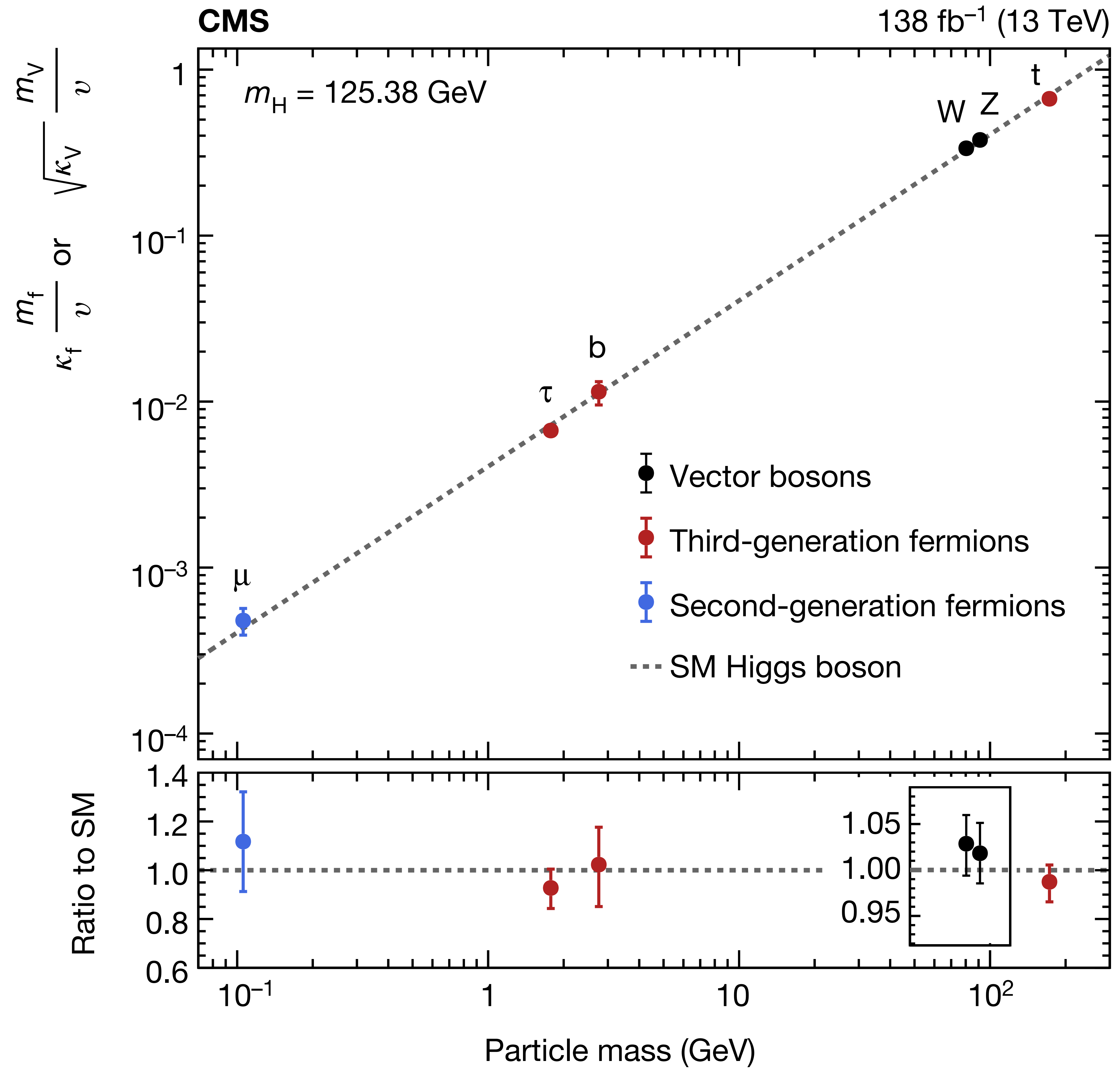
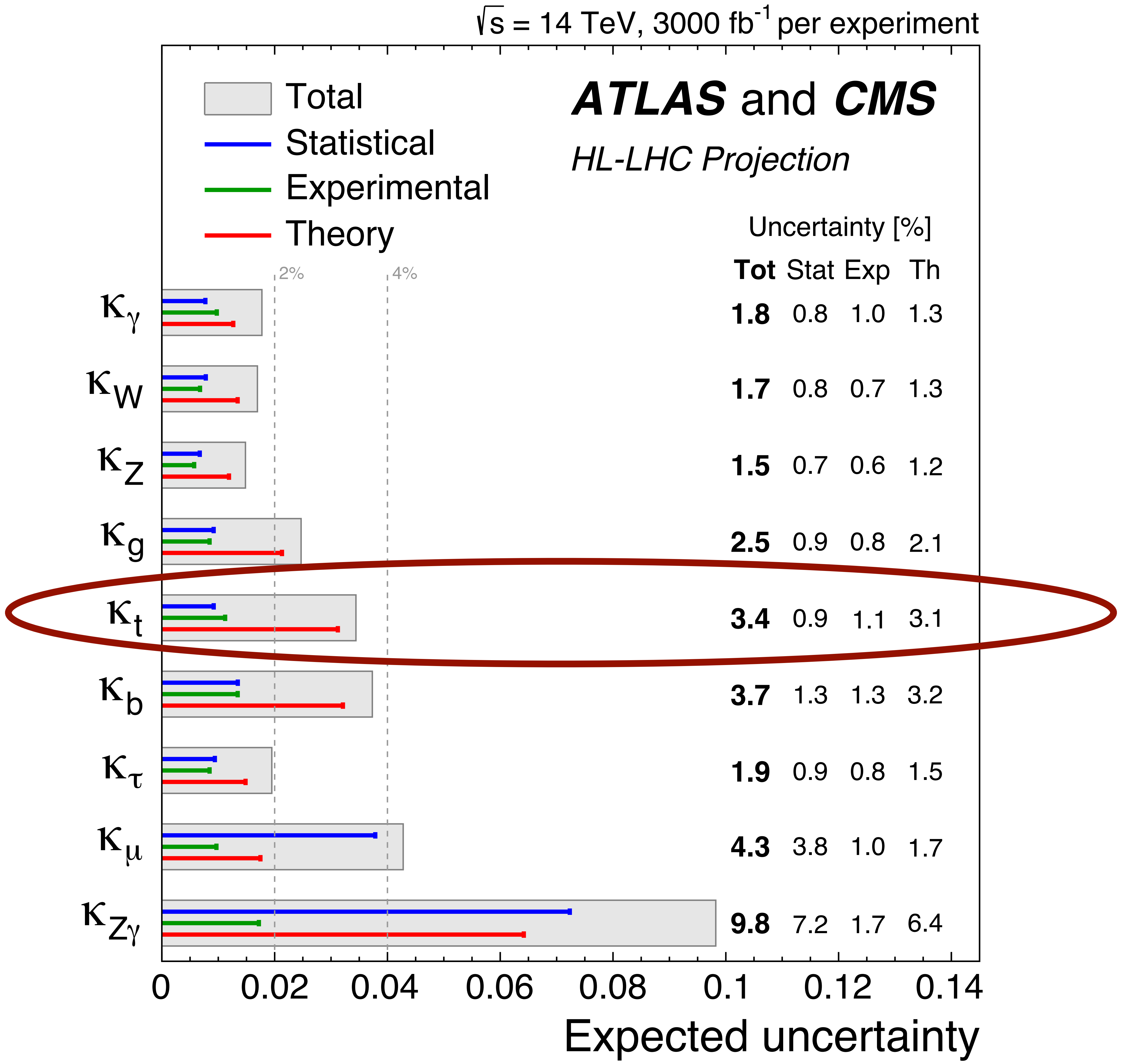
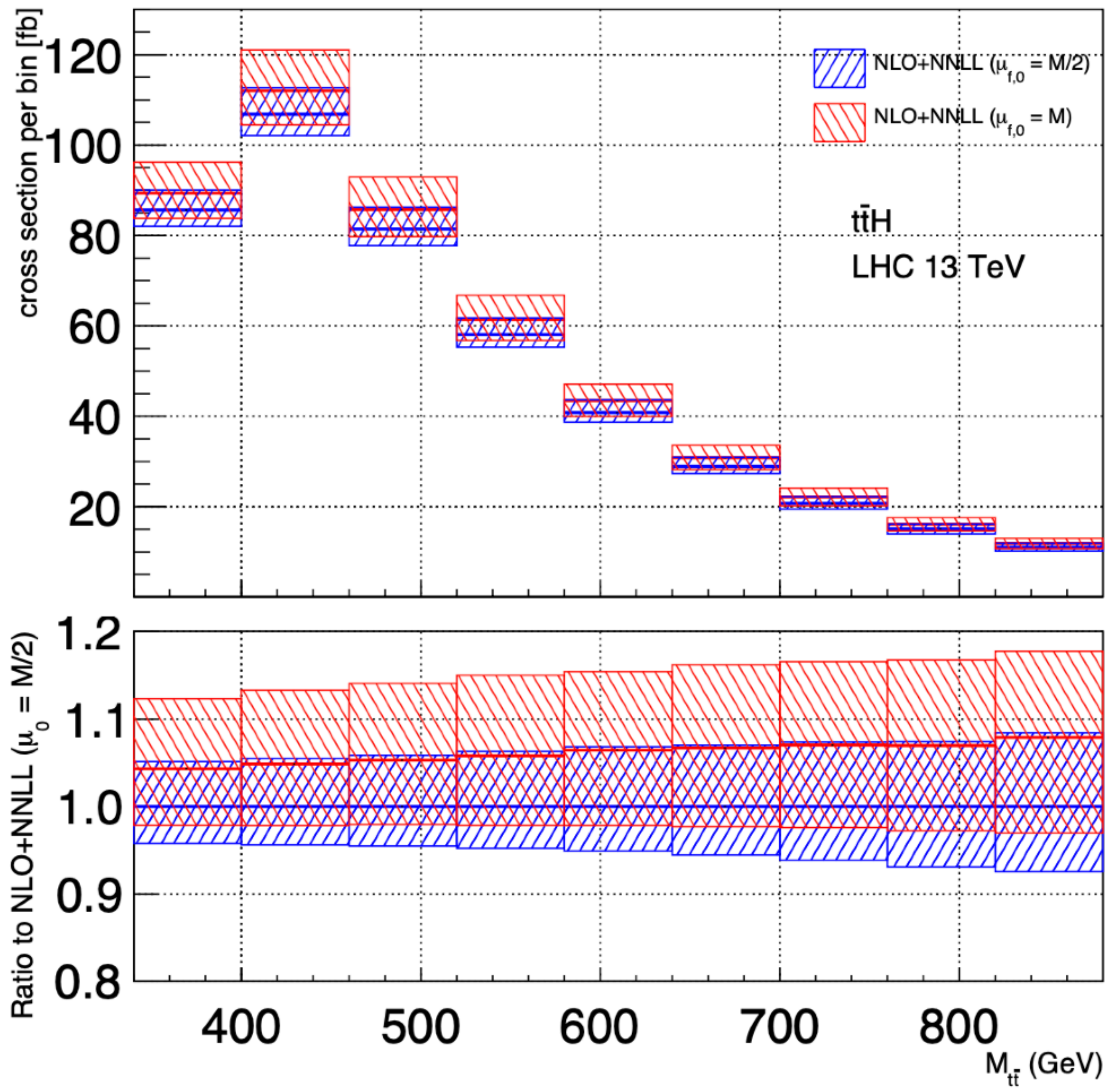




Theoretical status

- ➤ NLO + resummation — Broggio, Ferroglia, Pecjak, LLY: 1611.00049
- ➤ Coulomb corrections — Ju, LLY: 1904.08744

	13 TeV LHC (pb)	14 TeV LHC (pb)
NLO	$0.493^{+5.8\%}_{-9.2\%}$	$0.597^{+6.1\%}_{-9.2\%}$
NLL′+NLO	$0.521^{+1.9\%}_{-2.6\%}$	$0.630^{+2.3\%}_{-2.6\%}$
K-factor	1.06	1.06



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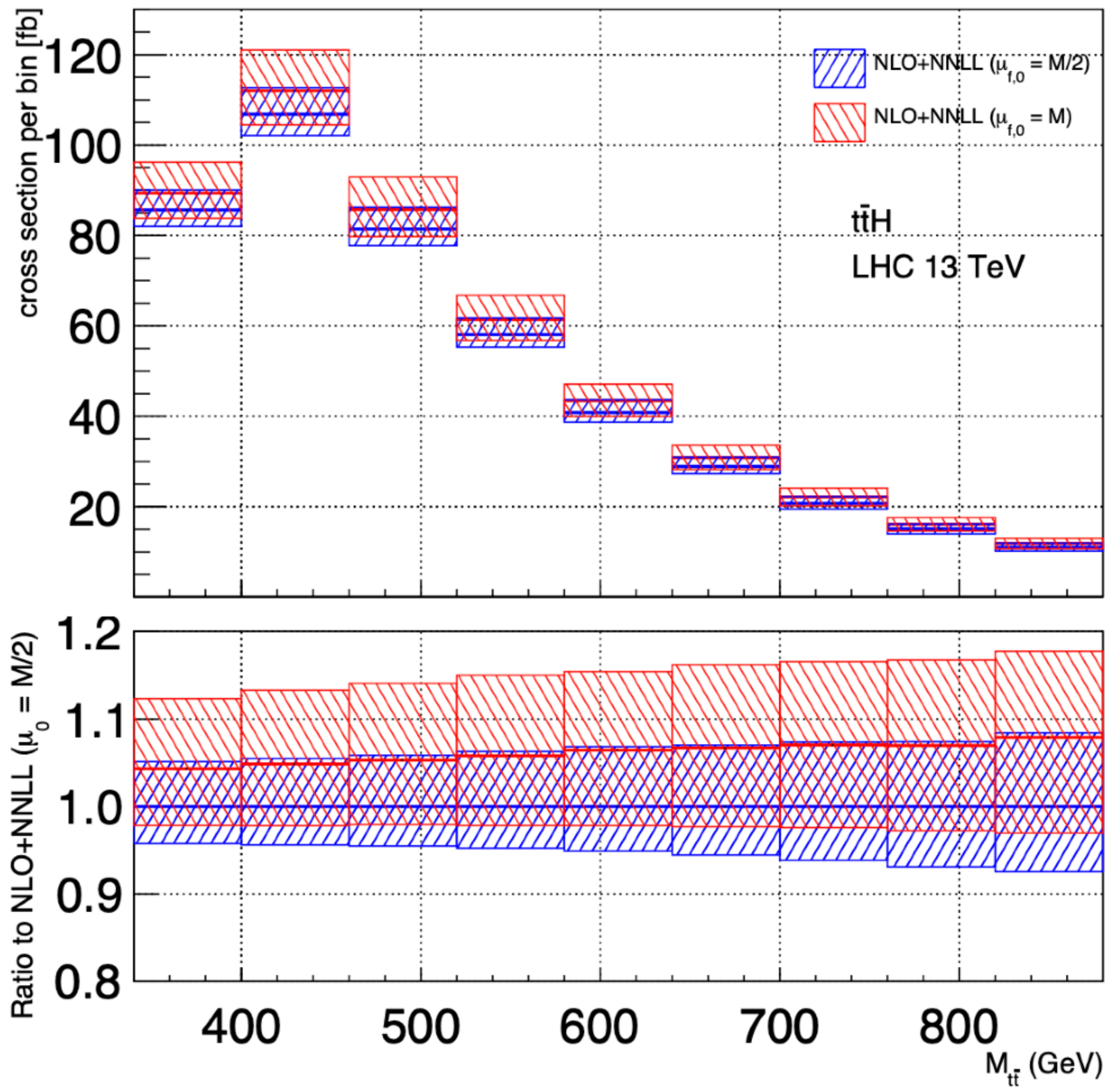
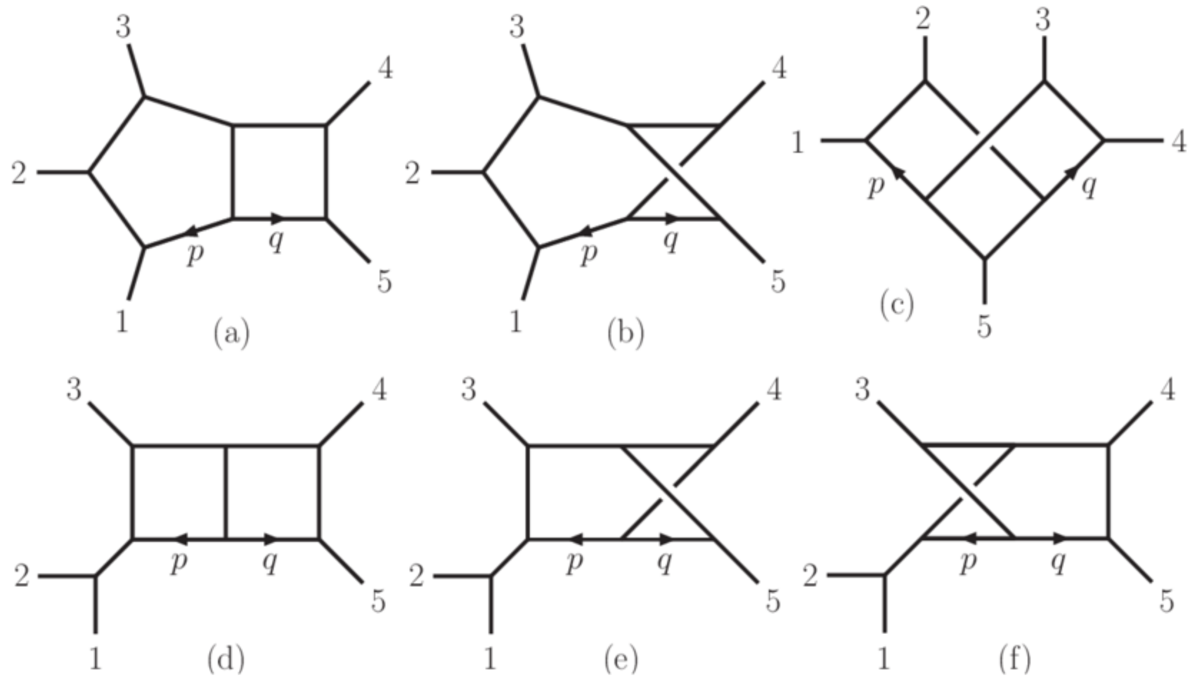
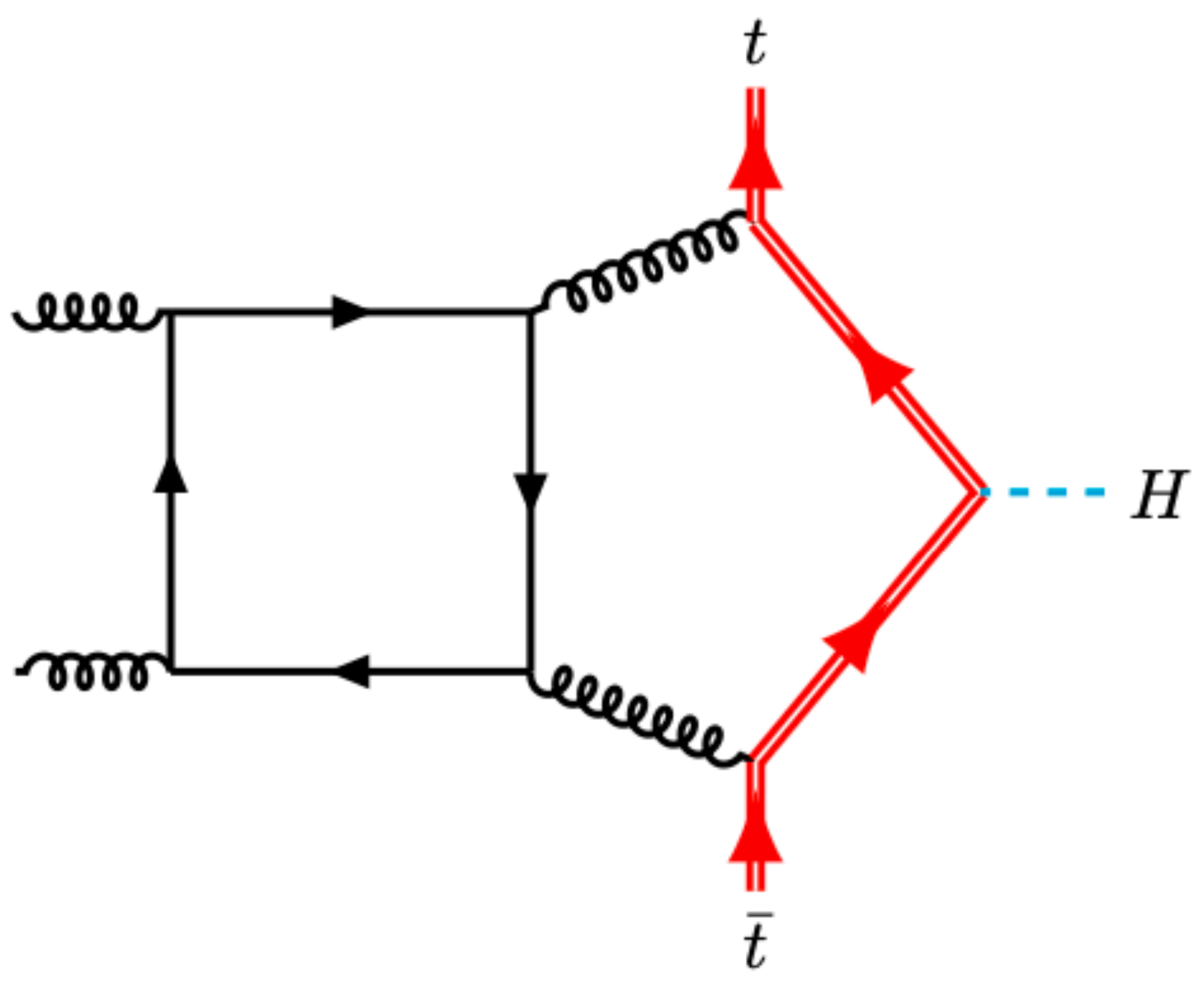


- Bottlenecks towards NNLO
  - Two-loop amplitudes
  - IR subtraction



Two-loop amplitudes for $t\bar{t}H$



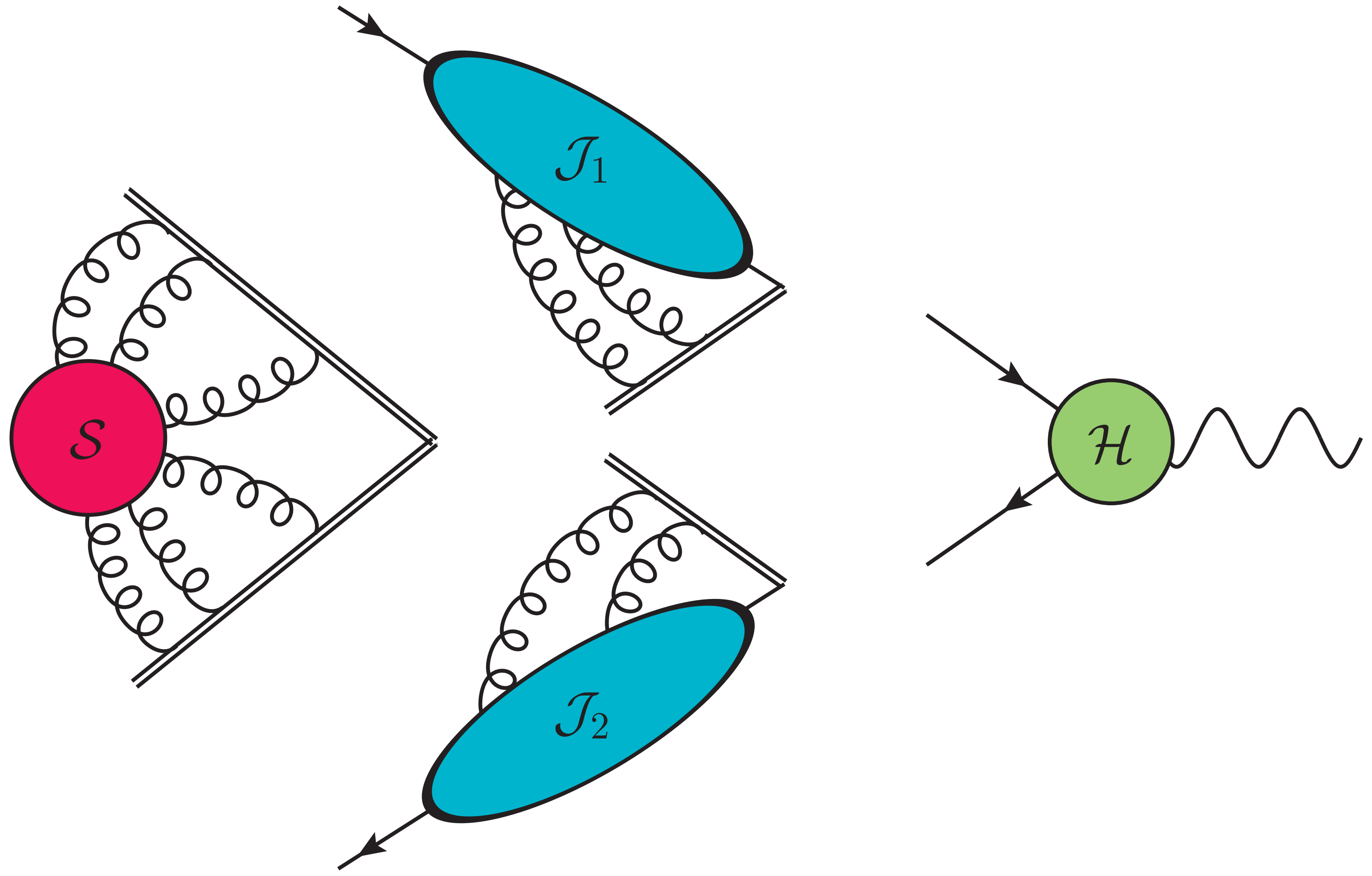
+ many more planar and non-planar families

- Two-loop five-point amplitudes with 7 scales
- Partial results for simpler families e.g.: 2312.08131, 2402.03301
- Full results require much more efforts (analytic + numeric methods)

Two-loop IR singularities

Chen, Ma, Wang, LLY, Ye: 2202.02913

IR singularities of QCD amplitudes admit a universal structure due to soft/collinear factorization



$$Z^{-1}(\epsilon)\,\mathscr{M}^{\text{UV renormalized}}(\epsilon) = \mathcal{O}(\epsilon^0)$$

Two-loop poles = Two-loop Z-factor × One-loop amplitude to ϵ^1

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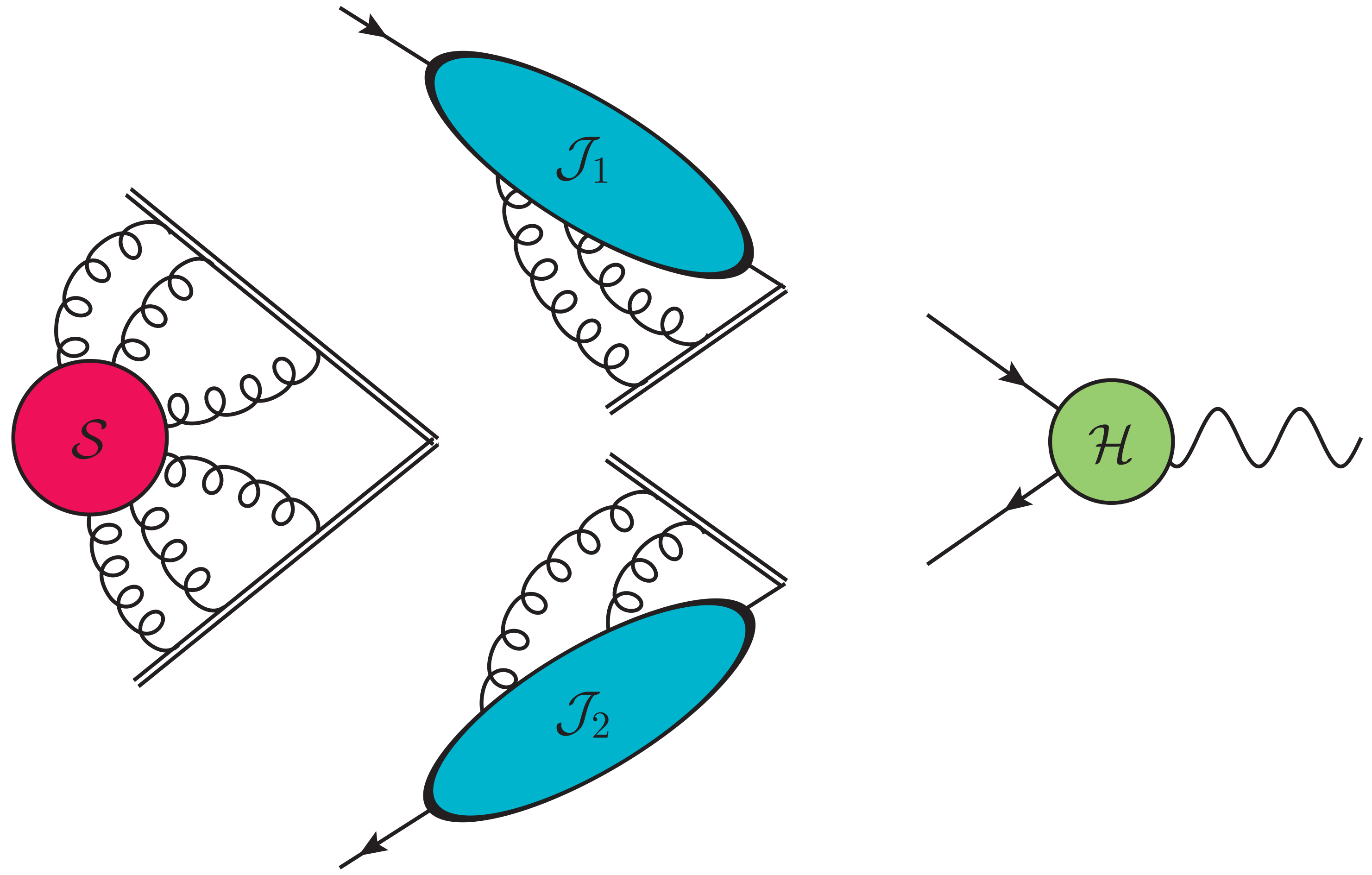

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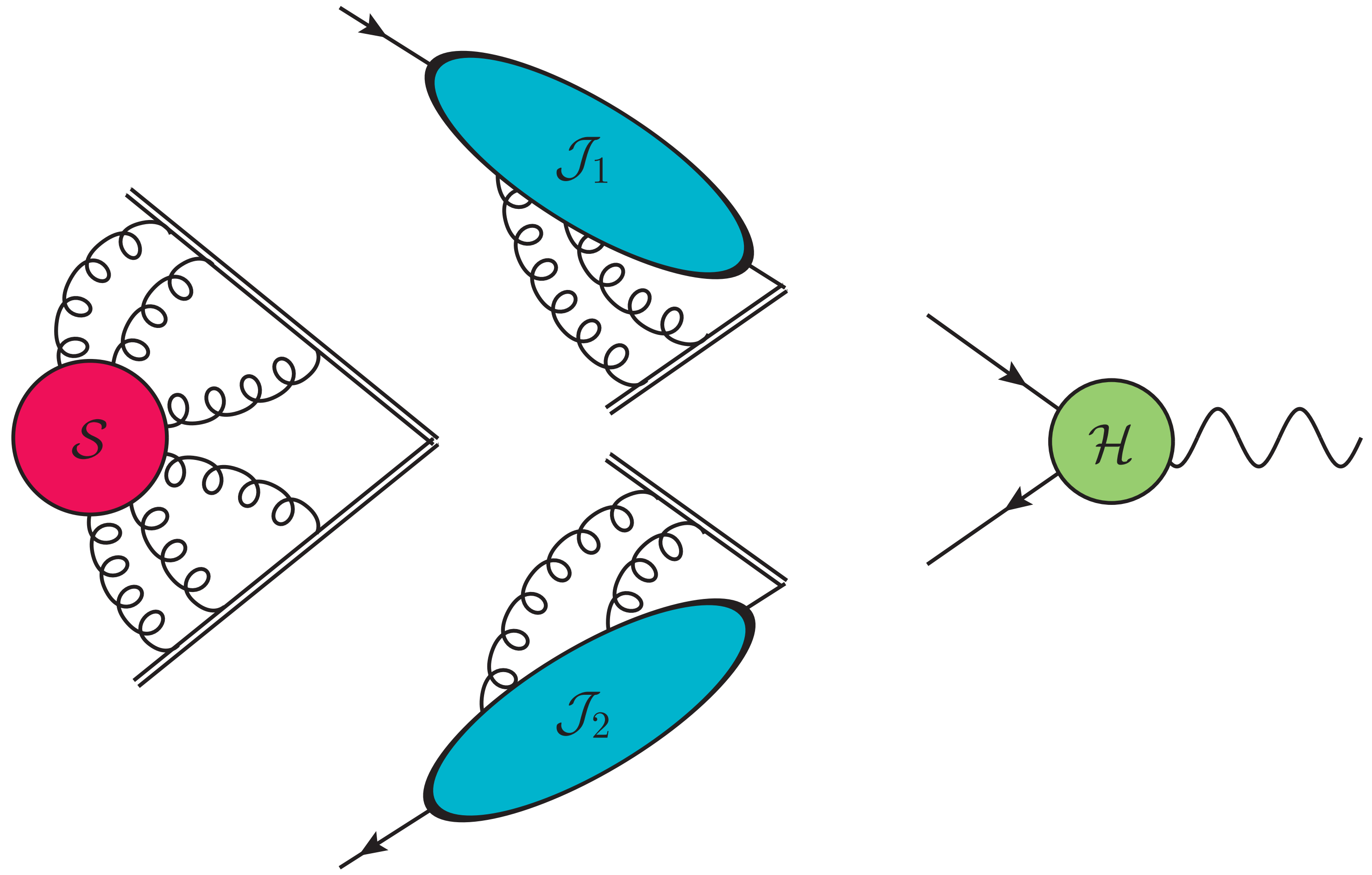

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Chen, Ma, LLY: 2201.12998
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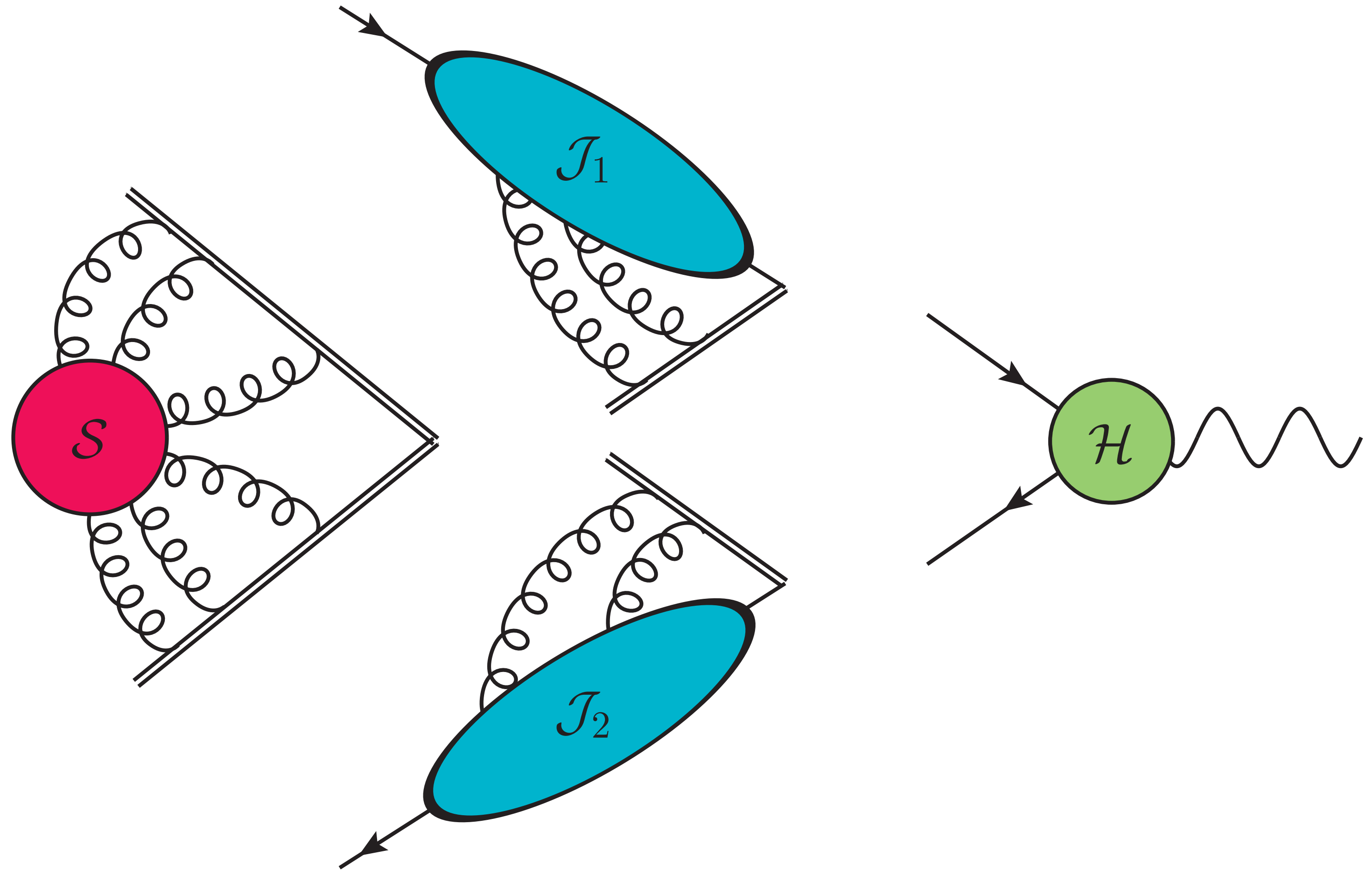

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- ➤ Predict two-loop IR poles for tTH
- ➤ Provide strong check on two-loop amplitudes
- ➤ Validate IR subtraction

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}
A^g	17.37022326	6.277797530	−162.1830217	559.8062598
B^g	−32.49510001	−34.75486260	−624.1343773	3901.332369
C^g		−9.463444735	−54.41556200	−497.5350517
D^g			143.6321997	−578.4857199
E^g_l		−20.26526047	46.54471184	−10.69967085
E^g_h			−24.23013938	79.68650479
F^g_l		37.91095001	−74.94866603	71.66904977
F^g_h			43.70151160	−132.3384924
G^g_l			4.731722368	85.25318119
G^g_h				6.363526190
H^g_l			3.860049613	−10.52987601
H^g_{lh}				8.076713126
H^g_h				
I^g_l			−7.221133335	19.49234494
I^g_{lh}				−14.56717053
I^g_h				
A^q	2.390051823	15.03938540	0.597121534	−34.95784899
B^q	−4.780103646	−22.69017086	49.54607207	106.0851578
C^q	2.390051823	7.650785464	−186.5751188	−21.39439443
D^q_l		−2.390051823	0.308675876	−6.605875838
D^q_h			6.244349191	4.860387981
E^q_l		2.390051823	1.610219156	77.52356965
E^q_h			−6.244349191	19.76269918
F^q_l				
F^q_{lh}				
F^q_h				

Table 1. IR poles decomposed as color coefficients for the phase-space point $x_{12} = 10$, $x_{13} = -1339/920$, $x_{14} = -2269/465$, $x_{23} = -1951/620$, $x_{24} = -1803/1810$ and $x_{34} = 5$.

Off-topic: symbol letters of Feynman integrals

It's very often that Feynman integrals can be written as iterated integrals

$$\int_{\boldsymbol{x}_0}^{\boldsymbol{x}} \mathrm{d}\alpha_{i_n}(\boldsymbol{x}_n) \cdots \int_{\boldsymbol{x}_0}^{\boldsymbol{x}_3} \mathrm{d}\alpha_{i_2}(\boldsymbol{x}_2) \int_{\boldsymbol{x}_0}^{\boldsymbol{x}_2} \mathrm{d}\alpha_{i_1}(\boldsymbol{x}_1)$$

Structure determined by symbol letters

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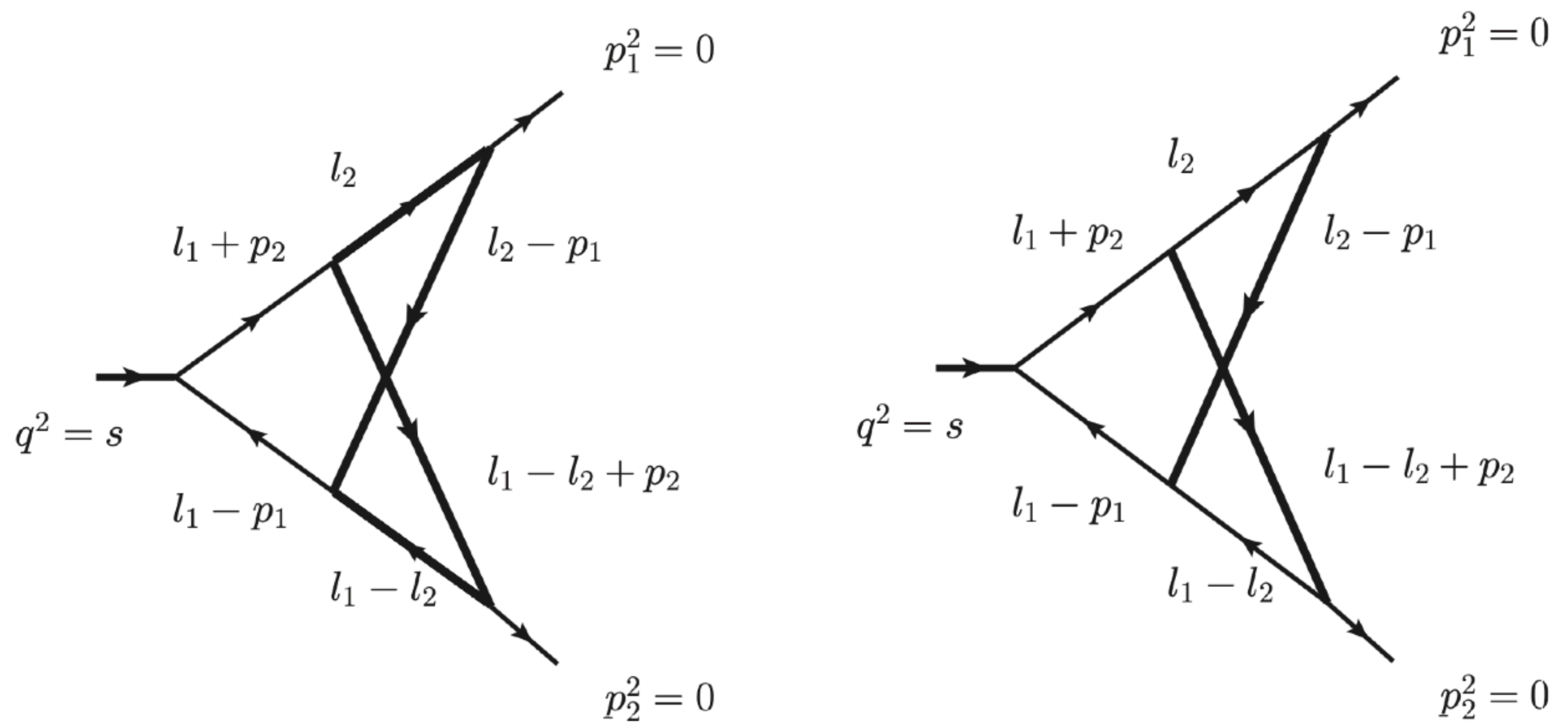
Structure determined by symbol letters

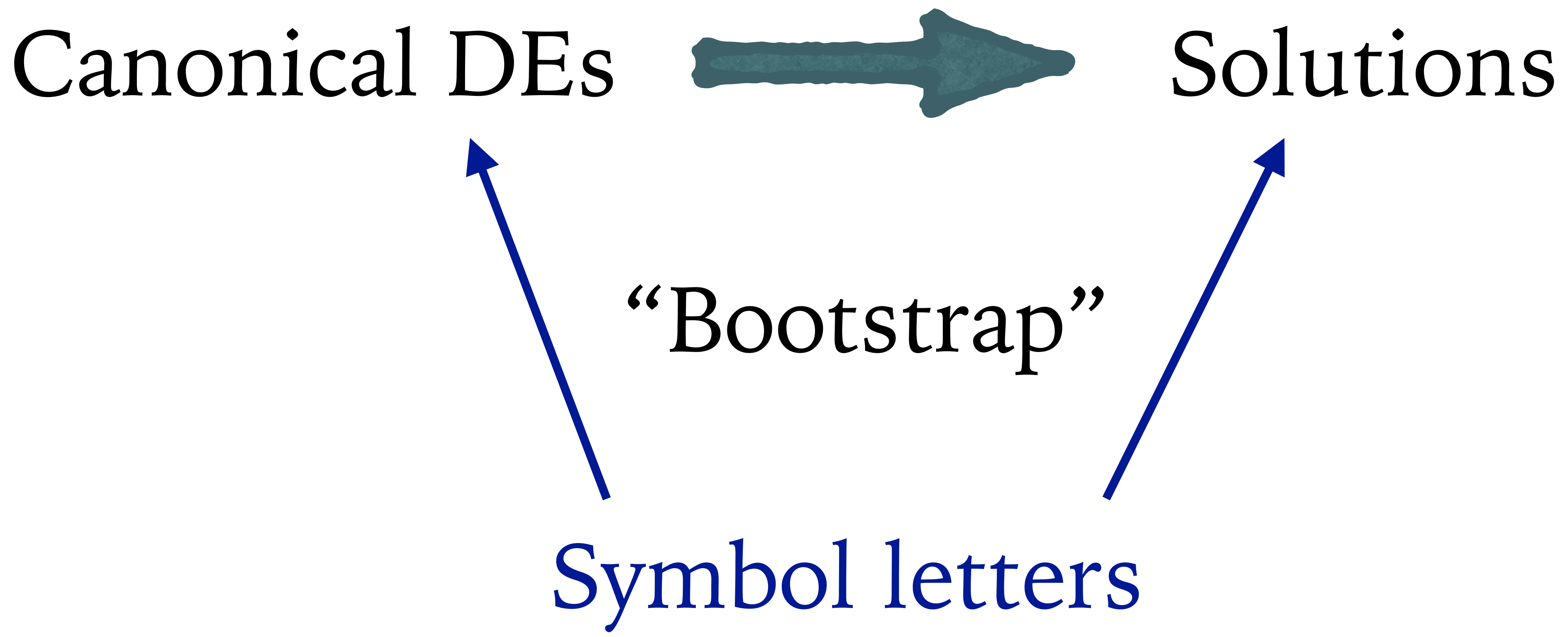
Works for elliptic integrals as well

Jiang, Wang, LLY, Zhao: 2305.13951



$$I(f_1,f_2,\ldots,f_n;\tau,\tau_0) = (2\pi i)^n \int_{\tau_0}^{\tau} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \cdots \int_{\tau_0}^{\tau_{n-1}} d\tau_n\, f_1(\tau_1)\, f_2(\tau_2) \ldots f_n(\tau_n)$$

Bottom-up approach: from symbol letters to Feynman integrals



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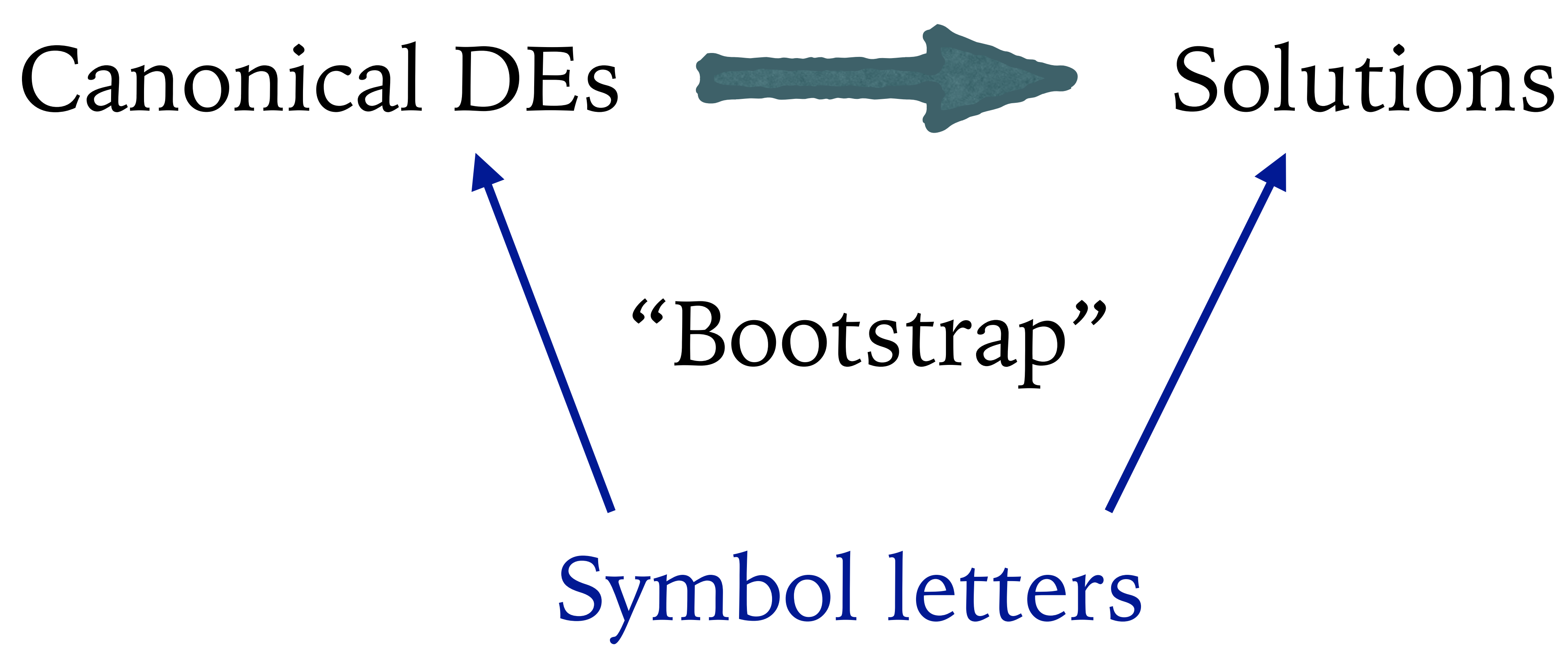

Many efforts trying to construct symbol letters, e.g.:

Chen, Jiang, Xu, LLY: 2008.03045
Chen, Ma, LLY: 2201.12998
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Jiang, LLY: 2303.11657
Chen, Feng, LLY: 2305.01283

Bottom-up approach: from symbol letters to Feynman integrals

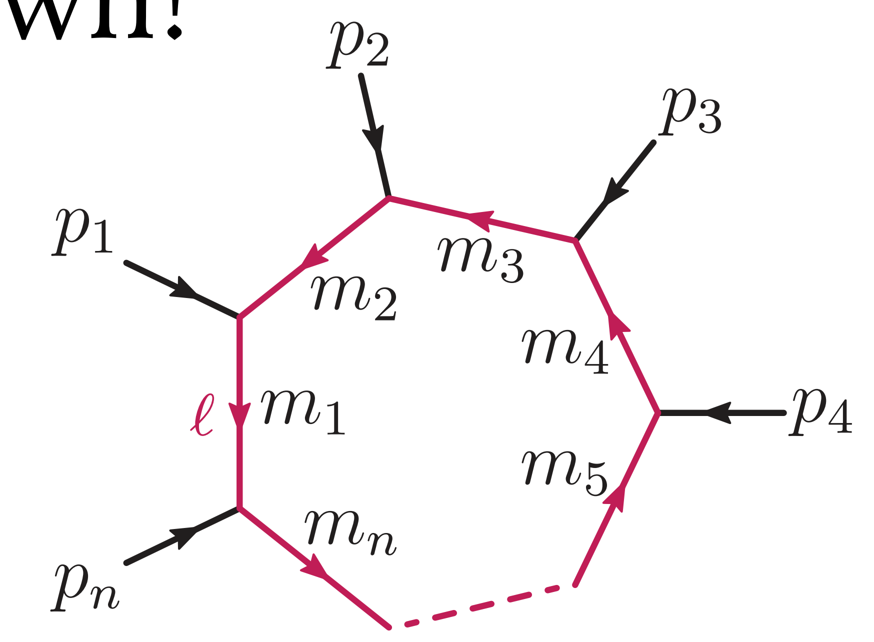
Canonical DEs → Solutions

"Bootstrap"

Symbol letters

Canonical bases and symbol letters of one-loop integrals completely known!

→ Arbitrary order in ϵ



$$g_N\big|_{N\ \text{odd}} = \frac{\epsilon^{(N+1)/2}}{(4\pi)^{(N-1)/2}\,\Gamma(1-\epsilon)} \int \left(-\frac{\mathcal{K}_N}{G_N(\boldsymbol{z})}\right)^{\epsilon} \prod_{i=1}^{N} \frac{dz_i}{z_i}\,,$$

$$g_N\big|_{N\ \text{even}} = \frac{\epsilon^{N/2}}{(4\pi)^{(N-1)/2}\,\Gamma(1/2-\epsilon)} \int \frac{\sqrt{G_N(\boldsymbol{0})}}{\sqrt{G_N(\boldsymbol{z})}} \left(-\frac{\mathcal{K}_N}{G_N(\boldsymbol{z})}\right)^{\epsilon} \prod_{i=1}^{N} \frac{dz_i}{z_i}\,,$$

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$$d\log \frac{G(\{q_1,q_2,\ldots,q_n,l\},\{q_1,q_2,\ldots,q_n,q_{n+1}\}) - \sqrt{-G(q_1,\ldots,q_n)G(q_1,\ldots,q_{n+1},l)}}{G(\{q_1,q_2,\ldots,q_n,l\},\{q_1,q_2,\ldots,q_n,q_{n+1}\}) + \sqrt{-G(q_1,\ldots,q_n)G(q_1,\ldots,q_{n+1},l)}}$$

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Input for two-loop IR poles!

A new algorithmic approach

Jiang, Liu, Xu, LLY: 2401.07632

https://github.com/windfolgen/Baikovletter

Based on:

- ➤ Recursive structure of Baikov representations
- ➤ Landau singularities for rational letters
- ➤ Generic ansatz for algebraic letters

$$d\log \frac{G(\{q_1,q_2,\ldots,q_n,l\},\{q_1,q_2,\ldots,q_n,q_{n+1}\}) - \sqrt{-G(q_1,\ldots,q_n)G(q_1,\ldots,q_{n+1},l)}}{G(\{q_1,q_2,\ldots,q_n,l\},\{q_1,q_2,\ldots,q_n,q_{n+1}\}) + \sqrt{-G(q_1,\ldots,q_n)G(q_1,\ldots,q_{n+1},l)}}$$

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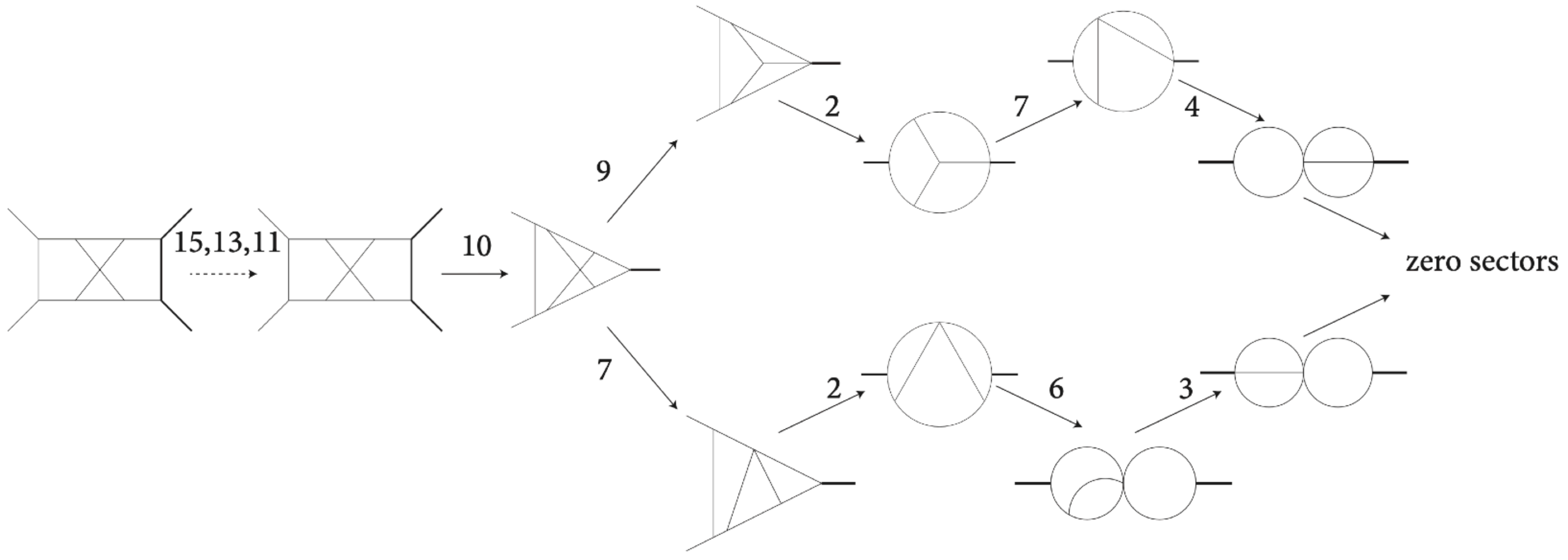

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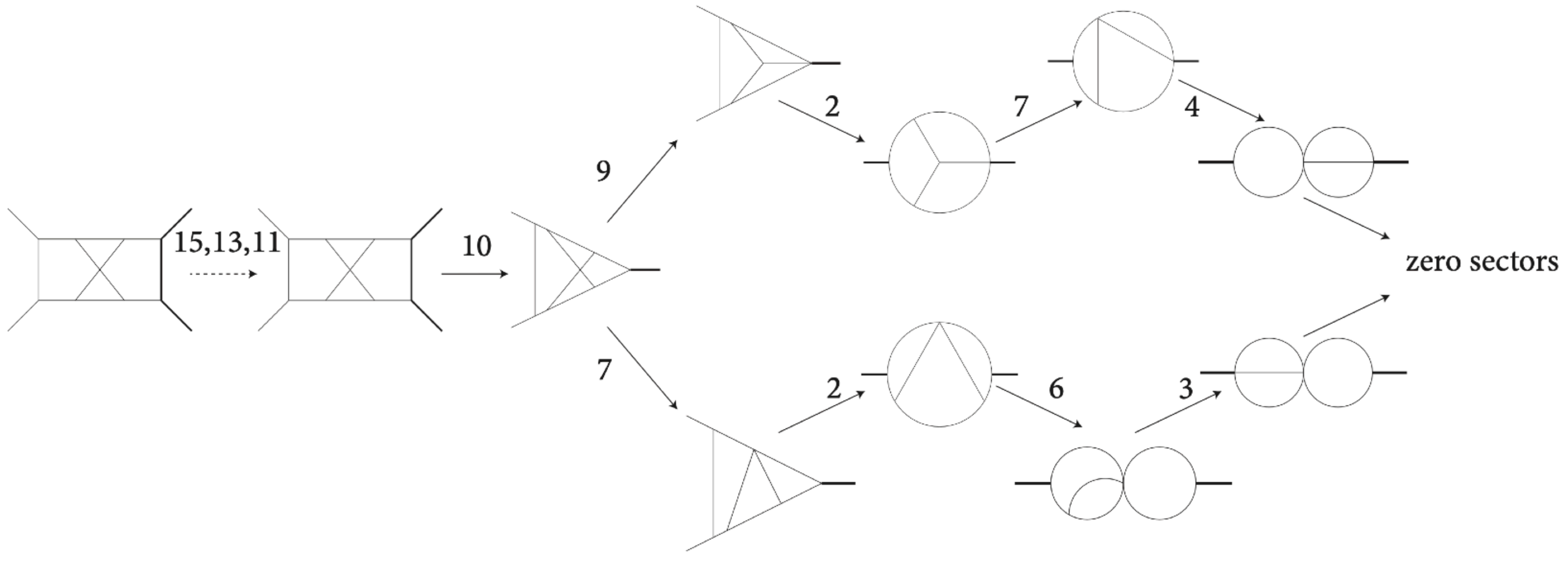

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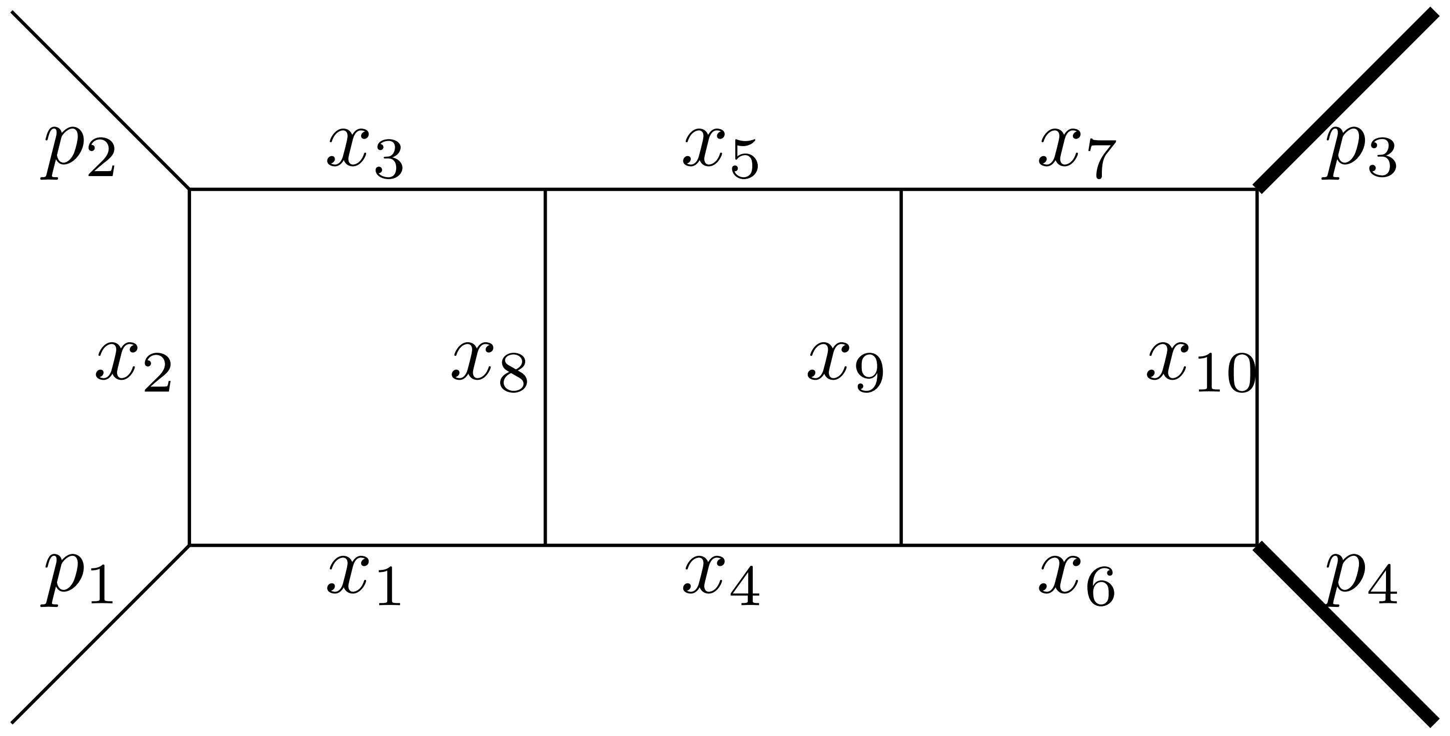
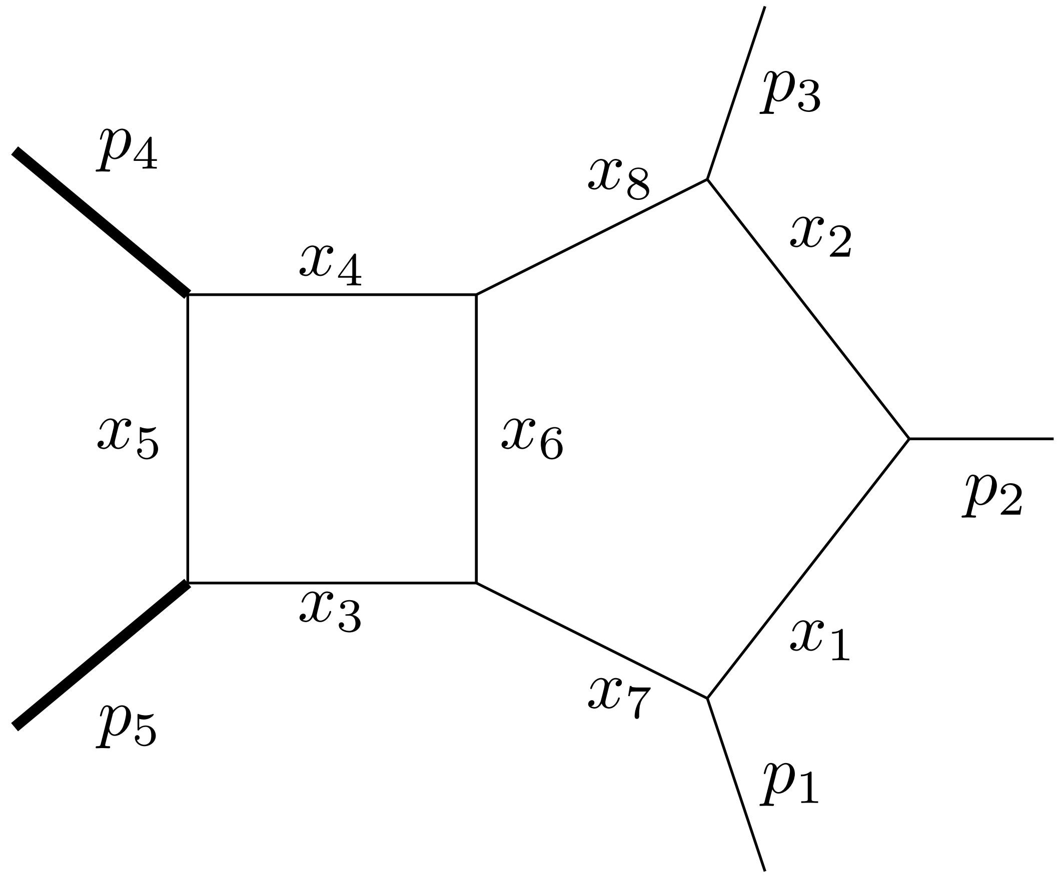
Tested in many non-trivial examples, providing new results not available in the literature!





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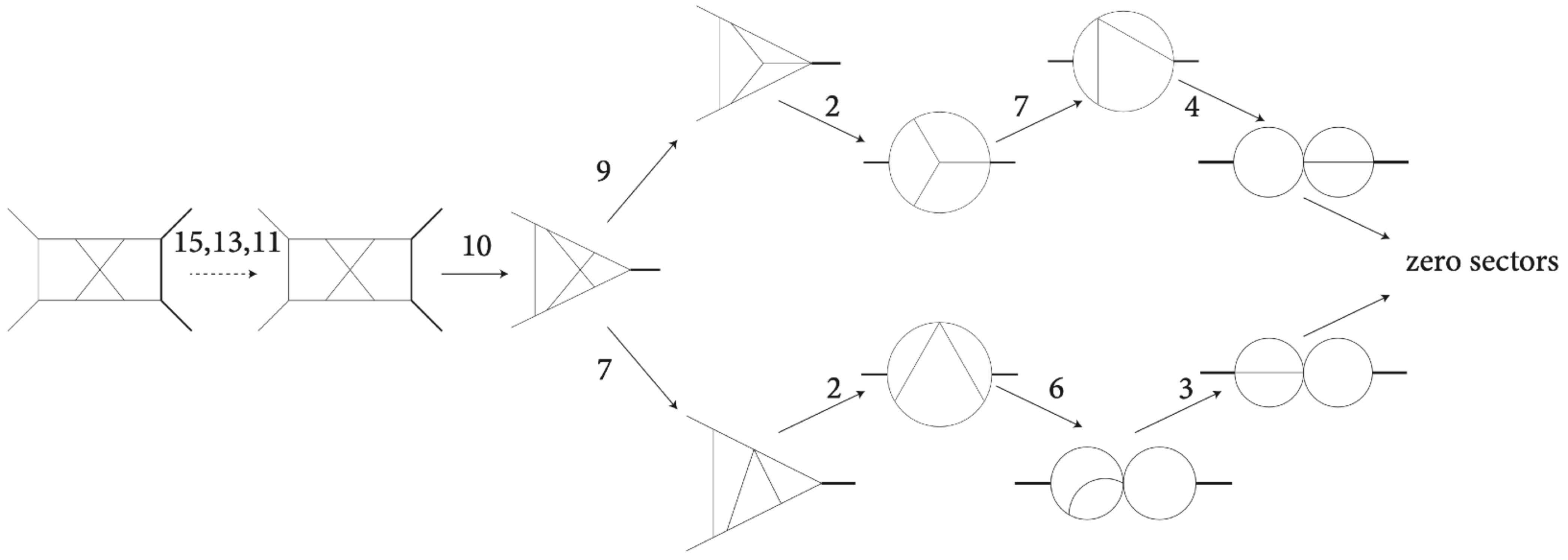

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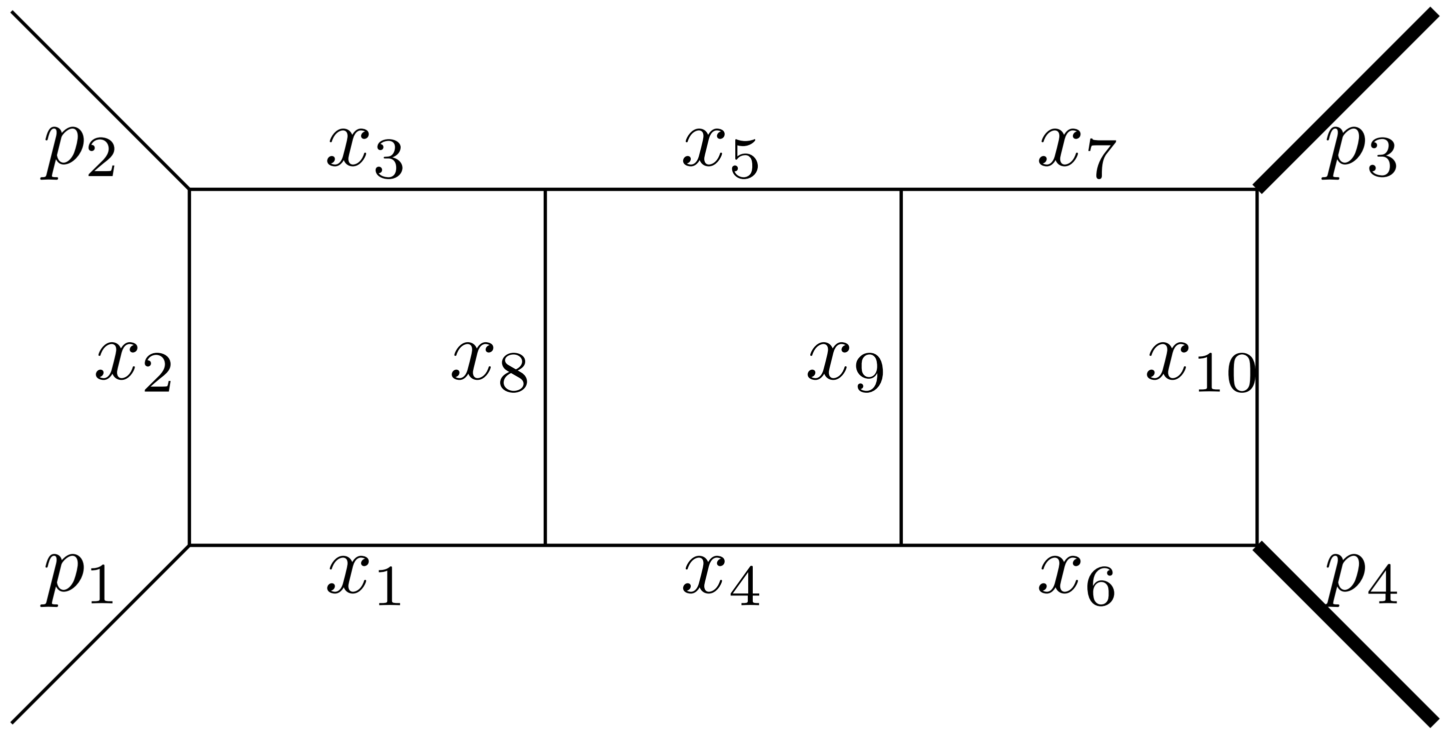
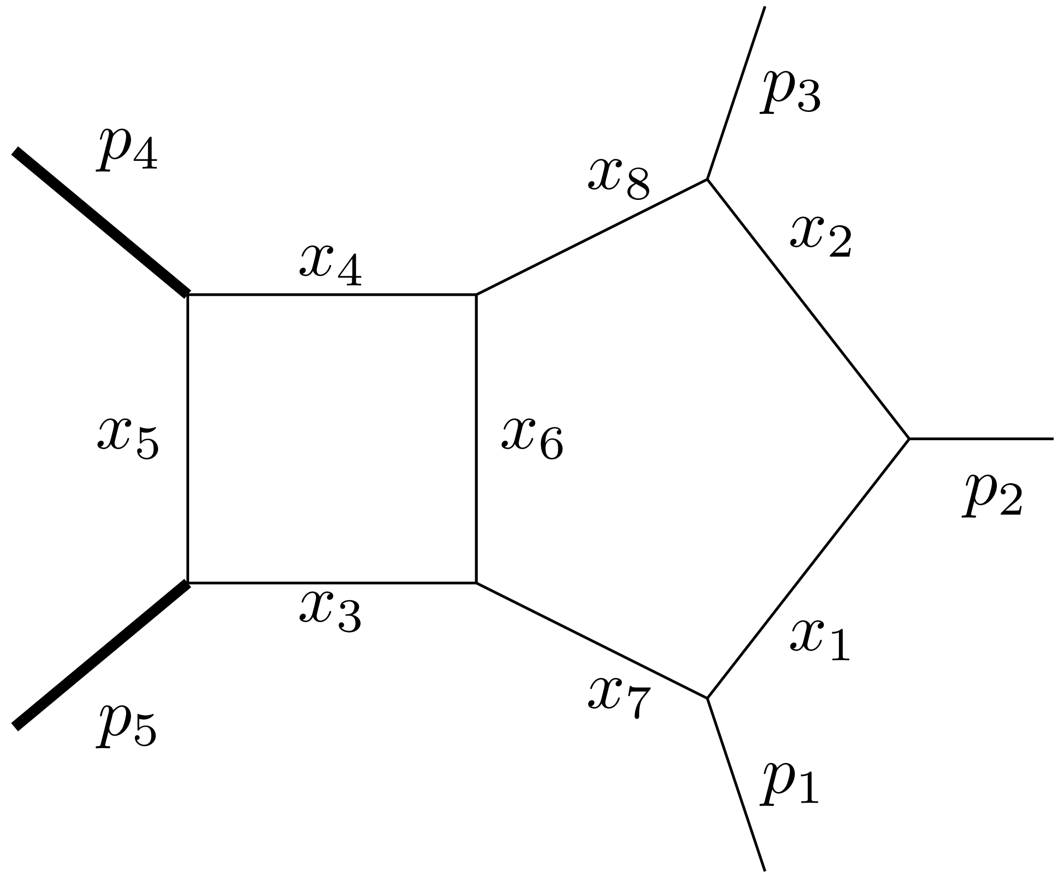




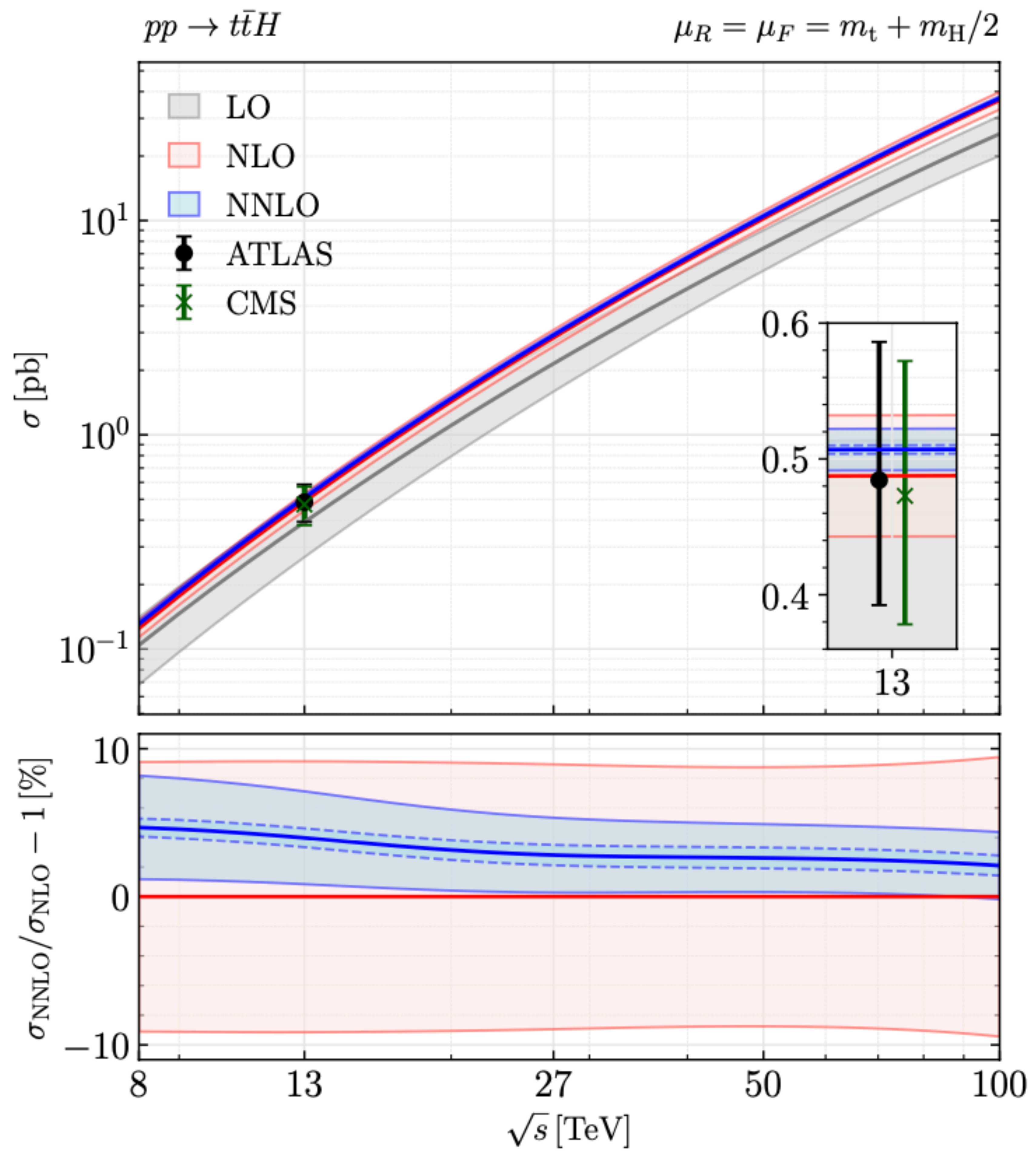
But, tTH is still difficult… seeking approximations

Approximation with soft Higgs

Catani et al.: 2210.07846

Eikonal approximation: 2 → 2 kinematics

$$\mathcal{M}(\{p_i\}, k) \simeq F(\alpha_S(\mu_R); \tfrac{m_t}{\mu_R}) \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}(\{p_i\})$$



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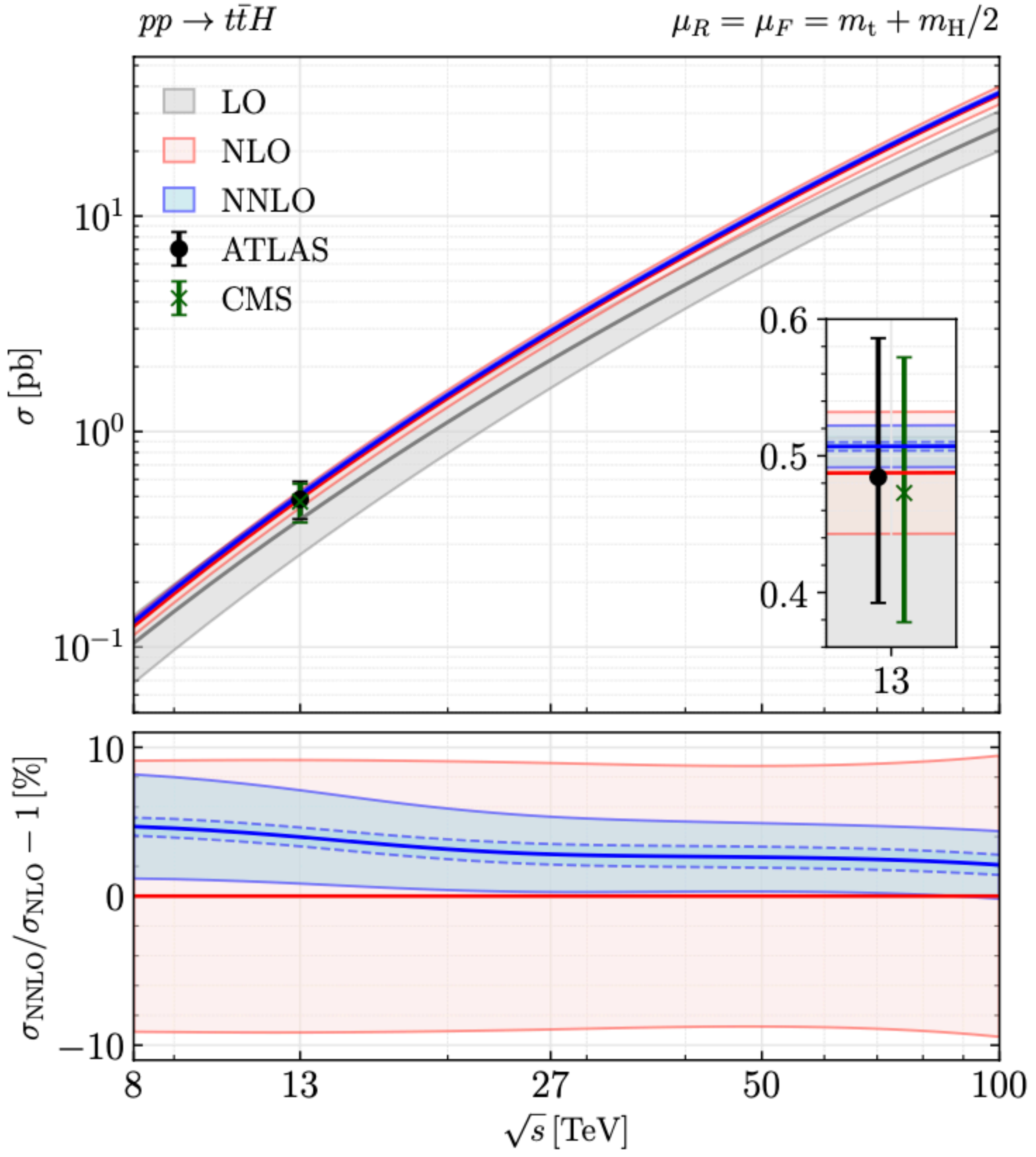
Not a good approximation for two-loop amplitudes:

- One-loop already 30% error
- Two-loop estimated 100% error



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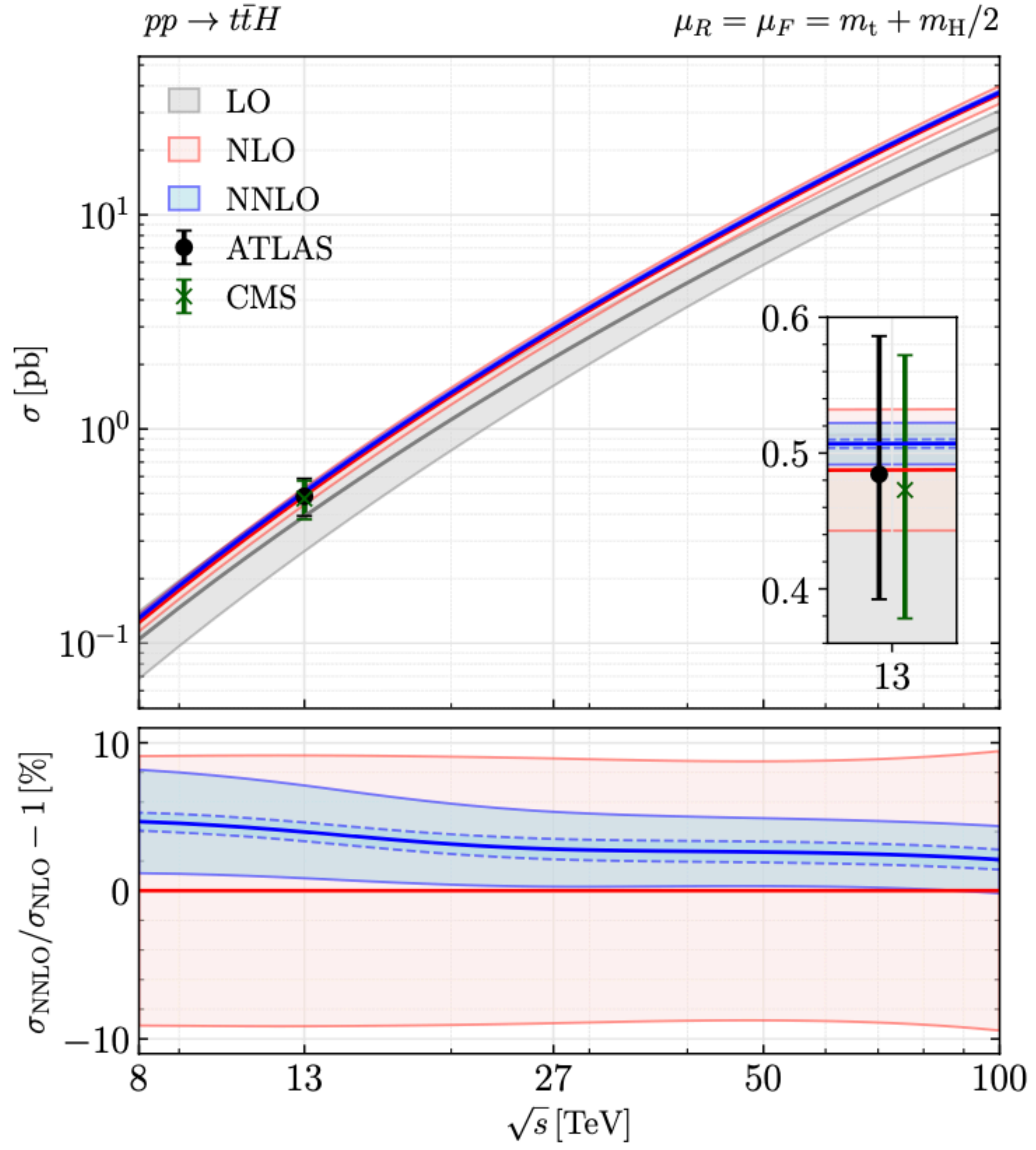

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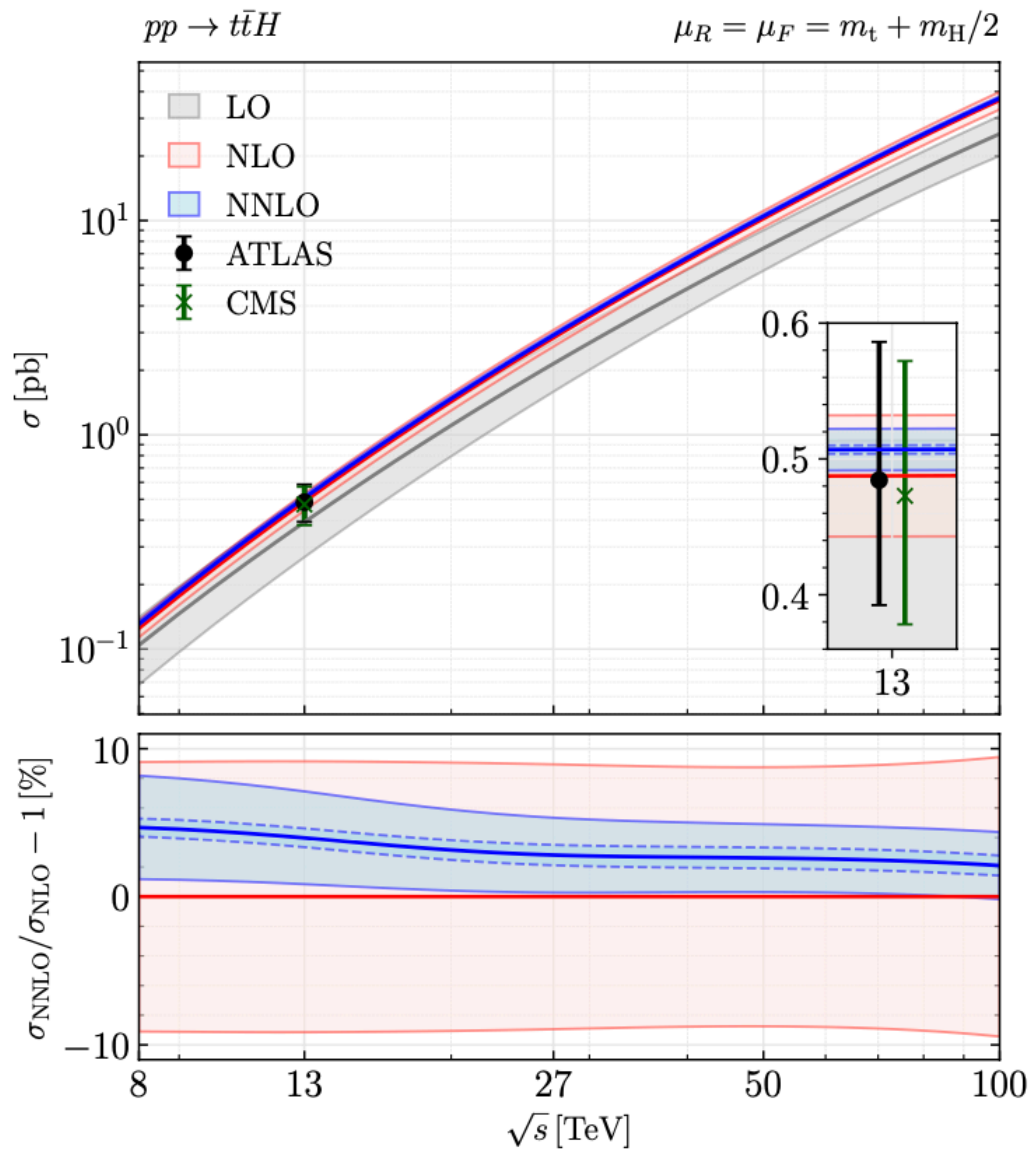

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What about differential cross sections?

Approximation in the high energy limit

It is known that a massive amplitude can be factorized into a massless amplitude and a collinear factor for each leg in the high-energy limit

Mitov, Moch: hep-ph/0612149

$$\mathcal{M}^{[p],(m)}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) =$$

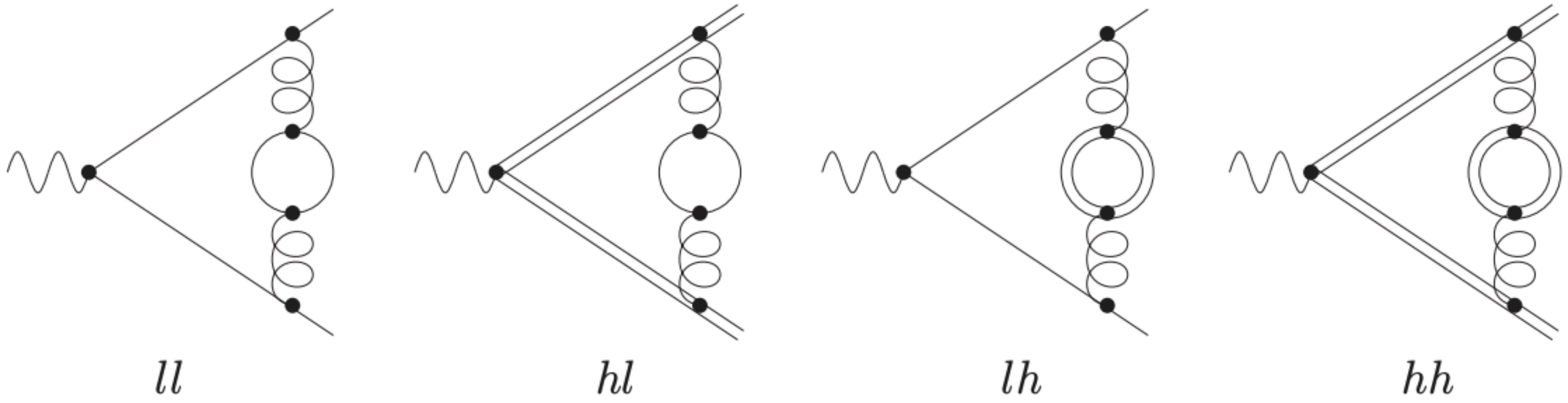
$$\prod_{i\in\,\{\text{all legs}\}} \left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right)\right)^{\frac{1}{2}} \times \mathcal{M}^{[p],(m=0)}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right)$$

Approximation in the high energy limit

It is known that a massive amplitude can be factorized into a massless amplitude and a collinear factor for each leg in the high-energy limit

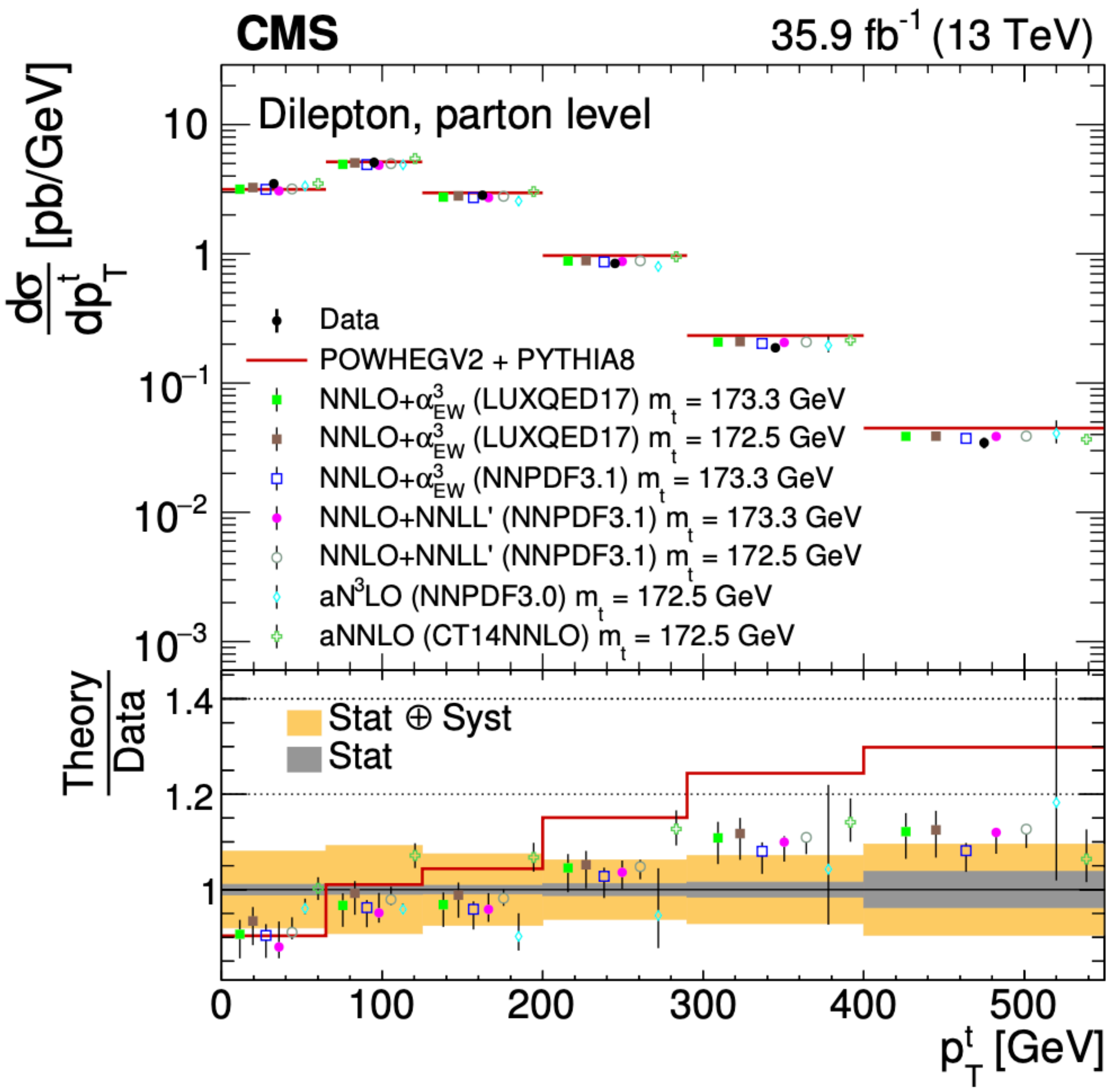
Mitov, Moch: hep-ph/0612149

$$\mathcal{M}^{[\mathrm{p}],(m)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right) = \prod_{i\in\,\{\text{all legs}\}}\left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)\right)^{\frac{1}{2}} \times \mathcal{M}^{[\mathrm{p}],(m=0)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)$$



But the heavy-quark bubbles are not included!

Top quark pair production

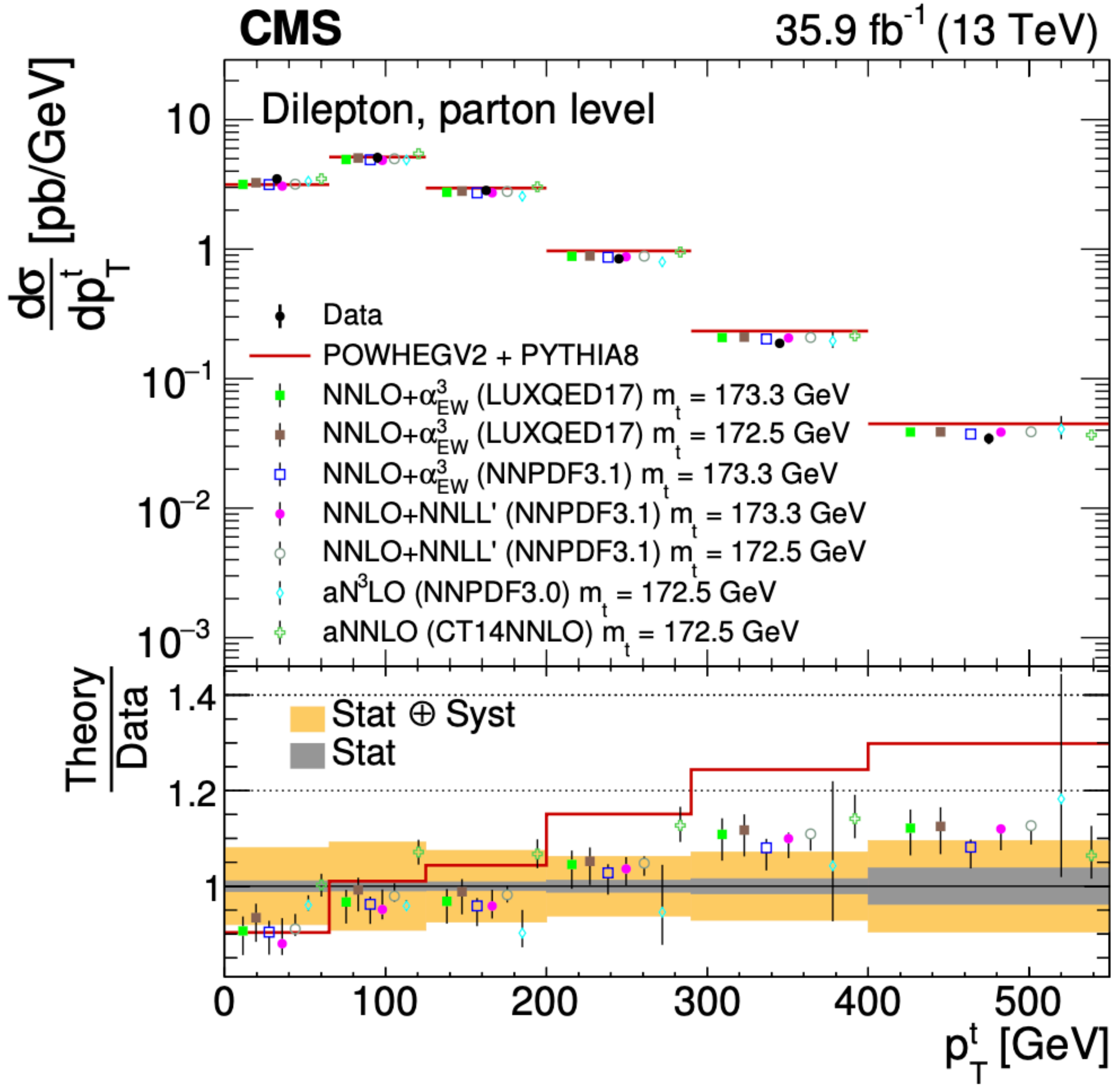


High energy factorization has been applied in the resummation for top quark pair production

1205.3662
1306.1537
1310.3836
1601.07020
1803.07623
1901.08281

Best precision:
NNLO+NNLL' in QCD + NLO in EW

Top quark pair production



High energy factorization has been applied in the resummation for top quark pair production

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Best precision:
NNLO+NNLL' in QCD + NLO in EW

But the factorization of heavy quark bubbles was not understood…

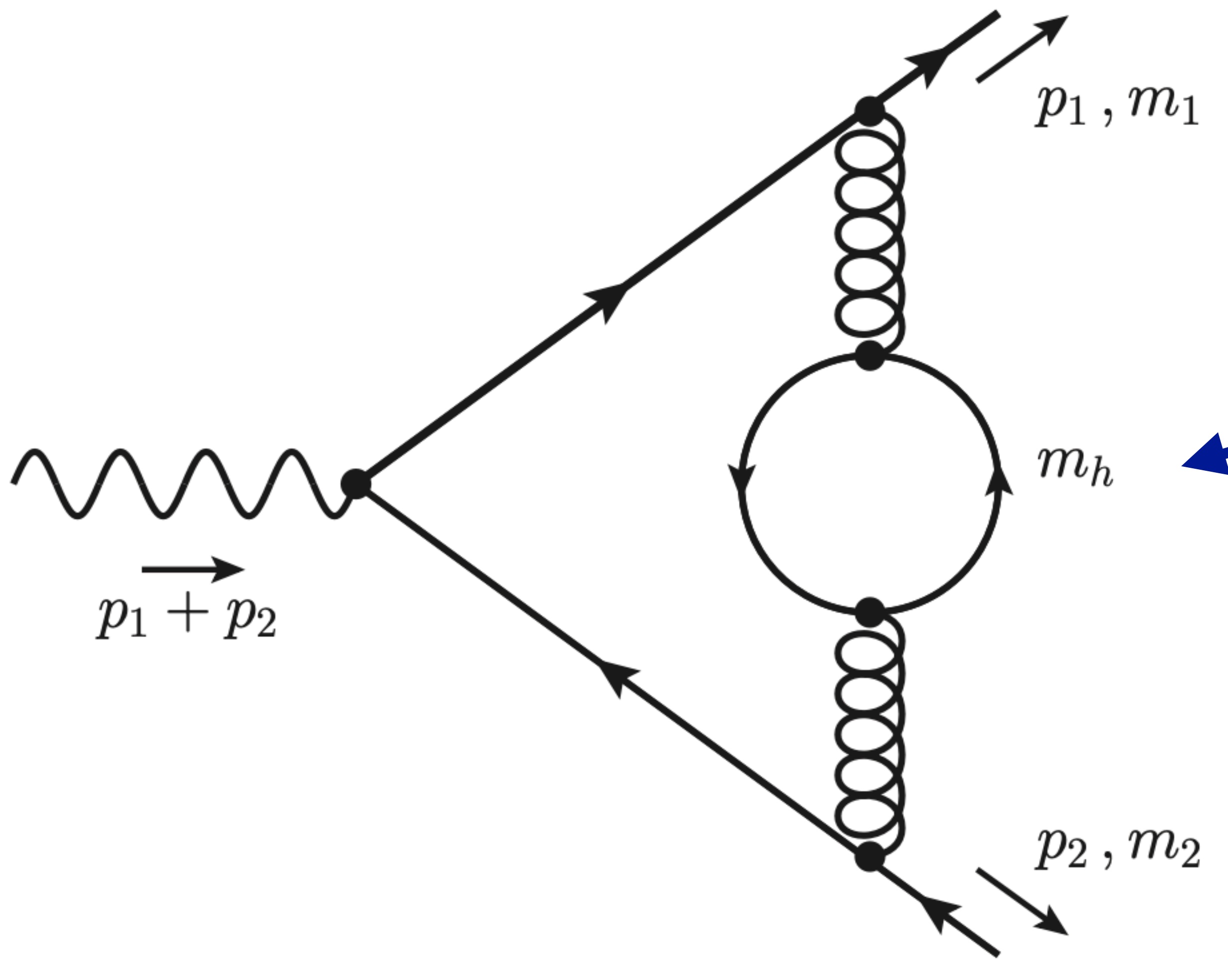
Heavy-quark bubbles

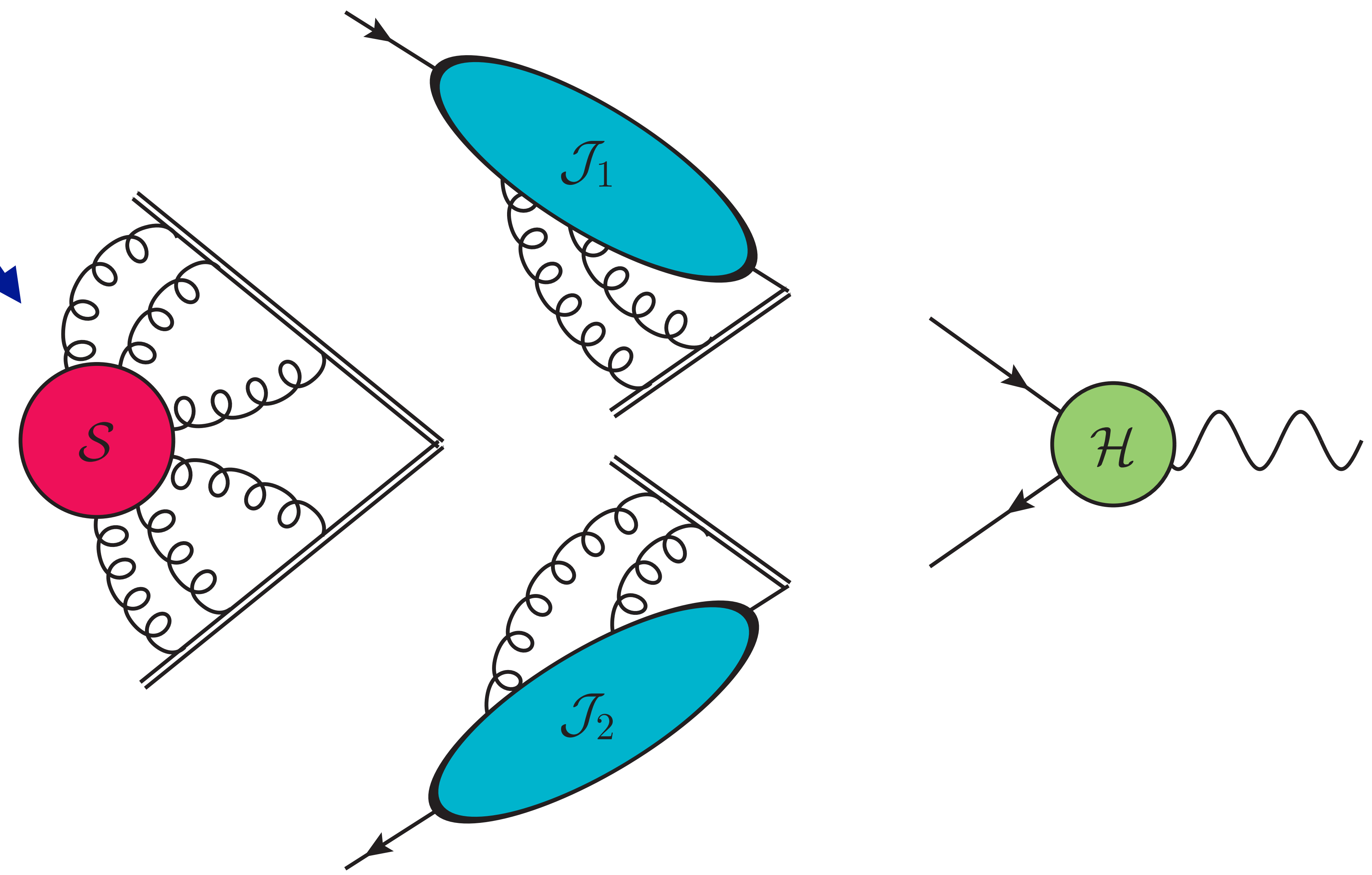
Wang, Xia, LLY, Ye: 2312.12242

A new factorization formula

$$\left|\mathcal{M}^{\mathrm{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\mathrm{massless}}(\{p\})\right\rangle$$

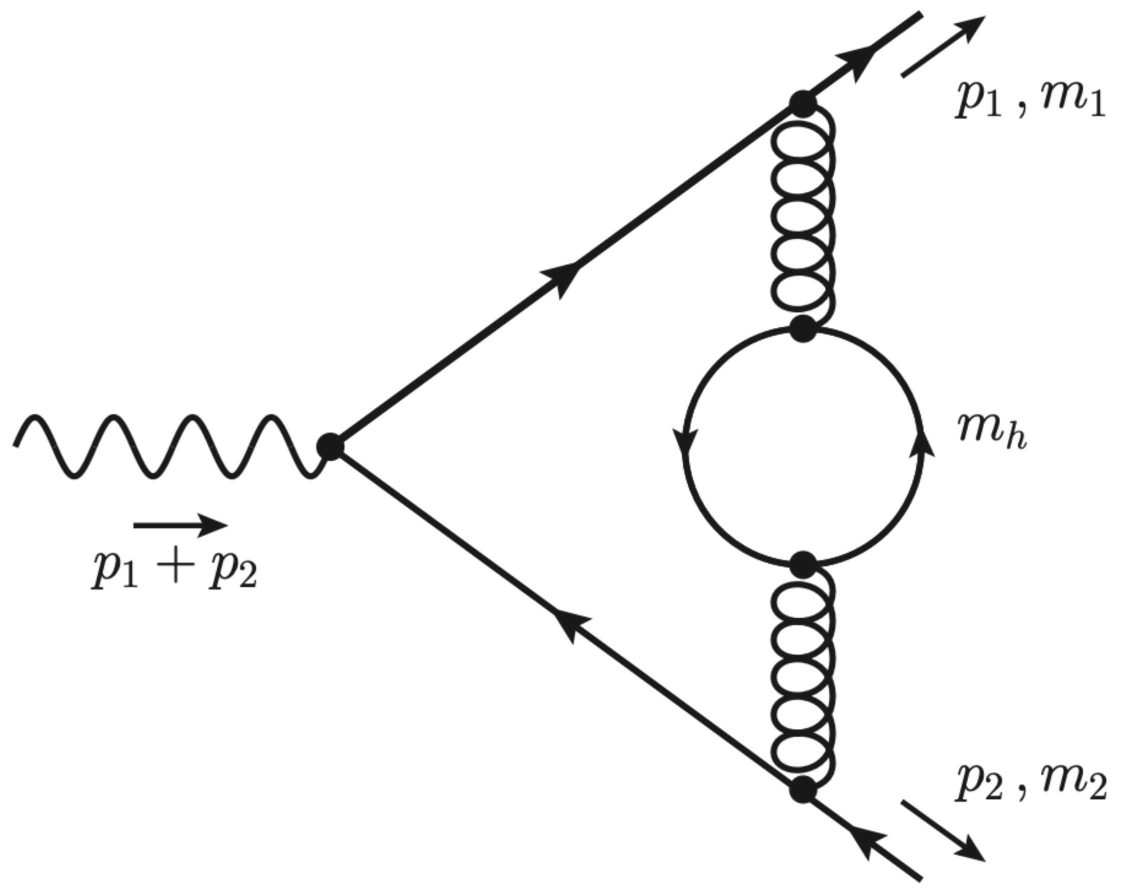
A new soft function





The new soft function

Wang, Xia, LLY, Ye: 2312.12242



$$\text{hard} : k^\mu \sim \sqrt{|s|}\,,$$
$$n_i\text{-collinear} : (n_i\cdot k,\, \bar{n}_i\cdot k,\, k_\perp) \sim \sqrt{|s|}\,(\lambda^2,\, 1,\, \lambda)$$
$$\text{soft} : k^\mu \sim \sqrt{|s|}\,\lambda\,.$$

Rapidity divergence: analytic regulator

$$I_{\{a_i\}} \equiv \mu^{4\epsilon}\int \frac{dk_1}{(2\pi)^d}\frac{dk_2}{(2\pi)^d}\frac{1}{[k_1^2-m_h^2]^{a_1}}\frac{1}{[k_2^2-m_h^2]^{a_2}}\frac{1}{[(k_1+k_2)^2]^{a_3}}\frac{1}{[(k_1+k_2-p_1)^2-m_1^2]^{a_4}}$$
$$\times\frac{(-\tilde{\mu}^2)^\nu}{[(k_1+k_2+p_2)^2-m_2^2]^{a_5+\nu}}\frac{1}{[(k_1-p_1)^2]^{a_6}}\frac{1}{[(k_1+p_2)^2]^{a_7}}\,, \quad (3.4)$$

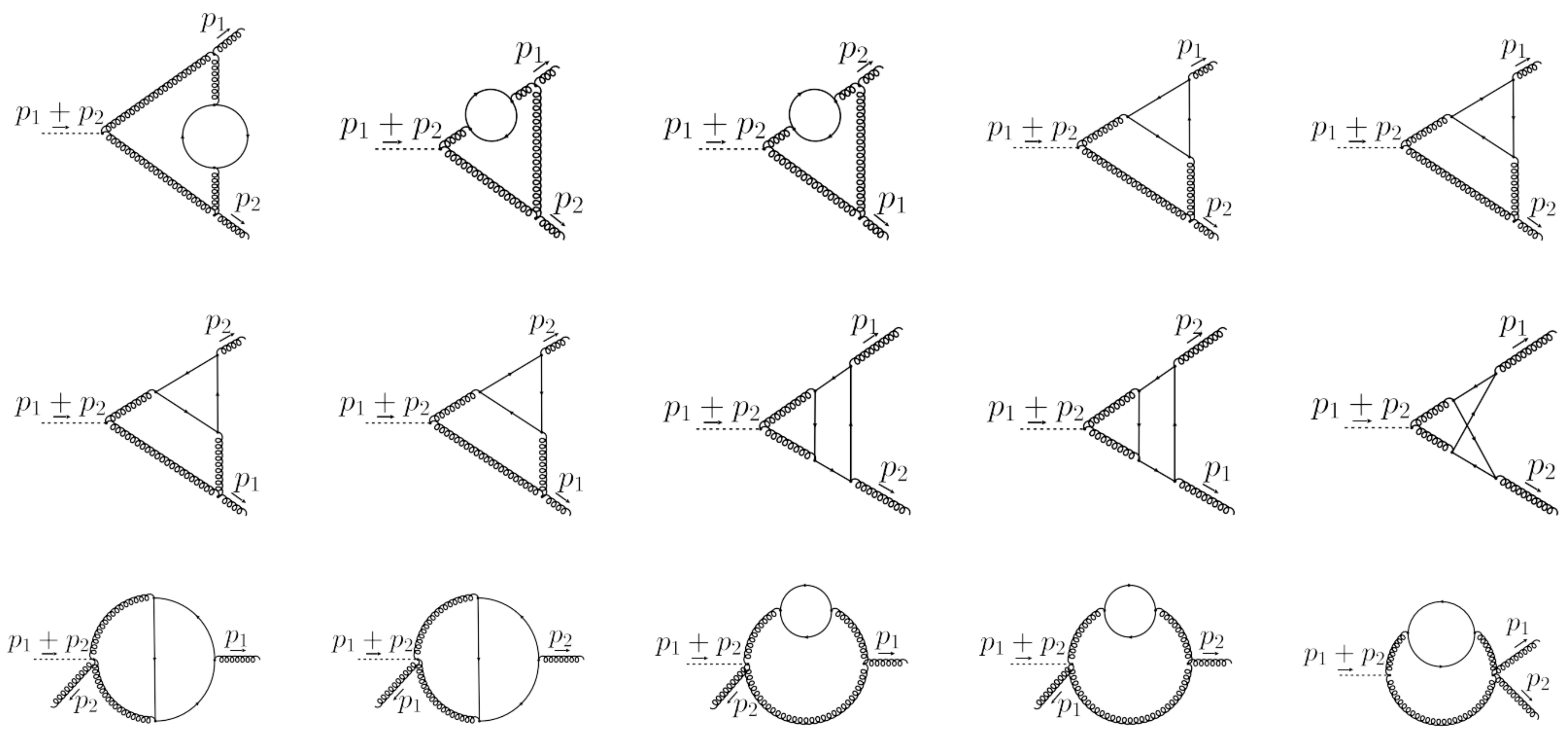
$$\boldsymbol{\mathcal{S}}(\{p\},\{m\}) = 1+\left(\frac{\alpha_s}{4\pi}\right)^2\sum_{\substack{i,j\\ i\neq j}}(-\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\sum_h \mathcal{S}^{(2)}(s_{ij},m_h^2)+\mathcal{O}(\alpha_s^3)$$

$$\mathcal{S}^{(2)}(s_{ij},m_h^2) = T_F\left(\frac{\mu^2}{m_h^2}\right)^{2\epsilon}\left(-\frac{4}{3\epsilon^2}+\frac{20}{9\epsilon}-\frac{112}{27}-\frac{4\zeta_2}{3}\right)\ln\frac{-s_{ij}}{m_h^2}$$

Validation of the new formula

Wang, Xia, LLY, Ye: 2312.12242

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$



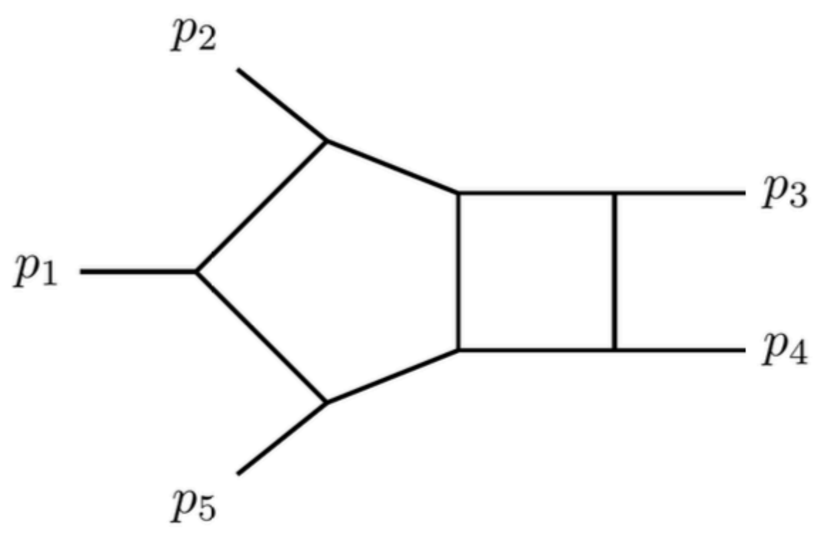
Checked in various situations:

- Quark form factors: heavy-heavy, heavy-light, light-light
- Gluon form factor
- Top quark pair amplitude

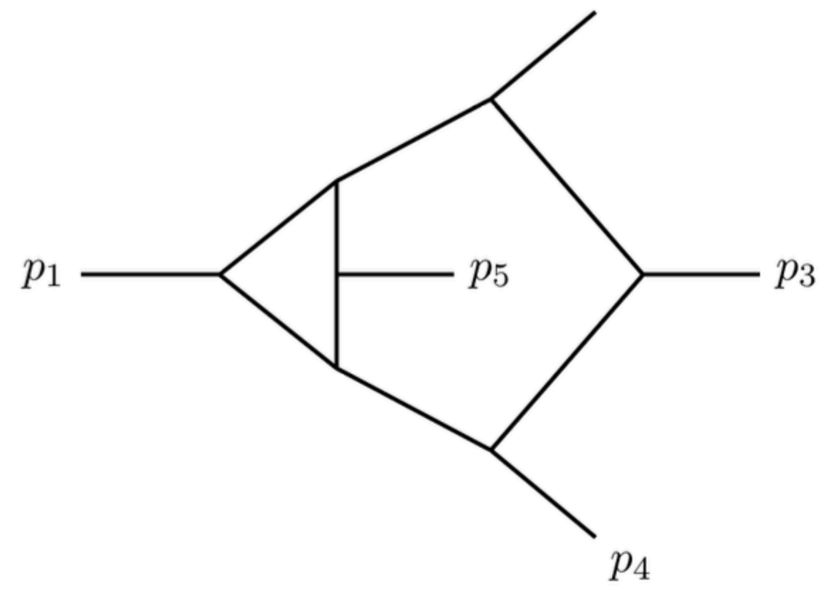
Two-loop amplitudes for tTH in the high-energy limit

Wang, Xia, LLY, Ye: 2402.00431

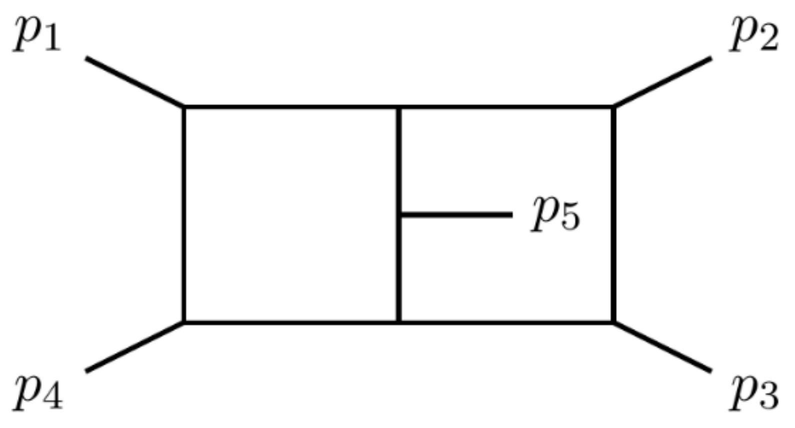
$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$



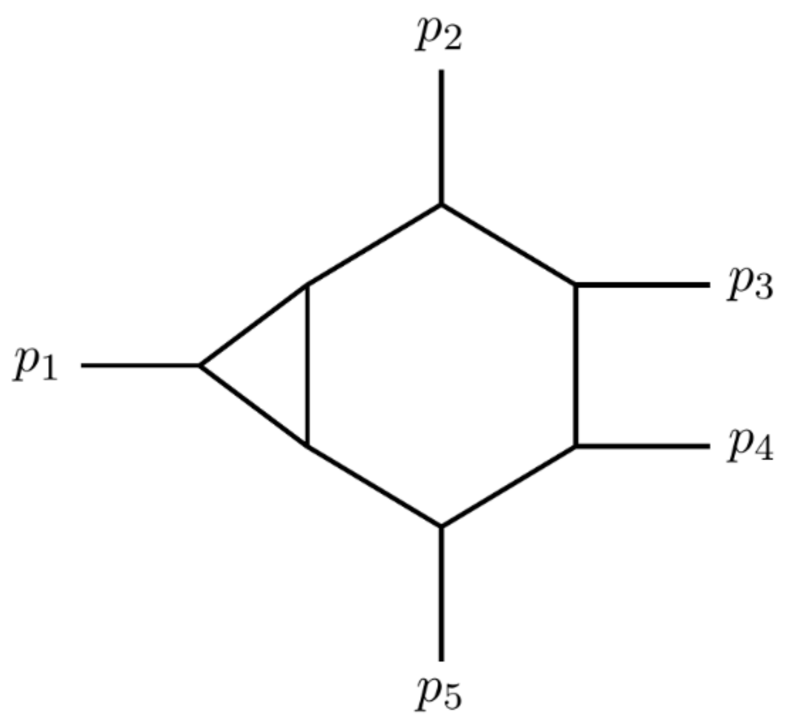
(a) planar pentagon-box (PB)



(b) non-planar hexagon-box (HB)



(c) non-planar double pentagon (DP)

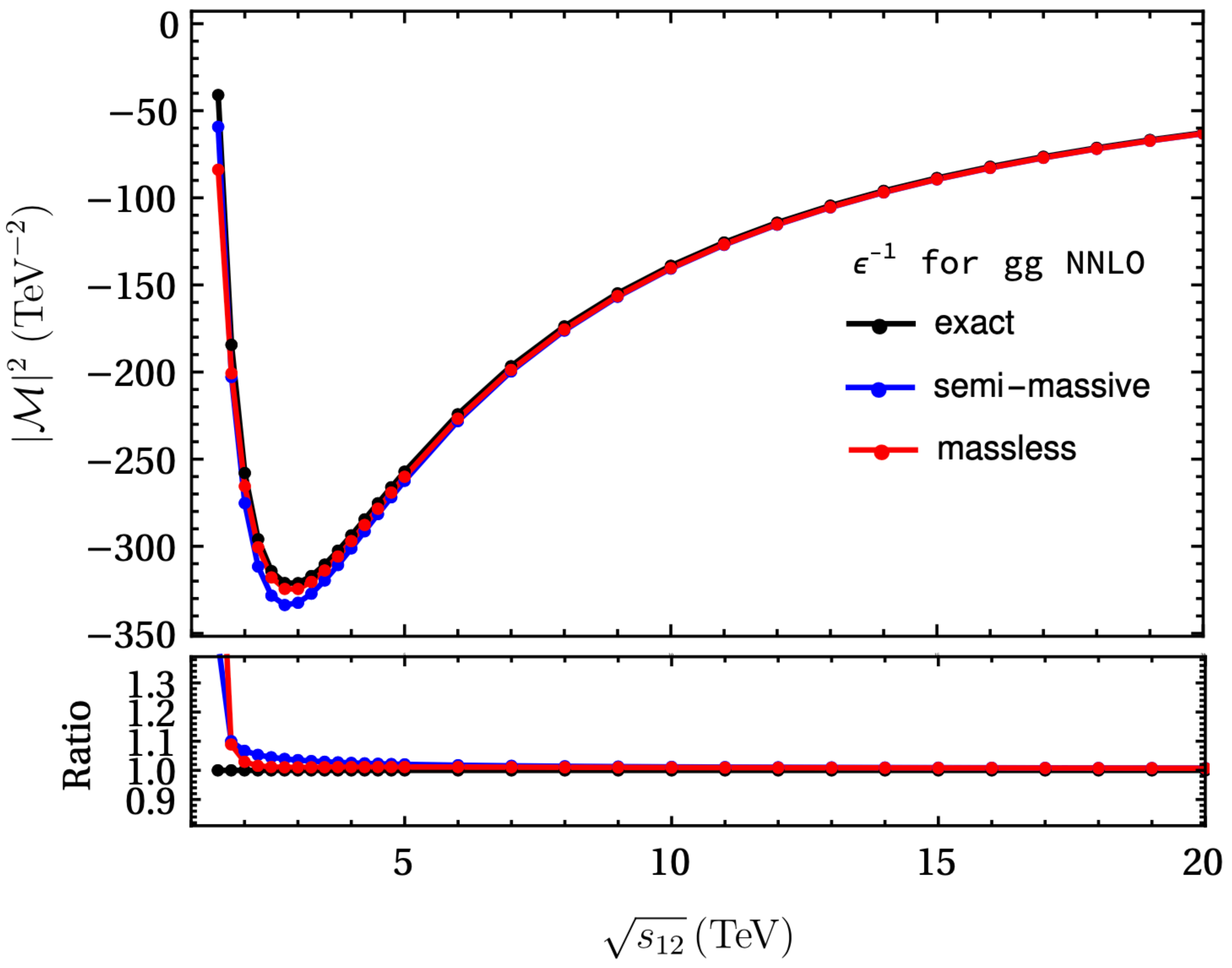


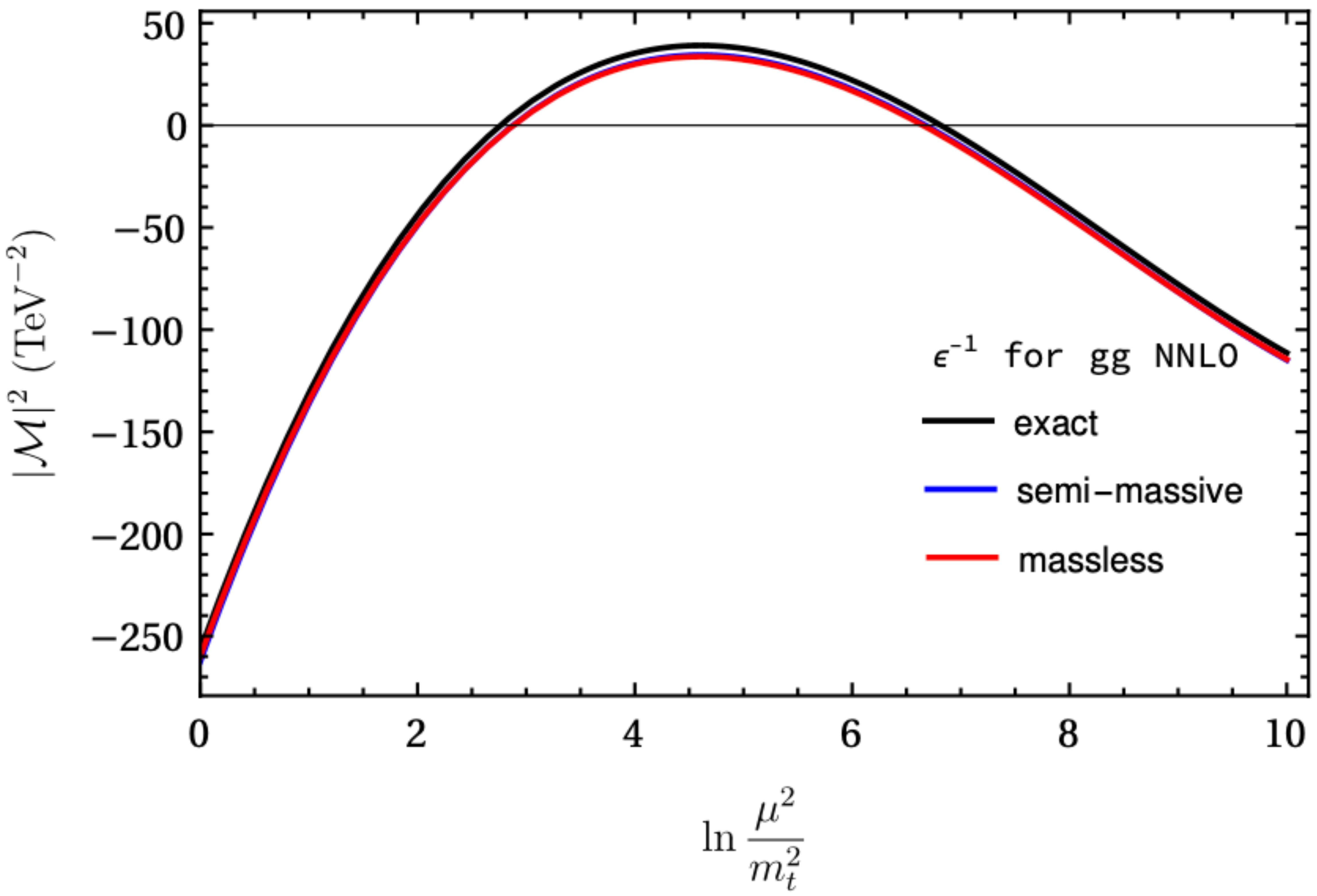
(d) planar hexagon-triangle (HT)

- Massless amplitudes computed using standard techniques
- Very large expressions, simplified using MultivariateApart
- Fast numeric evaluation with PentagonMI

Numerical results

Wang, Xia, LLY, Ye: 2402.00431



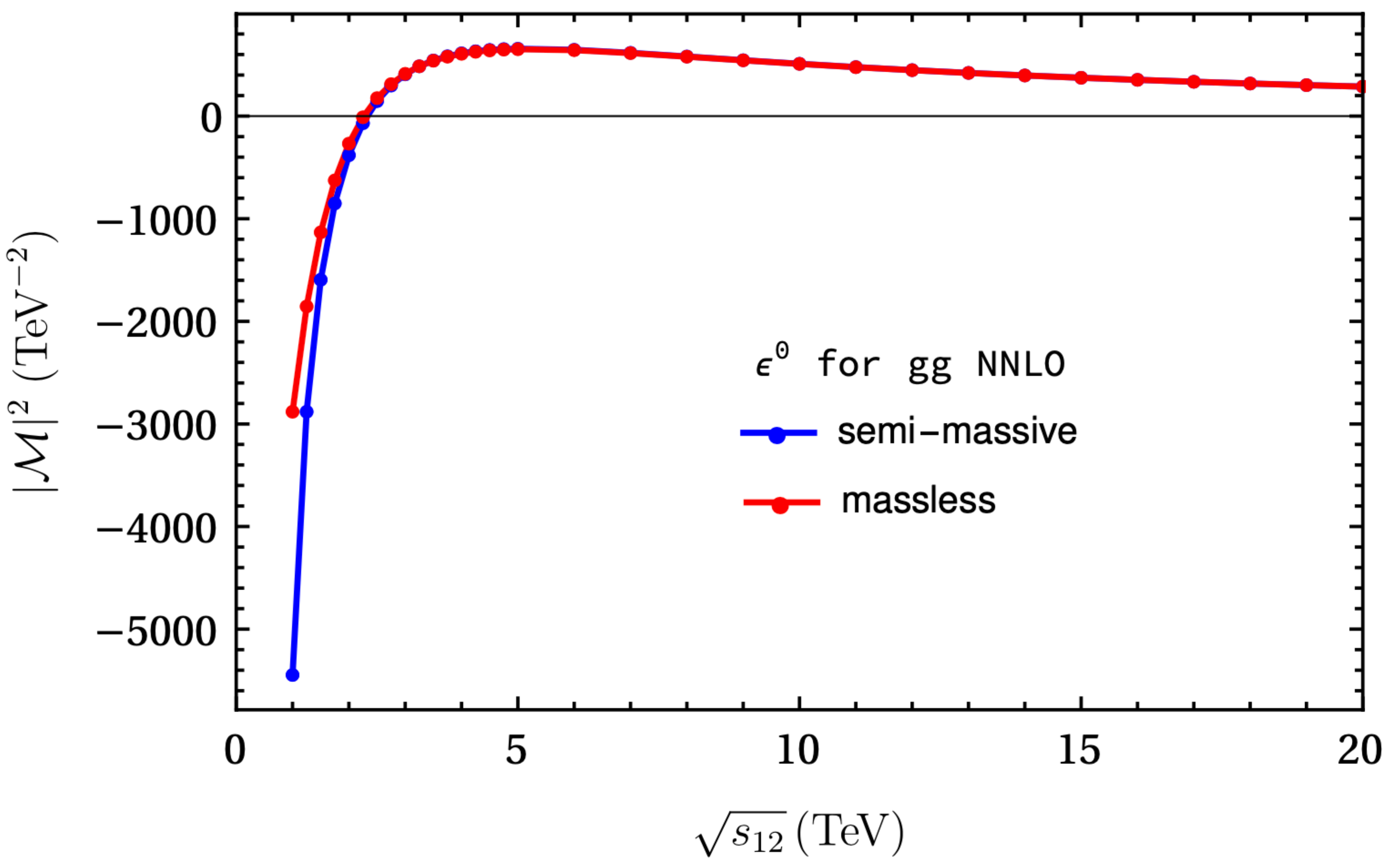


IR poles validated against exact results in Chen, Ma, Wang, LLY, Ye: 2202.02913

Note: without the heavy quark bubble, the scale-dependence would be wrong!

Numerical results

Wang, Xia, LLY, Ye: 2402.00431



- Two-loop amplitudes at high energies are ready
- Combine with low energy approximations (threshold / soft Higgs)?
- Differential cross sections (IR subtraction)?

Summary and outlook

- The tTH production is important for probing the top quark Yukawa coupling
- Theoretical status:
  - NLO+NNLL resummation for differential cross sections
  - NNLO with soft Higgs approximation for total cross section
  - Full NNLO not available (main bottleneck: two-loop amplitudes)
  - Two-loop IR poles computed
- Towards NNLO prediction at high energies
  - High energy factorization formula for QCD amplitudes
  - Applied to tTH production: approximate two-loop amplitudes now available
  - Future: combine with real emissions (IR subtraction) for differential cross sections

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Thank you!