# Light quark mediated Higgs boson production in associate with a jet at NNLO and beyond

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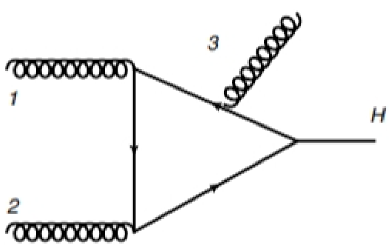
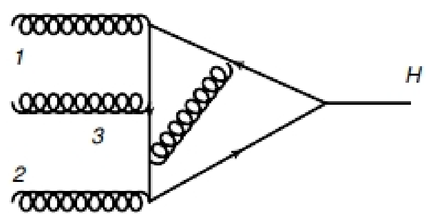



第六届重味物理与量子色动力学研讨会
2024年4月20日

# Peter Higgs (1929–2024): A Gentle Giant of Science

We have lost an important scientist; we have also lost a wonderful man.

Lawrence M. Krauss
14 Apr 2024 · 6 min read


Peter Higgs visiting the CMS detector at CERN, November 15, 2018. Photo by Marc Buehler via Flickr.

# Outline

1. **Background and motivation**

2. **Resummation of the abelian corrections** [Melnikov, Penin 16]

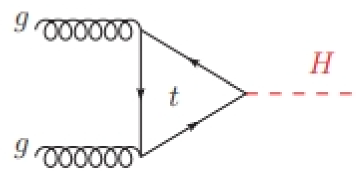
3. **Factorization and resummation for full QCD**

4. **Phenomenological applications**

5. **Conclusion**

*Based on: Tao Liu, Alexander Penin, Abdur Rehman arXiv:2402.18625*

# Higgs production via gluon fusion




$$\sigma_{\text{ggF}}^{\text{N3LO}} = 48.6\,\text{pb}^{+2.2\,\text{pb}\,(+4.6\%)}_{-3.3\,\text{pb}\,(-6.7\%)}\,(\text{theory}) \pm 1.6\,\text{pb}\,(3.2\%)\,(\text{PDF}+\alpha_s).$$

[PDG, 2020]

**Obtained in heavy top limit;**
**Improvements: finite top mass effects, threshold resummation, EW corrections …**

**Light quark amplitudes: enhanced by non-Sudakov double logarithms**

*11. Status of Higgs Boson Physics* **207**

boson $p_T$ distribution is known at NNLO [60, 61] (see Ref. [62] for a recent reappraisal) and heavy quark mass effects, including top-bottom interferences, have been computed at NLO [63], revealing a non-trivial logarithmic structure that will make resummation difficult [64]. A programatic approach for a fixed-order/resummation matching of the top-bottom interferences has been proposed [65]. Many search modes for the Higgs boson are

[64] S. Forte and C. Muselli, JHEP **03**, 122 (2016), [arXiv:1511.05561]; K. Melnikov and A. Penin, JHEP **05**, 172 (2016), [arXiv:1602.09020]; T. Liu and A. Penin, JHEP **11**, 158 (2018), [arXiv:1809.04950].

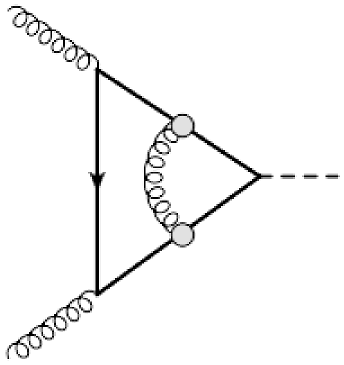
[65] F. Caola *et al.*, JHEP **09**, 035 (2018), [arXiv:1804.07632].

# Light quark mediated Higgs production

## Next-to-leading power:

$\rho = m_q^2/Q^2$

$x = \frac{\alpha_s}{4\pi} \ln^2 \rho, \quad z = (C_A - C_F)x,$

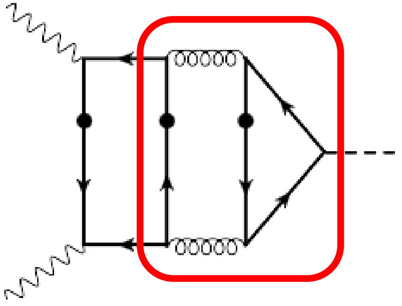


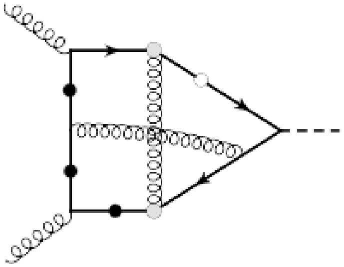
$$Z_g^2 = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\right]$$

$$\mathcal{M}^b_{gg\to H} = -Z_g^2 g(z)\left(\frac{3}{2}\ln^2\rho\,\rho\right)\mathcal{M}^{(0)}_{gg\to H}$$

$$g(z) = 2\int_0^1 d\xi \int_0^{1-\xi} d\eta e^{2z\eta\xi} = {}_2F_2\left(1,1;3/2,2;z/2\right)$$

## New contributions at next-to-next-to-leading power:





# Higgs production in associate with jet

$$g(p_1) + g(p_2) \to g(p_3) + H(p_H)$$

**Near threshold region:** $m_q^2 \ll p_\perp^2 \ll s, m_H^2$

⇒ **Two different kind of logarithms**

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→ **Two different kind of logarithms**

**Gauge conditions:** $\epsilon_i p_i = 0 \quad (i = 1,2,3)\,, \qquad \epsilon_1 p_2 = 0\,, \qquad \epsilon_2 p_1 = 0\,, \qquad \epsilon_3 p_2 = 0$

$$M_{+++} = -\sqrt{2} f^{a_1 a_2 a_3} \frac{g_s}{v} \frac{\alpha_s}{4\pi} \frac{\langle 12\rangle^2}{[12]\langle 23\rangle\langle 13\rangle} Z_{3g} \sum_q A^{(q)}_{+++}\,,$$

$$M_{++-} = -\sqrt{2} f^{a_1 a_2 a_3} \frac{g_s}{v} \frac{\alpha_s}{4\pi} \frac{\langle 12\rangle}{[23][13]} Z_{3g} \sum_q A^{(q)}_{++-}\,,$$

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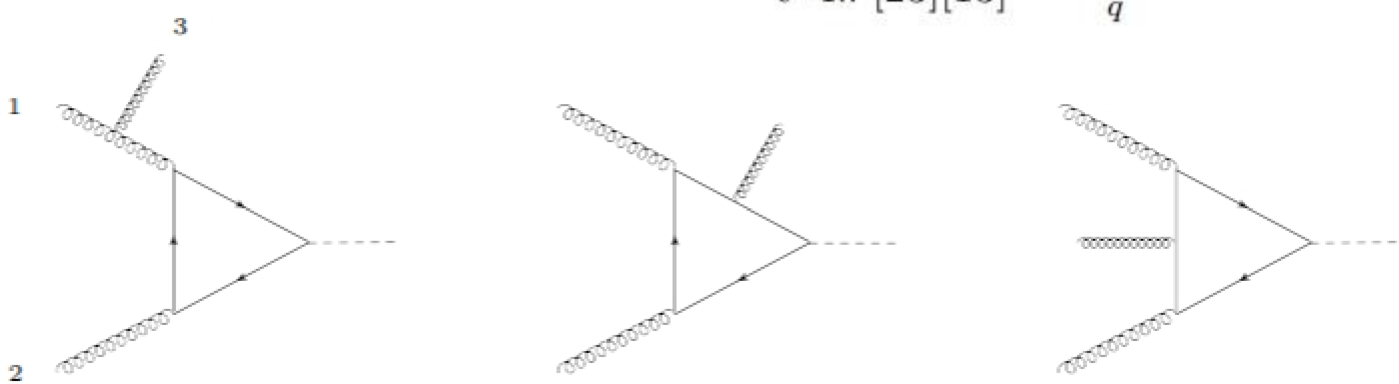
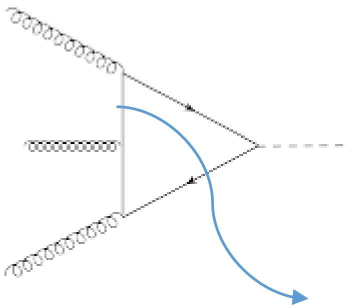


# Higgs production in associate with jet



Off-shell quark propagators:

soft loop momentum $l$

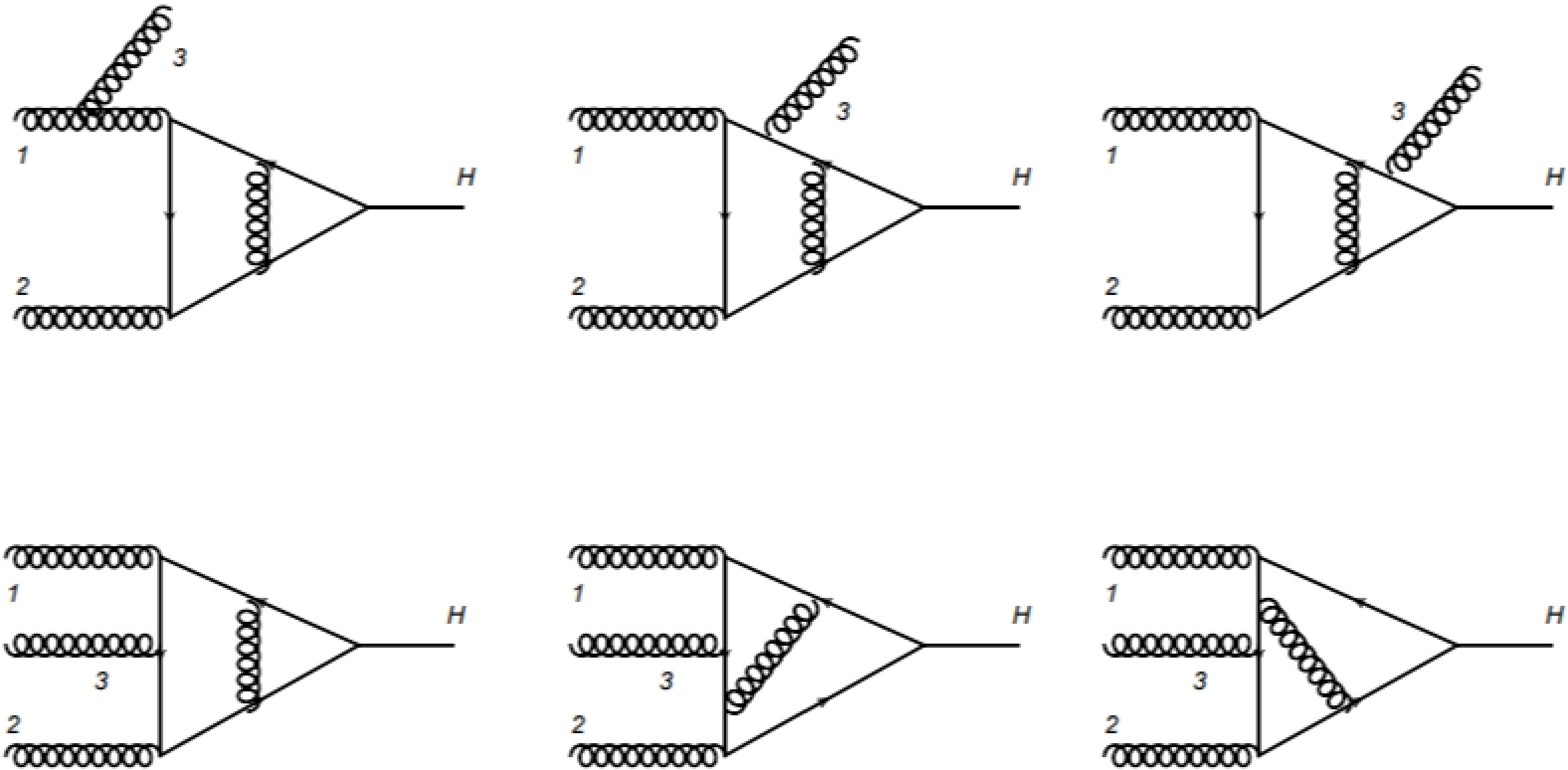
$$S(p_1 + l) \to \frac{\not{p}_1}{2p_1 l},$$

$$S(p_3 + l) \to \frac{\not{p}_3 + \not{l}}{2p_3 l},$$

$$S(p_3 - p_2 + l) \to \frac{\not{p}_2}{2p_2 p_3 + 2p_2 l}.$$

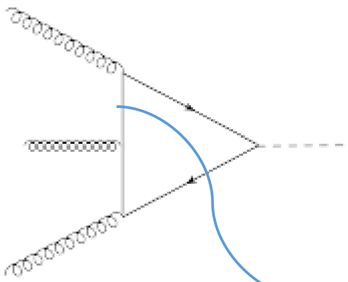
Two choices for double logarithmic scaling

# Resummation of the Abelian part




[Melnikov, Penin 2016]

# Factorization of one-loop amplitudes



Off-shell quark propagators:

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$$S(p_1+l) \to \frac{\not{p}_1}{2p_1 l},$$

$$S(p_3+l) \to \frac{\not{p}_3+\not{l}}{2p_3 l},$$

$$S(p_3-p_2+l) \to \frac{\not{p}_2}{2p_2p_3+2p_2 l}.$$

**Two choices for double logarithmic scaling**

---

$$S(p_3+l)\not{\epsilon}_2 S(p_3-p_2+l) \to \theta\left(|p_2p_3|-|p_2l|\right)\frac{\not{p}_3}{2p_3l}\not{\epsilon}_2\frac{\not{p}_2}{2p_2p_3} + \theta(|p_2l|-|p_2p_3|)\frac{\not{l}}{2p_3l}\not{\epsilon}_2\frac{\not{p}_2}{2p_2l}.$$

$$\to \theta\left(|p_2p_3|-|p_2l|\right)\left(\frac{\not{p}_3}{2p_3l}\not{\epsilon}_2\frac{\not{p}_2}{2p_2p_3} - \frac{\not{l}}{2p_3l}\not{\epsilon}_2\frac{\not{p}_2}{2p_2l}\right) + \frac{\not{l}}{2p_3l}\not{\epsilon}_2\frac{\not{p}_2}{2p_2l},$$

Soft dipole contribution

Symmetric structure G^3

Unconstrained dipole contribution

# Factorization of one-loop amplitudes

$$L = \ln(s/m_q^2)$$
$$\tau_t = \ln\left(|t|/m_q^2\right)/L$$
$$\tau = \ln\left(p_\perp^2/m_q^2\right)/L$$

$$\left[A^{(1)}_{++\pm}\right]_{\text{s.d.}} = \mp L^2 \int_{\tau_t-\tau}^{\tau_t} \mathrm{d}\eta \int_{1-\tau_t}^{1-\eta} \mathrm{d}\xi = \mp L^2 \frac{\tau^2}{2}\,,$$

$$\left[A^{(1)}_{+++}\right]_{\text{sym.}} = 0\,,$$

$$\left[A^{(1)}_{++-}\right]_{\text{sym.}} = -L^2 \left(\int_{\tau_t-\tau}^{\tau_t} \mathrm{d}\eta \int_{1-\tau_t}^{1-\eta} \mathrm{d}\xi + (\tau_t \leftrightarrow \tau_u)\right) = -L^2\tau^2$$

$$\left[A^{(1)}_{++\pm}\right]_{\text{s.d.}} + \left[A^{(1)}_{++\pm}\right]_{\text{sym.}} = -\frac{\tau^2}{2}L^2\,.$$

**All the others(eikonal dipole) sum to**

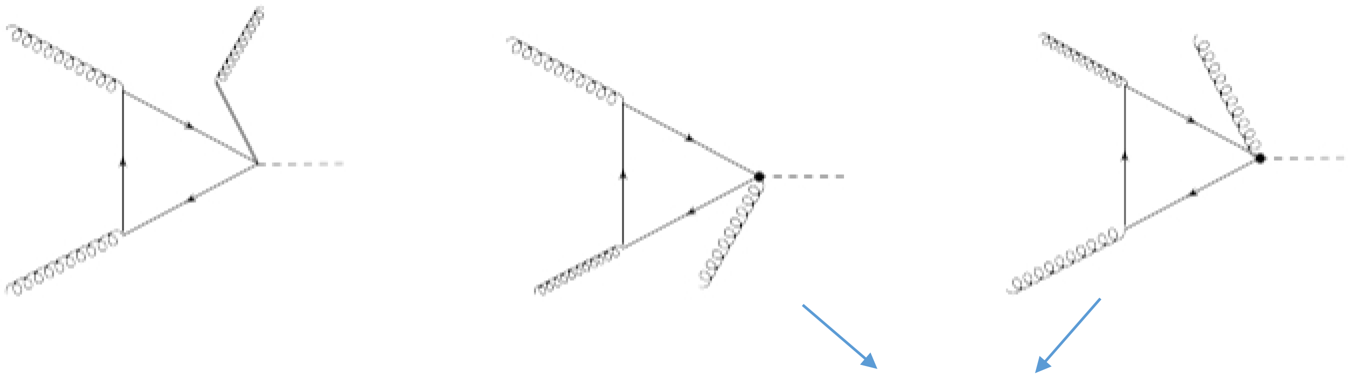
$$\left[A^{(1)}_{++\pm}\right]_{\text{e.d.}} = \pm 2L^2 \int_0^1 \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi = \pm L^2\,,$$

→ one-loop $g(p_1)g(p_2)H$ form factor

**Jet decoupled from the soft quark loop!**

# Factorization of one-loop amplitudes

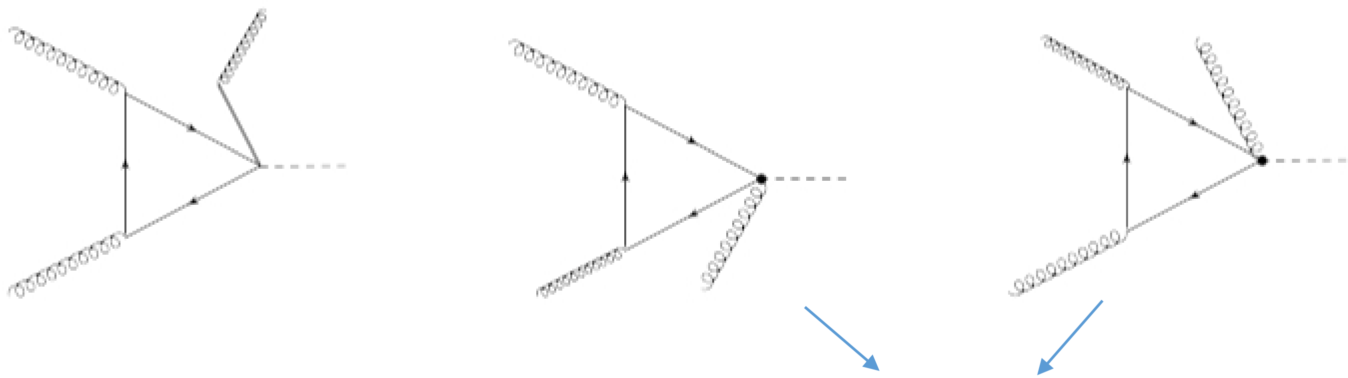
**Effective diagrams:**



Soft dipole

Symmetric contribution

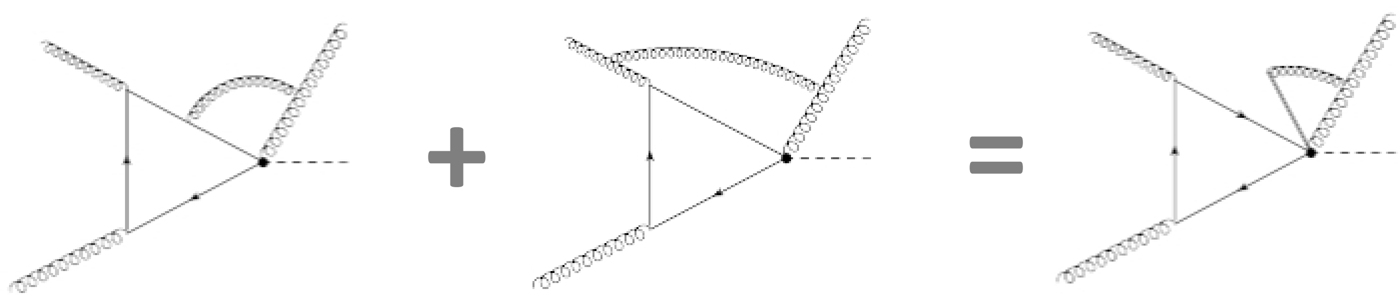
# Factorization and all-order analysis



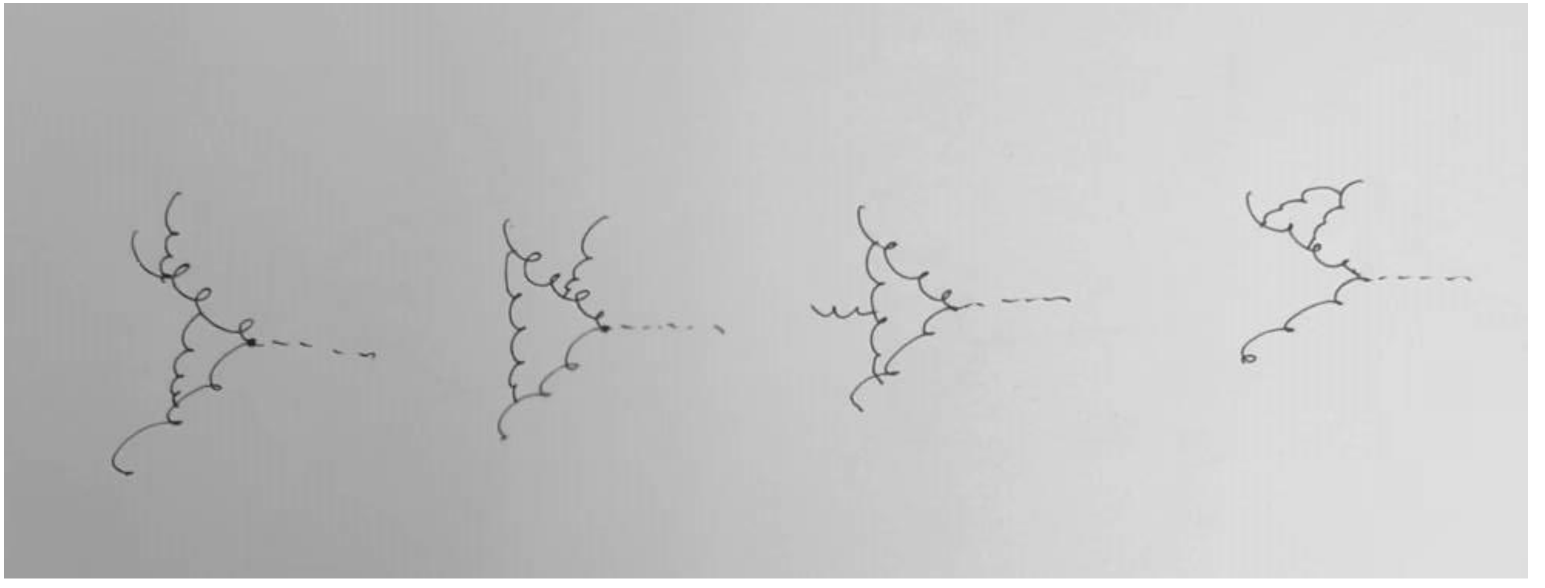
Soft dipole

Symmetric contribution

## Example:

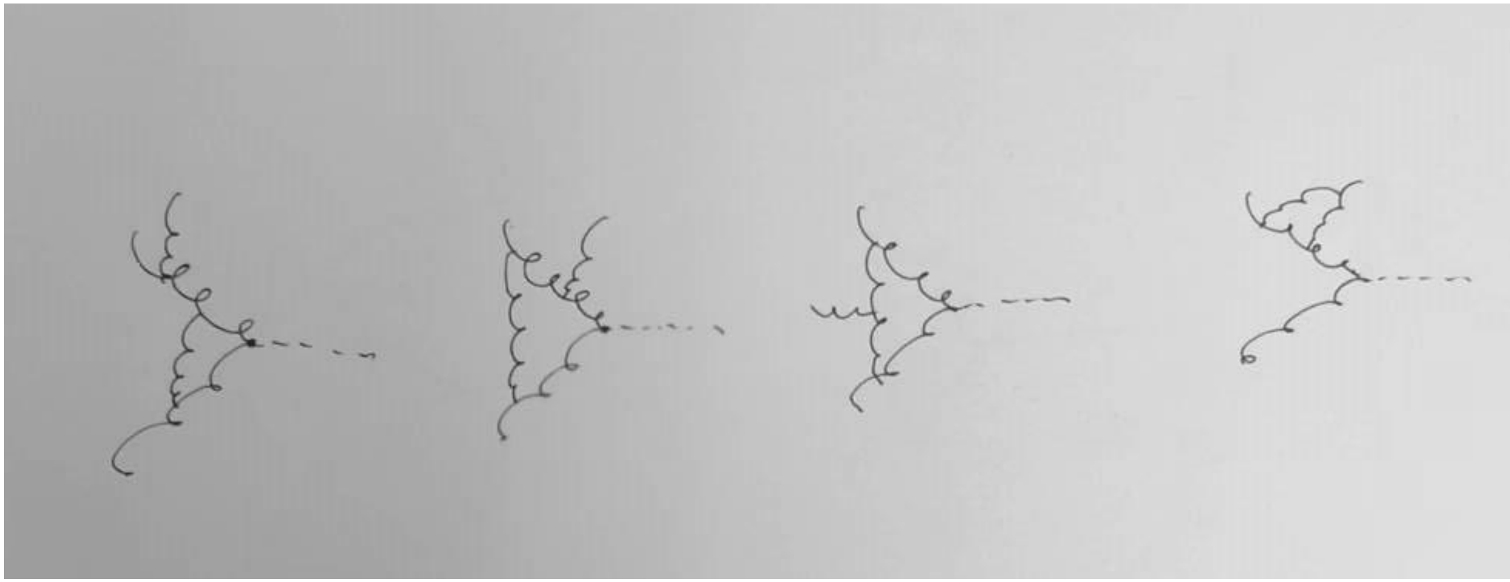


# Factorization and all-order analysis



**Same gauge conditions.**

# Factorization and all-order analysis



$$I^{(1)}_{\rm ph} = -\frac{C_A \alpha_s}{2\epsilon^2}\left[2\left(\frac{-s}{\mu^2}\right)^{-\epsilon} + \left(\frac{-tu}{s\mu^2}\right)^{-\epsilon}\right].$$

$$I^{(1)}_{\rm sym} = -\frac{C_A}{2\epsilon^2}\left[\left(\frac{-s}{\mu^2}\right)^{-\epsilon} + \left(\frac{-t}{\mu^2}\right)^{-\epsilon} + \left(\frac{-u}{\mu^2}\right)^{-\epsilon}\right],$$

**Differs from Catani's IR operator by finite double logarithmic terms.**

# Factorization and all-order analysis

$$z = (C_A - C_F)x$$

$$x = \frac{\alpha_s(\mu_s)}{4\pi} L^2.$$

$$\zeta = \ln(t/u)/L$$

$$\left[A^{\rm LL}_{++\pm}\right]_{\rm e.d.} = \pm 2L^2 \int_0^1 {\rm d}\eta \int_0^{1-\eta} {\rm d}\xi\, e^{2z\eta\xi} = \pm L^2 \int_0^1 \frac{{\rm d}\eta}{z\eta} \left(e^{2z\eta(1-\eta)} - 1\right) = \pm L^2 g(z)$$

$$\left[A^{\rm LL}_{++\pm}\right]_{\rm s.d.} = \mp L^2 \int_{\tau_t-\tau}^{\tau_t} {\rm d}\eta \int_{1-\tau_t}^{1-\eta} {\rm d}\xi\, e^{2z\eta\xi} = \mp L^2 \int_{\tau_t-\tau}^{\tau_t} \frac{{\rm d}\eta}{2z\eta} \left(e^{2z\eta(1-\eta)} - e^{2z\eta(1-\tau_t)}\right)$$

$$\begin{aligned}\left[A^{\rm LL}_{++-}\right]_{\rm sym.} &= -L^2 \left[\int_{\tau_t-\tau}^{\tau_t} {\rm d}\eta \int_{1-\tau_t}^{1-\eta} {\rm d}\xi\, e^{2z(\eta-\tau_t+\tau)(\xi-1+\tau_t)} e^{2z(\tau_t-\tau)\xi} + (\tau_t \to \tau_u)\right] \\ &= -L^2 \left[\int_{\tau_t-\tau}^{\tau_t} \frac{{\rm d}\eta}{2z\eta} e^{2z(\tau_t-\tau)(1-\tau_t)} \left(e^{2z\eta(\tau_t-\eta)} - 1\right) + (\tau_t \to \tau_u)\right].\end{aligned}$$

**Agree with explicit calculations after IR subtraction .**

# Phenomenological application

$$L = \ln(s/m_q^2)$$
$$\tau = \ln\left(p_\perp^2/m_q^2\right)/L$$
$$\zeta = \ln(t/u)/L$$

**Top-bottom interference and extra soft emissions:**

$$d\sigma^{tb}_{gg\to Hg+X} = -\frac{3m_b^2}{m_H^2} L^2 C_t C_b(\tau,\zeta) d\tilde{\sigma}^{\rm eff}_{gg\to Hg+X}\,.$$

$$\begin{aligned} C_b(\tau,\zeta) = \frac{A_{+++}-A_{++-}}{2L^2} &= 1 + \frac{z}{6}\left(1-\tau^3+\tau^4\right) \\ &+ z^2\left[\frac{1}{45} - \frac{\tau^3}{12} + \frac{\tau^4}{6} - \frac{7\tau^5}{60} + \frac{\tau^6}{30} + \underline{\frac{\zeta^2}{12}(\tau^3-\tau^4)}\right] \\ &+ z^3\left[\frac{1}{420} - \frac{\tau^3}{48} + \frac{\tau^4}{16} - \frac{\tau^5}{12} + \frac{23\tau^6}{360} - \frac{143\tau^7}{5040} + \frac{31\tau^8}{5040}\right. \\ &\left. + \underline{\zeta^2\left(\frac{\tau^3}{24} - \frac{\tau^4}{12} + \frac{\tau^5}{20} - \frac{\tau^6}{180}\right) - \frac{\zeta^4}{48}(\tau^3-\tau^4)}\right] + \ldots \end{aligned}$$

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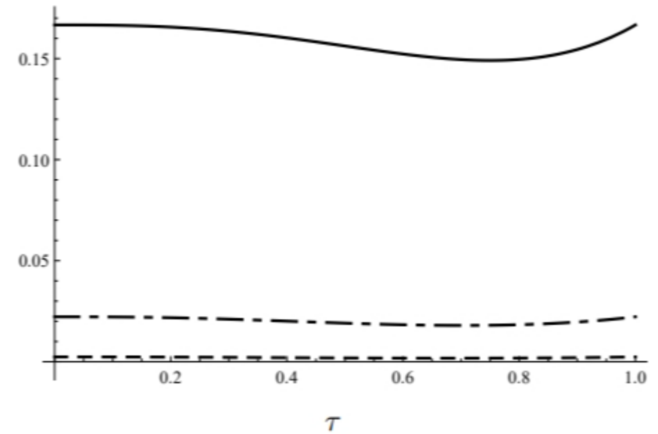
**Approximately** $C_b = C_b(0,\zeta) = C_b(1,0)$

**Dominate contribution come from the eikonal dipole part, where jet decouples from the quark loop.**




**All-order NLL?**

# Conclusion

1. All-order double logarithmic analysis for light quark mediated Higgs production in associate with a jet.

2. To our knowledge, this is the first asymptotic result for a QCD amplitude which captures all-order dependence on two kinematical variables.

3. Same color factor CF-CA which are observed on three-point amplitudes is obtained after physical IR subtraction.

4. Partonic cross section shows very weak dependence on the rapidity and transverse momentum of the jet.

Thanks for your attention!