Understanding the Energy Momentum Distribution with the Weizsäcker-Williams Method

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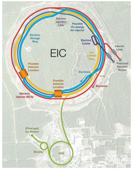
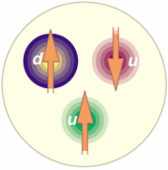
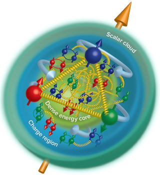
Emergent Phenomena in QCD



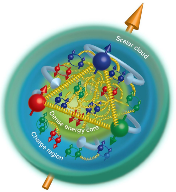


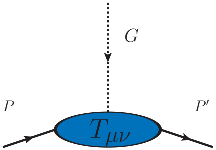


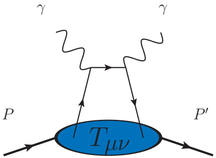
Three pillars of EIC Physics:

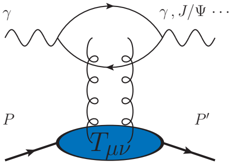
- How does proton mass arise/distribute? Mass gap: million dollar question.
- How does the spin of proton arise? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- This talk: Mass distribution? quark model VS Gluon Core?
- Determining the gluonic GFFs of the proton B. Duran, *et al.*, Nature **615** (2023) no.7954, 813-816.
  Determined a mass radius that is notably smaller than the electric charge radius!

Energy-Momentum Tensor and Gravitational Form Factors







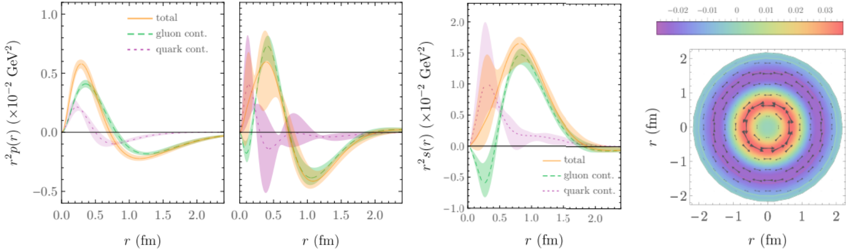


- Impossible to use graviton (spin 2) to probe proton mass distribution (GFF).
- [Ji, 97]; [Kharzeev, 96] Use two photons/gluons (spin 1) to study GPDs and GFFs, and probe quark and gluon parts, respectively.
- Usually near the production threshold, but what about high energy limit?

Pressure and Shear forces inside proton

[Shanahan, Delmold, Phys. Rev. Lett. **122**, 072003 (2019)] Link

$$T^{ij}(r) = \left(\frac{r^i r^i}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij} p(r)$$

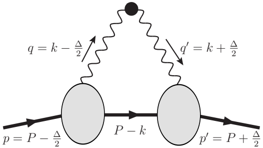


- The spatial of static EMT define the stress tensor. It can be decomposed in a traceless part associated with shear forces $s(r)$ and a trace associated with the pressure $p(r)$.
- $s(r)$ and $p(r)$ are computed in LQCD recently.

Energy-Momentum Tensor and Gravitational Form Factors

Consider the photon/gluon GFFs, defined from the associated EMT:

$$T_{\gamma}^{\mu\nu} = F^{\mu\alpha} F_{\alpha}^{\ \nu} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}. \quad \text{N.B.} \quad T^{++} = F^{+\alpha} F_{\alpha}^{+}$$



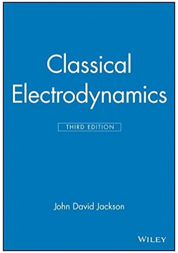
The photon/gluon GFFs for a spin-zero hadron [Polyakov, Schweitzer, 2018]:

$$\text{spin-0}: \quad \langle p' | T_{\gamma}^{\mu\nu} | p \rangle = 2P^{\mu} P^{\nu} A_{\gamma}(t) + C_{\gamma}(t) \frac{\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2}{2} + 2m^2 \bar{C}_{\gamma}(t) g^{\mu\nu}.$$

- A: mass/momentum distribution; C (or D): shear and pressure information.
- B (or J) are related to spin. $\bar{C}$ is due to non-conservation of individual part.

Classical Electrodynamics and Virtual Quanta

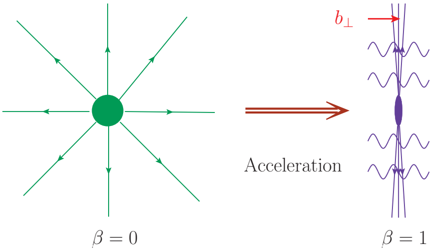




- Following Fermi[24], Weizsäcker [34] and Williams [35] discovered that the EM fields of a relativistically moving charged particle are almost transverse. Equivalent to Say:
- Charged particles carry a cloud of quasi-real photons, ready to be radiated if perturbed.
- Weizsäcker-Williams method of virtual quanta (Equivalent Photon Approximation).
- Application in QCD: WW gluon distribution. [McLerran, Venugopalan, 94; Kovchegov, 96; Jalilian-Marian, Kovner, McLerran and Weigert, 97]

EPA and Weizsäcker-Williams Photon Distribution

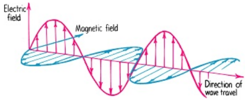
Boost Static Field to infinite momentum frame ($\gamma \to \infty$): [Jackiw, Kabat and Ortiz, 92; Jackson]



$$A^+_{Cov} = -\frac{q}{\pi}\ln(\lambda b_\perp)\delta(t-z),$$

$$\vec{E} = \frac{q}{2\pi}\frac{\vec{b}_\perp}{b_\perp^2}\delta(t-z), \quad \vec{B} = \frac{q}{2\pi}\frac{\hat{v}\times\vec{b}_\perp}{b_\perp^2}\delta(t-z),$$

$$\vec{A}^{LC}_\perp = -\frac{q}{2\pi}[\vec{\nabla}_\perp \ln(\lambda b_\perp)]\theta(t-z).$$



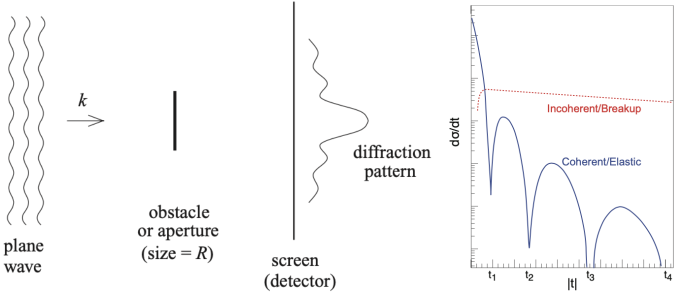
Static E fields $\Rightarrow$ EM Wave

$\Rightarrow$ EM pulses are equivalent to photons

- A_μ in Covariant gauge and LC gauge are related by a gauge transformation.
- Classical EM: transverse EM fields $\Leftrightarrow$ QM: Co-moving Quasi-real photons.
- Quantization $\Rightarrow$ photon distribution

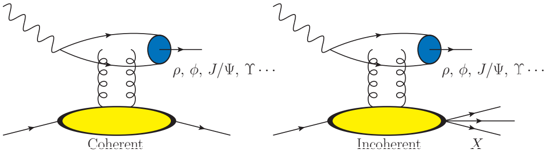
An analogy to Fraunhofer Diffraction in Optics

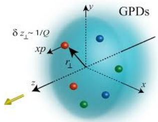
[QCD at high energy, Kovchegov and Levin, 12]

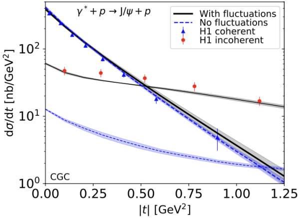


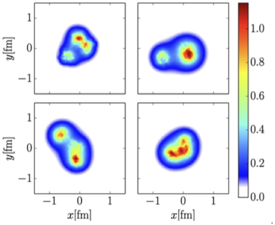
- Treat the hadron target in DIS as a black disk. [Joseph von Fraunhofer, 1821]
- Similar pattern in optics ($\theta_i^{\min} \sim 1/(kR)$) and high energy QCD $t_i \sim \frac{1}{R^2}$.
- Two difference: 1. σ sensitive to gluon distribution; 2. Breakup of the target.
- Diffractive scattering $\Rightarrow$ gluon spatial distribution.

Diffractive vector meson production





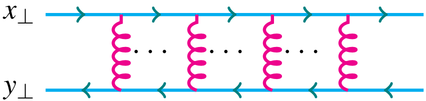




Wilson Lines in Color Glass Condensate Formalism

The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$S^{(2)}(r_\perp) = \frac{1}{N_c}\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\rangle = e^{-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4}}$$



- IP-Sat Model and Glauber-Mueller formula for $S^{(2)}(r_\perp)$

$$Q_s^2(x, b_\perp) = \frac{2\pi^2}{N_c}\alpha_s x g(x, \mu^2) T(b_\perp), \quad \text{with} \quad T(b_\perp) = \frac{1}{2\pi B_G} e^{-b_\perp^2/(2B_G)}.$$

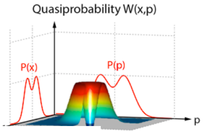
- MV model for large nuclei with uniform nucleon distribution

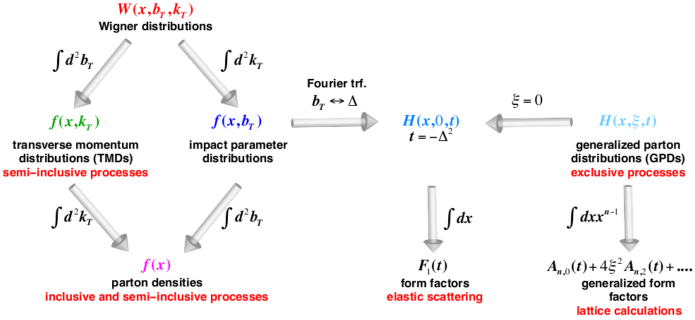
$$T_A(b_\perp) = \frac{3A}{2\pi R_A^3}\sqrt{R_A^2 - b_\perp^2}.$$

- In general, unitarity and color transparency imply $S^{(2)}(r_\perp) = 1 - Q_s^2 r_\perp^2/4 + \cdots$.

3D Tomography of Proton

Wigner distributions [Belitsky, Ji, Yuan, 04] ingeniously encode all quantum information of how partons are distributed inside hadrons.



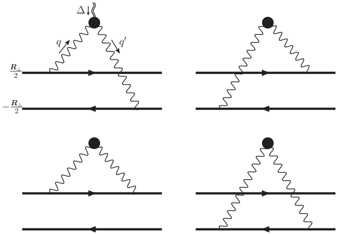


Photon Gravitational Form Factors

- For a pointlike charge, the photon A-GFF at small-$|t|$

$$A_\gamma(t) = \frac{\alpha}{\pi}\left[\text{U.V.} + \frac{t}{m^2}\left(\frac{3}{16}\frac{m\pi^2}{\sqrt{-t}} - \frac{1}{3}\right) + \cdots\right], \quad \langle b_\perp^2\rangle_\gamma = \frac{4}{A_\gamma(0)}\frac{dA_\gamma(t)}{dt}\bigg|_{t=0}.$$

  Divergent radius due to the long-range tail of the Coulomb field.
- IR and UV divergences disappear for a charge neutral dipole with a distribution.



- Compute GFF-A from two methods: the WW method directly or from GTMD.

Understanding GFFs with the WW Method

Extract Momentum GFF from the $++$ component

$$\begin{aligned} A_g(t) &= \int_0^1 dx \int d^2k_\perp x\mathcal{G}_x(k_\perp, \Delta_\perp), \\ &= \frac{N_c}{\alpha_s}\int_0^1 dx \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot b_\perp} \vec{\nabla}^2_{r_\perp} \left[1 - S_x(b_\perp, r_\perp)\right]\Big|_{r_\perp=0}, \\ &= \frac{N_c}{\alpha_s}\int_0^1 dx \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot b_\perp} Q_s^2(x, b_\perp), \qquad \text{Gaussian Ansatz.} \end{aligned}$$

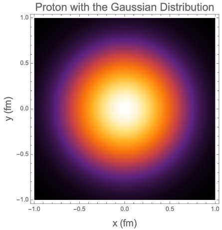
- New Relation between GFF and Dipole Scattering Amplitude.
- Two GTMDs (WW and Dipole) reduce to the same GPD and GFF.
- Gaussian Ansatz. $S_x(b_\perp, r_\perp) = \exp[-\frac{r_\perp^2}{4}Q_s^2(x, b_\perp)] \Rightarrow$ Probe Q_s.

Understanding GFFs with the WW Method

Recall IP-Sat: $Q_s^2(x, b_\perp) = \frac{2\pi^2}{N_c}\alpha_s xg(x, \mu^2)T(b_\perp)$, with $T(b_\perp) = \frac{1}{2\pi B_G}e^{-b_\perp^2/(2B_G)}$.

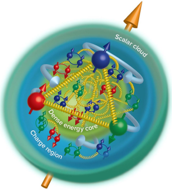
$$
\begin{aligned}
A_g(t) &= A_g(0)\int d^2b_\perp e^{-i\Delta_\perp\cdot b_\perp}T(b_\perp) \\
&= A_g(0)e^{-B_G|t|/2},
\end{aligned}
$$

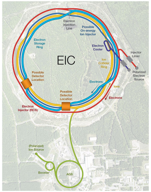
with $$A_g(0) = \int_0^1 dxxg(x, \mu^2) \to \frac{4C_F}{4C_F + n_f}.$$



- Exclusive diffraction at HERA: $B_G = 4.0 \pm 0.4$ GeV^{-2} (IP-Sat). [Caldwell and Kowalski, 2010] [Rezaeian, Siddikov, Van de Klundert, Venugopalan, 13]
- Gluon Radius in the proton $\sqrt{\langle b_\perp^2\rangle_g} \approx 0.56$fm and $\sqrt{\langle r^2\rangle_g} \approx 0.61$fm.
- Agree with B. Duran, *et al.*, ▸ Nature **615** (2023) no.7954, 813-816. and lattice (MIT). Gluon core!

Summary





- WW method provides analytic insights into gluon GFFs and radii.
- A new relation between the gluon A-GFF and the Laplacian of dipole amplitude.
- Understanding dense gluon core inside Proton! B. Duran, *et al.* Nature **615** (2023) no.7954, 813-816.
- A-GFF of nuclei ⇒ nuclear gluon distribution – the charge distribution ⇒ neutron distribution for large nuclei.
- Measurements of GFFs at the upcoming EIC and EicC.